Bayesian Nonparametric Mixture Models

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政大 E **計畫課程: 貝氏資料分析 (楊立行/李玉麟)**

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Outline

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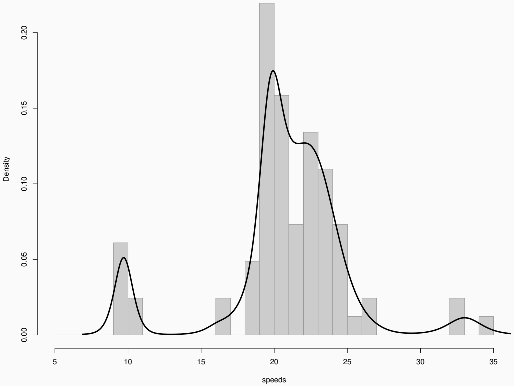
Simple Worked Examples

- `library("DPpackage")`:[1] dataset `galaxy`
- Receding velocities from 82 galaxies into 4 clusters
- Escobar & West (1995). *JASA*, 90, 577–88

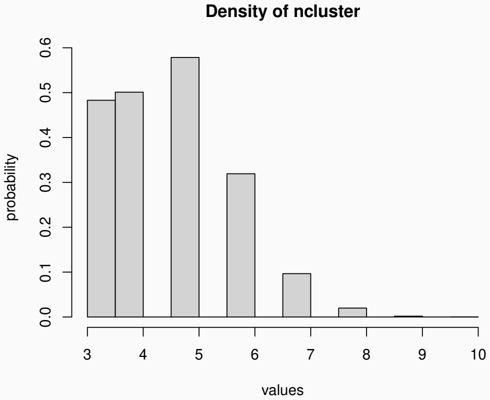



```
> # source('galaxy.R')
> library("DPpackage")
> prior2 <- list(alpha=1,m1=rep(0,1),
    psiinv2=solve(diag(0.5,1)),
    nu1=4,nu2=4,tau1=1,tau2=100)
> fit1.2 <- DPdensity(y=speeds,
    prior=prior2,mcmc=mcmc,
    state=state,status=TRUE)
> hist(speeds, probability = T,
    breaks = seq(5, 35, by =1),
    col = "grey80", border = "grey60")
> lines(cbind(fit1.2$x1,fit1.2$dens),
    lwd = 3)
```

[1] Jara et al. (2011), *J Stats Software*, 40, 1–30

Posterior distribution of K

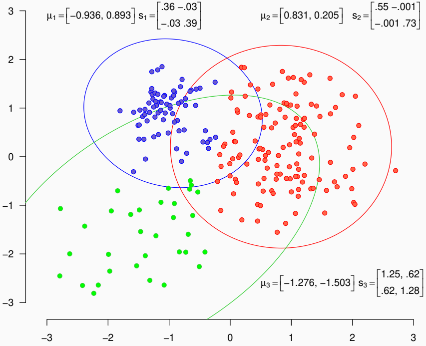
```
> plot(fit1.2, output = 'param')
```



- Mixture of up to $K = 10$ distributions

Mixture of Bivariate/Multivariate Normals

- `scikit-learn.org`: Machine Learning in Python
- Variational `BayesGaussianMixture()`[2]



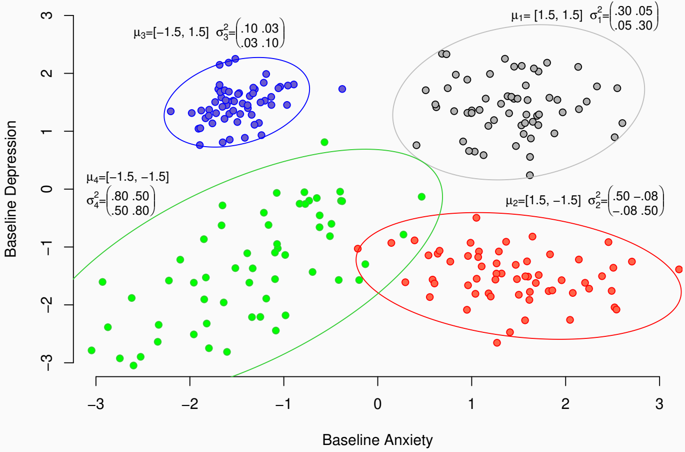

```
import numpy as np
from sklearn.mixture import BayesianGaussianMixture

K = 10
DProc = BayesianGaussianMixture(n_components=K,
  weight_concentration_prior_type="dirichlet_process",
  init_params='kmeans',
  covariance_prior = 10 * np.eye(n_cols),
  random_state=random_seed
  ).fit(csv_data)
results = DProc.predict(csv_data)
probs = DProc.predict_proba(csv_data)
print DProc.means_          # print cluster means
print DProc.covariances_    # and covariances
```

[2]scikit-learn.org/stable/modules/mixture.htm#bgmm

Theory

Hypothetical Data to Illustrate the Theory

- Hypothetical psychotherapy clinical trial for patients with advanced or terminal cancer



Conventional Clustering vs. BNP Clustering

- Conventional clustering
  - E.g., `kmeans()`, `hclust()`, or finite mixtures
  - You must specify what K is
  - Try $k = 1, 2, 3, \ldots, K$, then determine K by model comparison metrics
- Bayesian nonparametric clustering does things fundamentally differently
  - Posterior distribution of $k = 1, 2, 3, \ldots, \infty$ to a theoretically infinite number of clusters
  - A specific K may emerge out of a dataset
  - K is unbounded and may grow as the model encounters more data
  - Sometimes called an *infinite* mixture model [3]

---

[3] Rasmussen (2000).
http://papers.nips.cc/paper/1745-the-infinite-gaussian-mixture-model.pdf

- Neal (2000): one of the most cited papers on BNP[4]

$$
\begin{aligned}
y_i | c_i, \phi &\sim \mathcal{N}(\phi_k) \\
c_i | \boldsymbol{\pi} &\sim \mathrm{Discrete}(\pi_1, \pi_2, \ldots, \pi_k) \\
\phi_k &\sim G_0 \\
\boldsymbol{\pi} &\sim \mathrm{DP}(\alpha/K, \ldots, \alpha/K).
\end{aligned}
$$

[4] Neal (2000), J Compu and Graph Stats, 9(2): 249-265.

- Neal (2000): one of the most cited papers on BNP[4]

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\end{aligned}
$$

[4] Neal (2000), J Compu and Graph Stats, 9(2): 249-265.

Dirichlet Process Mixture Modeling

- Neal (2000): one of the most cited papers on BNP[4]

$$
\begin{aligned}
y_i | c_i, \phi &\sim \mathcal{N}(\phi_k) \\
c_i | \boldsymbol{\pi} &\sim \text{Discrete}(\pi_1, \pi_2, \ldots, \pi_k) \\
\phi_k &\sim G_0 \\
\boldsymbol{\pi} &\sim \text{DP}(\alpha/K, \ldots, \alpha/K). \quad \text{“Dirichlet Process Prior”}
\end{aligned}
$$

- Last two lines allow DP to spawn new clusters, look simple
  - But how? We went through many tutorial papers, books, & videos
  - Did not get very far with them
  - Details omitted entirely, assumed understood, or explained in abstract and unfamilar concepts

[4] Neal (2000), J Compu and Graph Stats, 9(2): 249-265.

