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Skewness in financial returns

Amado Peiró *

Universitat de València, Department of d'Anàlisi Econòmica, Av Dels Tarongers s/n, 46011 Valencia, Spain

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Abstract

In this paper the symmetry of daily returns is addressed in eight international stock markets and three spot exchange rates. Tests of symmetry with the sample skewness seem of little value, due to the non-normality of the returns. Under alternative non-normal distributions, the symmetry of the returns cannot be rejected for most markets. Distribution-free procedures do not detect strong asymmetries in most of the series either; however, some differences between returns below the mean and returns over the mean are observed in several markets © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

In recent decades, the unconditional distribution of price variations in financial markets has been studied in depth. A common conclusion to these studies is that the normal distribution inadequately represents the short period returns of financial assets. For this reason, many authors (e.g., Mandelbrot, 1963; Fama, 1965; Press, 1967; Ball and Torous, 1983; Kon, 1984; Praetz, 1972; Blattberg and Gonedes, 1974; Smith, 1981; Gray and French, 1990) have

* Tel.: 34 6 382 82 56; fax: 34 6 382 82 49; e-mail: amado.peiro@uv.es

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proposed different statistical distributions for price changes of these assets. All these distributions try to give account of the main departure from the normal distribution: the high kurtosis existent in the empirical distribution of the returns, more peaked and with fatter tails than the normal distribution.

While the high kurtosis of the returns is a well-established fact, the situation is much more obscure with regard to the symmetry of the distribution. Many authors do not observe anything special on this point, but other researchers (e.g., Simkowitz and Beedles, 1980; Kon, 1984; So, 1987) have drawn the attention to the asymmetry of the distribution. One important point, as will be seen later, is that most of these conclusions hinge on the sample skewness (third central moment divided by the cube of standard deviation).

The relevance of symmetry analysis transcends the pure determination of the statistical distribution. The Capital Asset Pricing Model (CAPM) assumes that only the mean and variance of returns mind in asset pricing, and, therefore, higher-order moments are unimportant. This implies that upside and downside risks are considered equally by investors, but this assumption is not reasonable given that most investors have a preference for positive skewness. Brennan (1979) and He and Leland (1993) have shown that if the market's portfolio rate of return has constant mean and volatility, the average investor has a power utility function. As this function has a positive third derivative, it implies skewness preference that is positively valued by investors. In fact, several attempts have been made to capture this skewness preference (see Sortino and Vandermeer, 1991; Leland, 1996), and many financial models allow or even incorporate an asymmetric behavior of financial returns. In this way, Arditti and Levy (1975) build a three-parameter multi-period model. Kraus and Litzenberger (1976) extend the capital asset pricing model to include the effect of skewness on valuation, and present empirical evidence consistent with their extension.

Portfolio selection can also be affected by the skewness of the distribution of returns. Simkowitz and Beedles (1978), Conine and Tamarkin (1981) explain the low diversification of many investors' portfolios by the preference for positive skewness. Recently, Lai (1991) and Chunhachinda et al. (1997) have analysed the problem of portfolio selection taking into account the skewness of returns. Chunhachinda et al. (1997) find that skewness is very important, to the extent that 'the incorporation of skewness into the investor's portfolio decision causes a major change in the construction of the optimal portfolio'. Furthermore, they also find that investors trade expected return for skewness.

The issue of skewness in financial returns is also important for option pricing theories. The widely used Black–Scholes option pricing model frequently misprices deep-in-the-money and deep-out-the-money options. Hull (1993) has explained this anomaly, known as volatility skew, as a consequence of non-normality, and Corrado and Su (1996, 1997) attribute this fact to the skewness and kurtosis of returns distribution. They find significant non-normal skewness and kurtosis implied by option prices and show that when skewness- and kurtosis-adjustment terms are added to the Black–Scholes formula, improved accuracy is obtained for pricing options.

