24 YR 9 PATH MATHS: LESSON OUTLINE: INDICES A-C (MATHSPACE – M.S)

**Instructions: Every lesson: Remember to work off "Lesson Instructions" you glued in the front of your book (also on GC).**

1. **Mathspace :** Complete "Check-in" date check-in's table on Lesson Outline.
2. **Copy ALL the "Theory Notes"** supplied by Teacher in class.
3. **Mathspace Lesson Activity**: Read through the "Lesson" Add any additional notes to your "Theory Notes" from the class.
4. **Practical Exercises**:
   - Write Heading for Exercise with date.
   - Complete set work showing ALL working out in your Exercise Book.
   - Any work not completed in class AUTOMATICALLY becomes homework.
   - When finished, entering answers into M.S and marking your answers in your exercise book.
   - Upload working out on G.C. This is considered part of homework.

**MATHSPACE/PRACTICAL EXERCISE: MATHSPACE (M.S) ACTIVITY: Set by teacher: Custom Task (C.T), Adaptive Task (A.T), Worksheet (W.S) ALL FROM YEAR 9 2024 TEXTBOOK**

First lesson: Draw up a table with a small column called Word and larger column called Definition: This is your Definition Table (**D.T**). You are to add the definitions of words from LESSONS throughout the topic.

| Subtopics | DATE | LESSON | MATHSPACE/PRACTICAL EX: (M.S) ACTIVITY: Set by teacher | M.S C.T/WS |
|---|---|---|---|---|
| | | **INDICES A: CORE** | | |
| Multiplication law | | • Complete "Check-in" (record date on table below)<br>• Read "Lesson 1.01C Multiplication law "& complete the practice questions.<br>• **ADD:** Definitions to D.T | ❖Complete **M.S**: "Activity: 1.01C Multiplication law". Remember follow instructions above for "**4. Practical Exercise".** | |
| Division Law | | • Complete "Check-in"<br>• Read "Lesson 1.02C Division Law " | ❖Complete **M.S**: "Activity: 1.02C Division Law". | |
| Power of a power law | | • Complete "Check-in"<br>• Read "Lesson 1.03C Power of a power law | ❖Complete **M.S**: "Activity: 1.03C Power of a power law". | |
| Zero and negative indices with numeric bases | | • Complete "Check-in"<br>• Read "Lesson 1.04C Zero and negative indices with numeric bases " | ❖Complete **M.S**: "Activity: 1.04C Zero and negative indices with numeric bases". | |
| | | • Complete "Check-in"<br>• Read "Lesson (YR 10) 2.05 Negative indices " | ❖Complete **M.S**: "Activity: (YR 10) 2.05 Negative indices". | |
| | | **INDICES B: CORE** | | |
| Index laws with algebraic bases | | • Complete "Check-in" Read "Lesson 1.05C/P Index laws with algebraic bases " | ❖Complete **M.S**: "Activity: 1.05C/P Index laws with algebraic bases". | |
| | | **INDICES C: PATH** | | |
| Irrational numbers/ simplifying surds | | • Complete "Check-in" (record date on table below)<br>• Read "Lesson 2.01P Simplifying surds " | ❖Complete **M.S**: "Activity: 2.01P Simplifying surds". | |
| Add & subtract surds | | • Complete "Check-in"<br>• Read "Lesson 2.02P Add & subtract surds | ❖Complete **M.S**: "Activity: 2.02P Add & subtract surds". | |
| • Multiply & divide surds | | • Complete "Check-in" Read "Lesson 2.03P Multiply & divide surds " | ❖Complete **M.S**: "Activity: 2.03P Multiply & divide surds". | |
| • Binomial expansions with surds | • | • Complete "Check-in"<br>• Read "Lesson 2.04P Binomial expansions with surds " | ❖Complete **M.S**: "Activity: 2.04P Binomial expansions with surds". | |
| Rationalise the denominator | • | • Read "Lesson 2.05P Rationalise the denominator | ❖Complete **M.S**: "Activity: 2.05P Rationalise the denominator". | |

REMEMBER:

- Pre-set Mathspace is to be continually completed throughout the topic.
- Work not completed in class **automatically** becomes homework.
- Challenge yourself by completing other topic tasks from Mathspace when finished set work.
- Always date your work in your Exercise Book with ALL working out & your Lesson Outline.