BNP by Dirichlet Process Mixtures: Our How-To Guide

- We worked out these missing details for non-technical readers

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Tutorial

A tutorial on Dirichlet process mixture modeling

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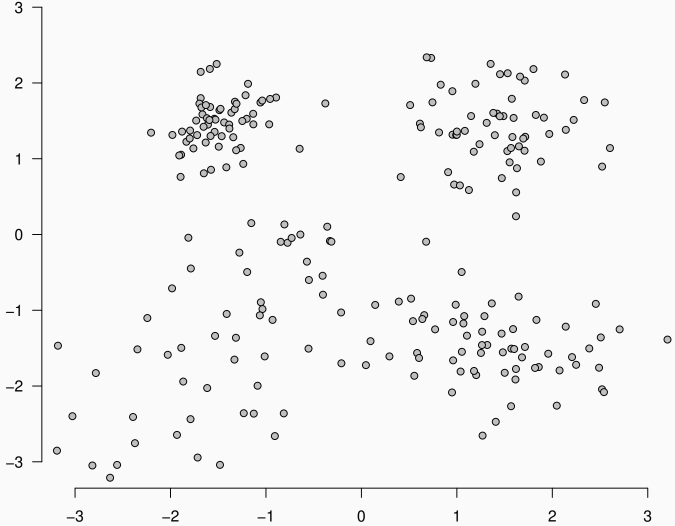
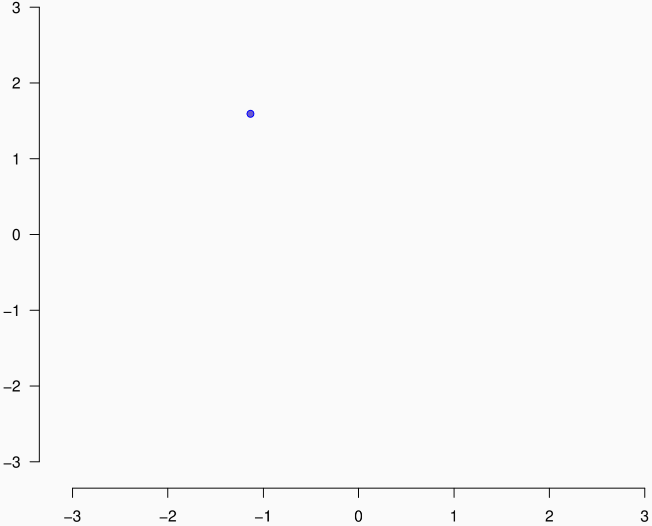
- Important equations carefully derived in great detail
- It may also help pave the way to other BNP methods

What DPMM Looks Like



What DPMM Looks Like



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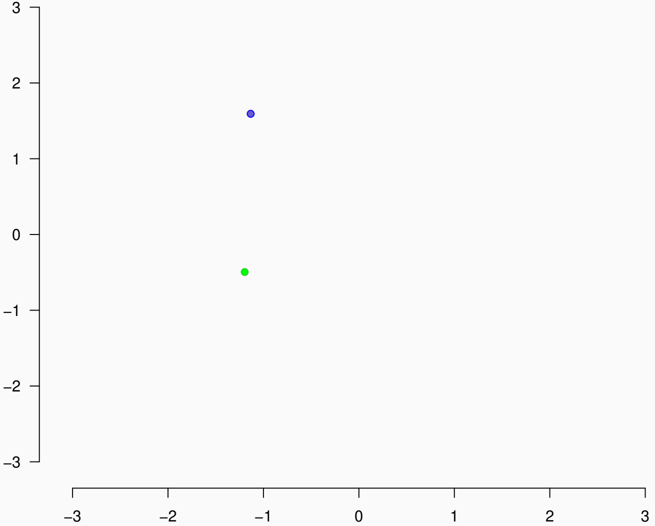

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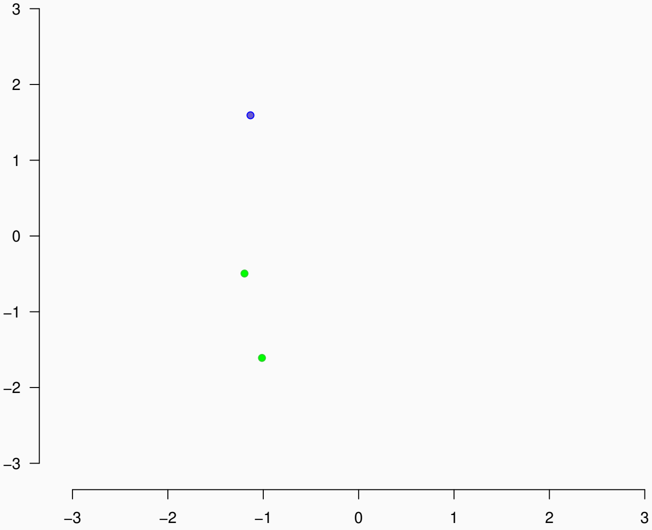
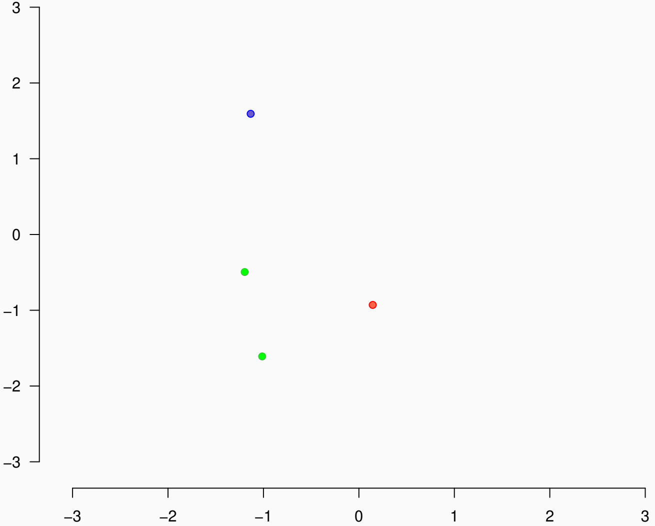
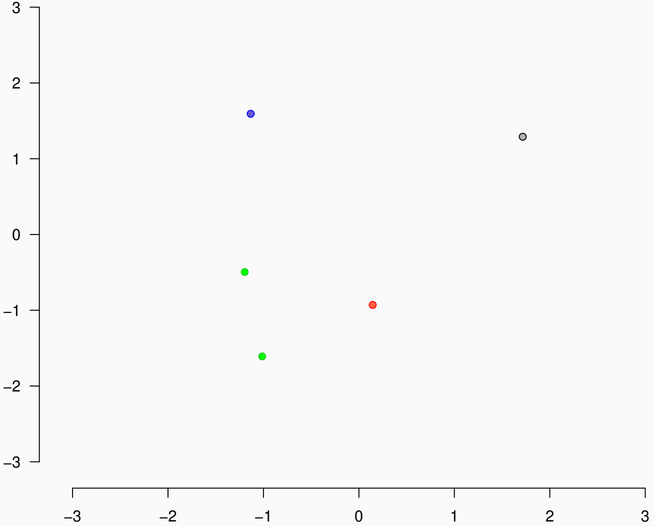


