

On hybrid approach in microwave scattering theory for wire-filled composites

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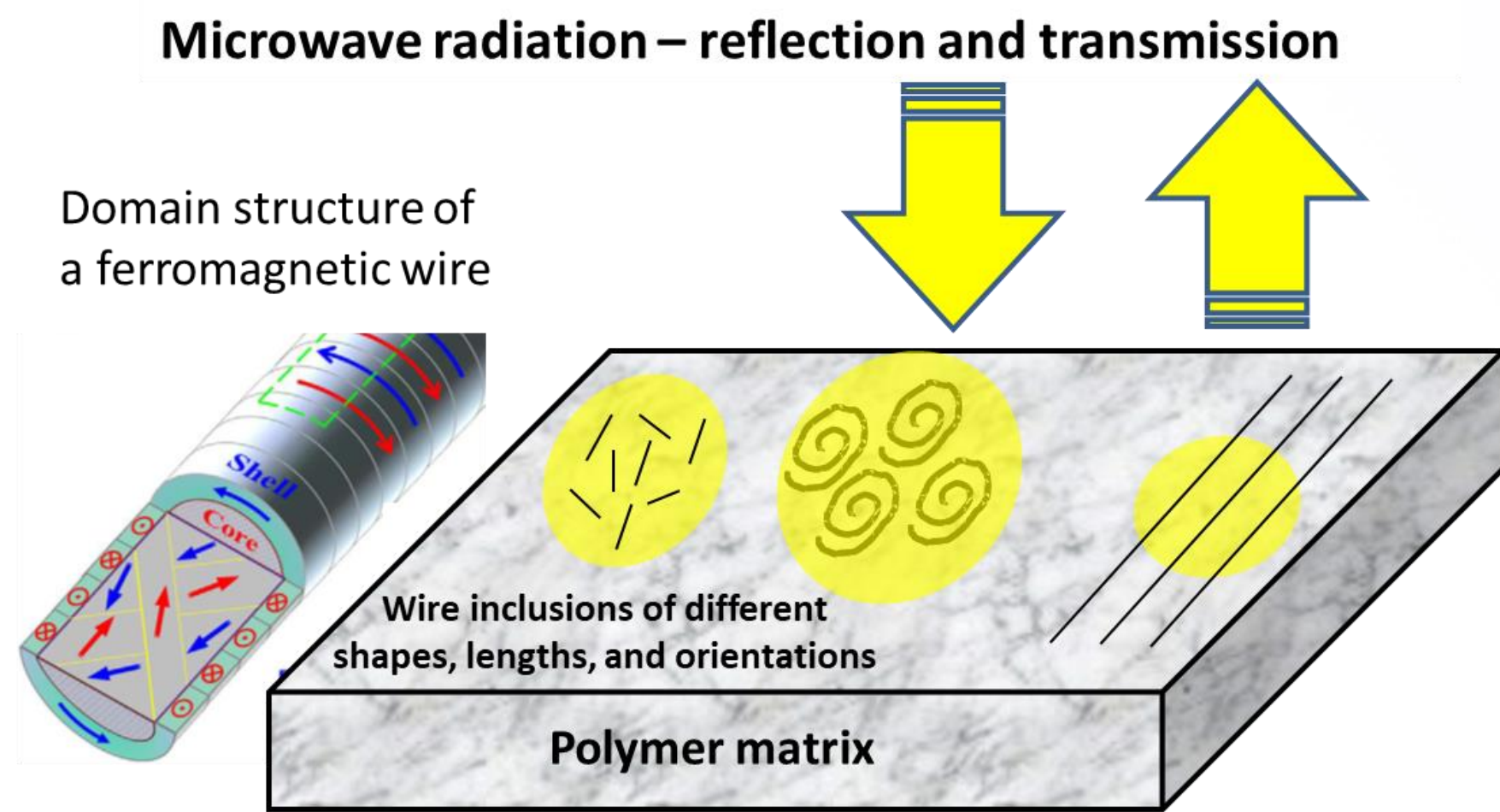
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Abstract: Within the framework of the proposed hybrid approach, combining metrological measurements of individual properties of wires and accurate solution of an external electrodynamic problem, there is no need to consider the most complex internal problem. The developed approach has been used to calculate the dipole moment induced by an external wave in a short ferromagnetic wire. The surface impedance of wire in the presence of a longitudinal DC magnetic field and tensile stress was measured in a specially designed PCB cell. The numerical algorithm for solving the antenna equation with the impedance boundary condition was implemented in PyCharm IDE. The single particle scattering problem is the key stage to constructing a general theory that considers the collective response of many particles, as well as their EM coupling.

