Introspection dynamics:
A simple model of counterfactual learning in asymmetric games

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Motivation

Social behavior through a mathematical lens

- Evolution of cooperation and coordination
- How individuals adopt strategies (learning processes), in the context of social interactions (strategic decision-making)

Motivation

Symmetric interactions

- If individuals are equal, they can learn by imitation (social learning)
- Symmetric payoff matrix:

		player 2	
		S_1	S_2
player 1	S_1	3, 3	0, 4
	S_2	4, 0	1, 1

Motivation

Asymmetric interactions

- But individuals can be different
- Asymmetric payoff matrix:

	S_1'	S_2'	S_3'
S_1	3, 2	0, 1	2, 4
S_2	4, 1	1, 0	1, 2

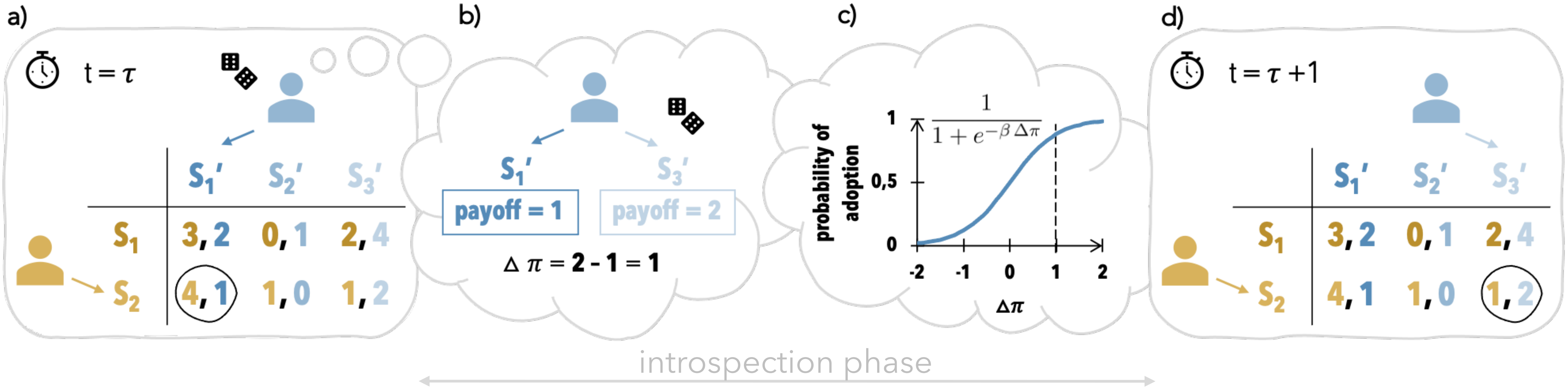
Motivation

Questions

- In the heterogeneous case, what learning process would be suitable?
- We propose *introspection dynamics*, a model where players don't look at other players' payoffs or strategies, but only at their own
- What outcomes result from it?