**YR 9 STAGE 5 SKILLS COMPLETE "CHECK-IN" (10 Q's per check-in).**

| TOPIC | First Topic Check – ins | SUB-TOPICS | Additional Sub-topic Check-in date | **Understanding** E- excellent G- Good S- Satisfactory X – need extra help |
|---|---|---|---|---|
| **Algebra** 2 Check - ins | Check: Date: __ Check: Date: __ | Patterns & Algebra | | |
| | | Indices | | |
| | | Algebraic Techniques | | |

## TOPIC: INDICES A

**Outcomes: A student:**

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- simplifies algebraic expressions involving positive-integer and zero indices, and establishes the meaning of negative indices for numerical bases **MA5-IND-C-01**

**Content:** ***Extend and apply the index laws to variables, using positive-integer indices and the zero index***

- Apply the index laws for numerical bases with positive-integer indices to develop the index laws in algebraic form

**Example(s):**

Generalising numerical expressions, such as $2^2 \times 2^3 = 2^{2+3} = 2^5$ to $a^m \times a^n = a^{m+n}$.

Generalising numerical expressions, such as $2^5 \div 2^3 = 2^{5-3} = 2^2$ to $a^m \div a^n = a^{m-n}$.

- Generalising numerical expressions, such as $(2^2)^3 = 2^{2\times 3} = 2^6$ to $(a^m)^n = a^{mn}$.
- Establish that $x^0 = 1$ algebraically using index laws

**Example(s):**

By considering $\frac{x^4}{x^4} = x^{4-4} = x^0$ and $\frac{x^4}{x^4} = \frac{x\times x\times x\times x}{x\times x\times x\times x} = 1$, establish that $x^0 = 1$.

- Simplify algebraic expressions that involve the zero index

**Example(s):**

- Recognising and explaining that $7m^0 = 7 \times 1 = 7$ and $(3x)^0 + 5 = 6$.

**Content:** ***Simplify algebraic products and quotients using index laws***

- Simplify algebraic expressions that involve powers, products and quotients of simple algebraic terms containing positive-integer indices

**Example(s):** Simplifying $(3x^2)^3 = 27x^6$ and $15a^6 \div 3a^2 = 5a^4$.

**Content:** ***Apply index laws to numerical expressions with negative-integer indices***

- Apply index notation, patterns and index laws to establish the meaning of negative indices for numerical bases

**Example(s):**

Using numerical patterns:

| $3^3$ | $3^2$ | $3^1$ | $3^0$ | $3^{-1}$ | $3^{-2}$ |
|---|---|---|---|---|---|
| 27 | 9 | 3 | 1 | $\frac{1}{3}$ | $\frac{1}{3^2} = \frac{1}{9}$ |

*Image long description*: 6 columns in a table showing descending powers with a base of 3 on the top row and their calculations on the bottom row. On the top row, 3 cubed, 3 squared, 3 to the power of 1, 3 to the power of 0, 3 to the power of minus 1 and 3 to the power of minus 2. The bottom row is 27, 9, 3, 1, one-third, and 1 over 3 squared equals 1 over 9.

Applying index laws: $4^3 \div 4^5 = 4^{-2}$ and $\frac{4^3}{4^5} = \frac{1}{4^2}$ to establish that $4^{-2} = \frac{1}{4^2}$.

- Evaluate numerical expressions involving a negative index by first representing them with a positive index

**Example(s):** Evaluating $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$.

- Represent given numbers in index form (integer indices and bases only) and vice versa

**Example(s):** Representing $\frac{1}{125}$ as $\frac{1}{5^3}$ and $5^{-3}$.

## *TOPIC: INDICES B*

**Outcomes: A student:**

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- applies the index laws to operate with algebraic expressions involving negative-integer indices *(Path: Adv)* **MA5-IND-P-01**

***Content:*** ***Apply index laws to algebraic expressions involving negative-integer indices***

- Apply index notation, patterns and index laws to establish $a^{-1} = \frac{1}{a}$, $a^{-2} = \frac{1}{a^2}$, $a^{-3} = \frac{1}{a^3}$ and $a^{-n} = \frac{1}{a^n}$

**Example(s):** Explaining the difference between $x^{-2}$, $-x^2$ and $-2x$ by first examining the differences numerically using a calculator.

- Represent expressions involving negative-integer indices as expressions involving positive-integer indices and vice versa

**Example(s):** Examining expressions such as $x^{-3} = \frac{1}{x^3}$ , $2x^{-1} = \frac{2}{x^1} = \frac{2}{x}$ , $\frac{4}{x} = \frac{4}{x^1} = 4x^{-1}$ and $\frac{1}{x^2} = x^{-2}$.

