Analysis of datasheets of brushed DC motors

Maxon motors: https://www.maxongroup.com/en/drives-and-systems/brushed-dc-motors

Case study: datasheet for Faulhaber 4.2 mNm DC motors

FAULHABER

DC-Micromotors

Precious Metal Commutation

4,2 mNm

For combination with
Gearheads:
15A, 16/7, 16A
Encoders:
IE2-1024, IE2-16

Series 1724 ... SR

No.		1724 T		003 SR	006 SR	012 SR	018 SR	024 SR	
1	Nominal voltage	U_N		3	6	12	18	24	V
2	Terminal resistance	R		0,78	3,41	16,2	32,1	54,6	Ω
3	Output power	$P_{2\,max.}$		2,83	2,58	2,17	2,47	2,58	W
4	Efficiency, max.	$\eta_{max.}$		82	81	80	81	81	%
5	No-load speed	n_0		8 200	8 600	7 900	8 400	8 600	rpm
6	No-load current (with shaft ø 1,5 mm)	I_0		0,038	0,02	0,009	0,006	0,005	A
7	Stall torque	M_H		13,2	11,5	10,5	11,2	11,5	mNm
8	Friction torque	M_R		0,13	0,13	0,13	0,12	0,13	mNm
9	Speed constant	k_n		2 760	1 450	666	472	362	rpm/V
10	Back-EMF constant	k_E		0,362	0,69	1,5	2,12	2,76	mV/rpm
11	Torque constant	k_M		3,46	6,59	14,3	20,2	26,3	mNm/A
12	Current constant	k_I		0,289	0,152	0,07	0,049	0,038	A/mNm
13	Slope of n-M curve	$\Delta n/\Delta M$		621	748	752	750	748	rpm/mNm
14	Rotor inductance	L		21	75	360	710	1 200	µH
15	Mechanical time constant	τ_m		8	8	8	8	8	ms
16	Rotor inertia	J		1,2	1	1	1	1	gcm²
17	Angular acceleration	$\alpha_{max.}$		110	110	100	100	100	·10³rad/s²
18	Thermal resistance	R_{th1} / R_{th2}	4 / 24,5						K/W
19	Thermal time constant	τ_{w1} / τ_{w2}	2,6 / 270						s
20	Operating temperature range:								
	– motor		-30 ... +85 (optional version -55 ... +125)						°C
	– rotor, max. permissible		+125						°C
21	Shaft bearings		sintered bearings (standard)		ball bearings (optional version)		ball bearings, preloaded (optional version)		
22	Shaft load max.:								
	– with shaft diameter		1,5		1,5		1,5		mm
	– radial at 3 000 rpm (3 mm from bearing)		1,2		5		5		N
	– axial at 3 000 rpm		0,2		0,5		0,5		N
	– axial at standstill		20		10		10		N
23	Shaft play								
	– radial	≤	0,03		0,015		0,015		mm
	– axial	≤	0,2		0,2		0		mm
24	Housing material		steel, black coated						
25	Weight		27						g
26	Direction of rotation		clockwise, viewed from the front face						
	Recommended values - mathematically independent of each other								
27	Speed up to	$n_{e\,max.}$		8 000	8 000	8 000	8 000	8 000	rpm
28	Torque up to	$M_{e\,max.}$		4,2	4,2	4,2	4,2	4,2	mNm

General model for a brushed DC motor with a stator “s” and armature “a” (rotor) coils:

$$\begin{cases} V_a(t) = R_a \times I_a(t) + L_a \times \frac{\partial I_a(t)}{\partial t} + K_b \times I_s(t) \times \frac{\partial \theta(t)}{\partial t} \\ V_s(t) = R_s \times I_s(t) + L_s \times \frac{\partial I_s(t)}{\partial t} \\ J\frac{\partial^2\theta(t)}{\partial t^2} + B\frac{\partial \theta(t)}{\partial t} + k\theta(t) = T_{mag}(t) + T_{ex}(t) \\ T_{mag}(t) = K_T \times I_s(t) \times I_a(t) \\ K_b = const \\ K_T = const \end{cases} \quad (1)$$

where:

