

## Max 3-SAT-CNF

Date \_\_\_\_\_  
Page \_\_\_\_\_

Ex: Find Expected Number of clauses satisfied for 3-SAT expressions having  $m$  clauses and  $n$  variables.

Ans: Say there are  $m$  clauses in 3-SAT formula  $\phi$ .

→ Each clause is OR of 3 literals.

→  $X_i$ : Indicator Random Variable denoting that  $i^{\text{th}}$  clause is satisfied.

i.e.  $X_i = 1$  if  $i^{\text{th}}$  clause is satisfied  
 $= 0$  otherwise

$$\begin{aligned} \rightarrow E[X_i] &= \Pr\{\text{\textit{i}^{\text{th}} clause satisfied}\} \\ &= 1 - \Pr\{\text{\textit{i}^{\text{th}} clause is not satisfied}\} \end{aligned}$$

The clause will not be satisfied if each of its literal evaluates to false. i.e. all 3 literals say false

$\therefore \Pr$ . of  $i^{\text{th}}$  clause is not satisfied

$$= \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$$

$$= \frac{1}{8}$$



$$\begin{aligned} \therefore E[X_i] &= 1 - \Pr\{\text{clause is not satisfied}\} \\ &= 1 - \frac{1}{8} \\ &= \frac{7}{8} \end{aligned}$$

Total Number of clauses satisfied is Random Variable, say  $X$

$$\begin{aligned} \text{i.e. } X &= X_1 + X_2 + \dots + X_m \\ &= \sum_{i=1}^m X_i \end{aligned}$$

$$\therefore E[X] = E\left[\sum_{i=1}^m X_i\right]$$

$$= \sum_{i=1}^m E[X_i]$$

$$= \sum_{i=1}^m \frac{7}{8}$$

$$\therefore E[X] = \frac{7}{8}m$$

Expected Number of clauses Satisfied in 3-SAT.