MATHSPACE

CUSTOM TASKS

YEAR: 9

TOPIC:

INDICES A, B & C

Understanding

1

a Using the expanded form, $3^5 = \square \times \square \times \square \times \square \times \square$

b Using the expanded form, $3^4 = \square \times \square \times \square \times \square$

c

$$3^5 \times 3^4 = (\square \times \square \times \square \times \square \times \square) \times (\square \times \square \times \square \times \square)$$

$$= 3^{\square}$$

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d Complete the statement: When you multiply terms that have the same bases you $\square$ the indices.

2

a Write 5^3 in expanded form.

b Write $2 \times 2 \times 2 \times 2$ in index form.

c Write a^5 in expanded form.

d Write $b \times b \times b \times b \times b \times b$ in index form.

3 Trent stated that $2^2 + 3^2$ is equal to 5^2.

a Use expanded form for each term to explain why Trent is incorrect.

b Is there a simplification that can be made for $2^2 + 3^2$ using indices?

Fluency

4 Simplify:

a $2^3 \times 2^4$	b $3^5 \times 3^3$	c 2×2^5	d $5^4 \times 5$
e $3^2 \times 3^2$	f $2^{10} \times 2^5$	g $3^4 \times 3^5$	h $7^3 \times 7^7$
i $3^x \times 3^y$	j $2^a \times 2^b$	k $3^c \times 3^d$	l $7^u \times 7^v$
m $(-2)^3 \times (-2)^5$	n $(-3)^a \times (-3)^7$	o $(-2)^x \times (-2)^y$	p $(-9)^u \times (-9)^3$
q $2^3 \times 2^5 \times 2$	r $3^2 \times 3 \times 3^5$	s $7^x \times 7^y \times 7^z$	t $5^u \times 5^3 \times 5^u$

Example 1a

5 Simplify:

a $y^3 \times y^2$	b $x^6 \times x^3$	c $y^6 \times y$	d $a^7 \times a^5$
e $a^4 \times a$	f $b^7 \times b^6$	g $y^5 \times y^3$	h $u^3 \times u$

i $b^6 \times b^7$ j $x^y \times x^z$ k $a^b \times a^c$ l $x^a \times x^b \times x^c$

m $a^6 \times a^{-3}$ n $x^{-5} \times x^8$ o $d^7 \times d^{-4}$ p $e^{-10} \times e^8$

Example 1b

6 Simplify:

a $5y^2 \times y^3$ b $y^3 \times 3y$ c $2x^5 \times 2x^4$ d $3a^4 \times 2a^6$

e $-b^7 \times -b^5$ f $7a^4 \times 3a^2$ g $u^3 \times u$ h $5y^2 \times 3y^2$

Example 1c

7 Simplify:

a $3x^2y^2 \times 2xy^3$ b $5c^5d^4 \times 4c^2d^3$

c $7a^3b^4 \times 2ab^5$ d $-2u^4v^2 \times 3u^6v^9$

e $-3t^6u^3 \times -4t^5u^3$ f $5a^3b^2c^4 \times -5a^2b^4c^6$

g $5w^5x^3y^9 \times -3wx^5y$ h $3abc \times -3a^2b^3c^4 \times -3a^2bc^3$

Example 1d

Reasoning

8 Explain why the expression $a^4 \times b^4$ cannot be simplified using the multiplication law.

9 Marvin is asked to solve the expression $(b^2) \times (b^3) \times (b^7)$. He claims the result is b^{12}. Is Marvin answer correct? Justify your answer.

10

a Are $(-2)^3$ and -2^3 equal? Explain.

b Are $(-2)^4$ and -2^4 equal? Explain.

c Generalise a rule for when a negative is raised to an odd or even power.

Problem solving

11 Fill in the missing number:

a $a^2 \times a^{\square} = a^{11}$ b $c^{\square} \times c^7 = c^8$

c $\square^5 \times \square^6 = b^{11}$ d $2a^2b^{\square} \times \square a^{\square}b = 6a^4b^3$

e $3mc^{\square} \times \square bm^{\square} \times 2b^{\square}c^3 = -6b^4c^5m^7$

f $\frac{1}{2}a^{\square}b^2c^4 \times \square a^{\square}b^{\square}c^{\square} = \frac{1}{4}a^2b^3c^6$

12 A rectangle has a length a^2 cm and width a^3 cm. Write an expression for the area of the rectangle.

13 If $2^x \times 2^y = 2^{10}$, $x \geq y$.

a How many positive integer values of x and y satisfy the inequality?

b Why can we not list the possible solutions if we allow negative integers?

Worksheet 1.02C - Division Law

Understanding

1 Complete the expression:

$$2^7 \div 2^4 = \frac{\square \times \square \times \square \times \square \times \square \times \square \times \square}{\square \times \square \times \square \times \square}$$

$$= 2^{\square}$$

2 Is each statement true or false?

a $a^m \div a^n = a^{m \div n}$ b $a^m \div b^n = a^{m-n}$ c $a^m \div a^n = a^{m-n}$ d $\dfrac{a^m}{a^n} = a^{m-n}$

e $\dfrac{(-2)^8}{(-2)^5} = (-2)^3$ f $\dfrac{5^7}{2^7} = 3^7$ g $\dfrac{30x^8}{5x^2} = 6x^6$ h $\dfrac{10y^6}{3y^4} = 7x^2$

3 Evaluate:

a 5^2 b 4^3 c $(-3)^3$ d $(-2)^6$

e $5(-1)^4$ f $\left(\dfrac{2}{5}\right)^3$ g $\left(-\dfrac{3}{4}\right)^2$ h $6\left(\dfrac{1}{3}\right)^3$

Fluency

4 Simplify:

a $3^{10} \div 3^3$ b $7^7 \div 7$

c $9^{20} \div 9^9$ d $\dfrac{3^7}{3^5}$

e $\dfrac{6^{15}}{6^7}$ f $\dfrac{2^{12}}{2^6}$

g $(-5)^8 \div (-5)^2$ h $(-2)^6 \div (-2)^4$

i $(-8)^{10} \div (-8)^3$ j $\left(\dfrac{1}{2}\right)^4 \div \left(\dfrac{1}{2}\right)$

k $\left(\dfrac{6}{7}\right)^{11} \div \left(\dfrac{6}{7}\right)^3$ l $\left(\dfrac{3}{2}\right)^6 \div \left(\dfrac{3}{2}\right)^5$

m $\left(-\dfrac{5}{9}\right)^{15} \div \left(-\dfrac{5}{9}\right)^7$ n $3^a \div 3^b$

o $\dfrac{5^w}{5^x}$ p $\dfrac{2^y}{2^z}$

Example 1a

5 Simplify and evaluate as a basic numeral:

a $5^{13} \div 5^{12}$ b $2^9 \div 2^5$ c $4^8 \div 4^5$ d $(-2)^{11} \div (-2)^{10}$

e $\dfrac{3^{22}}{3^{20}}$ f $\dfrac{(-1)^{16}}{(-1)^{11}}$ g $\dfrac{(-2)^{18}}{(-2)^{13}}$ h $\left(\dfrac{2}{3}\right)^7 \div \left(\dfrac{2}{3}\right)^5$

6 Simplify:

a $a^9 \div a^5$ b $\dfrac{x^{11}}{x^9}$ c $\dfrac{y^{15}}{y^9}$ d $b^6 \div b^2$

e $\dfrac{j^9}{j^2}$ f $x^7 \div x^4$ g $\dfrac{f^8}{f^5}$ h $s^{13} \div s^4$

i $\dfrac{p^{13}}{p^{10}}$ j $\dfrac{h^{10}}{h}$ k $c^7 \div c^2$ l $d^{20} \div d^{14}$

Example 1b

7 Simplify:

a $8b^{10} \div 2b^6$ b $\dfrac{27y^{12}}{9y}$ c $\dfrac{6m^{15}}{m^4}$ d $36c^8 \div 9c^3$

e $\dfrac{25b^{11}}{5b^3}$ f $30y^{11} \div 6y^8$ g $\dfrac{28k^{13}}{7k^8}$ h $32t^{13} \div 4t^{11}$

i $\dfrac{4k^{11}}{2k^6}$ j $\dfrac{8g^{12}}{g^5}$ k $6p^{13} \div 2p^{10}$ l $15l^8 \div 3l^2$

Example 1c

8 Simplify:

a $\dfrac{x^7 \times x^4}{x^5}$ b $\dfrac{y^{10} \times y^5}{y^9}$ c $\dfrac{t^9 \times t^9}{t^{13}}$ d $\dfrac{k^6 \times k^{10}}{k^2}$

e $\dfrac{a^{10}}{a^2 \times a^3}$ f $\dfrac{b^{16}}{b^9 \times b^2}$ g $\dfrac{u^9 \times u^7}{u^8 \times u^3}$ h $\dfrac{w^7 \times w^5}{w^6 \times w^4}$

Example 1d

9 Simplify:

a $f^4 g^{10} \div f^2 g^2$ b $\dfrac{c^5 d^{16}}{cd^9}$ c $m^8 p^{13} \div mp^3$ d $\dfrac{w^3 v^{14}}{w^2 v^7}$

e $40h^{11} j^9 \div 8h^7 j^6$ f $\dfrac{12x^7 y^5}{4x^6 y^4}$ g $\dfrac{100r^4 s^{15}}{5rs^{12}}$ h $\dfrac{52k^9 l^{11}}{13l^2 k^5}$

Example 1e

10 Simplify:

a $\dfrac{x^5y \times y^3}{x^2}$ b $\dfrac{a^3b^5 \times ab^2}{ab^4}$ c $\dfrac{j^2t^9}{jt^3 \times t^5}$ d $\dfrac{k^{10}m^6}{k^2m \times km}$

e $\dfrac{n^2p \times n^3p^2}{p^2}$ f $\dfrac{cd^4 \times c^5}{c^3d^2}$ g $\dfrac{a^2z^9 \times a^4z}{z^3 \times a^3z^5}$ h $\dfrac{q^6w^3}{q^2} \times \dfrac{q^2w^5}{qw^2}$

Reasoning

11 Jane simplifies the expression $\dfrac{3d^{12}}{15d^6}$ and gets $5d^2$ as the result. Is Jane's solution correct? Explain your reasoning.