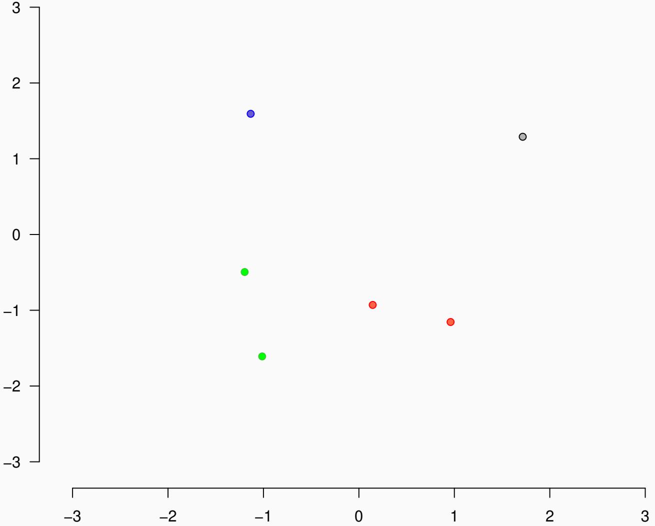
What DPMM Looks Like



What DPMM Looks Like



What DPMM Looks Like



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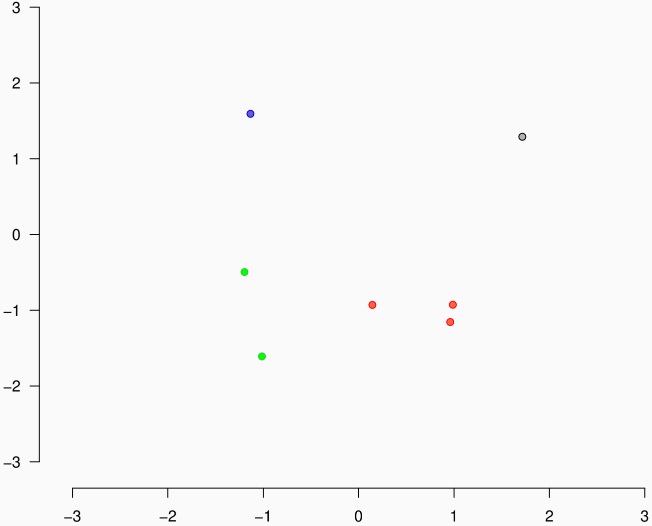

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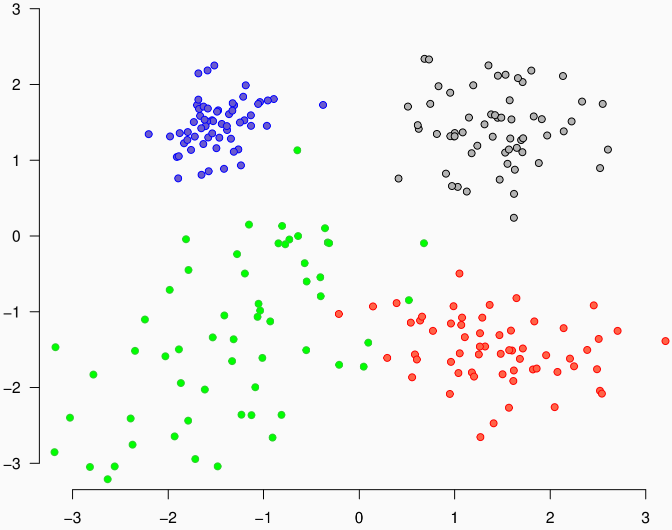

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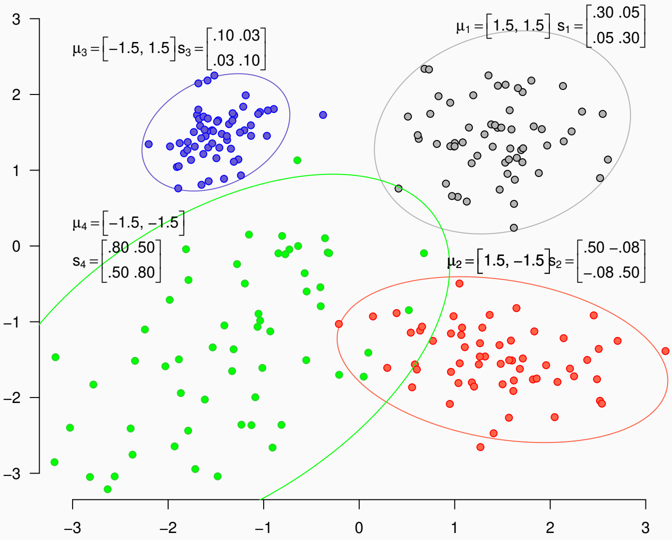
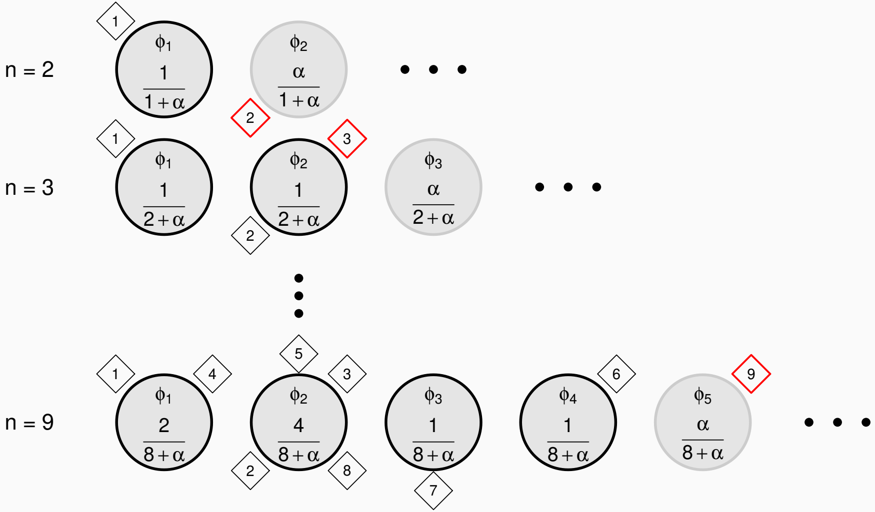


The Chinese Restaurant Process (CRP) [5]



[5] Gershman & Blei (2012), *J Math Psych*, 56, 1–12.

Finite Mixture Model

Data likelihood on mixture of Gaussians when k is fixed [6]

$$p(y|\mu_1, \ldots, \mu_k, s_1, \ldots, s_k, \pi_1, \ldots, \pi_k) = \sum_{j=1}^{k} \pi_j \, \mathcal{N}(\mu_j, s_j),$$

[6] Rasmussen (2000), The Infinite Gaussian Mixture Model.

Finite Mixture Model

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Priors on cluster means and precisions:

$$p(\mu_j) \sim \mathcal{N}(\mu_0, s_0)$$

$$p_0(s_j|\gamma, \beta) \sim \mathcal{G}(\gamma, \beta) \propto s^{\gamma-1} \exp(-\beta s)$$

[6] Rasmussen (2000), The Infinite Gaussian Mixture Model.