The objective of this paper is to investigate the symmetry of the unconditional distribution (probability density function) of daily financial returns. To carry out this study, Section 2 presents the data used, relative changes in stock prices and in spot exchange rates. Section 3 examines some preliminary statistical and graphical evidence. Section 4 develops the analysis of symmetry under the perspective of two distributions frequently proposed for financial returns: discrete mixtures of normal distributions and Student's t distributions. To avoid the problems of distribution-based methods, distribution-free methods will be used in Section 5 to test for skewness. Finally Section 6 summarizes the main results and conclusions.

2. Returns series

In the following paragraphs nine daily stock indexes and three daily spot exchange rates have been considered. The indexes are the Standard & Poor's 500 Composite (SP), Dow–Jones Industrial (DJ), Nikkei (NI), Financial Times 100 (FT), Commerzbank (CB), CAC General (CA), Composite (CI), Banca Commerciale Italiana (BC) and General (IG) of the stock exchanges of New York (the first two), Tokyo, London, Frankfurt, Paris, Toronto, Milan and Madrid. The exchange rates are the Japanese Yen vs the US Dollar (YD), the British Pound vs the US Dollar (PD) and the German Mark vs the US Dollar (MD). These exchange rates are measured in non-US currency units per US Dollar. Daily returns were obtained by logarithmic differernces; that is by $R_t = \log(I_t/I_{t-1})$, where R_t is the return for day t and I_t is the daily index or the exchange rate for the same day. So, all the observations are one-day returns, excluding the Monday ones which are three-day returns. [1] Except for FT, the returns are from 3 January, 1980 to 27 September, 1993, and they provide from 2862 to 3392 observations. FT returns cover from 3 January, 1984 to 27 September, 1993, and they imply 2408 observations.

Some important features regarding this sample information must be stressed. The methodology used in the building of these stock price indices is not homogeneous. Some indices are value weighted while others are equally weighted, but all of them adjust for share issues. Exchange rates were obtained from the London market. As the original exchange rates were quoted against the British Pound, cross rates were calculated to get YD and MD returns. In order to analyse the symmetry of the unconditional distribution of the returns,

[1] When excluding Monday returns, results similar to those of the following pages were obtained.

these returns will be considered to be independent and identically distributed. Although this hypothesis has been questioned in recent years, the analysis of symmetry would be virtually impossible without this assumption since there would then be several financial series, each of them with several distributions (corresponding to the different days of the week or to different subperiods) and with different relationships of dependence. Finally, these different samples cannot be considered as completely independent of each other. In the case of stock returns, this is due to the integration of these markets, as is clearly indicated by their common movements. In the case of exchange rate returns, it is due basically to the fact that they are all quoted against the US Dollar.

3. Preliminary evidence

According to the usual concept of symmetry, the returns are symmetric about μ if (for any k)

$$f(\mu + k) = f(\mu - k), \tag{1}$$

where f is the density function of the returns. If (1) is true, then μ is the mean of the distribution and coincides with the median. To test the symmetry of the returns, most researchers have used the sample skewness,

$$\hat{\alpha} = \frac{\sum_{t=1}^{T} (R_t - \overline{R})^3 / T}{\hat{\alpha}^3}, \tag{2}$$

where T is the number of observations, R_t is the return at date t, $\overline{R}$ is the sample mean and $\hat{\sigma}$ is the sample standard deviation. Under normality, the asymptotic distribution of $\hat{\alpha}$ is given by

$$\hat{\alpha} \rightarrow \mathrm{N}(0, 6/T). \tag{3}$$

In Table 1 some interesting statistics are shown. The kurtosis, Studentized range, Kolmogorov–Smirnov and Jarque–Bera statistics indicate a clear rejection of the normality of daily returns. In all cases, skewness is negative [2] and, with the only exception of PD, tests of symmetry with Eqs. (2) and (3) imply negligible marginal significance levels. These values admit the following two alternative interpretations: (i) the rejection of symmetry of the distribution (and, therefore, the rejection of the hypothesis of normality); (ii) the mere rejection of normality (but not necessarily the rejection of symmetry). As previously mentioned, the first interpretation is very frequent in existing literature.

