Advanced Algorithms

Geometric Algorithm

Determining whether any pair of segments intersects

Our algorithm for determining whether any two of *n* line segments intersect makes two simplifying assumptions.

First, we assume that no input segment is vertical.

Second, we assume that no three input segments intersect at a single point.

This Shamos-Hoey Algorithm is developed in 1976

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The algorithm uses a technique known as "sweeping," which is common to many computational-geometry algorithms.

In ***sweeping***, an imaginary vertical ***sweep line*** passes through the given set of geometric objects, usually from left to right. The spatial dimension that the sweep line moves across, in this case the *x*-dimension, is treated as a dimension of time.

The line-segment-intersection algorithm considers all the line-segment endpoints in left-to-right order and checks for an intersection each time it encounters an endpoint.

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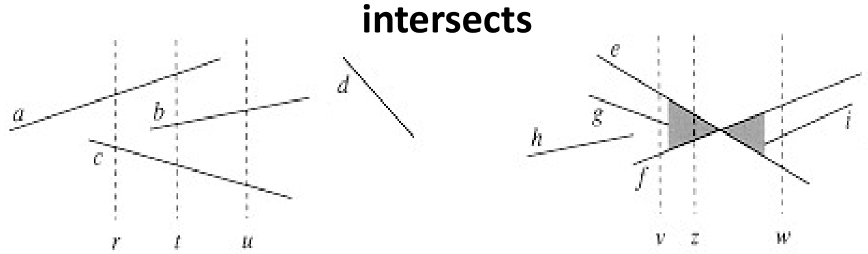

(a) (b)

a >r c

a >t b

a >t c

b >t c

b >u c

e >v f

f >w e

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Since we assume that there are no vertical segments, any input segment that intersects a given vertical sweep line intersects it at a single point.

We can thus order the segments that intersect a vertical sweep line according to the *y*-coordinates of the points of intersection.

Consider two nonintersecting segments s_1 and s_2 We say that these segments are ***comparable*** at x if the vertical sweep line with x-coordinate x intersects both of them.

We say that s_l is **above** s_2 at x, written s_1 >x s_2, if s_1 and s_2 are comparable at x and the intersection of s_1 with the sweep line at x is higher than the intersection of s_2 with the same sweep line.

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Sweeping algorithms typically manage two sets of data:

1. The ***sweep-line status*** gives the relationships among the objects intersected by the sweep line.

2. The ***event-point schedule*** is a sequence of *x*-coordinates, ordered from left to right, that defines the halting positions of the sweep line. We call each such halting position an ***event point***. Changes to the sweep-line status occur only at event points.

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The sweep-line status is a total order *T*, for which we require the following operations:

`INSERT`(*T, s*): insert segment *s* into *T.*

`DELETE`(*T*, *s*): delete segment *s* from *T.*

`ABOVE`(*T*, *s*): return the segment immediately above segment *s* in *T.*

`BELOW`(*T*, *s*): return the segment immediately below segment *s* in *T.*

If there are *n* segments in the input, we can perform each of the above operations in *O(*lg *n)* time using red-black trees **OR** AVL Tree.