Model

Strategy update rule



Model

Adoption probability

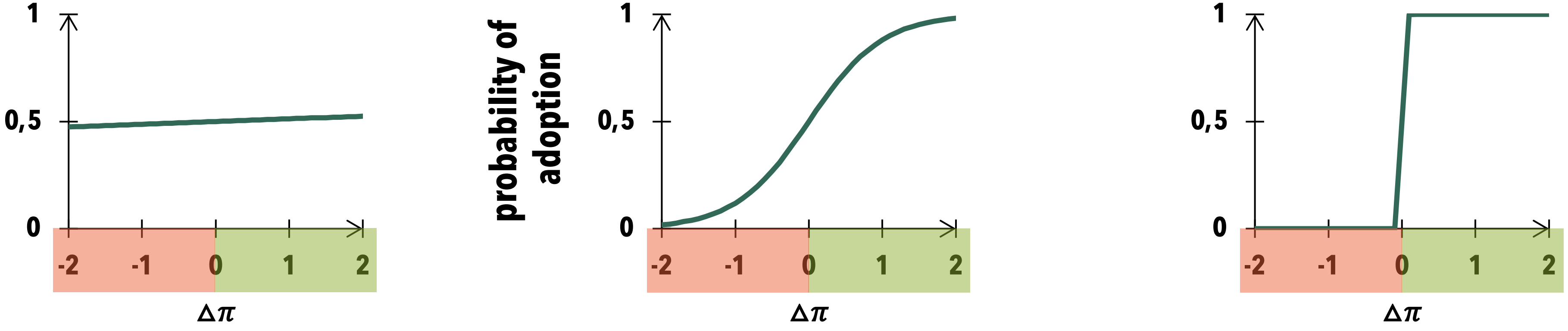
$$\varphi_\beta(\Delta\pi) = \frac{1}{1 + e^{-\beta\,\Delta\pi}}$$

Intensity of selection β

Weak selection $\beta \to 0$

Intermediate selection

Strong selection $\beta \to \infty$



Analytical properties

Markov process

- Average abundance of each state in the long run (stationary distribution)
- Special cases (simplifications):
  - Weak selection ($\beta \to 0$)
  - 2-strategy

	S_1'	S_2'	S_3'
S_1	3, 2	0, 1	2, 4
S_2	4, 1	1, 0	1, 2

states (strategy profiles)

