Advanced Algorithms

Geometric Algorithm

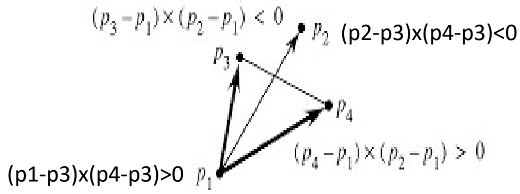
Intersection of Two Line Segments

To determine whether two line segments intersect, we check whether each segment straddles the line containing the other. A segment $\overline{p_1p_2}$ ***straddles*** a line if point p_1 lies on one side of the line and point p_2 lies on the other side. A boundary case arises if p_1 or p_2 lies directly on the line. Two line segments intersect if and only if either (or both) of the following conditions holds:

1. Each segment straddles the line containing the other.
2. An endpoint of one segment lies on the other segment. (This condition comes from the boundary case.)

Intersection of Two Line Segments

$P_1(1,1)$
$P_2(4,8)$
$P_3(2,6)$
$P_4(6,3)$

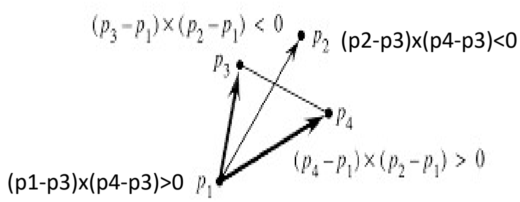


$$\underbrace{(P_3-P_1)\times(P_2-P_1)}_{d3} = (x_3-x_1)(y_2-y_1)-(x_2-x_1)(y_3-y_1)$$
$$= (2-1)(8-1) - (4-1)(6-1)$$
$$= 1\times 7 - 3\times 5$$
$$= 7-15 = \boxed{-8<0}$$

$$\underbrace{(P_4-P_1)\times(P_2-P_1)}_{d4} = (x_4-x_1)(y_2-y_1)-(x_2-x_1)(y_4-y_1)$$
$$= 5\times 7 - 3\times 2$$
$$= 35-6 = \boxed{29>0}$$

Intersection of Two Line Segments

$P_1(1,1)$
$P_2(4,8)$
$P_3(2,6)$
$P_4(6,3)$

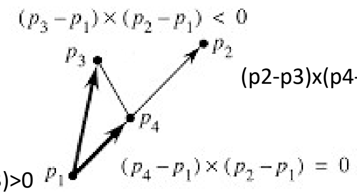


$$\underbrace{(P_1-P_3)\times(P_4-P_3)}_{d1} = (x_1-x_3)(y_4-y_3)-(x_4-x_3)(y_1-y_3)$$
$$= (1-2)(3-6) - (6-2)(1-6)$$
$$= (-1)\times(-3) - 4\times(-5)$$
$$= 3+20 = \boxed{23>0}$$

$$\underbrace{(P_2-P_3)\times(P_4-P_3)}_{d_2} = (x_2-x_3)(y_4-y_3)-(x_4-x_3)(y_2-y_3)$$
$$= (4-2)(3-6) - (6-2)(8-6)$$
$$= 2\times(-3) - 4\times 2$$
$$= -6-8 = \boxed{-14}$$

Intersection of Two Line Segments

$P_1(1,1)$
$P_2(8,8)$
$P_3(3,6)$
$P_4(5,5)$



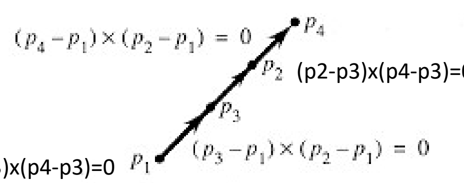
(p2-p3)x(p4-p3)<0

(p1-p3)x(p4-p3)>0

$d_1 > 0$
$d_2 < 0$
$d_3 < 0$

$$\underbrace{(P_4 - P_1) \times (P_2 - P_1)}_{d_4} = (x_4 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_4 - y_1)$$
$$= (5-1)(8-1) - (8-1)(5-1)$$
$$= 4 \times 7 - 7 \times 4$$
$$= 0$$

Intersection of Two Line Segments

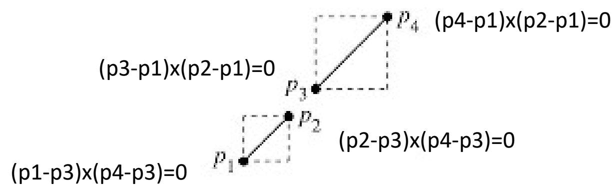


(p2-p3)x(p4-p3)=0

(p1-p3)x(p4-p3)=0

$P_1(1,1)$	$d_1 = 0$
$P_2(5,5)$	$d_2 = 0$
$P_3(3,3)$	$d_3 = 0$
$P_4(8,8)$	$d_4 = 0$

Intersection of Two Line Segments



$P_1\ (1,1)$ $\qquad d_1 = 0$

$P_2\ (3,3)$ $\qquad d_2 = 0$

$P_3\ (5,5)$ $\qquad d_3 = 0$

$P_4\ (8,8)$ $\qquad d_4 = 0$

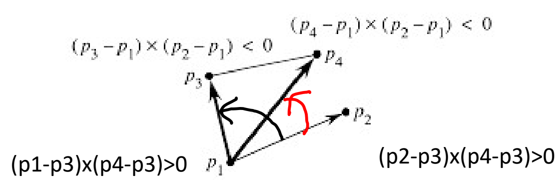
Intersection of Two Line Segments

$P_1(2,2)$

$P_2\ (8,3)$

$P_3\ (1,4)$

$P_4\ (6,5)$



$$(P_3 - P_1)(P_2 - P_1) = (x_3 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_3 - y_1)$$
$$= (1-2)(3-2) - (8-2)(4-2)$$
$$= (-1)\times 1 - 6 \times 2$$
$$= -1 - 12 = \boxed{-13 < 0}$$

$$(P_4 - P_1)(P_2 - P_1)$$
$$= (x_4 - x_1)(y_2 - y_1) - (x_2 - x_1)(y_4 - y_1)$$
$$= (6-2)(3-2) - (8-2)(5-2)$$
$$= 4 \times 1 - 6 \times 3 = 4 - 18 = \boxed{-14}$$

Algorithm

```
SEGMENTS-INTERSECT(p1, p2, p3, p4)
  d1 ← DIRECTION(p3, p4, p1)
  d2 ← DIRECTION(p3, p4, p2)
  d3 ← DIRECTION(p1, p2, p3)
  d4 ← DIRECTION(p1, p2, p4)
  if ((d1 > 0 and d2 < 0) or (d1 < 0 and d2 > 0)) and
          ((d3 > 0 and d4 < 0) or (d3 < 0 and d4 > 0))
     then return TRUE
  elseif d1 = 0 and ON-SEGMENT(p3, p4, p1)
     then return TRUE
  elseif d2 = 0 and ON-SEGMENT(p3, p4, p2)
     then return TRUE
  elseif d3 = 0 and ON-SEGMENT(p1, p2, p3)
     then return TRUE
  elseif d4 = 0 and ON-SEGMENT(p1, p2, p4)
     then return TRUE
  else return FALSE
DIRECTION(pi, pj, pk)
  return (pk - pi) × (pj - pi)
ON-SEGMENT(pi, pj, pk)
  if min(xi, xj) ≤ xk ≤ max(xi, xj) and min(yi, yj) ≤ yk ≤ max(yi, yj)
     then return TRUE
     else return FALSE
```

Time Complexity

- Time complexity of the above algorithm is Ɵ (c)

Try This