[2] Simkowitz and Beedles (1987) observed that skewness in monthly stock returns decreases and becomes negative with diversification. On the other hand, if exchange rates were defined inversely (US dollars per foreign currency unit) the signs of sample skewness would be the opposite.

Table 1
Returns statistics[a]

	Obs.	Mean (%)	S. dev.	Skewness	Kurtosis	Stud. range	Kolm–Smirnov	Jarque–Bera
SP	3373	0.045	0.010	−3.408 (0.042)	79.565	30.611	0.073	830,411
DJ	3373	0.043	0.011	−4.100 (0.042)	105.172	32.932	0.075	1,476,579
NI	3232	0.032	0.012	−0.352 (0.043)	23.899	24.734	0.106	58,884
FT	2408	0.046	0.010	−1.632 (0.050)	26.637	20.917	0.048	57,125
CB	3331	0.028	0.011	−1.089 (0.042)	18.335	18.736	0.069	33,295
CA	3319	0.054	0.010	−0.527 (0.042)	8.450	12.855	0.069	4,261
CI	3368	0.023	0.008	−1.028 (0.042)	24.159	24.487	0.082	82,187
BC	3327	0.052	0.014	−0.879 (0.042)	10.327	16.290	0.085	15,151
IG	2862	0.065	0.010	−0.399 (0.046)	9.221	15.951	0.084	10,165
YD	3392	−0.021	0.007	−0.356 (0.042)	5.258	10.513	0.063	792
PD	3392	0.014	0.007	−0.019 (0.042)	5.525	12.008	0.047	901
MD	3392	−0.002	0.007	−0.137 (0.042)	4.849	9.984	0.050	494

[a] Skewness: $= m_3/s^3$, Kurtosis: $= m_4/s^4$, Studentized range (Stud. range): $= (\max\{R_t\} - \min\{R_t\})/s$ and Jarque–Bera: $= T\,(\text{Skewness}^2/6 + (\text{Kurtosis} - 3)^2/24)$, where $m_k = \Sigma(R_t - \overline{R})^k/T$, $s^2 = \Sigma(R_t - \overline{R})^2/(T-1)$ and T is the number of observations. Kolmogorov–Smirnov (Kolm.–Smirnov) is the usual Kolmogorov–Smirnov statistic for testing normality. The values in parentheses are the standard errors of the coefficients of skewness under the hypothesis of normality, $(6/T)^{1/2}$.

Nevertheless, this asymmetry is not clearly observed in the empirical distributions. As the mean of the returns is not zero, excess returns have been obtained by subtracting the mean. The histograms corresponding to excess returns seem to be approximately symmetric about zero, or, as is equivalent, returns seem to be approximately symmetric about the mean. In order to achieve a deeper graphic insight, two sub-samples have been constructed for each series: one formed by negative excess returns in absolute values,

$$|R^-| = \{\overline{R} - R_t | R_t < \overline{R}\}, \tag{4}$$

and one formed by positive excess returns,

$$R^+ = \{R_t - \overline{R} | R_t > \overline{R}\}. \tag{5}$$

If returns are symmetric (about the mean) these two sub-samples should have the same distribution. The relative frequencies of SP excess returns are graphed in Fig. 1. Significant differences are not observed between the two types of excess returns, and similar figures are obtained for the remaining series. Therefore, while the results reflected in Table 1 have often been judged as a clear exponent of strong asymmetry, this is not appreciated in the graphs of empirical distributions.