12 Explain why $m^5 \div z^3$ is not equal to $\left(\dfrac{m}{z}\right)^2$.

13 Aidan says we can't evaluate $\dfrac{(-4)^2}{5^3}$. Becky says we can. Who is correct? Explain.

14 If $m^6 = m \times m \times m \times m \times m \times m$ then show using the division law why $m^0 = 1$.

Problem solving

15 Complete these statements:

a $b^9 \div b^{\square} = b^5$ b $x^{\square} \div x^3 = x^2$

c $\dfrac{g^{\square}}{g^4} = g^7$ d $\dfrac{k^7}{k^{\square}} = k^3$

e $\dfrac{m^9}{m^{\square}} = {\square}^7$ f $\dfrac{{\square}a^{\square}b^{\square}}{2a^4} = \dfrac{2b}{a^2}$

g $\dfrac{15m^3p^{\square}}{{\square}m^{\square}p} = m^2p$ h $\dfrac{{\square}f^8g^{\square}k^{12}}{8f^8g^3k^{\square}} = 3gk^5$

16 Using digits 1 to 9 at most once, complete the equation.

$$\frac{2^{\square} \times 2^{\square}}{2^{\square}} = 2^{\square} = 8$$

17 A rectangle has an area of $8a^6$ cm^2. If one of the sides is of length $2a^2$ cm, find an expression for the other side.

18 Simplify $\dfrac{3x^4y \times 8y^2}{12y^7}$

19 Write 3 different expressions using the division law that are all equivalent to:

a $4g^4$

b $16a^2b^{12}c^3$

Understanding

1

a Are the expressions equivalent to $(p^3)^2$?

i $(p \times p \times p) \times (p \times p \times p)$

ii $p^3 \times p^3$

iii $(p \times p \times p)^2$

iv $p^3 \times p^2$

v $(p \times p \times p) \times (p \times p)$

b True or false:

i $(p^3)^2 = p^{3+2}$

ii $(p^3)^2 = p^{3\times 2}$

c Complete the equation:

$$(p^3)^2 = p^{\square}$$

d Complete the equation:

$$(p^m)^n = p^{\square}$$

Fluency

2 Simplify:

a $(2^3)^7$ b $(3^4)^4$ c $(7^2)^9$ d $(5^3)^2$

e $(9^8)^3$ f $(11^5)^5$ g $(2^{10})^3$ h $(5^{12})^5$

i $(7^7)^6$ j $((-3)^9)^3$ k $((-2)^7)^4$ l $(13^6)^5$

Example 1a

3 Simplify:

a $(j^3)^6$ b $(w^2)^4$ c $(t^4)^3$ d $(a^7)^3$

e $(h^7)^6$ f $(p^2)^9$ g $(a^5)^5$ h $(b^4)^7$

i $(c^{10})^3$ j $(e^{11})^4$ k $(g^x)^y$ l $(w^a)^b$

Example 1b

4 Simplify:

a $(ab^2)^3$ b $(wx^3)^4$ c $(t^2u^3)^5$ d $(a^7b^2)^4$

e $(2h^5)^2$ f $(3a^2)^3$ g $(5c^7)^2$ h $(ab^4c^3)^5$

i $\left(\frac{j^5}{2}\right)^3$ j $\left(\frac{3}{h^7}\right)^2$ k $\left(\frac{a^4}{b^5}\right)^7$ l $\left(\frac{2x^3}{3y^4}\right)^2$

m $(-2a^3)^5$ n $(-3a^2)^3$ o $\left(\frac{-2g^3}{-3h^5}\right)^2$ p $\left(\frac{3w^3}{-5x^4}\right)^3$

Example 1c, 1d

5 Simplify:

a $(2^5)^3 \times 2^7$ b $(3^2)^4 \times (3^3)^2$ c $(a^3)^5 \times (a^7)^4$ d $(x^2)^{10} \div (x^4)^2$

e $\frac{(y^2)^4 \times y^3}{(y^3)^2}$ f $\frac{(f^7)^3 \times (f^2)^2}{(f^3)^2}$ g $\frac{2t^4 \times (3t^3)^3}{6t^2}$ h $\frac{(2u^2)^5}{(2u^5)^4 \times (u^5)^5}$

Example 1e

Reasoning

6 Is $(w^2)^3 = (w^3)^2$? Explain.

7 Find a value of x so that $a^x \times a^x = (a^x)^x$. Explain.

8 A student simplifies the expression $\frac{(z^4)^3}{z^3}$ and gets z^4 as the result. Explain the student's mistake. What should the correct simplification be?

Problem solving

9 Complete each statement:

a $(abc)^{\square} = a^3b^3c^3$ b $(\square m^{\square}n^{\square})^3 = 8m^6n^9$

c $\left(\frac{1}{4}a^3m^4\right)^{\square} = \frac{1}{16}a^6m^8$ d $\left(\frac{\square j^{\square}}{k^2}\right)^3 = \frac{8j^{15}}{k^6}$

e $\left(\frac{v^{\square}}{m^4}\right)^{\square} = \frac{v^{18}}{m^{24}}$ f $(4m^2)^{\square} \times (\square b^{\square})^2 = 256m^6b^4$

g $(0.1a^{\square})^2 \times (10b^4)^{\square} = a^6b^4$ h $\left[(2a^{\square})^3\right]^2 = 64a^{12}$

10 Write $(16^p)^4$ in the form a^b, where a is a prime number.

11 Write $\frac{27^a \times 25^b}{9^c \times 125^d}$ as a product of prime numbers in index form.

12 If the side length of a cube is $5a^4$, what is the volume?

Worksheet 1.04C Zero and Negative Indices with numerical bases.

Understanding

1 Look at the pattern:

$$2^3 = 8$$
$$2^2 = 4$$
$$2^1 = 2$$
$$2^0 = \square$$
$$2^{-1} = \square$$

a Complete the sentence:

Each time the power of 2 decreases by 1, the number on the right is divided by $\square$.

b Complete the pattern.

2 Identify the base and the power.

a 10^{-7} b 2^{-4} c 13^{-10} d $(-5)^{-8}$

3 When you have a fraction raised to a negative index, what happens?

A The fraction gets inverted. **B** The fraction becomes a decimal.

C The numerator and denominator both become negative.

D The fraction becomes zero.

4 Is 2^{-3} less than or greater than 1?

Fluency

5 Evaluate:

a 6^0 b $3^3 \times 3^0$ c $7^2 \div 7^2$ d $(4 \times 15)^0$

e $(-3)^0$ f -4^0 g $\left(\frac{2}{3}\right)^0$ h $7\,(14 \times 18)^0$

i -100^0 j $-9\,(12 \times 15)^0$ k $2^5 \times 11^0$ l $\frac{(10^5)^3}{10^{15}}$

m $\frac{13^{10}}{(13^5)^2}$ n $\frac{9^7}{9^2 \times 9^5}$ o $\frac{15^4 \times 15^2}{15^6}$ p $\frac{(14^3)^4}{14^4 \times 14^8}$

Example 1

6 Complete:

a $\frac{6^8}{6^8} = 6^{\square}$ b $\frac{7^{\square}}{7^7} = 7^0$ c $\frac{4^{10}}{4^{\square}} = 4^0$ d $7^{\square} = 1$

7 Complete the table:

a

2^5	2^4	2^3	2^2	2^1	2^0	2^{-1}
32	16					

b

10^5	10^4	10^3	10^2	10^1	10^0	10^{-1}
100 000	10 000					

c

3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}
27	9					

8 Evaluate:

a $2^2 + 2 + 2^0$ b $7 \times 3^0 + 3^2$ c $\left(2 + 2^0\right) \times 2^0$ d $5 \times 5^0 - 3 \times 2^0$

e $4 \times 3^{-2} + 8^0$ f $6 \div 2^{-1} + 4$ g $\left(2 + 3^{-2}\right) \times 2^{-1}$ h $\left(\frac{2}{3}\right)^{-3} \div 6^{-1}$

Example 2

9 Complete:

a $\frac{1}{3^2} = 3^{\square}$ b $\frac{1}{6^5} = 6^{\square}$ c $\frac{1}{27} = 3^{\square}$ d $\frac{1}{5^3} = 5^{\square}$

e $\frac{1}{7^{11}} = 7^{\square}$ f $-\frac{1}{64} = -4^{\square}$ g $-\frac{1}{9^4} = -9^{\square}$ h $-\frac{1}{32} = -2^{\square}$

