Week 5. Sequential games with complete information I: Subgame perfection

Exercise 1: Sequential prisoner's dilemma with remorse

$$\begin{array}{c c} & \begin{array}{cc} \text{Silence} & \text{Confess} \end{array} \\ \begin{array}{c} \text{Silence} \\ \text{Confess} \end{array} & \begin{pmatrix} (3,3) & (0,0) \\ (4,0) & (1,1) \end{pmatrix} \end{array}$$

We already know that if both players move simultaneously, then the only Nash equilibrium is (Confess, Confess).

Now solve this game with backward induction, assuming that

1. the row player moves first,
2. the column player moves first,

and interpret the result.

Exercise 2: Ultimatum bargain in 2 rounds

There is a good that is worth 1€ to the buyer and 0€ to the seller. Sequence of moves:

1. Seller names a price $p \in \{0.01, 0.02, \ldots, 0.99\}$.
2. Buyer accepts or rejects the offer. If buyer accepts, the payoffs are p for the seller, $1 - p$ for the buyer and the game is over.
3. Otherwise, the buyer names a price $p \in \{0.01, 0.02, \ldots, 0.99\}$.
4. Seller accepts or rejects the offer. If seller accepts, the payoffs are $p - \delta$ for the seller, $1 - p - \delta$ where $\delta = 0.045$ reflects the cost of having to go through a long negotiation. If seller rejects, payoffs are $-\delta$.

Solve by backward induction, and interpret the result.

Exercise 3: Stackelberg Duopoly

Similar to the Cournot duopoly game consider the following situation:

- Players: $N = \{\text{Firm } 1, \text{Firm } 2\}$
- Actions: Amount of good produced, $x^{(i)} \in [0, \infty)$ for $i \in \{1, 2\}$
- Payoffs: $\pi^{(i)}(x^{(1)}, x^{(2)}) = [a - b(x^{(1)} + x^{(2)})]x^{(i)} - cx^{(i)}$

However, now assume that Firm 1 decided first, and Firm 2 observes Firms 1's decision before choosing an action.

Solve the game by backwards induction (for $a = 10, b = 1, c = 1$) and compare the result to the Nash equilibrium of the Cournot duopoly ($\hat{x}^{(1)} = \hat{x}^{(2)} = 3$).

[Hint: First you should find the best response of Firm 2 and use this to calculate what Firm 1 should do.]

Bonus Exercise 1: Properties of backwards induction

Prove that for any finite game with perfect information (i.e. in any game in which backwards induction can be applied) the solution defined by backwards induction is a Nash equilibrium.