## Chapter 10

**10-1 (a)** ***K* = 5** $\omega_n = \sqrt{5} = 2.24 \text{ rad/sec} \qquad \zeta = \dfrac{6.54}{4.48} = 1.46 \qquad M_r = 1 \qquad \omega_r = 0 \text{ rad/sec}$

**(b)** ***K* = 21.39** $\omega_n = \sqrt{21.39} = 4.62 \text{ rad/sec} \quad \zeta = \dfrac{6.54}{9.24} = 0.707 \quad M_r = \dfrac{1}{2\zeta\sqrt{1-\zeta^2}} = 1$

$$\omega_r = \omega_n\sqrt{1-\varsigma^2} = 3.27 \text{ rad/sec}$$

**(c)** ***K* = 100** $\omega_n = 10 \text{ rad/sec} \quad \zeta = \dfrac{6.54}{20} = 0.327 \quad M_r = 1.618 \quad \omega_r = 9.45 \text{ rad/sec}$

**10-2**

**MATLAB code:**

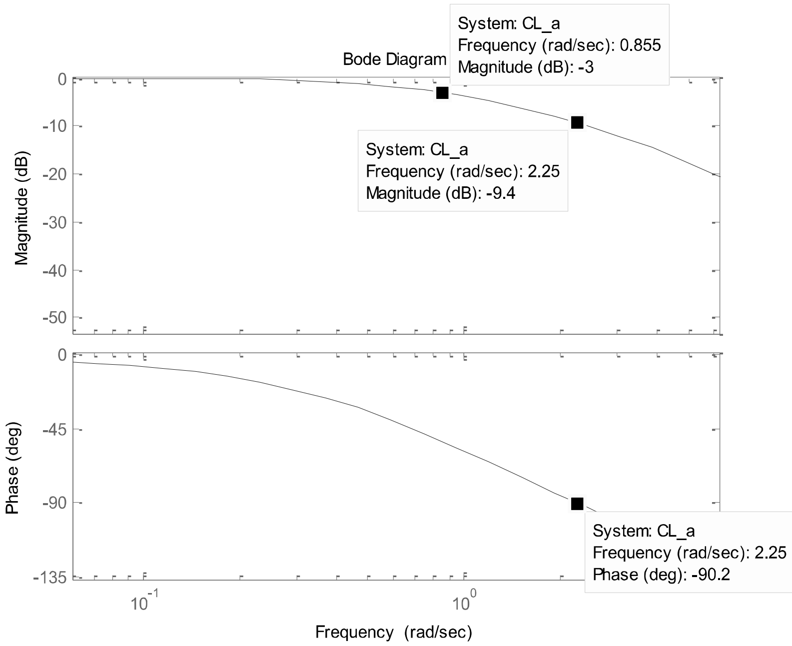
```
% Question 10-2,
clear all;
close all;
s = tf('s')

%a)
num_G_a= 5;
den_G_a=s*(s+6.54);
G_a=num_G_a/den_G_a;
CL_a=G_a/(1+G_a)
BW = bandwidth(CL_a)
bode(CL_a)

%b)
figure(2);
num_G_b=21.38;
den_G_b=s*(s+6.54);
G_b=num_G_b/den_G_b;
CL_b=G_b/(1+G_b)
BW = bandwidth(CL_b)
bode(CL_b)

%c)
figure(3);
num_G_c=100;
den_G_c=s*(s+6.54);
G_c=num_G_c/den_G_c;
```

**Bode diagram (a) – k=5:** data points from top to bottom indicate bandwidth BW, resonance peak $M_r$, and resonant frequency $\omega_r$.




**Bode diagram (b) – k=21.38:** data points from top to bottom indicate bandwidth BW, resonance peak $M_r$, and resonant frequency $\omega_r$.

<table>
<tr>
<td>

```
CL_c=G_c/(1+G_c)
BW = bandwidth(CL_c)
bode(CL_c)
```

</td>
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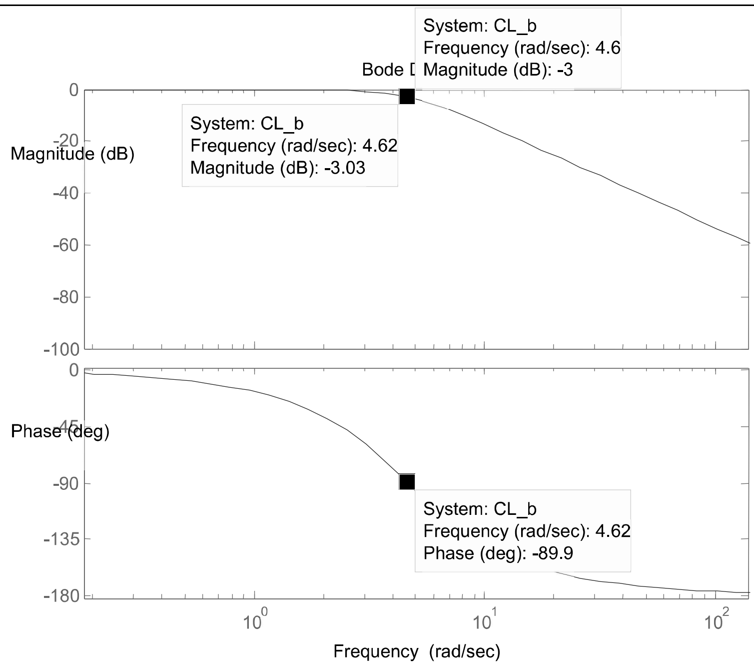
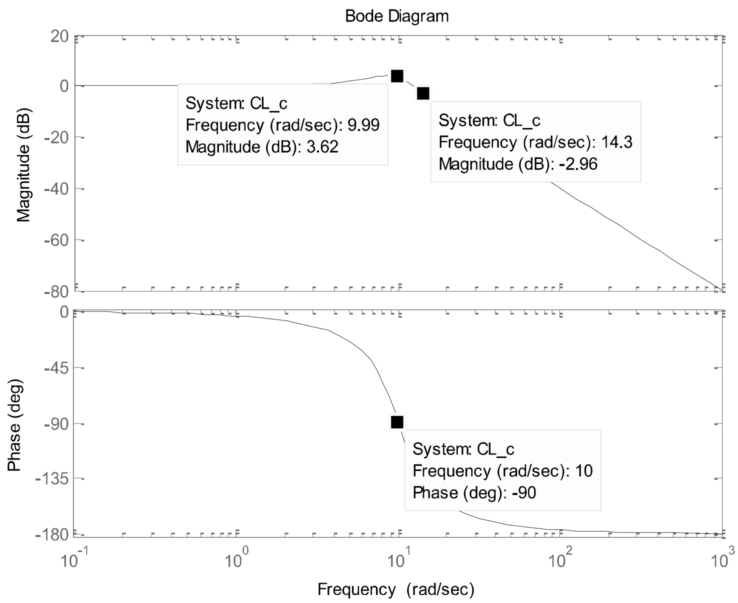



**Bode diagram (c) – k=100:** data points from top to bottom indicate resonance peak $M_r$, bandwidth BW, and resonant frequency $\omega_r$.