The reason for this discrepancy is probably due to the sample distribution of the skewness statistic when the underlying distribution is non-normal. Under normality this statistic does follow the asymptotic distribution reflected in (3). However, as there is abundant literature on the non-normality of daily returns,

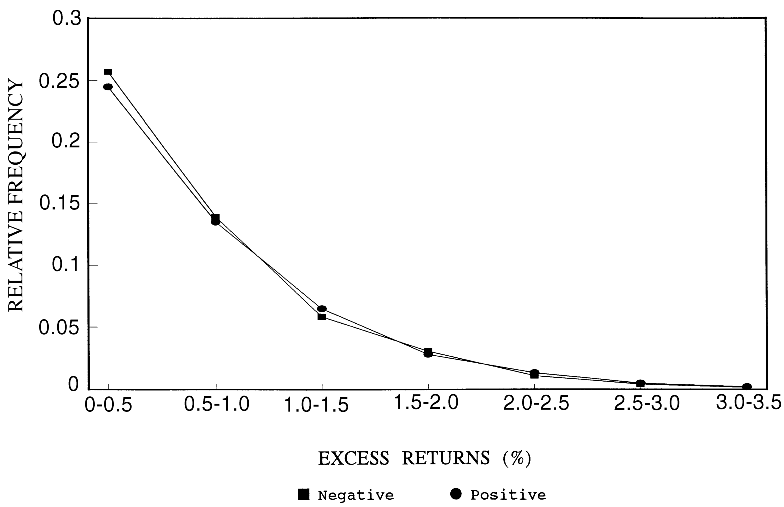

Fig. 1. Standard & Poor's excess returns

it would be desirable to examine the symmetry of returns under alternative distributions.

4. The symmetry of returns under non-normal distributions

Numerous distributions have been proposed for daily financial returns. Although there is no agreement on the statistical distribution that best represents these returns, the discrete mixture of normal distributions and the Student's t distribution are among the widest accepted. While the discrete mixture can be symmetric or asymmetric, the Student's t distribution is necessarily symmetric.

In the case of the discrete mixture of normal distributions, two or three normal distributions seem to be sufficient. For two normal distributions, the density function is given by

$$f(x) = \frac{\delta}{\sqrt{2\pi\sigma_1^2}} \exp\left(\frac{-(x-\mu_1)^2}{2\sigma_1^2}\right) + \frac{1-\delta}{\sqrt{2\pi\sigma_2^2}} \exp\left(\frac{-(x-\mu_2)^2}{2\sigma_2^2}\right), \tag{6}$$

where $0 < \delta < 1$. This distribution is symmetric if either of the two following conditions occur (Johnson, et al., 1994):

$$\mu_1 = \mu_2 \tag{7}$$

or

$$\sigma_1 = \sigma_2, \quad \delta = \frac{1}{2}. \tag{8}$$

Then, under the hypothesis that the returns follow a discrete mixture of two normal distributions, we can test the symmetry of the distribution through a likelihood ratio test. The evidence allows a clear rejection of (8), but the situation with respect to (7) is not so clear. Table 2 shows the results of the likelihood ratio tests. At the 1% significance level, restriction (7) can only be rejected for YD returns. At the 5% significance level, restriction (7) is also rejected for the Japanese and German stock markets. But for these markets the fit of the mixture distribution is not especially good (judging by Pearson's adherence statistic). In particular, it is much worse than the Student's t distribution. For the rest of the series, Eq. (7) cannot be rejected. The distributions seem to have the same expectation and, as a result, the symmetry of returns cannot be rejected.

