

## Week 3. Static games with complete information II: Nash equilibrium

### Exercise 1: Iterated elimination of dominated strategies I

Construct a 2-player game such that:

1. Both players have 3 actions.
2. The game cannot be solved by elimination of dominated strategies.
3. The game can be solved by iterated elimination of dominated strategies.

### Exercise 2: Iterated elimination of dominated strategies II

Consider the following 3-players game. Here the first player chooses a row, the second player chooses a column, and the third player chooses a matrix.

Can you solve this game using iterated elimination of strategies?

Matrix 1			Matrix 2		
	Col 1	Col 2		Col 1	Col 2
Row 1	$(2, 1, 6)$	$(3, 2, 3)$	Row 1	$(1, -1, 4)$	$(2, 1, 4)$
Row 2	$(0, 4, 0)$	$(1, 0, 0)$	Row 2	$(-1, 4, 0)$	$(0, 0, 3)$

### Exercise 3: Nash equilibrium vs dominance solvability

Prove the following statements:

- (i) If a pure strategy  $s_j^{(i)}$  is dominated by a pure strategy  $s_k^{(i)}$  and  $\sigma = (\sigma^{(1)}, \dots, \sigma^{(n)})$  is a Nash equilibrium, then  $\sigma_j^{(i)} = 0$ .
- (ii) If the game is dominance solvable such that the unique outcome of iterated elimination of dominated strategies is some pure strategy  $s = (s^{(1)}, \dots, s^{(n)})$ , then  $s$  is a Nash equilibrium.

[Suggestion: One could use contradiction to prove the above statements. For example, for (i) assume that there was a Nash equilibrium with  $\sigma_j^{(i)} > 0$ , and show that this would yield some contradiction.]

#### Exercise 4: Best responses

Consider the stag hunt game:

		<u>player 2</u>	
		Stag	Hare
<u>player 1</u>	Stag	$(10, 10)$	$(0, 6)$
	Hare	$(6, 0)$	$(6, 6)$

Suppose player 1 uses the mixed strategy  $(x, 1 - x)$ , where  $x$  is player 1's probability to Stag. Similarly, player 2's strategy is  $(y, 1 - y)$ .

- (i) For given  $x, y$  compute the players' payoffs  $\pi^{(1)}(x, y), \pi^{(2)}(x, y)$  (see Remarks 2.6, 2.7).
- (ii) For a given  $y$  compute player 1's best response ( $BR(y)$ ). In particular, show that there is some  $y^*$  such that all  $x \in [0, 1]$  are a best response.
- (iii) Draw the two best response correspondences  $BR(x), BR(y)$  into a  $x - y$  plane. How often do they intersect? What does it mean if they intersect.