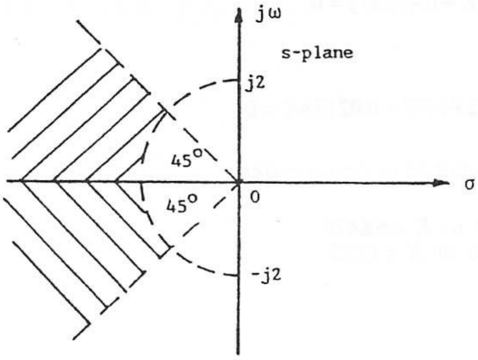
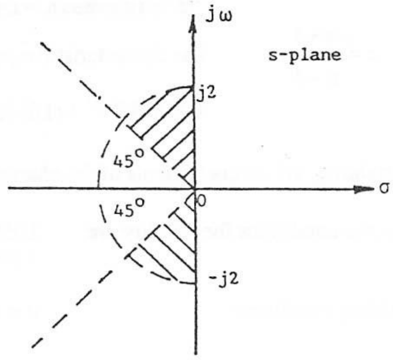
Chapter 7

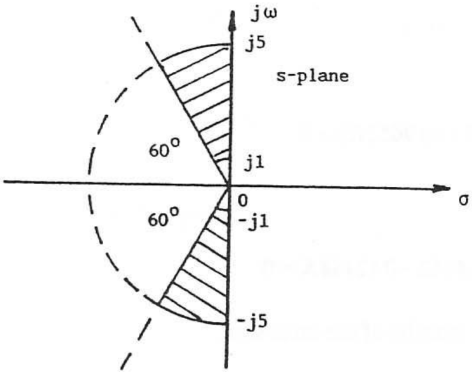
7-1 (a) $\zeta \geq 0.707 \quad \omega_n \geq 2 \text{ rad/sec}$



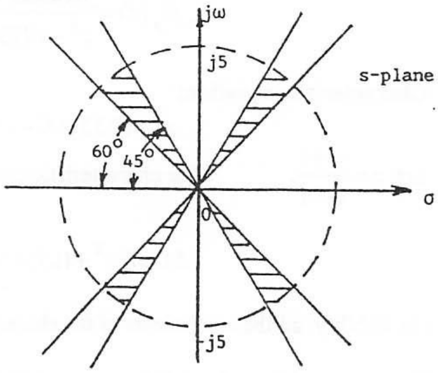
(b) $0 \leq \zeta \leq 0.707 \quad \omega_n \leq 2 \text{ rad/sec}$



(c) $\zeta \leq 0.5 \quad 1 \leq \omega_n \leq 5 \text{ rad/sec}$



(d) $0.5 \leq \zeta \leq 0.707 \quad \omega_n \leq 0.5 \text{ rad/sec}$



7-2 (a) Type 0 **(b)** Type 0 **(c)** Type 1 **(d)** Type 2 **(e)** Type 3 **(f)** Type 3

(g) type 2 **(h)** type 1

7-3 (a) $K_p = \lim_{s\to 0} G(s) = 1000 \qquad K_v = \lim_{s\to 0} sG(s) = 0 \qquad K_a = \lim_{s\to 0} s^2G(s) = 0$

(b) $K_p = \lim_{s\to 0} G(s) = \infty \qquad K_v = \lim_{s\to 0} sG(s) = 1 \qquad K_a = \lim_{s\to 0} s^2G(s) = 0$

(c) $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = K$ $K_a = \lim_{s\to 0} s^2 G(s) = 0$

(d) $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = \infty$ $K_a = \lim_{s\to 0} s^2 G(s) = 1$

(e) $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = 1$ $K_a = \lim_{s\to 0} s^2 G(s) = 0$

(f) $K_p = \lim_{s\to 0} G(s) = \infty$ $K_v = \lim_{s\to 0} sG(s) = \infty$ $K_a = \lim_{s\to 0} s^2 G(s) = K$

7-4 (a)

Input	Error Constants	Steady-state Error
$u_s(t)$	$K_p = 1000$	1/1001
$tu_s(t)$	$K_v = 0$	∞
$t^2 u_s(t)/2$	$K_a = 0$	∞

(b)

Input	Error Constants	Steady-state Error
$u_s(t)$	$K_p = \infty$	0
$tu_s(t)$	$K_v = 1$	1
$t^2 u_s(t)/2$	$K_a = 0$	∞

(c)

Input	Error Constants	Steady-state Error
$u_s(t)$	$K_p = \infty$	0

$tu_s(t)$	$K_v = K$	$1/K$
$t^2u_s(t)/2$	$K_a = 0$	∞

The above results are valid if the value of *K* corresponds to a stable closed-loop system.

(d) The closed-loop system is unstable. It is meaningless to conduct a steady-state error analysis.

(e)

Input	Error Constants	Steady-state Error
$u_s(t)$	$K_p = \infty$	0
$tu_s(t)$	$K_v = 1$	1
$t^2u_s(t)/2$	$K_a = 0$	∞

(f)

Input	Error Constants	Steady-state Error
$u_s(t)$	$K_p = \infty$	0
$tu_s(t)$	$K_v = \infty$	0
$t^2u_s(t)/2$	$K_a = K$	$1/K$

The closed-loop system is stable for all positive values of *K*. Thus the above results are valid.

7-5 (a) $K_H = H(0) = 1$ $$M(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{s+1}{s^3+2s^2+3s+3}$$

$$a_0 = 3, \quad a_1 = 3, \quad a_2 = 2, \quad b_0 = 1, \quad b_1 = 1.$$

Unit-step Input:

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = \frac{2}{3}$$

Unit-ramp input:

$$a_0 - b_0 K_H = 3 - 1 = 2 \neq 0. \text{ Thus } e_{ss} = \infty.$$

Unit-parabolic Input:

$$a_0 - b_0 K_H = 2 \neq 0 \text{ and } a_1 - b_1 K_H = 1 \neq 0. \text{ Thus } e_{ss} = \infty.$$

(b) $K_H = H(0) = 5$

$$M(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{1}{s^2 + 5s + 5} \qquad a_0 = 5, \quad a_1 = 5, \quad b_0 = 1, \quad b_1 = 0.$$

Unit-step Input:

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = \frac{1}{5}\left(1 - \frac{5}{5}\right) = 0$$

Unit-ramp Input:

$$i = 0: \quad a_0 - b_0 K_H = 0 \qquad i = 1: \quad a_1 - b_1 K_H = 5 \neq 0$$

$$e_{ss} = \frac{a_1 - b_1 K_H}{a_0 K_H} = \frac{5}{25} = \frac{1}{5}$$

Unit-parabolic Input:

$$e_{ss} = \infty$$

(c) $K_H = H(0) = 1/5$

$$M(s) = \frac{G(s)}{1 + G(s)H(s)} = \frac{s + 5}{s^4 + 15s^3 + 50s^2 + s + 1}$$ The system is stable.

$$a_0 = 1, \quad a_1 = 1, \quad a_2 = 50, \quad a_3 = 15, \quad b_0 = 5, \quad b_1 = 1$$

Unit-step Input:

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = 5\left(1 - \frac{5/5}{1}\right) = 0$$

Unit-ramp Input:

$$i=0\text{:}\quad a_0-b_0K_H=0\qquad i=1\text{:}\quad a_1-b_1K_H=4/5\neq 0$$

$$e_{ss}=\frac{a_1-b_1K_H}{a_0K_H}=\frac{1-1/5}{1/5}=4$$

Unit-parabolic Input:

$$e_{ss}=\infty$$

(d) $K_H=H(0)=10$

$$M(s)=\frac{G(s)}{1+G(s)H(s)}=\frac{1}{s^3+12s^2+5s+10}$$ The system is stable.

$$a_0=10,\quad a_1=5,\quad a_2=12,\quad b_0=1,\quad b_1=0,\quad b_2=0$$

Unit-step Input:

$$e_{ss}=\frac{1}{K_H}\left(1-\frac{b_0K_H}{a_0}\right)=\frac{1}{10}\left(1-\frac{10}{10}\right)=0$$

Unit-ramp Input:

$$i=0\text{:}\quad a_0-b_0K_H=0\quad i=1\text{:}\quad a_1-b_1K_H=5\neq 0$$

$$e_{ss}=\frac{a_1-b_1K_H}{a_0K_H}=\frac{5}{100}=0.05$$

Unit-parabolic Input:

$$e_{ss}=\infty$$

7-6 (a) $M(s)=\dfrac{s+4}{s^4+16s^3+48s^2+4s+4}\quad K_H=1$ The system is stable.

$$a_0=4,\quad a_1=4,\quad a_2=48,\quad a_3=16,\quad b_0=4,\quad b_1=1,\quad b_2=0,\quad b_3=0$$

Unit-step Input:

$$e_{ss}=\frac{1}{K_H}\left(1-\frac{b_0K_H}{a_0}\right)=\left(1-\frac{4}{4}\right)=0$$

Unit-ramp input:

$$i=0\text{:}\quad a_0-b_0K_H=0\qquad i=1\text{:}\quad a_1-b_1K_H=4-1=3\neq 0$$

$$e_{ss} = \frac{a_1 - b_1 K_H}{a_0 K_H} = \frac{4-1}{4} = \frac{3}{4}$$

Unit-parabolic Input:

$$e_{ss} = \infty$$

(b) $M(s) = \dfrac{K(s+3)}{s^3 + 3s^2 + (K+2)s + 3K}$ $K_H = 1$ The system is stable for $K > 0$.

$$a_0 = 3K, \quad a_1 = K+2, \quad a_2 = 3, \quad b_0 = 3K, \quad b_1 = K$$

Unit-step Input:

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = \left(1 - \frac{3K}{3K}\right) = 0$$

Unit-ramp Input:

$$i = 0: \quad a_0 - b_0 K_H = 0 \qquad i = 1: \quad a_1 - b_1 K_H = K + 2 - K = 2 \neq 0$$

$$e_{ss} = \frac{a_1 - b_1 K_H}{a_0 K_H} = \frac{K+2-K}{3K} = \frac{2}{3K}$$

Unit-parabolic Input:

$$e_{ss} = \infty$$

The above results are valid for $K > 0$.

(c) $M(s) = \dfrac{s+5}{s^4 + 15s^3 + 50s^2 + 10s}$ $H(s) = \dfrac{10s}{s+5}$ $K_H = \lim_{s \to 0} \dfrac{H(s)}{s} = 2$

$$a_0 = 0, \quad a_1 = 10, \quad a_2 = 50, \quad a_3 = 15, \quad b_0 = 5, \quad b_1 = 1$$

Unit-step Input:

$$e_{ss} = \frac{1}{K_H}\left(\frac{a_2 - b_1 K_H}{a_1}\right) = \frac{1}{2}\left(\frac{50 - 1 \times 2}{10}\right) = 2.4$$

Unit-ramp Input:

$$e_{ss} = \infty$$

Unit-parabolic Input:

$$e_{ss} = \infty$$

(d) $M(s) = \dfrac{K(s+5)}{s^4 + 17s^3 + 60s^2 + 5Ks + 5K}$ $\quad K_H = 1$ The system is stable for 0 < *K* < 204.

$$a_0 = 5K, \quad a_1 = 5K, \quad a_2 = 60, \quad a_3 = 17, \quad b_0 = 5K, \quad b_1 = K$$

Unit-step Input:

$$e_{ss} = \frac{1}{K_H}\left(1 - \frac{b_0 K_H}{a_0}\right) = \left(1 - \frac{5K}{5K}\right) = 0$$

Unit-ramp Input:

$$i = 0: \quad a_0 - b_0 K_H = 0 \qquad i = 1: \quad a_1 - b_1 K_H = 5K - K = 4K \neq 0$$

$$e_{ss} = \frac{a_1 - b_1 K_H}{a_0 K_H} = \frac{5K - K}{5K} = \frac{4}{5}$$

Unit-parabolic Input:

$$e_{ss} = \infty$$

The results are valid for 0 < *K* < 204.

7-7)

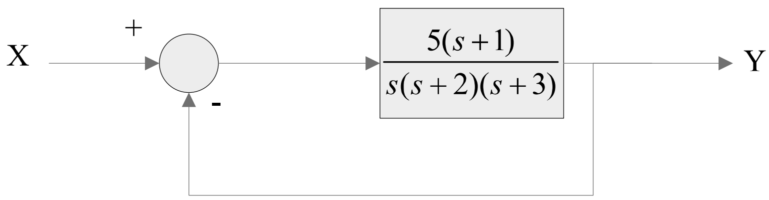
$$\frac{Y(s)}{X(s)} = \frac{\left(\left(\frac{s+1}{s+3}\right)\frac{s}{s(s+2)}\right)}{1+\frac{5(s+1)}{s(s+2)(s+3)}} = \frac{5(s+1)}{s^3+5s^2+11s+5}$$

⇨ Type of the system is zero

Pole: s = -2.2013 + 1.8773i, s = -2.2013 - 1.8773i, and s = -0.5974

Zero: s = -1

7-8)



$$G(s) = \frac{5(s+1)}{s(s+2)(s+3)}$$

a) Position error: $K_p = \lim_{s\to 0} \mathrm{G(s)} = \lim_{s\to 0} \frac{5(s+1)}{s(s+2)(s+3)} = \infty$

b) Velocity error: $K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} \frac{5(s+1)}{(s+2)(s+3)} = \frac{5}{6}$

c) Acceleration error: $K_a = \lim_{s\to\infty} s^2G(s) = \lim_{s\to\infty} \frac{5s(s+1)}{(s+2)(s+3)} = 0$

7-9) a) Steady state error for unit step input:

$e_{ss} = \frac{1}{1+K_p}$

Referring to the result of problem 7-8, $K_p = \infty$ ➔ $e_{ss} = 0$

b) Steady state error for ramp input:

$e_{ss} = \frac{1}{K_v}$

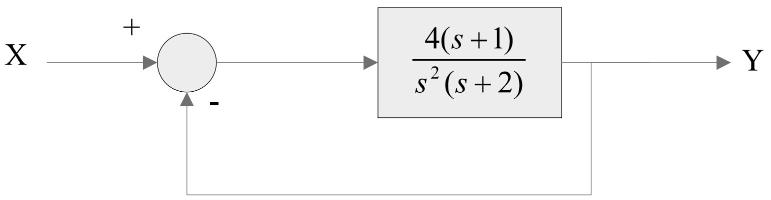
Regarding the result of problem 7-8, $K_v = \frac{5}{6}$ ➔ $e(\infty) = \frac{6}{5}$

c) Steady state error for parabolic input:

$e_{ss} = \frac{1}{K_a}$

Regarding the result of problem 7-8, $K_a = 0$ ➔ $e(\infty) = \infty$

7-10)



a) Step error constant: $K_p = \lim_{s \to 0} \frac{4(s+1)}{s^2(s+2)} = \infty$

b) Ramp error constant: $K_v = \lim_{s \to 0} \frac{4(s+1)}{s(s+2)} = \infty$

c) Parabolic error constant: $K_a = \lim_{s \to 0} \frac{4(s+1)}{s+2} = 2$

7-11) $X = \frac{5}{2s} - \frac{3}{s^2} + \frac{4}{s^3} = \frac{5}{2}X_1(s) - 3X_2(s) + 4X_3(s)$

where x_1 is a unit step input, x_2 is a ramp input, and x_3 is a unit parabola input. Since the system is linear, then the effect of X(s) is the summation of effect of each individual input.

That is: $e(\infty) = \frac{5}{2}e_1(\infty) - 3e_2(\infty) + 4e_3(\infty)$

So:

$$\begin{cases} e_{step} = \frac{1}{1+K_p} = 0 \\ e_{ramp} = \frac{1}{Kv} = 0 \\ e_{parabolic} = \frac{1}{K_a} = \frac{1}{2} \end{cases}$$

⇨ $e_{ss} = 4\left(\frac{1}{2}\right) = 2$

7-12) The step input response of the system is:

$$Y(s) = G(s)U(s) = \frac{1-k}{s(s-k)} = \frac{1}{1+k}\left[\frac{1}{s} - \frac{1}{s-k}\right]$$

Therefore:

$$y(t) = \frac{1}{1+k}[e^{Kt} + 1]u(t)$$

The rise time is the time that unit step response value reaches from 0.1 to 0.9. Then:

$$t_r = \frac{1}{1+k}[e^{0.9k} - e^{0.1k}]$$

It is obvious that $t_r > 0$, then:

$$\frac{1}{1+k}[e^{0.9k} - e^{0.1k}] > 0$$

As $|k| < 1$, then $\frac{1}{1+k} > 0$

Therefore $e^{0.9k} - e^{0.1k} > 0$ or $e^{0.9k} > e^{0.1k}$

which yields: $k > 0$

7-13)

$$G(s) = \frac{Y(s)}{E(s)} = \frac{KG_p(s)/20s}{1+K_tG_p(s)} = \frac{100K}{20s(1+0.2s+100K_t)}$$ Type-1 system.

Error constants: $K_p = \infty, \quad K_v = \frac{5K}{1+100K_t}, \quad K_a = 0$

(a) $r(t) = u_s(t)$: $e_{ss} = \frac{1}{1+K_p} = 0$

(b) $r(t) = tu_s(t)$: $e_{ss} = \frac{1}{K_v} = \frac{1+100K_t}{5K}$

(c) $r(t) = t^2u_s(t)/2$: $e_{ss} = \frac{1}{K_a} = \infty$

7-14

$$G_p(s) = \frac{100}{(1+0.1s)(1+0.5s)} \qquad G(s) = \frac{Y(s)}{E(s)} = \frac{KG_p(s)}{20s\left[1+K_tG_p(s)\right]}$$

$$G(s) = \frac{100K}{20s\left[(1+0.1s)(1+0.5s)+100K_t\right]}$$

Error constants: $K_p = \infty, \quad K_v = \dfrac{5K}{1+100K_t}, \quad K_a = 0$

(a) $r(t) = u_s(t)$: $\quad e_{ss} = \dfrac{1}{1+K_p} = 0$

(b) $r(t) = tu_s(t)$: $\quad e_{ss} = \dfrac{1}{K_v} = \dfrac{1+100K_t}{5K}$

(c) $r(t) = t^2u_s(t)/2$: $e_{ss} = \dfrac{1}{K_a} = \infty$

Since the system is of the third order, the values of *K* and K_t must be constrained so that the system is stable. The characteristic equation is

$$s^3 + 12s^2 + \left(20 + 2000K_t\right)s + 100K = 0$$

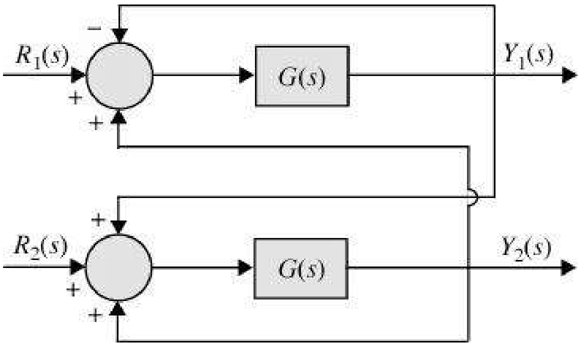
Routh Tabulation:

s^3	1	$20+2000K_t$
s^2	12	$100K$
s^1	$\dfrac{240+24000K_t-100K}{12}$	
s^0	$100K$	

Stability Conditions: *K* > 0 $\qquad 12\left(1+100K_t\right)-5K>0 \quad \text{or} \quad \dfrac{1+100K_t}{5K} > \dfrac{1}{12}$

Thus, the minimum steady-state error that can be obtained with a unit-ramp input is 1/12.

7-15 (a) From Figure 3P-29,



$$\frac{\Theta_o(s)}{\Theta_r(s)} = \frac{1+\dfrac{K_1K_2}{R_a+L_as}+\dfrac{K_iK_b+KK_1K_iK_t}{(R_a+L_as)(B_t+J_ts)}}{1+\dfrac{K_1K_2}{R_a+L_as}+\dfrac{K_iK_b+KK_1K_iK_t}{(R_a+L_as)(B_t+J_ts)}+\dfrac{KK_sK_1K_iN}{s(R_a+L_as)(B_t+J_ts)}}$$

$$\frac{\Theta_o(s)}{\Theta_r(s)} = \frac{s\left[(R_a+L_as)(B_t+J_ts)+K_1K_2(B_t+J_ts)+K_iK_b+KK_1K_iK_t\right]}{L_aJ_ts^3+(L_aB_t+R_aJ_t+K_1K_2J_t)s^2+(R_aB_t+K_iK_b+KK_iK_1K_t+K_1K_2B_t)s+KK_sK_1K_iN}$$

$$\theta_r(t)=u_s(t), \quad \Theta_r(s)=\frac{1}{s} \qquad \lim_{s\to 0} s\Theta_e(s)=0$$

Provided that all the poles of $s\Theta_e(s)$ are all in the left-half *s*-plane.

(b) For a unit-ramp input, $\Theta_r(s)=1/s^2$.

$$e_{ss}=\lim_{t\to\infty}\theta_e(t)=\lim_{s\to 0}s\Theta_e(s)=\frac{R_aB_t+K_1K_2B_t+K_iK_b+KK_1K_iK_t}{KK_sK_1K_iN}$$

if the limit is valid.

7-16 (a) **Forward-path transfer function:** [$n(t)$ = 0]:

$$G(s)=\frac{Y(s)}{E(s)}=\frac{\dfrac{K(1+0.02s)}{s^2(s+25)}}{1+\dfrac{KK_t s}{s^2(s+25)}}=\frac{K(1+0.02s)}{s\left(s^2+25s+KK_t\right)}$$

Type-1 system.

Error Constants: $K_p=\infty, \quad K_v=\dfrac{1}{K_t}, \quad K_a=0$

For a unit-ramp input, $r(t)=tu_s(t), \quad R(s)=\dfrac{1}{s^2}, \quad e_{ss}=\lim\limits_{t\to\infty}e(t)=\lim\limits_{s\to 0}sE(s)=\dfrac{1}{K_v}=K_t$

Routh Tabulation:

s^3	1	$KK_t+0.02K$
s^2	25	K
s^1	$\dfrac{25K\left(K_t+0.02\right)-K}{25}$	
s^0	K	

Stability Conditions: $K>0 \qquad 25\left(K_t+0.02\right)-K>0 \quad \text{or} \quad K_t>0.02$

(b) With $r(t)$ = 0, $n(t)=u_s(t), \quad N(s)=1/s.$

System Transfer Function with $N(s)$ as Input:

$$\frac{Y(s)}{N(s)}=\frac{\dfrac{K}{s^2(s+25)}}{1+\dfrac{K(1+0.02s)}{s^2(s+25)}+\dfrac{KK_t s}{s^2(s+25)}}=\frac{K}{s^3+25s^2+K\left(K_t+0.02\right)s+K}$$

Steady-State Output due to $n(t)$:

$$y_{ss}=\lim_{t\to\infty}y(t)=\lim_{s\to 0}sY(s)=1$$

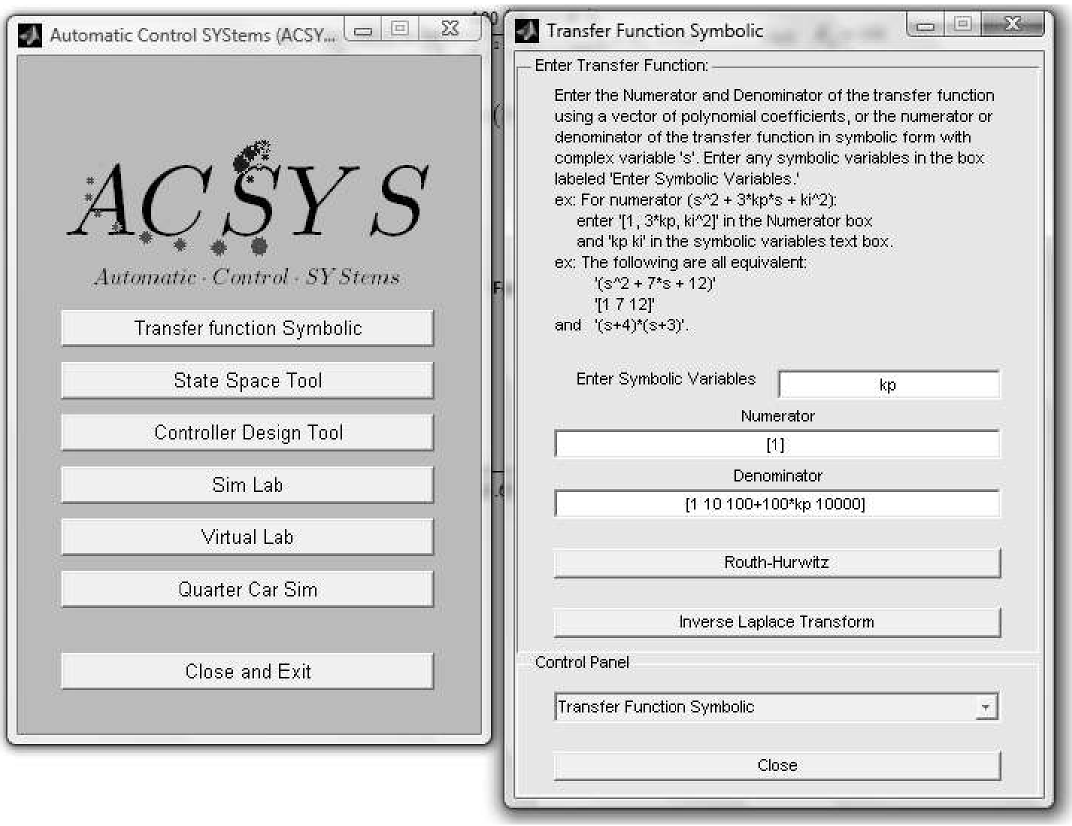
if the limit is valid.

7-17 You may use MATLAB in all Routh Hurwitz calculations.

1. Activate MATLAB
2. Go to the directory containing the ACSYS software.
3. Type in

Acsys

4. Then press the "transfer function Symbolic" and enter the Characteristic equation
5. Then press the "Routh Hurwitz" button
6. For example look at below Figures



(a) $n(t) = 0, \quad r(t) = tu_s(t).$

Forward-path Transfer function:

$$G(s) = \left.\frac{Y(s)}{E(s)}\right|_{n=0} = \frac{K(s+\alpha)(s+3)}{s\left(s^2-1\right)}$$ Type-1 system.

Ramp-error constant: $K_v = \lim_{s\to 0} sG(s) = -3K\alpha$

Steady-state error: $e_{ss} = \frac{1}{K_v} = -\frac{1}{3K_v}$

Characteristic equation: $s^3 + Ks^2 + [K(3+\alpha)-1]s + 3\alpha K = 0$

Routh Tabulation:

s^3	1	$3K+\alpha K-1$
s^2	K	$3\alpha K$
s^1	$\frac{K(3K+\alpha K-1)-3\alpha K}{K}$	
s^0	$3\alpha K$	

Stability Conditions: $3K+\alpha K-1-3\alpha > 0$ or $K > \frac{1+3K}{3+\alpha}$

$\alpha K > 0$

(b) When *r*(*t*) = 0, $n(t) = u_s(t), \quad N(s) = 1/s.$

Transfer Function between *n*(*t*) and *y*(*t*): $$\left.\frac{Y(s)}{N(s)}\right|_{r=0} = \frac{\frac{K(s+3)}{s^2-1}}{1+\frac{K(s+\alpha)(s+3)}{s\left(s^2-1\right)}} = \frac{Ks(s+3)}{s^3+Ks^2+[K(s+\alpha)-1]s+3\alpha K}$$

Steady-State Output due to *n*(*t*):

$$y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0$$ if the limit is valid.

7-18

$$\text{Percent maximum overshoot} = 0.25 = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$$

Thus

$$\pi\zeta\sqrt{1-\zeta^2} = -\ln 0.25 = 1.386 \qquad \pi^2\zeta^2 = 1.922\left(1-\zeta^2\right)$$

Solving for ζ from the last equation, we have ζ = 0.404.

Peak Time $t_{\max} = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 0.01$ sec. Thus, $\omega_n = \dfrac{\pi}{0.01\sqrt{1-(0.404)^2}} = 343.4$ rad/sec

Transfer Function of the Second-order Prototype System:

$$\frac{Y(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} = \frac{117916}{s^2 + 277.3s + 117916}$$

7-19 **Closed-Loop Transfer Function:** **Characteristic equation:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + \left(5 + 500K_t\right)s + 25K}$$

$$s^2 + \left(5 + 500K_t\right)s + 25K = 0$$

For a second-order prototype system, when the maximum overshoot is 4.3%, $\zeta = 0.707$.

$$\omega_n = \sqrt{25K}, \qquad 2\zeta\omega_n = 5 + 500K_t = 1.414\sqrt{25K}$$

Rise Time:

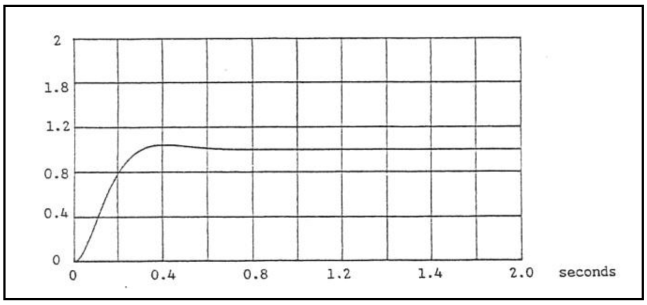
$$t_r = \frac{1 - 0.4167\zeta + 2.917\zeta^2}{\omega_n} = \frac{2.164}{\omega_n} = 0.2 \quad \text{sec} \qquad \text{Thus} \quad \omega_n = 10.82 \quad \text{rad/sec}$$

Thus, $K = \frac{\omega_n^2}{25} = \frac{(10.82)^2}{25} = 4.68 \qquad 5 + 500K_t = 1.414\omega_n = 15.3$ Thus $K_t = \frac{10.3}{500} = 0.0206$

With $K = 4.68$ and $K_t = 0.0206$, the system transfer function is

$$\frac{Y(s)}{R(s)} = \frac{117}{s^2 + 15.3s + 117}$$

Unit-step Response:



y = 0.1 at *t* = 0.047 sec.

y = 0.9 at *t* = 0.244 sec.