Another statistical distribution proposed by different researchers is Student's t distribution, whose density function is given by

$$f(x) = \frac{\Gamma((\upsilon+1)/2)}{\Gamma(\upsilon/2)\sqrt{\pi(\upsilon-2)\sigma^2}} \left(1 + \frac{(x-\mu)^2}{(\upsilon-2)\sigma^2}\right)^{-(\upsilon+1)/2} \tag{9}$$

Table 2
Likelihood ratio tests of equal expectations in the discrete mixtures

	χ^2_1	(P-value)
SP	1.307	(0.253)
DJ	0.129	(0.719)
NI	4.194	(0.041)
FT	1.184	(0.277)
CB	4.592	(0.032)
CA	2.925	(0.087)
CI	3.245	(0.072)
BC	0.963	(0.326)
IG	1.375	(0.241)
YD	14.819	(<0.001)
PD	2.538	(0.111)
MD	1.993	(0.158)

where Γ denotes the gamma function, μ is a location parameter equal to the expectation if $\upsilon > 1$, σ is a scale parameter equal to the standard deviation if $\upsilon > 2$, and υ is a parameter usually referred to as degrees of freedom. As this distribution is symmetric about μ, the assumption of this distribution implies the symmetry of the returns.

Nevertheless, in this case the sample distribution of $\hat{\alpha}$ can be quite different from (3), especially for low values of υ. In order to analyse this possibility, 100,000 replications were generated with sample size T of Student's t variables with υ degrees of freedom. T was fixed in 100, 1000 and 3400 (this last value being close to the sample size of the series considered here). On the other hand, to choose the value of υ, this parameter was estimated by maximum-likelihood for all the series. The estimates range from 2.4 (for the Japanese stock market) to 5.8 (for the British stock market). These values are similar to those obtained by other researchers (see, for example, Blattberg and Gonedes, 1974; Boothe and Glassman, 1987; or Peiró, 1994). For these reasons, in the simulations υ was fixed at 2, 4 and 6. It is interesting to remember that the Student's t distribution has no moments of order lower than or equal to υ. Therefore, the population coefficient of skewness for the Student's t distribution does not exist when $\upsilon \leqslant 3$.

The 0.95, 0.975 and 0.995 quantiles of the sample distribution of $\hat{\alpha}$ in these simulations are shown in Table 3. For the purpose of comparison, the same quantiles of the asymptotic distribution under normality, obtained through (3), are also shown. Several conclusions are obtained from Table 3. Firstly, when the population coefficient of symmetry does not exist (the case $\upsilon = 2$), the quantiles are extremely high, rising up as T increases. Secondly, as T increases, the difference between the quantiles of t distributions and the quantiles of the normal distribution also increases. Thus, for example, for $T = 100$, the quantile 0.975 of the Student's t distribution with four degrees of freedom is approxi-

Table 3
Sample distribution of skewness[a]

	$T=100$				$T=1000$				$T=3400$			
	$\upsilon=2$	$\upsilon=4$	$\upsilon=6$	Normal	$\upsilon=2$	$\upsilon=4$	$\upsilon=6$	Normal	$\upsilon=2$	$\upsilon=4$	$\upsilon=6$	Normal
0.950	5.57	1.63	0.95	0.40	13.93	1.22	0.46	0.13	22.84	0.98	0.29	0.07
0.975	7.18	2.30	1.26	0.48	19.76	1.89	0.61	0.15	33.88	1.55	0.38	0.08
0.995	9.30	4.44	2.26	0.63	28.58	4.93	1.19	0.20	51.46	4.48	0.71	0.11

[a] The table shows the quantiles of the sample distribution of skewness in Student's t distribution with υ degrees of freedom and in normal distributions with sample size T.

mately five times that of the normal distribution, but for $T = 3400$ it is more than 18 times greater. The main conclusion is clear: the quantiles are much higher for Student's t distributions with low degrees of freedom than for the normal distribution and the difference increases with the sample size. [3] In empirical research with daily financial data, the sample size is usually quite high. Then, if financial daily returns follow Student's t distribution, tests of symmetry with (2) and (3) will incorrectly reject the null of symmetry very often.

Now, taking into account the estimates of the degrees of freedom, only SP, DJ and FT returns exceed the 95% critical value of Table 3. But just excluding the observation corresponding to the crash of October 1987 (19 October, for SP and DJ returns and 20 October, for FT returns) the values of $\hat{\alpha}$ change from -3.408, -4.100 and -1.632 to -0.227, -0.058 and -0.761 for SP, DJ and FT returns respectively. These dramatic changes originate from one single observation out of 3373 (2408 in the case of FT returns).