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**10-3)** If $u(t) = U\sin(\omega t)$ is the input, then

$$G(j\omega) = \frac{j\omega + \frac{1}{A_1}}{j\omega + \frac{1}{A_2}} = \frac{A_2(1 + A_1 j\omega)}{A_1(1 + A_2 j\omega)}$$

where

$$|G(j\omega)| = \frac{A_2}{A_1}\sqrt{\frac{1 + A_1^2\omega^2}{1 + A_2^2\omega^2}}$$

and

$$\angle G(j\omega) = \tan^{-1} A_1\omega - \tan^{-1} T_2\omega \; = \Phi$$

Therefore:

$$y(t) = \frac{UA_2}{A_1}\sqrt{\frac{1 + A_1^2\omega^2}{1 + A_2^2\omega^2}}\sin(\omega t + \tan^{-1} A_1\omega - \tan^{-1} A_2\omega)$$

As a result:

$$\begin{cases} if\ A_1 > A_2 \rightarrow \Phi > 0 \rightarrow the\ network\ is\ a\ lead\ network \\ if\ A_1 < A_2 \rightarrow \Phi < 0 \rightarrow the\ network\ is\ a\ lag\ network \end{cases}$$

**10-4 (a)** $M_r = 2.944\ (9.38\ \text{dB})$ $\omega_r = 3\ \text{rad/sec}$ $\text{BW} = 4.495\ \text{rad/sec}$

**(b)** $M_r = 15.34\ (23.71\ \text{dB})$ $\omega_r = 4\ \text{rad/sec}$ $\text{BW} = 6.223\ \text{rad/sec}$

**(c)** $M_r = 4.17\ (12.4\ \text{dB})$ $\omega_r = 6.25\ \text{rad/sec}$ $\text{BW} = 9.18\ \text{rad/sec}$

**(d)** $M_r = 1\ (0\ \text{dB})$ $\omega_r = 0\ \text{rad/sec}$ $\text{BW} = 0.46\ \text{rad/sec}$

**(e)** $M_r = 1.57\ (3.918\ \text{dB})$ $\omega_r = 0.82\ \text{rad/sec}$ $\text{BW} = 1.12\ \text{rad/sec}$

**(f)** $M_r = \infty$ (unstable) $\omega_r = 1.5$ rad/sec BW = 2.44 rad/sec

**(g)** $M_r = 3.09$ (9.8 dB) $\omega_r = 1.25$ rad/sec BW = 2.07 rad/sec

**(h)** $M_r = 4.12$ (12.3 dB) $\omega_r = 3.5$ rad/sec BW = 5.16 rad/sec

**10-5)**

Maximum overshoot = 0.1 Thus, $\zeta = 0.59$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.05 \qquad t_r = \frac{1-0.416\zeta+2.917\zeta^2}{\omega_n} = 0.1 \text{ sec}$$

Thus, minimum $\omega_n = 17.7$ rad/sec Maximum $M_r = 1.05$

Minimum BW = $\omega_n\left(\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}\right)^{1/2} = 20.56$ rad/sec

**10-6)**

Maximum overshoot = 0.2 Thus, $0.2 = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$ $\zeta = 0.456$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.232 \qquad t_r = \frac{1-0.416\zeta+2.917\zeta^2}{\omega_n} = 0.2$$

Thus, minimum $\omega_n = 14.168$ rad/sec

Maximum $M_r = 1.232$ Minimum BW = $\left(\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}\right)^{1/2} = 18.7$ rad/sec

**10-7)** Maximum overshoot = 0.3 Thus, $0.3 = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}}$ $\zeta = 0.358$

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 1.496 \qquad t_r = \frac{1-0.416\zeta+2.917\zeta^2}{\omega_n} = 0.2$$

Thus, minimum $\omega_n = 6.1246$ rad/sec

Maximum $M_r$ = 1.496 Minimum BW = $\left(\left(1-2\zeta^2\right)+\sqrt{4\zeta^4-4\zeta^2+2}\right)^{1/2} = 1.4106$ rad/sec

**10-8)** (a)

$$G(j\omega) = \frac{0.5K}{-0.375\ \omega^2 + j(\omega - 0.25\ \omega^2)}$$

At the gain crossover:

$$|G(j\omega)| = \frac{0.5K}{\sqrt{(0.375^2\omega^4) + (\omega^2 - 0.25\ \omega^3)^2}} = 1$$

Therefore:

$$(0.375)^2\omega^4 + (\omega - 0.25\ \omega^3)^2 - 0.25K^2 = 0$$

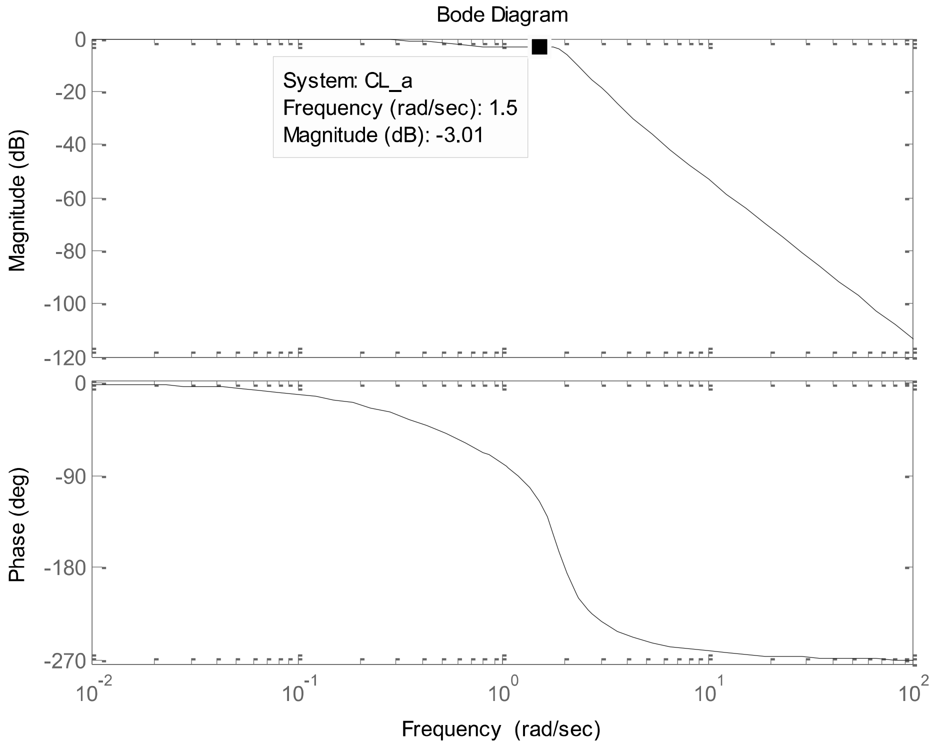
at ω = 1.5 ⇨ $K = 2.138$

**(b)**

**MATLAB code:**

```
%solving for k:
syms kc
omega=1.5
sol=eval(solve('0.25*kc^2=0.7079^2*((-0.25*omega^3+omega)^2+(-
0.375*omega^2+0.5*kc)^2)',kc))
%ploting bode with K=1.0370
s = tf('s')
K=1.0370;
num_G_a= 0.5*K;
den_G_a=s*(0.25*s^2+0.375*s+1);
G_a=num_G_a/den_G_a;
CL_a = G_a/(1+G_a)
BW = bandwidth(CL_a)
bode(CL_a);
```