Applications

Asymmetric prisoner's dilemma

		C	D
Cooperate	**C**	$b-c_1, b-c_2$	$-c_1, b$
Defect	**D**	$b, -c_2$	$0, 0$

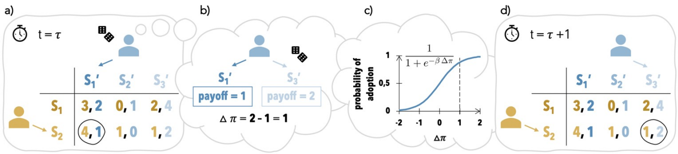
$b > c_1 > c_2 > 0$

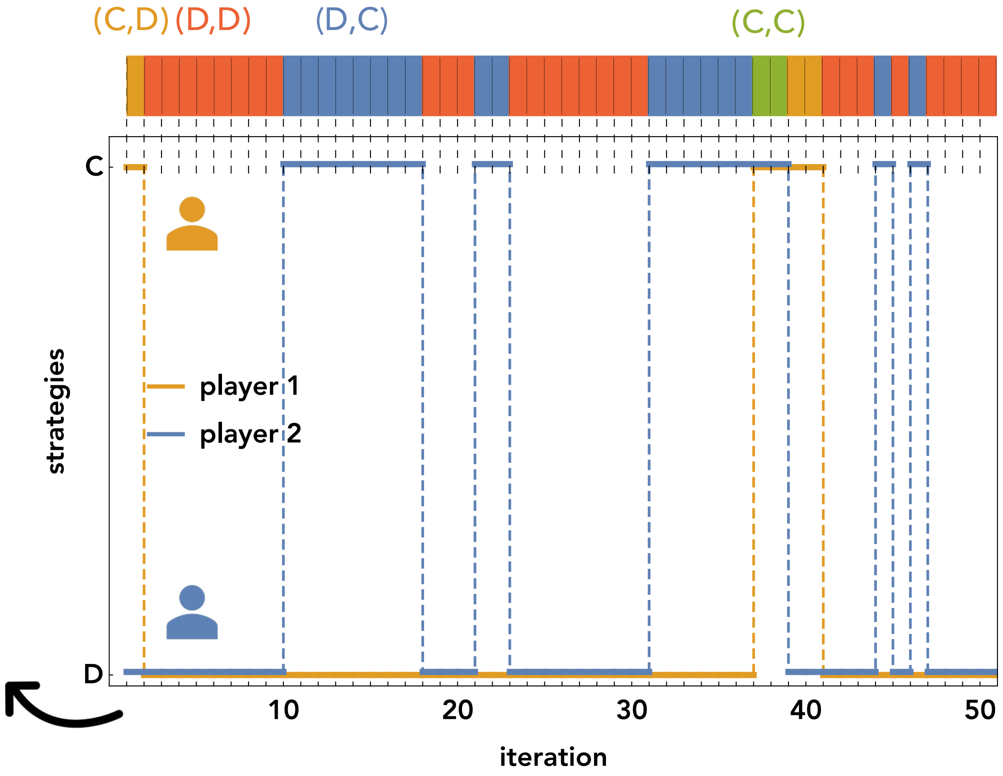
Applications

Asymmetric prisoner’s dilemma

	C	D
C	$b-c_1, b-c_2$	$-c_1, b$
D	$b, -c_2$	$0, 0$

$b = 1,\ c_1 = 0.6,\ c_2 = 0.1,\ \beta = 5$





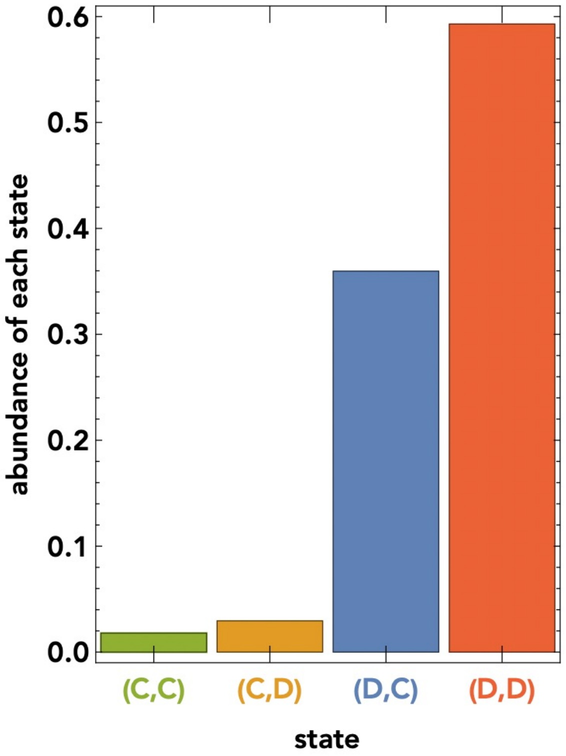
Applications

Asymmetric prisoner's dilemma

	C	**D**
C	$b-c_1, b-c_2$	$-c_1, b$
D	$b, -c_2$	$0, 0$

$b = 1$, $c_1 = 0.6$, $c_2 = 0.1$, $\beta = 5$

$$\mathbf{u} = \left(u_{\mathbf{CC}}, u_{\mathbf{CD}}, u_{\mathbf{DC}}, u_{\mathbf{DD}}\right)$$
$$\propto \left(1,\ \mathrm{e}^{\beta c_2},\ \mathrm{e}^{\beta c_1},\ \mathrm{e}^{\beta(c_1+c_2)}\right)$$