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```
ANY-SEGMENTS-INTERSECT(S)
T ← Ø
sort the endpoints of the segments in S from left to right,
          breaking ties by putting left endpoints before right endpoints
          and breaking further ties by putting points with lower
          y-coordinates first
for each point p in the sorted list of endpoints
     do if p is the left endpoint of a segment s
           then INSERT(T, s)
                if (ABOVE(T, s) exists and intersects s)
                       or (BELOW(T, s) exists and intersects s)
                   then return TRUE
        if p is the right endpoint of a segment s
           then if both ABOVE(T, s) and BELOW(T, s) exist
                        and ABOVE(T, s) intersects BELOW(T, s)
                   then return TRUE
                DELETE(T, s)
return FALSE
```

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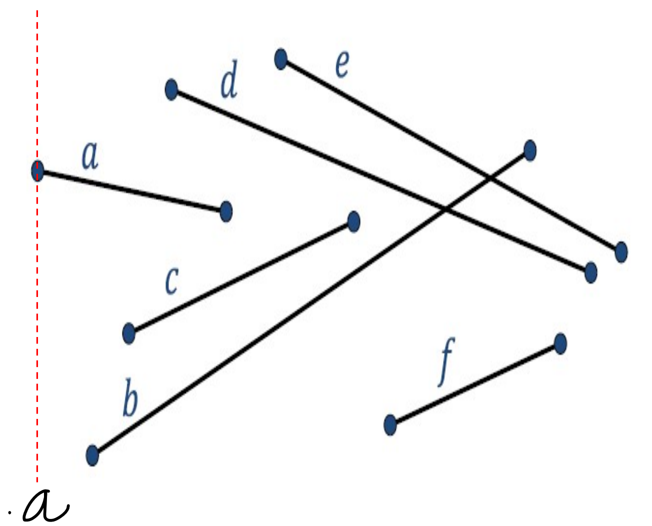

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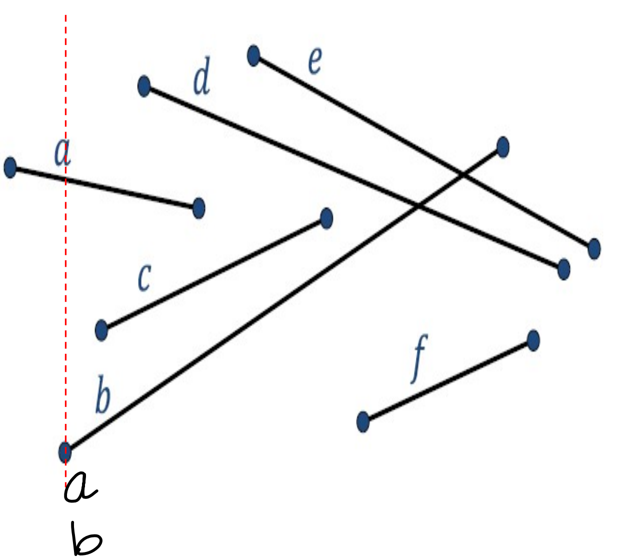

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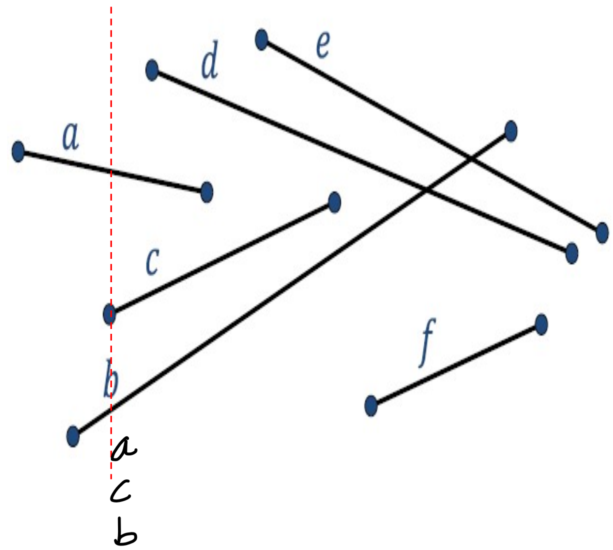