Example 3

10 Express with a negative index:

a $\frac{1}{3}$ b $\frac{1}{37}$ c $\frac{1}{5}$ d $\frac{1}{4^7}$

e $\frac{1}{-15^3}$ f $\frac{1}{10^5}$ g $\frac{1}{(-24)^{10}}$ h $\frac{1}{25^3}$

i $\frac{1}{13^{11}}$ j $-\frac{1}{7^8}$ k $\frac{1}{16^{12}}$ l $\frac{1}{(-45)^7}$

11 Express with a positive index:

a 8^{-15} b 73^{-14} c $(-9)^{-7}$ d 9^{-1}

e 17^{-6} f 55^{-1} g $(-12)^{-8}$ h -45^{-5}

i -8^{-11} j $(-20)^{-3}$ k 7^{-6} l $(-5)^{-1}$

Example 4

12 Simplify, leaving your answer in index form:

a $5^{11} \div 5^{-3}$ b $7^{-7} \div 7^5$ c $7^{-3} \times 7^{-4}$ d $5^{-4} \div 5^{-9}$

e $9^0 \times 9^{-12}$ f $5^{13} \div 5^{-9}$ g $10^2 \div 10^3$ h $100^{25} \div 100^{26}$

i $\frac{2^3}{2^5}$ j $\frac{\left(5^2\right)^9 \times 5^6}{5^{40}}$ k $\frac{\left(19^2\right)^3}{19^{-3} \times 19^{-9}}$ l $\frac{-\left(8^5\right)^2}{8^{-2} \times 8^{-12}}$

13 Evaluate:

a $6^7 \times 6^{-7}$ b $4^8 \times 4^{-6}$ c $3^{-8} \times 3^{11}$ d $5^5 \times 5^{-7}$

e $4^{-4} \times 2^{-4}$ f $12^{13} \div 12^7 \div 12^8$ g $5^{-2} \times 3^{-2}$ h $(-4)^3 \times (-4)^{-7}$

i $7^{12} \div 7^{-7} \div 7^{17}$ j $\dfrac{3^{-9} \times 3^{-7}}{3^{-14}}$ k $\dfrac{5^7 \times 5^{-8}}{5^{-4}}$ l $\dfrac{(-7)^{-11} \times (-7)^{-4}}{(-7)^{-14}}$

Example 5

Reasoning

14

a What is 0^4 equal to?

b Explain why 0^{-4} is undefined.

15 When asked to find an expression that is equivalent to -2^0, a student responded 1. Is this answer correct? Explain why or why not.

16 Can zero be raised to a negative index? Explain why or why not.

Problem-solving

17 Write $\left(\dfrac{1}{2}\right)^{-2}$ at least three different ways.

18 Complete each statement:

a $\dfrac{2^{\square}}{5^{\square}} = 2^3 \times 5^{-2}$

b $\dfrac{8^3 \times 11^{\square}}{8^{\square} \times 11^2} = 8^{-1} \times 11^4$

c $((-2) \times 3) \times \left(\dfrac{1}{(-2) \times 3}\right)^{\square} = (-2)^4 \times 3^4$

d $\left(10^4 \times 18^{\square}\right)^{\square} = \dfrac{18^{12}}{10^{16}}$

19 Simplify, leaving your answers with positive indices:

a $\left(5^{-1} + 3^{-1}\right)^{-1}$ b $\dfrac{4^{400}}{8^{200}}$ c $\left[\left(25^2 \times 12\right)^2\right]^{-2}$ d $7^4 \div \left(\dfrac{7}{9}\right)^{-5}$

20 If the surface area of a cube is $6s^2$, where s is the length of a side in centimetres, find the

surface area when $s = 3^{-1}$ cm (without using a calculator).

21 Without using a calculator, what is the simplified form of $(4^{-1} \times 2^3)^2$?

22 It is estimated there were 7×10^8 jellyfish in the largest bloom ever observed. If each jellyfish can consume 5 grams of plankton in a day, how long would it take the bloom to consume 10 tonnes (1 tonne $= 1\,000\,000$ g) of plankton?

23 Why is $2^{-2} < 2$ but $\left(\frac{1}{2}\right)^{-2} > \frac{1}{2}$? Give another example where this pattern applies.

Worksheet 1.05C Index Laws with Algebraic Bases

Understanding

1 True or false:

a $8^{-2} = \frac{1}{8^2}$ b $(-3)^2 = \frac{1}{3^2}$ c $a^{-4} = -a^4$ d $(-c)^{-3} = -\frac{1}{c^3}$

2 Is 4^{-2} less than or greater than 1?

3 Write $\frac{1}{a}$ using a negative index.

Fluency

4 Rewrite with positive indices:

a a^{-9} b $\dfrac{1}{a^{-n}}$ c $\dfrac{a^{-9}}{4}$ d $\dfrac{a^{-n}}{b^{-m}}$

e p^{-2} f $3x^{-4}$ g $7x^{-9}$ h $p^{-2}q^{3}$

i $8p^{-3}$ j $2x^{-8}y^{3}$ k $\dfrac{b^{-7}}{6c^{-3}}$ l $m^{-5}n^{-4}p^{4}$

m $3a^{4}b^{-5}c^{-7}$ n $4^{-2}k^{-3}l^{7}$ o $\dfrac{f^{-7}}{g^{-2}h^{8}}$ p $\dfrac{s^{4}t^{-9}}{5u^{-6}}$

5 Rewrite without fractions:

a $\dfrac{1}{u^{4}}$ b $\dfrac{2}{r^{6}}$ c $\dfrac{x}{y^{2}}$ d $\dfrac{4}{p^{6}q^{7}}$

6 Simplify:

a $\dfrac{4a^{3}}{2a^{4}}$ b $\dfrac{-8x^{11}}{64x^{15}}$ c $\dfrac{s^{2}t^{4}}{5s^{3}t^{2}}$ d $\dfrac{100p^{6}q^{9}r^{12}}{10p^{12}q^{5}r^{17}}$

Example 1

7 Simplify:

a $5y^{9} \times 4y^{-3}$ b $7a^{4} \times 4a^{-2}$ c $5x^{4} \times \left(-3x^{-8}\right)$ d $3y^{-2} \times 4y^{-3}$

e $2h^{-4} \times 4h^{11}$ f $3y^{-2} \times 2y^{-5}$ g $-4y^{2} \times \left(-4y^{-5}\right)$ h $(5mp)^{2} \times mp^{-2}$

Example 2

8 Simplify:

a $2y^{6} \times 4y^{7} \times 4y^{-5}$ b $6y^{7} \times 2y^{-5} \times 5y^{3}$

c $2x^{-2} \times 5x^{5} \times 4x^{-5}$ d $12x^{-9} \times 2x^{4} \times 3x^{-2}$

e $4y^{3} \times 3y^{8} \div 2y^{-1}$ f $8y^{10} \div 2y^{-4} \div y^{3}$

g $40x^{-2} \div 5x^{8} \div 4x^{-9}$ h $14x^{-8} \div 2x \times 3x^{6}$

Example 3

9 Simplify:

a $(2m)^{-3}$ b $\left(4m^{-6}\right)^{4}$ c $\left(3p^{-4}\right)^{-2}$ d $\left(3y^{2}\right)^{-2}$

e $\left(\frac{k}{5}\right)^{-3}$ f $\left(\frac{3x^{5}}{2y^{4}}\right)^{-1}$ g $\left(\frac{x^{7}}{y^{9}}\right)^{-4}$ h $\frac{2x^{3}}{18x^{-2}}$

i $\frac{4x^{-7}}{3x^{-3}}$ j $\left(\frac{z}{3}\right)^{-4}$ k $\left(\frac{6p^{3}}{14q^{7}}\right)^{-1}$ l $\left(\frac{x^{-4}}{y^{-8}}\right)^{-2}$

m $\frac{15h^{2}}{12h^{-7}}$ n $\frac{12x^{5}}{4x^{-3}}$ o $\frac{28k^{-6}}{12k^{-4}}$ p $\frac{36w^{-6}}{16w^{4}}$

Example 4

10 Simplify:

a $\left(3x^{-4}\times 2x^{2}\right)^{2}$ b $\left(10x^{3}y^{-4}\right)^{2}$

c $\left(4x^{3}\right)^{-1}\times\left(y^{-2}\right)^{-3}$ d $\left(10x^{3}\right)^{3}\times\left(2x^{4}y^{3}\right)^{-2}$

e $\left(\frac{27x^{-3}y^{6}}{3x^{2}y^{-2}}\right)^{2}$ f $\left(\frac{18n^{3}m^{7}}{3mn^{2}}\right)^{-2}$

Reasoning

11 Oreste and Donovan are writing an equivalent expression for $\left(\frac{x^{-2}}{x^{3}}\right)^{2}$ as a single term.

Oreste writes down $\frac{1}{x^{10}}$ and Donovan writes down x^{-10}.

Who is correct and why?

12 A student wanted to express $5a^{-1}$ without negative indices and wrote $5a^{-1}=\frac{1}{5a}$. Explain why their answer is incorrect, and write the correct answer.

13 For $x>1$, what happens when we raise x to a very large negative index? That is, what happens to x^{-a} as a gets closer to 0. Explain your answer.