T_{mag} – magnetic torque

V_a – armature voltage

V_s – stator voltage

I_a – armature current

I_s – stator current

R_a – armature resistance

R_s – stator resistance

L_a – armature self-inductance

L_s – stator self-inductance

J – moment of inertia

$\omega = \frac{\partial \theta(t)}{\partial t}$ – angular speed

$\alpha = \frac{\partial^2\theta(t)}{\partial t^2}$ – angular acceleration

$B\frac{\partial \theta(t)}{\partial t}$ – torque proportional to the angular speed and directed against T_{mag}

$k\theta(t)$ – Hook’s torque directed against T_{mag}

For a permanent magnet DC motor, there are no parameters associated with the stator:

$$\begin{cases} V_a(t) = R_a \times I_a(t) + L_a \times \frac{\partial I_a(t)}{\partial t} + K_F \times \frac{\partial \theta(t)}{\partial t} \\ J\frac{\partial^2\theta(t)}{\partial t^2} + B\frac{\partial \theta(t)}{\partial t} + k\theta(t) = T_{mag}(t) + T_{ex}(t) + T_f \\ T_{mag}(t) = K_M \times I_a(t) \end{cases} \quad (2)$$

where T_f is the friction torque directed against $T_{mag}(t) + T_{ex}(t)$.

For a steady rotation, we have:

$$\omega = const$$

$$\alpha = 0 \quad (3)$$

$$V_a = const$$

$$\frac{\partial I_a(t)}{\partial t} = 0$$

Then:

$$V_a = R_a \times I_a + K_F \times \omega \quad (4)$$

$$T_{mag} = K_M \times I_a \quad (5)$$

Expressing I_a through T_{mag} in (5) and substituting it to (4), we obtain:

$$I_a = \frac{T_{mag}}{K_M} \quad (6)$$

$$T_{mag} = \frac{V_a K_M}{R_a} - \frac{K_F K_M}{R_a} \times \omega \text{ (speed-to-torque equation)} \quad (7)$$

$$\omega = \frac{V_a}{K_F} - \frac{R_a}{K_F K_M} \times T_{mag} \text{ (torque-to-speed equation)} \quad (8)$$

Both Eqs. (7) and (8) are the straight lines with negative slopes. From Eqs. (7) and (8), we can derive the maximum torque and speed:

$$max\, T_{mag}\big|_{\omega=0} = \frac{V_a K_M}{R_a} \quad (9)$$

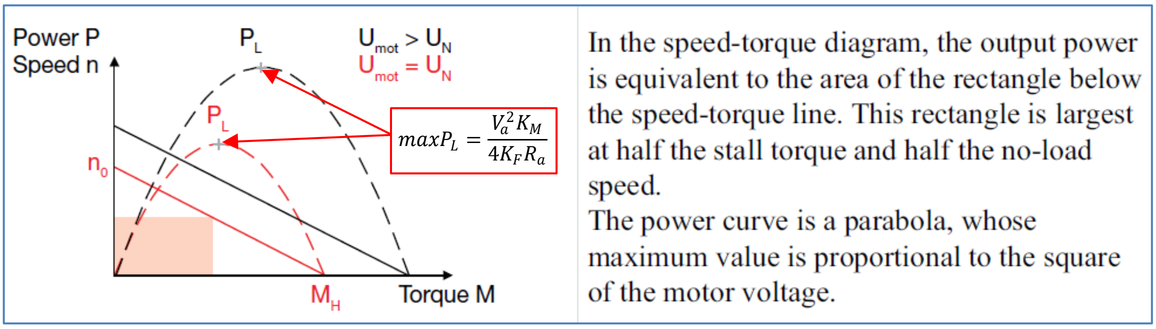
$$max\omega\big|_{T_{mag}=T_f} = \frac{V_a}{K_F} - \frac{T_f R_a}{K_F K_M} \quad (10)$$

If $B = T_f \equiv 0$ (no frictions) and $T_{ex} = 0$, then $T_{mag} = 0$ and $I_a = 0$. In practice, I_a may be very small for a free rotation, but it is never zero due to a residual friction that always presents in a motor.

