Task: Find all currents and voltages in the bridge circuit shown below.

In this educational project, we demonstrate how to solve DC resistance networks using Python through matrix equations. The Python calculations are validated through simulations in LTspice. Additionally, we delve into the discussion of solving overdetermined systems of linear equations.

Terms: Kirchhoff's Current Law (KCL) and Kirchhoff's Voltage Law (KVL). Parameters used in the circuit below: $R_1 = 4\ \Omega$, $R_2 = 7\ \Omega$, $R_3 = 9\ \Omega$, $R_4 = 5\ \Omega$, $R_5 = 20\ \Omega$, and $V = 32$ V.

Answers:

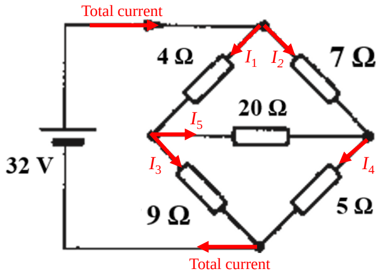
```
1. I1 =  2.699 A
2. I2 =  2.524 A
3. I3 =  2.356 A
4. I4 =  2.867 A
5. I5 =  0.343 A
6. Total current =  5.223 A
7.
8. V1 =  10.796 V
9. V2 =  17.668 V
10. V3 =  21.204 V
11. V4 =  14.335 V
12. V5 =  6.86 V
```

Solution:

KCL:

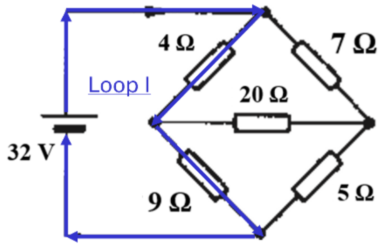
$$I_1 + I_2 = I_3 + I_4$$

$$I_1 = I_3 + I_5$$



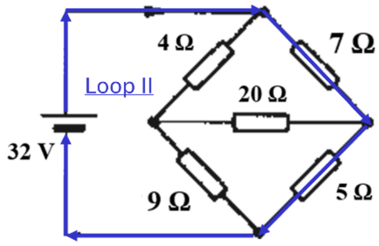
KVL, Loop I:

$$R_1 I_1 + R_3 I_3 = V$$



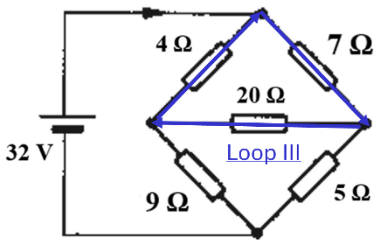
KVL, Loop II:

$$R_2 I_2 + R_4 I_4 = V$$



KVL, Loop III:

$$-R_1 I_1 + R_2 I_2 - R_5 I_5 = 0$$



System of equations derived from KCL and KVL:

$$\begin{aligned} & I_1 + I_2 - I_3 - I_4 = 0 \\ & I_1 - I_3 - I_5 = 0 \\ & R_1 I_1 + R_3 I_3 = V \\ & R_2 I_2 + R_4 I_4 = V \\ & -R_1 I_1 + R_2 I_2 - R_5 I_5 = 0 \end{aligned} \qquad (1)$$

Eq. (1) can be written in the matrix form:

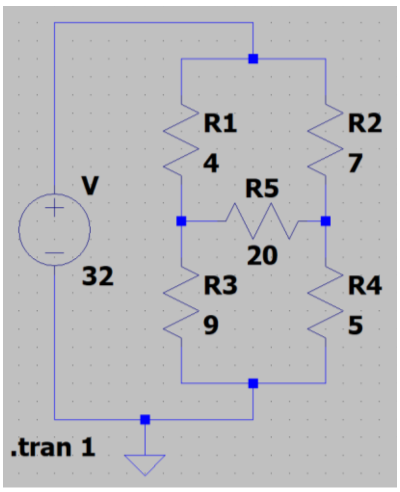
$$\hat{\mathbf{R}} \times \vec{\mathbf{I}} = \vec{\mathbf{E}} \quad \Rightarrow \quad \vec{\mathbf{I}} = \hat{\mathbf{R}}^{-1} \vec{\mathbf{E}} \qquad (2)$$

$$\begin{pmatrix} 1 & 1 & -1 & -1 & 0 \\ 1 & 0 & -1 & 0 & -1 \\ R_1 & 0 & R_3 & 0 & 0 \\ 0 & R_2 & 0 & R_4 & 0 \\ -R_1 & R_2 & 0 & 0 & -R_5 \end{pmatrix} \times \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ V \\ V \\ 0 \end{pmatrix} \qquad (3)$$