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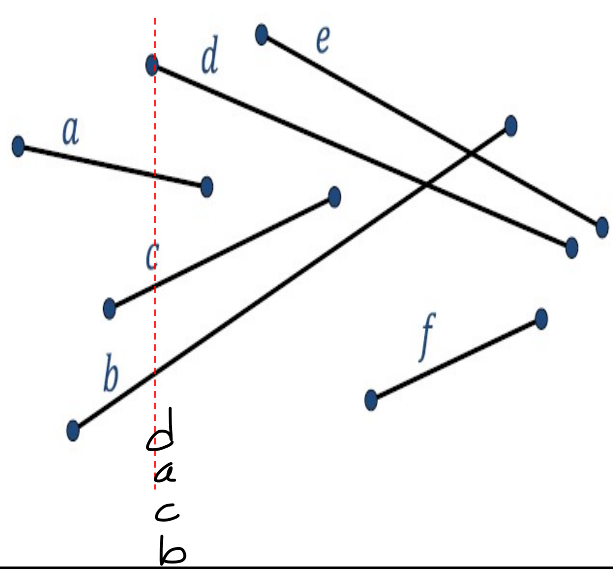

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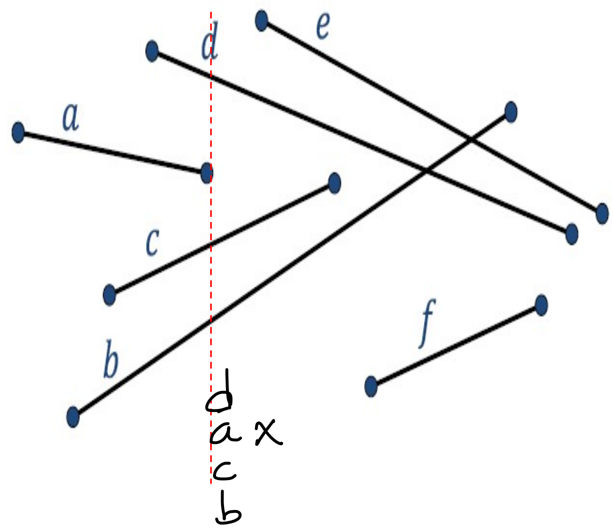

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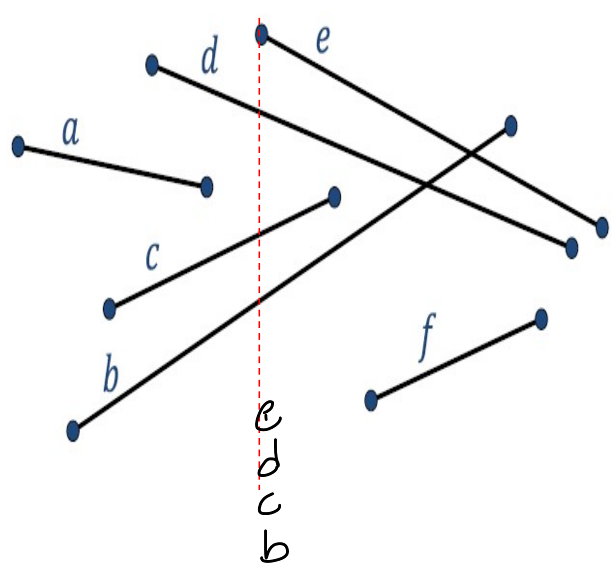

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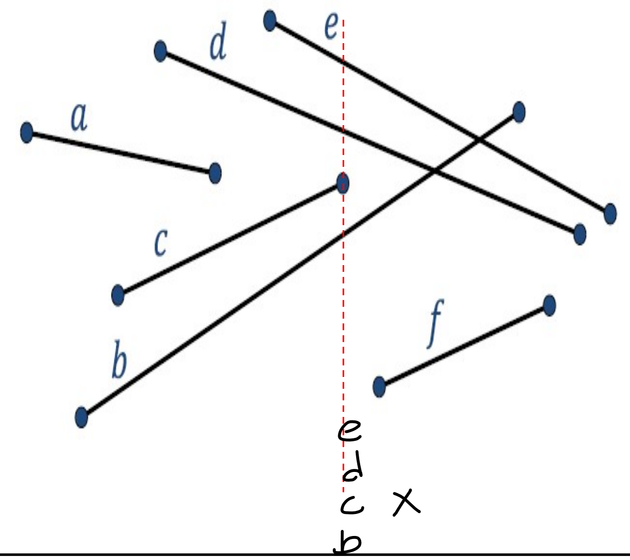

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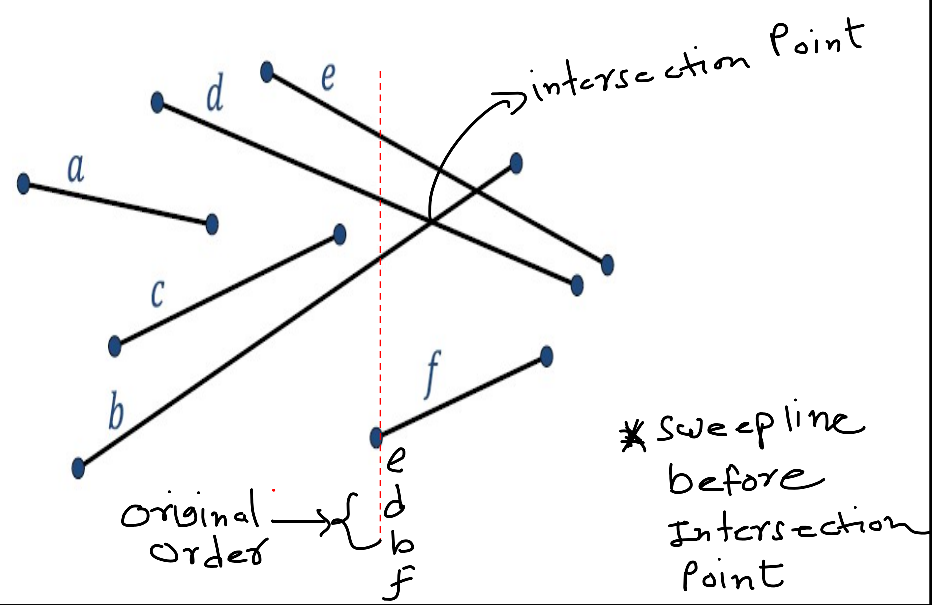

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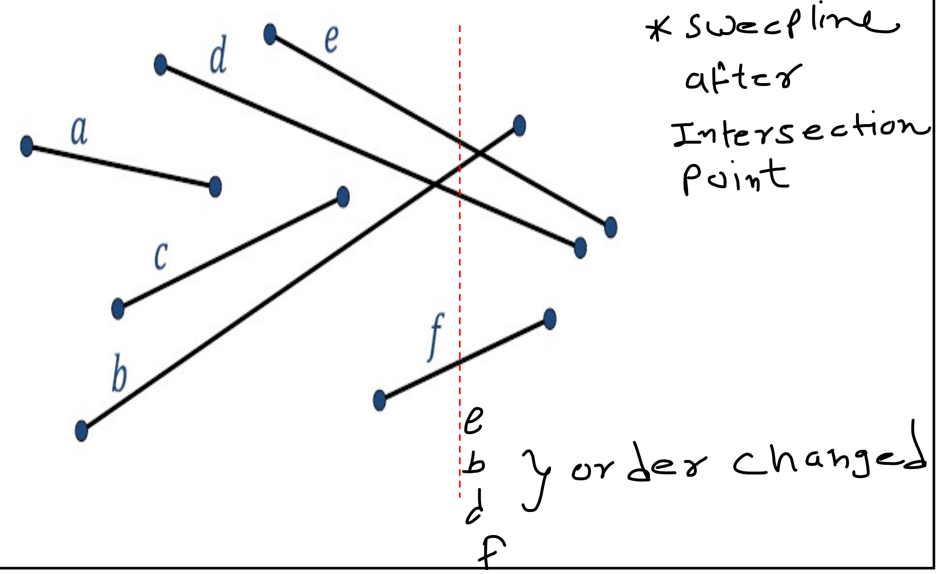