14

a For variable p, calculate the following expressions for $p = -2, 0, 4, 10$:

- $2p$
- $4p$
- $2^{-p} + 4p$
- $2^{-p} \times 4p$

b What do you predict will happen to the value of each expression as p increases significantly?

Problem-solving

15 Complete:

$$\left(x^4 y^{\square}\right)^{\square} = \frac{y^{12}}{x^{16}}$$

16 Solve for n:

a $\frac{1}{8} = 2^n$

b $\left(x^3 y^{-5}\right)^n = x^{-12} y^{20}$

c $\left(a^{-5} b^3\right)^n = a^{15} b^{-9}$

d $\left(\frac{y^7}{x^2}\right)^{-n} = \frac{y^{21}}{x^6}$

17 In electrical engineering, the formula used for calculating the capacitance of a capacitor is $C = \frac{Q}{V}$, where C is the capacitance measured in farads, Q is the charge on the capacitor measured in coulombs and V is the voltage across the capacitor measured in volts.

Use this formula to answer the following questions.

a A capacitor has a charge (Q) of $15 \times 10^{-6} C$ (coulombs) and a voltage (V) of 5 volts. What is the capacitance (C) of the capacitor?

b 1 microcoulomb is equivalent to 1×10^{-6} coulombs. A capacitor has a charge of 20 microcoulombs, and a voltage of 4 volts, what is the capacitance of the capacitor? Remember to first convert the charge to coulombs.

c A capacitor has a capacitance (C) of $10 \times 10^{-4} F$ (farads), and a charge of 30 microcoulombs. What voltage was applied across the capacitor?

Worksheet 2.01P Simplifying Surds

Understanding

1 Which statements are true and which are false?

a All surds are irrational.

b All square roots are surds.

c The cube root of a perfect square is always rational

d Some square roots are rational and some are irrational.

e The cube root of a perfect cube is a surd.

2 Are these surds?

a $\sqrt{4}$ b $\sqrt{14}$ c $2\sqrt{3}$ d $\sqrt[3]{4^3}$

3 Fill in the boxes:

a $\sqrt{20} = \sqrt{\square \times 5}$ b $\sqrt{50} = \sqrt{\square \times 2}$

c $\sqrt{18} = \sqrt{\square \times 2}$ d $-\sqrt{17} = -\sqrt{\square \times 17}$

e $\sqrt[3]{32} = \sqrt[3]{\square \times 4}$ f $\sqrt[3]{192} = \sqrt[3]{\square \times 3}$

Fluency

4 Is each number rational, irrational, or neither?

a $\sqrt{16}$ b $0.\overline{3}$ c $\sqrt{-3}$ d $\sqrt{125}$

e $\sqrt{33}$ f π g $\dfrac{0}{\sqrt{6}}$ h ∞

i $\sqrt[3]{64}$ j $\dfrac{\sqrt{4}}{\sqrt{25}}$ k $\sqrt[3]{47}$ l $\dfrac{1}{0}$

m $\sqrt[3]{125}$ n $\sqrt[3]{1000}$ o $\sqrt[3]{-1}$ p $\sqrt[3]{49}$

Example 1

5 Rewrite each root as the product of the highest perfect square or cube factor and a whole number:

a $\sqrt{72}$ b $\sqrt{48}$ c $\sqrt[3]{270}$ d $\sqrt{500}$

6 Simplify:

a $\sqrt{16 \times 5}$ b $\sqrt{49 \times 2}$ c $3\sqrt{4 \times 5}$ d $-\sqrt{9 \times 6}$

7 Simplify, and then determine if the value is rational or irrational:

a $\sqrt{24}$ b $\sqrt{18}$ c $\sqrt{1}$ d $\sqrt{121}$

e $\sqrt{147}$ f $\sqrt{81}$ g $\sqrt{2}$ h $\sqrt{112}$

i $\sqrt{50}$ j $\sqrt[3]{27}$ k $\sqrt{7}$ l $\sqrt{45}$

m $\sqrt[3]{125}$ n $\sqrt[3]{9}$ o $-\sqrt{19}$ p $\sqrt[3]{8}$

q $\sqrt{80}$ r $\sqrt{32}$ s $-\sqrt{144}$ t $\sqrt[3]{16}$

u $\sqrt{99}$ v $-\sqrt{180}$ w $\sqrt[3]{108}$ x $\sqrt{31}$

Example 2

8 Simplify:

a $\sqrt{180}$ b $\sqrt{125}$ c $3\sqrt{54}$ d $7\sqrt{32}$

e $6\sqrt{100}$ f $\sqrt{25 \times 6}$ g $\sqrt[3]{24 \times 9}$ h $\frac{1}{3}\sqrt[3]{54}$

Example 3

9 Simplify:

a $\frac{1}{\sqrt{25}}$ b $\frac{16}{\sqrt{16}}$ c $\frac{\sqrt{64}}{8}$ d $\frac{36}{\sqrt[3]{216}}$

Reasoning

10

a Is $\sqrt{290}$ an exact value?

b A calculator states that $\sqrt{290}$ is 17.029386366. Is this still exact? Explain.

11 During a physics assignment, Jordan tried to evaluate the cube root of 27 on their scientific calculator. They ended up with an answer of $19\,683$. Identify and correct Jordan's error.

12 Are these expressions written in their simplest surd form? Explain your reasoning.

a $\sqrt{17}$ b $\sqrt{50}$ c $7\sqrt{125}$ d $11\sqrt{21}$

13 Heiko is asked to simplify $\sqrt{80}$. He notes that $80 = 4 \times 20$, and simplifies to $2\sqrt{20}$.

a Has he fully simplified the surd? If yes, explain how you know. If not, further simplify Heiko's answer.

b Use another pair of factors for simplifying $\sqrt{80}$. Explain your choice.

14 If p is a rational number and $p < 0$, which statements are true and which are false? Explain your answer.

a $\sqrt{p}$ is negative

b $\sqrt{p^2}$ is rational

c $\sqrt{p^2}$ is positive

d $\left(\sqrt{p}\right)^2$ is positive

Problem-solving

15 The volume V of a regular tetrahedron with edge length a is given by:

$$V = \frac{a^3}{6\sqrt{2}}$$

a Find the volume of a tetrahedron with a side length of 2 cm.

b Rearrange the formula to isolate a.

c Now, find the side length of a tetrahedon with a volume of 72 cm^3.

16 Write the expression $3\sqrt{5}$ as a single surd.

17 Simplify $\sqrt{75 - 25\sqrt{6}}$

18 Simplify:

a $\sqrt{64x^2}$

b $\sqrt{81z^{10}}$

c $\sqrt{16v^{12}}$

d $\sqrt{144b^6}$

e $\sqrt[3]{216h^6}$

f $\sqrt[3]{8x^6}$

g $\sqrt[3]{27a^6}$

h $\sqrt{25a^{14}b^6}$

Worksheet 2.02P Add & Subtract surds

Understanding

1 Fill in the blanks:

a To add or subtract two surds they must be ⬚.

b For a surd to be in its simplest form, the radicand must not have any factors that are a ⬚.

2 Are each pair "like surds" or not?

a $\sqrt{5}$ and $\sqrt{15}$ b $2\sqrt{3}$ and $5\sqrt{3}$ c $-10\sqrt{7}$ and $4\sqrt{7}$ d $5\sqrt{11}$ and $5\sqrt{10}$

e $2\sqrt{3}$ and $3\sqrt{2}$ f $\sqrt{2}$ and $\sqrt{8}$ g $\sqrt{6}$ and $\sqrt{150}$ h $\sqrt[3]{2}$ and $\sqrt{2}$

Example 1

3 Are these equations true or false?

a $\sqrt{11}+\sqrt{5}=\sqrt{16}$ b $\sqrt{4}+\sqrt{4}=4$

c $5-\sqrt{5}=\sqrt{5}$ d $\sqrt{1}+\sqrt{4}=\sqrt{25}$

e $\sqrt{16}+\sqrt{9}=7$ f $\sqrt{32}+\sqrt{2}=5\sqrt{2}$

g $\sqrt{25}-\sqrt{9}=\sqrt{16}$ h $\sqrt{100}-\sqrt{64}=2$

Fluency

4 Simplify:

a $10\sqrt{2}+14\sqrt{2}$ b $\sqrt{6}+14\sqrt{6}$

c $\sqrt[3]{10}-20\sqrt[3]{10}$ d $12\sqrt{2}-3\sqrt{2}$

e $4\sqrt{3}-12\sqrt{3}$ f $18\sqrt{5}-15\sqrt{5}$

g $6\sqrt{7}-8\sqrt{7}$ h $6\sqrt{2}+13\sqrt{2}$

i $5\sqrt[3]{5}+9\sqrt[3]{5}-7\sqrt[3]{5}$ j $10\sqrt{6}-3\sqrt{6}+20\sqrt{6}$

k $7\sqrt{6}+18\sqrt{6}-9\sqrt{6}$ l $19\sqrt{6}-4\sqrt{6}-2\sqrt{6}$

m $8\sqrt{5}-9\sqrt{5}-14\sqrt{5}$ n $18\sqrt[3]{10}-9\sqrt[3]{10}-7\sqrt[3]{10}$