- Apply the index laws to simplify algebraic products and quotients involving negative-integer indices

**Example(s):** Justifying why the following statements of equality are true or false:

$$\frac{9x^5}{3x^{-5}} = 3x^{10}$$

$$9x^{-7} \div 3x^5 = 3x^2$$

$$a^{-5} \times a^3 = a^8$$
$$a^5 \times a^{-7} = a^{-2}.$$

- Describe and use $x^{-1}$ as the reciprocal of $x$ and generalise this relationship to expressions of the form $\left(\frac{a}{b}\right)^{-1}$
- Use knowledge of the reciprocal to simplify expressions of the form $\left(\frac{a}{b}\right)^{-n}$

**Example(s):** Simplifying $\left(\frac{2x^2}{3}\right)^{-2}$ and expressing without a negative index.

## *TOPIC: INDICES C*

**Outcomes: A student:**

- develops understanding and fluency in mathematics through exploring and connecting mathematical concepts, choosing and applying mathematical techniques to solve problems, and communicating their thinking and reasoning coherently and clearly **MAO-WM-01**
- describes and performs operations with surds and fractional indices *(Path: Adv)* **MA5-IND-P-02**

***Content:*** ***Describe surds***

- Describe a real number as a number that can be represented by a point on the number line
- Examine the differences between rational and irrational numbers and recognise that all rational and irrational numbers are real

- Convert between recurring decimals and their fractional form using digital tools
- Describe the term *surd* as referring to irrational expressions of the form $\sqrt[n]{x}$ where $x$ is a rational number and $n$ is an integer such that $n \geq 2$, and $x > 0$ when $n$ is even
- Recognise that a surd is an exact value that can be approximated by a rounded decimal
- Demonstrate that $\sqrt{x}$ is undefined for $x < 0$ and that $\sqrt{x} = 0$ when $x = 0$ using digital tools

**Example:** Explaining why an error occurs when attempting to find a value of $\sqrt{-4}$ as opposed to $\sqrt{4}$.

- Describe $\sqrt{x}$ as the positive square root of $x$ for $x > 0$ and $\sqrt{0} = 0$

**Example:** Reasoning why the negative solution of the relevant quadratic equation is not feasible when solving problems involving Pythagoras' theorem.

**Content**: **Apply knowledge of surds to solve problems**

* Establish and apply the following results for $x > 0$ and $y > 0$:
$\left(\sqrt{x}\right)^2 = x = \sqrt{x^2}$, $\sqrt{xy} = \sqrt{x} \times \sqrt{y}$ and $\sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}}$

- Apply the 4 operations to simplify expressions involving surds

**Example(s):** Demonstrating why $\sqrt{6} + \sqrt{8}$ and $\sqrt{14}$ are different.

- Expand and simplify expressions involving surds

**Example(s):**

Expanding and simplifying: $\left(\sqrt{15} + 2\sqrt{3}\right)\left(5\sqrt{8} - \sqrt{10}\right)$

$$= 5\sqrt{120} - \sqrt{150} + 10\sqrt{24} - 2\sqrt{30}$$

$$= 5\sqrt{4 \times 30} - \sqrt{25 \times 6} + 10\sqrt{4 \times 6} - 2\sqrt{30}$$

$$= 10\sqrt{30} - 5\sqrt{6} + 20\sqrt{6} - 2\sqrt{30}$$
$$= 8\sqrt{30} + 15\sqrt{6}.$$

- Rationalise the denominators of surds of the form $\frac{a\sqrt{b}}{c\sqrt{d}}$

**Example:** Rationalising the denominator of $\frac{2\sqrt{3}}{3\sqrt{5}}$ by multiplying the numerator and denominator by $\sqrt{5}$.

**Content**: **Describe and use fractional indices**

- Apply index laws to describe fractional indices as: $a^{\frac{1}{n}} = \sqrt[n]{a}$ and $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$

**Example(s):** Explaining why finding the square root of an expression is the same as raising the expression to the power of a half by considering that if $\left(\sqrt{9}\right)^2 = 9$ and $\left(9^{\frac{1}{2}}\right)^2 = 9$, then $\sqrt{9} = 9^{\frac{1}{2}}$.

- Translate expressions in surd form to expressions in index form and vice versa

Evaluate numerical expressions involving fractional indices including using digital tools