5. Distribution-free tests

It would be desirable to rely on a more robust measure of symmetry that does not depend to such an extent on extreme returns. Besides, there are serious differences in the sample distribution of $\hat{\alpha}$ under alternative distributions. Given these problems, other inference techniques must be considered. The distribution-free methods are specially suitable in these cases because the distribution of the test statistic does not depend on the specific distribution function of the population; these methods only require minimal assumptions about the underlying distribution.

To check the symmetry of daily financial returns, these returns will be considered symmetric about their mean when: (1) the probability of occurrence of a negative excess return is equal to the probability of occurrence of a positive excess return, and (2) the statistical distribution of the negative excess returns in absolute values is equal to the statistical distribution of positive excess returns; that is, the equality of the statistical distributions of $|R^-|$ and $|R^+|$.

If the probability of occurrence of a negative excess return is equal to the probability of occurrence of a positive excess return, then, when excluding zero excess returns, both the number of negative and positive excess returns follow a binomial distribution with parameters equal to the number of returns and 1/2. Using this distribution, the equal probability of negative and positive excess returns is rejected for IG and YD returns at the 5% level, and only for YD

[3] An additional surprising fact is that the 0.995 quantile for $v = 4$ and $T = 1000$ is higher than for $T = 100$ and $T = 3400$. Similar results were obtained in new simulations.

returns at the 1% level; for the other ten samples, the equal probability of both kinds of returns cannot be rejected with marginal significance levels higher than 10%. [4]

To test for the equality of the distributions, three distribution-free methods will be used: the Kolmogorov–Smirnov two sample test, the Wilcoxon rank-sum test and the Siegel–Tukey test. These three tests are two-sample tests and will allow the comparison of the distributions of $|R^-|$ and $|R^+|$. In all of them the null hypothesis establishes the equality of the populations underlying the two samples. But, while the Kolmogorov–Smirnov test is sensitive to any difference in the distribution of the two samples, the Wilcoxon rank-sum test is especially appropriate to detect differences in location and the Siegel–Tukey test is especially appropriate to detect differences in dispersion. (see Gibbons and Chakraborti, 1992).

The results of these distribution-free tests are shown in Table 4. Analysing the results of Table 4 by markets, the equality of the distributions of $|R^-|$ and $|R^+|$cannot be rejected for SP, DJ, CB, CI, PD and MD returns at the 5% significance level in any test. Although the Wilcoxon and the Siegel–Tukey tests do not allow the rejection of the equality of the distributions for CA returns, the Kolmogorov–Smirnov allows the rejection at the 5% significance level but not at the 1% level (the P-value is 3.2%). Differences between negative and

Table 4
Distribution-free tests[a]

	KS	W	ST
SP	0.0437 (0.093)	2,819,223 [−1.960] (0.050)	2,860,964 [−0.394] (0.693)
DJ	0.0284 (0.503)	2,830,510 [0.050] (0.960)	2,872,661 [1.571] (0.116)
NI	0.0307 (0.431)	2,548,244 [−0.403] (0.687)	2,481,446 [−2.921] (0.003)
FT	0.0452 (0.171)	1,412,325 [−0.103] (0.918)	1,372,521 [−2.438] (0.015)
CB	0.0338 (0.298)	2,709,247 [−1.249] (0.212)	2,697,999 [−1.625] (0.104)
CA	0.0500 (0.032)	2,702,850 [−1.610] (0.107)	2,711,186 [−1.249] (0.212)
CI	0.0407 (0.123)	2,762,479 [−1.019] (0.308)	2,746,477 [−1.588] (0.225)
MI	0.0556 (0.012)	2,741,013 [−2.989] (0.003)	2,878,413 [1.973] (0.097)
IG	0.0545 (0.029)	2,079,634 [−2.413] (0.016)	2,158,129 [−1.141] (0.254)
YD	0.0469 (0.048)	2,780,545 [1.429] (0.153)	2,679,873 [−2.105] (0.035)
PD	0.0414 (0.109)	2,904,661 [−0.705] (0.481)	2,959,889 [1.232] (0.218)
MD	0.0356 (0.233)	2,840,853 [1.401] (0.161)	2,726,626 [−0.847] (0.397)