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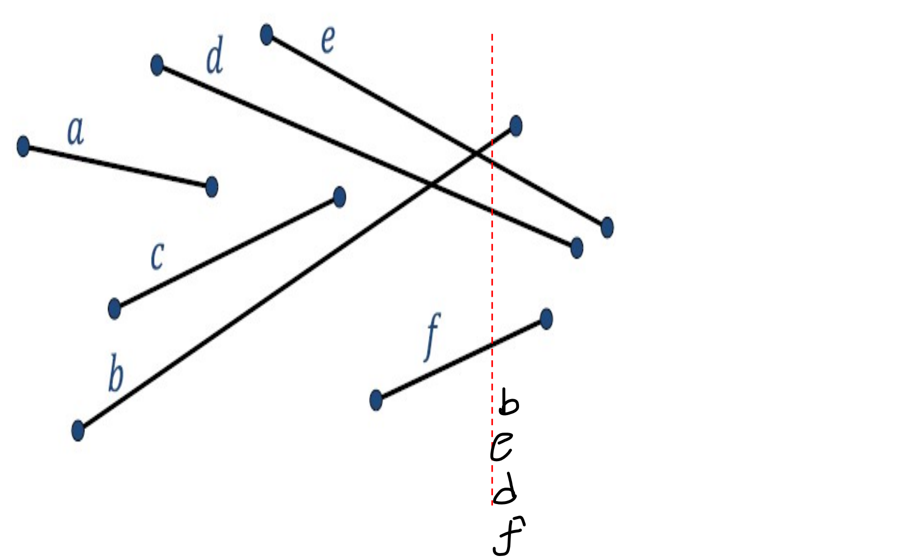

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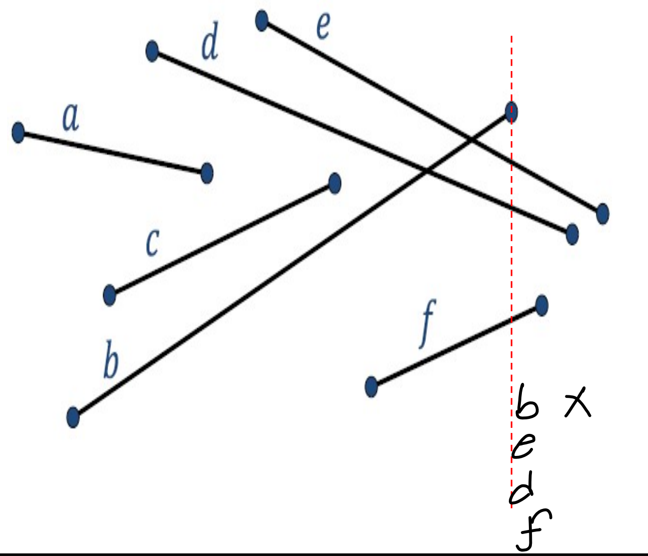

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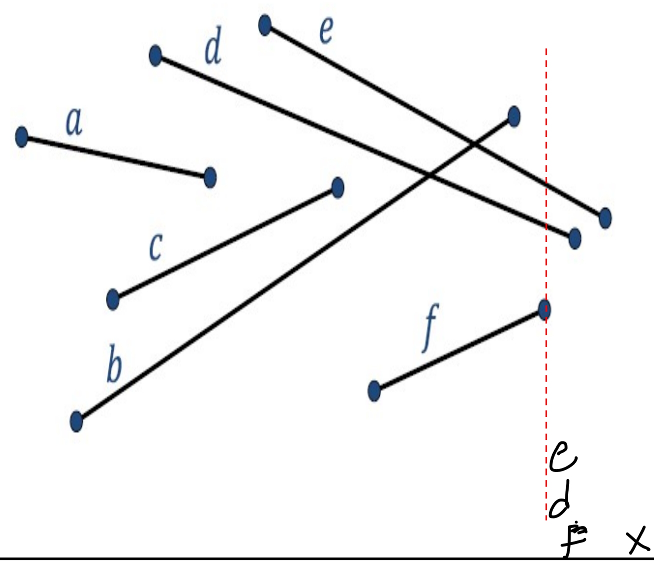

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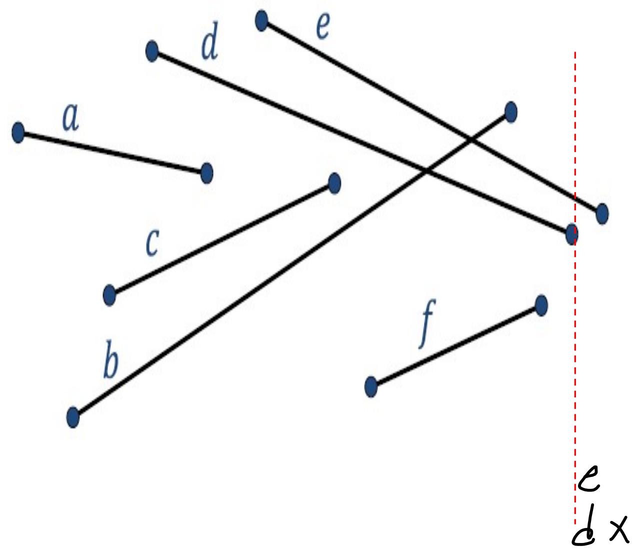

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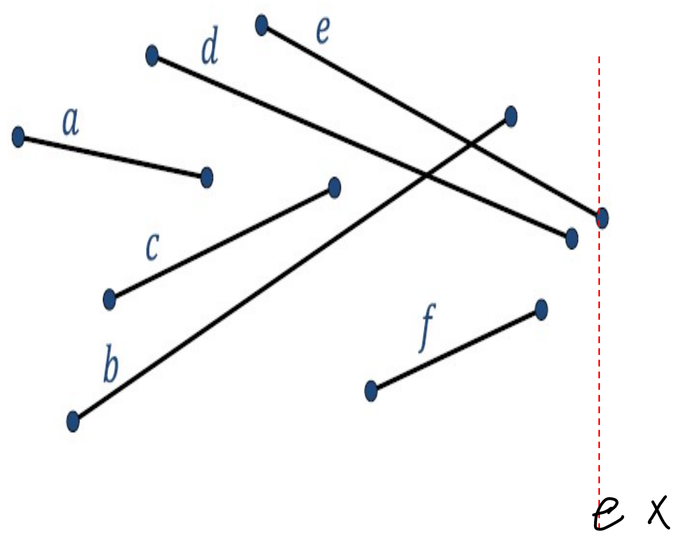
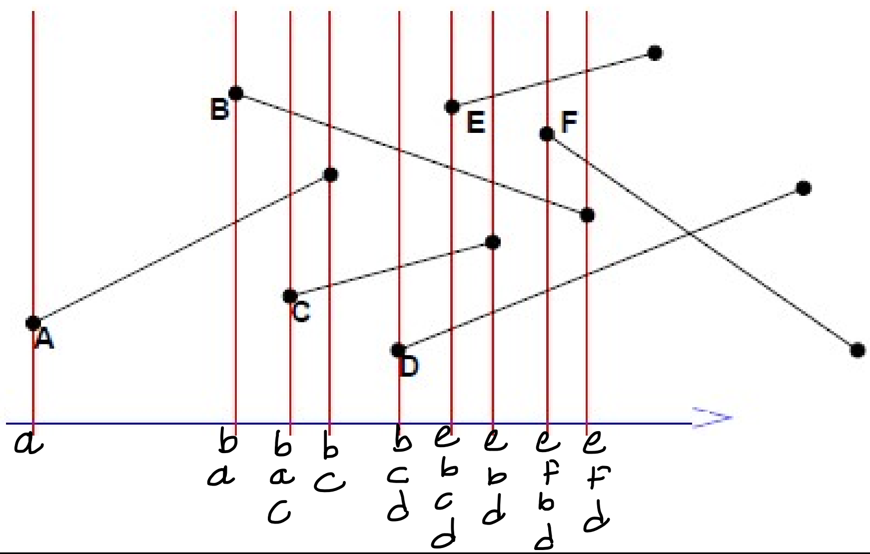


Example-2



Determining whether any pair of segments intersects

If there are *n* segments in set *S*, then ANY-SEGMENTS-INTERSECT runs in time *O(n* lg *n*).

Line 1 takes *O*(1) time.
Line 2 takes *O(n* lg *n*) time, using merge sort or heapsort.

Since there are 2*n* event points, the **for** loop of lines 3-11 iterates at most 2*n* times.

Each iteration takes *O(*lg *n*) time, since each red-black-tree operation takes *O*(lg *n*) time and, using INTERSECTION OF TWO LINE SEGMENTS METHOD, each intersection test takes *O*(1) time.

The total time is thus *O*(*n* lg *n*).