



**DEPARTMENT OF COMPUTER SCIENCE & ENGINEERING
(ARTIFICIAL INTELLIGENCE & MACHINE LEARNING)**

② Apply PCA on the following data and find the principal component.

x 2.5 0.5 2.2 1.9 3.1 2.3 2 1 1.5 1.1

y 2.4 0.7 2.9 2.2 3 2.7 1.6 1.1 1.6 0.9

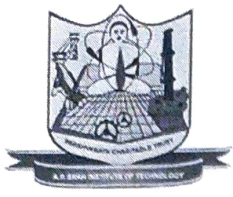
→ calculate mean:

$$\bar{x} = 1.81$$

$$\bar{y} = 1.91$$

calculate covariance:

$$\begin{aligned} \text{cov}(x, y) &= \frac{1}{9} (2.5-1.81)^2 + (0.5-1.81)^2 + (2.2-1.81)^2 \\ &\quad + (1.9-1.81)^2 + (3.1-1.81)^2 + (2.3-1.81)^2 \\ &\quad + (2-1.81)^2 + (1-1.81)^2 + (1.5-1.81)^2 \\ &\quad + (1.1-1.81)^2 \\ &= \frac{1}{9} (0.4554 + 1.7161 + 0.1521 + 0.0081 \\ &\quad + 1.6641 + 0.2401 + 0.0361 + 0.6561 \\ &\quad + 0.0961 + 0.5041) \\ &= 0.6165 \end{aligned}$$



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$$\text{cov}(y, y) = \cancel{1} \quad 0.7165$$

$$\text{cov}(x, y) = 0.61544$$

$$\text{cov}(y, x) = 0.61544$$

Now find the eigenvalues for covariance matrix.

$$\text{cov}(x, y) = \begin{bmatrix} 0.6165 & 0.61544 \\ 0.61544 & 0.7165 \end{bmatrix}$$

$$\begin{bmatrix} 0.6165 - \lambda & 0.61544 \\ 0.61544 & 0.7165 - \lambda \end{bmatrix}$$

$$= (0.6165 - \lambda)(0.7165 - \lambda) - (0.61544 \times 0.61544)$$

$$= \lambda^2 - 1.333\lambda + 0.4417 - 0.3788$$

$$= \lambda^2 - 1.333\lambda - 0.0629$$

$$\therefore \lambda_1 = 0.0489$$

$$\lambda_2 = 1.284$$



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calculate eigenvector for highest eigenvalue.
 i.e. for $\lambda = 1.284$

$$\therefore \text{eigenvector} = \begin{bmatrix} 0.922 \\ 1 \end{bmatrix}$$

$$\begin{aligned} \text{length of eigenvector} &= \sqrt{(0.922)^2 + (1)^2} \\ &= \sqrt{1.85} \\ &= 1.3602 \end{aligned}$$

$$\begin{aligned} \therefore \text{Unit Vector} &= \begin{bmatrix} 0.922 / 1.3602 \\ 1 / 1.3602 \end{bmatrix} \\ &= \begin{bmatrix} 0.6778 \\ 0.7352 \end{bmatrix} \end{aligned}$$

Now compute principal components



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x	y	First Principal Component
2.5	2.4	0.68
0.5	0.7	-1.30
2.2	2.9	0.389
1.9	2.2	0.0899
3.1	3	0.6121
2.3	2.7	0.4899
2	1.6	0.1899
1	1.1	-0.8099
1.5	1.6	-0.3099
1.1	0.9	-0.7099