Ex

Min $2x_1 + 7x_2 + x_3$

S.t.

$x_1 - x_3 = 7$

$3x_1 + x_2 \geq 24$

$x_2 \geq 0$

$x_3 \leq 0$

⇓

Max $-2x_1 - 7x_2 - x_3$

S.t.

$x_1 - x_3 = 7$

$3x_1 + x_2 \geq 24$

$x_2 \geq 0$

$x_3 \leq 0$

⇓

x_1 is unrestricted

$\therefore x_1 = x_1' - x_1''$

Where $x_1' \geq 0$

$x_1'' \geq 0$

Question: Convert the Program to the standard Linear program.

E.g. $x_1 = -3$

$= 2 - 5$

$= x_1' - x_1''$

E.g. $x_1 = 13$

$= 18 - 5$

$= x_1' - x_1''$

$x_1' = 18 \geq 0$

$x_1'' = 5 \geq 0$

∴ Max $-2x_1' + 2x_1'' - 7x_2 - x_3$

S.t. $x_1' - x_1'' - x_3 = 7$

$3x_1' - 3x_1'' + x_2 \geq 24$

$x_1' \geq 0$

$x_1'' \geq 0$

$x_2 \geq 0$

$x_3 \leq 0$

→ Since $x_3 \leq 0$, we want ≥ 0 for standard form

Let's take $x_4 = -x_3$

As $x_3 \leq 0 \Rightarrow x_4 = -x_3 \geq 0$

Max $-2x_1' + 2x_1'' - 7x_2 + x_4$

S.t. $x_1' - x_1'' + x_4 = 7$

$3x_1' - 3x_1'' + x_2 \geq 24$

$x_1' \geq 0$

$x_1'' \geq 0$

$x_2 \geq 0$

$x_4 \geq 0$

→ In standard form of LP, we have $\leq$ type of constraint.

∴ We need to Modify

$$3x_1' - 3x_1'' + x_2 \geq 24$$

by Multiplying both the sides by (-1)

$$\therefore -3x_1' + 3x_1'' - x_2 \leq -24$$

→ In standard form all the constraints are Inequalities only. We need to Modify equality

$$x_1' - x_1'' + x_4 = 7$$

by rewriting it as

(i) $x_1' - x_1'' + x_4 \geq 7$

(ii) $x_1' - x_1'' + x_4 \leq 7$

(ii) is $\leq$ but for (i) we need to Make it $\leq$ by Multiplying both the sides by (-1)

$$\therefore -x_1' + x_1'' - x_4 \leq -7$$

and $$x_1' - x_1'' + x_4 \leq 7$$

∴ Finally we can write standard LP as:

Max $-2x_1' + 2x_1'' - 7x_2 + x_4$

Subject to

$$-x_1' + x_1'' - x_4 \leq -7$$
$$x_1' - x_1'' + x_4 \leq 7$$
$$-3x_1' + 3x_1'' - x_2 \leq -24$$
$$x_1', x_1'', x_2, x_4 \geq 0$$

→ Rewrite $x_1' = x_1$, $x_1'' = x_3$ and we get

Max $-2x_1 + 2x_3 - 7x_2 + x_4$

S.t. $-x_1 + x_3 - x_4 \leq -7$
$x_1 - x_3 + x_4 \leq 7$
$-3x_1 + 3x_3 - x_2 \leq -24$
$x_1, x_3, x_2, x_4 \geq 0$

Standard LP

Objective: Max

Constraint: → All Inequality Constraints of type $\leq$

→ All decision variables ≥ 0 i.e. Non-negative