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Prior on mixing proportions:

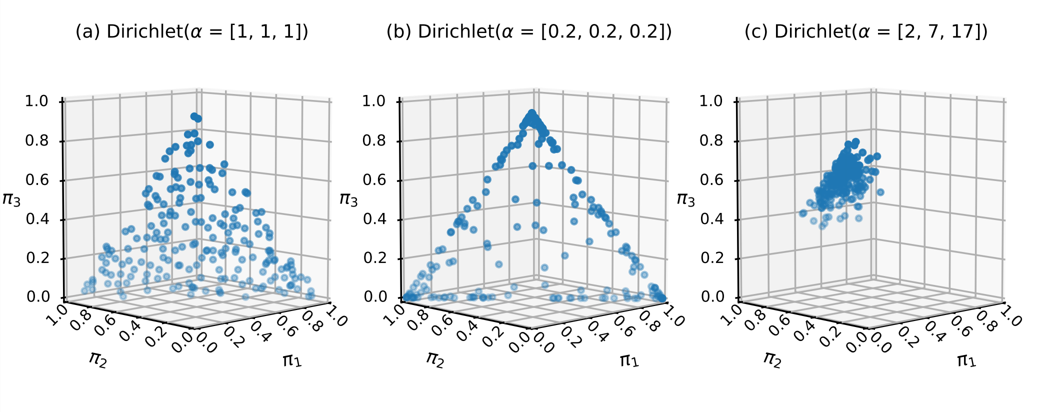
$$\begin{aligned} p(\pi_1, \ldots, \pi_k|\alpha) &\sim \mathrm{Dirichlet}(\alpha/k, \ldots, \alpha/k) \\ &= \frac{\Gamma(\alpha)}{\Gamma(\alpha/k)^k} \prod_{j=1}^{k} \pi_j^{\alpha/k-1}, \end{aligned}$$

where $\alpha/k = 1$ for a flat Dirichlet distribution for simplicity.

[6] Rasmussen (2000), The Infinite Gaussian Mixture Model.

Dirichlet Distribution

- Dirichlet distribution when $k = 3$ is fixed, e.g., $\mathrm{Dirichlet}(\alpha/k = [1, 1, 1])$ as a conjugate prior for a 3-category solution



Posteriors: [7]

$$
\begin{aligned}
p(\boldsymbol{\mu}_j|\boldsymbol{c}, \boldsymbol{y}, s_j, \mu_0, s_0) &\propto p(\boldsymbol{y}|\boldsymbol{\mu}, \boldsymbol{c}, s_j)p(\boldsymbol{\mu_0}) \\
&\sim \mathcal{N}\left(\frac{\bar{y}_j n_j s_j + \mu_0 s_0}{n_j s_j + s_0}, \frac{1}{n_j s_j + s_0}\right), \\
p(s_j|\boldsymbol{c}, \boldsymbol{y}, \boldsymbol{\mu}_j, \alpha, \beta) &\propto s^{n_j/2} \exp\left(-n_j \sum_{[i]\, c_j = j} (y_i - \bar{y}_j)^2\right) \times s^{\alpha-1} \exp(-\beta s) \\
&= s^{\alpha-1+n_j/2} \exp\left(-s(\beta + \frac{1}{2} \sum_{[i]\, c_j = j} (y_i - \bar{y}_j)^2)\right) \\
&= \mathcal{G}\Big(\gamma + n_j/2, \beta + \frac{\sum_{i=1}^{n_j} (y_i - \mu_j)^2}{2}\Big).
\end{aligned}
$$

---

[7] See Li, Schofield & Gönen (2019) in handouts for step by step detailed derivations.

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&= \mathcal{G}\left(\gamma + n_j/2, \beta + \frac{\sum_{i=1}^{n_j} (y_i - \mu_j)^2}{2}\right).
\end{aligned}
$$

Prior parameters updated by data likelihood when k is fixed.

[7] See Li, Schofield & Gönen (2019) in handouts for step by step detailed derivations.

Let an indicator variable c_i represent each person's latent cluster membership.

$$
\begin{aligned}
p(c_1, \ldots, c_k | \pi_1, \ldots, \pi_k) &= \prod_{j=1}^{k} \pi_j^{n_j}. \\
p(\pi_1, \ldots, \pi_k | \alpha) &\sim \mathrm{Dirichlet}(\alpha/k, \ldots, \alpha/k) = \frac{\Gamma(\alpha)}{\Gamma(\alpha/k)^k} \prod_{j=1}^{k} \pi_j^{\alpha/k-1}. \\
p(c_1, \ldots, c_k | \alpha) &= \int p(c_1, \ldots, c_k | \pi_1, \ldots, \pi_k) p(\pi_1, \ldots, \pi_k) \, d\pi_1 \cdots d\pi_k \\
&= \frac{\Gamma(\alpha)}{\Gamma(\alpha/k)^k} \int \prod_{j=1}^{k} \pi_j^{n_j+\alpha/k-1} \, d\pi_j \\
&= \frac{\Gamma(\alpha)}{\Gamma(n+\alpha)} \prod_{j=1}^{k} \frac{\Gamma(n_j+\alpha/k)}{\Gamma(\alpha/k)}.
\end{aligned}
$$

But How to Get k to Grow with Data?

Let an indicator variable c_i represent each person's latent cluster membership.

$$
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p(c_i = j | \boldsymbol{c}_{-i}, \alpha) &= \frac{n_{-i,j} + \alpha/k}{n - 1 + \alpha}.
\end{aligned}
$$

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&= \frac{\Gamma(\alpha)}{\Gamma(\alpha/k)^k} \int \prod_{j=1}^{k} \pi_j^{n_j+\alpha/k-1} \, d\pi_j \\
&= \frac{\Gamma(\alpha)}{\Gamma(n+\alpha)} \prod_{j=1}^{k} \frac{\Gamma(n_j+\alpha/k)}{\Gamma(\alpha/k)}. \\
p(c_i = j | \boldsymbol{c}_{-i}, \alpha) &= \frac{n_{-i,j} + \alpha/k}{n-1+\alpha}. \iff \textit{exchangeability!}
\end{aligned}
$$

Letting $k \to \infty$

Continue from the previous slide,

$$p(c_i = j | \boldsymbol{c}_{-i}, \alpha) = \frac{n_{-i,j} + \alpha/k}{n - 1 + \alpha}, \quad \text{letting } k \to \infty,$$

$$\lim_{k \to \infty} \frac{n_{-i,j} + \alpha/k}{n - 1 + \alpha} = \frac{n_{-i,j}}{n - 1 + \alpha},$$

Letting $k \to \infty$

Continue from the previous slide,

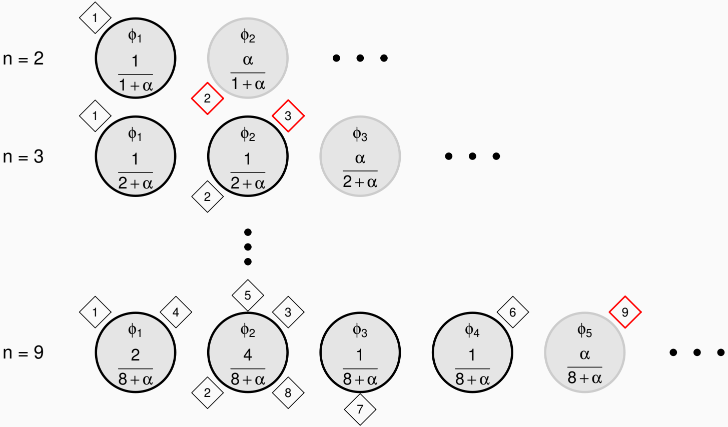
$$p(c_i = j | \boldsymbol{c}_{-i}, \alpha) = \frac{n_{-i,j} + \alpha/k}{n - 1 + \alpha}, \quad \text{letting } k \to \infty,$$

$$\lim_{k\to\infty} \frac{n_{-i,j} + \alpha/k}{n - 1 + \alpha} = \frac{n_{-i,j}}{n - 1 + \alpha},$$

$$\begin{cases} \text{already occupied clusters} : & p(c_i | \boldsymbol{c}_{-i}, \alpha) & = \dfrac{n_{-i,j}}{n - 1 + \alpha}, \\ \text{a new cluster} : & p(c_i \neq c_j \, \forall j \neq i | \boldsymbol{c}_{-i}, \alpha) & = \dfrac{\alpha}{n - 1 + \alpha}. \end{cases}$$