**Bode diagram:** data point shows -3dB point at 1.5 rad/sec frequency which is the closed loop bandwidth




**10-9)** $\theta = \sin^{-1}\left(\frac{1}{Mp}\right) = \sin^{-1}\left(\frac{1}{2.2}\right) \approx 27^o$

$\alpha = 90 - \theta = 63^{\circ}$

$$OA = -\frac{M^2}{M^2-1} = -1.26$$

Therefore:

$$\begin{aligned} AB &= \frac{M}{M^2-1}\cos\alpha \\ &= \left(\frac{M}{M^2-1}\right)\cos(90-\theta) \\ &= \frac{M}{M^2-1}\sin\alpha \\ &= \frac{M}{M^2-1}\left(\frac{1}{M}\right) = \frac{1}{M^2-1} \end{aligned}$$

As a result:

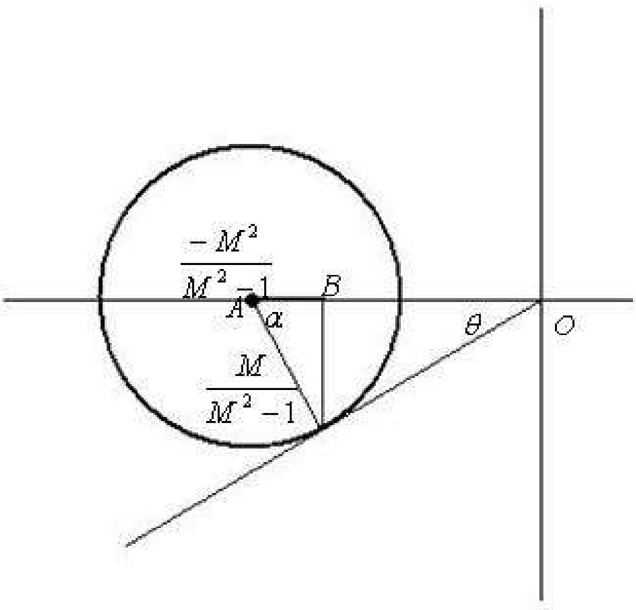
$$OB = -\frac{M^2}{M^2-1} - \frac{1}{M^2-1} = -1$$

Therefore:

$$|G(j\omega)|_{\substack{s=-1 \\ k=1}} = -0.54$$

To change the crossover frequency requires adding gain as:

$$K = -\frac{1}{-0.54} = 1.85$$

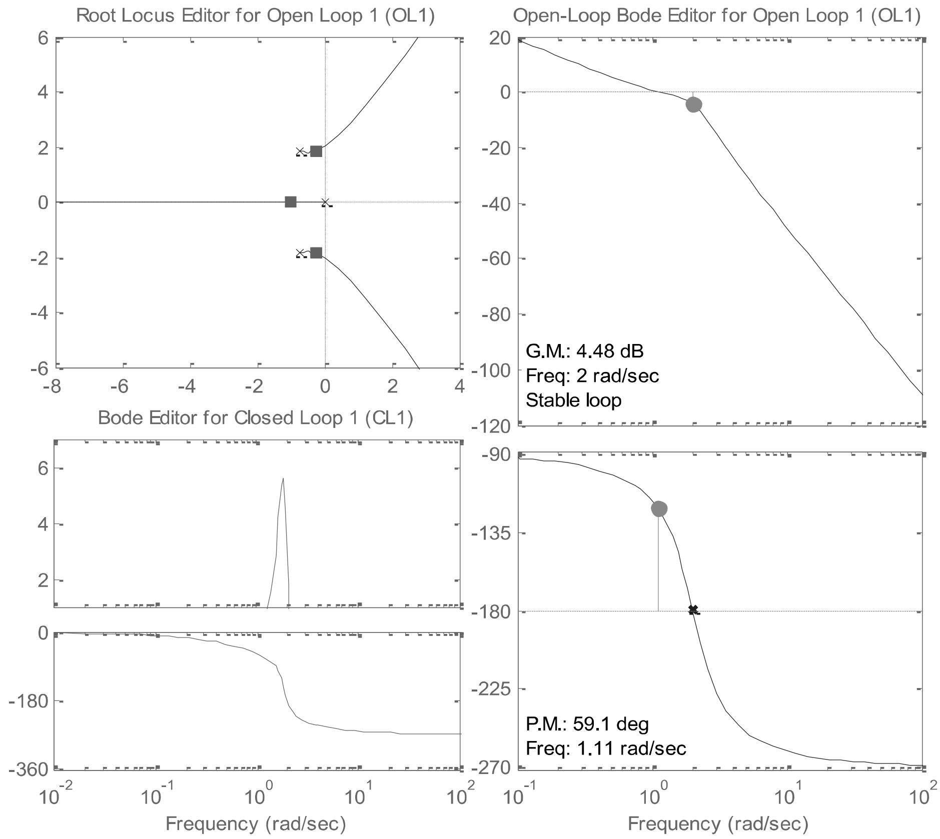



**(b) MATLAB code:**

```
s = tf('s')
%(b)
K = 0.95*2;
num_G_a = 0.5*K;
den_G_a = s*(0.25*s^2+0.375*s+1);
G_a = num_G_a/den_G_a;
CL_a = G_a/(1+G_a)
bode(CL_a);
figure(2);
sisotool
```

Peak mag = 2.22 can be converted to dB units by: 20*Log(2.22)= 6.9271 dB

By using sisotool and importing the loop transfer function, the overall gain (0.5K) was changed until the magnitude of the resonance in Bode was about 6.9 dB. At 0.5K=~0.95 or K=1.9, this resonance peak was achieved as can be seen in the BODE diagram of the following figure:




**10-10)**

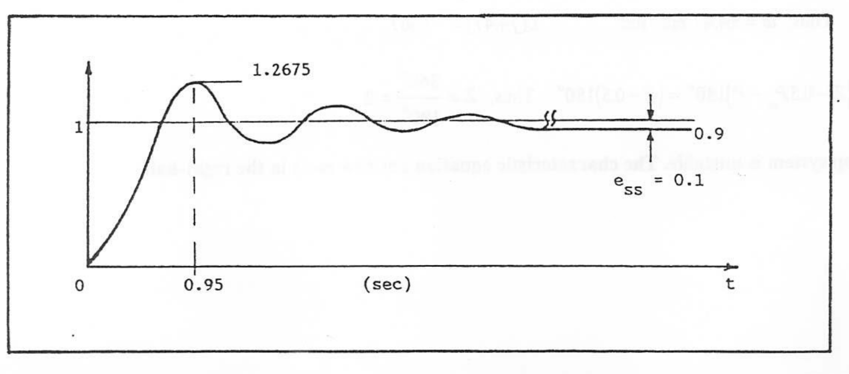
$$M_r = 1.4 = \frac{1}{2\zeta\sqrt{1-\zeta^2}} \qquad \text{Thus,} \quad \zeta = 0.387 \qquad \text{Maximum overshoot} = e^{\frac{-\pi\zeta}{\sqrt{1-\zeta^2}}} = 0.2675 \ (26.75\%)$$

$$\omega_r = 3 \text{ rad/sec} = \omega_n\sqrt{1-2\zeta^2} = 0.8367\omega_n \text{ rad/sec} \qquad \omega_n = \frac{3}{0.8367} = 3.586 \text{ r ad/sec}$$

$$t_{\max} = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}} = \frac{\pi}{3.586\sqrt{1-(0.387)^2}} = 0.95 \text{ sec} \qquad \text{At } \omega = 0, \quad |M| = 0.9.$$

This indicates that the steady-state value of the unit-step response is 0.9.

