Appendix - Emergent Naming System in an Unknown Dynamic Environment: a Collective Perception Scenario

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This appendix demonstrates how the quality estimation strategy presented in the paper favours recent experiences and has a rubberband-like behaviour.

Let x_i be options present in the environment; in our scenario, $i \in \{0, 1\}$.
Let $q_T[x_i]$ denotes the estimated quality for option i at T, we have:

$$q_T[x_0] = 1 - q_T[x_1]$$

At time T, suppose that the agent senses option x_0, or received an opinion signal for option x_0, the agent increments the quality of the option by ϵ, the qualities are then normalized:

$$q_{t+1}[x_0] = \frac{q_t[x_0] + \epsilon}{q_t[x_0] + \epsilon + q_t[x_1]} = \frac{q_t[x_0] + \epsilon}{q_t[x_0] + \epsilon + 1 - q_t[x_0]} = \frac{q_t[x_0] + \epsilon}{\epsilon + 1}$$

$$q_{t+1}[x_1] = \frac{q_t[x_1]}{q_t[x_0] + \epsilon + q_t[x_1]} = \frac{q_t[x_1]}{q_t[x_0] + \epsilon + 1 - q_t[x_0]} = \frac{q_t[x_1]}{\epsilon + 1}$$

Let r_{x_i} be the absolute difference between qualities of the previous and current timestep:

$$r_{x_0} = q_{t+1}[x_0] - q_t[x_0] = \frac{q_t[x_0] + \epsilon - (q_t[x_0] * \epsilon) - q_t[x_0]}{\epsilon + 1} = \frac{\epsilon - (q_t[x_0] * \epsilon)}{\epsilon + 1}$$

$$\implies \frac{d(r_{x_0})}{d(q_t[x_0])} = \frac{-\epsilon}{\epsilon + 1} < 0, \forall \epsilon > 0 \tag{1}$$

$$r_{x_1} = q_t[x_1] - q_{t+1}[x_1] = \frac{(q_t[x_1] * \epsilon) + q_t[x_1] - q_t[x_1]}{\epsilon + 1} = \frac{(q_t[x_1] * \epsilon)}{\epsilon + 1}$$

$$\implies \frac{d(r_{x_1})}{d(q_t[x_1])} = \frac{\epsilon}{\epsilon + 1} > 0, \forall \epsilon > 0 \tag{2}$$

(1) $\Rightarrow$ the higher a quality is, the smaller subsequent increases to it is.
(2) $\Rightarrow$ the lower a quality is, the smaller subsequent decreases to it is.

At time $T + 1$, suppose now that the agent senses/receives signal for option x_1:

$$q_{t+2}[x_0] = \frac{q_{t+1}[x_0]}{q_{t+1}[x_0] + q_{t+1}[x_1] + \epsilon} = \frac{q_{t+1}[x_0]}{\epsilon + 1} = \frac{q_t[x_0]}{(\epsilon + 1)^2} + \frac{\epsilon}{(\epsilon + 1)^2} \tag{3}$$

$$q_{t+2}[x_1] = \frac{q_{t+1}[x_1] + \epsilon}{q_{t+1}[x_0] + q_{t+1}[x_1] + \epsilon} = \frac{q_{t+1}[x_1] + \epsilon}{\epsilon + 1} = \frac{q_t[x_1]}{(\epsilon + 1)^2} + \frac{\epsilon}{(\epsilon + 1)} \tag{4}$$

Observe that, compared to time T, by $T + 2$, the amount that the quality of the option x_0 sensed at $T + 1$ gets incremented by is $\epsilon + 1$ times less than that of the option x_1 sensed at $T + 2$ (4,3) $\Rightarrow$ gives more weight to the more recently sensed/received options.