$t_r = 0.244 - 0.047 = 0.197$ sec.

$$y_{\max} = 0.0432 \quad (4.32\% \text{ max. overshoot})$$

7-20 **Closed-loop Transfer Function:** **Characteristic Equation:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + (5 + 500K_t)s + 25K} \qquad s^2 + (5 + 500K_t)s + 25K = 0$$

When Maximum overshoot = 10%, $\frac{\pi\zeta}{\sqrt{1-\zeta^2}} = -\ln 0.1 = 2.3 \qquad \pi^2\zeta^2 = 5.3\left(1-\zeta^2\right)$

Solving for ζ, we get ζ = 0.59.

The Natural undamped frequency is $\omega_n = \sqrt{25K}$ Thus, $5+500K_t = 2\zeta\omega_n = 1.18\omega_n$

Rise Time:

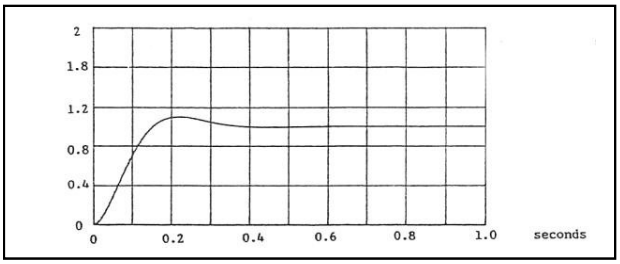
$$t_r = \frac{1-0.4167\zeta + 2.917\zeta^2}{\omega_n} = 0.1 = \frac{1.7696}{\omega_n} \text{ sec.} \quad \text{Thus } \omega_n = 17.7 \text{ rad/sec}$$

$$K = \frac{\omega_n^2}{25} = 12.58 \qquad \text{Thus } K_t = \frac{15.88}{500} = 0.0318$$

With K = 12.58 and $K_t = 0.0318$, the system transfer function is

$$\frac{Y(s)}{R(s)} = \frac{313}{s^2 + 20.88s + 314.5}$$

Unit-step Response:



y = 0.1 when t = 0.028 sec.

y = 0.9 when t = 0.131 sec.

$t_r = 0.131 - 0.028 = 0.103$ sec.

$y_{max} = 1.1 \quad (10\% \quad \text{max. overshoot})$

7-21 **Closed-Loop Transfer Function:** **Characteristic Equation:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + (5+500K_t)s + 25K} \qquad s^2 + \left(5+500K_t\right)s + 25K = 0$$

When Maximum overshoot = 20%, $\dfrac{\pi\zeta}{\sqrt{1-\zeta^2}} = -\ln 0.2 = 1.61 \qquad \pi^2\zeta^2 = 2.59\left(1-\zeta^2\right)$

Solving for ζ, we get $\zeta = 0.456$.

The Natural undamped frequency $\omega_n = \sqrt{25K} \qquad 5+500K_t = 2\zeta\omega_n = 0.912\omega_n$

Rise Time:

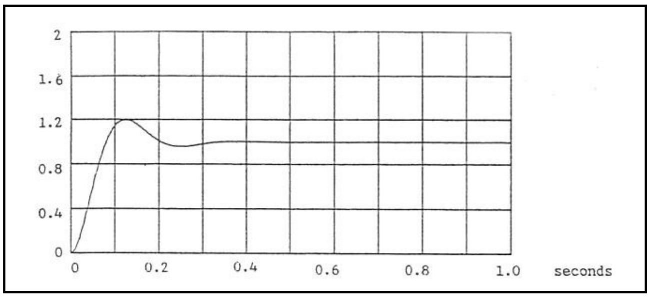
$$t_r = \frac{1-0.4167\zeta+2.917\zeta^2}{\omega_n} = 0.05 = \frac{1.4165}{\omega_n} \text{ sec. Thus, } \omega_n = \frac{1.4165}{0.05} = 28.33$$

$$K = \frac{\omega_n^2}{25} = 32.1 \qquad 5+500K_t = 0.912\omega_n = 25.84 \qquad \text{Thus, } K_t = 0.0417$$

With K = 32.1 and $K_t = 0.0417$, the system transfer function is

$$\frac{Y(s)}{R(s)} = \frac{802.59}{s^2 + 25.84s + 802.59}$$

Unit-step Response:



y = 0.1 when t = 0.0178 sec.

y = 0.9 when t = 0.072 sec.

$t_r = 0.072 - 0.0178 = 0.0542$ sec.

$y_{\max} = 1.2$ (20% max. overshoot)

7-22 **Closed-Loop Transfer Function:** **Characteristic Equation:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + (5 + 500K_t)s + 25K} \qquad s^2 + (5 + 500K_t)s + 25K = 0$$

Delay time $t_d \cong \dfrac{1.1 + 0.125\zeta + 0.469\zeta^2}{\omega_n} = 0.1$ sec.

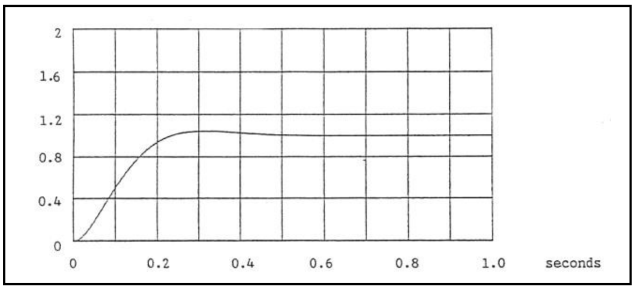
When Maximum overshoot = 4.3%, $\zeta = 0.707$. $t_d = \dfrac{1.423}{\omega_n} = 0.1$ sec. Thus $\omega_n = 14.23$ rad/sec.

$$K = \left(\frac{\omega_n}{5}\right)^2 = \left(\frac{14.23}{5}\right)^2 = 8.1 \quad 5 + 500K_t = 2\zeta\omega_n = 1.414\omega_n = 20.12 \quad \text{Thus} \quad K_t = \frac{15.12}{500} = 0.0302$$

With K = 20.12 and $K_t = 0.0302$, the system transfer function is

$$\frac{Y(s)}{R(s)} = \frac{202.5}{s^2 + 20.1s + 202.5}$$

Unit-Step Response:



When y = 0.5, t = 0.1005 sec.

Thus, $t_d = 0.1005$ sec.

$y_{\max} = 1.043 \quad (4.3\% \quad \text{max. overshoot})$

7-23 **Closed-Loop Transfer Function:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + \left(5 + 500K_t\right)s + 25K}$$

Characteristic Equation:

$$s^2 + \left(5 + 500K_t\right)s + 25K = 0$$

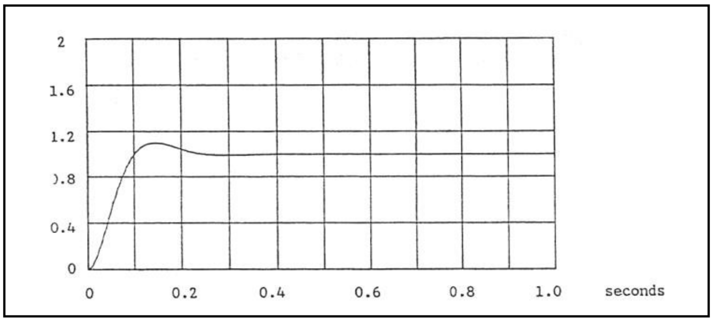
Delay time $t_d \cong \dfrac{1.1 + 0.125\zeta + 0.469\zeta^2}{\omega_n} = 0.05 = \dfrac{1.337}{\omega_n}$ Thus, $\omega_n = \dfrac{1.337}{0.05} = 26.74$

$$K = \left(\frac{\omega_n}{5}\right)^2 = \left(\frac{26.74}{5}\right)^2 = 28.6 \qquad 5 + 500K_t = 2\zeta\omega_n = 2 \times 0.59 \times 26.74 = 31.55 \quad \text{Thus } K_t = 0.0531$$

With K = 28.6 and $K_t = 0.0531$, the system transfer function is

$$\frac{Y(s)}{R(s)} = \frac{715}{s^2 + 31.55s + 715}$$

Unit-Step Response:



y = 0.5 when t = 0.0505 sec.

Thus, $t_d = 0.0505$ sec.

$y_{\max} = 1.1007$ (10.07% max. overshoot)

7-24 **Closed-Loop Transfer Function:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + \left(5 + 500K_t\right)s + 25K}$$

Characteristic Equation:

$$s^2 + \left(5 + 500K_t\right)s + 25K = 0$$

For Maximum overshoot = 0.2, $\zeta = 0.456$.

Delay time $t_d = \dfrac{1.1 + 0.125\zeta + 0.469\zeta^2}{\omega_n} = \dfrac{1.2545}{\omega_n} = 0.01$ sec.

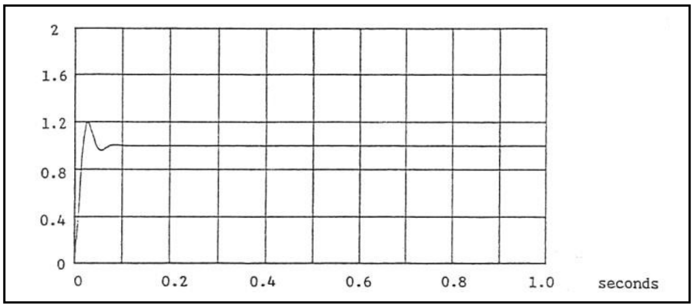
Natural Undamped Frequency $\omega_n = \dfrac{1.2545}{0.01} = 125.45$ rad/sec. Thus, $K = \left(\dfrac{\omega_n}{5}\right)^2 = \dfrac{15737.7}{25} = 629.5$

$5 + 500K_t = 2\zeta\omega_n = 2 \times 0.456 \times 125.45 = 114.41$ Thus, $K_t = 0.2188$

With *K* = 629.5 and $K_t = 0.2188$, the system transfer function is

$$\frac{Y(s)}{R(s)} = \frac{15737.7}{s^2 + 114.41s + 15737.7}$$

Unit-step Response:



y = 0.5 when *t* = 0.0101 sec.

Thus, $t_d = 0.0101$ sec.

$y_{\max} = 1.2 \quad (20\% \text{ max. overshoot})$

7-25 **Closed-Loop Transfer Function:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + \left(5 + 500K_t\right)s + 25K}$$

Characteristic Equation:

$$s^2 + \left(5 + 5000K_t\right)s + 25K = 0$$

$$\zeta = 0.6 \qquad 2\zeta\omega_n = 5 + 500K_t = 1.2\omega_n$$

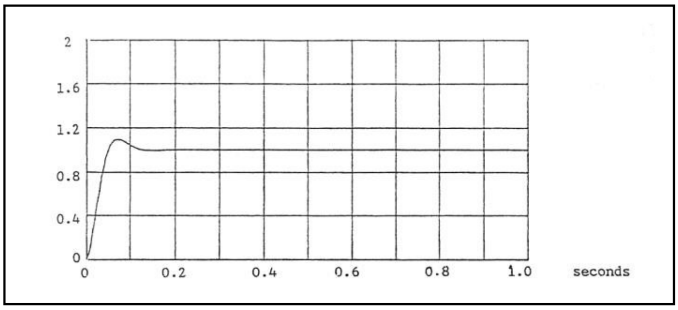
Settling time $t_s \cong \frac{3.2}{\zeta\omega_n} = \frac{3.2}{0.6\omega_n} = 0.1$ sec. Thus, $\omega_n = \frac{3.2}{0.06} = 53.33 \text{ rad/sec}$

$$K_t = \frac{1.2\omega_n - 5}{500} = 0.118 \qquad K = \frac{\omega_n^2}{25} = 113.76$$

System Transfer Function:

$$\frac{Y(s)}{R(s)} = \frac{2844}{s^2 + 64s + 2844}$$

Unit-step Response:



$y(t)$ reaches 1.00 and never exceeds this value at t = 0.098 sec.

Thus, $t_s = 0.098$ sec.

7-26 (a) **Closed-Loop Transfer Function:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + (5 + 500K_t)s + 25K}$$

Characteristic Equation:

$$s^2 + (5 + 500K_t)s + 25K = 0$$

For maximum overshoot = 0.1, $\zeta = 0.59$. $5 + 500K_t = 2\zeta\omega_n = 2 \times 0.59\omega_n = 1.18\omega_n$

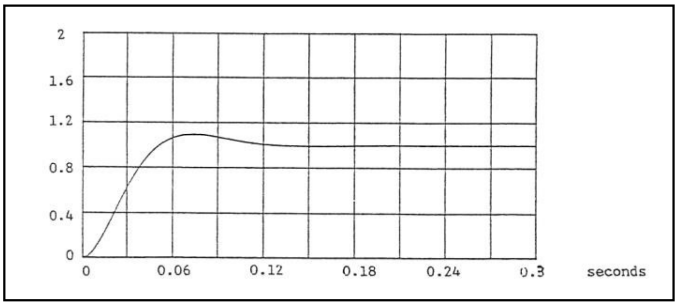
Settling time: $t_s = \dfrac{3.2}{\zeta\omega_n} = \dfrac{3.2}{0.59\omega_n} = 0.05$ sec. $\omega_n = \dfrac{3.2}{0.05 \times 0.59} = 108.47$

$$K_t = \frac{1.18\omega_n - 5}{500} = 0.246 \qquad K = \frac{\omega_n^2}{25} = 470.63$$

System Transfer Function:

$$\frac{Y(s)}{R(s)} = \frac{11765.74}{s^2 + 128s + 11765.74}$$

Unit-Step Response:



$y(t)$ reaches 1.05 and never exceeds this value at t = 0.048 sec.

Thus, $t_s = 0.048$ sec.

(b) For maximum overshoot = 0.2, $\zeta = 0.456$. $5 + 500K_t = 2\zeta\omega_n = 0.912\omega_n$

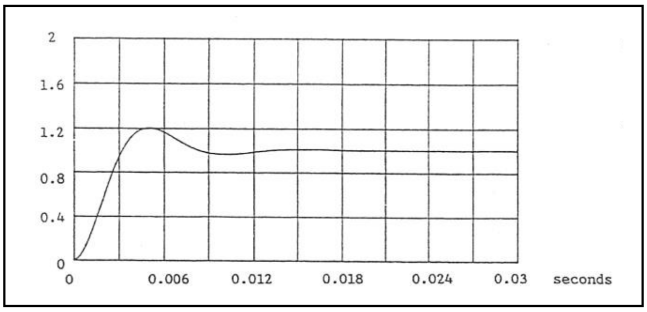
Settling time $t_s = \dfrac{3.2}{\zeta\omega_n} = \dfrac{3.2}{0.456\omega_n} = 0.01$ sec. $\omega_n = \dfrac{3.2}{0.456 \times 0.01} = 701.75$ rad / sec

$$K_t = \frac{0.912\omega_n - 5}{500} = 1.27$$

System Transfer Function:

$$\frac{Y(s)}{R(s)} = \frac{492453}{s^2 + 640s + 492453}$$

Unit-Step Response:



$y(t)$ reaches 1.05 and never exceeds this value at $t = 0.0074$ sec.

Thus, $t_s = 0.0074 \quad \text{sec.}$ This is less than the calculated value of 0.01 sec.

7-27 **Closed-Loop Transfer Function:**

$$\frac{Y(s)}{R(s)} = \frac{25K}{s^2 + \left(5 + 500K_t\right)s + 25K}$$

Characteristic Equation:

$$s^2 + \left(5 + 500K_t\right)s + 25K = 0$$

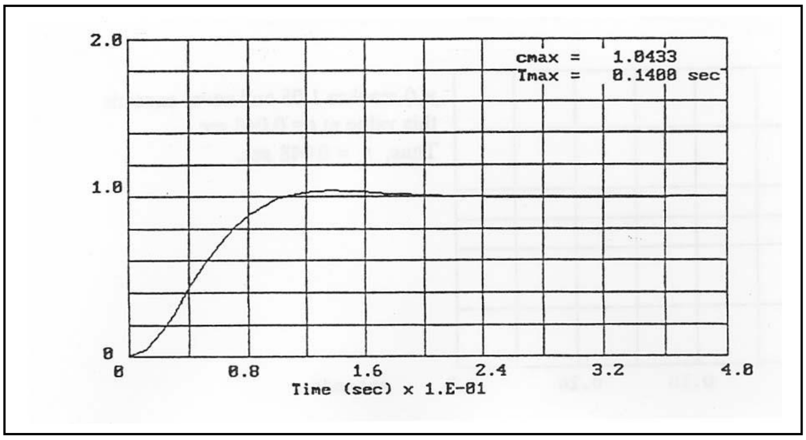
Damping ratio $\zeta = 0.707$. Settling time $t_s = \frac{4.5\zeta}{\omega_n} = \frac{3.1815}{\omega_n} = 0.1$ sec. Thus, $\omega_n = 31.815$ rad/sec.

$$5 + 500K_t = 2\zeta\omega_n = 44.986 \quad \text{Thus, } K_t = 0.08 \qquad K = \frac{\omega_n^2}{2\zeta} = 40.488$$

System Transfer Function:

$$\frac{Y(s)}{R(s)} = \frac{1012.2}{s^2 + 44.986s + 1012.2}$$

Unit-Step Response: The unit-step response reaches 0.95 at t = 0.092 sec. which is the measured t_s.



7-28 (a) When $\zeta = 0.5$, the rise time is

$$t_r \cong \frac{1 - 0.4167\zeta + 2.917\zeta^2}{\omega_n} = \frac{1.521}{\omega_n} = 1 \text{ sec.} \quad \text{Thus } \omega_n = 1.521 \text{ rad/sec.}$$

The second-order term of the characteristic equation is written

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 1.521s + 2.313 = 0$$

The characteristic equation of the system is $s^3 + (a+30)s^2 + 30as + K = 0$

Dividing the characteristic equation by $s^2 + 1.521s + 2.313$, we have

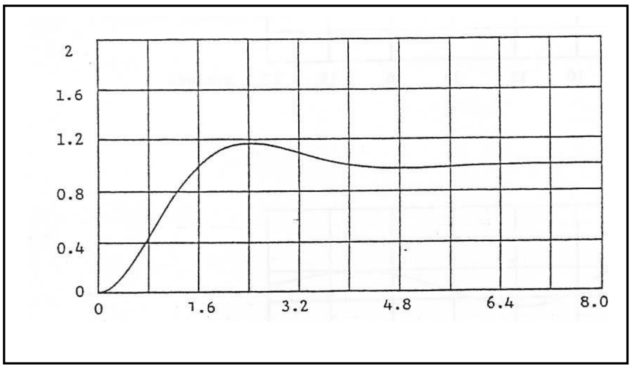
$$\begin{array}{r|l}
 & s + (28.48 + a) \\
\hline
s^2 + 1.521s + 2.313 & s^3 + (a+30)s^2 + 30as + K \\
 & s^3 + 1.521s^2 + 2.313s \\
\hline
 & (28.48 + a)s^2 + (30a - 2.323)s + K \\
 & (28.48 + a)s^2 + (1.521a + 43.32)s + 65.874 + 2.313a \\
\hline
 & (28.48a - 45.63)s + K - 0.744 - 2.313a
\end{array}$$

For zero remainders, $28.48a = 45.63$ Thus, $a = 1.6$ $K = 65.874 + 2.313a = 69.58$

Forward-Path Transfer Function:

$$G(s) = \frac{69.58}{s(s+1.6)(s+30)}$$

Unit-Step Response:



$y = 0.1$ when $t = 0.355$ sec.

$y = 0.9$ when $t = 1.43$ sec.

Rise Time:

$t_r = 1.43 - 0.355 = 1.075$ sec.

(b) The system is type 1.

(i) For a unit-step input, $e_{ss} = 0.$

(ii) For a unit-ramp input, $K_v = \lim_{s \to 0} sG(s) = \dfrac{K}{30a} = \dfrac{60.58}{30 \times 1.6} = 1.45 \qquad e_{ss} = \dfrac{1}{K_v} = 0.69$

7-29 (a) Characteristic Equation:

$$s^3 + 3s^2 + (2+K)s - K = 0$$

Apply the Routh-Hurwitz criterion to find the range of *K* for stability.

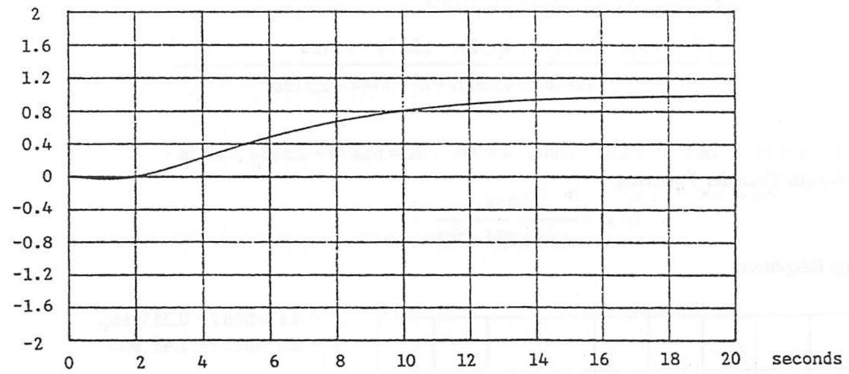
Routh Tabulation:

s^3	1	$2+K$
s^2	3	$-K$
s^1	$\dfrac{6+4K}{3}$	
s^0	$-K$	

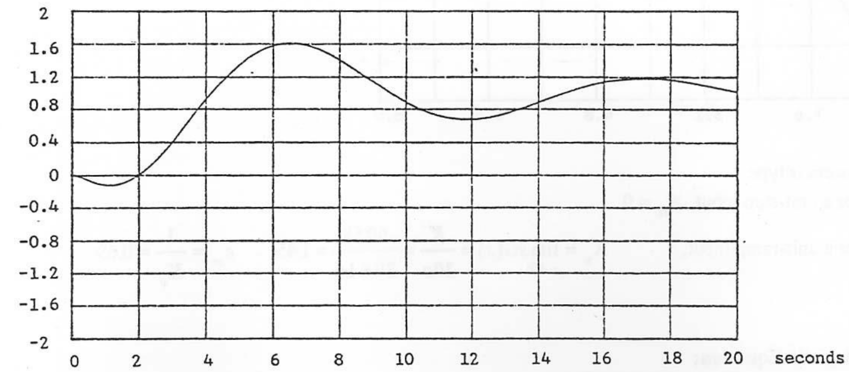
Stability Condition: -1.5 < *K* < 0 This simplifies the search for *K* for two equal roots.

When *K* = –0.27806, the characteristic equation roots are: –0.347, –0.347, and –2.3054.

(b) Unit-Step Response: (*K* = –0.27806)



(c) Unit-Step Response ($K = -1$)



The step responses in (a) and (b) all have a negative undershoot for small values of t. This is due to the zero of $G(s)$ that lies in the right-half s-plane.

7-30 (a) The state equations of the closed-loop system are:

$$\frac{dx_1}{dt} = -x_1 + 5x_2 \qquad \frac{dx_2}{dt} = -6x_1 - k_1x_1 - k_2x_2 + r$$

The characteristic equation of the closed-loop system is

$$\Delta = \begin{vmatrix} s+1 & -5 \\ 6+k_1 & s+k_2 \end{vmatrix} = s^2 + (1+k_2)s + (30+5k_1+k_2) = 0$$

For $\omega_n = 10 \text{ rad/sec}, \quad 30 + 5k_1 + k_2 = \omega_n^2 = 100.$ Thus $5k_1 + k_2 = 70$

(b) For $\zeta = 0.707, \; 2\zeta\omega_n = 1 + k_2.$ Thus $\omega_n = 1 + \frac{k_2}{1.414}.$

$$\omega_n^2 = \frac{(1+k_2)^2}{2} = 30 + 5k_1 + k_2 \qquad \text{Thus} \quad k_2^2 = 59 + 10k_1$$

(c) For $\omega_n = 10 \text{ rad/sec}$ and $\zeta = 0.707,$

$$5k_1 + k_2 = 100 \quad \text{and} \quad 1 + k_2 = 2\zeta\omega_n = 14.14 \quad \text{Thus} \quad k_2 = 13.14$$

Solving for k_1, we have $k_1 = 11.37.$

(d) The closed-loop transfer function is

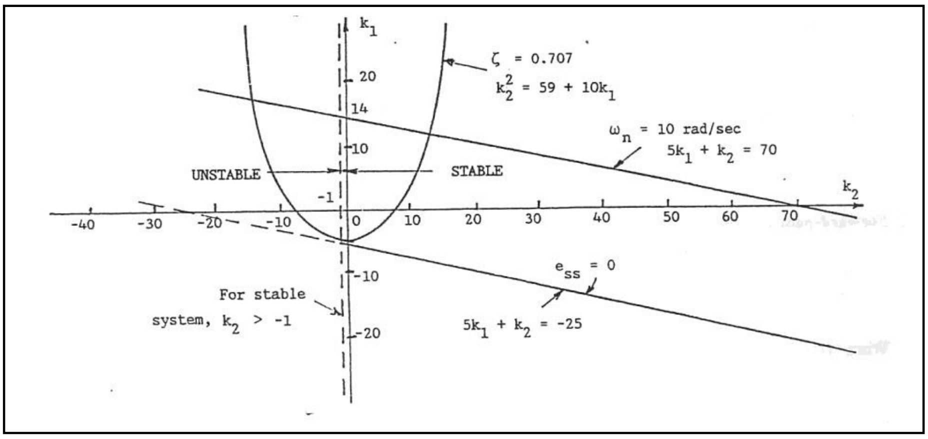
$$\frac{Y(s)}{R(s)} = \frac{5}{s^2 + (k_2+1)s + (30+5k_1+k_2)} = \frac{5}{s^2 + 14.14s + 100}$$

For a unit-step input, $\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \frac{5}{100} = 0.05$

(e) For zero steady-state error due to a unit-step input,

$$30 + 5k_1 + k_2 = 5 \qquad \text{Thus} \qquad 5k_1 + k_2 = -25$$

Parameter Plane k_1 versus k_2:



7-31 (a) Closed-Loop Transfer Function

$$\frac{Y(s)}{R(s)} = \frac{100\left(K_P + K_D s\right)}{s^2 + 100K_D s + 100K_P}$$

The system is stable for $K_P > 0$ and $K_D > 0$.

(b) Characteristic Equation:

$$s^2 + 100K_D s + 100K_P = 0$$

(b) For $\zeta = 1, \quad 2\zeta\omega_n = 100K_D$.

$$\omega_n = 10\sqrt{K_P} \quad \text{Thus} \quad 2\omega_n = 100K_D = 20\sqrt{K_P} \qquad K_D = 0.2\sqrt{K_P}$$

(c) See parameter plane in part **(g)**.

(d) See parameter plane in part **(g)**.

(e) Parabolic error constant $K_a = 1000\ \text{sec}^{-2}$

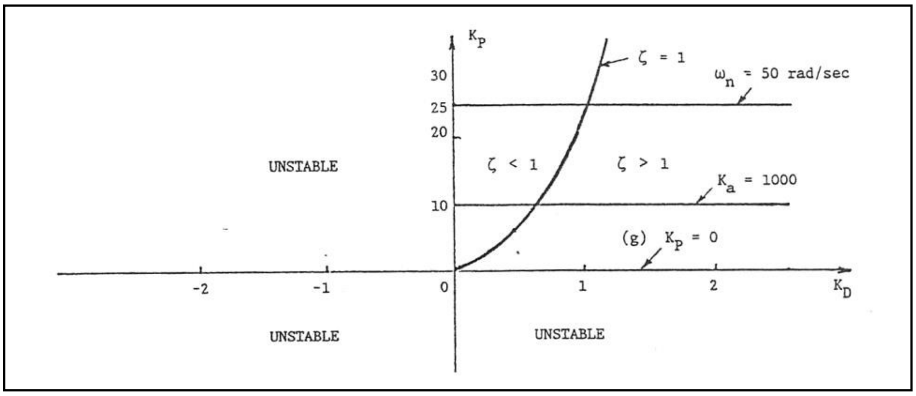
$$K_a = \lim_{s\to 0} s^2 G(s) = \lim_{s\to 0} 100\left(K_P + K_D s\right) = 100K_P = 1000 \qquad \text{Thus} \quad K_P = 10$$

(f) Natural undamped frequency $\omega_n = 50$ rad/sec.

$$\omega_n = 10\sqrt{K_P} = 50 \qquad \text{Thus} \qquad K_P = 25$$

(g) When $K_P = 0$,

$$G(s) = \frac{100K_D s}{s^2} = \frac{100K_D}{s} \qquad \text{(pole-zero cancellation)}$$



7-32 (a) Forward-path Transfer Function:

$$G(s) = \frac{Y(s)}{E(s)} = \frac{KK_i}{s\left[Js(1+Ts)+K_iK_t\right]} = \frac{10K}{s\left(0.001s^2+0.01s+10K_t\right)}$$

When $r(t) = tu_s(t)$, $\quad K_v = \lim_{s\to 0} sG(s) = \frac{K}{K_t} \qquad e_{ss} = \frac{1}{K_v} = \frac{K_t}{K}$

(b) When $r(t) = 0$

$$\frac{Y(s)}{T_d(s)} = \frac{1+Ts}{s\left[Js(1+Ts)+K_iK_t\right]+KK_i} = \frac{1+0.1s}{s\left(0.001s^2+0.01s+10K_t\right)+10K}$$

For $T_d(s) = \frac{1}{s}$ $\quad \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = \frac{1}{10K}$ $\qquad$ if the system is stable.