Introduction

Difficulties in constructing a predictive scattering theory

- Difficulties for predicting the microwave scattering from conductive inclusions with complex domain structures and shapes
- In the microwave range, radiation penetrates the entire depth of an inclusion. Therefore, its internal domain structure may strongly affect the high-frequency conductivity and, as a consequence, the scattering properties.
- Wire inclusions may have a complex 2D or 3D shape that makes it difficult to solve the external electrodynamic problem of microwave scattering.



External stimuli: magnetic field, mechanical stress, and heating

Antenna equation with the impedance boundary condition (cgs units)

An external EM field will induce the prevailing (\vec{e}_x, \vec{h}_ϕ) field polarization on the wire surface – longitudinal electric and circular magnetic fields. The same polarization can be induced by a linear current flowing along the wire axis with the volume density $j(x)\delta_{\vec{s}}$, where $\delta_{\vec{s}}(y, z)$ is two-dimensional Dirac's function. The function $j(x)$ will be further referred to as "linear current".

Antenna equation in the frequency domain:

$$\frac{\partial^2}{\partial x^2} (G * j) + k^2 (G * j) = \frac{i\omega\epsilon}{4\pi} \vec{e}_{0x}(x) - \frac{i\omega\epsilon\zeta_{xx}}{2\pi ac} (G_\phi * j) + \frac{i\omega\epsilon\zeta_{x\phi}}{4\pi} \vec{h}_{0x}(x)$$

Hybrid approach to solving the antenna equation

The surface impedance includes both conductive and magnetic properties of the wire sample thus freeing us from the need to solve the EM internal problem.

ζ_{xx}^{cgs} – wire longitudinal surface impedance (cgs units)
 Z – wire impedance in Ω
 l – wire length
 a – wire radius
 c – speed of light (cm/s)

$$\zeta_{xx}^{cgs} = \frac{10^9 a Z [Q]}{2cl} \quad \text{Measured in experiment}$$

Used in numerical calculations

Can be neglected

$$\frac{\partial^2}{\partial x^2} (G * j) + k^2 (G * j) = \frac{i\omega\epsilon}{4\pi} \vec{e}_{0x}(x) - \frac{i\omega\epsilon\zeta_{xx}}{2\pi ac} (G_\phi * j) + \frac{i\omega\epsilon\zeta_{x\phi}}{4\pi} \vec{h}_{0x}(x)$$

This (neglected) term with the off-diagonal impedance component $\zeta_{x\phi}$ is non-zero only for some specific ferromagnetic wires. And even for them the main contribution will be from the longitudinal impedance.

The solution of the integro-differential antenna equation for the linear current will allow calculating the scattered electromagnetic field. It will take into account both the resistive and radiative losses. A similar antenna equation can be written for a wire of any spatial shape.

Technique for measuring the high-frequency impedance of ferromagnetic wires

The method for wideband measurements of the wire impedance, subjected to the external DC magnetic field and tensile stress, was developed in Ref. 1. The measurement setup, shown in Fig. 1, includes a 2-port Vector Network Analyser (VNA), stress machine, Helmholtz coil, dog-bone PCB cell, calibration PCB cell with the surface mount SOLT terminations (SHORT, OPEN, LOAD, THRU), and a stress control sensor. The PCB, made of Rogers' material, contains 50 Ω microstrips on one side and a continuous ground plane on the opposite side.