CRP Revisited

$$\begin{cases} \text{already occupied clusters} \; : & p(c_i | \boldsymbol{c}_{-i}, \alpha) & = \frac{n_{-i,j}}{n-1+\alpha}, \\ \text{a new cluster} \; : & p(c_i \neq c_j \, \forall j \neq i | \boldsymbol{c}_{-i}, \alpha) & = \frac{\alpha}{n-1+\alpha}. \end{cases}$$



CRP Equivalent to Stick-Breaking Process Construction [8]

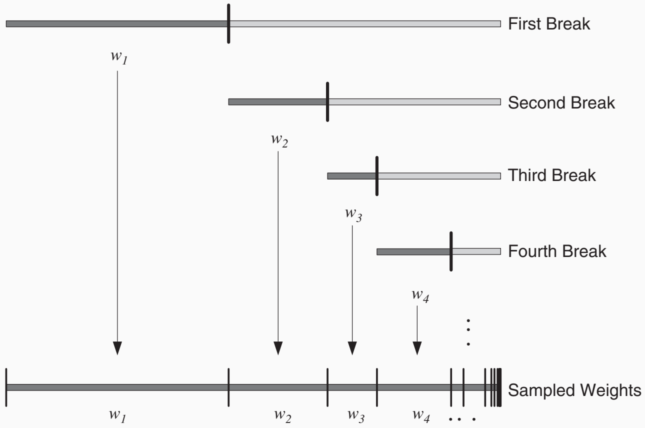


Fig. 5. A graphical depiction of the stick-breaking process, showing successive breaks of a stick with starting length one, and how the lengths of the pieces correspond to sampled weights.

[8] See Navarro et al., (2006) for a brief history and explanation

$p(c_i|\boldsymbol{c}_{-i})$ has to be weighted by the posterior probability of *newly observed* values given the data you have already seen:

$$\bullet \text{ clusters where } n_{-i,k} > 0 :$$

$$\begin{aligned} p(c_i|\boldsymbol{c}_{-i}, \mu_k, \tau_k, \alpha) &\propto p(c_i|\boldsymbol{c}_{-i}, \alpha) p(\tilde{y}_i|\mu_k, \tau_k, \boldsymbol{c}_{-i}) \\ &\propto \frac{n_{-i,k}}{n-1+\alpha} \mathcal{N}\left(\tilde{y}_i; \frac{\bar{y}_k n_k \tau_k + \mu_0 \tau_0}{n_k \tau_k + \tau_0}, \frac{1}{n_k \tau_k + \tau_0} + \sigma_y^2\right). \end{aligned}$$

$$\bullet \text{ all other clusters combined} :$$

$$\begin{aligned} &p(c_i \neq c_k \, \forall j \neq i | \boldsymbol{c}_{-i}, \mu_0, \tau_0, \gamma, \beta, \alpha) \propto \\ &\qquad p(c_i \neq c_k \, \forall j \neq i | \boldsymbol{c}_{-i}, \alpha) \int p(\tilde{y}_i|\mu_k, \tau_k) p(\mu_k, \tau_k|\mu_0, \tau_0, \gamma, \beta) \, d\mu_k d\tau_k \\ &\qquad \propto \frac{\alpha}{n-1+\alpha} \mathcal{N}(\tilde{y}_i; \mu_0, \sigma_0^2 + \sigma_y^2). \end{aligned}$$

Algorithm 1: DPMM algorithm

Let the state of the Markov chain consist of $\boldsymbol{c} = (c_1, \cdots, c_n)$ and $\boldsymbol{\phi} = (\phi_c \ : \ c \in \{c_1, \cdots, c_n\})$. Repeatedly sample:

for $i \leftarrow 1$ **to** n **do**

- Remove y_i from cluster c_i because we are going to sample a new c_i.
- draw $c_i | c_{-i}, y$ from:

if $c = c_j$ for some $j \neq i$ **then**

$p(c_i = c | c_{-i}, y_i) \propto \frac{n_{-i,c}}{n-1+\alpha} \int \mathcal{N}(\tilde{y}_i, \phi) dH_{-i,c}(\phi)$

else

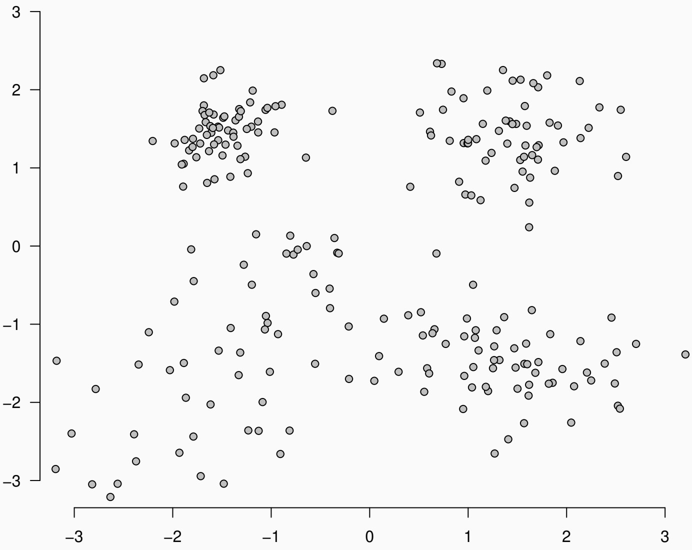
$p(c_i \neq c_j \, \forall j \neq i | c_i, y_i) \propto \frac{\alpha}{n-1+\alpha} \int \mathcal{N}(\tilde{y}_i, \phi) dG_0(\phi)$

end

return c_i

[9] Neal (2000), algorithm 3. *J Compu Graph Stats*, 9(2), 249–265.