(c) The characteristic equation of the closed-loop system is

$$0.001s^3 + 0.01s^2 + 0.1s + 10K = 0$$

The system is unstable for $K > 0.1$. So we can set K to just less than 0.1. Then, the minimum value of the steady-state value of $y(t)$ is

$$\left.\frac{1}{10K}\right|_{K=0.1^-} = 1^+$$

However, with this value of K, the system response will be very oscillatory. The maximum overshoot will be nearly 100%.

(d) For K = 0.1, the characteristic equation is

$$0.001s^3 + 0.01s^2 + 10K_t s + 1 = 0 \quad \text{or} \quad s^3 + 10s^2 + 10^4 K_t s + 1000 = 0$$

For the two complex roots to have real parts of –2/5. we let the characteristic equation be written as

$$(s+a)\left(s^2+5s+b\right)=0 \qquad \text{or} \qquad s^3+(s+5)s^2+(5a+b)s+ab=0$$

Then, $a+5=10$ $\quad a=5$ $\quad ab=1000$ $\quad b=200$ $\quad 5a+b=10^4K_t$ $\quad K_t=0.0225$

The three roots are: $s=-a=-5$ $\quad s=-a=-5$ $\quad s=-2.5\pm j13.92$

7-33) Rise time: $t_r \cong \frac{0.8+2.5\,\xi}{\omega_n} = \frac{0.8+2.5*0.6}{5} = 0.56\ sec$

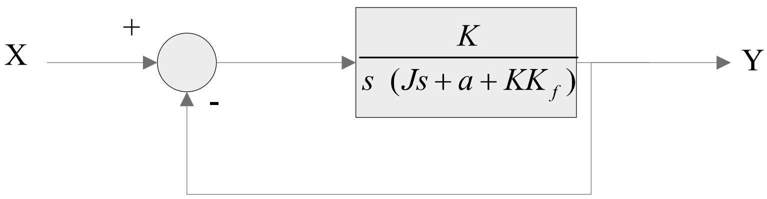
Peak time: $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{3.14}{5\sqrt{0.64}} = 0.785\ sec$

Maximum overshoot: $M_p = e^{-\frac{\pi\xi}{\sqrt{1-\xi^2}}} = e^{-\frac{0.6\pi}{0.8}} = 0.095$

Settling time: $t_s \cong \frac{3.2}{\xi\omega_n} \qquad 0<\xi<0.69$

⇨ $t_s \cong \frac{3.2}{0.615} \cong 1.067\ sec$

7-34)



- ⇨ $\frac{Y(s)}{X(s)} = \frac{K}{Js^2+(a+KK_f)s+K}$
- ⇨ $M_p = \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right) = 0.2 \Rightarrow \xi = 0.456$
- ⇨ $t_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = 0.1 \Rightarrow \omega_n = 0.353$
- ⇨ $G(s) = \frac{\omega_n^2}{s(s+2\,\xi\omega_n)} = \frac{\frac{K}{J}}{s\left(s+\frac{a+KK_f}{J}\right)}$
- ⇨ $\begin{cases} \omega_n = \sqrt{\frac{K}{J}} & \rightarrow \quad K = 0.125 \\ 2\,\xi\omega_n = \frac{a+KK_f}{J} & \rightarrow \quad K_f = \frac{2J\,\xi\omega_n - a}{K} \cong -5.42 \end{cases}$
- ⇨ $t_r = \frac{0.8+2.5f}{\omega_n} \cong 5.49\ sec$
- ⇨ $t_s = \frac{3.2}{\xi\omega_n} \cong 19.88\ sec$

7-35) a)

$$\begin{cases} \dot{x}_1 = -x_1 - x_2 + u_1 + u_2 \\ \dot{x}_2 = 6.5x_1 + u_1 \\ y_1 = x_1 \\ y_2 = x_2 \end{cases}$$

$$\begin{cases} sX_1(s) = -X_1(s) - X_2(s) + U_1(s) + U_2(s) \quad (1) \\ sX_2(s) = 6.5X_1(s) + U_1(s) \quad (2) \\ Y_1(s) = X_1(s) \\ Y_2(s) = X_2(s) \end{cases}$$

⇨ $(s+1)x_1(s) = -\frac{6.5}{s}X(s) + \frac{U_1(s)}{s} + U_1(s) + U_2(s)$

⇨ $(s^2 + s + 6.5)X_1(s) = (s-1)U_1(s) + U_2(s)$

⇨ $Y_1(s) = X_1(s) = \frac{s-1}{s^2+s+6.5}U_1(s) + \frac{5}{s^2+s+6.5}U_2(s)$

Substituting into equation (2) gives:

$$Y_2(s) = X_2(s) = \frac{s+7.5}{s^2+s+6.5}U_1(s) + \frac{6.5}{s^2+s+6.5}U_2(s)$$

Since the system is multi input and multi output, there are 4 transfer functions as:

$$\left[\frac{Y_1(s)}{U_1(s)}\right]_{U_2=0}, \left[\frac{Y_1(s)}{U_2(s)}\right]_{U_1=0}, \left[\frac{Y_2(s)}{U_1(s)}\right]_{U_2=0}, \left[\frac{Y_2(s)}{U_2(s)}\right]_{U_1=0}$$

To find the unit step response of the system, let's consider

$$\frac{Y_2(s)}{U_2(s)} = \frac{6.5}{s^2+s+6.5} = \frac{\omega_n^2}{s(s^2 + 2\,\xi\omega_n + \omega_n^2)}$$

$$where \begin{cases} \omega_n^2 = 6.5 \rightarrow \omega_n = \sqrt{6.5} \\ 2\,\xi\omega_n = 1 \rightarrow \xi = \frac{1}{2\,\omega_n} = \frac{1}{2\sqrt{6.5}} \end{cases}$$

By looking at the Laplace transform function table:

$$y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2}\,t + \theta\right)$$

where $\theta = \cos^{-1}\xi$

b)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - x_2 + u \\ y = x_1 \end{cases}$$

Therefore:

$$\begin{cases} sX_1(s) = X_2(s) \\ sX_2(s) = -X_1(s) - X_2(s) + U(s) \\ Y(s) = X_1(s) \end{cases}$$

As a result:

$$s^2X_1(s) = -X_1(s) - sX_1(s) + U(s)$$

which means:

$$X_1(s) = \frac{1}{s^2 + s + 1}U(s)$$

The unit step response is:

$$Y(s) = X_1(s) = \frac{1}{s(s^2 + s + 1)}$$

Therefore as a result:

$$y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin \omega_n\sqrt{1-\xi^2}\,t$$

where $\omega_n = 1$ and $\xi\omega_n = 1/2$

c)

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 - x_2 + 4 \\ \dot{x}_3 = x_1 \\ y = x_3 \end{cases}$$

Therefore:

$$\begin{cases} sX_1(s) = X_2(s) \\ (s+1)X_2(s) = -X_1(s) + U(s) \rightarrow s(s+1)X_1(s) = -x_1(s) + U(s) \\ sX_3(s) = X_1(s) \\ Y(s) = X_3(s) \quad \rightarrow \quad Y(s) = \dfrac{X_1(s)}{s} \end{cases}$$

As a result, the step response of the system is:

$$Y(s) = \frac{1}{s^2(s^2 + s + 1)}$$

By looking up at the Laplace transfer function table:

$$Y(s) = \frac{\omega_n^2}{s^2(s^2+2\,\xi\omega_n+\,\omega_n^2)}$$

where $\omega_n = 1$, and $2\xi\,\omega_n = 1 \Rightarrow \xi = 1/2$

$$y(t) = t - \frac{2\xi}{\omega_n} + \frac{1}{\omega_n\sqrt{1-\xi^2}}e^{-\xi\omega_n t}\sin\left(\omega_n\sqrt{1-\xi^2} + \theta\right)$$

where $\theta = \cos^{-1}(2\xi^2 - 1) = \cos^{-1}(0.5)$, therefore,

$$y(t) = t - 1 + \frac{2}{\sqrt{3}}e^{-\xi\omega_n t}\sin\left(\frac{\sqrt{3}}{2}t + \theta\right)$$

7-36) MATLAB CODE

```
(a)

clear all
Amat=[-1 -1;6.5 0]
Bmat=[1 1;1 0]
Cmat=[1 0;0 1]
Dmat=[0 0;0 0]
disp(' State-Space Model is:')
Statemodel=ss(Amat,Bmat,Cmat,Dmat)
[mA,nA]=size(Amat);
rankA=rank(Amat);
disp(' Characteristic Polynomial:')
chareq=poly(Amat);

%p = poly(A) where A is an n-by-n matrix returns an n+1 element
%row vector whose elements are the coefficients of the characteristic
%polynomialdet(sI-A). The coefficients are ordered in descending powers.

[mchareq,nchareq]=size(chareq);
syms 's';

poly2sym(chareq,s)
disp(' Equivalent Transfer Function Model is:')

Hmat=Cmat*inv(s*eye(2)-Amat)*Bmat+Dmat
```

Since the system is multi input and multi output, there are 4 transfer functions as:

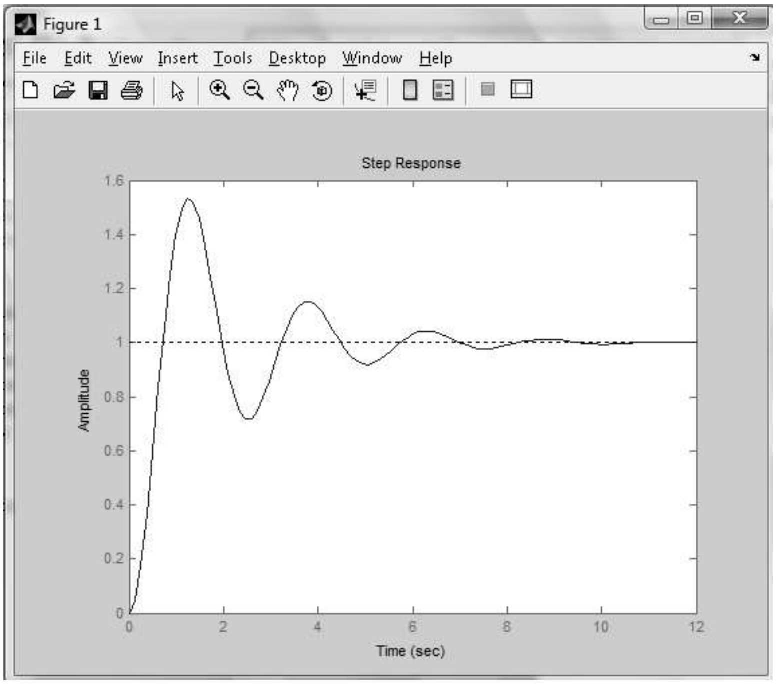
$$\left[\frac{Y_1(s)}{U_1(s)}\right]_{U_2=0},\left[\frac{Y_1(s)}{U_2(s)}\right]_{U_1=0},\left[\frac{Y_2(s)}{U_1(s)}\right]_{U_2=0},\left[\frac{Y_2(s)}{U_2(s)}\right]_{U_1=0}$$

To find the unit step response of the system, let's consider

$$\frac{Y_2(s)}{U_2(s)} = \frac{6.5}{s^2+s+6.5}$$

Let's obtain this term and find $Y_2(s)$ time response for a step input.

```
H22=Hmat(2,2)
ilaplace(H22/s)
Pretty(H22)
H22poly=tf([13/2],chareq)
step(H22poly)

H22 =

13/(2*s^2+2*s+13)

ans =

1-1/5*exp(-1/2*t)*(5*cos(5/2*t)+sin(5/2*t))

                                                  13
                                            ---------------
                                                2
                                            2 s  + 2 s + 13

Transfer function:
     6.5
-------------
s^2 + s + 6.5
```




```
To find the step response H11, H12, and H21 follow the same procedure.
```

Other parts are the same.

7-37) Impulse response:

a) $Y(s) = \frac{6.5}{s^2+s+6.5}$ and $U(s) = 1$, therefore,

$$y(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n\sqrt{1-\xi^2}\, t\right)$$

b) $Y(s) = \frac{1}{s^2+s+1}$

$$y(t) = \frac{\omega_n}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n\sqrt{1-\xi^2}\, t\right)$$

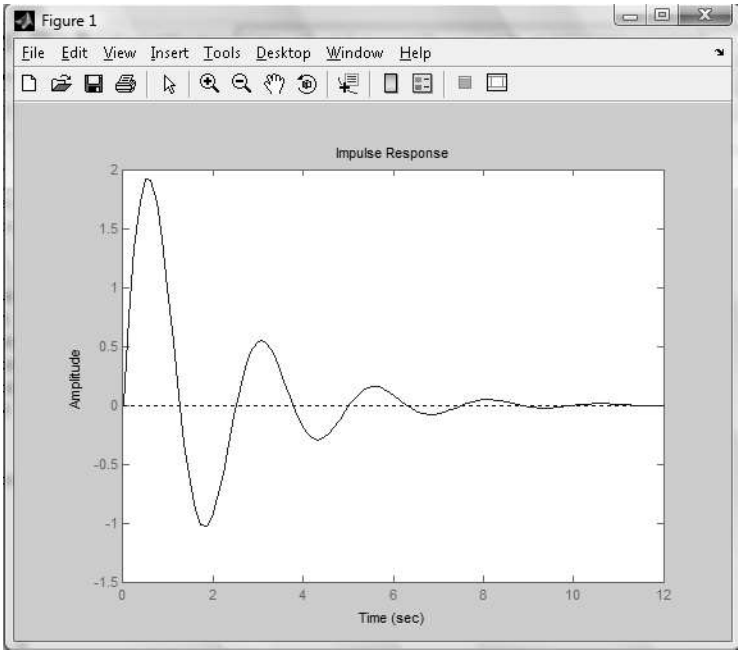
c) $Y(s) = \frac{1}{s(s^2+s+1)}$

$$y(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_n\sqrt{1-\xi^2}\, t + \alpha\right)$$

where $\alpha = \cos^{-1} \xi$

7-38) Use the approach in 7-36 except:

```
H22=Hmat(2,2)
ilaplace(H22)
Pretty(H22)
H22poly=tf([13/2],chareq)
impulse(H22poly)

H22 =

13/(2*s^2+2*s+13)

ans =

13/5*exp(-1/2*t)*sin(5/2*t)

                             13
                      ---------------
                         2
                      2 s  + 2 s + 13

Transfer function:
     6.5
-------------
s^2 + s + 6.5
```



Other parts are the same.

7-39) a) The displacement of the bar is:

$$x = L\sin\theta$$

Then the equation of motion is:

$$B\left(\frac{dy}{dt} - \frac{dx}{dt}\right) - K\tau = 0$$

As x is a function of θ and changing with time, then

$$\frac{dx}{dt} = L\frac{d\theta}{dt}\cos\theta$$

If θ is small enough, then sin θ ≈ θ and cos θ ≈ 1. Therefore, the equation of motion is rewritten as:

$$B(\dot{y} - L\dot{\theta}) - KL\,\theta = 0$$

$$L(B\dot{\theta} + K\,\theta) = B\dot{y}$$

$$L(Bs + K)\theta(s) = BsY(s)$$

$$\frac{\theta(s)}{Y(s)} = \frac{Bs}{L(Bs+K)}$$

(b) To find the unit step response, you can use the symbolic approach shown in Toolbox 2-1-1:

```
clear all
%s=tf('s');
syms s B L K
Theta=B*s/s/L/(B*s+K)
ilaplace(Theta)

Theta =
B/L/(B*s+K)

ans =
1/L*exp(-K*t/B)
```

Alternatively, assign values to B L K and find the step response – see solution to problem 7-36.

7-40 (a) $K_t = 10000 \text{ oz-in/rad}$

The Forward-Path Transfer Function:

$$G(s) = \frac{9\times10^{12}K}{s\left(s^4+5000s^3+1.067\times10^7 s^2+50.5\times10^9 s+5.724\times10^{12}\right)}$$
$$= \frac{9\times10^{12}K}{s(s+116)(s+4883)(s+41.68+j3178.3)(s+41.68-j3178.3)}$$

Routh Tabulation:

s^5	1	1.067×10^7	5.724×10^{12}
s^4	5000	50.5×10^9	$9\times10^{12}K$
s^3	5.7×10^5	$5.72\times10^{12}-1.8\times10^9 K$	0
s^2	$2.895\times10^8+1.579\times10^7 K$	$9\times10^{12}K$	
s^1	$\dfrac{16.6\times10^{13}+8.473\times10^{12}K-2.8422\times10^9 K^2}{29+1.579K}$		
s^0	$9\times10^{12}K$		

From the s^1 row, the condition of stability is $165710+8473K-2.8422K^2>0$

or $K^2-2981.14K-58303.427<0$ or $(K+19.43)(K-3000.57)<0$

Stability Condition: $0 < K < 3000.56$

The critical value of *K* for stability is 3000.56. With this value of *K*, the roots of the characteristic equation are: −4916.9, −41.57 + *j*3113.3, −41.57 + *j*3113.3, −*j*752.68, and *j*752.68

(b) $K_L = 1000$ oz-in/rad. The forward-path transfer function is

$$G(s) = \frac{9\times 10^{11}K}{s\left(s^4+5000s^3+1.582\times 10^6 s^2+5.05\times 10^9 s+5.724\times 10^{11}\right)}$$
$$= \frac{9\times 10^{11}K}{s(1+116.06)(s+4882.8)(s+56.248+j1005)(s+56.248-j1005)}$$

(c) Characteristic Equation of the Closed-Loop System:

$$s^5+5000s^4+1.582\times 10^6 s^3+5.05\times 10^9 s^2+5.724\times 10^{11}s+9\times 10^{11}K=0$$

Routh Tabulation:

s^5	1	1.582×10^6	5.724×10^{11}
s^4	5000	5.05×10^9	$9\times 10^{11}K$
s^3	5.72×10^5	$5.724\times 10^{11}-1.8\times 10^8 K$	0
s^2	$4.6503\times 10^7+1.5734\times 10^6 K$	$9\times 10^{11}K$	
s^1	$\dfrac{26.618\times 10^{18}+377.43\times 10^{15}K-2.832\times 10^{14}K^2}{4.6503\times 10^7+1.5734\times 10^6 K}$		
s^0	$9\times 10^{11}K$		

From the s^1 row, the condition of stability is $26.618\times 10^4 + 3774.3K - 2.832K^2 > 0$

Or, $K^2 - 1332.73K - 93990 < 0$ or $(K-1400)(K+67.14) < 0$

Stability Condition: $0 < K < 1400$

The critical value of *K* for stability is 1400. With this value of K, the characteristic equation root are:

–4885.1, –57.465 + *j*676, –57.465 – *j*676, *j*748.44, and –*j*748.44

(c) $K_L = \infty$.

Forward-Path Transfer Function:

$$G(s) = \frac{nK_s K_i K}{s\left[L_a J_T s^2 + \left(R_a J_T + R_m L_a\right)s + R_a B_m + K_i K_b\right]} \qquad J_T = J_m + n^2 J_L$$

$$= \frac{891100K}{s\left(s^2 + 5000s + 566700\right)} = \frac{891100K}{s(s+116)(s+4884)}$$

Characteristic Equation of the Closed-Loop System:

$$s^3 + 5000s^2 + 566700s + 891100K = 0$$

Routh Tabulation:

s^3	1	566700
s^2	5000	$891100K$
s^1	$566700 - 178.22K$	
s^0	$8991100K$	

From the s^1 row, the condition of *K* for stability is $566700 - 178.22K > 0$.

Stability Condition: $0 < K < 3179.78$

The critical value of K for stability is 3179.78. With $K = 3179.78$, the characteristic equation roots are

-5000, $j752.79$, and $-j752.79$.

When the motor shaft is flexible, K_L is finite, two of the open-loop poles are complex. As the shaft becomes stiffer, K_L increases, and the imaginary parts of the open-loop poles also increase. When $K_L = \infty$, the shaft is rigid, the poles of the forward-path transfer function are all real. Similar effects are observed for the roots of the characteristic equation with respect to the value of K_L.

7-41 (a)

$$G_c(s) = 1 \qquad G(s) = \frac{100(s+2)}{s^2 - 1} \qquad K_p = \lim_{s \to 0} G(s) = -200$$

When $d(t) = 0$, the steady-state error due to a unit-step input is

$$e_{ss} = \frac{1}{1+K_p} = \frac{1}{1-200} = -\frac{1}{199} = -0.005025$$

(b)

$$G_c(s) = \frac{s+\alpha}{s} \qquad G(s) = \frac{100(s+2)(s+\alpha)}{s\left(s^2-1\right)} \qquad K_p = \infty \qquad e_{ss} = 0$$

(c)

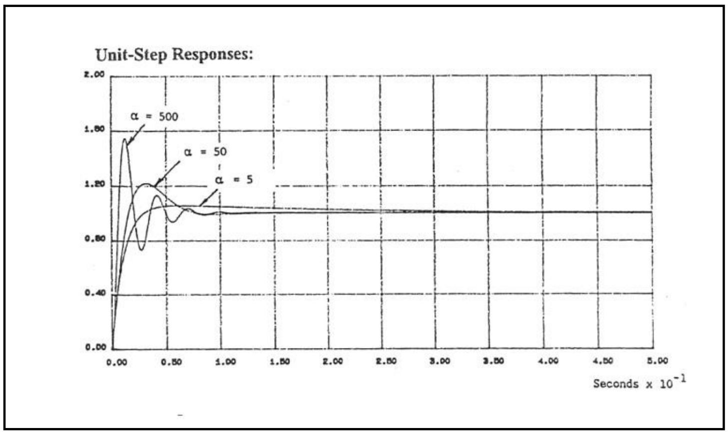
$\alpha = 5$ maximum overshoot = 5.6%

$\alpha = 50$ maximum overshoot = 22%

$\alpha = 500$ maximum overshoot = 54.6%

As the value of α increases, the maximum overshoot increases because the damping effect of the zero at $s=-\alpha$ becomes less effective.

Unit-Step Responses:



(d) $r(t)=0$ and $G_c(s)=1$. $d(t)=u_s(t)$ $\quad D(s)=\dfrac{1}{s}$

System Transfer Function: (*r* = 0)

$$\left.\frac{Y(s)}{D(s)}\right|_{r=0}=\frac{100(s+2)}{s^3+100s^2+(199+100\alpha)s+200\alpha}$$

Output Due to Unit-Step Input:

$$Y(s)=\frac{100(s+2)}{s\left[s^3+100s^2+(199+100\alpha)s+200\alpha\right]}$$

$$y_{ss}=\lim_{t\to\infty}y(t)=\lim_{s\to 0}sY(s)=\frac{200}{200\alpha}=\frac{1}{\alpha}$$

(e) $r(t)=0,\quad d(t)=u_s(t)$

$$G_c(s)=\frac{s+\alpha}{s}$$

System Transfer Function [*r*(*t*) = 0]

$$\left.\frac{Y(s)}{D(s)}\right|_{r=0} = \frac{100s(s+20}{s^3+100s^2+(199+100\alpha)s+200\alpha} \qquad D(s)=\frac{1}{s}$$

$$y_{ss} = \lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) = 0$$

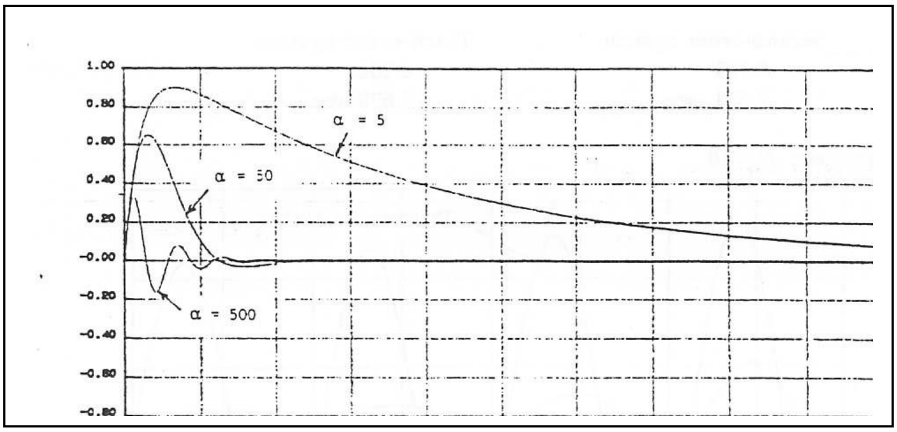
(f)

$$\alpha = 5 \qquad \left.\frac{Y(s)}{D(s)}\right|_{r=0} = \frac{100s(s+2)}{s^3+100s^2+699s+1000}$$

$$\alpha = 50 \qquad \left.\frac{Y(s)}{D(s)}\right|_{r=0} = \frac{100s(s+2)}{s^3+100s^2+5199s+10000}$$

$$\alpha = 5000 \qquad \left.\frac{Y(s)}{D(s)}\right|_{r=0} = \frac{100s(s+2)}{s^3+100s^2+50199s+100000}$$

Unit-Step Responses:



(g) As the value of α increases, the output response *y*(*t*) due to *r*(*t*) becomes more oscillatory, and the overshoot is larger. As the value of α increases, the amplitude of the output response *y*(*t*) due to *d*(*t*) becomes smaller and more oscillatory.

7-42 (a) Forward-Path Transfer function: **Characteristic Equation:**

$$G(s) = \frac{H(s)}{E(s)} = \frac{10N}{s(s+1)(s+10)} \cong \frac{N}{s(s+1)} \qquad s^2 + s + N = 0$$

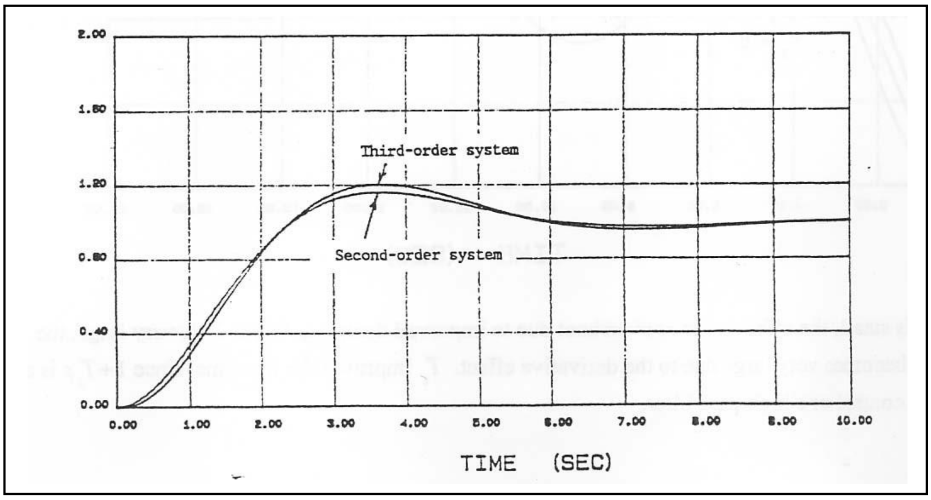
N=1: Characteristic Equation: $s^2 + s + 1 = 0$ $\qquad \zeta = 0.5 \qquad \omega_n = 1$ rad/sec.

$\text{Maximum overshoot} = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.163$ (16.3%) Peak time $t_{\max} = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 3.628$ sec.

N=10: Characteristic Equation: $s^2 + s = 10 = 0$ $\qquad \zeta = 0.158 \qquad \omega_n = 10$ rad/sec.

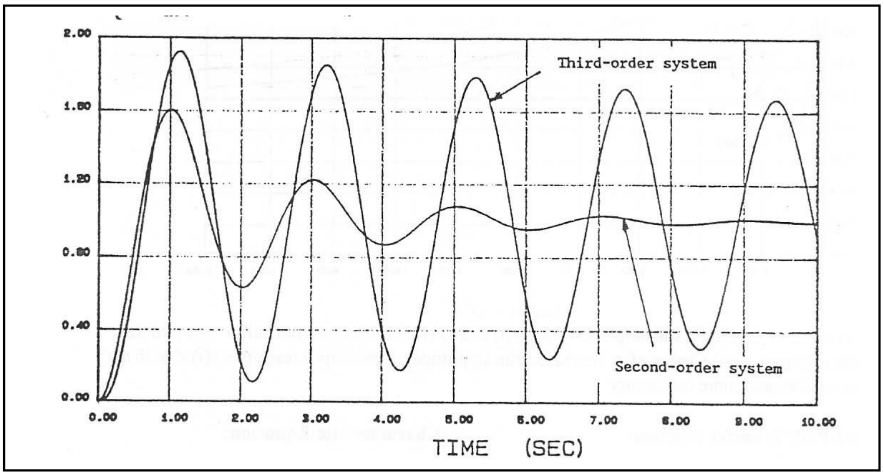
Maximum overshoot $= e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.605$ (60.5%) Peak time $t_{\max} = \dfrac{\pi}{\omega_n\sqrt{1-\zeta^2}} = 1.006$ sec.