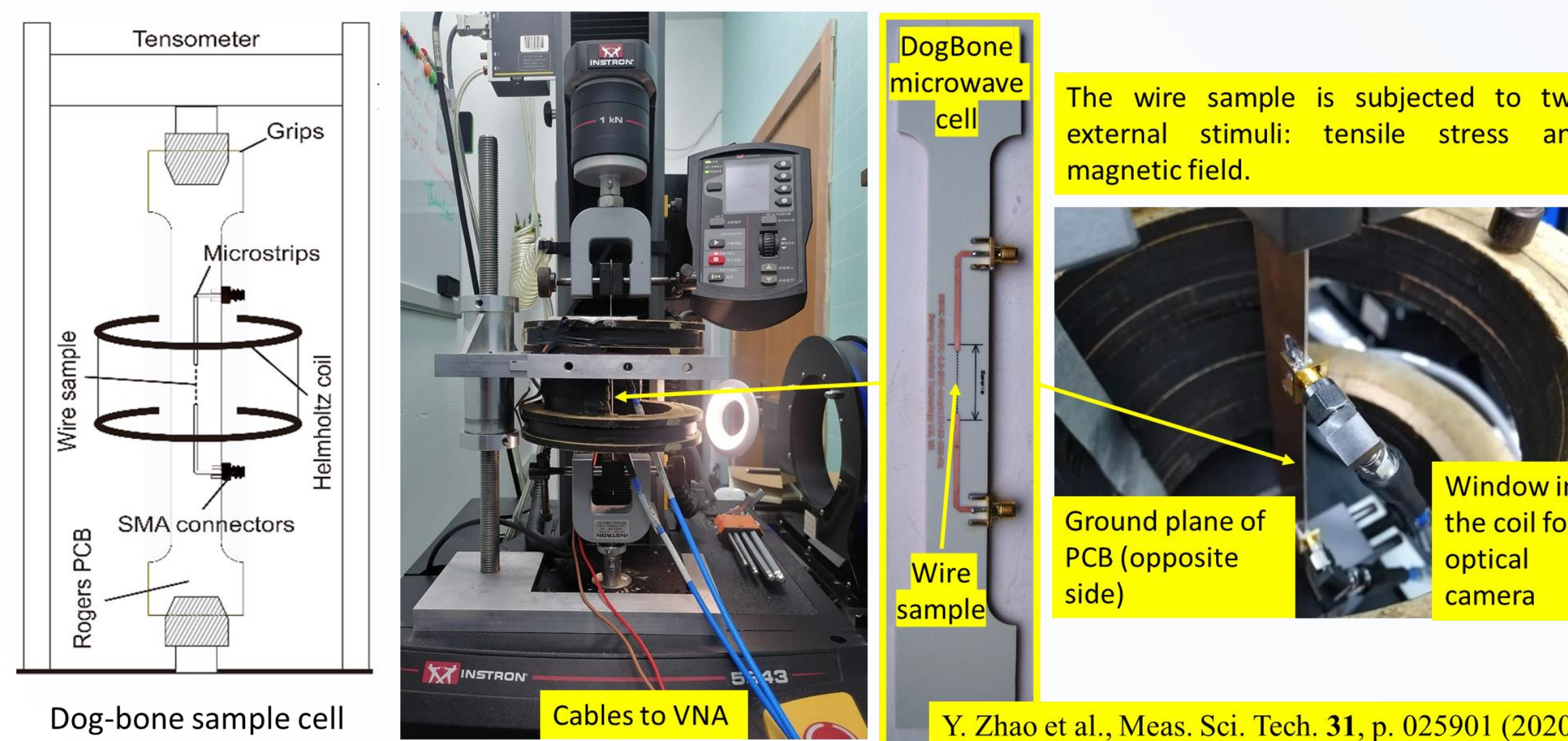


Figure 1. Experimental setup for the broadband measurement of the wire impedance subjected to the external DC magnetic field and tensile stresses.

Two stage microwave error correction

De-embedding of the PCB microstrips by means of the 3-term model with the self-made non-coaxial terminations: SHORT, OPEN, and LOAD as shown in Fig. 2.