Chinese Restaurant Process Construction


3
2
1
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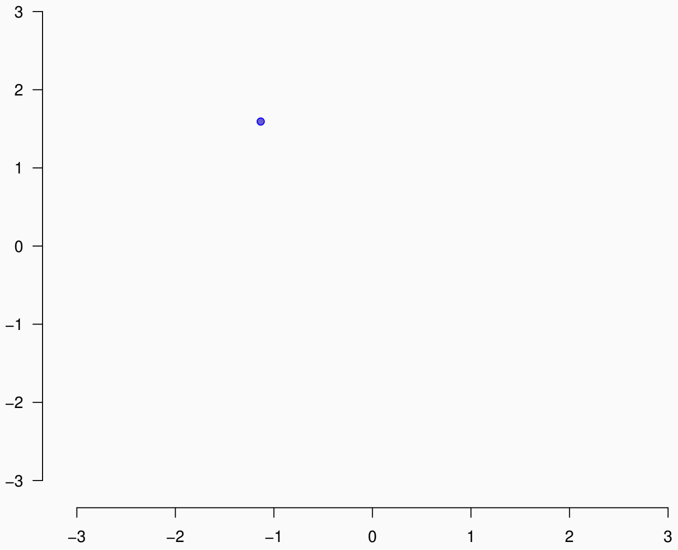
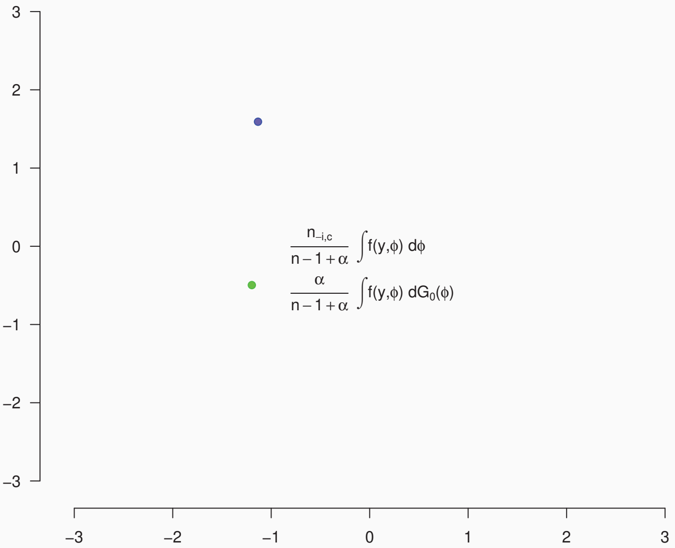
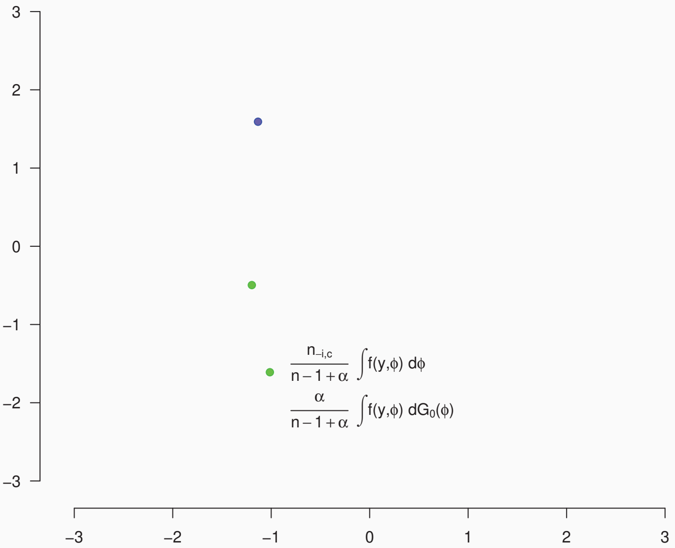
−2
−3
−3
−2
−1
0
1
2
3

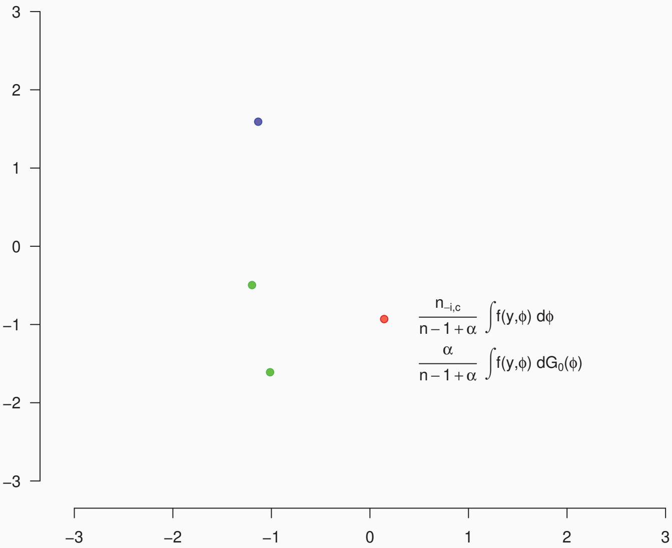


3
2
1
0
–1
–2
–3
–3
–2
–1
0
1
2
3

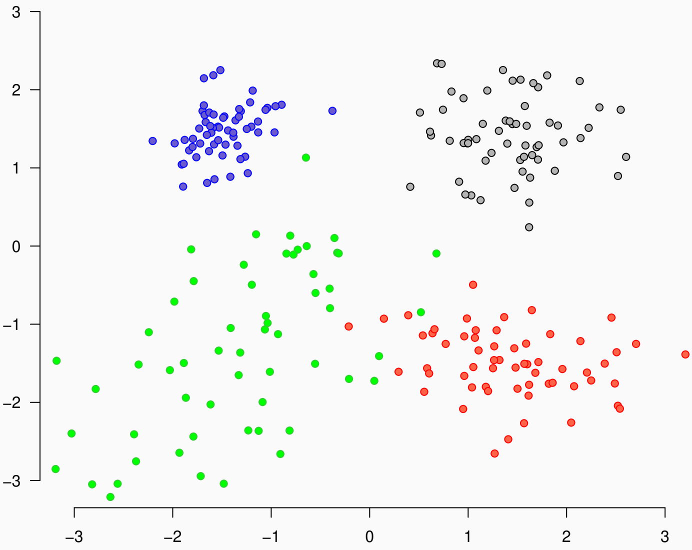

3
2
1
0
–1
–2
–3
$\frac{n_{-i,c}}{n-1+\alpha} \int f(y,\phi)\, d\phi$
$\frac{\alpha}{n-1+\alpha} \int f(y,\phi)\, dG_0(\phi)$
–3
–2
–1
0
1
2
3



3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3
$\frac{n_{-i,c}}{n-1+\alpha}\int f(y,\phi)\,d\phi$
$\frac{\alpha}{n-1+\alpha}\int f(y,\phi)\,dG_0(\phi)$



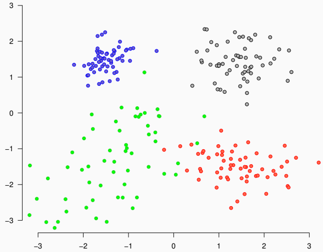
$\frac{n_{-i,c}}{n-1+\alpha} \int f(y,\phi)\, d\phi$
$\frac{\alpha}{n-1+\alpha} \int f(y,\phi)\, dG_0(\phi)$


3
2
1
0
−1
−2
−3
−3
−2
−1
0
1
2
3

R Implementation of the CRP [10]

```
....
# already occupied table, n/(N + alpha) * mvnorm() density
log_weights[c.idx] <- log(counts[c.idx]) +
      dmvnorm(data[n, ], mean = c_mean,
              sigma = c_Sig + Sig, log = TRUE)
....
# new table, alpha/(N + alpha) weighted by mvnorm() density
log_weights[Nclust + 1] <- log(alpha) +
      dmvnorm(data[n, ], mean = mu0, sigma = Sig0 + Sig,
      log = TRUE)
....
```

[10] Modified from the R program by Tamara Broderick
https://people.csail.mit.edu/tbroderick/tutorial_2016_mlss_cadiz.html



		k-means				crp_gibbs()				Python			
		$k = 4$				$k = 10, \alpha = 1/k$				$k = 10, \alpha = 1/k$			
		●	●	●	●	●	●	●	●	●	●	●	●
Estimated Cluster	1	0	0	60	13	0	60	0	8	0	0	60	1
	2	0	60	0	4	0	0	0	36	0	1	0	55
	3	0	0	0	43	0	0	60	16	0	59	0	4
	4	60	0	0	0	60	0	0	0	60	0	0	0
		$k = 4$				$k = 5, \alpha = 1/k$				$k = 5, \alpha = 1/k$			
Estimated Cluster	1					0	0	60	18	0	0	60	1
	2	same as above				0	0	0	35	60	0	0	0
	3					0	60	0	7	0	60	0	4
	4					60	0	0	0	0	0	0	55

Why BNP in PRO Research?