The mechanical power generated by the motor to compensate for the load torque is $P_L = T_{mag} \times \omega$, where $\omega = \frac{V_a}{K_F} - \frac{R_a}{K_F K_M} \times T_{mag}$ in Eq. (8):

$$P_L(T_{mag}) = \left(\frac{V_a}{K_F} - \frac{T_{mag} R_a}{K_F K_M}\right) \times T_{mag} \quad (11)$$

where $T_{mag} \in \left[T_f, \frac{V_a K_M}{R_a}\right]$. This is a quadratic function shown below.



U_N – nominal armature voltage, n – angular speed in Maxon Formulae Handbook.

The maximum mechanical power P_L is found from the condition $\frac{\partial P_L(T_{mag})}{\partial T_{mag}} = 0$:

$$\frac{\partial P_L(T_{mag})}{\partial T_{mag}} = 0 \rightarrow T_{mag} = \frac{V_a K_M}{2R_a} \rightarrow \omega = \frac{V_a}{2K_F}$$

$$maxP_L = \frac{V_a^2 K_M}{4K_F R_a} \approx \frac{V_a^2}{4R_a} \quad (12)$$

The motor efficiency η is defined as the ratio of the mechanical power $(T_{mag} - T_f) \times \omega$ to the electrical power $P_E = V_a \times I_a$ supplied to the motor. Using Eqs. (6) and (8), we obtain:

$$\eta(T_{mag}) = \frac{(T_{mag} - T_f)\times\omega}{V_a \times I_a} = \frac{K_M}{K_F}\left(1 - \frac{R_a}{V_a K_M} T_{mag}\right) \times \left(1 - \frac{T_f}{T_{mag}}\right) \quad (13)$$

The maximum efficiency is found from the condition $\frac{\partial \eta(T_{mag})}{\partial T_{mag}} = 0$:

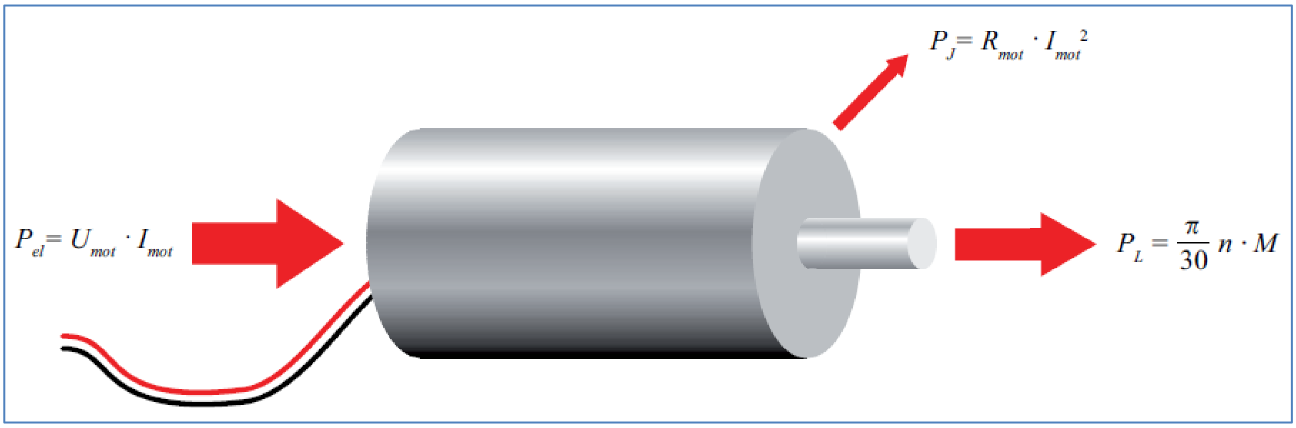
$$\frac{\partial \eta}{\partial T_{mag}} = 0 \rightarrow T_{mag} = \sqrt{\frac{V_a K_M T_f}{R_a}} = \sqrt{maxT_{mag} \times T_f} \quad (14)$$

Substituting this value of T_{mag} to Eq. (13), we obtain the maximum efficiency:

$$max\eta = \frac{K_M}{K_F}\left(1 - \sqrt{\frac{R_a T_f}{V_a K_M}}\right)^2 = \frac{K_M}{K_F}\left(1 - \sqrt{\frac{I_a^{no-load}}{maxI_a}}\right)^2 \quad (15)$$

where $I_a^{no-load}$ is the no-load armature current and $maxI_a = \frac{V_a}{R_a}$ is the maximum armature current.