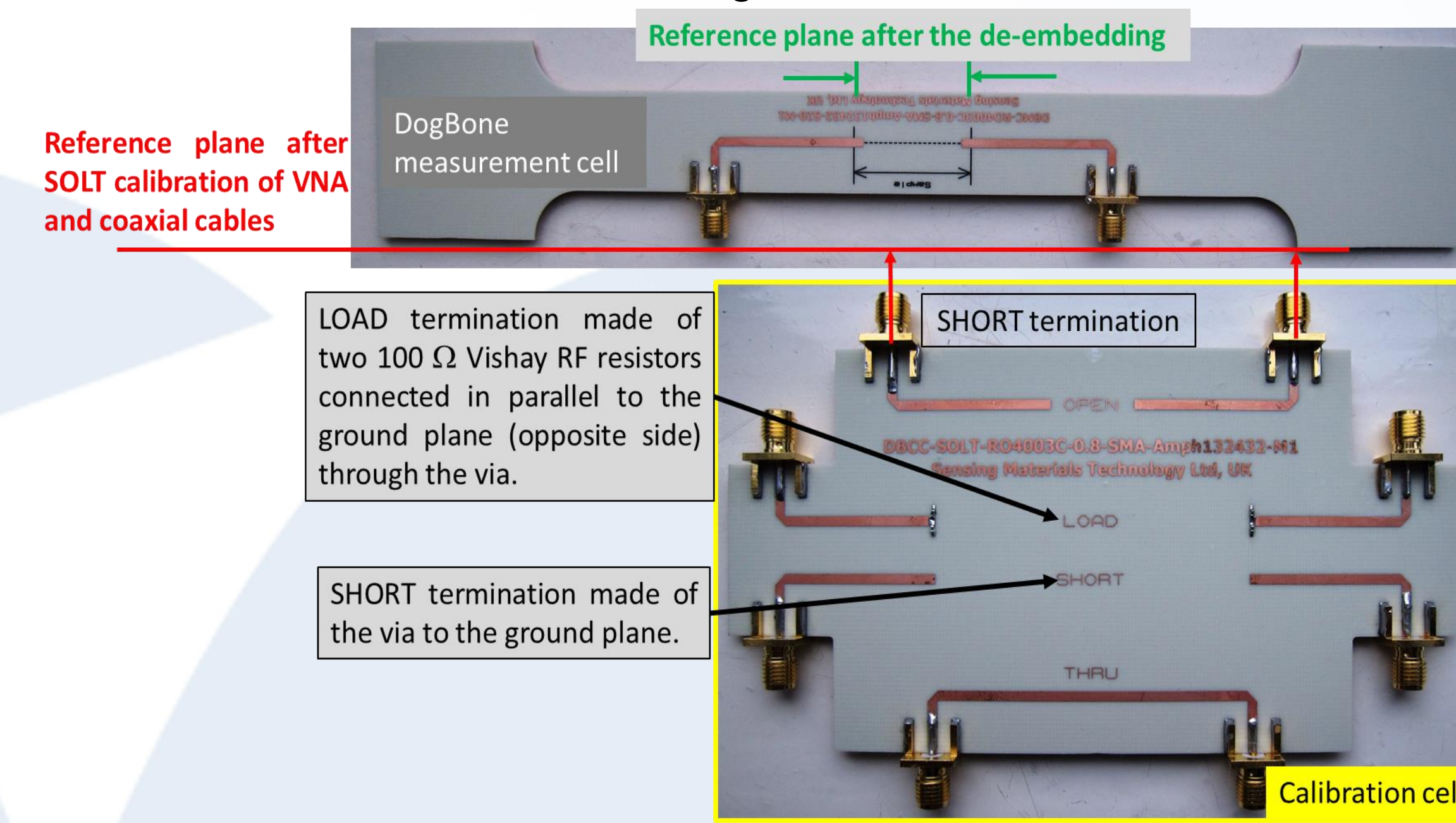
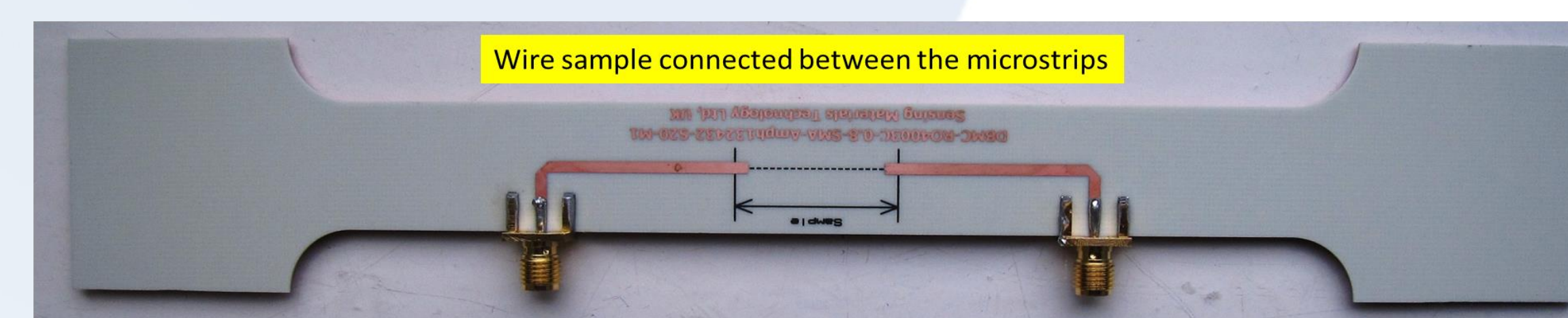


