"श्री"

Lecture - 2

★ **Hiring Problem:** You decide to use an employment agency to hire one candidate. The agency sends you one candidate each day.

→ You interview that person and then decide to hire that person or not.

→ You must pay to agency

- Small fee to interview a candidate
- Substantial amount of fee to hire a candidate because we need to fire existing candidate and then hire new.

★ **Deterministic algorithm**

```
Hire_Assistant(n)
{
    best = 0  //Candidate 0 is a dummy candidate
    for i = 1 to n
    {
        Interview Candidate i;
        if Candidate i is better than the best
        {
            best = i;
            Hire Candidate i;
        }
    }
}
```

Say, Cost to Interview One Candidate = C_i

Cost to Hire One Candidate = C_h

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→ Let's say out of n, m number of people are hired.
→ The total cost $= O(c_i n + c_h m)$
→ We need to interview each of the n candidate, so that can't be changed
∴ We concentrate on analyzing $c_h m$, the hiring cost.

★ Worst-Case Analysis: In the worst-case, we actually hire every candidate that we interview.
→ This occurs if the candidates come in strictly increasing order of quality, in which case we hire n times, for a total hiring cost of $O(c_h n)$
→ In reality this may not occur.

★ Let's Modify the strategy:
→ Rather than relying on agency for candidate they send, we will modify the strategy.
→ We will ask for a list of candidates.
→ Then we will randomly permute the list of n candidates
→ So, control is now in our hands. We actually know that candidates arrive in random order.

☆ Randomized_Hire_Assistant (n)

```
{
  → Randomly permute the list of Candidates
  → best = 0
  → for i = 1 to n
    {
      Interview Candidate i
      If Candidate i is better than the best
      {
        best = i;
        Hire Candidate i;
      }
    }
}
```

☆ Analysis:

$$X_i = I\{\text{candidate } i \text{ is hired}\}$$

$$= \begin{cases} 1, & \text{if Candidate } i \text{ is hired} \\ 0, & \text{if Candidate } i \text{ is not hired} \end{cases}$$

X = R.V. for total number of Candidates hired

$$= X_1 + X_2 + \cdots + X_n$$

$$= \sum_{i=1}^{n} X_i$$

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→ $E[X_i] = Pr\{\text{Candidate } i \text{ is hired}\}$
($\because X_i$ is Indicator Random Variable)
$= \frac{1}{i}$ (Any of these first i Candidates is equally likely to be the best qualified so far)

→ We want to find $E[X]$

$$X = \sum_{i=1}^{n} X_i$$

$$\therefore E[X] = E\left[\sum_{i=1}^{n} X_i\right]$$

$$= \sum_{i=1}^{n} E[X_i] \quad (\because \text{Linearity of Expectation})$$

$$= \sum_{i=1}^{n} \frac{1}{i}$$

$$= \ln(n) + O(1)$$

$\therefore$ Even though we interview n people, we hire only approximately $\ln(n)$ of them.

→ Thus, average total hiring cost $= C_h \ln(n)$