(b) Unit-Step Response: *N* = 1



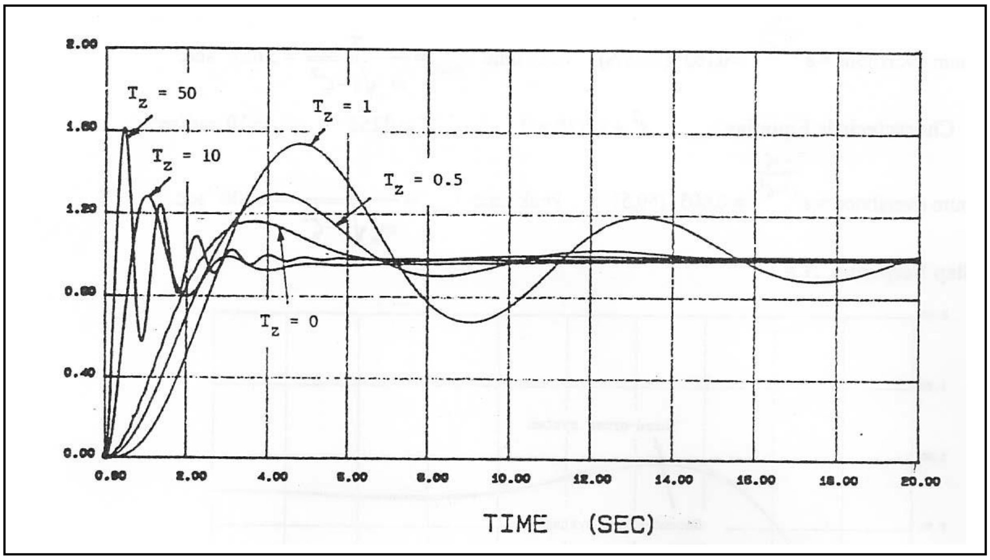
	Second-order System	Third-order System
Maximum overshoot	0.163	0.206
Peak time	3.628 sec.	3.628 sec.

Unit-Step Response: *N* = 10



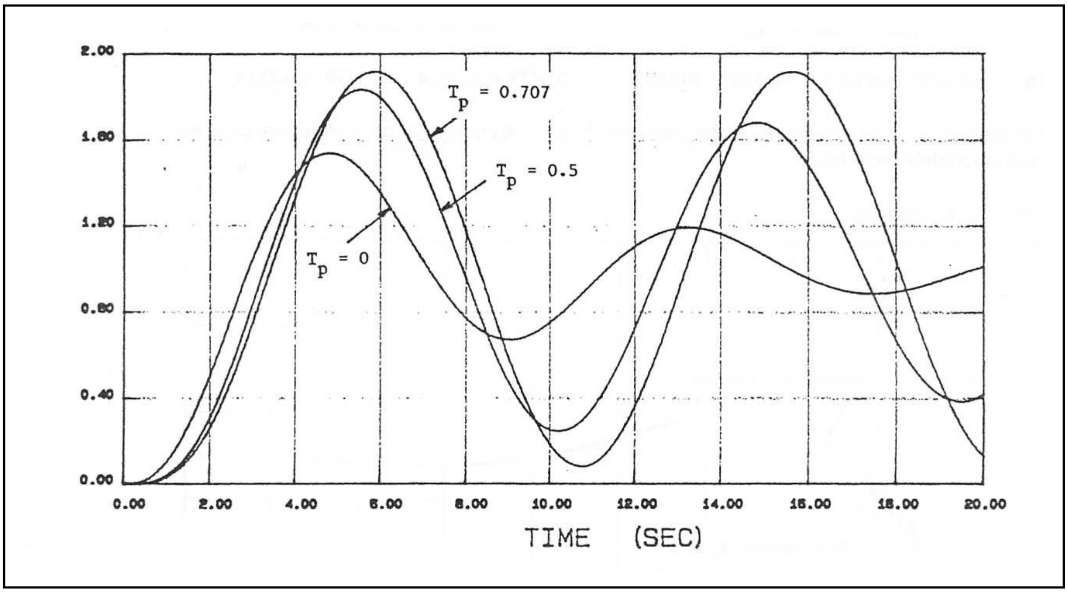
	Second-order System	Third-order System
Maximum overshoot	0.605	0.926
Peak time	1.006 sec.	1.13 sec.

7-43 **Unit-Step Responses:**



When T_z is small, the effect is lower overshoot due to improved damping. When T_z is very large, the overshoot becomes very large due to the derivative effect. T_z improves the rise time, since $1+T_z s$ is a derivative control or a high-pass filter.

7-44 **Unit-Step Responses**

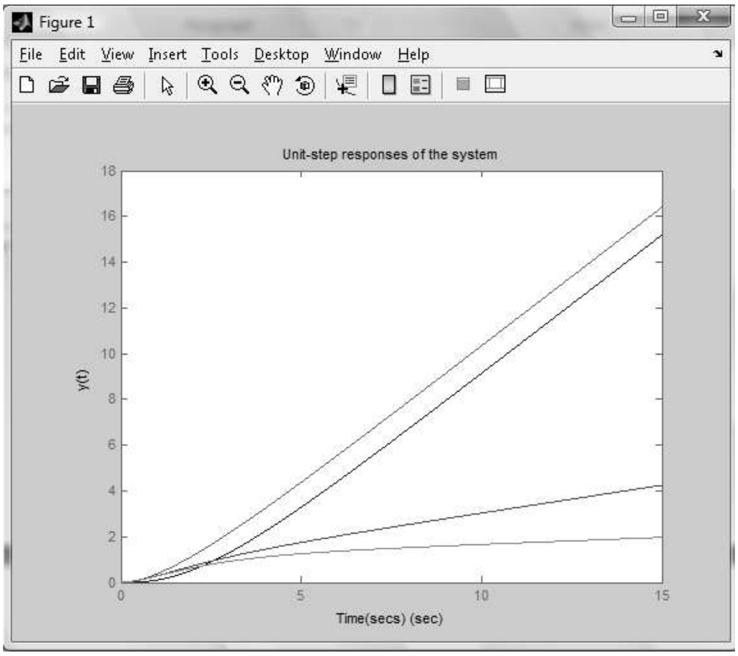


The effect of adding the pole at $s = -\dfrac{1}{T_p}$ to $G(s)$ is to increase the rise time and the overshoot. The system is less stable. When $T_p > 0.707$, the closed-loop system is stable.

7-45) You may use the ACSYS software developed for this book. For description refer to Chapter 9. **We use a MATLAB code similar to toolboxes in Chapter 7 to solve this problem.**

(a) **Using MATLAB**

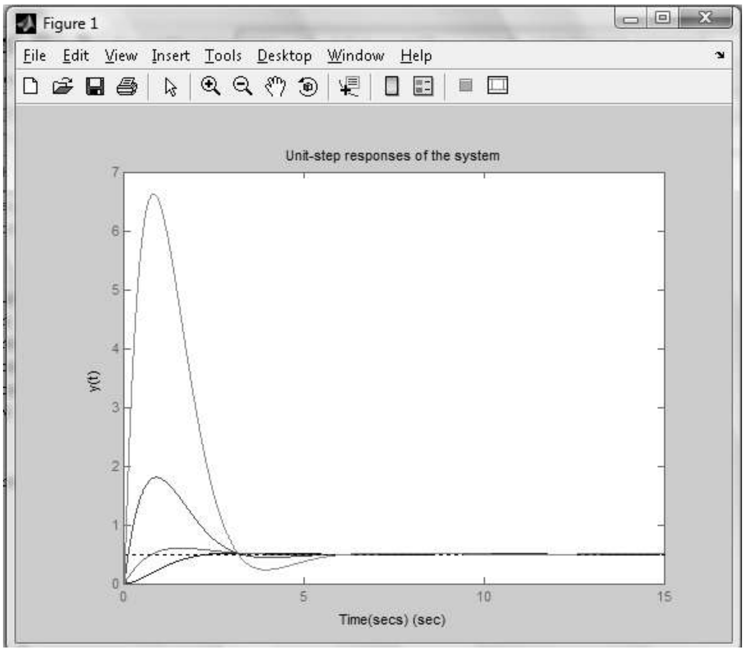
```
clear all
num = [];
den = [0 -0.55 -1.5];
G=zpk(num,den,1)
t=0:0.001:15;
step(G,t);
hold on;
for Tz=[1 5 20];
t=0:0.001:15;
num = [-1/Tz];
den = [0 -0.55 -1.5];
G=zpk(num,den,1)
step(G,t);
hold on;
end
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')


Zero/pole/gain:
        1
------------------
s (s+0.55) (s+1.5)


Zero/pole/gain:
      (s+1)
------------------
s (s+0.55) (s+1.5)


Zero/pole/gain:
     (s+0.2)
------------------
s (s+0.55) (s+1.5)


Zero/pole/gain:
     (s+0.05)
------------------
s (s+0.55) (s+1.5)
```



(b)

```
clear all
for Tz=[0 1 5 20];
t=0:0.001:15;
num = [Tz 1];
den = [1 2 2];
G=tf(num,den)
step(G,t);
hold on;
end
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')

Transfer function:
      1
-------------
s^2 + 2 s + 2


Transfer function:
    s + 1
-------------
s^2 + 2 s + 2


Transfer function:
   5 s + 1
-------------
s^2 + 2 s + 2
```

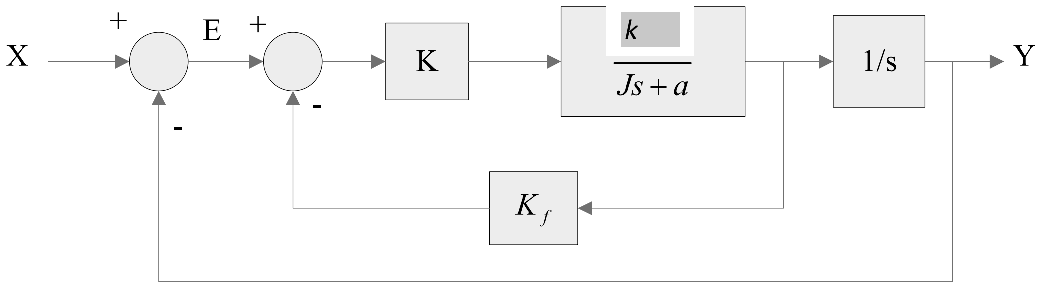
```
Transfer function:
  20 s + 1
-------------
s^2 + 2 s + 2
```



Follow the same procedure for other parts.

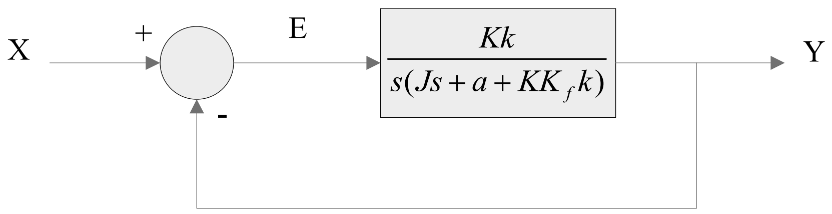
7-46) Since the system is linear we use superposition to find Y, for inputs X and D

First, consider D = 0



Then

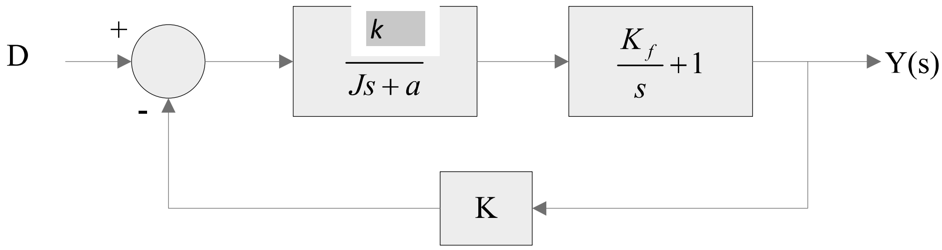
$$\frac{Y}{X} = G(s) = \frac{G_1}{1+G_1} = \frac{Kk}{s(Js+a+KkK_f)+Kk}\ ;\ G_1 = \frac{Kk}{s(Js+a+KkK_f)}$$



According to above block diagram:

$$E(s) = X(s) - Y(s) = X(s) - \frac{X(s)G_1(s)}{1+G_1(s)} = \frac{1}{1+G_1(s)}X(s)$$

Now consider X = 0, then:



Accordingly,

$$G_2(s) = \frac{k\left(K_f + s\right)}{s(Js+a)}$$

and

$$Y(s) = \frac{G_2(s)}{1+KG_2(s)}D(s) = \frac{k\left(K_f + s\right)}{s(Js+a) + k\left(K_f + s\right)K}D(s)$$

In this case, *E(s) = D-KY(s)*

$$E(s) = D(s) - \frac{KG_2(s)}{1+KG_2(s)}D(s) = -\frac{1}{1+KG_2(s)}D(s)$$

Now the steady state error can be easily calculated by:

$$\begin{cases} for\ unit\ step\ input\ X = 1/s:\ \ e_{ss} = \lim_{s\to 0}\dfrac{1}{1+G_1(s)} = \dfrac{Kk}{s\left(Js+a+KkK_f\right)} \\ \lim_{s\to 0}\dfrac{1}{1+\dfrac{Kk}{s\left(Js+a+KkK_f\right)}} = \lim_{s\to 0}\dfrac{s\left(Js+a+KkK_f\right)}{s\left(Js+a+KkK_f\right)+Kk} = 0 \\ for\ ramp\ input\ D = \dfrac{1}{s^2}:\ \ e_{ss} = \lim_{s\to 0} -\dfrac{1}{s\left(1+KG_2(s)\right)} \\ = \lim_{s\to 0}\dfrac{-1}{s\left(1+K\dfrac{k\left(K_f+s\right)}{s(Js+a)}\right)} \\ = \lim_{s\to 0}\dfrac{-s(Js+a)}{s\left(s(Js+a)+Kk\left(K_f+s\right)\right)} = -\dfrac{-a}{KkK_f} \end{cases}$$

(c) The overall response is obtained through superposition

$$Y(s) = Y(s)\big|_{D=0} + Y(s)\big|_{X=0}$$
$$y(t) = y(t)\big|_{d(t)=0} + y(t)\big|_{x(t)=0}$$

$$Y(s) = \frac{Kk}{s\left(Js+a+KkK_f\right)+Kk}X(s) + \frac{k\left(K_f+s\right)}{s(Js+a)+k\left(K_f+s\right)K}D(s)$$

MATLAB

```
clear all
syms s K k J a Kf
X=1/s;
D=1/s^2
Y=K*k*X/(s*(J*s+a+K*k*Kf)+K*k)+k*(Kf+s)*D/(s*(J*s+a)+k*(Kf+s)*K)
ilaplace(Y)
D =
1/s^2
Y =
K*k/s/(s*(J*s+a+K*k*Kf)+K*k)+k*(Kf+s)/s^2/(s*(J*s+a)+k*(Kf+s)*K)
ans =

1+t/K+1/k/K^2/Kf/(a^2+2*a*K*k+K^2*k^2-
4*J*K*k*Kf)^(1/2)*sinh(1/2*t/J*(a^2+2*a*K*k+K^2*k^2-4*J*K*k*Kf)^(1/2))*exp(-
1/2*(a+K*k)/J*t)*(a^2+a*K*k-2*J*K*k*Kf)-cosh(1/2*t/J*(a^2+2*a*K*k*Kf+K^2*k^2*Kf^2-
4*J*K*k)^(1/2))*exp(-1/2*(a+K*k*Kf)/J*t)-(a+K*k*Kf)/(a^2+2*a*K*k*Kf+K^2*k^2*Kf^2-
4*J*K*k)^(1/2)*sinh(1/2*t/J*(a^2+2*a*K*k*Kf+K^2*k^2*Kf^2-4*J*K*k)^(1/2))*exp(-
1/2*(a+K*k*Kf)/J*t)+1/k/K^2/Kf*a*(-1+exp(-
1/2*(a+K*k)/J*t)*cosh(1/2*t/J*(a^2+2*a*K*k+K^2*k^2-4*J*K*k*Kf)^(1/2)))
```

7-47) (a) Find the $\int_0^\infty e(t)dt$ when *e(t)* is the error in the unit step response.

As the system is stable then $\int_0^\infty e(t)dt$ will converge to a constant value:

$$\int_0^\infty e(t)dt = \lim_{s\to 0} s\frac{E(s)}{s} = \lim_{s\to 0} E(s)$$

$$\frac{Y(s)}{X(s)} = \frac{G(s)}{1+G(s)} = \frac{(A_1 s+1)(A_2 s+1)\ldots(A_n s+1)}{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)} \qquad n \le m$$

$$\mathrm{E}(\mathrm{s}) = X(s) - Y(s) = X(s) - \frac{G(s)}{1+G(s)}X(s) = \frac{1}{1+G(s)}X(s)$$

$$= \left(1 - \frac{(A_1 s+1)(A_2 s+1)\ldots(A_n s+1)}{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)}\right)X(s) = \left(\frac{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)-(A_1 s+1)(A_2 s+1)\ldots(A_n s+1)}{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)}\right)X(s)$$

$$\lim_{s\to 0} E(s) = \lim_{s\to 0}\frac{1}{s}\left(\frac{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)-(A_1 s+1)(A_2 s+1)\ldots(A_n s+1)}{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)}\right)$$

$$= \left(B_1 + B_2 + \cdots + B_m\right) - \left(A_1 + A_2 + \cdots + A_n\right)$$

$$G(s) = \left(\frac{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)}{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)-(A_1 s+1)(A_2 s+1)\ldots(A_n s+1)}\right) - 1$$

$$G(s) = \left(\frac{(A_1 s+1)(A_2 s+1)\ldots(A_n s+1)}{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)-(A_1 s+1)(A_2 s+1)\ldots(A_n s+1)}\right)$$

$$e_{ss} = \lim_{s\to 0} s\frac{1}{s}\left(1 - \frac{(A_1 s+1)(A_2 s+1)\ldots(A_n s+1)}{(B_1 s+1)(B_2 s+1)\ldots(B_m s+1)}\right) = 0$$

(b) Calculate $\frac{1}{K} = \frac{1}{\lim\limits_{s \to 0} sG(s)}$

Recall

$$\mathrm{E}(\mathrm{s}) = X(s) - Y(s) = X(s) - \frac{G(s)}{1+G(s)} X(s) = \frac{1}{1+G(s)} X(s)$$

Hence

$$\lim_{s \to 0} \mathrm{E}(\mathrm{s}) = \lim_{s \to 0} \frac{1}{1+G(s)} X(s) = \lim_{s \to 0} \frac{1}{s+sG(s)} = \lim_{s \to 0} \frac{1}{sG(s)} = \frac{1}{K_v}$$

Ramp Error Constant

7-48)

$$\frac{C(s)}{R(s)} = \frac{10(s+K)}{(s+p)(s+25)+(s+K)10} = \frac{10(s+K)}{s^2+(35+p)s+(25\,p+10K)}$$

Comparing with the second order prototype system and matching denominators:

$$\begin{cases} 25\,p+10K=\omega_n^2 \\ 35+p=2\,\xi\omega_n \end{cases}$$

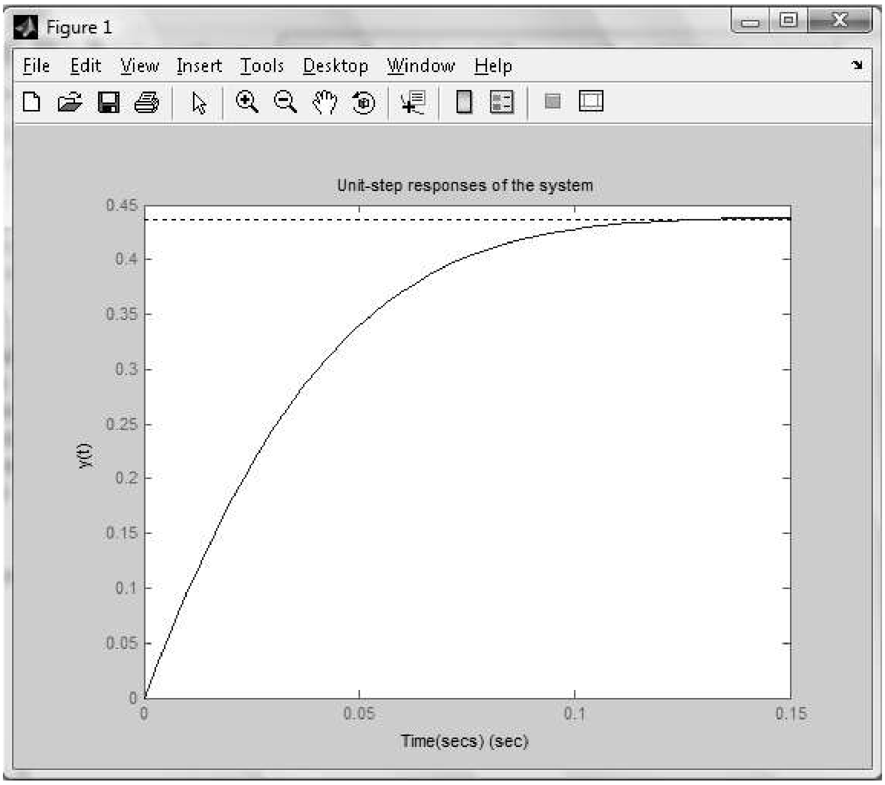
$$\begin{cases} M_\rho = \exp\left(-\dfrac{\pi\xi}{\sqrt{1-\xi^2}}\right) \geq 0.25 \ \rightarrow \ \dfrac{\pi\xi}{\sqrt{1-\xi^2}} \geq 1.386 \rightarrow \xi \geq 0.210 \\ t_s = \dfrac{3.2}{\xi\omega_n} \leq 0.1 \rightarrow \ \omega_n \leq 80, \ \ when\ 0<\xi<0.69 \end{cases}$$

Let $\xi = 0.4$ and $\omega_n = 80$

Then

$$\begin{cases} p=(2\,\xi\omega_n)(35)=29 \\ K=\frac{\omega_n^2-25\,p}{100}=56.25 \end{cases}$$

```
clear all
p=29;
K=56.25;
num = [10 10*K];
den = [1 35+p 25*p+10*K];
G=tf(num,den)
step(G);
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')

Transfer function:
  10 s + 562.5
-----------------
s^2 + 64 s + 1288
```

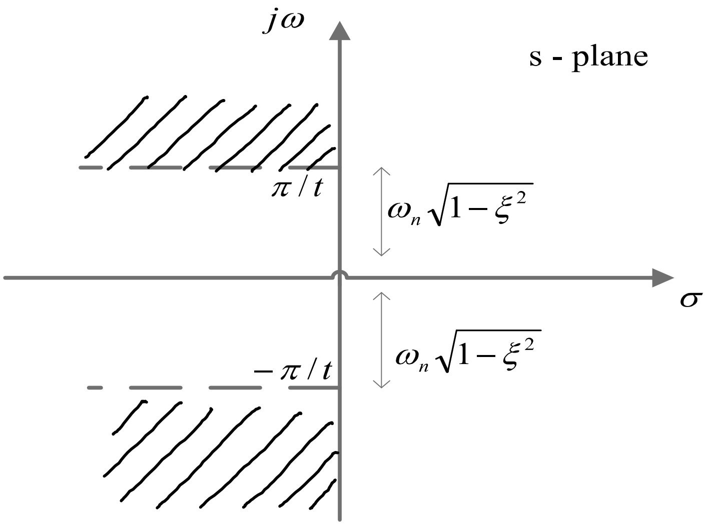


7-49) According to the maximum overshoot:

$t_{max} = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

which should be less than t, then

$\frac{\pi}{\omega_n\sqrt{1-\xi^2}} < t$ or $\omega_n\sqrt{1-\xi^2} > \frac{\pi}{t}$



7-50) Using a 2nd order prototype system format, from Figure 7-9, ω_n is the radial distance from the complex conjugate roots to the origin of the s-plane, then ω_n with respect to the origin of the shown region is $\omega_n \approx 3.6$.

Therefore the natural frequency range in the region shown is around $2.6 \leq \omega_n \leq 4.6$

On the other hand, the damping ratio ζ at the two dashed radial lines is obtained from:

$$\begin{cases} \zeta_1 = \cos(\pi/2 - \alpha_1) = \sin\alpha_1 \\ \zeta_2 = \cos(\pi/2 - \alpha_2) = \sin\alpha_2 \end{cases}$$

The approximation from the figure gives:

$$\begin{cases} \zeta_1 \approx 0.56 \\ \zeta_2 \approx 0.91 \end{cases}$$

Therefore $0.56 \leq \zeta \leq 0.91$

b)

$$\frac{C(s)}{R(s)} = \frac{KK_p(s+K_I)}{s^2 + (p + KK_p)s + KK_pK_I}$$

As $K_p=2$, then:

$$\frac{C(s)}{R(s)} = \frac{2K(s+K_I)}{s^2 + 2(K+1)s + 2KK_I}$$

If the roots of the characteristic equations are assumed to be lied in the centre of the shown region:

$$\begin{cases} P_1 = s - 3 - 2j \\ P_2 = s - 3 + 2j \end{cases} \Rightarrow s^2 + 6s + 13 = 0$$

Comparing with the characteristic equation:

$$\begin{cases} 2(K+1) = 6 \rightarrow \ K = 2 \\ 2KK_I = 13 \rightarrow K_I = 3.25 \end{cases}$$

c) The characteristic equation

$$s^2 + 2(p + KK_p)s + KK_pK_I = 0$$

is a second order polynomial with two roots. These two roots can be determined by two terms $2(p+KK_p)$ and KK_pK_I which includes four parameters. Regardless of the p and K_p values, we can always choose K and K_I so that to place the roots in a desired location.

7-51) a) $J_m s^2\theta_m(s) + \left(B + \frac{K_1K_2}{R}\right) s\theta(s) = \frac{K_1}{R} V(s)$

$$\frac{\theta_m(s)}{V(s)} = \frac{\frac{K_1}{R}}{s\left(s + \left(B + \frac{K_1K_2}{R}\right)\right)}$$

By substituting the values:

$$\frac{\theta_m(s)}{V(s)} = \frac{0.2}{s(s+0.109)}$$

b) Speed of the motor is $\frac{d\theta}{dt} = \omega$

$$\frac{\omega(s)}{V(s)} = \frac{s\theta(s)}{V(s)} = \frac{0.2}{s+0.109}$$

$e_{ss} = V \lim_{s\to 0} G(s) = 10\frac{0.2}{0.109} = 19.23$

(c)

$$\frac{\theta_m(s)}{V(s)} = \frac{0.2}{s(s+0.109)}$$

d)

$$\theta_m(s) = \frac{0.2}{s(s+0.109)}V(s)$$
$$= \frac{0.2}{s(s+0.109)}K\left(\theta_p(s) - \theta_m(s)\right)$$

As a result:

$$\frac{\theta_m(s)}{\theta_p(s)} = \frac{0.2K}{s^2 + 0.109s + 0.2K}$$

e) As $M_p = \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right) = 0.2s$, then, $\xi = 0.404$

According to the transfer function, $2\,\xi\omega_n = 0.109$, then $\omega_n = \frac{0.109}{(2)(0.404)} \approx 0.14 rad/sec$

where $\omega_n^2 = 0.2K$ ⇨ K<0.0845

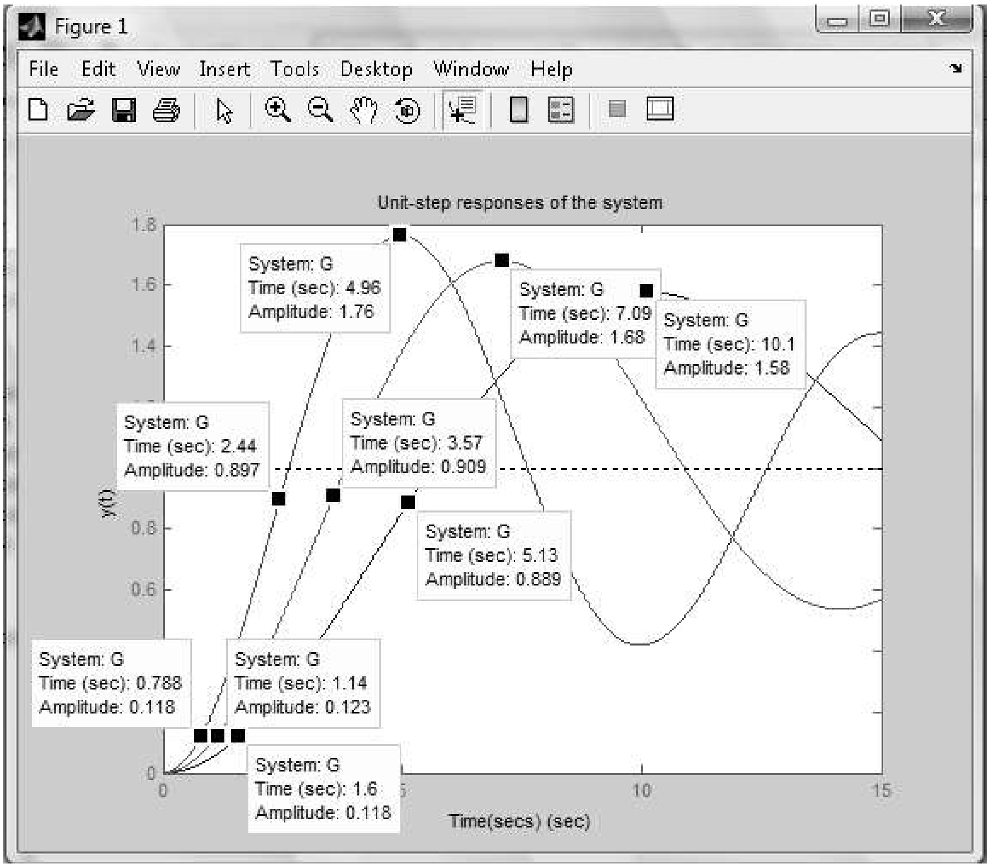
f) As $t_r = \frac{0.8+2.5\,\xi}{\omega_n} \approx \frac{1.8}{\omega_n}$, then $\omega_n \geq 0.6$,

as $\omega_n^2 = 0.2K$, therefore; $K \geq 1$

g) MATLAB

```
clear all
for K=[0.5 1 2];
t=0:0.001:15;
num = [0.2*K];
den = [1 0.109 0.2*K];
G=tf(num,den)
step(G,t);
hold on;
end
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')
```

```
Transfer function:
        0.1
-------------------
s^2 + 0.109 s + 0.1


Transfer function:
        0.2
-------------------
s^2 + 0.109 s + 0.2


Transfer function:
        0.4
-------------------
s^2 + 0.109 s + 0.4
```



Rise time decreases with *K* increasing.

