Week 7. Static games with incomplete information: Bayesian Nash equilibrium

Exercise 1: Win-Stay Lose-Shift

The strategy Win-Stay Lose-Shift (WSLS) cooperates in the first round, and if either both players cooperated or defected in the previous round.

Show that for the infinitely repeated prisoner's dilemma with payoffs

$$\begin{array}{c c} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} (3,3) & (0,4) \\ (4,0) & (1,1) \end{pmatrix} \end{array}$$

the strategy profile (WSLS, SWLS) is a subgame perfect equilibrium if $\delta \geq \frac{1}{2}$.

[Hint: Similarly to the examples covered in class, to prove that the strategy profile (WSLS, SWLS) is a subgame perfect equilibrium we need to check different cases. For case one consider that at h_t both strategies either defected or cooperated. For case two consider that one strategy defected.]

Exercise 2: Mini-Max I

Consider the matching pennies games

$$\begin{array}{c c} & \begin{array}{cc} \text{Left} & \text{Right} \end{array} \\ \begin{array}{c} \text{Up} \\ \text{Down} \end{array} & \begin{pmatrix} (0.8, 0.4) & (0.4, 0.8) \\ (0.4, 0.8) & (0.8, 0.4) \end{pmatrix} \end{array}$$

Show that in the definition of minimax, it is important to allow for mixed strategies of the opponent.

Specifically show that:

$$\min_{s^{(2)}} \max_{s^{(1)}} u^{(1)}(s^{(1)}, s^{(2)}) = 0.8, \text{ but}$$
$$\min_{\sigma^{(2)}} \max_{s^{(1)}} u^{(1)}(s^{(1)}, \sigma^{(2)}) = 0.6.$$

Bonus 1: Mini-Max II

Show that the minimax payoff of a player can be lower than what this player could get in a Nash equilibrium. Specifically, consider the game

$$\begin{array}{c} \\ \text{Up} \\ \text{Medium} \\ \text{Down} \end{array} \begin{array}{c} \begin{array}{cc} \text{Left} & \text{Right} \end{array} \\ \begin{pmatrix} (-2,2) & (1,-2) \\ (1,-2) & (-2,2) \\ (0,1) & (0,1) \end{pmatrix} \end{array}$$

- **Show that** the Nash equilibria of this game are of the form

$$\sigma^{(1)} = (0,0,1)$$
$$\sigma^{(2)} = (q, 1-q) \text{ with } q \in \left[\frac{1}{3}, \frac{2}{3}\right],$$

  and the resulting payoffs are $\hat{u}^{(1)} = 0, \hat{u}^{(2)} = 1$.

- **Show that** player 2's minimax payoff $u^{(2)} = \min\limits_{\sigma^{(2)}} \max\limits_{s^{(1)}} u^{(2)}(s^{(1)}, \sigma^{(2)}) = 0 < \hat{u}^{(2)}$.

Now that you have shown that the minimax payoff of a player can be lower than their Nash equilibrium payoff, **conclude that in repeated games with a sufficient large δ, players may be worse off than in the one shot game.**

Bonus Exercise 2: Folk Theorem

Consider the battle of the sexes

$$\begin{array}{c} \\ a_1 \\ a_2 \end{array} \begin{array}{c} \begin{array}{cc} a_1 & a_2 \end{array} \\ \begin{pmatrix} (3,1) & (0,0) \\ (0,0) & (1,3) \end{pmatrix} \end{array}$$

What is the set of feasible and individual rational payoffs?

<u>Bonus:</u> Construct a strategy $\hat{\sigma}$ for the repeated battle of sexes that for a sufficiently large δ satisfies the following 2 conditions.

(i) When both player adopt the strategy, they obtain a payoff of approximately $\pi^{(1)} = \pi^{(2)} = 2$.

(ii) $(\hat{\sigma}, \hat{\sigma})$ is a subgame perfect equilibrium. You do not need to show this rigorously, but give a convincing argument.