Power conservation law for the motor: **Electrical power = Joule Loss + Mechanical Work**



In [Maxon's Formulae Handbook](): $\frac{\pi}{30} \times n$ is the angular speed n in rpm recalculated into rad/s. $I_{mot} = I_a$, $U_{mot} = V_a$, $R_{mot} = R_a$, and $M = T_{mag}$. Due to the absence of a magnetic core in the rotor, Maxon motors have a negligible magnetic loss.

Let us extract the model parameters from the datasheet for Faulhaber 006 SR:

- **(2)** Terminal (armature) resistance $R_a = 3.41\ \Omega$
- **(14)** Rotor's (armature) inductance $L_a = 7.5 \times 10^{-5}$ H
- **(10)** Back-EMF constant $K_F = 0.69$ mV/rpm or $K_F = 0.69 \times 10^{-3}/(2\pi/60) = 6.589 \times 10^{-3}$ V×s/rad
- **(16)** Rotor's moment of inertia $J = 1$ g×cm^2 or $J = 10^{-7}$ kg×m^2
- **(11)** Torque constant $K_M = 6.59 \times 10^{-3}$ N×m/A
- **(8)** Friction torque $T_f = 1.3 \times 10^{-4}$ N×m

Note that K_F and K_M have the same values but different dimensions.

Checking the validity of the model

1. Calculate the slop $\Delta\omega/\Delta T$ and compare with (13) in the datasheet:

The graph of $\omega(T_{mag})$ in Eq. (8) is a straight line with the slop $\Delta\omega/\Delta T = R_a/(K_F K_M) \approx 78533$ rad/(s×N×m) – theoretical value. The experimental slop in (13) is 748 rpm/(mN×m) = 748×π×10^3/30 rad/(s×N×m) ≈ 78330 rad/(s×N×m). Good agreement with (13) in the datasheet.

2. Calculate the no-load speed $max\ \omega$ in Eq. (10) and compare it with (5) in the datasheet:

$max\omega|_{T_{mag}=T_f} = \frac{V_a}{K_F} - \frac{T_f R_a}{K_F K_M} \approx 900.4$, where $V_a = 6$ V. The experimental $max\ \omega$ is 8600 rpm = 8600×π/30 rad/s ≈ 900.6 rad/s. Good agreement with (5) in the datasheet.

3. Calculate $max\, T_{mag}$ in Eq. (9) and compare it with (7) in the datasheet:

$max\, T_{mag}\big|_{\omega=0} = \frac{V_a K_M}{R_a} \approx 1.1595 \times 10^{-2}$ N×m, where $V_a = 6$ V. The experimental $max\, T_{mag}$ (stall torque) is 11.5 mN×m = 1.15×10^{-2} N×m. Good agreement with (7) in the datasheet.

4. Calculate the speed constant and compare it with (9) in the datasheet:

Using Eq. (10), $max\omega \approx \frac{V_a}{K_F}$. So, the speed constant is $\frac{1}{K_F} \approx 151.7681$ rad/(V×s). The experimental value is 1450 rpm/V = 1450×π/30 rad/(V×s) ≈ 151.8436 rad/(V×s). Good agreement with (9) in the datasheet.

5. Express the friction torque (8) through other constants in the datasheet:

> For the free rotation (no-load) with $T_{mag} \approx |T_f|$. On the other hand, $T_{mag} = K_M \times I_a$. From these two equations, we obtain for the friction torque: $T_f = K_M \times I_a^{no-load}$. Using the no-load current (6) from the datasheet $I_a^{no-load} = 2 \times 10^{-2}$ A and $K_M = 6.59 \times 10^{-3}$ N×m/A, we obtain: $T_f \approx 6.59 \times 10^{-3} \times 2 \times 10^{-2} = 1.318 \times 10^{-4}$ N×m. Good agreement with (8) in the datasheet.