Example 2

5 Simplify:

a $8\sqrt[3]{2}+2\sqrt{11}+2\sqrt[3]{2}+4\sqrt{11}$ b $10\sqrt{2}+5\sqrt{3}+4\sqrt{2}-7\sqrt{3}$

c $6\sqrt{7}+7\sqrt{5}-3\sqrt{7}+8\sqrt{5}$ d $18\sqrt[3]{7}-9\sqrt{3}+20\sqrt[3]{7}+11\sqrt{3}$

e $20\sqrt{7}+7\sqrt{11}+\sqrt{7}+25\sqrt{11}$ f $7\sqrt{11}+28\sqrt{5}-25\sqrt{11}-11\sqrt{5}$

g $20\sqrt{11} + 26\sqrt{5} + 22\sqrt{5} - 15\sqrt{11}$

Example 3

6 Simplify:

a $\sqrt{3} + \sqrt{48}$ b $\sqrt{180} + \sqrt{5}$ c $\sqrt{45} + \sqrt{80}$ d $\sqrt{245} - \sqrt{5}$

e $\sqrt{48} - \sqrt{12}$ f $3\sqrt{27} + 2\sqrt{12}$ g $3\sqrt{192} - 2\sqrt{108}$ h $\frac{3\sqrt{2}}{2} + \frac{\sqrt{2}}{6}$

i $\sqrt[3]{16} + \sqrt[3]{54}$ j $7\sqrt[3]{3} + \sqrt[3]{24}$ k $5\sqrt[3]{2} - \sqrt[3]{128}$ l $10\sqrt[3]{5} - \sqrt[3]{40}$

Example 4

7 Simplify:

a $x\sqrt{5} + 7x\sqrt{5}$ b $10\sqrt{y} + 3\sqrt{y}$

c $\sqrt{5b} + 3\sqrt{5b}$ d $\sqrt{6x} + 5\sqrt{6x}$

e $8\sqrt{7y} + 4\sqrt{7y} + 4\sqrt{7y}$ f $6\sqrt{6a} + 4\sqrt{10a} + 2\sqrt{6a} + 6\sqrt{10a}$

g $9\sqrt{7z} - \sqrt{7z}$ h $\sqrt{42t} - 3\sqrt{42t}$

i $\sqrt{9x} - \sqrt{16x}$ j $\sqrt{18y} - \sqrt{8y} - w\sqrt{5} + \sqrt{20w^2}$

Example 5

Reasoning

8 Explain why we simplify surds.

9 For $4\sqrt{5} + 5\sqrt{7} + \sqrt{5} - 2\sqrt{7}$:

a Simplify $4x + 5y + x - 2y$.

b How does $4x + 5y + x - 2y$ relate to $4\sqrt{5} + 5\sqrt{7} + \sqrt{5} - 2\sqrt{7}$?

c Simplify $4\sqrt{5} + 5\sqrt{7} + \sqrt{5} - 2\sqrt{7}$.

10 For $\sqrt{k} + \sqrt{m} = \sqrt{k+m}$:

a Choose some values of k and m to show that the equation is not generally true.

b Are there values of k and m that would make the equation true?

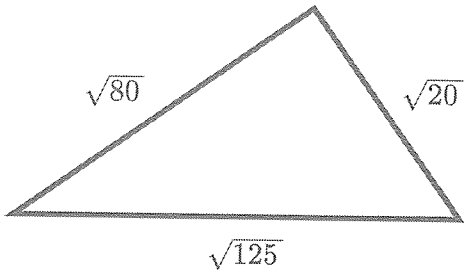
c Apply the same reasoning to $\sqrt{k} - \sqrt{m} = \sqrt{k-m}$. What are your conclusions?

11 Identify and explain the error in each equation:

a $5\sqrt[3]{5} - 2\sqrt{5} = 3\sqrt{5}$ b $\sqrt{7} + \sqrt{3} = \sqrt{10}$

Problem-solving

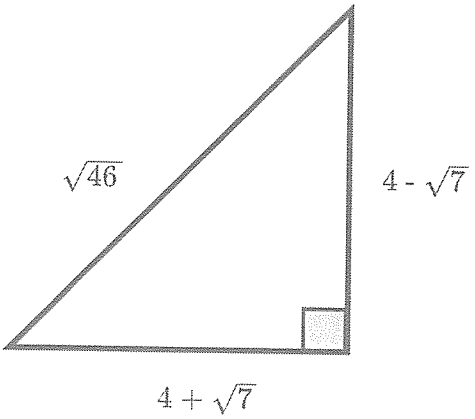
12 Find the perimeter of the triangle in simplified surd form.



13 An athlete is participating in a race around the track shown in the diagram, which has dimensions in kilometres.

a Determine the exact distance of one lap around this track.

b If the track distance is rounded to the nearest kilometre, determine by how much this approximation would overestimate the distance. Give your answer to the nearest metre.



14 A rectangle has length $\sqrt{44}$ cm and width $\sqrt{99}$ cm. Katie estimates the perimeter by using the nearest whole number to each surd.

a What was Katie's answer?

b Is Katie's answer an overestimate or an underestimate? Explain.

c Write an expression for the exact perimeter.

d What is the percentage difference between Katie's estimate and the true value? Round your answer to two decimal places.

15 Emma found a piece of paper on which another student had written the first four terms of a sequence. It read $(1+\sqrt{2})$, $(2-\sqrt{2})$, $(3+\sqrt{2})$, $(4-\sqrt{2})$.

a Emma wanted to continue the sequence. What would the next three terms be?

b The sum of the first n integers is $\frac{n(n+1)}{2}$. Find the sum of the first seven terms of this sequence as an exact value.

c If the last term of the sequence that Emma wrote was $(60-\sqrt{2})$, find the sum of all the terms in the sequence as an exact value.

Worksheet 2.03P MULTIPLY & DIVIDE SURDS

Understanding

1 Is each expression defined or undefined in the real numbers?

a $\sqrt{5}$ b $\sqrt{-5}$ c $\sqrt{0}$ d $\dfrac{1}{\sqrt{0}}$

2 Which of these statements are true and which are false? Assume all variables are positive.

a $\sqrt{8^2} = (\sqrt{8})^2$ b $\sqrt{5^2} = (\sqrt{5 \times 5})^2$

c $\sqrt{2^2} = \sqrt{2+2}$ d $\sqrt{8^2} = \sqrt{16} \times \sqrt{4}$

e $\sqrt{a} \times \sqrt{b} = \sqrt{a+b}$ f $(\sqrt{p})^2 = p$

g $\sqrt{x} \times \sqrt{y} = \sqrt{xy}$ h $\dfrac{\sqrt{c}}{\sqrt{d}} = \sqrt{\dfrac{c}{d}}$

i $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{a} - \sqrt{b}$ j $\sqrt{p}(\sqrt{p} - 2) = p - 2\sqrt{p}$

3 Complete these statements using this example:

$$\sqrt{9 \times 4} = \sqrt{9} \times \sqrt{4} = 3 \times 2 = 6$$

a $\sqrt{36 \times 25} = \sqrt{\square} \times \sqrt{\square} = \square \times \square = \square$ b $\sqrt{9 \times 11} = \sqrt{\square} \times \sqrt{\square} = \square\sqrt{\square}$

c $\sqrt{49 \times 5} = \sqrt{\square} \times \sqrt{\square} = \square\sqrt{\square}$ d $\sqrt{64 \times 3} = \sqrt{\square} \times \sqrt{\square} = \square\sqrt{\square}$

Fluency

4 Simplify:

a $(\sqrt{75})^2$ b $(\sqrt{5})^2 \times (\sqrt[3]{7})^3$ c $\left(6\sqrt{8}\right)^2$ d $(6\sqrt{3})^2$

e $\sqrt[3]{3} \times \sqrt[3]{6} \times \sqrt[3]{12}$ f $(4\sqrt{5})^2$ g $\sqrt[3]{5} \times (\sqrt[3]{5})^2$ h $(5\sqrt{2})^2$

5 Simplify:

a $\sqrt{5} \times \sqrt{7}$ b $8 \times 10\sqrt{5}$ c $\sqrt{7} \times \sqrt{3} \times \sqrt{11}$ d $\sqrt{55} \times \sqrt{11}$

e $4\sqrt{11} \times 5$ f $2\sqrt{5} \times 15\sqrt{11}$ g $7\sqrt{22} \times \sqrt{2}$ h $\sqrt{180} \times \sqrt{48}$

i $8\sqrt{15} \times 8\sqrt{5}$ j $5\sqrt{17} \times 8\sqrt{3}$ k $17\sqrt{35} \times 4\sqrt{5}$ l $8\sqrt{51} \times 9\sqrt{3}$

Example 1, 2

6 Simplify:

a $\sqrt{11}\left(\sqrt{7}+4\right)$ b $\sqrt{7}\left(3+\sqrt{3}\right)$ c $\sqrt{2}\left(\sqrt{11}-6\right)$ d $3\sqrt{3}\left(\sqrt{13}-5\right)$

e $\sqrt{3}\left(\sqrt{11}+\sqrt{13}\right)$ f $4\sqrt{7}\left(\sqrt{2}-\sqrt{11}\right)$ g $3\sqrt{5}\left(\sqrt{55}+\sqrt{11}\right)$ h $8\sqrt{2}\left(\sqrt{3}-3\sqrt{7}\right)$

i $5\sqrt{2}\left(3\sqrt{5}+4\sqrt{7}\right)$ j $7\sqrt{3}\left(\sqrt{15}+\sqrt{60}\right)$ k $11\sqrt{3}\left(3\sqrt{5}-\sqrt{20}\right)$ l $8\sqrt{11}\left(3\sqrt{7}-4\sqrt{5}\right)$