**Unit-step Response:**




**10-11)** a) The closed loop transfer function is:

$$\frac{Y(s)}{X(s)} = \frac{GH}{1+GH} = \frac{K}{10s^2+s+K} = \frac{K}{s^2+0.1s+0.1K}$$

as $\frac{1}{2\,\xi\sqrt{1-\xi^2}} = 1.4$ ,which means $\xi = 0.387$

According to the transfer function: $\xi\,\omega_n = 0.1 \Rightarrow \omega_n = 0.129$ rad/s
As $\omega_n^2 = 0.1K$ ; then, $K = 10\,\omega_n^2 = 0.1669$

b) $\omega_R = \omega_0\sqrt{1-2\,\xi^2} = 0.108\,\frac{rad}{s}$

$M_p = \exp\left(-\frac{\pi\xi}{\sqrt{1-\xi^2}}\right) = 0.268$

$PM = \angle GH(j\omega_g) - 180^o$

$|GH(j\omega)|_{\omega=\omega_g} = 1 \Rightarrow \frac{K}{|(j\omega)(10j\omega+1)|} = \frac{K}{\sqrt{100\,\omega_g^4 + \omega_g^2}} = 1$

As K = 0.1664, then $100\,\omega_g^4 + \omega_g^4 = 0.0277$
which means $\omega_g = 0.1104$
Accordingly PM = 42°
As $M(\omega) = \left|\frac{Y(j\omega)}{X(j\omega)}\right| > \left(\frac{\sqrt{2}}{2}\right)K$, then $\omega_b = 0.179\,\frac{rad}{s}$

**10-12)**

| ***T*** | **BW (rad/sec)** | $M_r$ |
|---|---|---|
| 0 | 1.14 | 1.54 |
| 0.5 | 1.17 | 1.09 |
| 1.0 | 1.26 | 1.00 |
| 2.0 | 1.63 | 1.09 |
| 3.0 | 1.96 | 1.29 |
| 4.0 | 2.26 | 1.46 |
| 5.0 | 2.52 | 1.63 |

**10-13)**

| ***T*** | **BW (rad/sec)** | $M_r$ |
|---|---|---|
| 0 | 1.14 | 1.54 |
| 0.5 | 1.00 | 2.32 |
| 1.0 | 0.90 | 2.65 |
| 2.0 | 0.74 | 2.91 |
| 3.0 | 0.63 | 3.18 |
| 4.0 | 0.55 | 3.37 |
| 5.0 | 0.50 | 3.62 |

**10-14)** The Routh array is:

$$\begin{array}{c|ll} S^3 & 0.25 & 1 \\ S^2 & 0.375 & 0.5K \\ S^1 & 1\text{-}1/3 & 0 \\ S^0 & 0.5K & \end{array}$$

Therefore:

$$GH \approx \frac{0.5K}{0.375s^2 + \left(1 - \frac{K}{3}\right)s + 0.5K}$$

As $H(\omega) = |GH(j\omega_b)| > \frac{\sqrt{2}}{k}K$, if GH is rearranged as:

$$GH \approx \frac{1}{\frac{0.75}{k}s^2 + 2\left(\frac{1}{k} - \frac{1}{3}\right)s + 1}$$

then

$$\frac{\omega_b^2}{\omega_n^2} = (1 - \xi^2) + \sqrt{2 - 4\,\xi^2(1 - \xi^2)}$$

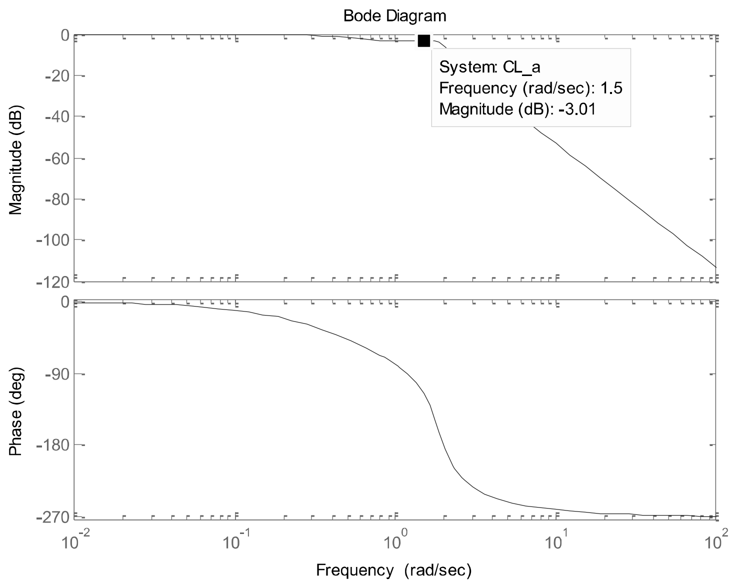
which gives

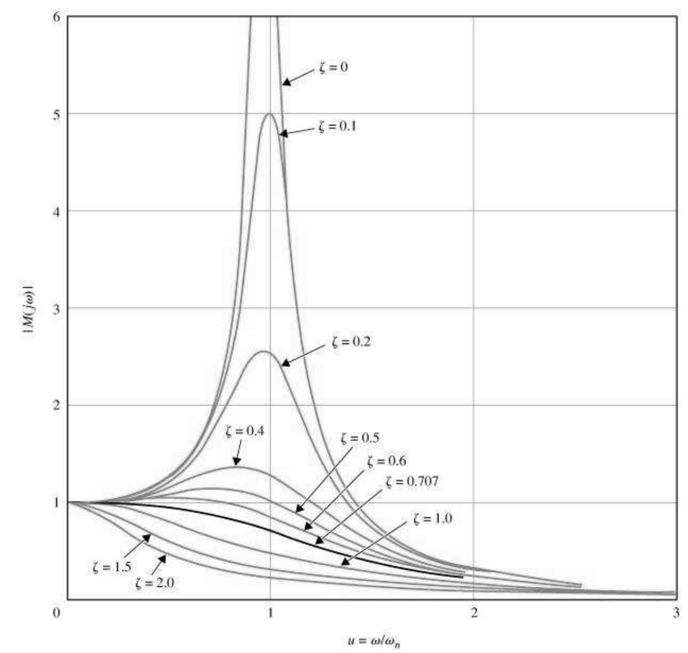
$$\omega_b^2 = \omega_n^2\left[(1 - 2\,\xi^2) + \sqrt{2 - 4\,\xi^2(1 - \xi^2)}\right] = (1.5)^2$$

where $\omega_n^2 = \frac{K}{0.75}\ and\ \xi^2 = \frac{1}{0.75k}$

therefore, $K = 2.146, \omega_n = 1.692\ and\ \xi = 0.6213$

**(c)**

| **MATLAB code:** | **Bode diagram:** |
| --- | --- |
| `s = tf('s')`<br>`%c)`<br>`K = 1.03697;`<br>`num_G_a = 0.5*K;`<br>`den_G_a = s*(0.25*s^2+0.375*s+1);`<br>`%create closed-loop system`<br>`G_a = num_G_a/den_G_a;`<br>`CL_a = G_a/(1+G_a)`<br>`bode(CL_a);`<br><br>**Notes:** |  |