Figure 2: Two stage microwave error correction.

Impedance can only be measured correctly from a lumped component. A wire sample is a waveguide: wire over the PCB ground on the opposite side. The phase incursion along the sample must be compensated. This can be done by evaluating the delay time Δt obtained from the unwrapped phase of the measured S_{21} (with applied SOLT calibration and microstrip de-embedding).

Compensation of the phase distortion caused by a wire sample



$$Z(\omega) = 100 \times \frac{1 - S_{21}(\omega)}{S_{21}(\omega)} \quad \text{Corrected} \quad \text{Measured} \quad \hat{S}_{21}(\omega) = S_{21}(\omega) \exp(i\omega\Delta t)$$

Since the scattering in the radiative near/far fields will be of a dipole nature, we recalculated the linear current density to the dipole moment, which will be used for numerical analysis. Using the continuity equation written in the frequency domain, $\partial j(x)/\partial x = i\omega \rho(x)$, where $\rho(x)$ is the linear charge density, we obtain for the dipole moment D :

$$D = \int_{-l/2}^{l/2} \rho(x) dx = \frac{i}{\omega} \int_{-l/2}^{l/2} j(x) dx$$

Typical impedance dispersion under stress and magnetic field

For 640 MHz (ferromagnetic resonance) and bias field 5.5 Oe, microwire impedance shows the largest strain sensitivity ($\approx 40\%$) as shown in Fig. 3.