Overshoot increases with *K*.

7-52) $M_p = \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right) = 0.1$, therefore, $\xi = 0.59$

As $t_s = \frac{3.2}{\xi\omega_n}$, then, $\omega_n = \frac{3.2}{(0.59)(1.5)} \approx 3.62$

$$\frac{Y(s)}{X(s)} = \frac{G(s)H(s)}{1+G(s)H(s)} = \frac{K(s+a)}{s(s+3)(s+b)+K(s+a)}$$
$$= \frac{K(s+a)}{s^3+(3+b)s^2+(3b+K)s+Ka}$$

Therefore:

$$s^3+(3+b)s^2+(3b+K)s+Ka = (s+p)(s^2+2\,\xi\omega_n s+\omega^2)$$

If p is a non-dominant pole; therefore comparing both sides of above equation and:

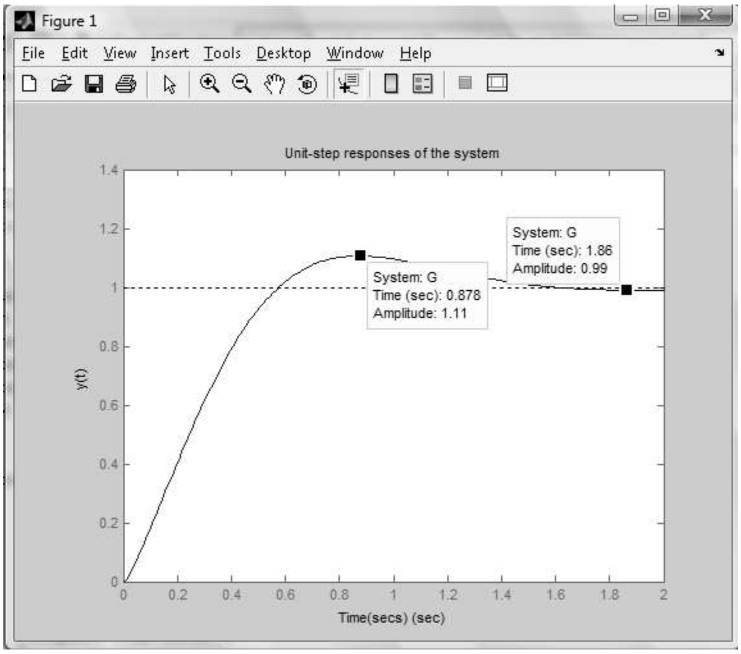
$$\begin{cases} 2\,\xi\omega_n + p = 3+b \\ 2\xi\omega_n p + \omega_n^2 = 3b+k \\ \omega_n^2 p = Ka \end{cases}$$

If we consider p = 10a (non-dominant pole), $\xi = 0.6$ and $\omega_n = 4$, then:

$$\begin{cases} 4.8+10a = 3+b \\ 48a+16 = 3b+K \\ 160a = Ka \;\; \rightarrow \;\; K = 160 \end{cases}$$

$$\begin{cases} a = 8.3 \\ b = 89.8 \\ p = 83 \end{cases}$$

```
clear all
K=160;
a=8.3;
b=89.8;
p=83;
num = [K K*a];
den = [1 3+b 3*b+K K*a];
G=tf(num,den)
step(G);
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')

Transfer function:
         160 s + 1328
-------------------------------
s^3 + 92.8 s^2 + 429.4 s + 1328
```



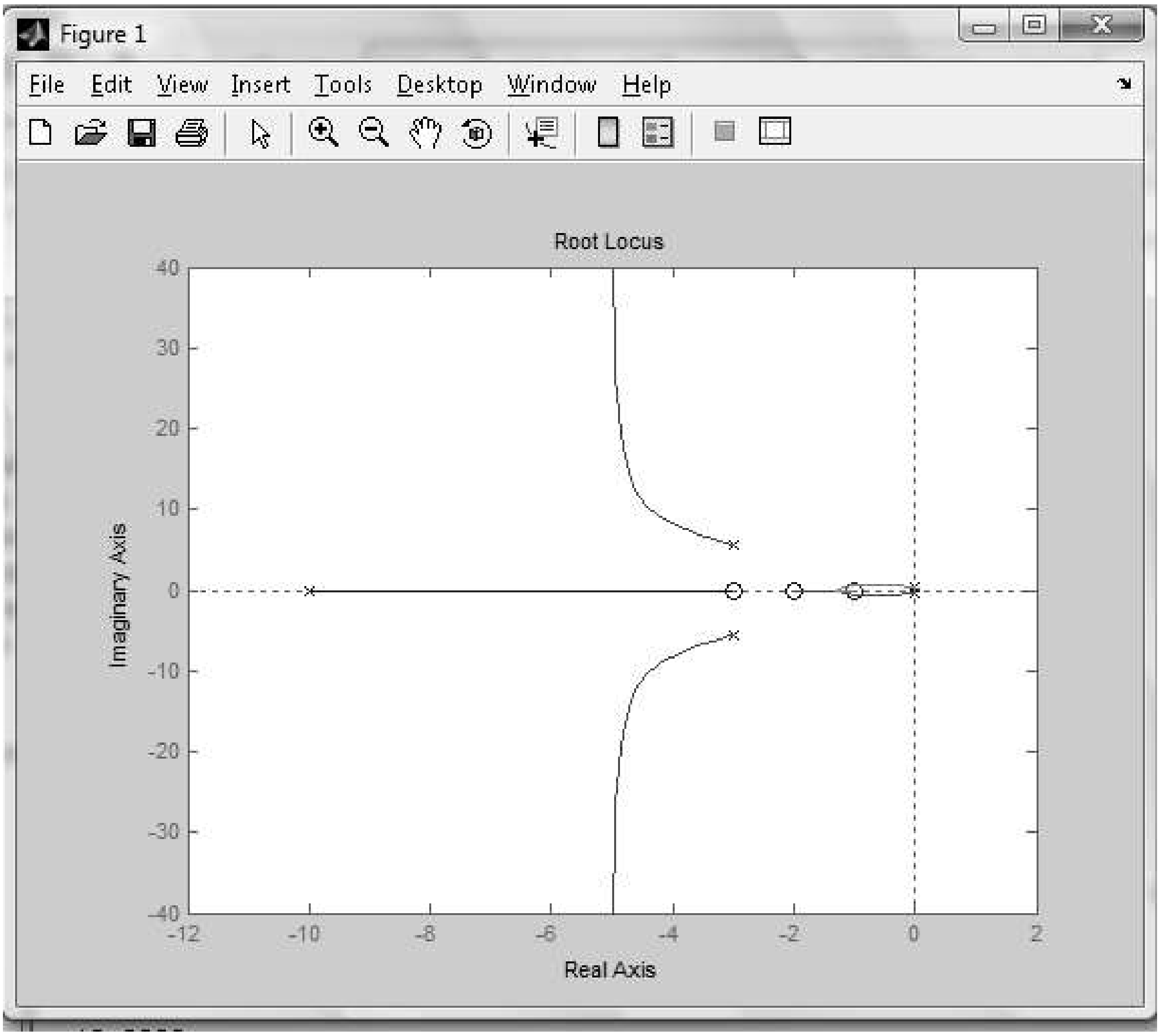
Both Overshoot and settling time values are met. No need to adjust parameters.

7-53) For the controller and the plant to be in series and using a unity feedback loop we have:

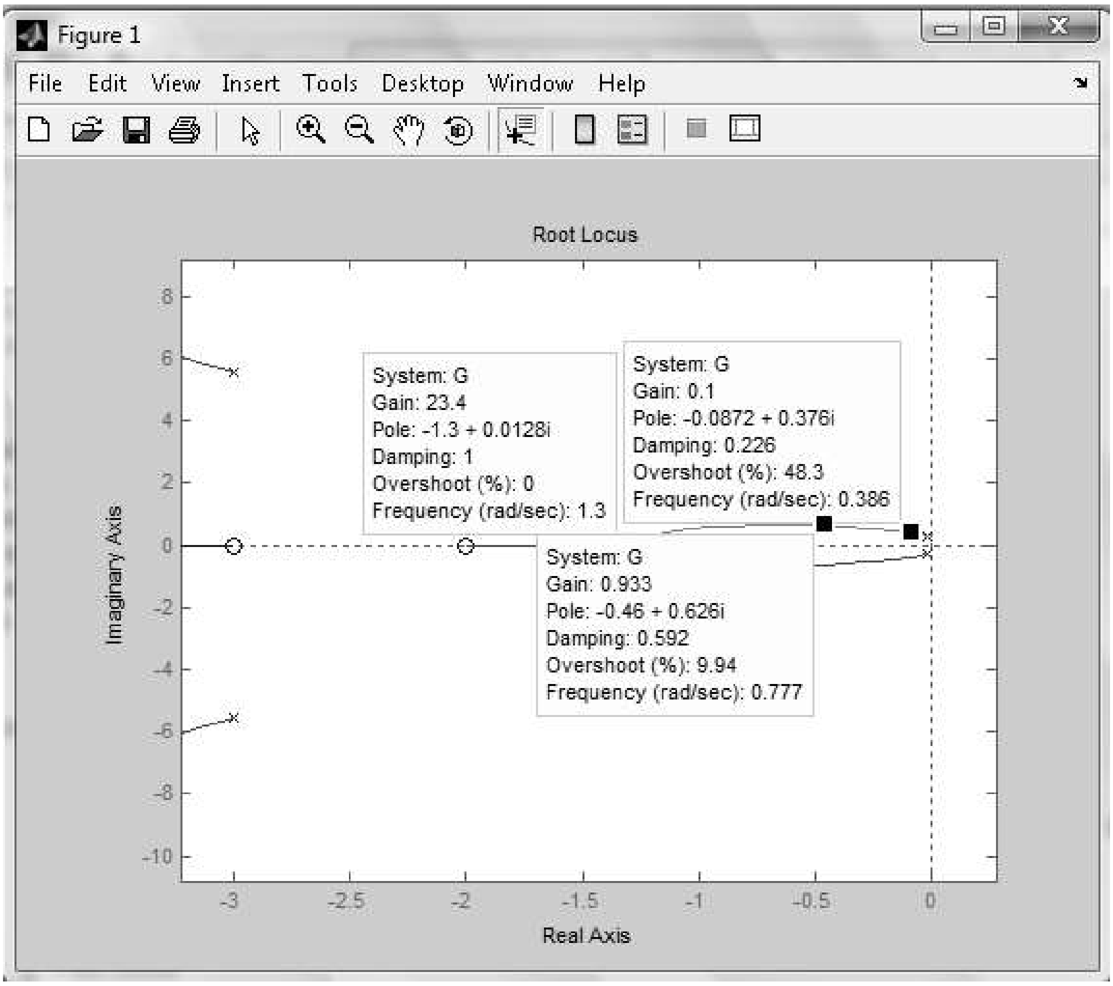
MATLAB

USE toolbox

```
clear all
num=[-1 -2 -3];
denom=[-3+sqrt(9-40) -3-sqrt(9-40) -0.02+sqrt(.004-.07) -0.02-sqrt(.004-.07) -10];
G=zpk(num,denom,60)
rlocus(G)

Zero/pole/gain:
         60 (s+1) (s+2) (s+3)
----------------------------------------------
(s+10) (s^2  + 0.04s + 0.0664) (s^2  + 6s + 40)
```



Note the system has two dominant complex poles close to the imaginary axis. Lets zoom in the root locus diagram and use the cursor to find the parameter values.



As shown for K=0.933 the dominant closed loop poles are at -0.46±j 0.626 AND OVERSHOOT IS ALMOST 10%.

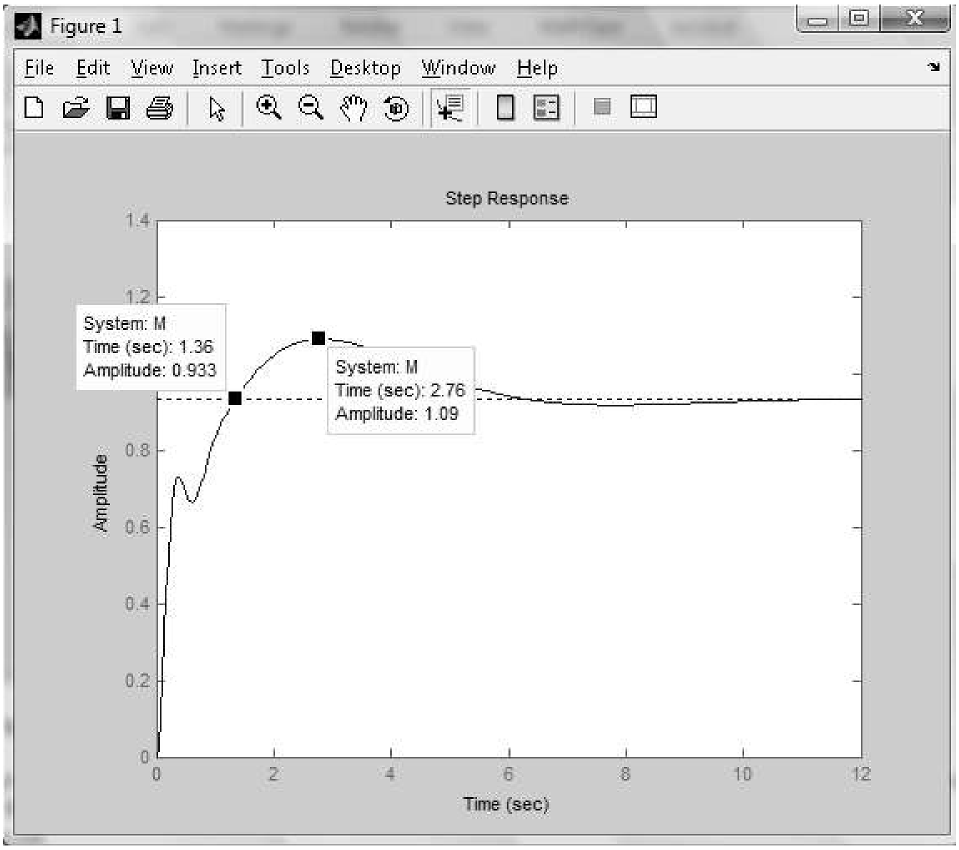
Increasing K will push the poles closer towards less dominant zeros and poles. As a process the design process becomes less trivial and more difficult.

To confirm use

M=feedback(G*.933,1) %See toolbox in problem 7-53
step(M)
Zero/pole/gain:

55.98 (s+3) (s+2) (s+1)

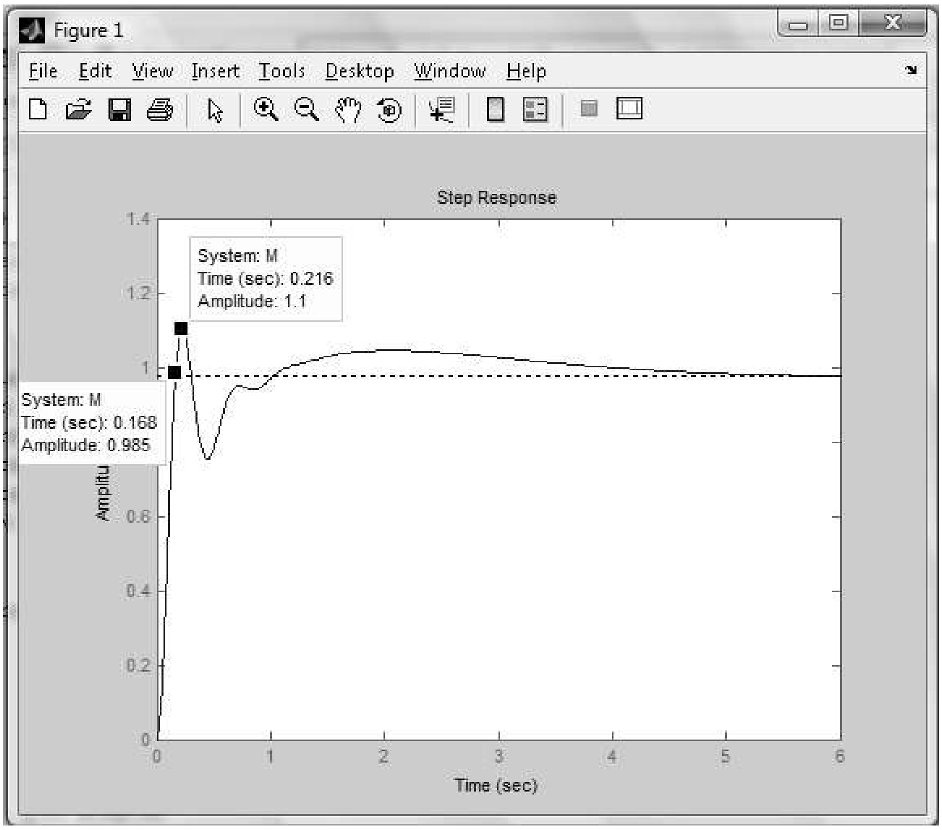
(s+7.048) (s^2 + 0.9195s + 0.603) (s^2 + 8.072s + 85.29)



To reduce rise time, the poles have to move to left to make the secondary poles more dominant. As a result the little bump in the left hand side of the above graph should rise. Try K=3:

```
>> M=feedback(G*3,1)

Zero/pole/gain:
            180 (s+3) (s+2) (s+1)
-------------------------------------------------------
(s+5.01) (s^2  + 1.655s + 1.058) (s^2  + 9.375s + 208.9)

>> step(M)
```

****Try a higher K value, but looking at the root locus and the time plots, it appears that the overshoot and rise time criteria will never be met simultaneously.**



K=5

M=feedback(G*5,1) %See toolbox in problem 7-53
step(M)

Zero/pole/gain:

$$\frac{300\ (s+3)\ (s+2)\ (s+1)}{(s+4.434)\ (s^2\ +1.958s+1.252)\ (s^2\ +9.648s+329.1)}$$

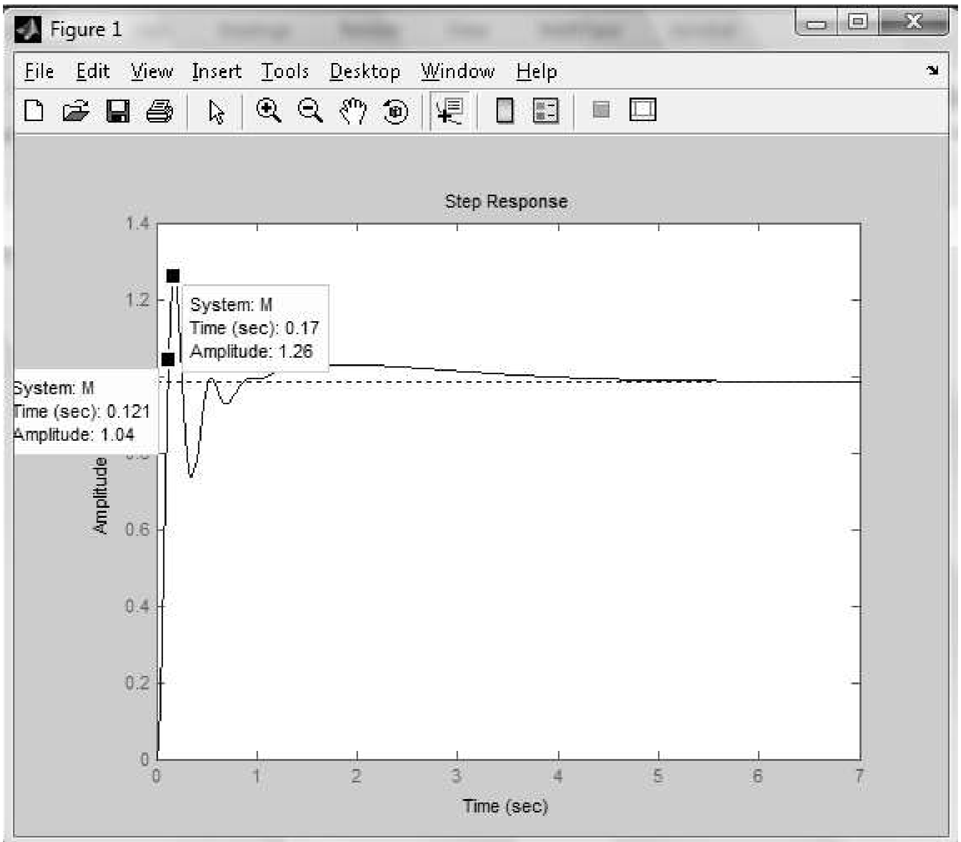


Figure 1
File Edit View Insert Tools Desktop Window Help
Step Response
System: M
Time (sec): 0.17
Amplitude: 1.26
System: M
Time (sec): 0.121
Amplitude: 1.04
Amplitude
Time (sec)
0
0.2
0.4
0.6
1.2
1.4
0
1
2
3
4
5
6
7

7-54) Forward-path Transfer Function:

$$G(s) = \frac{M(s)}{1 - M(s)} = \frac{K}{s^3 + (20 + a)s^2 + (200 + 20a)s + 200a - K}$$

For type 1 system, $200a - K = 0$ Thus $K = 200a$

Ramp-error constant:

$$K_v = \lim_{s \to 0} sG(s) = \frac{K}{200 + 20a} = \frac{200a}{200 + 20a} = 5$$ Thus $a = 10$ $K = 2000$

MATLAB Symbolic tool can be used to solve above. We use it to find the roots for the next part:

```
>> syms s a K

>>solve(5*200+5*20*a-200a)

ans =

10

>> D=(s^2+20*s+200)*s+a))

 D =

(s^2+20*s+200)*(s+a)

>> expand(D)

ans =

s^3+s^2*a+20*s^2+20*s*a+200*s+200*a

>> solve(ans,s)

ans =

      -a

 -10+10*i

 -10-10*i
```

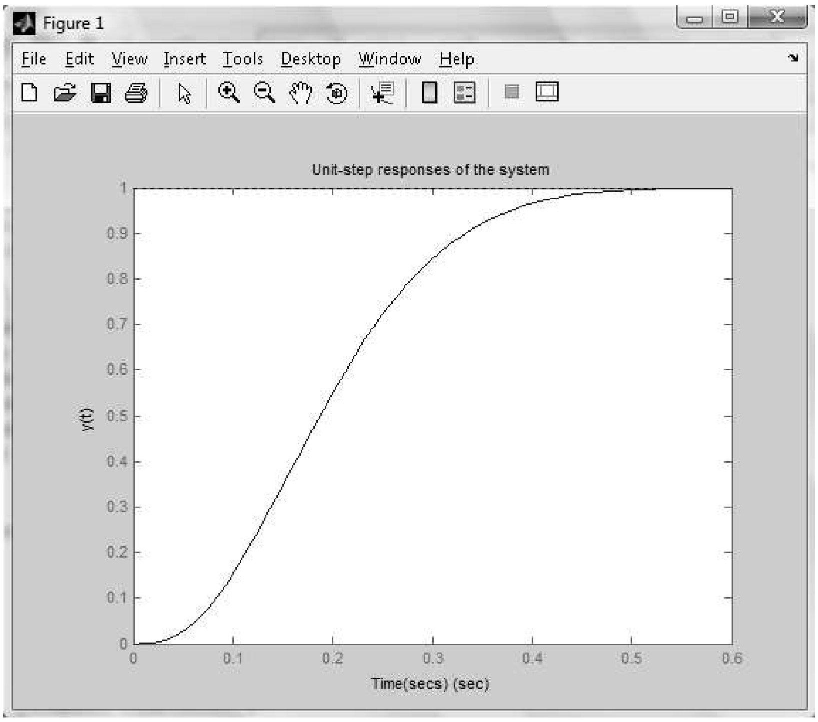
The forward-path transfer function is The controller transfer function is

$$G(s) = \frac{2000}{s\left(s^2 + 30s + 400\right)} \qquad G_c(s) = \frac{G(s)}{G_p(s)} = \frac{20\left(s^2 + 10s + 100\right)}{\left(s^2 + 30s + 400\right)}$$

The maximum overshoot of the unit-step response is 0 percent.

MATLAB

```
clear all
K=2000;
a=10;
num = [];
den = [-10+10i -10-10i -a];
G=zpk(num,den,K)
step(G);
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')


Zero/pole/gain:
          2000
-------------------------

(s+10) (s^2  + 20s + 200)
```



Clearly PO=0.

7-55)

Forward-path Transfer Function:

$$G(s) = \frac{M(s)}{1 - M(s)} = \frac{K}{s^3 + (20 + a)s^2 + (200 + 20a)s + 200a - K}$$

For type 1 system, $200a - K = 0$ Thus $K = 200a$

Ramp-error constant:

$$K_v = \lim_{s \to 0} sG(s) = \frac{K}{200 + 20a} = \frac{200a}{200 + 20a} = 9$$ Thus $a = 90$ $K = 18000$

MATLAB Symbolic tool can be used to solve above. We use it to find the roots for the next part:

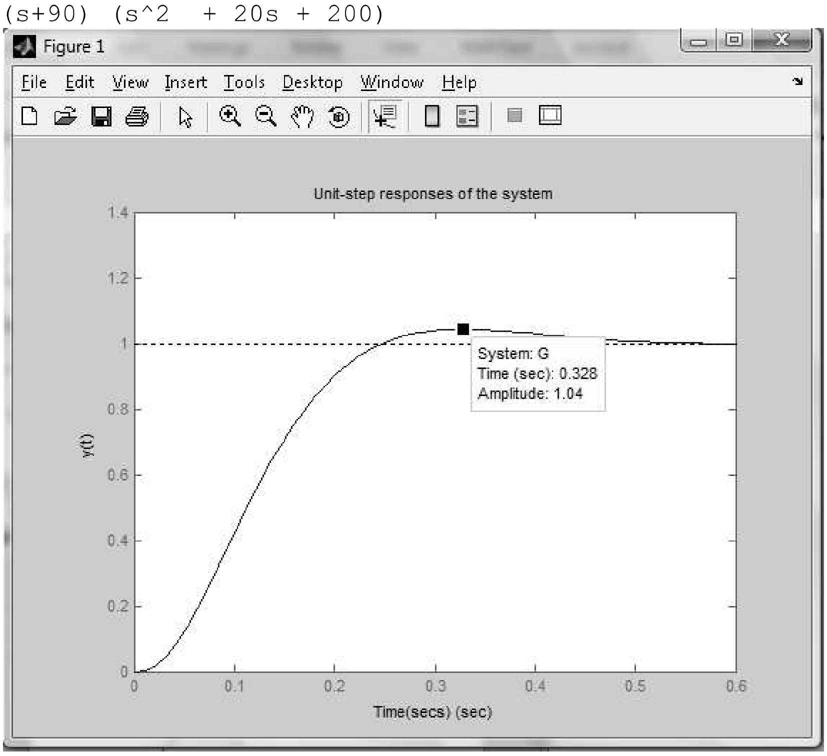
```
>> syms s a K

solve(9*200+9*20*a-200*a)

 ans =

 90

>>D=(s^2+20*s+200)*s+a))

 D =

(s^2+20*s+200)*(s+a)

>> expand(D)

ans =

s^3+s^2*a+20*s^2+20*s*a+200*s+200*a

>> solve(ans,s)

ans =

      -a

 -10+10*i

 -10-10*i
```

The forward-path transfer function is The controller transfer function is

$$G(s) = \frac{18000}{s\left(s^2 + 110s + 2000\right)} \qquad G_c(s) = \frac{G(s)}{G_p(s)} = \frac{180\left(s^2 + 10s + 100\right)}{\left(s^2 + 110s + 2000\right)}$$

The maximum overshoot of the unit-step response is 4.3 percent.
From the expression for the ramp-error constant, we see that as *a* or *K* goes to infinity, K_v approaches 10. Thus the maximum value of K_v that can be realized is 10. The difficulties with very large values of *K* and *a* are that a high-gain amplifier is needed and unrealistic circuit parameters are needed for the controller.

```
clear all
K=18000;
a=90;
num = [];
den = [-10+10i -10-10i -a];
G=zpk(num,den,K)
step(G);
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')

Zero/pole/gain:
          18000
-------------------------
(s+90) (s^2  + 20s + 200)
```



PO is less than 4.