<table>
<tr>
<td>1 - BW is verified by finding -3dB point at Freq = 1.5 rad/sec in the Bode graph at calculated k.<br><br>2- By comparison to diagram of typical 2nd order poles with different damping ratios, damping ratio is approximated as:<br>ξ = ~ .707</td>
<td>
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**10-15 (a)**

$$L(s) = \frac{20}{s(1+0.1s)(1+0.5s)} \qquad P_\omega = 1, \quad P = 0$$

When $\omega = 0$: $\angle L(j\omega) = -90^o \quad |L(j\omega)| = \infty$ When $\omega = \infty$: $\angle L(j\omega) = -270^o \quad |L(j\omega)| = 0$

$$L(j\omega) = \frac{20}{-0.6\omega^2 + j\omega\left(1-0.05\omega^2\right)} = \frac{20\left[-0.6\omega^2 - j\omega\left(1-0.05\omega^2\right)\right]}{0.36\omega^4 + \omega^2\left(1-0.05\omega^2\right)^2}$$

Setting $\mathrm{Im}\left[L(j\omega)\right] = 0$

$1 - 0.05\omega^2 = 0$ Thus, $\omega = \pm 4.47 \text{ rad/sec}$ $\qquad L(j4.47) = -1.667$

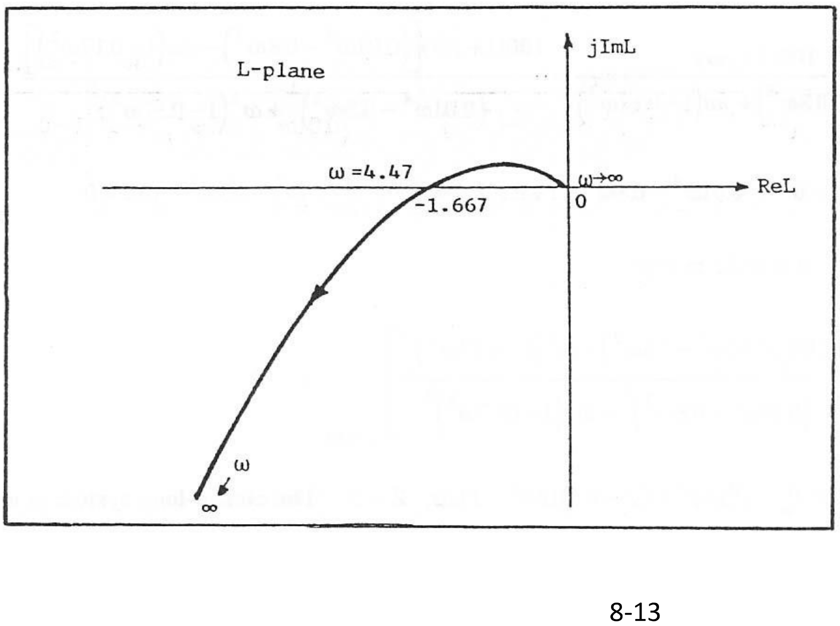
$$\Phi_{11} = 270^o = \left(Z - 0.5P_\omega - P\right)180^o = \left(Z - 0.5\right)180^o \quad \text{Thus, } Z = \frac{360^o}{180^o} = 2$$

**The closed-loop system is unstable. The characteristic equation has two roots in the right-half *s*-plane.**

## MATLAB code:

```
s = tf('s')
%a)
figure(1);
num_G_a= 20;
den_G_a=s*(0.1*s+1)*(0.5*s+1);
G_a=num_G_a/den_G_a;
nyquist(G_a)
```

**Nyquist Plot of** $L(j\omega)$:



**(b)**

$$L(s) = \frac{10}{s(1+0.1s)(1+0.5s)}$$

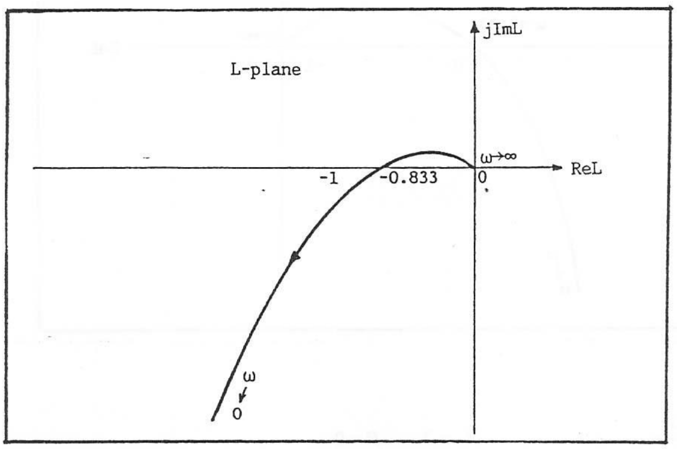
Based on the analysis conducted in part (a), the intersect of the negative real axis by the $L(j\omega)$ plot is at –0.8333, and the corresponding $\omega$ is 4.47 rad/sec.

$\Phi_{11} = -90^o = (Z - 0.5P_\omega - P)180^o = 180Z - 90^o$ Thus, $Z = 0$. **The closed-loop system is stable.**

## MATLAB code:

```
s = tf('s')
%b)
figure(1);
num_G_a= 10;
den_G_a=s*(0.1*s+1)*(0.5*s+1);
G_a=num_G_a/den_G_a;
nyquist(G_a)
```

**Nyquist Plot of** $L(j\omega)$:



**(c)**

$$L(s) = \frac{100(1+s)}{s(1+0.1s)(1+0.2s)(1+0.5s)} \qquad P_\omega = 1, \quad P = 0.$$

When $\omega = 0$: $\angle L(j0) = -90^o$ $\quad |L(j0)| = \infty$ When $\omega = \infty$: $\angle L(j\infty) = -270^o$ $\quad |L(j\infty)| = 0$

When $\omega = \infty$: $\angle L(j\omega) = -270^o$ $\quad |L(j\omega)| = 0$ When $\omega = \infty$: $\angle L(j\omega) = -270^o$ $\quad |L(j\omega)| = 0$

$$L(j\omega) = \frac{100(1+j\omega)}{\left(0.01\omega^4 - 0.8\omega^2\right) + j\omega\left(1-0.17\omega^2\right)} = \frac{100(1+j\omega)\left[\left(0.01\omega^4 - 0.8\omega^2\right) - j\omega\left(1-0.17\omega^2\right)\right]}{\left(0.01\omega^4 - 0.8\omega^2\right)^2 + \omega^2\left(1-0.17\omega^2\right)^2}$$

Setting $\text{Im}\left[L(j\omega)\right] = 0 \qquad 0.01\omega^4 - 0.8\omega^2 - 1 + 0.17\omega^2 = 0 \qquad \omega^4 - 63\omega^2 - 100 = 0$

Thus, $\omega^2 = 64.55 \quad \omega = \pm 8.03$ rad/sec

$$L(j8.03) = \left|\left(\frac{100\left[\left(0.01\omega^4 - 0.8\omega^2\right) + \omega^2\left(1-0.17\omega^2\right)\right]}{\left(0.01\omega^2 - 0.8\omega^2\right)^2 + \omega^2\left(1-0.17\omega^2\right)^2}\right)\right|_{\omega=8.03} = -10$$

$\Phi_{11} = 270^o = \left(Z - 0.5P_\omega - P\right)180^o = \left(Z - 0.5\right)180^o \quad$ Thus, $Z = 2$ **The closed-loop system is unstable.**