Example 3

7 Simplify:

a $\sqrt{5x} \times 2\sqrt{3}$ b $\sqrt{6x} \times \sqrt{7x}$ c $7 \times 9\sqrt{3x}$ d $\sqrt{10y} \times \sqrt{11y}$

e $6\sqrt{5a} \times 7$ f $5\sqrt{7x} \times 6\sqrt{3x}$ g $8\sqrt{14p} \times 5\sqrt{5p}$ h $3\sqrt{2y} \times 5\sqrt{6}$

i $\sqrt{2}\left(x\sqrt{2}+5\right)$ j $4\sqrt{3}\left(x\sqrt{3}+\sqrt{6}\right)$ k $\sqrt{x}\left(3\sqrt{x}-1\right)$ l $2\sqrt{y}\left(3\sqrt{y}-\sqrt{5}\right)$

8 Simplify:

a $\dfrac{\sqrt{15}}{\sqrt{5}}$ b $\dfrac{\sqrt{55}}{\sqrt{5}}$ c $\dfrac{\sqrt{51}}{\sqrt{17}}$ d $\dfrac{\sqrt{21}}{\sqrt{3}}$

e $\dfrac{\sqrt{91}}{\sqrt{7}}$ f $\sqrt{\dfrac{28}{7}}$ g $\sqrt{\dfrac{9}{45}}$ h $\sqrt{\dfrac{64}{4}}$

i $\sqrt{\dfrac{48}{144}}$ j $\dfrac{\sqrt{12}}{\sqrt{36}}$ k $\dfrac{\sqrt{56}}{\sqrt{14}}$ l $\dfrac{\sqrt{36}}{\sqrt{81}}$

m $\dfrac{\sqrt{72}}{\sqrt{32}}$ n $\dfrac{40\sqrt{7}}{8}$ o $\dfrac{10\sqrt{55}}{\sqrt{11}}$ p $\dfrac{15\sqrt{22}}{\sqrt{11}}$

q $\dfrac{4\sqrt{35}}{2\sqrt{5}}$ r $\dfrac{\sqrt{27}}{\sqrt{3}}$ s $\dfrac{3\sqrt{20}}{\sqrt{5}}$ t $\dfrac{5\sqrt{8}}{\sqrt{2}}$

u $\dfrac{40\sqrt{96}}{10\sqrt{6}}$ v $\dfrac{50\sqrt{24}}{10\sqrt{6}}$ w $\dfrac{\sqrt{25}}{\sqrt{81}}$ x $\dfrac{\sqrt{162}}{\sqrt{8}}$

Example 4, 5

9 Simplify:

a $\dfrac{20\sqrt{x}}{4\sqrt{x}}$ b $\dfrac{\sqrt{10y}}{\sqrt{2y}}$ c $\dfrac{\sqrt{15m}}{\sqrt{5m}}$ d $\dfrac{4\sqrt{18b}}{\sqrt{3b}}$

e $\sqrt{\dfrac{49n}{9n}}$ f $\sqrt{\dfrac{52k}{13k}}$ g $\dfrac{\sqrt{35z}}{\sqrt{5}}$ h $\dfrac{\sqrt{125h}}{\sqrt{25h}}$

Reasoning

10 Identify whether the resulting values are always a surd, sometimes a surd, or never a surd. Justify your answers with examples.

a The quotient of two surds.

b The quotient of a surd and an integer.

c The quotient of two integers.

11 Eliora claimed that if a rectangle has sides whose lengths are surds, that the area must also be a surd. Is Eliora correct? Provide an example to justify your conclusion.

12 The cross-sectional area of a prism can be found by dividing the volume by the height. If both the volume and the height are surds, is it possible for the height to not be a surd? Provide an example to justify your conclusion.

13 Pythagoras' theorem states that $c^2 = a^2 + b^2$, where c is the length of the hypotenuse of a right-angled triangle, and a and b are the lengths of the shorter sides.

a Give an example where a and b are not surds, but c is a surd.

b Give an example where a and b are surds, but c is not a surd.

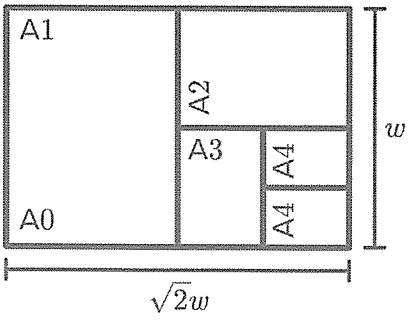
c Give an example where a and b are surds, and c is also a surd.

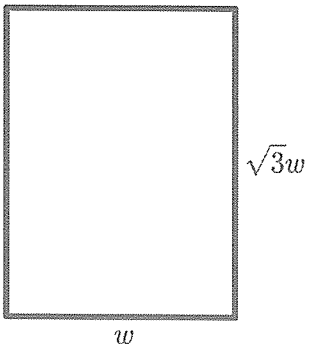
14 Suppose that a and b are different prime numbers.

a Explain why $\sqrt{a}$ is a surd, but $\sqrt{a^2}$ is not.

b Explain why $\sqrt{ab}$ is a surd?

15 ISO paper, for example A4 paper, always has a length that is $\sqrt{2}$ times the width.



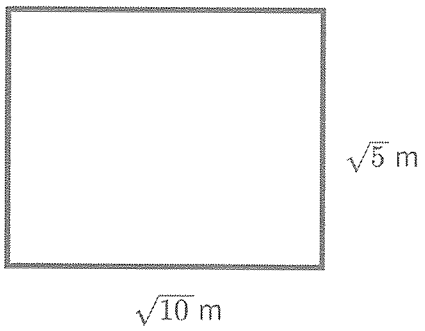


Scarlett is starting a new standard series of paper where the length is $\sqrt{3}$ times the width. The smallest size in the series has a width of $\sqrt{2}$ cm. Each subsequent member of the series has lengths that are $\sqrt{3}$ times the lengths of the previous size. Explain why all of the paper in this series would have an area which is a surd.

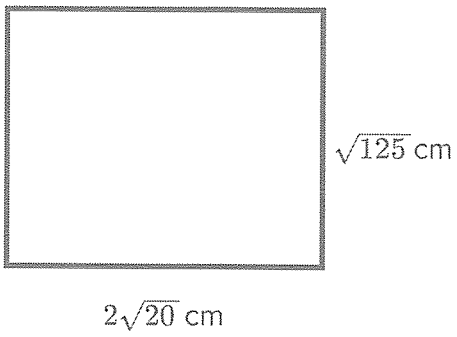
Problem-solving

16 Find the exact area of these rectangles. Give your answers as simplified surds.

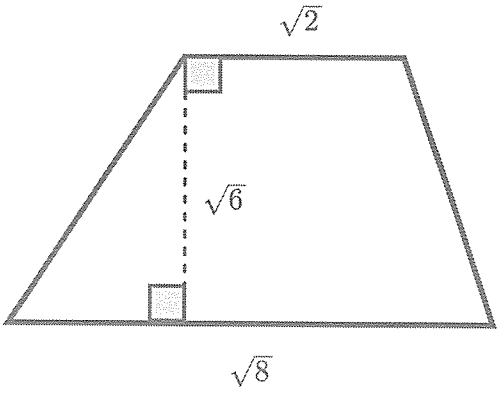
a



b



17 Find the area of the trapezium in simplified surd form.



18 The body surface area of a person in square metres can be approximately modelled by $A = \dfrac{\sqrt{h} \times \sqrt{w}}{60}$, where A is the surface area, h is the height of the person in cm, and w is the weight of the person in kg.

a Use the model to find the surface area of a person who is 164 cm tall and weighs 63 kg. Leave your answer in exact form.

b Find the approximate surface area of the person, to the nearest hundredth of a square metre.

c What are possible height and weight ranges for a person with a surface area of $2\ \text{m}^2$?

19 Find the exact perpendicular height of a triangle whose area is $40\sqrt{65}\ \text{cm}^2$ and whose base measures $10\sqrt{13}\ \text{cm}$.

20 A rectangle has a side length of $\left(8 + \sqrt{5}\right)\ \text{m}$ and a width of $2\sqrt{5}\ \text{m}$.

a Find the exact area of the rectangle.

b State the area of the rectangle to the nearest metre.

c Find an approximation for the area of the rectangle by rounding the length and width to the nearest metre before multiplying.

d Is there a significant difference between the estimated area found by rounding before and after the area calculation? Calculate the difference as a percentage of the exact area.

21 A big rectangular carpet is $\sqrt{20}+5$ metres long and $\sqrt{10}$ metres wide. A small rectangle, 4 metres long and $\sqrt{8}$ metres wide, is cut out of the carpet. Find the area of the carpet that is remaining in simplified surd form.

22 Ciera needs to perform each of these calculations, where $x=\sqrt{2}$. In her calculations Ciera uses x rounded to one decimal place as an approximation.