7-56) (a) **Ramp-error Constant:**

MATLAB

```
clear all
syms s Kp Kd kv
Gnum=(Kp+Kd*s)*1000
Gden= (s*(s+10))
G=Gnum/Gden
Kv=s*G
s=0
eval(Kv)

Gnum =
 1000*Kp+1000*Kd*s

Gden =
 s*(s+10)

G =
 (1000*Kp+1000*Kd*s)/s/(s+10)

Kv =
 (1000*Kp+1000*Kd*s)/(s+10)

 s =
    0

ans =
 100*Kp
```

$$K_v = \lim_{s\to 0} s\frac{1000\left(K_P + K_D s\right)}{s(s+10)} = \frac{1000K_P}{10} = 100K_P = 1000 \qquad \text{Thus} \qquad K_P = 10$$

```
Kp=10

clear s

syms s

Mnum=(Kp+Kd*s)*1000/s/(s+10)

Mden=1+(Kp+Kd*s)*1000/s/(s+10)
```

Kp =

10

Mnum =

(10000+1000*Kd*s)/s/(s+10)

Mden =

1+(10000+1000*Kd*s)/s/(s+10)

ans =

(s^2+10*s+10000+1000*Kd*s)/s/(s+10)

Characteristic Equation: $s^2 + \left(10 + 1000K_D\right)s + 1000K_P = 0$

Match with a 2nd order prototype system

$\omega_n = \sqrt{1000\,K_P} = \sqrt{10000} = 100$ rad/sec $\qquad 2\zeta\omega_n = 10 + 1000K_D = 2 \times 0.5 \times 100 = 100$

solve(10+1000*Kd-100)

ans =

9/100

Thus $\quad K_D = \dfrac{90}{1000} = 0.09$

Use the same procedure for other parts.

(b) For $K_v = 1000$ and $\zeta = 0.707$, and from part (a), $\omega_n = 100$ rad/sec,

$2\zeta\omega_n = 10 + 1000K_D = 2 \times 0.707 \times 100 = 141.4$ Thus $\quad K_D = \dfrac{131.4}{1000} = 0.1314$

(c) For $K_v = 1000$ and $\zeta = 1.0$, and from part (a), $\omega_n = 100$ rad/sec,

$2\zeta\omega_n = 10 + 1000K_D = 2 \times 1 \times 100 = 200$ Thus $\quad K_D = \dfrac{190}{1000} = 0.19$

7-57) **The ramp-error constant:**

$$K_v = \lim_{s \to 0} s \frac{1000\left(K_P + K_D s\right)}{s(s+10)} = 100K_P = 10,000 \qquad \text{Thus} \quad K_P = 100$$

The forward-path transfer function is: $G(s) = \dfrac{1000\left(100 + K_D s\right)}{s(s+10)}$

```
clear all
for KD=0.2:0.2:1.0;
num = [-100/KD];
den = [0 -10];
G=zpk(num,den,1000);
M=feedback(G,1)
step(M);
hold on;
end
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')
```

Zero/pole/gain:

1000 (s+500)

(s^2 + 1010s + 5e005)

Zero/pole/gain:

1000 (s+250)

(s+434.1) (s+575.9)

Zero/pole/gain:

1000 (s+166.7)

(s+207.7) (s+802.3)

Zero/pole/gain:

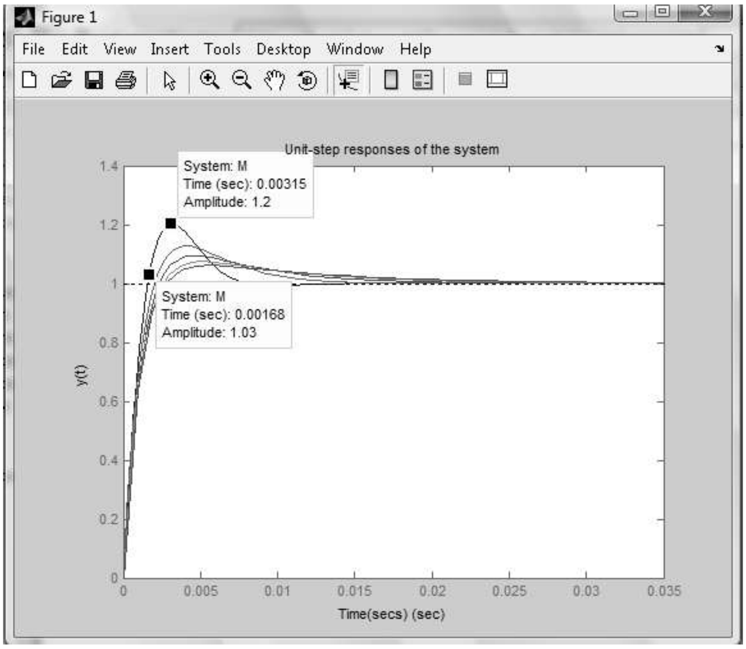
1000 (s+125)

(s+144.4) (s+865.6)

Zero/pole/gain:

1000 (s+100)

(s+111.3) (s+898.7)



Use the cursor to obtain the PO and tr values.

For part b the maximum value of KD results in the minimum overshoot.

7-58) (a) **Forward-path Transfer Function:**

$$G(s) = G_c(s)G_p(s) = \frac{4500K\left(K_P + K_D s\right)}{s(s+361.2)}$$

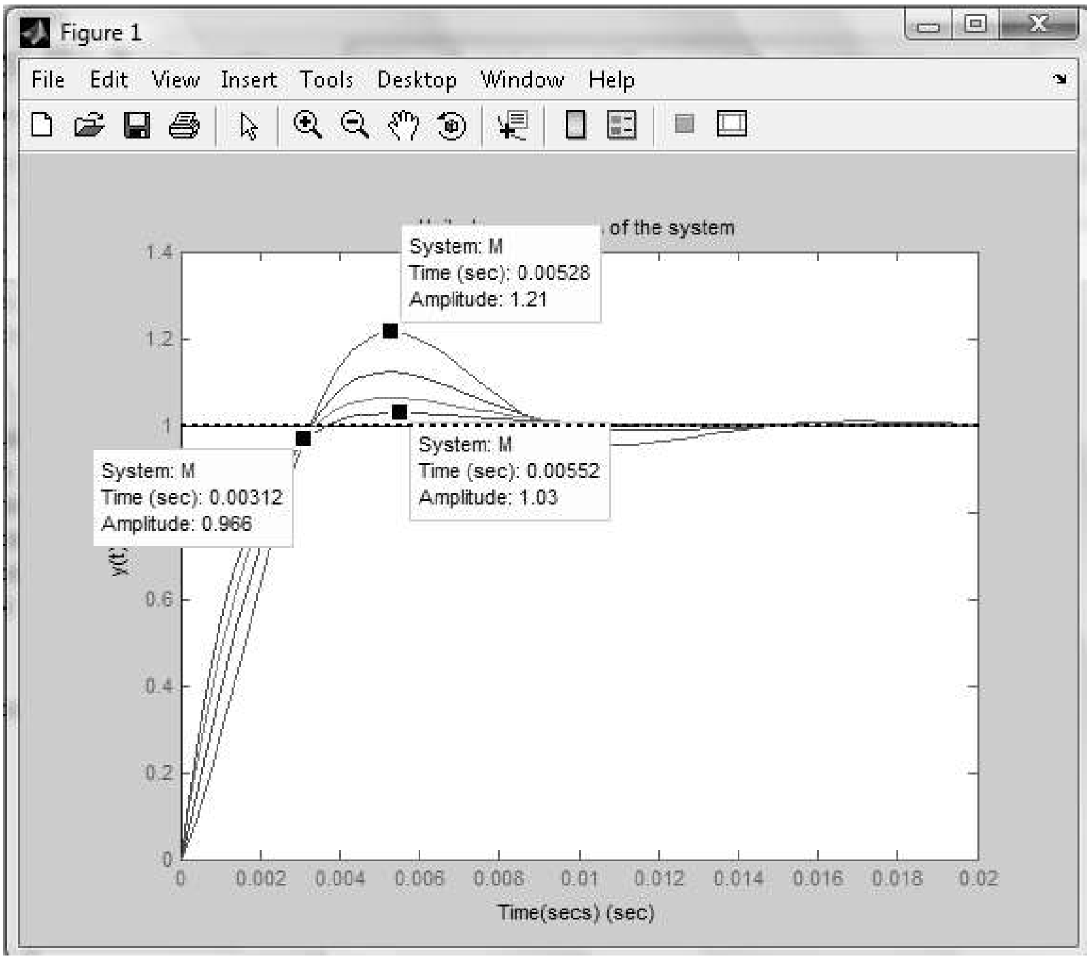
Ramp Error Constant: $K_v = \lim_{s\to 0} sG(s) = \frac{4500KK_P}{361.2} = 12.458KK_P$

$e_{ss} = \frac{1}{K_v} = \frac{0.0802}{KK_P} \le 0.001$ Thus $KK_P \ge 80.2$ Let $K_P = 1$ and $K = 80.2$

```
clear all
KP=1;
K=80.2;
figure(1)
num = [-KP];
den = [0 -361.2];
G=zpk(num,den,4500*K)
M=feedback(G,1)
step(M)
hold on;
for KD=00.0005:0.0005:0.002;
num = [-KP/KD];
den = [0 -361.2];
G=zpk(num,den,4500*K*KD)
M=feedback(G,1)
step(M)
end
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')
```

```
Zero/pole/gain:
360900 (s+1)
------------
s (s+361.2)


Zero/pole/gain:
   360900 (s+1)
----------------------
(s+0.999) (s+3.613e005)


Zero/pole/gain:
180.45 (s+2000)
---------------
 s (s+361.2)


Zero/pole/gain:
    180.45 (s+2000)
```

```
--------------------------
(s^2  + 541.6s + 3.609e005)


Zero/pole/gain:
360.9 (s+1000)
--------------
 s (s+361.2)


Zero/pole/gain:
     360.9 (s+1000)
--------------------------
(s^2  + 722.1s + 3.609e005)


Zero/pole/gain:
541.35 (s+666.7)
----------------
  s (s+361.2)


Zero/pole/gain:
    541.35 (s+666.7)
--------------------------
(s^2  + 902.5s + 3.609e005)


Zero/pole/gain:
721.8 (s+500)
-------------
 s (s+361.2)


Zero/pole/gain:
     721.8 (s+500)
-------------------------
(s^2  + 1083s + 3.609e005)
```



K_D	t_r (sec)	t_s (sec)	Max Overshoot (%)
0	0.00221	0.0166	37.1
0.0005	0.00242	0.00812	21.5
0.0010	0.00245	0.00775	12.2
0.0015	0.0024	0.0065	6.4
0.0016	0.00239	0.00597	5.6
0.0017	0.00238	0.00287	4.8
0.0018	0.00236	0.0029	4.0
0.0020	0.00233	0.00283	2.8

7-59) The forward-path Transfer Function: *N* = 20

$$G(s) = \frac{200\left(K_P + K_D s\right)}{s(s+1)(s+10)}$$

To stabilize the system, we can reduce the forward-path gain. Since the system is type 1, reducing the gain does not affect the steady-state liquid level to a step input. Let $K_P = 0.05$

$$G(s) = \frac{200\left(0.05 + K_D s\right)}{s(s+1)(s+10)}$$

ALSO try other Kp values and compare your results.

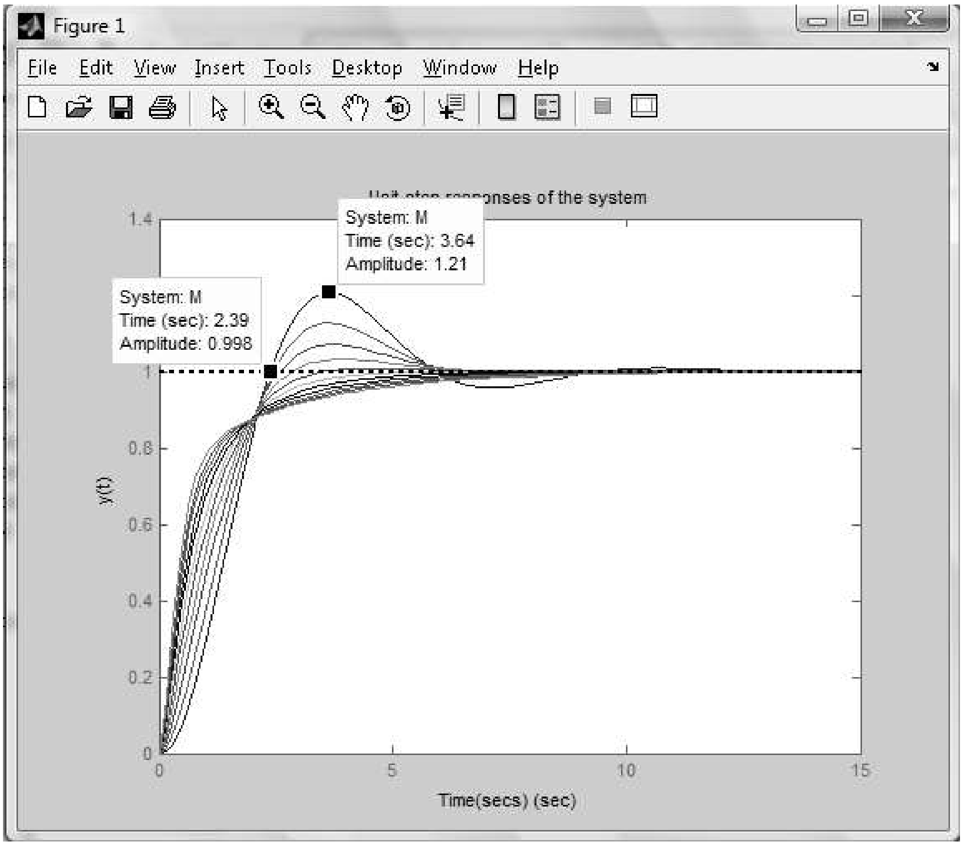
```
clear all
figure(1)
KD=0
num = [];
den = [0 -1 -10];
G=zpk(num,den,200*0.05)
M=feedback(G,1)
step(M)
hold on;
for KD=0.01:0.01:0.1;
KD
num = [-0.05/KD];
G=zpk(num,den,200*KD)
M=feedback(G,1)
step(M)
end
xlabel('Time(secs)')
ylabel('y(t)')
title('Unit-step responses of the system')
```

```
KD =
   0

Zero/pole/gain:
    10
--------------
s (s+1) (s+10)

Zero/pole/gain:
          10
-----------------------------------
(s+10.11) (s^2  + 0.8914s + 0.9893)

KD =
  0.0100
```

```
Zero/pole/gain:
  2 (s+5)
--------------
s (s+1) (s+10)

Zero/pole/gain:
          2 (s+5)
---------------------------------
(s+9.889) (s^2  + 1.111s + 1.011)

KD =
   0.0200

Zero/pole/gain:
 4 (s+2.5)
--------------
s (s+1) (s+10)


Zero/pole/gain:
         4 (s+2.5)
---------------------------------
(s+9.658) (s^2  + 1.342s + 1.035)


KD =
   0.0300

Zero/pole/gain:
 6 (s+1.667)
--------------
s (s+1) (s+10)

Zero/pole/gain:
        6 (s+1.667)
---------------------------------
(s+9.413) (s^2  + 1.587s + 1.062)

KD =
   0.0400


Zero/pole/gain:
 8 (s+1.25)
--------------
s (s+1) (s+10)


Zero/pole/gain:
        8 (s+1.25)
---------------------------------
(s+9.153) (s^2  + 1.847s + 1.093)
```

```
KD =
    0.0500

Zero/pole/gain:
   10 (s+1)
--------------
s (s+1) (s+10)

Zero/pole/gain:
       10 (s+1)
-------------------------
(s+8.873) (s+1.127) (s+1)

KD =
    0.0600

Zero/pole/gain:
12 (s+0.8333)
--------------
s (s+1) (s+10)

Zero/pole/gain:
       12 (s+0.8333)
------------------------------
(s+8.569) (s+1.773) (s+0.6582)

KD =
    0.0700

Zero/pole/gain:
14 (s+0.7143)
--------------
s (s+1) (s+10)


Zero/pole/gain:
       14 (s+0.7143)
-----------------------------
(s+8.232) (s+2.221) (s+0.547)

KD =
    0.0800

Zero/pole/gain:
 16 (s+0.625)
--------------
s (s+1) (s+10)

Zero/pole/gain:
       16 (s+0.625)
-----------------------------
(s+7.85) (s+2.673) (s+0.4765)
```

```
KD =
    0.0900

Zero/pole/gain:
18 (s+0.5556)
--------------
s (s+1) (s+10)


Zero/pole/gain:
      18 (s+0.5556)
------------------------------
(s+7.398) (s+3.177) (s+0.4255)

KD =
    0.1000

Zero/pole/gain:
  20 (s+0.5)
--------------
s (s+1) (s+10)

Zero/pole/gain:
        20 (s+0.5)
------------------------------
(s+0.3861) (s+3.803) (s+6.811)
```



Unit-step Response Attributes:

K_D	t_s (sec)	Max Overshoot (%)
0.01	5.159	12.7
0.02	4.57	7.1
0.03	2.35	3.2
0.04	2.526	0.8
0.05	2.721	0
0.06	3.039	0
0.10	4.317	0

When $K_D = 0.05$ the rise time is 2.721 sec, and the step response has no overshoot.

7-60) (a) For $e_{ss} = 1,$

$$K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{200\left(K_P + K_D s\right)}{s(s+1)(s+10)} = 20K_P = 1 \qquad \text{Thus} \quad K_P = 0.05$$

Forward-path Transfer Function:

$$G(s) = \frac{200\left(0.05 + K_D s\right)}{s(s+1)(s+10)}$$

```
Because of the choice of Kp this is the same as previous part.
```

7-61)

(a) Forward-path Transfer Function:

$$G(s) = \frac{100\left(K_P + \dfrac{K_I}{s}\right)}{s^2 + 10s + 100} \qquad \text{For } K_v = 10, \qquad K_v = \lim_{s\to 0} sG(s) = \lim_{s\to 0} s\frac{100\left(K_P s + K_I\right)}{s\left(s^2 + 10s + 100\right)} = K_I = 10$$

Thus $K_I = 10.$

$$G_{cl}(s) = \frac{100\left(K_P s + K_I\right)}{s^3 + 10s^2 + 100s + 100\left(K_P s + K_I\right)} = \frac{100\left(K_P s + 10\right)}{s^3 + 10s^2 + 100(1+K_P)s + 1000}$$

(b) Let the complex roots of the characteristic equation be written as $s = -\sigma + j15$ and $s = -\sigma - j15.$

The quadratic portion of the characteristic equation is $s^2 + 2\sigma s + \left(\sigma^2 + 225\right) = 0$

The characteristic equation of the system is $s^3 + 10s^2 + \left(100 + 100K_P\right)s + 1000 = 0$

The quadratic equation must satisfy the characteristic equation. Using long division and solve for zero remainder condition.

$$\begin{array}{r|l} & s+(10-2\sigma) \\ s^2+2\sigma s+\sigma^2+225 & s^3+10s^2+\left(100+100K_P\right)s+1000 \\ & s^3+2\sigma s^2+\left(\sigma^2+225\right)s \\ \hline & (10-2\sigma)s^2+\left(100K_P-\sigma^2-125\right)s+1000 \\ & (10-2\sigma)s^2+\left(20\sigma-4\sigma^2\right)s+(10-2\sigma)\left(s^2+225\right) \\ \hline & \left(100K_P+3\sigma^2-20\sigma-125\right)s+2\sigma^3-10\sigma^2+450\sigma-1250 \end{array}$$

For zero remainder, $2\sigma^3-10\sigma^2+450\sigma-1250=0$ (1)

and $100K_P+3\sigma^2-20\sigma-125=0$ (2)

The real solution of Eq. (1) is $\sigma=2.8555$. From Eq. (2),

$$K_P=\frac{125+20\sigma-3\sigma^2}{100}=1.5765$$

The characteristic equation roots are: $s=-2.8555+j15,\quad -2.8555-j15,$ and $s=-10+2\sigma=-4.289$

(c) Root Contours:

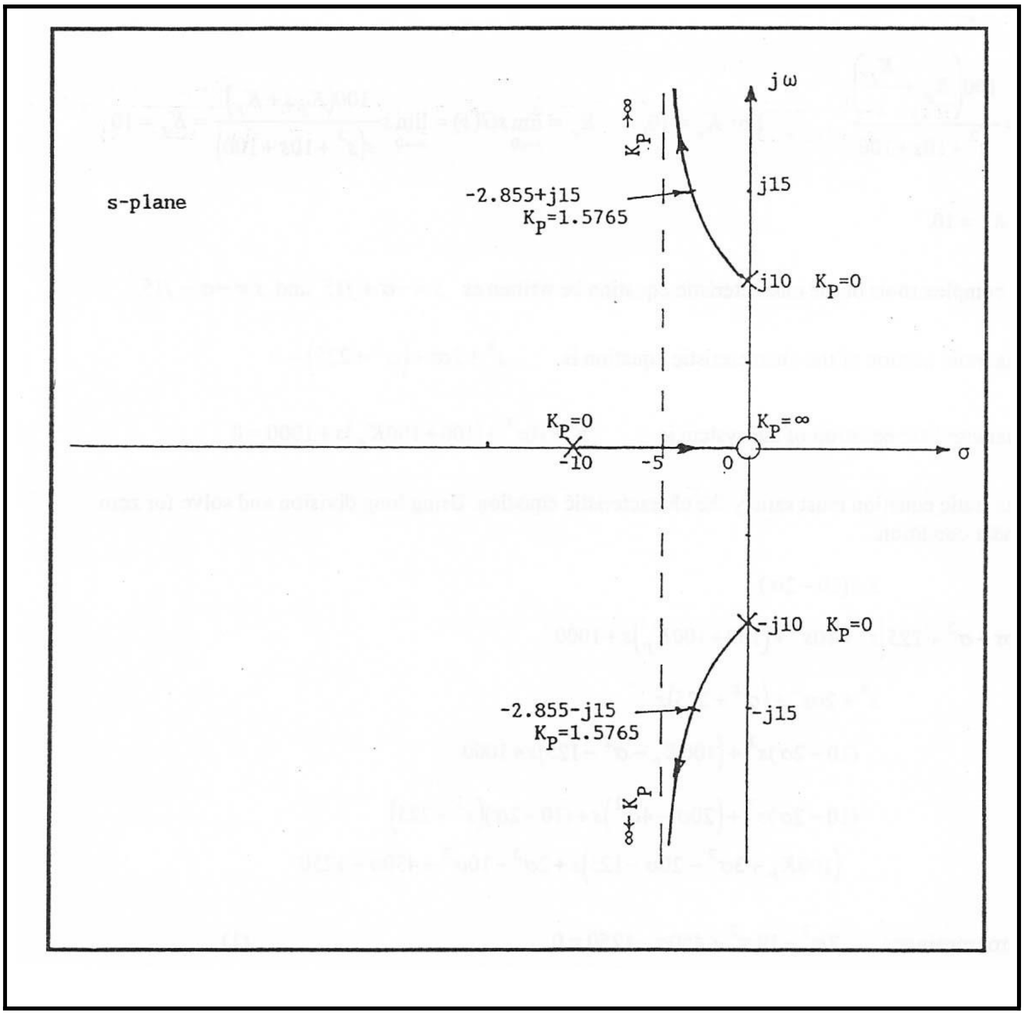
Dividing both sides of $s^3+10s^2+\left(100+100K_P\right)s+1000=0$ by the terms that do not contain K_P **we have:**

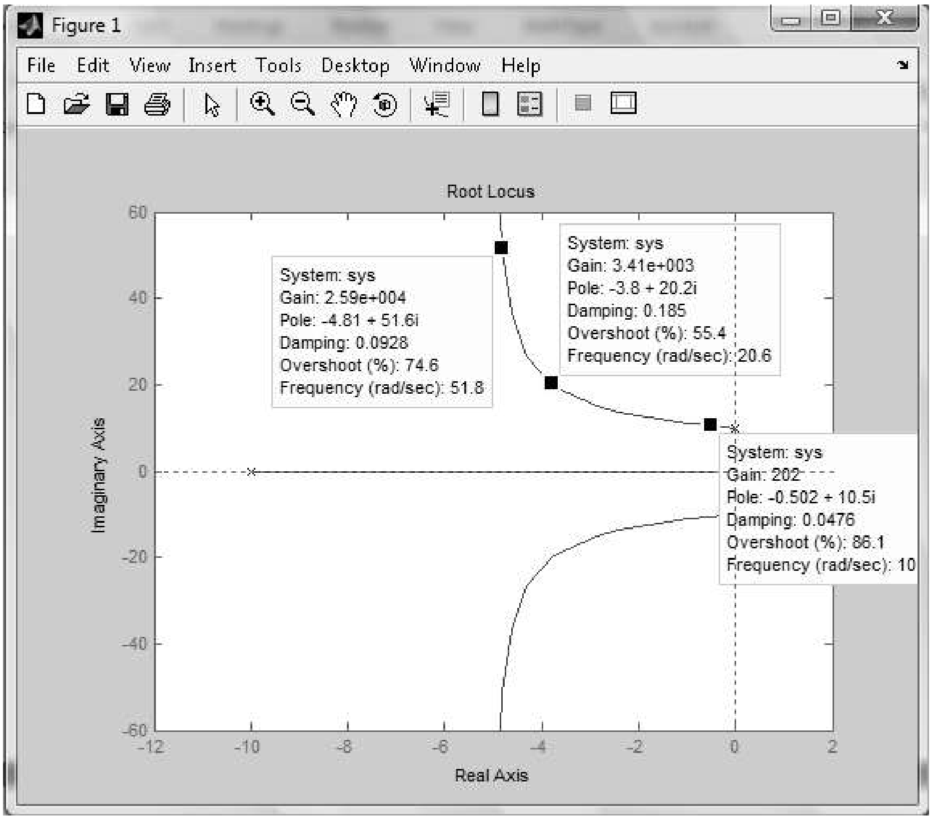
$$1+\frac{100K_P s}{s^3+10s^2+100s+1000}=1+G_{eq}$$

$$G_{eq}(s)=\frac{100K_P s}{s^3+10s^2+100s+1000}=\frac{100K_P s}{(s+10)\left(s^2+100\right)}$$

Root Contours: See Chapter 10 for more information

```
clear all
Kp =.001;
num = [100*Kp 0];
den = [1 10 100 1000];
rlocus(num,den)
```


jω
s-plane
$K_p \to \infty$
-2.855+j15
K_p=1.5765
j15
j10 K_p=0
K_p=0
K_p=∞
σ
-10
-5
0
-j10 K_p=0
-2.855-j15
K_p=1.5765
-j15
$K_p \to \infty$



7-62) (a) Forward-path Transfer Function:

$$G(s)=\frac{100\left(K_P+\frac{K_I}{s}\right)}{s^2+10s+100} \qquad \text{For } K_v=10, \qquad K_v=\lim_{s\to 0} sG(s)=\lim_{s\to 0} s\frac{100\left(K_P s+K_I\right)}{s\left(s^2+10s+100\right)}=K_I=10$$

Thus the forward-path transfer function becomes

$$G(s)=\frac{100\left(10+K_P s\right)}{s\left(s^2+10s+100\right)}$$

$$G_{cl}(s)=\frac{100\left(K_P s+K_I\right)}{s^3+10s^2+100s+100\left(K_P s+K_I\right)}=\frac{100\left(K_P s+10\right)}{s^3+10s^2+100(1+K_P)s+1000}$$

```
clear all
for Kp=.4:0.4:2;
num = [100*Kp 1000];
den =[1 10 100 0];
[numCL,denCL]=cloop(num,den);
GCL=tf(numCL,denCL);
step(GCL)
hold on;
end
```

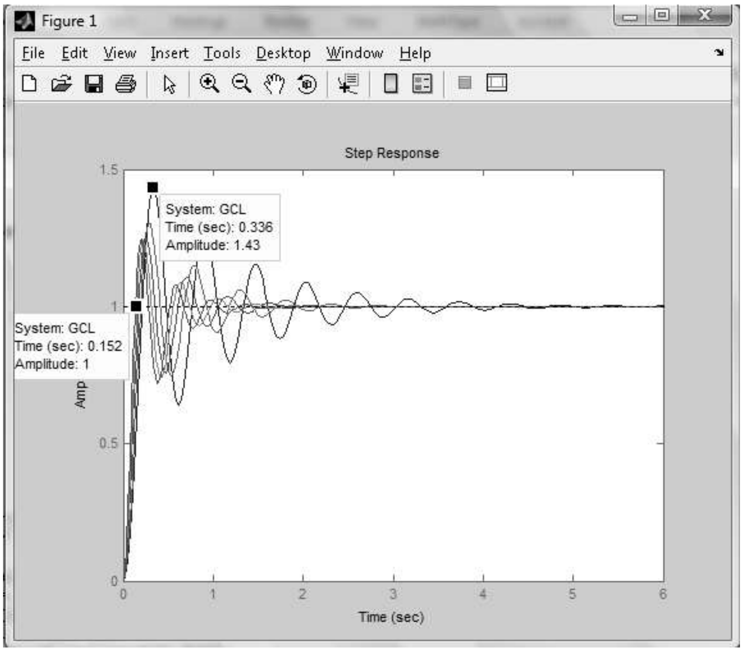
Use the curser to find the maximum overshoot and rise time. For example when $K_p = 2$, PO=43 and tr100%=0.152 sec.

