Week 3. Static games with complete information II: Nash equilibrium

Exercise 1: Iterated elimination of dominated strategies I

Construct a 2-player game such that:

1. Both players have 3 actions.
2. The game cannot be solved by elimination of dominated strategies.
3. The game can be solved by iterated elimination of dominated strategies.

Exercise 2: Iterated elimination of dominated strategies II

Consider the following 3-players game. Here the first player chooses a row, the second player chooses a column, and the third player chooses a matrix.

Can you solve this game using iterated elimination of strategies?

$$\text{Matrix 1} \qquad \begin{array}{cc} & \begin{array}{cc} \text{Col 1} & \text{Col 2} \end{array} \\ \begin{array}{c} \text{Row 1} \\ \text{Row 2} \end{array} & \begin{pmatrix} (2,1,6) & (3,2,3) \\ (0,4,0) & (1,0,0) \end{pmatrix} \end{array}$$

$$\text{Matrix 2} \qquad \begin{array}{cc} & \begin{array}{cc} \text{Col 1} & \text{Col 2} \end{array} \\ \begin{array}{c} \text{Row 1} \\ \text{Row 2} \end{array} & \begin{pmatrix} (1,-1,4) & (2,1,4) \\ (-1,4,0) & (0,0,3) \end{pmatrix} \end{array}$$

Exercise 3: Nash equilibrium vs dominance solvability

Prove the following statements:

(i) If a pure strategy $s_j^{(i)}$ is dominated by a pure strategy $s_k^{(i)}$ and $\sigma = (\sigma^{(1)}, \dots, \sigma^{(n)})$ is a Nash equilibrium, then $\sigma_j^{(i)} = 0$.

(ii) If the game is dominance solvable such that the unique outcome of iterated elimination of dominated strategies is some pure strategy $s = (s^{(1)}, \dots, s^{(n)})$, then s is a Nash equilibrium.

[Suggestion: One could use contradiction to prove the above statements. For example, for (i) assume that these was a Nash equilibrium with $\sigma_j^{(i)} > 0$, and show that this would yield some contradiction.]

Exercise 4: Best responses

Consider the stag hunt game:

$$\begin{array}{ccc} & & \underline{\text{player 2}} \\ & & \begin{array}{cc} \text{Stag} & \text{Hare} \end{array} \\ \underline{\text{player 1}} & \begin{array}{c} \text{Stag} \\ \text{Hare} \end{array} & \begin{pmatrix} (10,10) & (0,6) \\ (6,0) & (6,6) \end{pmatrix} \end{array}$$

Suppose player 1 uses the mixed strategy $(x, 1-x)$, where x is player 1's probability to Stag. Similarly, player 2's strategy is $(y, 1-y)$.

(i) For given x, y compute the players' payoffs $\pi^{(1)}(x,y), \pi^{(2)}(x,y)$ (see Remarks 2.6, 2.7).

(ii) For a given y compute player 1's best response $(\mathrm{BR}(y))$. In particular, show that there is some y^* such that all $x \in [0,1]$ are a best response.

(iii) Draw the two best response correspondences $\mathrm{BR}(x)$, $\mathrm{BR}(y)$ into a $x-y$ plane. How often do they intersect? What does it mean if they intersect?