Applications

Asymmetric prisoner's dilemma

	C	D
C	$b-c_1, b-c_2$	$-c_1, b$
D	$b, -c_2$	$0, 0$

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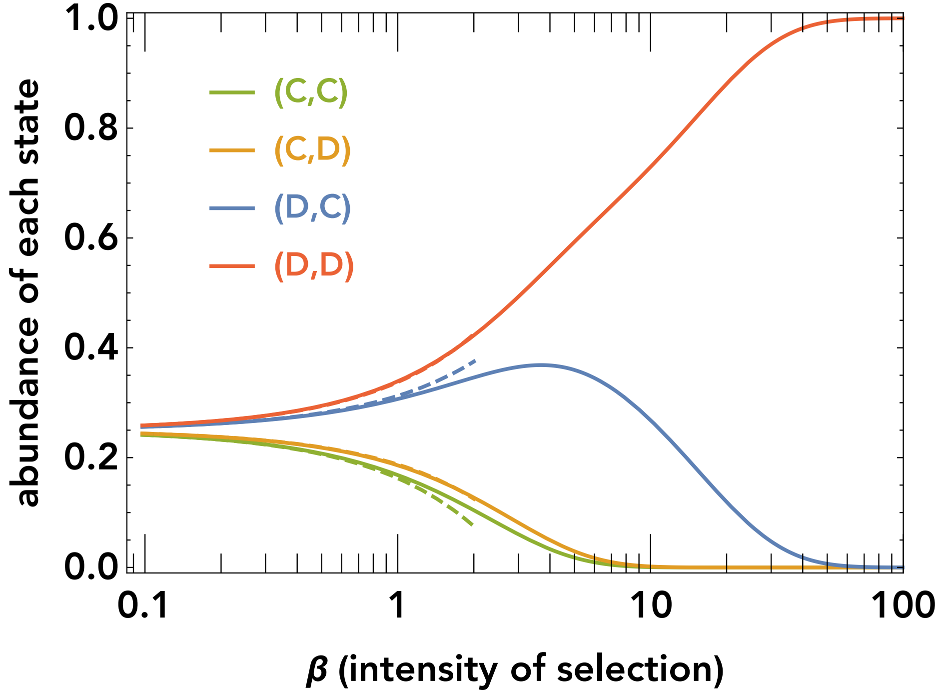
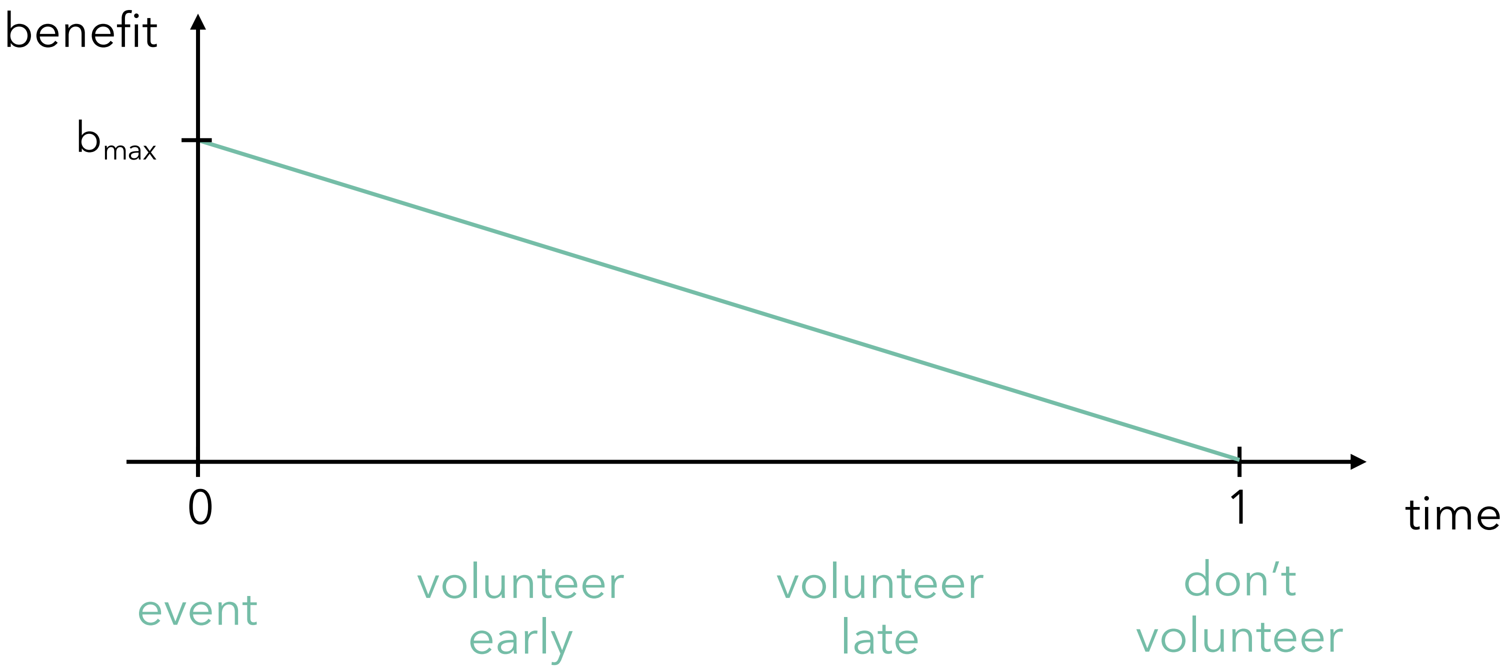