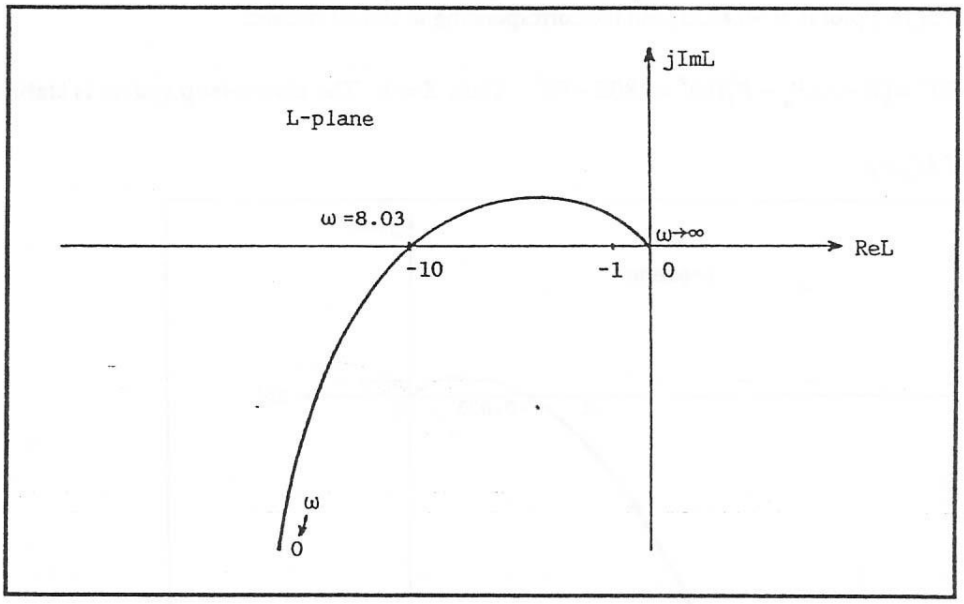
The characteristic equation has two roots in the right-half *s*-plane.

## MATLAB code:

```
s = tf('s')
%c)
figure(1);
num_G_a= 100*(s+1);
```

```
den_G_a=s*(0.1*s+1)*(0.2*s+1)*(0.5*s+1);
G_a=num_G_a/den_G_a;
nyquist(G_a)
```

**Nyquist Plot of** $L(j\omega)$:




**(d)**

$$L(s)=\frac{10}{s^2(1+0.2s)(1+0.5s)} \qquad P_\omega=2 \qquad P=0$$

When $\omega=0$: $\angle L(j\omega)=-180^o$ $\quad |L(j\omega)|=\infty$ $\quad$ When $\omega=\infty$: $\angle L(j\omega)=-360^o$ $\quad |L(j\omega)|=0$

$$L(j\omega)=\frac{10}{\left(0.1\omega^4-\omega^2\right)-j0.7\omega^3}=\frac{10\left(0.1\omega^4-\omega^2+j0.7\omega^3\right)}{\left(0.1\omega^4-\omega^2\right)^2+0.49\omega^6}$$

Setting $\text{Im}\left[L(j\omega)\right]=0$, $\omega=\infty$. The Nyquist plot of $L(j\omega)$ does not intersect the real axis except at the origin where $\omega = \infty$.

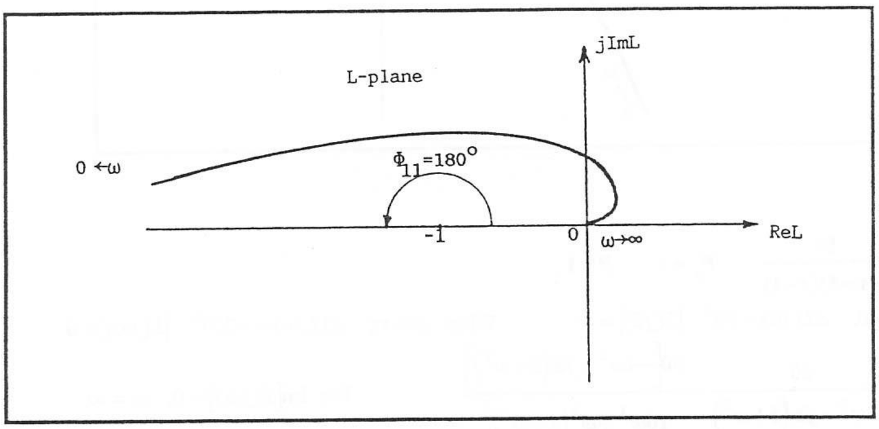
$$\Phi_{11} = (Z - 0.5P_\omega - P)180^o = (Z-1)180^o \quad \text{Thus, } Z = 2.$$

**The closed-loop system is unstable.** The characteristic equation has two roots in the right-half *s*-plane.

## MATLAB code:

```
s = tf('s')
%d)
figure(1);
num_G_a= 10;
den_G_a=s^2* (0.2*s+1)*(0.5*s+1);
G_a=num_G_a/den_G_a;
nyquist(G_a)
```

**Nyquist Plot of** $L(j\omega)$:



**10-15 (e)**

$$L(s)=\frac{3(s+2)}{s\left(s^3+3s+1\right)} \qquad P_\omega=1 \qquad P=2$$

When $\omega=0$: $\angle L(j0)=-90^o$ $\quad |L(j0)|=\infty$ $\qquad$ When $\omega=\infty$: $\angle L(j\infty)=-270^o$ $\quad |L(j\infty)|=0$

$$L(j\omega)=\frac{3(j\omega+2)}{\left(\omega^4-3\omega^2\right)+j\omega}=\frac{3(j\omega+2)\left[\left(\omega^4-3\omega^2\right)-j\omega\right]}{\left(4^4-3\omega^2\right)^2+\omega^2}$$

Setting $\mathrm{Im}\left[L(j\omega)\right]=0$,

$\omega^4-3\omega^2-2=0$ or $\omega^2=3.56$ $\quad \omega=\pm 1.89$ rad/sec. $\quad L(j1.89)=3$

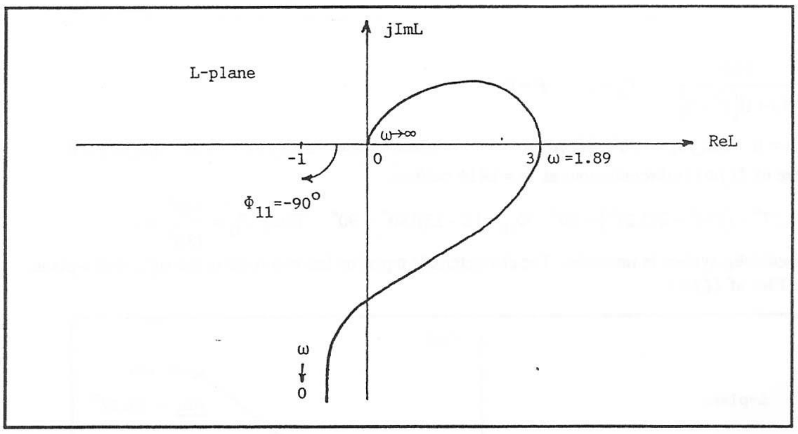
$$\Phi_{11}=\left(Z-0.5P_\omega-P\right)180^o=\left(Z-2.5\right)180^o=-90^o \qquad \text{Thus, } Z=2$$

**The closed-loop system is unstable.** The characteristic equation has two roots in the right-half *s*-plane.

## MATLAB code:

```
s = tf('s')
%e)
figure(1);
num_G_a= 3*(s+2);
den_G_a=s*(s^3+3*s+1);
G_a=num_G_a/den_G_a;
nyquist(G_a)
```

