Back Propagation - Numerical Example

Using back propagation network, find the new weights for the net as shown in below figure. It is presented with input pattern [0, 1] and the target output is 1. Use a learning rate $\alpha = 0.25$ and binary sigmoidal activation function.

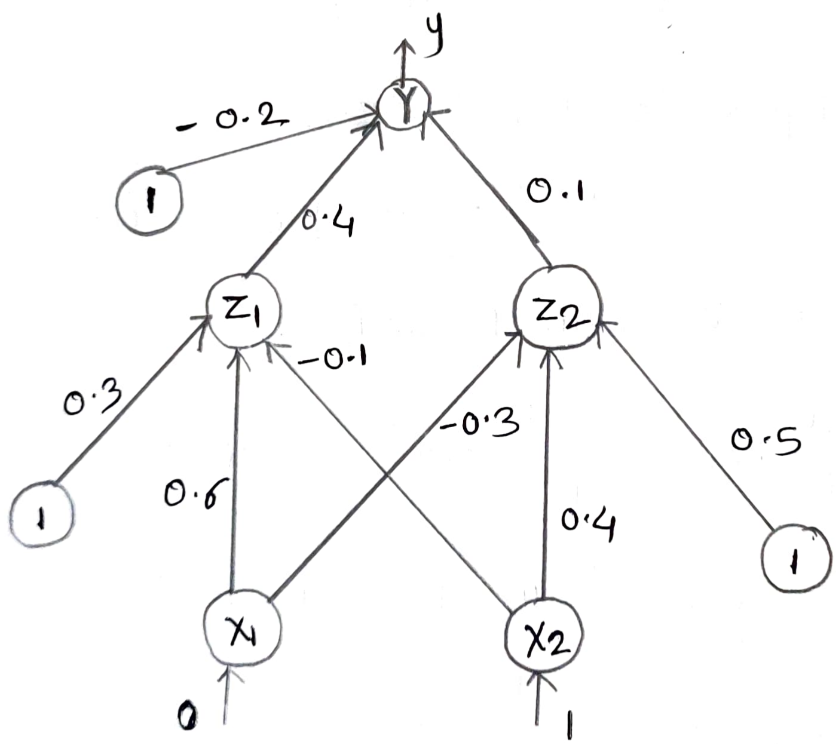

fig. Network

→ The new weights are calculated based on the back propagation training algorithm.

Binary sigmoidal function $= \frac{1}{1+e^{-x}}$

learning rate $= 0.25$

x_1	x_2	output
0	1	1

Calculate input to neuron z_1

$$Iz_1 = (0 * 0.6) + (1 * -0.1) + 0.3 = 0.2$$

Calculate input to neuron z_2

$$Iz_2 = (0 * (-0.3)) + (1 * 0.4) + 0.5 = 0.9$$

Calculate output from neuron z_1

$$Oz_1 = \frac{1}{1+e^{-Iz_1}} = \frac{1}{1+e^{-0.2}} = 0.5498$$

Calculate output from neuron z_2

$$Oz_2 = \frac{1}{1+e^{-Iz_2}} = \frac{1}{1+e^{-0.9}} = 0.7109$$

Now calculate net input entering into output neuron Y.

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$\therefore \; I_Y = (0.4 * O_{21}) + (0.1 * O_{22}) + \text{bias}$

$= (0.4 * 0.5498) + (0.1 * 0.7109) + (-0.2)$

$= 0.09101$

Now calculate output from neuron Y.

$$O_Y = \frac{1}{1+e^{-I_Y}} = \frac{1}{1+e^{-0.09101}} = 0.5227$$

Calculate error:

Error $=$ target $- O_Y$

$= 1 - 0.5227$

$= 0.4773$

Backpropagate this error to update the weights to minimize the error.

Backward Propagation:

For each unit k in the output layer,

$$\text{Error}_k = O_k(1-O_k)(\text{target}-O_k)$$

For each unit j in the hidden layer,

$$\text{Error}_j = O_j(1-O_j)\sum_k \text{Error}_k \, w_{jk}$$

For output unit Y, calculate Error_Y

$$\therefore \text{Error}_Y = O_Y(1-O_Y)(\text{target}-O_Y)$$

$$= 0.5227 * (1-0.5227)(1-0.5227)$$

$$= 0.5227 * (0.4773) * (0.4773)$$

$$= 0.1191$$

Update weights between hidden layer and output layer. (change in weight i.e. Δ value)

$$\Delta w_1 = \alpha \, \text{Error}_Y \, O_{21}$$

$$= 0.25 * 0.1191 * 0.5498$$

$$= 0.0164$$

$\Delta W_2 = \alpha \; Error_Y \; O_{22}$

$= 0.25 * 0.1191 * 0.7109$

$= 0.02117$

$\Delta W_0 \text{ (bias)} = \alpha \; Error_Y$

$= 0.25 * 0.1191 = 0.02978$

Now compute the error between input layer and the hidden layer.

$Error_{21} = O_{21} \, (1 - O_{21}) \, Error_Y * W_{21Y}$

$= 0.5498 * (1 - 0.5498) * 0.1191 * 0.4$

$= 0.5498 * 0.4502 * 0.1191 * 0.4$

$= 0.0118$

$Error_{22} = O_{22} \, (1 - O_{22}) * Error_Y * W_{22Y}$

$= 0.7109 * (1 - 0.7109) * 0.1191 * 0.1$

$= 0.7109 * 0.2891 * 0.1191 * 0.1$

$= 0.00245$

Now update the weights between input layer and output layer. (change in weight)

$\Delta V_{11} = \alpha * Error_{21} * x_1$

$= 0.25 * 0.0118 * 0 = 0$

$\Delta V_{21} = \alpha * Error_{21} * x_2$

$= 0.25 * 0.0118 * 1 = 0.00295$

ΔV_{01} (bias) $= \alpha * Error_{21}$

$= 0.25 * 0.0118 = 0.00295$

$\Delta V_{12} = \alpha * Error_{22} * x_1$

$= 0.25 * 0.00245 * 0 = 0$

$\Delta V_{22} = \alpha * Error_{22} * x_2$

$= 0.25 * 0.00245 * 1 = 0.0006125$

ΔV_{02} (bias) $= \alpha * Error_{22}$

$= 0.25 * 0.00245$

$= 0.0006125$

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Compute final weights of the network:

$V_{11}(new) = V_{11}(old) + \Delta V_{11}$

$= 0.6 + 0$

$= 0.6$

$V_{12}(new) = V_{12}(old) + \Delta V_{12}$

$= -0.3 + 0 = -0.3$

$V_{21}(new) = V_{21}(old) + \Delta V_{21}$

$= -0.1 + 0.00295$

$= -0.09705$

$V_{22}(new) = V_{22}(old) + \Delta V_{22}$

$= 0.4 + 0.0006125$

$= 0.4006125$

$V_{01}(new) = V_{01}(old) + \Delta V_{01}$

$= 0.3 + 0.00295$

$= 0.30295$

$V_{02}(new) = V_{02}(old) + \Delta V_{02}$

$= 0.5 + 0.0006125 = 0.5006125$