Why BNP in PRO Research?

- BNP offers solutions to enduring statistical challenges in the field of symptom science
    - Identification of symptom clusters (e.g., NINR Workshop; Nho et al, (2018))
- Informing personalized, patient-centered treatment decisions
    - Narrow down intervention options most likely to be successful for individual patients
    - NIH PA-18-139: Innovative Questions in Symptom Science and Genomics

Real-World PRO Examples

Individual Meaning-Centered Psychotherapy (IMCP)

- A randomized controlled trial (R01 CA128134, PI: Breitbart)
- Breitbart, Pessin, Rosenfeld, et al. (2018) *Cancer*, 124, 3231-3239
- Patients with advanced or terminal cancer were randomized
    1. Individual Meaning Centered Psychotherapy (IMCP, $n = 109$)
    2. Supportive Psychotherapy (SP, $n = 108$)
    3. Enhanced Usual Care (EUC, $n = 104$)
- Help patients develop/increase sense of meaning near end of life

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- Help patients develop/increase sense of meaning near end of life
- Psychological outcome measures
  1. Meaning Making, Hopelessness, Desire for Hastened Death, Anxiety and Depression
  2. Pre-intervention baseline, mid-intv (week 4), post-intv (week 7), and 2-months post-intv (week 15)

DP Mixtures on Baseline Psychosocial Profiles

- Before psychotherapy, do patients show different psychosocial characteristics?
- Can we cluster baseline characteristics into profiles?
- If so, do different clusters respond to IMCP differently?
- Notion of precision psychooncology
- Psychological interventions tailored to baseline profiles

Baseline Psychosocial Profiles by DPMM

- `BayesianGaussianMixture()` in Python
- Constraining $k \leq 5$ to control sparseness

Baseline psychosocial profiles identified by `BayesianGaussianMixture()`

	(n)	age	KPRS	Hopelessness	Hastened Death	Anxiety	Depression	Personal Meaning	Existential Transcendence
1	(22)	63.9	73.7	5.9	2.5	8.5	8.0	82.5	86.4
2	(17)	56.8	77.4	10.7	5.6	13.0	9.9	47.0	20.9
3	(131)	58.2	81.8	3.1	1.3	7.0	3.7	82.4	88.2
4	(10)	63.5	81.2	4.1	3.4	6.0	5.4	93.5	123.5
5	(72)	56.3	81.5	6.7	4.4	10.0	7.6	66.8	65.6

2: “Acutely Distressed” cluster

5: “Moderately Distressed” cluster

- Personal Meaning subscale scores at post-Tx

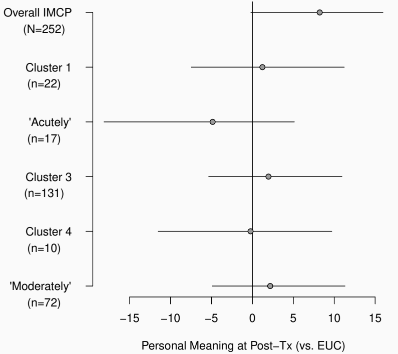
Post-Tx Personal Meaning subscale scores cf. baseline										
		Baseline		Post-Tx (week 7)						
		(N)		EUC	(n)	Meaning	(n)	Suprt	(n)
	1	(22)	82.5	78.5	6	88.8	5	83.3	11
"Acutely"	2	(17)	47.0	51.3	5	51.4	6	61.0	6
	3	(131)	82.4	81.9	46	92.0	48	87.5	37
	4	(10)	93.5	95.0	1	84.0	2	91.0	7
"Moderately"	5	(72)	66.8	71.4	16	84.1	31	75.8	25

Clusters Responded to IMCP Differently

- "random intervention effects" model [11]

$$y_{c[i]} = \beta_0 + \beta \mathrm{Tx}_{c[i]} + u_{0c} + u_{1c}\mathrm{Tx}_{c[i]} + \epsilon_{c[i]}, \qquad [u_{0c}, u_{1c}] \sim \mathcal{N}(0, \Sigma).$$

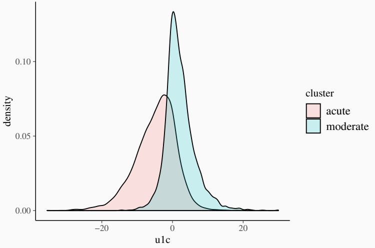
- `stan_lmer(PersMeaningT3 ~ Tx + (1 + Tx | clus), prior = NULL)`



[11] Lee & Thompson (2005), *Clin Trials*, 2(2), 163-73.

Probability of Intervention Fit

- Posterior $\Pr(u_{1c_i} > u_{1c_j} | c_i \neq c_j)$



clusters	1	2	3	4	5
1	-	0.81			
“Acutely” 2	0.19	-			
3	0.58	0.84	-		
4	0.42	0.74	0.34	-	
“Moderately” 5	0.58	0.88	0.52	0.65	-

Example 1: IMCP Summary

- Five baseline psychosocial profiles before therapy
- Most (n=131) appeared to be coping relatively well
- Few (n=17) appeared to be acutely distressed
- Some (n=72) moderately distressed but with meaning
- Heterogeneity of Treatment Effects (HTE)
- IMCP works better in moderately than acutely distressed
- Personalized interventions algorithm
    - We can derive probability of latent class, and
    - Probability of treatment response given latent class

Real-World Example 2: CHOICE Study

- A Comparative Effectiveness Trial (PCORI CER-1403-14292-IC, PI: Mao)
- Cancer survivors with Insomnia Severity Index > 7 (‘mild’), randomized
  1. Cognitive Behavioral Therapy for Insomnia (CBTI, $n = 79$)
  2. Acupuncture (Acupuncture, $n = 80$)
- Overall, Garland & Mao et al. (2019) reported that CBTI was better than acupuncture on insomnia

