Assignment 3: Quick Sort Revisited

Part 1

Files

1. quickSort.cpp

- My quickSort implementation that allows any index to be used.

2. driver.cpp

- A couple of tests to check my solution produces the correct results. Test cases written with the help of Claud Opus 4.6.

Reflection

This solution works by picking a random pivot point at the start and moving it to a[lo]. It then follows the standard quickSort implementation shown in the text book. Finally is swaps the pivot in the correct location by swapping it with a[j].

This works because after the initial swap a[lo] isn't moved again until the end. Once the array has been fully scanned it will be partitioned into two sections. Everything <= j is <= pivot (except the pivot which is still in a[lo]) and everything from j+1 to hi is >= pivot. since we know a[j] is <= pivot we can safely swap a[j] and a[lo], giving us a complete correct partitioning.

Part 2

please explain why it is k ln (N/k) + (n-k) ln (N/(N-k)) and solve the equation

The equation for the expected cost comes from two places.

1. The cost from pivots that are to the right of k

$$k \ln \frac{N}{k}$$

1. The cost of pivots to the that are to the left of k

$$(N-k)\ln\frac{N}{N-k}$$

This leaves us with the equation:

$$k\ln\frac{N}{k}+(N-k)\ln\frac{N}{N-k}$$

If we want to evaluate this function we can take its derivative with respect to k like so.

$$f(k)=N\ln N-k\ln k-(N-k)\ln(N-k)$$

$$\frac{d}{dk}[N\ln N]=0$$

$$\frac{d}{dk}[k\ln k]=-\ln(k)-1$$

$$\frac{d}{dk}[-(N-k)\ln(N-k)]=\ln(N-k)+1$$

$$f'(k)=\ln\frac{N-k}{k}$$

Now we can set the derivative to 0 and solve for k.

$$\ln\frac{N-k}{k}=0$$

$$\frac{N-k}{k}=1$$

$$N-k=k$$

$$k=\frac{N}{2}$$

After that we plug k back into f(k)

$$f(\frac{N}{2})=\frac{N}{2}\ln(\frac{N}{N/2})+(N-\frac{N}{2})\ln(\frac{N}{N-N/2})$$

Simplify

$$f(\frac{N}{2}) = \frac{N}{2}\ln 2 + \frac{N}{2}\ln 2$$

$$f(\frac{N}{2}) = N\ln 2 \approx 0.693N$$