**Nyquist Plot of** $L(j\omega)$:




**10-15 (f)**

$$L(s)=\frac{0.1}{s(s+1)(s^2+s+1)} \qquad P_\omega=1 \qquad P=0$$

When $\omega=0$: $\angle L(j0)=-90^o$ $|L(j0)|=\infty$ When $\omega=\infty$: $\angle L(j\infty)=-360^o$ $|L(j\infty)|=0$

$$L(j\omega)=\frac{0.1}{(\omega^4-2\omega^2)+j\omega(1-2\omega^2)}=\frac{0.1\left[(\omega^4-2\omega^2)-j\omega(1-2\omega^2)\right]}{(\omega^4-2\omega^2)^2+\omega^2(1-2\omega^2)^2}$$ Settiing $\text{Im}[L(j\omega)]=0$

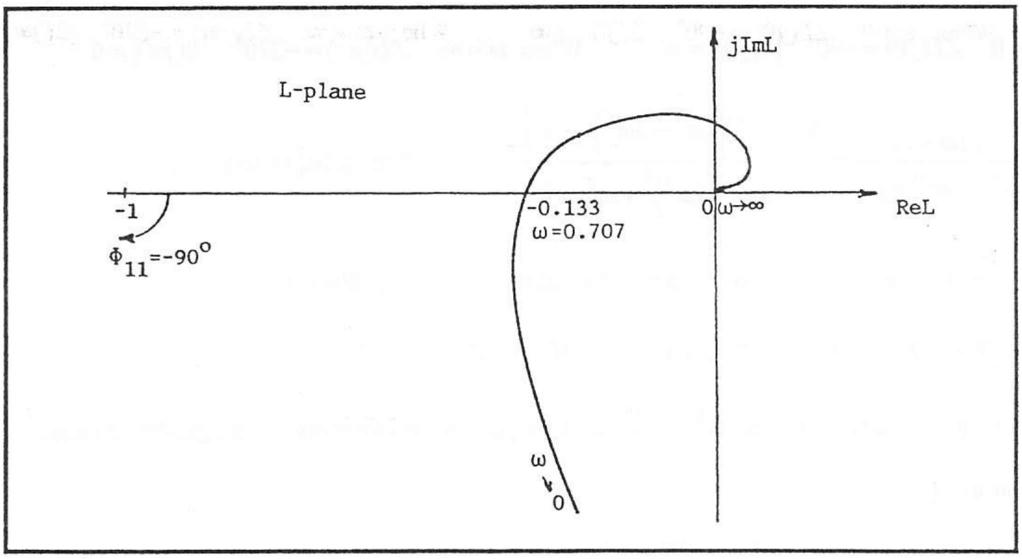
$\omega=\infty$ or $\omega^2=0.5$ $\omega=\pm 0.707$ rad/sec $L(j0.707)=-0.1333$

$\Phi_{11}=(Z-0.5P_\omega-P)180^o=(Z-0.5)180^o=-90^o$ Thus, $Z=0$ **The closed-loop system is stable.**

## MATLAB code:

```
s = tf('s')
%f)
figure(1);
num_G_a= 0.1;
den_G_a=s*(s+1)*(s^2+s+1);
G_a=num_G_a/den_G_a;
nyquist(G_a)
```

**Nyquist Plot of** $L(j\omega)$:




**10-15 (g)**

$$L(s) = \frac{100}{s(s+1)(s^2+2)} \qquad P_\omega = 3 \qquad P = 0$$

When $\omega = 0$: $\angle L(j0) = -90^o$ $|L(j0)| = \infty$ When $\omega = \infty$: $\angle L(j\infty) = -360^o$ $|L(j\infty)| = 0$

The phase of $L(j\omega)$ is discontinuous at $\omega = 1.414$ rad/sec.

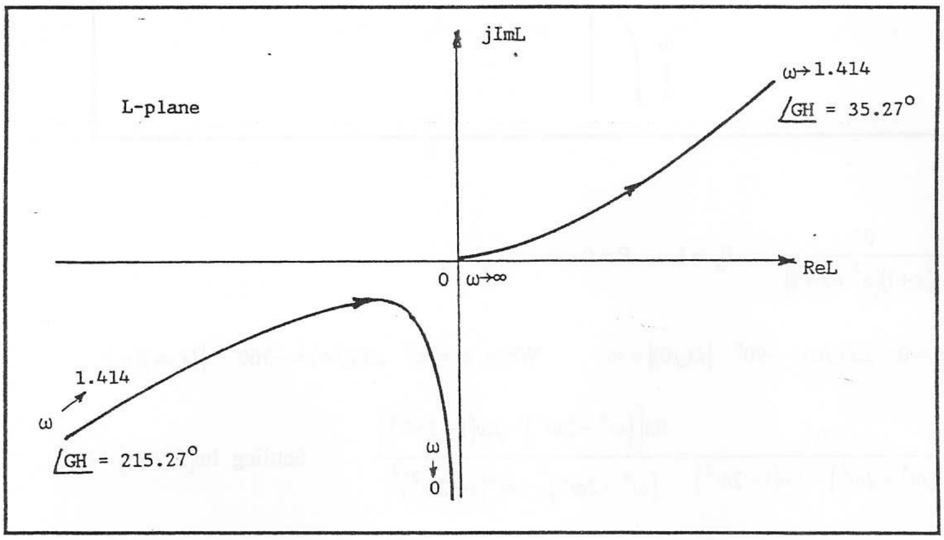
$\Phi_{11} = 35.27^o + (270^o - 215.27^o) = 90^o$ $\Phi_{11} = (Z - 1.5)180^o = 90^o$ Thus, $P_{11} = \frac{360^o}{180^o} = 2$

**The closed-loop system is unstable.** The characteristic equation has two roots in the right-half *s*-plane.

## MATLAB code:

```
s = tf('s')
%g)
figure(1);
num_G_a= 100;
den_G_a=s*(s+1)*(s^2+2);
G_a=num_G_a/den_G_a;
nyquist(G_a)
```

**Nyquist Plot of** $L(j\omega)$:




**10-15 (h)**

$$L(s) = \frac{10(s+10)}{s(s+1)(s+100)} \qquad P_\omega = 1 \qquad P = 0$$

When $\omega = 0$: $\angle L(j0) = -90^o$ $|L(j0)| = \infty$ When $\omega = \infty$: $\angle L(j\infty) = -180^o$ $|L(j\infty)| = 0$

$$L(j\omega) = \frac{10(j\omega+10)}{-101\omega^2 + j\omega(100-\omega^2)} = \frac{10(j\omega+10)\left[-101\omega^2 - j\omega\left(100-\omega^2\right)\right]}{10201\omega^4 + \omega^2\left(100-\omega^2\right)^2}$$

Setting $\mathrm{Im}\left[L(j\omega)\right] = 0$, $\omega = 0$ is the only solution. Thus, the Nyquist plot of $L(j\omega)$ does not intersect the real axis, except at the origin.

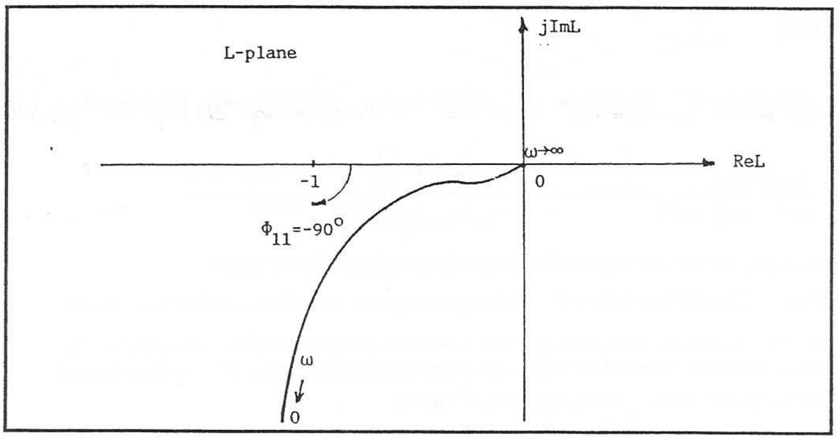
$\Phi_{11} = \left(Z - 0.5P_\omega - P\right)180^o = \left(Z - 0.5\right)180^o = -90^o$ Thus, $Z = 0$.

