# GAME THEORY

### Adminstrative oul

- 1) EXECTES
- d) Eille "Od exam as "find projed"

#### Sommon

- 1) Game theory is about stoday decipier-mahype (Rolle V, Ralette X)
- 2) Elements: Payor, Adias, order of moves, Payors, information
- 3) A diens: What each player can also
  Skategies: An which adian he pick pith he
  information he player currelly her

## [82 Stolic pomes with complete whomship (SGCI)

Définition 2.1 (SGCI, "one-shal panes", "nound-boin panes")

Elements of SGC1:

2) Adie of player i is on element of 
$$A^{(i)} = \{ \mathcal{D}_{i}^{(i)}, \dots, \mathcal{D}_{j_{i}}^{(i)} \}$$

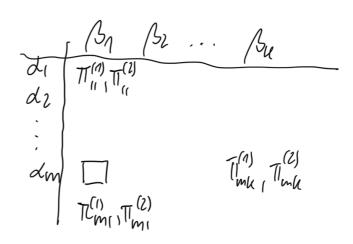
3) Payoffs 
$$\pi: A \to \mathbb{R}^n$$

Remark 2.2 (Two-player pames with Giriley many solar)

$$A^2 = \left\{ \beta_n, \beta_2, \dots, \beta_u \right\}$$

Tr: 
$$A^{(1)} \times A^{(2)} \rightarrow \mathbb{R}^2$$

as unally written as a payoff matrix



Similar way how to represed 3-player pamer N Exercises

Example 23 (Prisonos Oblemne)

Two player, ley can either confer or remain Siled

Payoffs = sould privar line

This pane has he Collowip paperher

1) 
$$A^{(1)} = A^{(2)}$$

2) 
$$\pi^{(n)}(a,a') = \pi^{(2)}(a,a)$$

2- player games with this proporty re colled symmetric"

n-player pames ? is Exercises

## Delimition 2.4 (Stratepier & SACI)

$$\bigcup_{i} A^{(i)} = \left\{ a_{i}^{(i)}, \dots, a_{m}^{(i)} \right\}$$

The a shalopy Cor player i is a probability Olichbution over the sel of socians

$$\widetilde{\mathcal{O}}^{(i)} = \left( \widetilde{\mathcal{O}}_{1}^{(i)}, \widetilde{\mathcal{O}}_{2}^{(i)}, \dots, \widetilde{\mathcal{O}}_{m}^{(i)} \right)$$

G;>0 SG;=1

Set of all sholepes  $\Sigma^{(i)}$ 

Set of shalpy profiles":  $Z = Z^{(1)} \times Z^{(2)} \times \times Z$ 

2) A sholepy is colled pure if there is a j such that  $G_j^{(i)}=1$ ,  $G_{l_i}^{(i)}=0$ a ell lyj.

If I would like to highlight that a study is price the limites [S=(0,...,0,1,0,...,0)] Set of all pue shapper of player is is so

Example 2.5 (Risono's dilemme) Robobility to remain

Skalpy & player 1: (1/2, 1/2)

Probability to confers

Pure Chalipier (1,0)

## Remort 2.6 (Payoffs les mixed stodepres)

(\*) If all players use pure shalories the repulling payoff is 
$$TT(a^{(1)}, a^{(2)}, \dots, a^{(n)})$$

(\*) Idea: Each player (andlownizes independently Shalepers are 
$$G^{(1)} = (G^{(1)})$$
  $G := (g^{(1)}, G^{(2)}, ..., G^{(n)}) \in \mathbb{Z}$ 

$$\pi(G) := \sum_{j_1, \dots, j_n} G^{(n)}_{j_2} \cdot G^{(2)}_{j_2} \dots G^{(n)}_{j_n} \pi(G^{(n)}_{j_n}, G^{(2)}_{j_2}, \dots, G^{(n)}_{j_n})$$

## Example 2.7 (Prisone's Olilemma)

$$A^{(i)} = \begin{cases} 86m1, 6nfess \end{cases} \frac{|SC|}{|S|}$$

$$6^{(n)} = (0.6, 0.4)$$

$$6^{(2)} = (0.3, 0.7)$$

Possible ovlosmes	Probability	Psyoff of
(Silel, Silel)	0.6 · 0.3	3
(Silen/, Confess)	0.6 · 0.7	O
(Canlers, Silent)	0.4.03	4
( Conless, Conless )	0.4.0.7	1

$$T^{(1)}(6^{(1)}, 6^{(1)}) = 0.6 \cdot 0.3 \cdot 3 + 0.6 \cdot 0.7 \cdot 0 + 0.4 \cdot 0.3 \cdot 4 + 0.4 \cdot 0.7 \cdot 1 + 0.4 \cdot 0.7 \cdot 1$$

$$0.6 \quad S \quad C \quad O.6 \quad S \quad C \quad O.7 \cdot 0.6 \quad$$

#### Notation 2.8

If I want to speak of playe i's strategy specifically

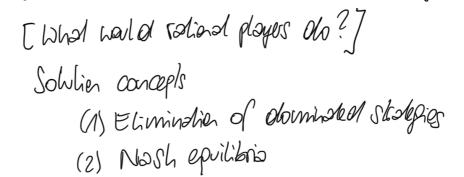
I sometimes write 
$$(\sigma^{(i)}, \sigma^{(-i)}) = (\sigma^{(n)}, \sigma^{(n)}, \dots, \sigma^{(n)})$$

Similarly  $T_i^{(i)}(\sigma^{(i)}, \dots, \sigma^{(n)}) = T_i^{(i)}(\sigma^{(i)}, \sigma^{(-i)})$ 

Remerk 2.9 (Solvhian caraple)

So for: Introduce madrinery to describe a game.

Question: What object I mean to solve a pome?



§ 2.1 Hooled climination of dominated stadges

Example 2.10 (Risone's Olilemma)

Siled confess

Siled confess

Siled confess

Siled confess

Siled confess

Siled confess

For rolional players, (Confess, Confess)
Seems to be the only topical outcome.

Definition 2.11 (Dominated statepes)

(1) A pre-sholepy s (i) is shirtly dominated

Supposition  $T(S^{(i)}, S^{(-i)}) < T(\widetilde{S}^{(i)}, S^{(-i)})$ 

if there is some  $S^{(i)} \in \mathbb{Z}^i$  such that G all  $S^{(-i)} \in S^{(-i)}$ :

$$\pi(s_{i}, s_{i}) < \pi(s_{i}) > \pi(s_{i})$$

(2) A prie 640lpy  $s^{(i)}$  is wealthy dominated if  $T^{(i)}(s^{(i)}, s^{(-i)}) \leq T^{(i)}(s^{(i)}, s^{(-i)}) \quad \forall s^{(-i)}$ 

$$\exists \, \widetilde{\mathcal{C}}^{(i)} \, \widetilde{\mathcal{C}}^$$

6 (i) := (i)

### Remark 2.12 (On Obminde skalepies)

- 1) In prisoner's Otilemme, "Silet" is stictly
- 2) Why do we explicitly allow to domination by mixed shalpper.

These are pames where no skalgery is Obourinated it you only allow domination by mixed shalpies is possible.

L R T 3,0 0,0 D 1,1 1,1 D 0,0 3,0

Skalepy M is obminated No Exercises

3) There is no odded advantage of replacing s<sup>(-i)</sup> by of (-i)

If slatement is true to all s<sup>(-i)</sup>

it is also true to all of (-i)

Example 2-13 (A prisoners obilemens with remote)

Sile I confess

Sile I 3,3 0,0 (1)

auless 4,0 1,1