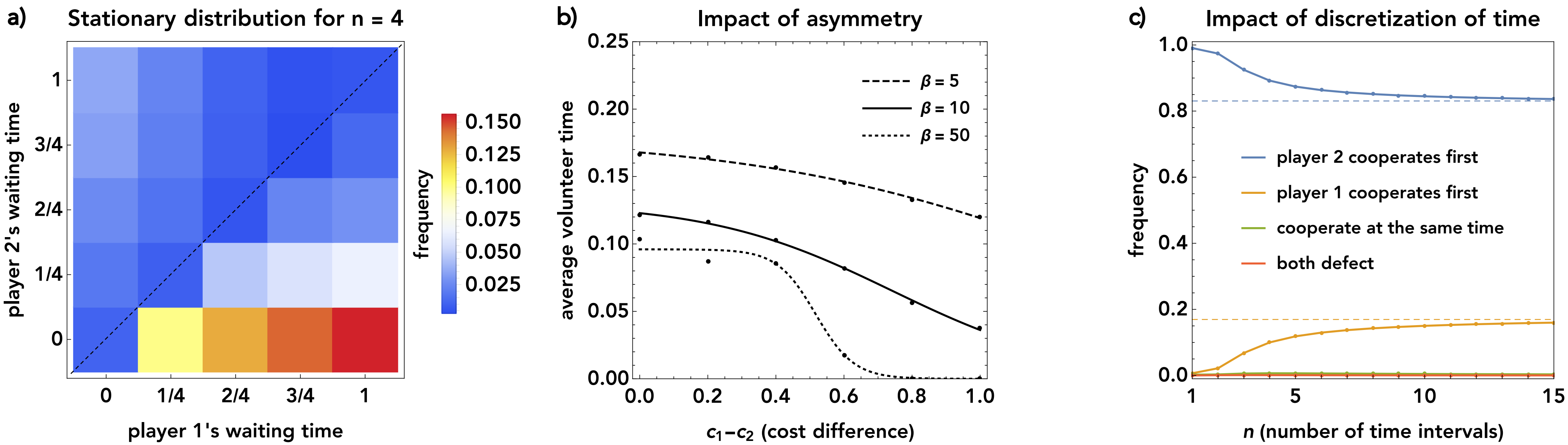
Applications

n-strategy game: volunteer's timing dilemma (Weesie, 1993)



Applications

n-strategy game: volunteer's timing dilemma



$b_{max} = 1$, $c_1 = 0.7$, $c_2 = 0.3$, $\beta = 10$

Conclusions

- A simple model of learning in social interactions
- Applicable to symmetric and asymmetric games alike
- Explicit formulas for the stationary distribution
- Agreement with other evolutionary processes
  - birth-death model for two co-evolving populations (Ohtsuki, 2010)
  - pairwise imitation (in the case of symmetric games)
- Asymmetry can help players coordinate more efficiently

Thank you!

http://web.evolbio.mpg.de/social-behaviour/

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