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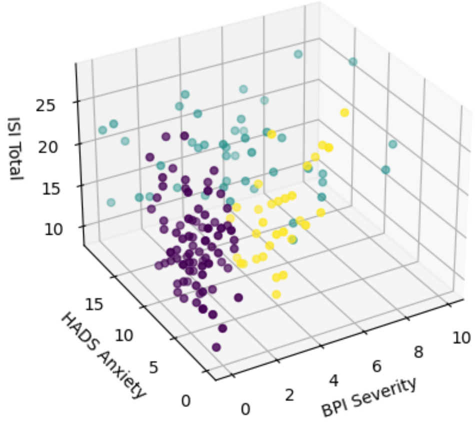
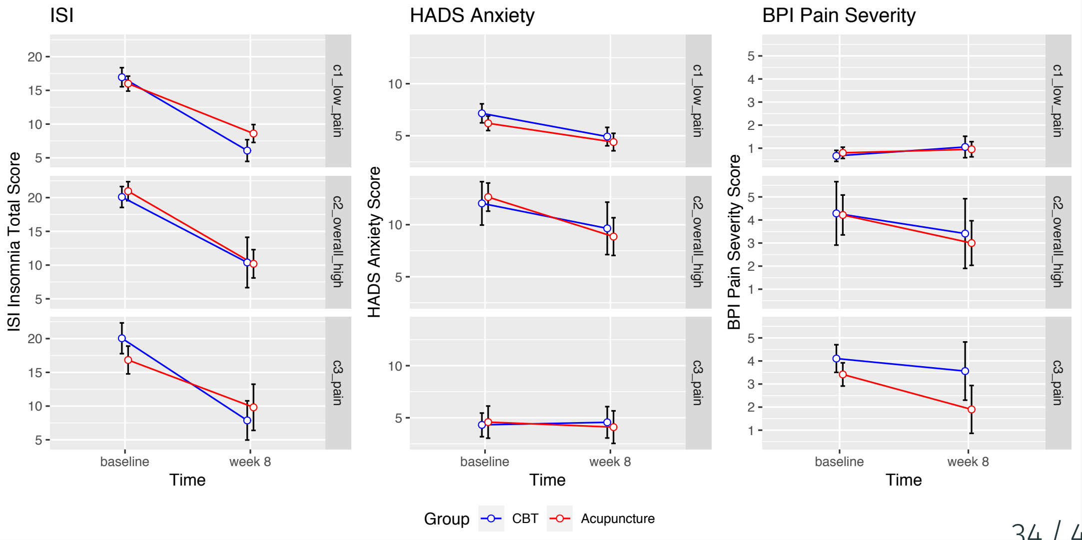
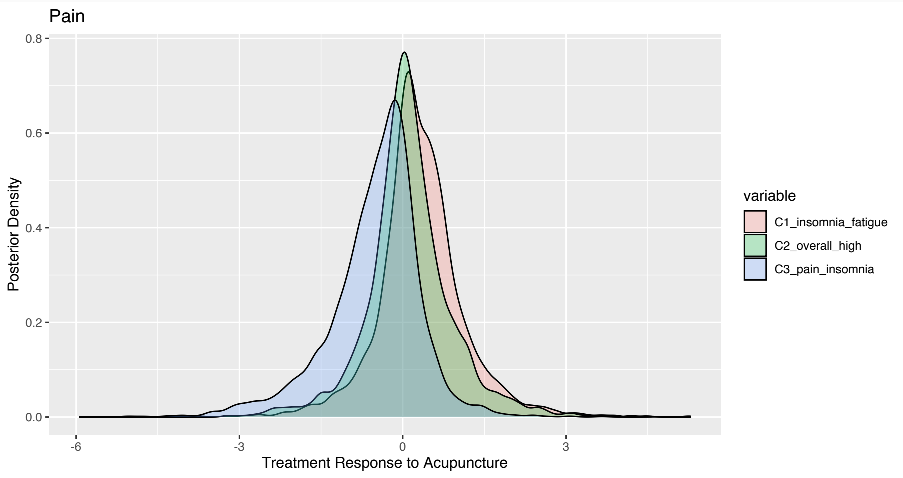
  1. Cognitive Behavioral Therapy for Insomnia (CBTI, $n = 79$)
  2. Acupuncture (Acupuncture, $n = 80$)
- Overall, Garland & Mao et al. (2019) reported that CBTI was better than acupuncture on insomnia
- However, were there symptom clusters that responded differently?
  1. We applied DPMM to baseline insomnia, fatigue, pain, and anxiety
  2. Pre-intervention baseline and post-intv (week 8)
  3. PCORI Methdology Standards: 5. Heterogeneity of Treatment Effects (HTE)


ISI Total
25
20
15
10
HADS Anxiety
15
10
5
0
BPI Severity
0
2
4
6
8
10

Which is Better and for What Symptoms?



Probability of Intervention Success



- $p(c_3 < c_1) = 0.87 \quad p(c_3 < c_2) = 0.79 \quad p(c_2 < c_1) = 0.64$

R Programs Explained Line by Line

R Programs Explained Line by Line

Key Features of BNP

Key Features of BNP

- Number of profiles should be able to grow as the model encounters more data,
- Model should introduce no more profiles than are necessary to explain the data
- Identify the subset of individuals who showed subtle and nuanced differences

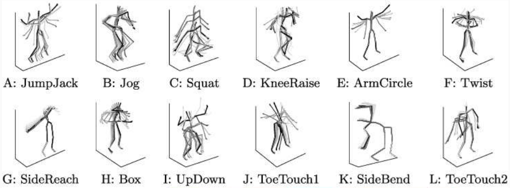
Numerous Other Applications of BNP and Future Directions (end at 10:00 CDT)

Bayesian Nonparametric (BNP) Methods

- Bayesian “nonparametric” methods?
- Not finite parameters (growing/infinite number of parameters)

Bayesian Nonparametric (BNP) Methods

- Bayesian “nonparametric” methods?
- Not finite parameters (growing/infinite number of parameters)
- Motion capture data (`https://arxiv.org/pdf/1308.4747.pdf`)



Bayesian Nonparametric (BNP) Methods

- Bayesian "nonparametric" methods?
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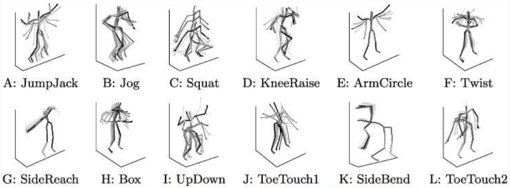


- Protein interactome data (Lloyd et al, *NIPS* 2012)

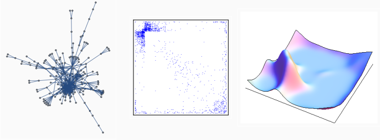
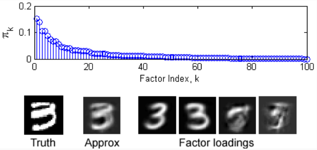


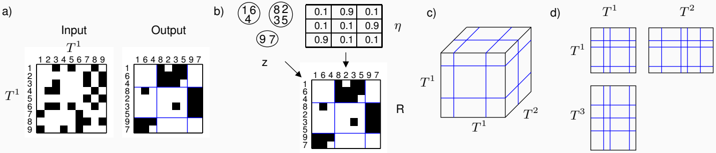
Figure 2: Protein interactome data. *Left:* Interactome network. *Middle:* Sorted adjacency matrix. The network exhibits stochastic equivalence (visible as block structure in the matrix) and homophily (concentration of points around the diagonal). *Right:* Maximum a posteriori estimate of the function Θ, corresponding to the function in Fig. 1 (middle).

Bayesian Nonparametric (BNP) Methods

- BNP as the engine for versatile analytic tools, e.g.,
  - Paisley et al. (2009): BNP factor analysis



  - Kemp et al. (2006): Infinite Relational Model



Bayesian Nonparametric (BNP) Methods

- Beyond infinite clusters, e.g.,
  - Navarro et al. (2006): web browsing habits
  - Karabatsos & Walker (2009): BNP Test Equating
  - Tanenbaum et al. (2011): “How to Grow a Mind”
  - Austerweil & Griffiths (2013): feature learning/representation
  - De Iorio et al. (2009) Biometrics, 65, 762–771.
  - Graziani et al. (2015) Biometrics, 71, 188–197.

Vast Literature on BNP

- Countless other points on BNP not covered here
- But I hope I have given you a fundamental intuition on BNP using the DP
- Make you feel more confident and efficient in learning more on your own
- Potentially rediscover finer details of BNP yourself

Thanks

- Funding
  - NIH P30 CA008748 for MSKCC
  - NIH R01 CA128134 (PI: Breitbart)
- DP mixture clustering computer program in R
  - Tamara Broderick at MIT[12]

[12] https://people.csail.mit.edu/tbroderick/tutorial_2016_mlss_cadiz.html