Numerical solution of Eqs. (2), (3):

```
 1. I1 =  2.699 A
 2. I2 =  2.524 A
 3. I3 =  2.356 A
 4. I4 =  2.867 A
 5. I5 =  0.343 A
 6. Total current =  5.223 A
 7.
 8. V1 =  10.796 V
 9. V2 =  17.668 V
10. V3 =  21.204 V
11. V4 =  14.335 V
12. V5 =  6.86 V
```

The calculated values were validated in LTspice simulator for the circuit shown below:



The Python code for solving Eqs. (2), (3) can be adjusted to accommodate any number of unknowns using the library scipy.linalg.lstsq. For instance, consider the following code tailored for a system with five unknowns:

```
#
# Two methods for solving a system of linear equations A * a = b, where a and b are vectors:
# (1) Matrix A is square (N, N)
# https://docs.scipy.org/doc/scipy/reference/generated/scipy.linalg.solve.html
#
# (2) Matrix A is not square (N, M), where the number of rows (N conditions) exceeds the number of columns (M unknowns)
# https://scipy.github.io/devdocs/reference/generated/scipy.linalg.lstsq.html#scipy.linalg.lstsq
#
# This algorithm computes the vector a such that the norm ||b - A * a|| is minimized.
#
# Dr. Dmitriy Makhnovskiy, City College Plymouth, England, 07.04.2024
#

from scipy.linalg import lstsq

# Parameters used in the A matrix
R1 = 4.0
R2 = 7.0
R3 = 9.0
R4 = 5.0
R5 = 20.0

# A matrix
A = [
     [1.0, 1.0, -1.0, -1.0, 0.0],
     [1.0, 0.0, -1.0, 0.0, -1.0],
     [R1, 0.0, R3, 0.0, 0.0],
     [0.0, R2, 0.0, R4, 0.0],
     [-R1, R2, 0.0, 0.0, -R5],
     ]

# b vector (right part of the equation)
b = [
     [0.0],
     [0.0],
     [32],
     [32],
     [0.0],
      ]

# Minimization of the norm ||b - A * a|| --> 0
a, res, rnk, s = lstsq(A, b)
a = a.flatten()  # 2D a-array was converted to a 1D array
a = [round(x, 3) for x in a]  # rounding to three significant figures after the dot

print('I1 = ', a[0], 'A')
print('I2 = ', a[1], 'A')
print('I3 = ', a[2], 'A')
print('I4 = ', a[3], 'A')
print('I5 = ', a[4], 'A')
print('Total current = ', a[0] + a[1], 'A')
print('')
print('V1 = ', a[0] * R1, 'V')
print('V2 = ', a[1] * R2, 'V')
print('V3 = ', a[2] * R3, 'V')
print('V4 = ', a[3] * R4, 'V')
print('V5 = ', a[4] * R5, 'V')
```

In the previous numerical solution, we employed a square matrix $(5, 5)$. However, it is also possible to utilize an excessive number of conditions, such as additional KCL and KVL equations. For instance, consider the following set of linear equations:

$I_1 + I_2 - I_3 - I_4 = 0$ (KCL)

$I_1 - I_3 - I_5 = 0$ (KCL)

$I_2 - I_4 + I_5 = 0$ (KCL)

$R_1 I_1 + R_3 I_3 = V$ (square KVL loop with the voltage source) (4)

$R_2 I_2 + R_4 I_4 = V$ (square KVL loop with the voltage source)

$-R_1 I_1 + R_2 I_2 - R_5 I_5 = 0$ (triangle KVL loop without the voltage source)

$-R_3 I_3 + R_4 I_4 + R_5 I_5 = 0$ (triangle KVL loop without the voltage source)

$R_1 I_1 - R_2 I_2 + R_3 I_3 - R_4 I_4 = 0$ (rhombic KVL loop without the voltage source)

In Eq. (4), we have five unknows and eight conditions. This system has the following matrix form $(8, 5)$:

$$\begin{pmatrix} 1 & 1 & -1 & -1 & 0 \\ 1 & 0 & -1 & 0 & -1 \\ 0 & 1 & 0 & -1 & 1 \\ R_1 & 0 & R_3 & 0 & 0 \\ 0 & R_2 & 0 & R_4 & 0 \\ -R_1 & R_2 & 0 & 0 & -R_5 \\ 0 & 0 & -R_3 & R_4 & R_5 \\ R_1 & -R_2 & R_3 & -R_4 & 0 \end{pmatrix} \times \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ V \\ V \\ 0 \\ 0 \\ 0 \end{pmatrix} \tag{5}$$

Numerical solution of Eq. (5):

```
1. I1 =  2.699 A
2. I2 =  2.524 A
3. I3 =  2.356 A
4. I4 =  2.867 A
5. I5 =  0.343 A
6. Total current =  5.223 A
```

These values align perfectly with those derived from the square matrix $(5, 5)$ in Eq. (3). Moreover, the overdetermined system may demonstrate increased resilience to inaccuracies in the input parameter values.