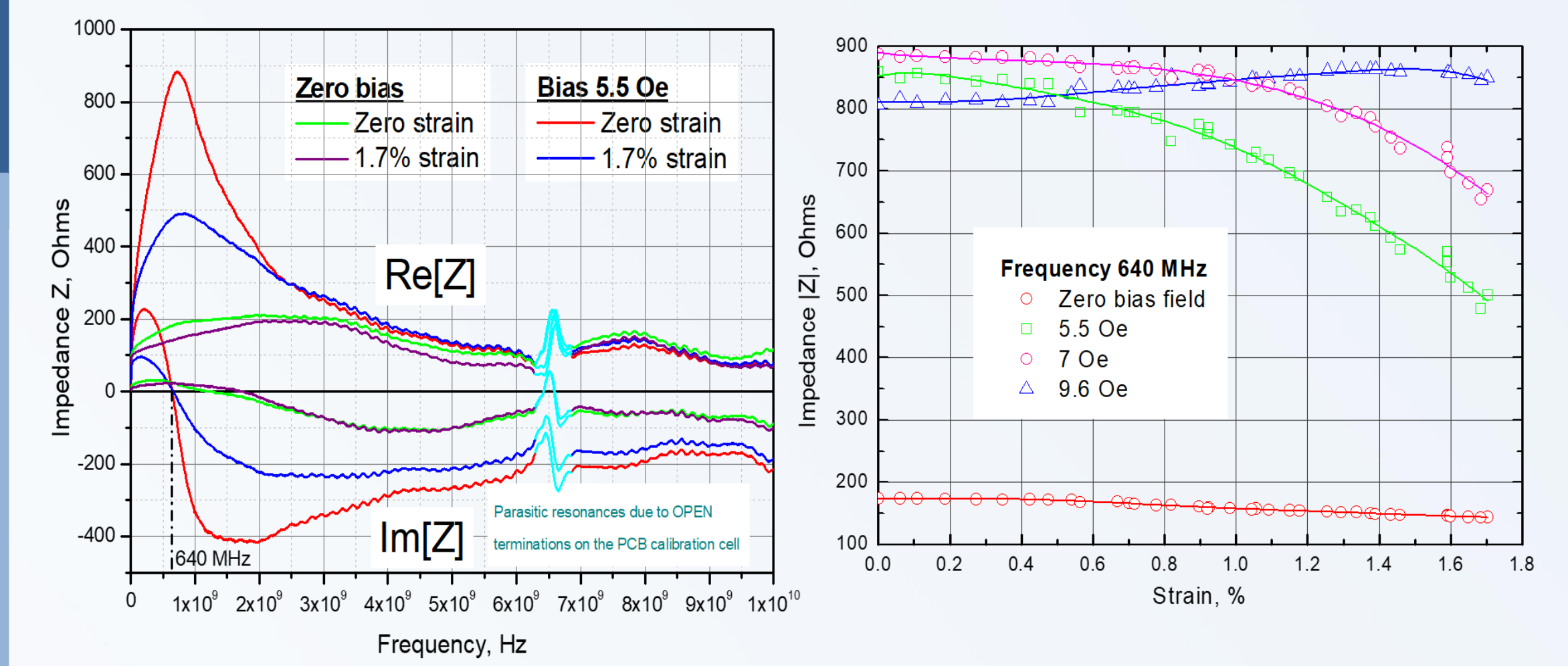


Figure 3. Frequency dispersion of impedance measured in the presence of bias fields (0 or 5 Oe) and tensile strains (0 or 1.7%).10.

Dispersions of the dipole moments calculated with the experimental Z

At higher GHz frequencies, the radiation losses become significant and the sensitivity of the impedance to the bias magnetic field and tensile stress decreases (Fig. 4).

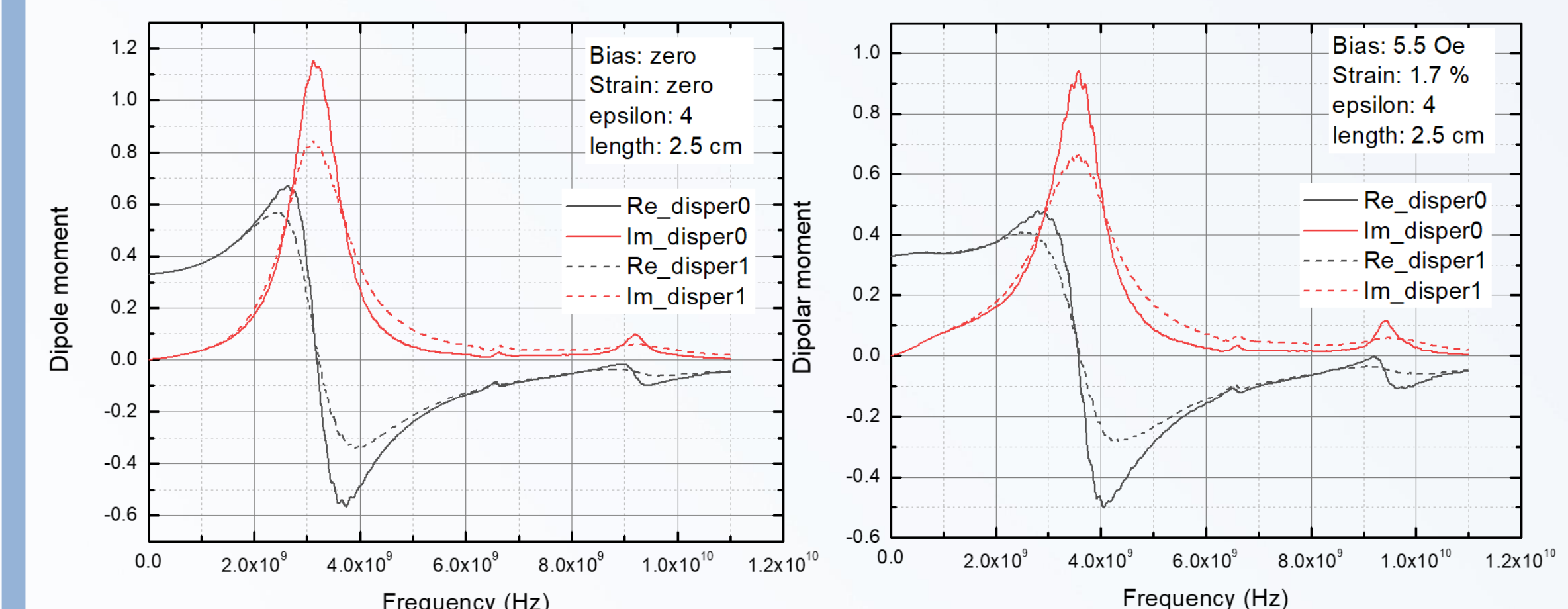


Figure 4: Frequency dispersion of the dipole moment calculated using experimentally measured for zero (disper0) and first (disper1) iterations of the current distribution at 0 – 12 GHz.