6. Express the current constant (12) through other constants in the datasheet:

> Using $I_a = \frac{T_{mag}}{K_M}$ in Eq. (6) and $K_M = 6.59 \times 10^{-3}$ N×m/A from (a), we can calculate the current constant $\frac{1}{K_M} \approx 151.7451$ A/(N×m). Good agreement with (12) in the datasheet.

7. Express the angular acceleration (17) through other constants in the datasheet:

> Using the mechanical equation $J\frac{\partial^2\theta(t)}{\partial t^2} + B\frac{\partial\theta(t)}{\partial t} + k\theta(t) = T_{mag}(t) + T_{ex}(t) + T_f$, for free rotation we obtain: $J\alpha(t) = T_{mag}(t) + T_f$, where $\alpha(t) = \frac{\partial^2\theta(t)}{\partial t^2}$ is the angular acceleration and T_f is directed against $T_{mag}(t)$. In the beginning of motion, when the magnetic torque takes its maximum value $\frac{V_a K_M}{R_a}$, we obtain: $\alpha = \frac{max\,T_{mag} - T_f}{J} = \frac{1.15\times10^{-2} - 1.318\times10^{-4}}{10^{-7}} = 113682$ rad/s^2. Here, $V_a = 6$ V. Good agreement with (17) in the datasheet.

8. Calculate the maximum motor efficiency in Eq. (15) and compare it with (4) in the datasheet:

> $max\eta = \frac{K_M}{K_F}\left(1 - \sqrt{\frac{I_a^{no-load}}{maxI_a}}\right)^2 \approx 0.8\ (80\%)$, where $I_a^{no-load} = 0.02$ A is the no-load armature current (see (6) in the datasheet) and $maxI_a = \frac{V_a}{R_a} = \frac{6}{3.41} \approx 1.76$ A. Good agreement with (4) in the datasheet.

9. Calculate the maximum motor power in Eq. (12) and compare it with (3) in the datasheet:

$maxP_L = \frac{V_a^2 K_M}{4K_F R_a} \approx$ 2.64 W. Good agreement with (3) in the datasheet.

$maxP_L = 2.64$ W $\rightarrow T_{mag} = \frac{V_a K_M}{2R_a} \approx 5.8$ mN×m $\qquad \omega = \frac{V_a}{2K_F} \approx 455$ rad/s

Efficiency $\eta(T_{mag}) \approx 0.5\ (50\%)$

Note that the recommended torque (28) from the datasheet is 4.2 mN×m, which is below the torque required for the maximum mechanical power. It is not clear from what criteria this torque value was selected. Maybe 50% efficiency?

Solver in Python: https://github.com/DmitriyMakhnovskiy/Brushed_DC_motor_solver

Output files: Speed-to-torque_characteristics.csv, Torque-to-speed_characteristics.csv, Torque-to-power_characteristics.csv, Torque-to-efficiency_characteristics.csv.