Determine how much each of her calculations differ from the exact value. Give your answer rounded to 2 decimal places.

a $10x$ b x^2+5 c $50x-2x^2$ d $\dfrac{200}{x}$

23 Vaughan needs 40 lengths of rope, each $\left(1+2\sqrt{3}\right)\text{ m}$ long. Vaughan rounds the individual rope length required up to the nearest metre and then multiplies by 40 to obtain an approximate amount to purchase from the hardware store.

Determine how much less Vaughan's estimate would be if he hadn't used rounding for each rope length.

Understanding

1 Expand and simplify:

a $3\left(2+\sqrt{5}\right)$ b $\sqrt{6}\left(8+\sqrt{5}\right)$ c $4\sqrt{7}\left(\sqrt{3}-2\right)$ d $\sqrt{3}\left(4-2\sqrt{6}\right)$

2 Expand and simplify:

a $(a+b)(c+d)$ b $(x+3)(x-2)$ c $(x-5)(x+5)$ d $(x+2)^2$

3 Simplify:

a $5+4\sqrt{5}+3\sqrt{5}+12$ b $7+3\sqrt{7}-2\sqrt{7}-6$

c $3-5\sqrt{7}-3\sqrt{7}+15$ d $2+2\sqrt{2}+2\sqrt{2}+4$

e $4-2\sqrt{3}-3\sqrt{5}+11$ f $\sqrt{10}-6\sqrt{2}+3\sqrt{5}-18$

4 When expanded and fully simplified, will these expressions have all rational terms, all surd terms, or a mix of rational and surds?

a $\left(2-\sqrt{3}\right)^2$ b $\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)$

c $\left(\sqrt{2}-\sqrt{3}\right)\left(\sqrt{5}+7\right)$ d $\left(\sqrt{2}-\sqrt{8}\right)\left(\sqrt{2}+\sqrt{8}\right)$

Fluency

5 Expand and fully simplify:

a $\left(\sqrt{11}+10\right)\left(\sqrt{3}-9\right)$ b $\left(\sqrt{11}-\sqrt{13}\right)\left(\sqrt{7}-\sqrt{2}\right)$

c $\left(11\sqrt{2}-\sqrt{7}\right)\left(13\sqrt{3}-\sqrt{5}\right)$ d $\left(4\sqrt{2}-\sqrt{7}\right)\left(3\sqrt{3}+\sqrt{8}\right)$

e $\left(17\sqrt{3}-8\sqrt{8}\right)\left(\sqrt{24}-\sqrt{5}\right)$ f $\left(\sqrt{90}-7\sqrt{7}\right)\left(\sqrt{72}-6\sqrt{5}\right)$

g $\left(5\sqrt{12}+\sqrt{7}\right)\left(3\sqrt{12}+2\sqrt{7}\right)$ h $\left(5\sqrt{11}-2\sqrt{3}\right)\left(4\sqrt{11}+2\sqrt{5}\right)$

Example 1

6 Expand and fully simplify:

a $\left(5-\sqrt{13}\right)\left(5+\sqrt{13}\right)$ b $\left(\sqrt{11}-11\right)\left(\sqrt{11}+11\right)$

c $\left(8\sqrt{5}-6\right)\left(8\sqrt{5}+6\right)$ d $\left(\sqrt{7}+\sqrt{5}\right)\left(\sqrt{7}-\sqrt{5}\right)$

e $\left(7\sqrt{11}-\sqrt{7}\right)\left(7\sqrt{11}+\sqrt{7}\right)$ f $\left(5\sqrt{3}+3\sqrt{5}\right)\left(5\sqrt{3}-3\sqrt{5}\right)$

g $\left(\sqrt{14}-\sqrt{7}\right)\left(\sqrt{14}+\sqrt{7}\right)$ h $\left(5\sqrt{11}-3\sqrt{7}\right)\left(5\sqrt{11}+3\sqrt{7}\right)$

Example 2

7 Expand and fully simplify:

a $\left(\sqrt{3}-13\right)^2$ b $\left(\sqrt{7}+\sqrt{3}\right)^2$ c $\left(3\sqrt{3}+8\right)^2$ d $\left(4\sqrt{2}-\sqrt{13}\right)^2$

e $\left(3\sqrt{2}+4\sqrt{13}\right)^2$ f $\left(5\sqrt{2}-\sqrt{32}\right)^2$ g $\left(2\sqrt{3}-\sqrt{2}\right)^2$ h $\left(5\sqrt{5}+2\sqrt{3}\right)^2$

Example 3

8 Fill in the boxes for each equation.

a $\left(3\sqrt{35}-2\sqrt{7}\right)^2 = \square - \square\sqrt{5}$ b $\left(\sqrt{\square}+\square\right)^2 = 16+6\sqrt{7}$

c $\left(\sqrt{\square}+7\right)^2 = 54+\square\sqrt{5}$ d $\left(\square\sqrt{\square}+5\right)^2 = 33+20\sqrt{2}$

Reasoning

9 Without expanding, explain whether or not $\left(a-\sqrt{b}\right)^3$ will have an $a\sqrt{b}$ term when it is expanded.

10 Explain why when expanding and simplifying $\left(a+\sqrt{b}\right)\left(a-\sqrt{b}\right)$ there would be two terms and for $\left(a+\sqrt{b}\right)\left(c+\sqrt{d}\right)$ there would be four terms, provided $c \neq a$ and $d \neq b$.

11 For:

$$\left(\sqrt{a}+\sqrt{b}\right)\left(\sqrt{c}+\sqrt{d}\right) = \sqrt{ac}+\sqrt{bc}+\sqrt{ad}+\sqrt{bd}$$

a Find values of a, b, c, and d, so that the expansion has four terms, none of which are like terms.

b Find values of a, b, c, and d, so that the expansion simplifies to two terms.

c Can a fully simplified product of two binomials involving surds ever have three terms? If so, give an example. If not, explain why not.

Problem-solving

12

a Expand and simplify $\left(\sqrt{7}+4\right)^2+\left(\sqrt{7}+m\right)^2$.

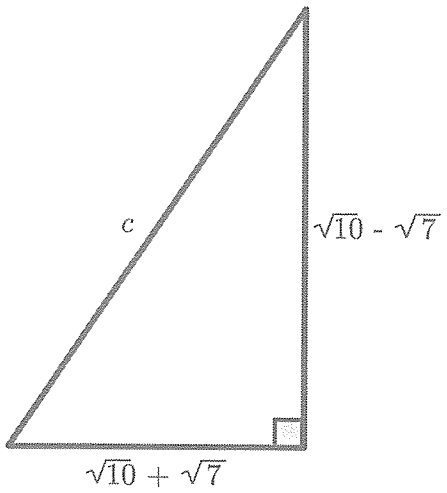
b What value of m can be substituted into $\left(\sqrt{7}+4\right)^2+\left(\sqrt{7}+m\right)^2$ so that the expression is rational?

13 For the given right triangle:

a Find c, the length of the hypotenuse of the triangle.

b Find the exact perimeter of the triangle.

c Find the exact area of the triangle.

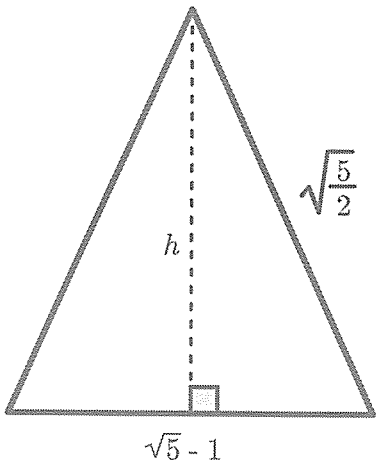


14 For the given isosceles triangle:

a Find h, the height of the isosceles triangle.

b Find the exact perimeter of the isosceles triangle.

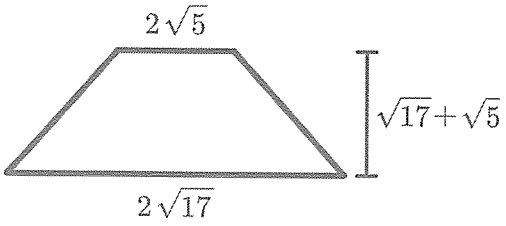
c Find the exact area of the isosceles triangle.



15 For the given isosceles trapezium:

a Find the exact area of the trapezium.

b Find the exact perimeter of the trapezium.



Worksheet 2.05P Rationalise the Denominator.

Understanding

1

a Simplify $\sqrt{5} \times \sqrt{5}$.

b Simplify $\frac{\sqrt{5}}{\sqrt{5}}$.

c Evaluate $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$.

d Why is $\frac{1}{\sqrt{5}}$ equal to $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}}$?

2 Determine whether each statement is true or false:

a The product of a conjugate pair will result in a rational number.

b A conjugate pair is a fraction with the same numerator and denominator.

c A fraction with the same numerator and denominator simplifies to 1.

d A product of conjugate pairs is similar to a difference of squares.