Transfer function:

200 s + 1000

----

s^3 + 10 s^2 + 300 s + 1000



7-63)

(a) Forward-path Transfer Function:

$$G(s) = \frac{100(K_P s + K_I)}{s(s^2 + 10s + 100)}$$

For $K_v = 100,$

$$K_v = \lim_{s \to 0} sG(s) = \lim_{s \to 0} s \frac{100(K_P s + K_I)}{s(s^2 + 10s + 100)} = K_I = 100 \qquad \text{Thus} \quad K_I = 100.$$

(b) The characteristic equation is $s^3 + 10s^2 + (100 + 100K_P)s + 100K_I = 0$

Routh Tabulation:

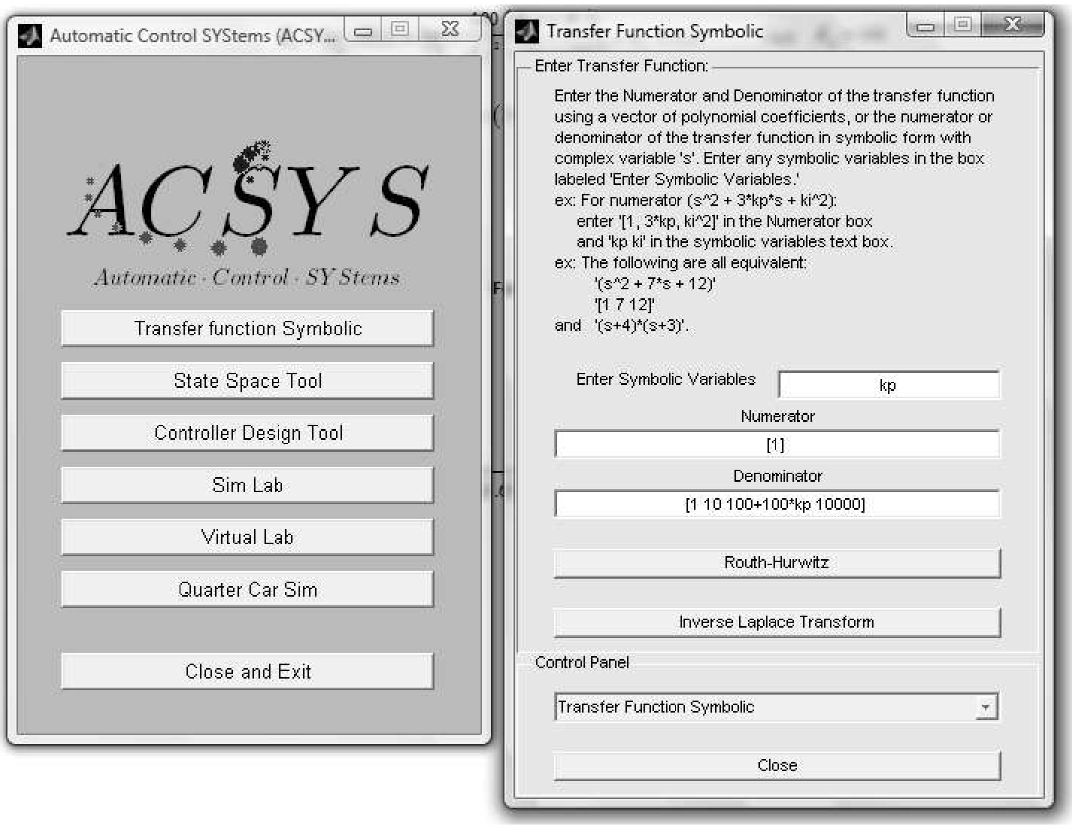
s^3	1	$100+100K_P$
s^2	10	$10{,}000$
s^1	$100K_P - 900$	0
s^0	$10{,}000$	

For stability, $100K_P - 900 > 0$ Thus $K_P > 9$

7. Activate MATLAB
8. Go to the directory containing the ACSYS software.
9. Type in

Acsys

10. Then press the “transfer function Symbolic” and enter the Characteristic equation
11. Then press the “Routh Hurwitz” button



RH =

[1, 100+100*kp]

[10, 10000]

[-900+100*kp, 0]

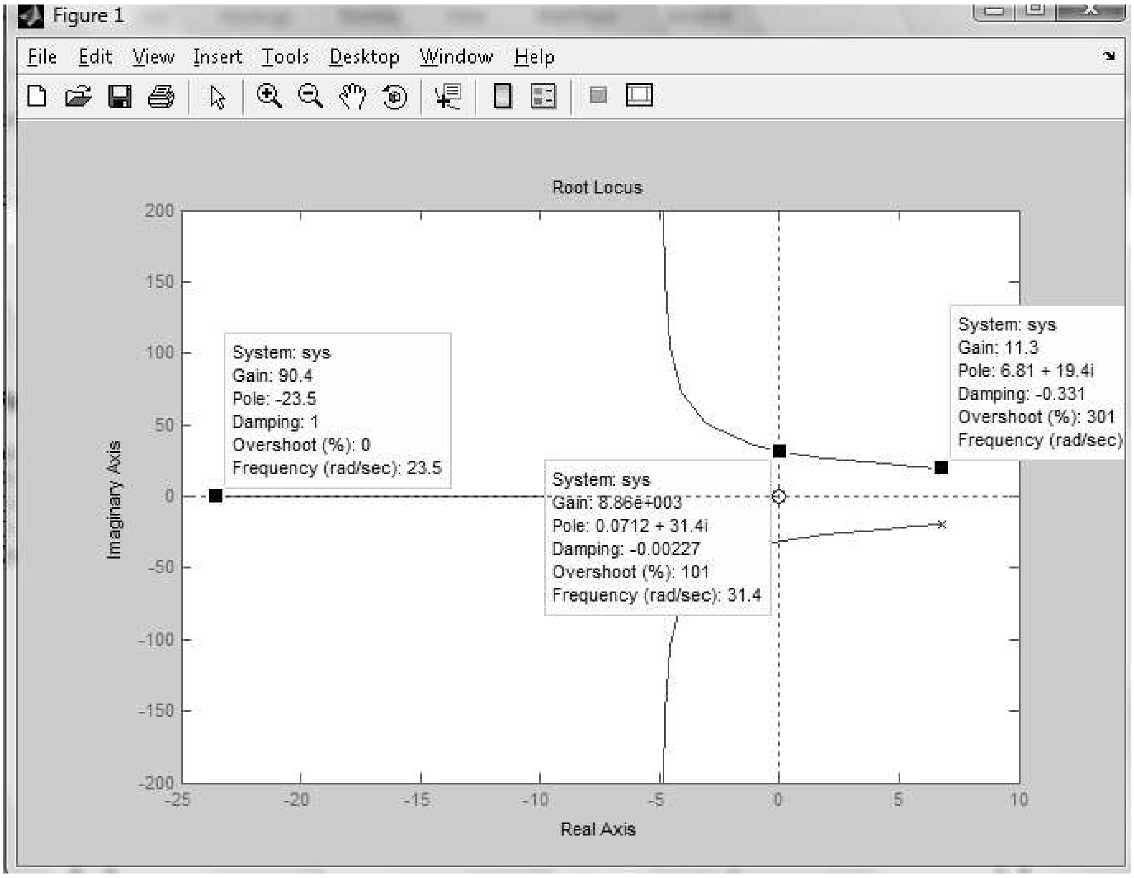
[(-9000000+1000000*kp)/(-900+100*kp), 0]

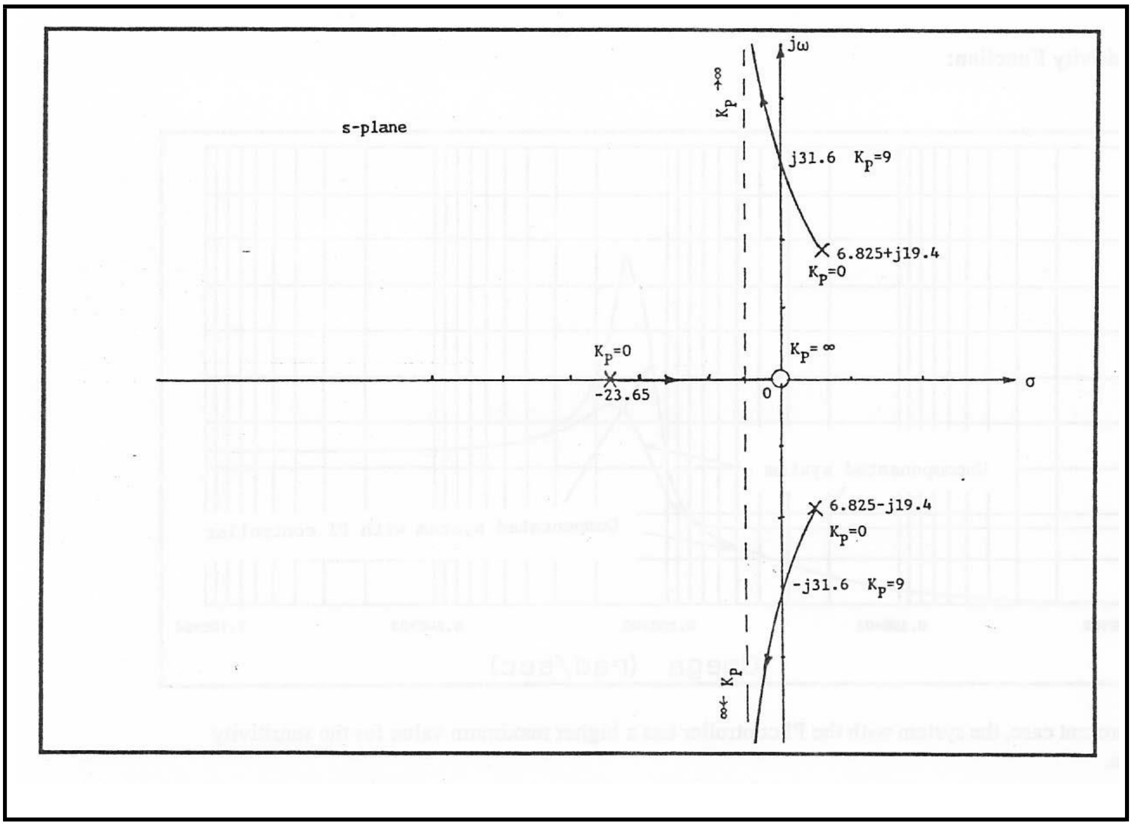
Root Contours:

$$G_{eq}(s) = \frac{100K_P s}{s^3 + 10s^2 + 100s + 10{,}000} = \frac{100K_P s}{(s+23.65)(s-6.825+j19.4)(s-6.825-j19.4)}$$

Root Contours: See Chapter 10 for more information

```
clear all
Kp =.001;
num = [100*Kp 0];
den = [1 10 100 10000];
rlocus(num,den)
```





(c) $K_I = 100$

$$G(s) = \frac{100\left(K_P s + 100\right)}{s\left(s^2 + 10s + 100\right)}$$

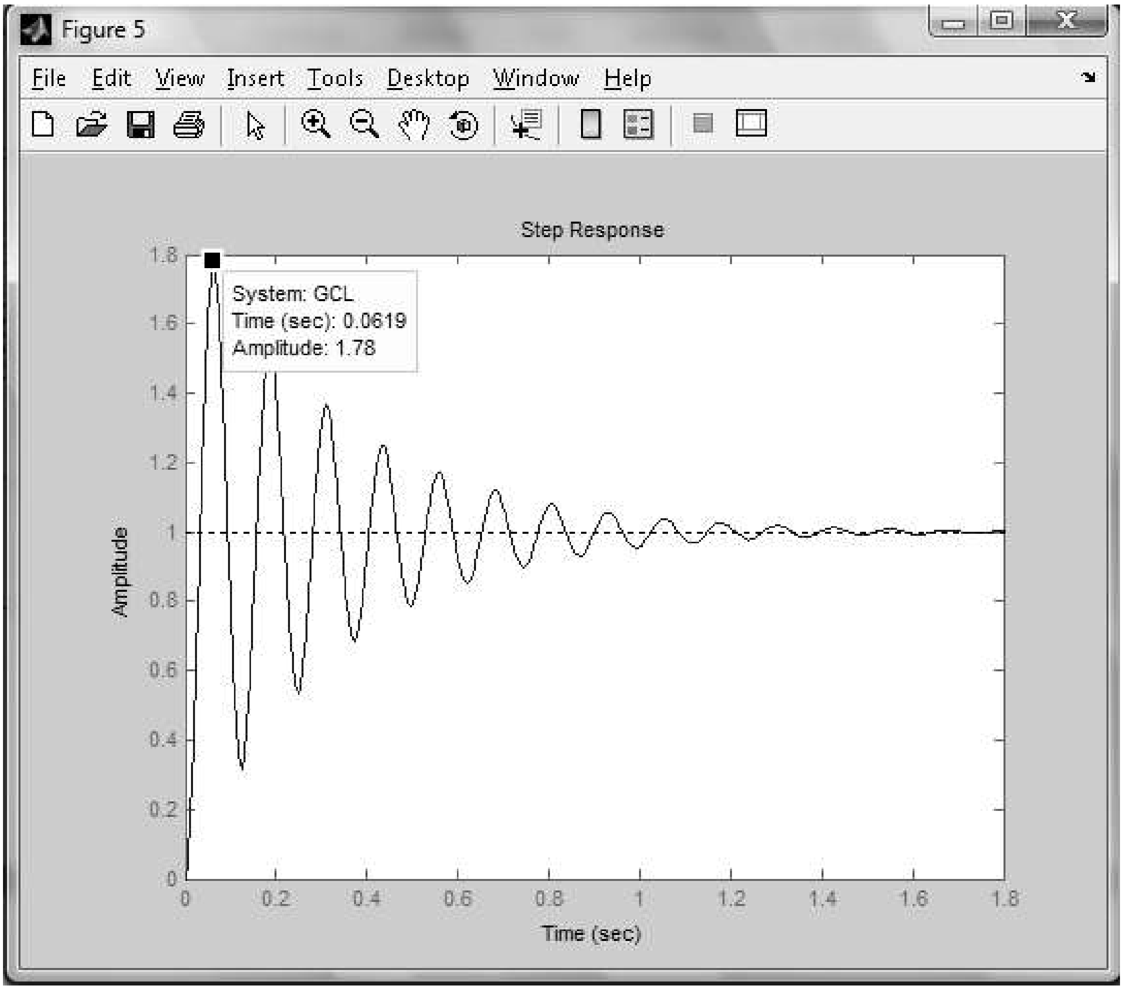
The following maximum overshoots of the system are computed for various values of K_P.

```
clear all
Kp=[15 20 22 24 25 26 30 40 100 1000];
[N,M]=size(Kp);
for i=1:M
num = [100*Kp(i) 10000];
den = [1 10 100 0];
[numCL,denCL]=cloop(num,den);
GCL=tf(numCL,denCL);
figure(i)
step(GCL)
end
```

K_P	15	20	22	24	25	26	30	40	100	1000

y_{max}	1.794	1.779	1.7788	1.7785	1.7756	1.779	1.782	1.795	1.844	1.859

When K_p = 25, minimum y_{max} = 1.7756



Use: **close all** to close all the figure windows.

7-64) MATLAB solution is the same as 7-63.

(a) Forward-path Transfer Function:

$$G(s) = \frac{100\left(K_P s + K_I\right)}{s\left(s^2 + 10s + 100\right)} \qquad \text{For } K_v = \frac{100K_I}{100} = 10, \qquad K_I = 10$$

(b) Characteristic Equation: $s^3 + 10s^2 + 100\left(K_P + 1\right)s + 1000 = 0$

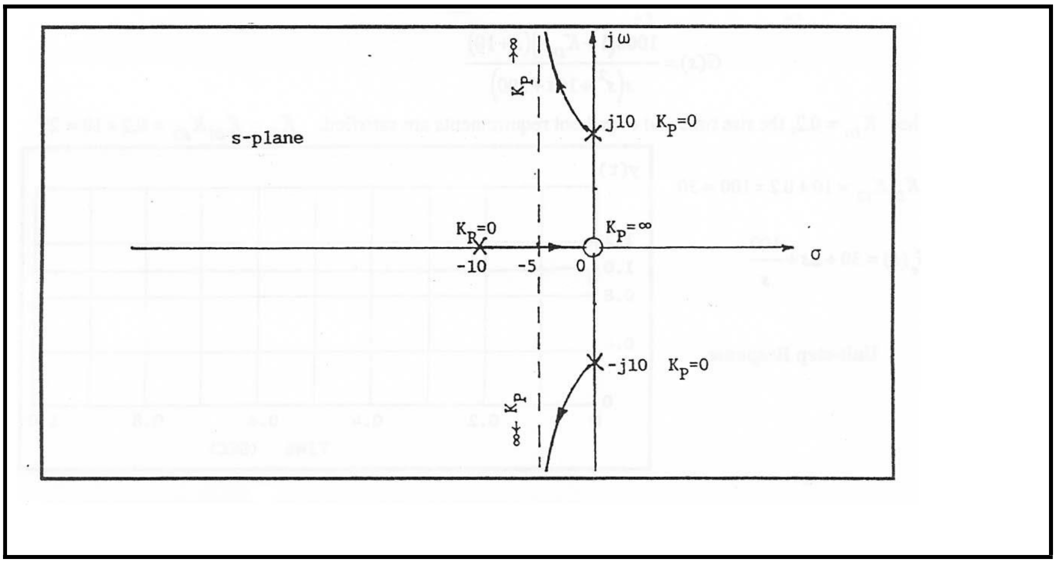
Routh Tabulation:

s^3	1	$100+100K_P$
s^2	10	1000
s^1	$100K_P$	0
s^0	1000	

For stability, $K_P > 0$

Root Contours:

$$G_{eq}(s) = \frac{100K_P s}{s^3 + 10s^2 + 100s + 1000}$$



(c) The maximum overshoots of the system for different values of K_P ranging from 0.5 to 20 are computed and tabulated below.

K_P	0.5	1.0	1.6	1.7	1.8	1.9	2.0	3.0	5.0	10	20
y_{max}	1.393	1.275	1.2317	1.2416	1.2424	1.2441	1.246	1.28	1.372	1.514	1.642

When K_P = 1.7, maximum y_{max} = 1.2416

7-65)

$$G_c(s) = K_P + K_D s + \frac{K_I}{s} = \frac{K_D s^2 + K_P s + K_I}{s} = \left(1 + K_{D1}s\right)\left(K_{P2} + \frac{K_{I2}}{s}\right)$$

where

$$K_P = K_{P2} + K_{D1}K_{I2} \qquad K_D = K_{D1}K_{P2} \qquad K_I = K_{I2}$$

Forward-path Transfer Function:

$$G(s) = G_c(s)G_p(s) = \frac{100\left(K_D s^2 + K_P s + K_I\right)}{s\left(s^2 + 10s + 100\right)}$$

And rename the ratios: $K_D / K_P = A, \quad K_I / K_P = B$

Thus

$$K_v = \lim_{s\to 0} sG(s) = 100\frac{K_I}{100} = 100$$
$$K_I = 100$$

For K_D being sufficiently small:

Forward-path Transfer Function:

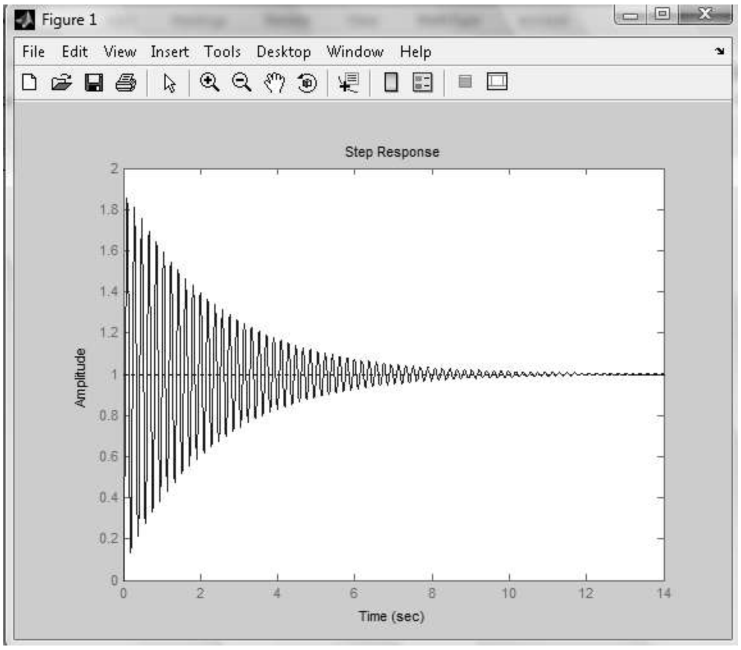
$$G(s) = \frac{100\left(K_p s + 100\right)}{s\left(s^2 + 10s + 100\right)}$$

Characteristic Equation:

$$s^3 + 10s^2 + \left(100 + 100K_p\right)s + 10,000 = 0$$

For stability, $K_p > 9$. Select $K_p = 10$ and observe the response.

```
clear all
Kp=10;
num = [100*Kp 10000];
den =[1 10 100 0];
[numCL,denCL]=cloop(num,den);
GCL=tf(numCL,denCL)
step(GCL)

Transfer function:
       1000 s + 10000
-----------------------------
s^3 + 10 s^2 + 1100 s + 10000
```

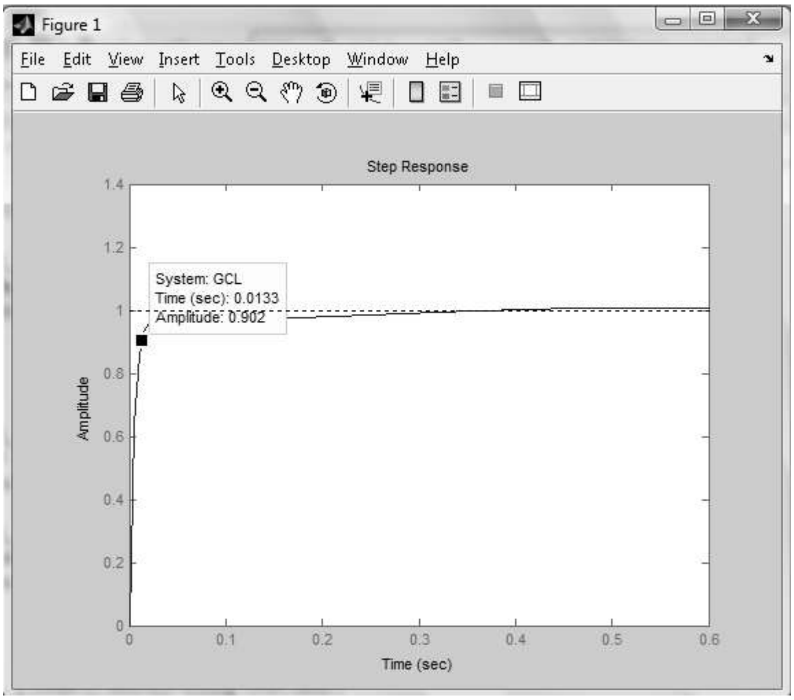


Obviously by increasing Kp more oscillations will occur. Add K_D to reduce oscillations.

```
clear all
Kp=10;
Kd=2;
num = [100*Kd 100*Kp 10000];
den =[1 10 100 0];
[numCL,denCL]=cloop(num,den);
GCL=tf(numCL,denCL)
step(GCL)
```

```
Transfer function:
   200 s^2 + 1000 s + 10000
------------------------------
s^3 + 210 s^2 + 1100 s + 10000
```

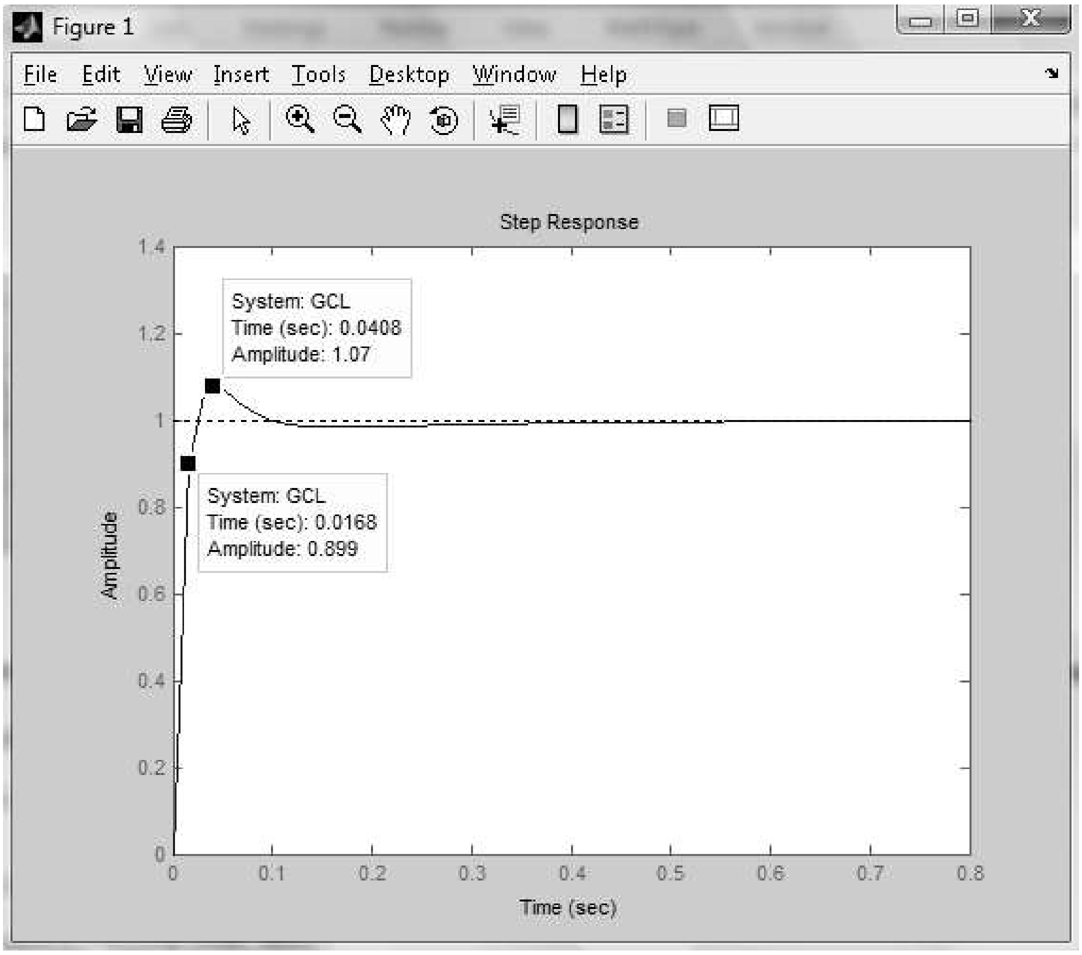
Unit-step Response



The rise time seems reasonable. But we need to increase Kp to improve approach to steady state.

Increase Kp to Kp=30.

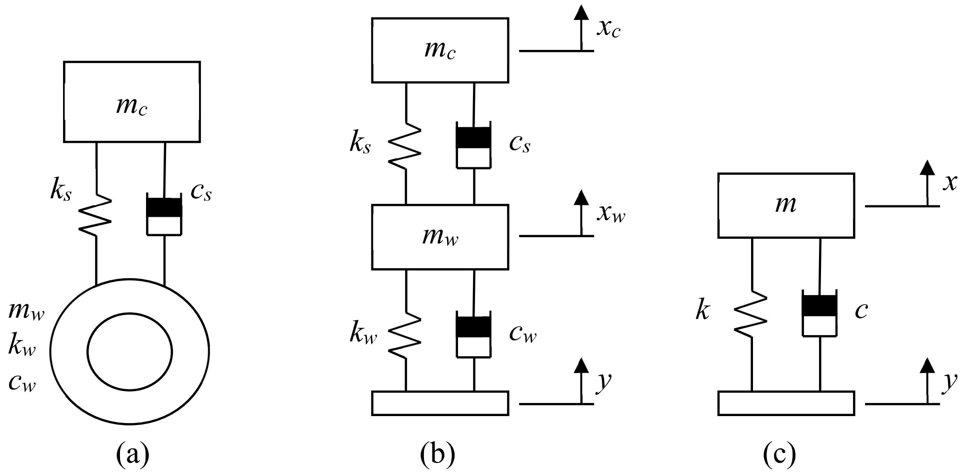
```
clear all
Kp=30;
Kd=1;
num = [100*Kd 100*Kp 10000];
den =[1 10 100 0];
[numCL,denCL]=cloop(num,den);
GCL=tf(numCL,denCL)
step(GCL)

Transfer function:
   100 s^2 + 3000 s + 10000
------------------------------
s^3 + 110 s^2 + 3100 s + 10000
```



To obtain a better response continue adjusting KD and KP.

7-66) For the sake simplicity, this problem we assume the control force *f(t)* is applied in parallel to the spring *K* and damper *B*. We will not concern the details of what actuator or sensors are used.

Lets look at



Quarter car model realization: (a) quarter car, (b) 2 degree of freedom, and (c) 1 degree of freedom model.

The equation of motion of the system is defined as follows:

$$m\ddot{x}(t)+c\dot{x}(t)+kx(t)=c\dot{y}(t)+ky(t)$$

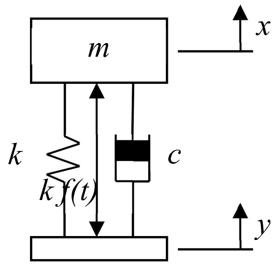
which can be simplified by substituting the relation *z(t) = x(t)-y(t)* and non-dimensionalizing the coefficients to the form

$$\ddot{z}(t)+2\zeta\omega_n\dot{z}(t)+\omega_n^{\,2}z(t)=-\ddot{y}(t)$$

The Laplace transform of Eq. (4-323) yields the input output relationship

$$\frac{Z(s)}{\ddot{Y}(s)}=\frac{-1}{s^2+2\zeta\omega_n s+\omega_n^{\,2}}$$

Now let's apply control.



