Appendix - Emergent Naming System in an Unknown Dynamic Environment: a Collective Perception Scenario

Dat Nguyen Tien Hoang[0009-0006-1159-6382] Nicolas Cambier[0000-0002-1165-286X]

February 4, 2025

This appendix demonstrates how the quality estimation strategy presented in the paper favours recent experiences and has a rubberband-like behaviour.

Let x_i be options present in the environment; in our scenario, $i \in \{0,1\}$.
Let $q_T[x_i]$ denotes the estimated quality for option i at T, we have:

$$q_T[x_0] = 1 - q_T[x_1]$$

At time T, suppose that the agent senses option x_0, or received an opinion signal for option x_0, the agent increments the quality of the option by ϵ, the qualities are then normalized:

$$q_{t+1}[x_0] = \frac{q_t[x_0]+\epsilon}{q_t[x_0]+\epsilon+q_t[x_1]} = \frac{q_t[x_0]+\epsilon}{q_t[x_0]+\epsilon+1-q_t[x_0]} = \frac{q_t[x_0]+\epsilon}{\epsilon+1}$$

$$q_{t+1}[x_1] = \frac{q_t[x_1]}{q_t[x_0]+\epsilon+q_t[x_1]} = \frac{q_t[x_1]}{q_t[x_0]+\epsilon+1-q_t[x_0]} = \frac{q_t[x_1]}{\epsilon+1}$$

Let r_{x_i} be the absolute difference between qualities of the previous and current timestep:

$$r_{x_0} = q_{t+1}[x_0] - q_t[x_0] = \frac{q_t[x_0]+\epsilon-(q_t[x_0]*\epsilon)-q_t[x_0]}{\epsilon+1} = \frac{\epsilon-(q_t[x_0]*\epsilon)}{\epsilon+1}$$

$$\implies \frac{d(r_{x_0})}{d(q_t[x_0])} = \frac{-\epsilon}{\epsilon+1} < 0, \forall\epsilon > 0 \tag{1}$$

$$r_{x_1} = q_t[x_1] - q_{t+1}[x_1] = \frac{(q_t[x_1]*\epsilon)+q_t[x_1]-q_t[x_1]}{\epsilon+1} = \frac{(q_t[x_1]*\epsilon)}{\epsilon+1}$$

$$\implies \frac{d(r_{x_1})}{d(q_t[x_1])} = \frac{\epsilon}{\epsilon+1} > 0, \forall\epsilon > 0 \tag{2}$$

(1) $\Rightarrow$ the higher a quality is, the smaller subsequent increases to it is.
(2) $\Rightarrow$ the lower a quality is, the smaller subsequent decreases to it is.

At time $T+1$, suppose now that the agent senses/receives signal for option x_1:

$$q_{t+2}[x_0] = \frac{q_{t+1}[x_0]}{q_{t+1}[x_0]+q_{t+1}[x_1]+\epsilon} = \frac{q_{t+1}[x_0]}{\epsilon+1} = \frac{q_t[x_0]}{(\epsilon+1)^2} + \frac{\epsilon}{(\epsilon+1)^2} \tag{3}$$

$$q_{t+2}[x_1] = \frac{q_{t+1}[x_1]+\epsilon}{q_{t+1}[x_0]+q_{t+1}[x_1]+\epsilon} = \frac{q_{t+1}[x_1]+\epsilon}{\epsilon+1} = \frac{q_t[x_1]}{(\epsilon+1)^2} + \frac{\epsilon}{(\epsilon+1)} \tag{4}$$

Observe that, compared to time T, by $T+2$, the amount that the quality of the option x_0 sensed at $T+1$ gets incremented by is $\epsilon+1$ less than that of the option x_1 sensed at $T+2$ (4,3) $\Rightarrow$ gives more weight to the more recently sensed/received options.