[a] KS is the Kolmogorov–Smirnov two sample test statistic. W is the Wilcoxon rank-sum test statistic. ST is the Siegel–Tukey test statistic. The standardized statistics are in square brackets. P-values are in parentheses. In all cases the first sample is formed by negative excess returns and the second sample is formed by positive excess returns.

[4] Evidently, the sum of positive excess returns is equal to the sum of negative excess returns in absolute values. This test can, therefore, be regarded as a test for the equality of means. In fact, the results are very similar to those obtained with the usual t-test.

positive excess returns are detected for both MI and IG returns; they seem to be due to differences in location as reflected by the Wilcoxon tests. It is interesting to note that this type of asymmetry occurs in Milan and Madrid, the markets with the lowest capitalizations and trading volumes from all the analysed. [5] For NI, FT and YD returns, the null hypothesis of equal distributions is rejected in the Siegel–Tukey tests. In spite of these rejections, the equality of the distributions for these returns is not rejected in the Wilcoxon and in the Kolmogorov–Smirnov tests, except for YD returns whose *P*-value in the Kolmogorov–Smirnov test is very close to 5%.

These results of the Siegel–Tukey tests and their contrast with the other tests suggest, for these three markets, the possibility of a different dispersion in negative excess returns and in positive ones. As the ST statistics are below their expected values, they seem to indicate that the dispersion of negative excess returns (first sample) is higher than the dispersion of positive excess returns (second sample). Given the nature of this test, it does not appear that this result is due to a few extreme returns (i.e. the crash of October 1987). On the contrary, it must be due to a substantial proportion of returns. To cast some light on this point, percentages of negative and positive excess returns have been calculated in the whole sample and in 10% of the excess returns with the highest absolute values. The results are shown in Table 5. As previously stated, the proportions in the whole samples only differ significantly for YD returns. In the

Table 5
Proportions of excess returns[a]

	Whole sample			10% highest absolute excess returns		
	% Neg.	% Pos.	*P*-value	% Neg.	% Pos.	*P*-value
SP	50.5	49.5	0.547	50.3	49.7	0.913
DJ	49.7	50.3	0.744	49.1	50.9	0.744
NI	49.0	51.0	0.246	55.9	44.1	0.035
FT	48.8	51.2	0.221	56.0	44.0	0.062
CB	49.4	50.6	0.521	52.4	47.6	0.381
CA	49.9	50.1	0.876	50.3	49.7	0.913
CI	49.2	50.8	0.352	52.8	47.2	0.301
MI	51.0	49.0	0.245	48.7	51.3	0.622
IG	52.1	47.9	0.027	47.6	52.4	0.408
YD	47.6	52.4	0.005	55.6	44.4	0.039
PD	50.8	49.2	0.336	46.2	53.8	0.159
MD	48.7	51.3	0.122	50.3	49.7	0.914

[a] Proportions of negative and positive excess returns in the whole sample and in the 10% highest excess returns in absolute values. The *P*-values correspond to the hypothesis that both types of excess returns are of equal probability.

[5] Alles and Kling (1994) find that smaller capitalized stock indices are more negatively skewed than larger stock indices.

sub-samples formed by the 10% excess returns with the highest absolute values, some differences are observed between the proportions of negative and positive excess returns. These differences are higher, precisely, for NI, FT and YD returns. In addition, in these three cases the proportion is higher for the negative excess returns as expected in the light of their negative standardized statistics in the Siegel–Tukey tests. An additional interesting fact occurs for YD returns. In the whole sample, the proportion of positive excess returns is significantly higher than the proportion of negative excess returns. But the reverse happens in the 10% extreme excess returns.