For simplicity and better presentation, we have scaled the control force as *kf(t)* we rewrite the above as:

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = c\dot{y}(t) + ky(t) + kf(t)$$
$$\ddot{z}(t) + 2\zeta\omega_n\dot{z}(t) + \omega_n{}^2 z(t) = -\ddot{y}(t) + \omega_n{}^2 f(t)$$
$$s^2 + 2\zeta\omega_n s + \omega_n{}^2 = -A(s) + \omega_n{}^2 F(s)$$
$$A(s) = \ddot{Y}(s)$$

Setting the controller structure such that the vehicle bounce $Z(s) = X(s) - Y(s)$ **is minimized:**

$$F(s) = 0 - \left(K_P + K_D s + \frac{K_I}{s}\right) Z(s)$$

$$\frac{Z(s)}{A(s)} = \frac{-1}{s^2 + 2\zeta\omega_n s + \omega_n{}^2\left(1 + K_P + K_D s + \frac{K_I}{s}\right)}$$

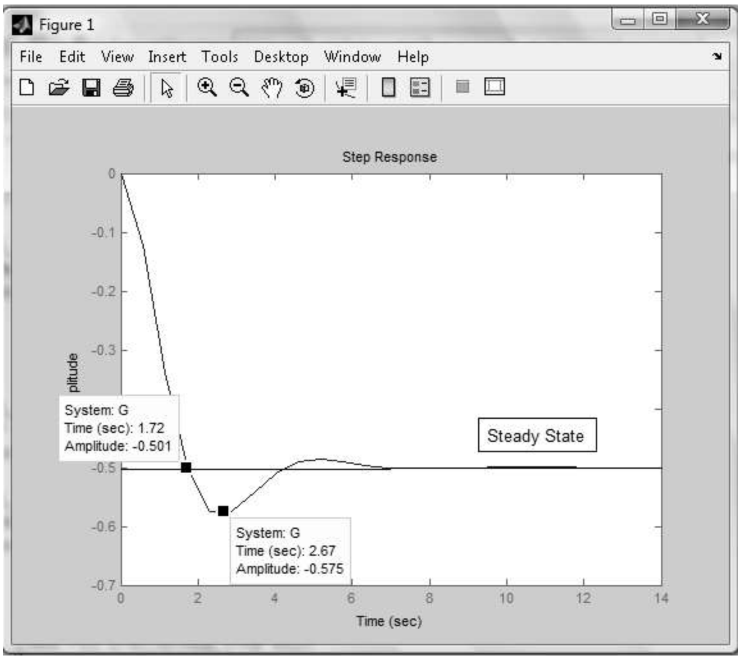
$$\frac{Z(s)}{A(s)} = \frac{-s}{s^3 + 2\zeta\omega_n s^2 + \omega_n{}^2\left(\left(1 + K_P\right)s + K_D s^2 + K_I\right)}$$

For proportional control K_D=K_I=0.

Pick $\varsigma = 0.707$ and $\omega_n = 1$ for simplicity. **This is now an underdamped system.**

Use MATLAB to obtain response now.

```
clear all
Kp=1;
Kd=0;
Ki=0;
num = [-1 0];
den =[1 2*0.707+Kd 1+Kp Ki];
G=tf(num,den)
step(G)

Transfer function:
         -s
---------------------
s^3 + 1.414 s^2 + 2 s
```



Adjust parameters to get the desired response if necessary.

The process is the same for parts b, c and d.

7-67) Replace F(s) with

$$F(s) = X_{ref} - \left(K_P + K_D s + \frac{K_I}{s}\right) X(s)$$

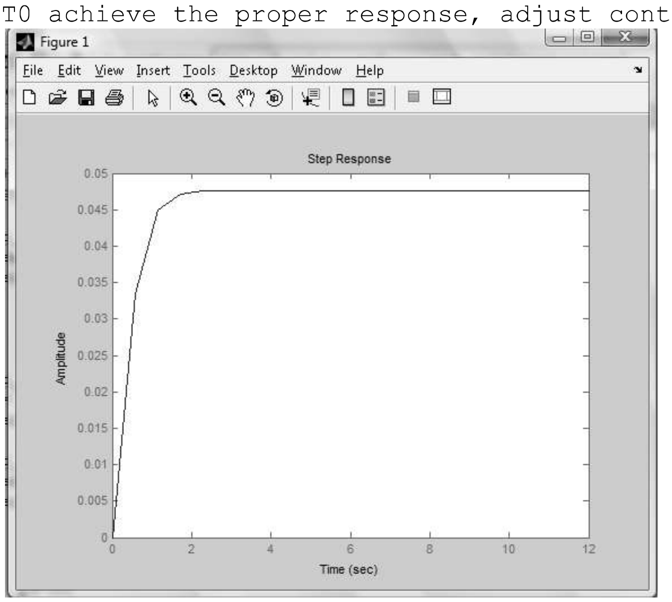
$$2\zeta\omega_n = \frac{B}{M}$$

$$\omega_n^{\ 2} = \frac{K}{M}$$

$$\frac{X(s)}{X_{ref}(s)} = \frac{1}{s^2 + 2\zeta\omega_n s + \left(\omega_n^{\ 2} + K_P + K_D s + \frac{K_I}{s}\right)}$$

Use MATLAB to obtain response now.

```
clear all
Kp=1;
Kd=0;
Ki=0;
B=10;
K=20;
M=1;
omega=sqrt(K/M);
zeta=(B/M)/2/omega;
num = [1 0];
den =[1 2*zeta*omega+Kd omega^2+Kp Ki];
G=tf(num,den)
step(G)
ransfer function:
          s
-------------------
s^3 + 10 s^2 + 21 s

T0 achieve the proper response, adjust controller gains accordingly.
```



7-68)

a) Rotational kinetic energy: $T_{rot} = \frac{1}{2} J \dot{\theta}^2$

Translational kinetic energy: $T_T = \frac{1}{2} m \dot{y}^2$

Relation between translational displacement and rotational displacement:

$$y = r\theta$$

$$\dot{y} = r\dot{\theta}$$

$$T_{Rot} = \frac{1}{2}\frac{J}{r^2}\dot{y}^2$$

Potential energy: $U = \frac{1}{2} K y^2$

As we know $T_{Rot} + T_T + U = constant$, then:

$$\frac{1}{2}\frac{J}{r^2}\dot{y}^2 + \frac{1}{2}\, m\dot{y}^2 + \frac{1}{2}\, Ky^2 = constant$$

By differentiating, we have:

$$\frac{J}{r^2}\dot{y}\ddot{y} + m\dot{y}\ddot{y} + Ky\dot{y} = 0$$

$$\dot{y}\left(\frac{J}{r^2}\ddot{y} + m\ddot{y} + Ky\right) = 0$$

Since $\dot{y}$ cannot be zero, then $J\frac{\ddot{y}}{r^2} + m\ddot{y} + Ky = 0$

b)

$\ddot{y} = r\ddot{\theta}$

$J\ddot{\theta}^2 + m\ddot{y} + Ky = 0$

$$\frac{Y(s)}{\theta(s)} = -\frac{J}{ms^2 + K}$$

c)

$$T_{max} = \frac{1}{2}\, m\, \dot{y}_{max}^2 + \frac{1}{2}\frac{J}{r^2}\dot{y}_{max}^2 = \frac{1}{2}\left(m + \frac{J}{r^2}\right)\dot{y}_{max}^2$$

$$\dot{y}_{max}^2 = \omega_n$$

where $\dot{y} = A$ at the maximum energy.

$$U_{max} = \frac{1}{2}Ky_{max}^2 = \frac{1}{2}KA^2$$

Then:

$$\frac{1}{2}\left(m + \frac{J}{r^2}\right)\omega_n^2 A^2 = \frac{1}{2}KA^2$$

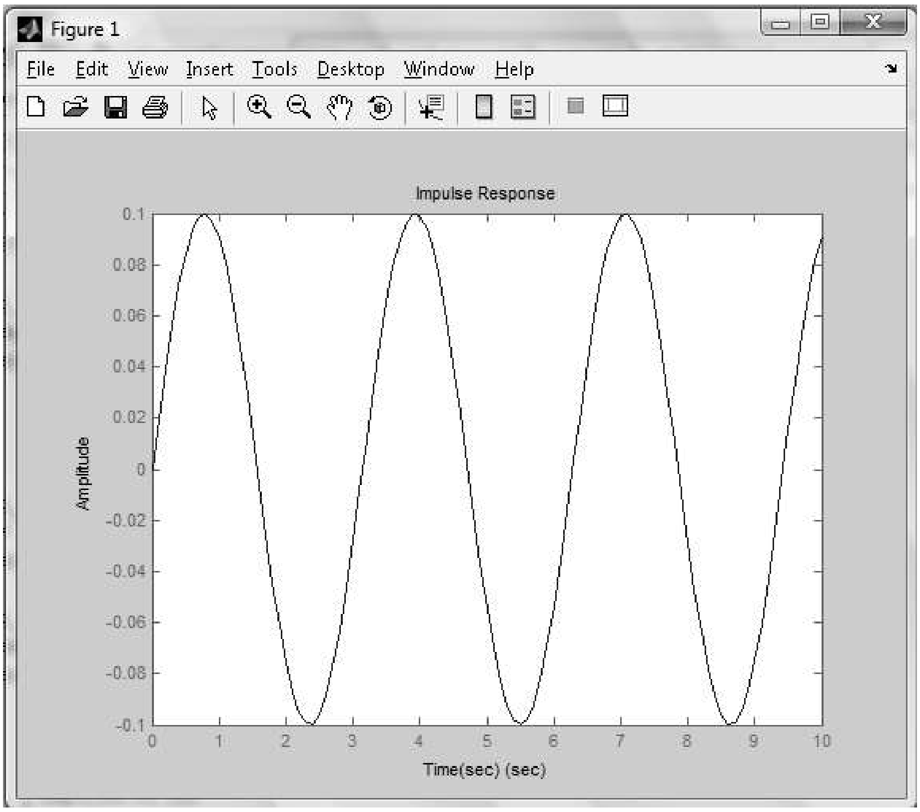
Or:

$$\omega_n = \sqrt{\frac{K}{m + \frac{J}{r^2}}} = r\sqrt{\frac{K}{r^m + J}}$$

d) $G(s) = \dfrac{J}{(ms^2 + K)}$

```
% select values of m, J and K
K=100;
J=5;
m=25;
G=tf([J],[m 0 K])
Pole(G)
impulse(G,10)
xlabel( 'Time(sec)');
ylabel('Amplitude');

Transfer function:
     5
------------
25 s^2 + 100

ans =
        0 + 2.0000i
        0 - 2.0000i
```

Uncontrolled

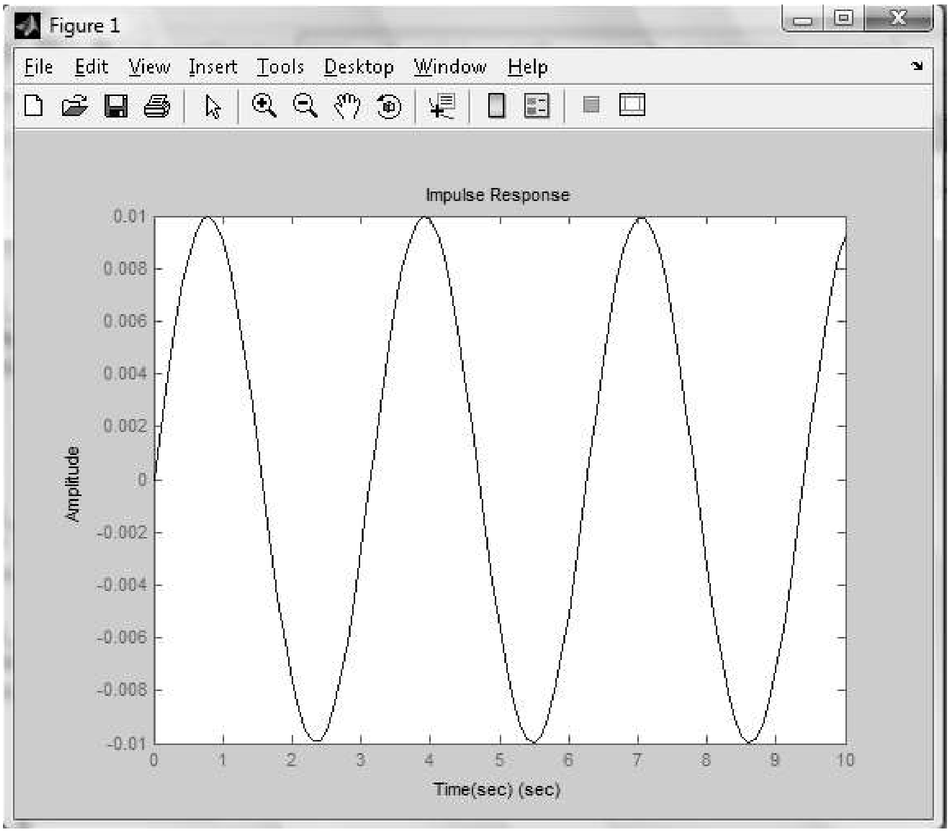


With a proportional controller one can adjust the oscillation amplitude the transfer function is rewritten as:

$$G_{cl}(s) = \frac{JK_p}{(ms^2 + K + JK_p)}$$

```
% select values of m, J and K
Kp=0.1
K=100;
J=5;
m=25;
G=tf([J*Kp],[m 0 (K+J*Kp)])
Pole(G)
impulse(G,10)
xlabel( 'Time(sec)');
ylabel('Amplitude');
```

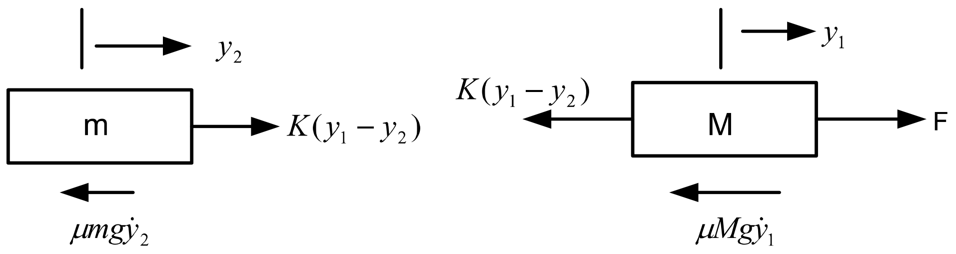
```
Kp =
    0.1000

Transfer function:
     0.5
--------------
25 s^2 + 100.5

ans =
        0 + 2.0050i
        0 - 2.0050i
```



A PD controller must be used to damp the oscillation and reduce overshoot. Use Example 7-7-1 as a guide.

7-69) Recall:

a)



b) From Newton's Law:

$M\ddot{y}_1 = F - K(y_1 - y_2) - \mu Mg\dot{y}_1$

$m\ddot{y}_2 = K(y_1 - y_2) - \mu mg\dot{y}_2$

If y_1 and y_2 are considered as a position and v_1 and v_2 as velocity variables

Then: $\begin{cases} \dot{y}_1 = v_1 \\ \dot{y}_2 = v_2 \\ M\dot{v}_1 = F - K(y_1 - y_2) - \mu Mgv_1 \\ m\dot{v}_2 = F - K(y_1 - y_2) - \mu mgv_2 \end{cases}$

The output equation can be the velocity of the engine, which means $z = v_2$

c)

$$\begin{cases} Ms^2Y_1(s) = F - K\big(Y_1(s) - Y_2(s)\big) - \mu MgsY_1(s) \\ ms^2Y_2(s) = K\big(Y_1(s) - Y_2(s)\big) - \mu mgsY_2(s) \\ Z(s) = V_2(s) = sY(s) \end{cases}$$

Obtaining $\frac{Z(s)}{F(s)}$ requires solving above equation with respect to $Y_2(s)$

From the first equation:

$$(Ms^2 + K + \mu Mgs)Y_1(s) = F + KY_2(s)$$

$$Y_1(s) = \frac{F + KY_2(s)}{Ms^2 + \mu Mgs + K}$$

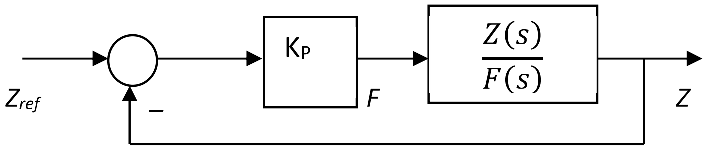
Substituting into the second equation:

$$ms^2Y_2(s) = \frac{KF + K^2Y_2(s)}{Ms^2 + \mu Mgs + K} - KY_2(s) - \mu mgsY_2(s)$$

By solving above equation:

$$\frac{Z(s)}{F(s)}=\frac{sY_2(s)}{F(s)}=\frac{ms^2+m\,\mu gs+1}{Mms^3+(2Mm\,\mu g)s^2+(Mk+Mm(\mu g)^2+mK)s+K\mu g(M+m)}$$

Replace Force F with a proportional controller so that $F=K(Z-Z_{ref})$:



$$\frac{Z(s)}{K(Z_{ref}(s)-Z(s))}=\frac{ms^2+m\,\mu gs+1}{Mms^3+(2Mm\,\mu g)s^2+(Mk+Mm(\mu g)^2+mK)s+K\mu g(M+m)}$$

$$\frac{Z(s)}{Z_{ref}(s)}=\frac{K_P(ms^2+m\,\mu gs+1)}{Mms^3+(2Mm\,\mu g)s^2+(Mk+Mm(\mu g)^2+mK)s+K\mu g(M+m)+\ K_P(ms^2+m\,\mu gs+1)}$$

7-70)

Here is an alternative representation including friction (damping) μ. In this case the angle *θ* is measured differently.

Let's find the dynamic model of the system:

1) $(M+m)\ddot{x} + \mu\dot{x} - ml\ddot{\theta}\cos\theta - ml\dot{\theta}^2\sin\theta = F$

2) $(I+mI^2)\ddot{\theta} + mgl\sin\theta = -ml\ddot{x}\cos\theta$

Let $\theta = \pi + \Phi$. If Φ is small enough then $\cos\Phi \to 1$ and $\sin\Phi \to \Phi$, therefore

$$\begin{cases}(M+m)\ddot{x} + \mu\dot{x} - ml\ddot{\Phi} = F \\ (l+ml^2)\ddot{\Phi} - mgl\Phi = ml\ddot{x}\end{cases}$$

which gives:

$$\frac{\Phi(s)}{F(s)} = \frac{mls^2}{[(M+m)(l+ml^2)-(ml)^2]s^3 + \mu(l+ml^2)s^2 - (M+m)mgls - \mu mgl}$$

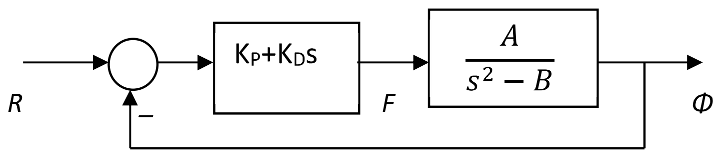
Ignoring friction $\mu = 0$.

$$\frac{\Phi(s)}{F(s)} = \frac{ml}{[(M+m)(l+ml^2)-(ml)^2]s^2 - (M+m)mgl} = \frac{A}{s^2 - B}$$

where

$$A = \frac{ml}{[(M+m)(I+ml^2)-(ml)^2]};\ B = \frac{(M+m)mgl}{[(M+m)(l+ml^2)-(ml)^2]}$$

Ignoring actuator dynamics (DC motor equations), we can incorporate feedback control using a series PD compensator and unity feedback. Hence,



$$F(s) = K_p\left(R(s) - \Phi\right) - K_D s\left(R(s) - \Phi\right)$$

The system transfer function is:

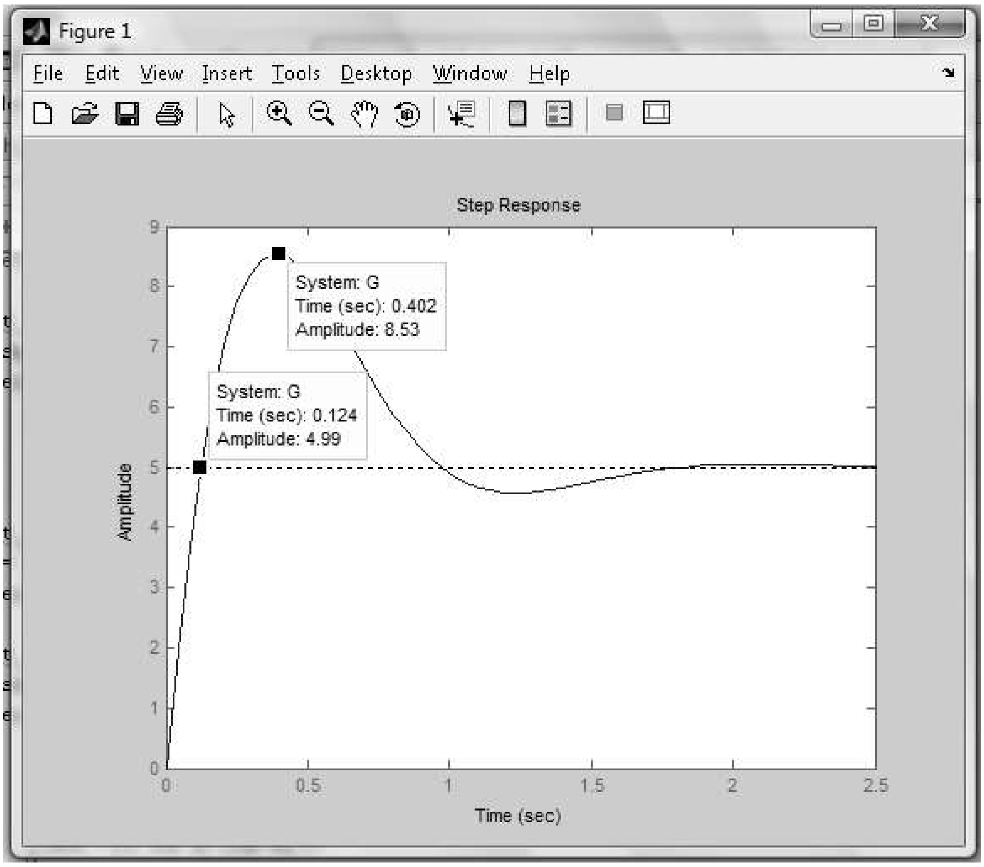
$$\frac{\Phi}{R} = \frac{A\left(K_p + K_D s\right)}{(s^2 + K_D s + A\left(K_p - B\right)}$$

Control is achieved by ensuring stability (K_p>B)

```
Use Routh Hurwitz to establish stability first. Use Acsys to do that as
demonstrated in this chapter problems. Also Chapter 2 has many examples.
```

Use MATLAB to simulate response:

```
clear all
Kp=10;
Kd=5;
A=10;
B=8;
num = [A*Kd A*Kp];
den =[1 Kd A*(Kp-B)];
G=tf(num,den)
step(G)

Transfer function:
  50 s + 100
--------------
s^2 + 5 s + 20
```



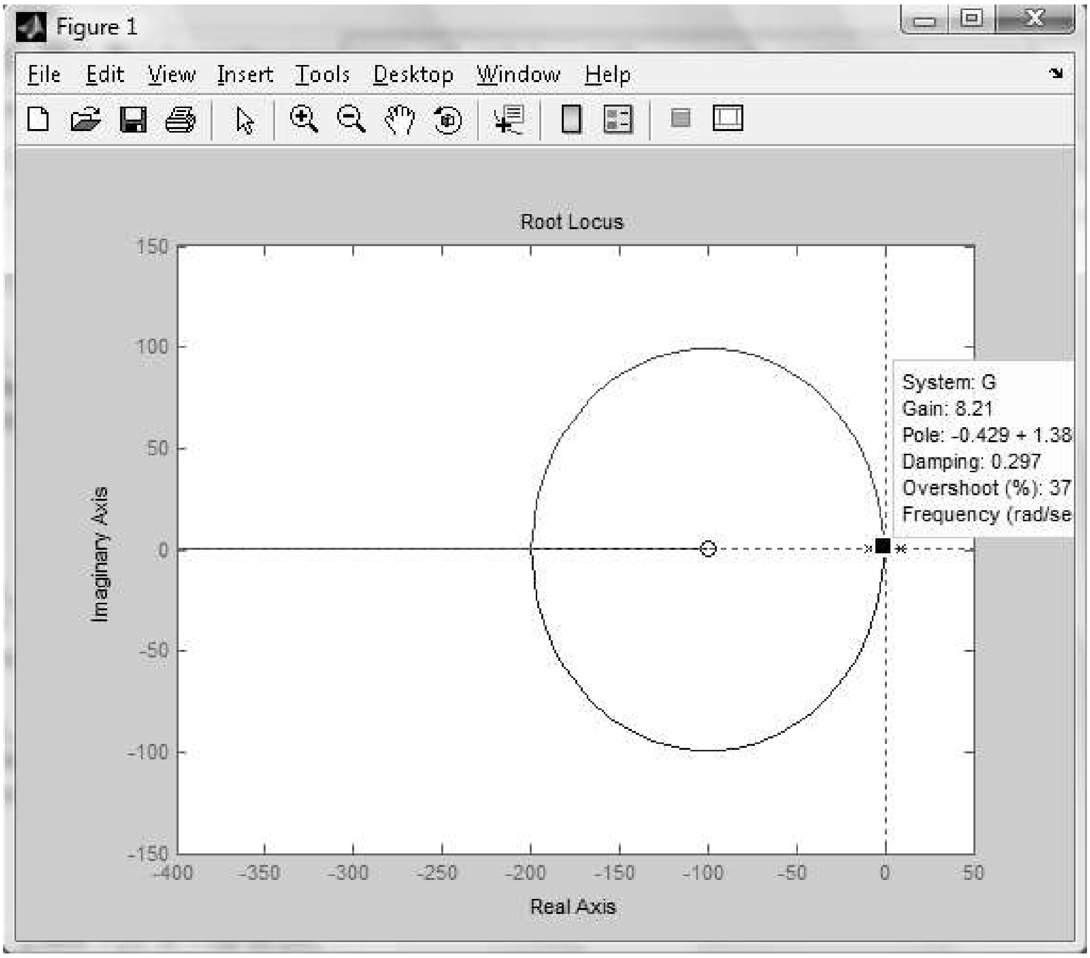
Adjust parameters to achieve desired response. Use THE PROCEDURE in Example 7-7-1.

You may look at the root locus of the forward path transfer function to get a better perspective.

$$\frac{\Phi}{E}=\frac{A\left(K_p+K_D s\right)}{s^2-AB}=\frac{AK_D\left(z+s\right)}{s^2-AB}$$

fix z and vary K_D.

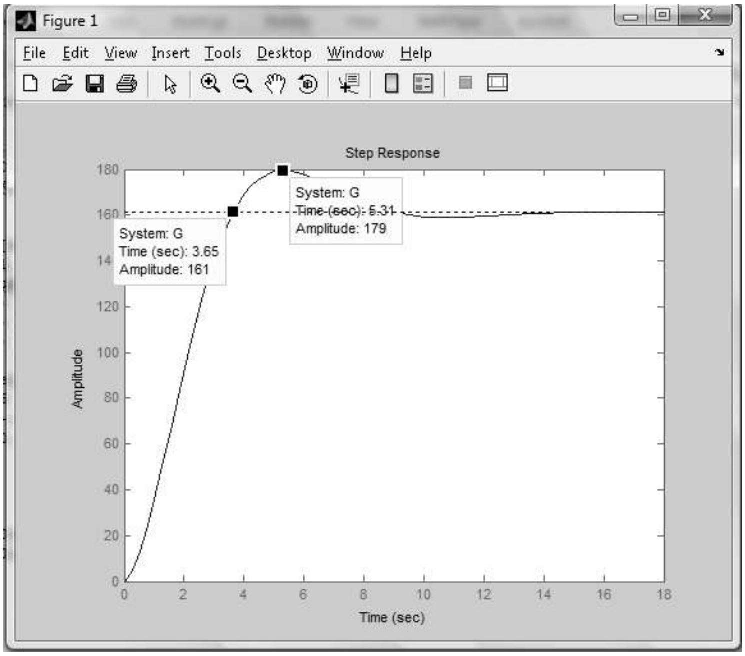
```
clear all
z=100;
Kd=0.01;
A=10;
B=8;
num = [A*Kd A*Kd*z];
den =[1 0 -(A*B)];
G=tf(num,den)
rlocus(G)

Transfer function:
0.1 s + 10
----------
 s^2 - 80
```



For z=10, a large K_D=0.805 results in:

```
clear all
Kd=0.805;
Kp=10*Kd;
A=10;
B=8;
num = [A*Kd A*Kp];
den =[1 Kd A*(Kp-B)];
G=tf(num,den)
pole(G)
zero(G)
step(G)

Transfer function:
   8.05 s + 80.5
-------------------
s^2 + 0.805 s + 0.5


ans =

  -0.4025 + 0.5814i
  -0.4025 - 0.5814i

ans =

   -10
```

Looking at dominant poles we expect to see an oscillatory response with overshoot close to desired values.



For a better design, and to meet rise time criterion, use Example 7-7-1.