Graphical Method:

Ex: Maximize $10x_1 + 9x_2$ Subject to

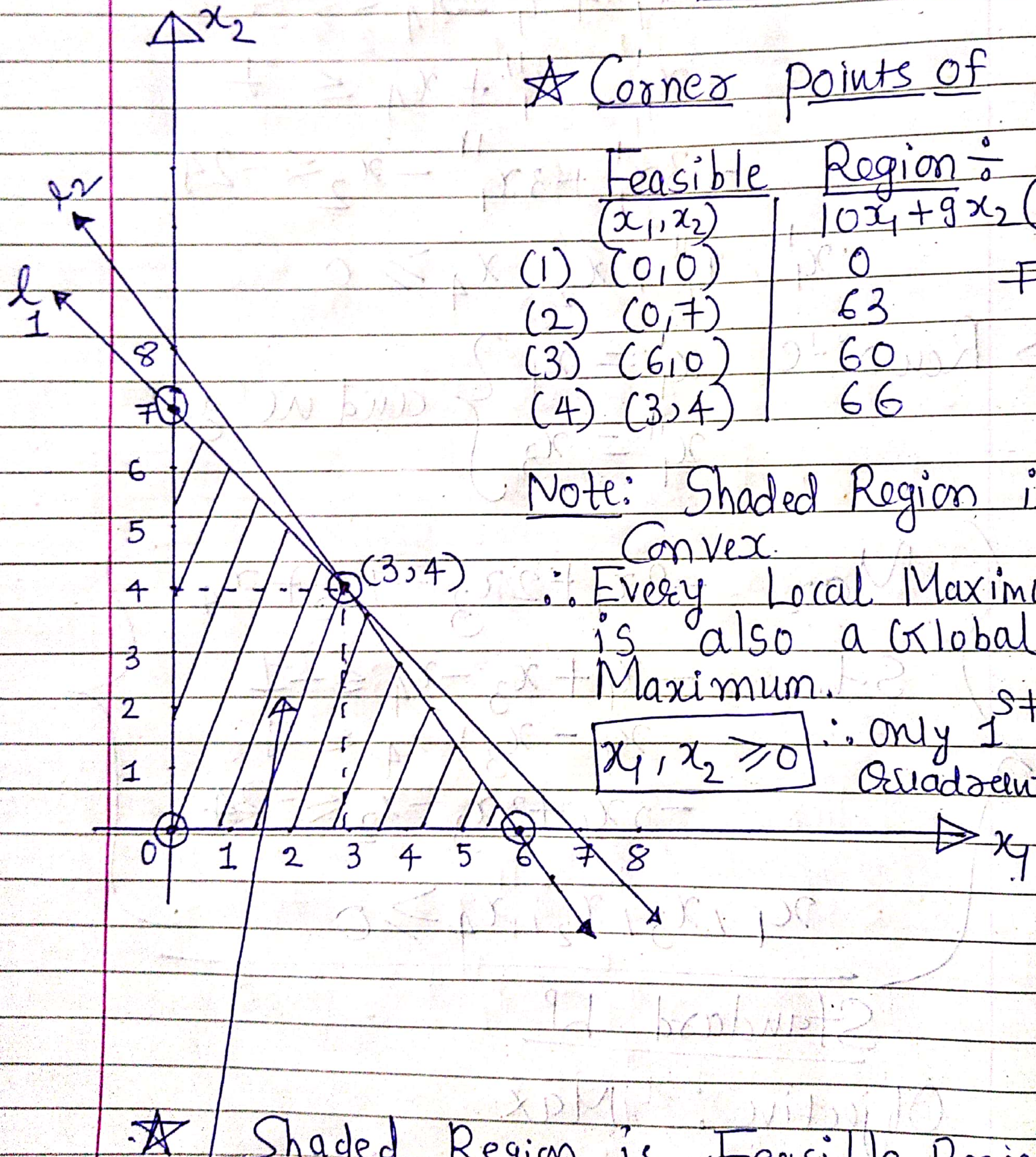
$3x_1 + 3x_2 \leq 21$

$4x_1 + 3x_2 \leq 24$

Where $x_1, x_2 \geq 0$

Ans: line l_1 : $3x_1 + 3x_2 = 21$

line l_2 : $4x_1 + 3x_2 = 24$



★ Corner points of Feasible Region:

(x_1, x_2)	$10x_1 + 9x_2$ (Value of Obj. Function)
(1) (0, 0)	0
(2) (0, 7)	63
(3) (6, 0)	60
(4) (3, 4)	66

Note: Shaded Region is Convex.

∴ Every Local Maximum is also a Global Maximum.

$\boxed{x_1, x_2 \geq 0}$ ∴ only 1^{st} Quadrant

★ Shaded Region is Feasible Region i.e. That Region satisfies both the constraints $3x_1 + 3x_2 \leq 21$ and $4x_1 + 3x_2 \leq 24$

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$3x_1 + 3x_2 = 21$
$4x_1 + 3x_2 = 24$
$- \quad - \quad -$

$-x_1 = -3$

$\therefore \boxed{x_1 = 3}$

$3 \cdot 3 + 3x_2 = 21$

$\therefore 3x_2 = 21 - 9 = 12$

$\therefore \boxed{x_2 = 4}$

$\therefore$ Point $(3,4)$ is point of Intersection

Point $(7,0)$ is Infeasible (Violates one constraint) while $(3,2)$ is Feasible. (Satisfies both constraints)

Every point in a feasible region is dominated by a boundary point.