At MHz and lower GHz frequencies, the resistive losses prevail over the radiation ones. The impedance becomes more sensitive to the bias magnetic field and tensile stress.

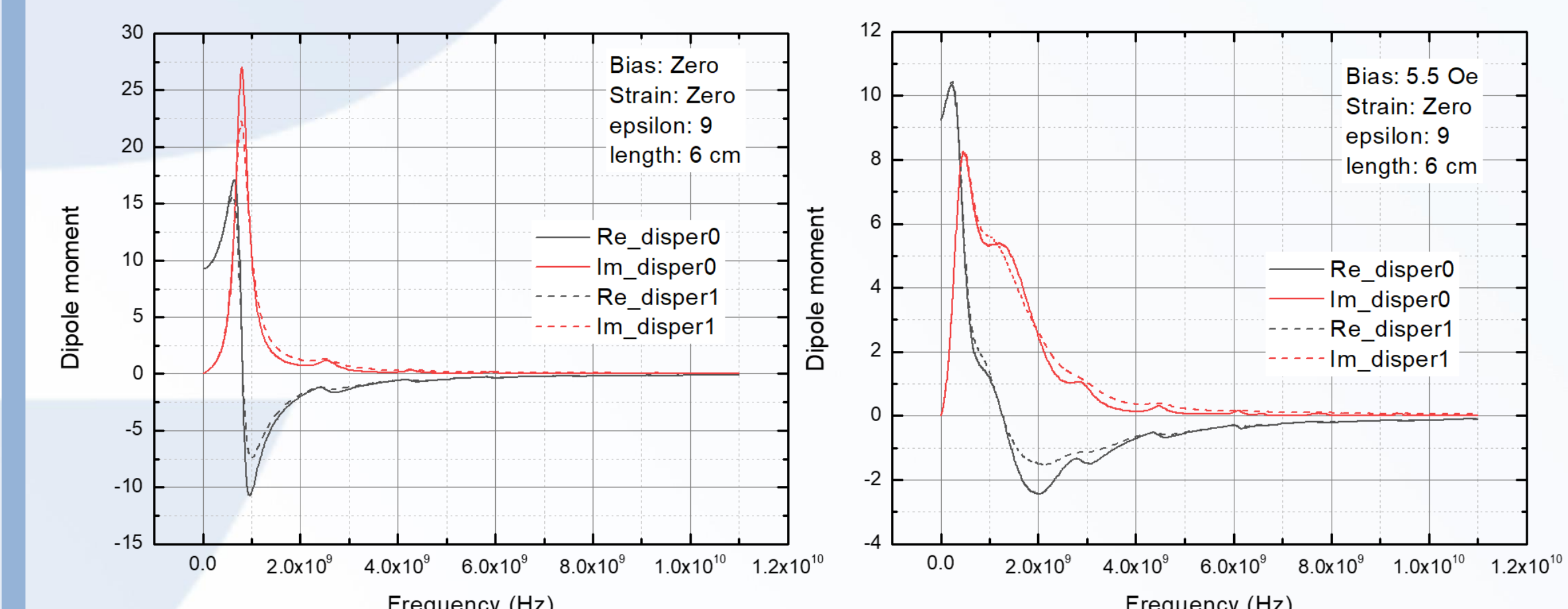


Figure 5. Frequency dispersion of the dipole moment calculated using experimentally measured when the antenna resonance is close to the ferromagnetic resonance at 0 – 12 GHz.

In this work, we have demonstrated a hybrid method for calculating microwave scattering from thin ferromagnetic wires. The proposed approach makes it possible to avoid solving the internal problem, reducing the consideration to only the external electrodynamic problem.

References

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