Output parameters (printed to console): Maximum speed, Maximum torque, Maximum efficiency, Maximum mechanical power, No-load armature current, Torque-to-current coefficient, Voltage-to-speed coefficient (no load), dw/dT slope, Maximum angular acceleration (no load).

```python
#
# Solver for brushed DC motors
# Further reading: https://support.maxongroup.com/hc/en-us/articles/360001900933-Formulae-Handbook
#
# Dr. Dmitriy Makhnovskiy, City College Plymouth, England
# 30.03.2024
#

import matplotlib.pyplot as plt
import csv

# Motor constants used in the model:
Va = 6.0 # Armature voltage, V
Ra = 3.41  # Armature resistance, Ohms
La = 7.5e-5  # Armature inductance, H
KF = 6.589e-3  # Back-EMF constant, V x s / rad
J = 1.0e-7  # Rotor moment of inertia, kg x m^2
KM = 6.59e-3  # Torque constant, N x m / A
Tf = 1.3e-4  # Friction torque, N x m
N = 1000  # Number of points in the graph

# Characteristic parameters:
maxw0 = Va / KF  # Maximum angular speed without the friction torque, rad / s
maxw = Va / KF - (Tf * Ra) / (KF * KM)  # Maximum angular speed with the friction torque, rad / s
maxTmag = (Va * KM) / Ra  # Maximum magnetic torque, N x m
maxh = (KM / KF) * (1.0 - ((Ra * Tf) / (Va * KM))**0.5)**2  # Maximum efficiency
maxPL = (Va**2 * KM) / (4.0 * KF * Ra)  # Maximum power, W
I0 = Tf / KM  # No-load armature current, A
TIa = 1.0 / KM  # Torque-to-current coefficient, A/(Nxm)
Vaw = 1.0 /KF  # Voltage-to-speed coefficient (no load), rad/(Vxs)
dwdT = Ra / (KF * KM)  # dw/dT slope
alfa = (maxTmag -Tf) / J  # Maximum angular acceleration (no load), rad/s^2
print('Maximum speed = ', format(maxw, ".3e"), ' rad/s' )
print('Maximum torque = ', format(maxTmag, ".3e"), ' Nm')
print('Maximum efficiency h = ', format(maxh * 100, ".3e"), '%')
print('Maximum mechanical power = ', format(maxPL, ".3e"), ' W')
```

```
print('No-load armature current = ', format(I0, ".3e"), ' A')
print('Torque-to-current coefficient = ', format(TIa, ".3e"), ' A/(Nm)')
print('Voltage-to-speed coefficient (no load) = ', format(Vaw, ".3e"), 'rad/(Vs)')
print('dw/dT slope = ', format(dwdT, ".3e"), ' rad/(sNm)')
print('Maximum angular acceleration (no load) = ', format(alfa, ".3e"), ' rad/s^2')

# Arrays for graphs
wx = [0.0 + maxw0 * i / (N - 1) for i in range(N)]  # Angular speed array
Tx = [Tf + (maxTmag - Tf) * i / (N - 1) for i in range(N)]  # Magnetic torque array
T_w = [(Va * KM) / Ra - (KF * KM * w) / Ra for w in wx]  # Speed-to-torque characteristics
w_T = [Va / KF - (Ra * T) / (KF * KM) for T in Tx]  # Torque-to-speed characteristics
PL_T = [(Va / KF -(T * Ra) / (KF * KM)) * T for T in Tx]  # Torque-to-power characteristics
h_T = [(KM / KF) * (1.0 - (Ra * T) / (Va * KM)) * (1.0 - Tf / T) for T in Tx]  # Torque-to-efficiency characteristics

# Function to plot graphs and write arrays to files
def plot_and_save_data(x, y, x_label, y_label, graph_title, filename):
    # Plotting the graph
    plt.plot(x, y)
    plt.xlabel(x_label)
    plt.ylabel(y_label)
    plt.title(graph_title)

    # Adding detailed grid
    plt.grid(True)

    # Saving data to CSV file
    with open(filename, 'w', newline='') as csvfile:
        csv_writer = csv.writer(csvfile)
        csv_writer.writerow([x_label, y_label])  # Write header
        for i in range(len(x)):
            csv_writer.writerow([x[i], y[i]])

    # Displaying the plot
    plt.show()

x_label = 'Speed, rad/s'
y_label = 'Torque, Nm'
graph_title = 'Speed-to-torque characteristics'
filename = 'Speed-to-torque_characteristics.csv'
plot_and_save_data(wx, T_w, x_label, y_label, graph_title, filename)

x_label = 'Torque, Nm'
y_label = 'Speed, rad/s'
graph_title = 'Torque-to-speed characteristics'
filename = 'Torque-to-speed_characteristics.csv'
plot_and_save_data(Tx, w_T, x_label, y_label, graph_title, filename)

x_label = 'Torque, Nm'
y_label = 'Power, W'
graph_title = 'Torque-to-power characteristics'
filename = 'Torque-to-power_characteristics.csv'
plot_and_save_data(Tx, PL_T, x_label, y_label, graph_title, filename)

x_label = 'Torque, Nm'
y_label = 'Efficiency, %'
graph_title = 'Torque-to-efficiency characteristics'
filename = 'Torque-to-efficiency_characteristics.csv'
h_T = [h_T * 100.0 for h_T in h_T]  # Transferring to %
plot_and_save_data(Tx, h_T, x_label, y_label, graph_title, filename)
```