To confirm that this 10% of extreme excess returns are the cause of the rejections in the Siegel–Tukey tests, this test was run again excluding these extreme values. The results obtained are very different to those displayed in Table 4. The new standardized statistics are -1.27, -1.06 and -0.34 for NI, FT and YD returns, respectively. These new values mean marginal significance levels of 0.20, 0.29 and 0.73. Therefore, with these sub-samples the null hypothesis of equal distribution can no longer be rejected. It seems reasonable to attribute the rejection of the equality between the distribution of $|R^-|$ and $|R^+|$ in these markets to the different dispersion caused by extreme excess returns, more frequent in $|R^-|$ than in $|R^+|$.

Consequently, to summarize the results of this section, there is no evidence against the property of symmetry in six of the 12 series analysed (SP, DJ, CB, CI, PD and MD returns). There are weak signs of asymmetry in CA returns. The other series (NI, FT, MI, IG and YD) present signs of asymmetry, but these are not very strong. The asymmetry of MI and IG returns seems to be due to the different location of negative and positive excess returns. The main source of asymmetry in the other three markets is the higher dispersion in negative excess returns than in positive ones. In turn, this higher dispersion seems to originate from the higher frequency of negative excess returns among the extreme ones.

As the stock series are stock index returns, the results really refer to portfolio returns, of equal composition and weights to those of the index. Therefore, these results do not question several authors' explanations of the low diversification of many investors' portfolios due to the preference for positive skewness. Conversely, if stock index returns are not skewed, the phenomenon known as 'volatility skew' in option pricing cannot be a consequence (at least with regard to index and foreign currency options) of the skewness in the distribution of returns. Finally, it should be taken into account that all the results in this study refer to daily exchange rate returns and, in the case of stock markets, to daily index returns. If the symmetry of daily returns is accepted, one should accept also the symmetry of weekly or monthly returns. Further research should extend the study of skewness to individual stocks and to shorter- and longer-period returns, as well as to other markets from different countries.

6. Conclusions

In statistical and financial literature, the issue of skewness in financial returns is not clear enough. Most of the studies rely on the sample skewness (third central moment divided by the cube of standard deviation) and use its asymptotic distribution under normality. With this test statistic and this asymptotic distribution, symmetry is rejected in eight of the nine series of stock returns and in the three series of exchange rate returns. But these results are worthless as they are very sensitive to the normality assumption, and there is abundant evidence on the non-normality of short-period returns.

Discrete mixtures of normal distributions and Student's t distributions have been frequently proposed for daily financial returns. Under the hypothesis of the discrete mixture of two normal distributions, the symmetry of returns can be tested by likelihood ratio tests. These tests do not allow the rejection of symmetry for most series. Under Student's t distributions, the sample distribution of the skewness statistic is very different to its distribution under normality. While under normality symmetry was rejected for 11 of the 12 series, under Student's t hypothesis symmetry cannot be rejected for most markets.

Tests on symmetry with sample skewness present serious problems: sensitivity with respect to extreme returns, ignorance of the true underlying distribution and, consequently, of the sample distribution of this statistic. To avoid these problems, distribution-free methods have been used. With these methods, the distribution of the absolute values of negative excess returns is compared with the distribution of positive excess returns. The results suggest that in most markets daily financial returns are symmetric or, at least, do not present strong evidence of skewness. Nevertheless, two features are observed. On the one hand, two markets present differences in location between negative and positive excess returns; these two markets are, interestingly, the least capitalised. On the other hand, in three markets a different dispersion is detected between both types of excess returns. This different dispersion seems to be due to the higher frequency of negative excess returns among those that are extreme.

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