3 Simplify:

a $\frac{1}{\sqrt{13}} \times \frac{1}{\sqrt{13}}$ b $2\sqrt{5} \times \sqrt{5}$ c $\sqrt{18} \times \sqrt{2}$ d $5\sqrt{27} \times \sqrt{3}$

4 For each surd, what is the simplest surd we could multiply it by to produce an integer result?

a $\sqrt{5}$ b $\sqrt{11}$ c $2\sqrt{3}$ d $9\sqrt{2}$

e $7\sqrt{5}$ f $9\sqrt{17}$ g $6\sqrt{8}$ h $2\sqrt{12}$

5

a Simplify $(3+\sqrt{2})(3-\sqrt{2})$.

b Simplify $\frac{(3-\sqrt{2})}{(3-\sqrt{2})}$.

c Evaluate $\frac{1}{(3+\sqrt{2})} \times \frac{(3-\sqrt{2})}{(3-\sqrt{2})}$.

6 Does each product show a conjugate pair or not?

a $(3-\sqrt{7})(3+\sqrt{7})$ b $(7-\sqrt{3})(7+\sqrt{3})$

c $(\sqrt{12}-\sqrt{8})(\sqrt{12}+\sqrt{8})$ d $(a-\sqrt{b})(a+\sqrt{b})$

e $(\sqrt{3}+\sqrt{10})(\sqrt{3}+\sqrt{10})$ f $(\sqrt{5}+\sqrt{12})(\sqrt{5}-\sqrt{12})$

g $(2\sqrt{11}-\sqrt{3})(4\sqrt{11}+2\sqrt{3})$ h $(2\sqrt{5}-\sqrt{12})(5\sqrt{5}-\sqrt{12})$

Fluency

7 Find the conjugate:

a $5+\sqrt{5}$ b $6-\sqrt{v}$ c $\sqrt{6}+\sqrt{3}$ d $\sqrt{n}-\sqrt{5}$

e $\sqrt{5}+y$ f $2-9\sqrt{3}$ g $3+6\sqrt{8}$ h $7-2\sqrt{9}$

i $3\sqrt{7}+4\sqrt{3}$ j $9\sqrt{2}-3\sqrt{r}$ k $4\sqrt{s}+8\sqrt{t}$ l $2\sqrt{w}-9\sqrt{x}$

8 Rationalise the denominator and simplify:

a $\frac{1}{\sqrt{7}}$ b $\frac{1}{\sqrt{6}}$ c $\frac{1}{\sqrt{13}}$ d $\frac{1}{\sqrt{2}}$

e $\frac{1}{\sqrt{5}}$ f $\frac{1}{\sqrt{30}}$ g $\frac{1}{\sqrt{8}}$ h $\frac{1}{13\sqrt{3}}$

i $-\frac{1}{5\sqrt{11}}$ j $-\frac{1}{6\sqrt{5}}$ k $\frac{1}{20\sqrt{7}}$ l $-\frac{1}{19\sqrt{8}}$

9 Rationalise the denominator and simplify:

a $\frac{3}{\sqrt{7}}$ b $\frac{2}{\sqrt{5}}$ c $\frac{5}{\sqrt{10}}$ d $\frac{\sqrt{7}}{\sqrt{14}}$

e $\frac{\sqrt{2}}{\sqrt{6}}$ f $\frac{\sqrt{5}}{\sqrt{30}}$ g $\frac{4\sqrt{12}}{\sqrt{6}}$ h $\frac{21}{\sqrt{7}}$

i $-\frac{4\sqrt{30}}{\sqrt{6}}$ j $-\frac{11\sqrt{7}}{13\sqrt{3}}$ k $-\frac{6\sqrt{22}}{5\sqrt{11}}$ l $-\frac{15\sqrt{18}}{19\sqrt{8}}$

Example 1

10 Rationalise the denominator and simplify:

a $\frac{\sqrt{5}+9}{\sqrt{7}}$ b $\frac{\sqrt{5}+3}{\sqrt{5}}$ c $\frac{4-\sqrt{12}}{\sqrt{10}}$ d $\frac{\sqrt{7}-3}{\sqrt{3}}$

e $\frac{10\sqrt{2}+7}{\sqrt{11}}$ f $\frac{15-2\sqrt{3}}{\sqrt{2}}$ g $\frac{3\sqrt{5}+12}{\sqrt{20}}$ h $\frac{6\sqrt{14}-11}{\sqrt{5}}$

i $\frac{\sqrt{39}+\sqrt{6}}{\sqrt{3}}$ j $\frac{\sqrt{7}-\sqrt{13}}{\sqrt{15}}$ k $\frac{6\sqrt{2}+10\sqrt{10}}{\sqrt{12}}$ l $\frac{-20\sqrt{5}+7\sqrt{11}}{\sqrt{6}}$

Example 2

11 Rationalise the denominator and simplify:

a $\dfrac{5}{\sqrt{7}-3}$ b $\dfrac{3}{5\sqrt{2}-4}$ c $\dfrac{5}{9+\sqrt{3}}$ d $\dfrac{27}{\sqrt{11}-\sqrt{2}}$

e $\dfrac{4}{\sqrt{6}-\sqrt{7}}$ f $\dfrac{3}{4\sqrt{7}+8\sqrt{2}}$ g $\dfrac{10}{5\sqrt{10}-5\sqrt{3}}$ h $\dfrac{2}{7\sqrt{3}+2\sqrt{6}}$

i $\dfrac{\sqrt{5}+\sqrt{2}}{\sqrt{5}-\sqrt{2}}$ j $\dfrac{9\sqrt{2}+3\sqrt{11}}{9\sqrt{2}-3\sqrt{11}}$ k $\dfrac{\sqrt{6}-3\sqrt{5}}{\sqrt{6}+3\sqrt{5}}$ l $\dfrac{3\sqrt{4}-4\sqrt{3}}{4\sqrt{4}+2\sqrt{3}}$

Example 3

12 Evaluate in simplest surd form with a rational denominator:

a $\dfrac{1}{\sqrt{3}}+\dfrac{3}{\sqrt{3}}$ b $\dfrac{2}{\sqrt{7}}+\dfrac{1}{\sqrt{7}}$ c $\dfrac{1}{\sqrt{2}}+\dfrac{1}{\sqrt{3}}$ d $\dfrac{1}{\sqrt{5}}-\dfrac{1}{\sqrt{10}}$

e $\dfrac{2}{\sqrt{14}}+\dfrac{4}{\sqrt{7}}$ f $\dfrac{6}{\sqrt{3}}-\dfrac{3}{\sqrt{6}}$ g $\dfrac{3}{\sqrt{11}}-\dfrac{5}{\sqrt{22}}$ h $\dfrac{\sqrt{2}}{\sqrt{7}}+\dfrac{\sqrt{7}}{\sqrt{2}}$

i $\dfrac{\sqrt{10}}{\sqrt{20}}-\dfrac{\sqrt{30}}{\sqrt{40}}$ j $\dfrac{2\sqrt{5}}{\sqrt{5}}+\dfrac{\sqrt{3}}{\sqrt{15}}$ k $\dfrac{4}{\sqrt{6}}-\dfrac{1}{2\sqrt{3}}$ l $\dfrac{3\sqrt{2}}{4\sqrt{12}}+\dfrac{2\sqrt{10}}{3\sqrt{2}}$

Example 4

Reasoning

13 For x^2-y^2

a Explain why this expression is known as a difference of perfect squares.

b Factorise the expression.

c Explain how this relationship is useful in rationalising the denominator of an expression like $\dfrac{1}{(x-y)}$, if x and y are surds.

14 Explain why the rationalised form of $\dfrac{a}{(a-\sqrt{b})}$ will contain a term of $a\sqrt{b}$.

15 Xander says that to rationalise $\dfrac{2}{3-\sqrt{2}}$ we can multiply it by $\dfrac{\sqrt{2}+3}{\sqrt{2}+3}$. Is he correct or incorrect? Justify your answer.

Problem-solving

16 A rectangle has a length of $\frac{3}{1+\sqrt{2}}$ cm and width of $\left(1+\sqrt{2}\right)$ cm. Find the area.

17 Pythagoras' theorem states that in a right-angled triangle, $c^2 = a^2 + b^2$, where c is the length of the hypotenuse and a and b are the lengths of the two shorter sides.

The shorter sides of a right-angled triangle are $\frac{1}{3-\sqrt{2}}$ and $\frac{1}{3+\sqrt{2}}$. Find the length of the hypotenuse of this right-angled triangle.

18 The first term of a sequence is $\frac{1}{\left(5-\sqrt{2}\right)^7}$. Subsequent terms of the sequence are found by multiplying the current term by $\frac{1}{5+\sqrt{2}}$, so the second term would be $\frac{1}{\left(5-\sqrt{2}\right)^7\left(5+\sqrt{2}\right)}$ and so on.

a Find a term of the sequence that is rational.

b Is the term found in part (a) the only term that would be rational?

19 A rectangular garden has an area of $18\ \text{m}^2$ and width of $\sqrt{5}-1$ m. Find the length of the garden in simplest form with a rational denominator.

20 Rewrite $\frac{\sqrt{5}-2}{\sqrt{5}-4}$ in the form of $x\sqrt{y}+z$ where x, y, and z are real numbers.

21 Find the mean of $\frac{1}{1+\sqrt{x}}$ and $\frac{1}{1-\sqrt{x}}$, giving the answer in simplified rational form. Assume that x is positive.

22 Evaluate $\frac{\sqrt{7}+\sqrt{5}}{2\sqrt{5}+\sqrt{7}} - \frac{\sqrt{7}-\sqrt{5}}{2\sqrt{5}-\sqrt{7}}$, giving the answer in simplified rational form.