Every boundary point is dominated by a corner point.

$\therefore$ It is enough only to evaluate corner points only.

→ We can take $10x_1 + 9x_2 = 50$ and increase constant till it leaves a region (feasible)

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4 corner points

(0,0)	0
(6,0)	60
(0,7)	63
(3,4)	66

Z : Value of objective function

Week-2

Lecture-2

Graphical Method

Minimize $7x_1 + 5x_2$

Subject to

$$x_1 + x_2 \geq 4$$
$$5x_1 + 2x_2 \geq 10$$
$$x_1, x_2 \geq 0$$

$x_1 + x_2 = 4$

$x_1 = 4,\ x_2 = 0$

$x_1 = 0,\ x_2 = 4$

$5x_1 + 2x_2 = 10$

$x_1 = 0,\ x_2 = 5$

$x_2 = 2\ x_2 = 0$

Intersection Point

$x_1 + x_2 = 4$

$5x_1 + 2x_2 = 10$

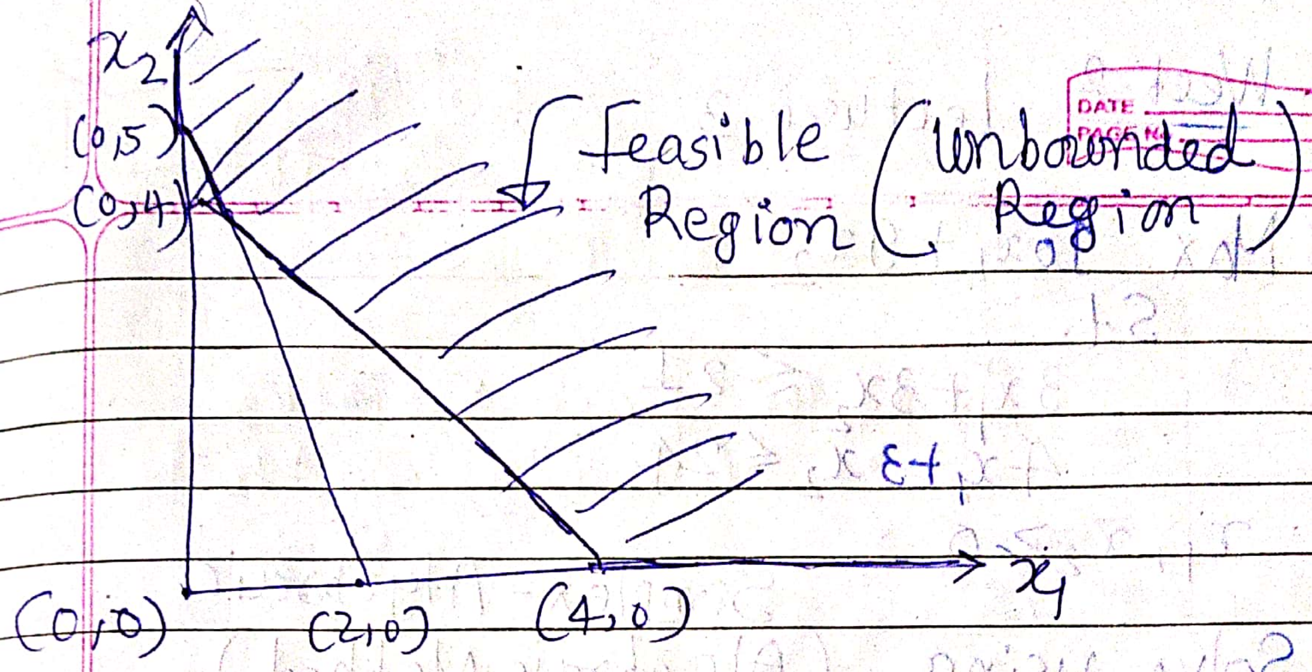
$5x_1 + 5x_2 = 20$

$5x_1 + 2x_2 = 10$

$-\quad -\quad -$

$3x_2 = 10 \Rightarrow x_2 = \frac{10}{3}$

$x_1 = 4 - \frac{10}{3} = \frac{2}{3}$ $\therefore (x_1, x_2) = \left(\frac{2}{3}, \frac{10}{3}\right)$



Corner Points

1. (4, 0) 28
2. (0, 5) 25
3. $\left(\frac{2}{3}, \frac{10}{3}\right)$ $7 \times \frac{2}{3} + 5 \times \frac{10}{3} = \frac{14+50}{3} = \frac{64}{3}$

$= 21.33$

Point $\left(\frac{2}{3}, \frac{10}{3}\right)$ has the smallest value

$\therefore$ It is the optimal point.

Due to convexity of feasible region, it is possible to find

a Boundary point

$\Big\downarrow$ Subsequently

a Corner point

which has better value of Objective function.