**The closed-loop system is stable.**

## MATLAB code:

```
s = tf('s')
%h)
figure(1);
num_G_a= 10*(s+10);
den_G_a=s*(s+1)*(s+100);
G_a=num_G_a/den_G_a;
nyquist(G_a)
```

**Nyquist Plot of** $L(j\omega)$:



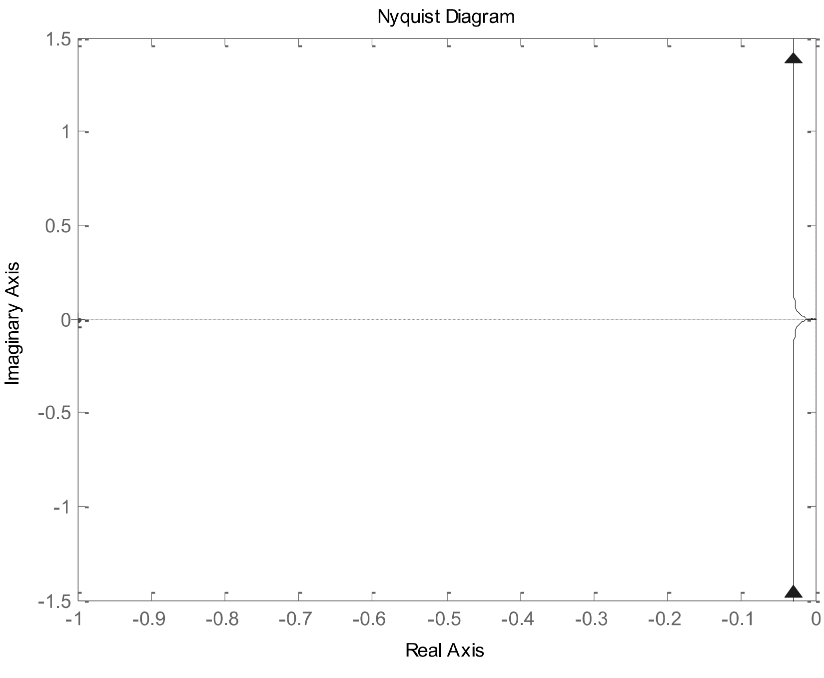

**10-16**

**MATLAB code:**

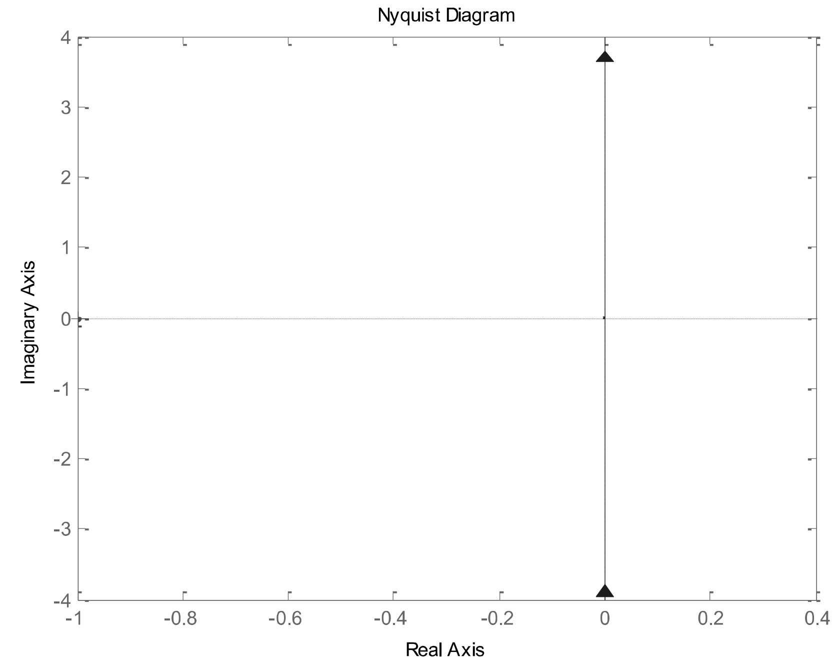
```
s = tf('s')
%a)
figure(1);
num_G_a= 1;
den_G_a=s*(s+2)*(s+10);
G_a=num_G_a/den_G_a;
nyquist(G_a)

%b)
figure(2);
num_G_b= 1*(s+1);
den_G_b=s*(s+2)*(s+5)*(s+15);
G_b=num_G_b/den_G_b;
nyquist(G_b)

%c)
figure(3);
num_G_c= 1;
den_G_c=s^2*(s+2)*(s+10);
G_c=num_G_c/den_G_c;
nyquist(G_c)

%d)
figure(4);
num_G_d= 1;
den_G_d=(s+2)^2*(s+5);
G_d=num_G_d/den_G_d;
```

```
nyquist(G_d)

%e)
figure(5);
num_G_e= 1*(s+5)*(s+1);
den_G_e=(s+50)*(s+2)^3;
G_e=num_G_e/den_G_e;
nyquist(G_e)
```

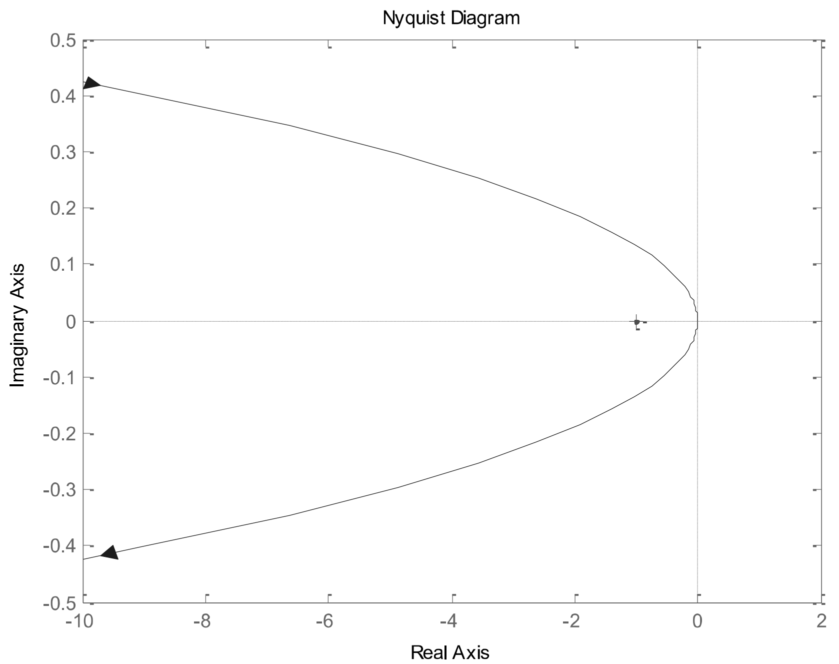
**Nyquist graph, part(a):**



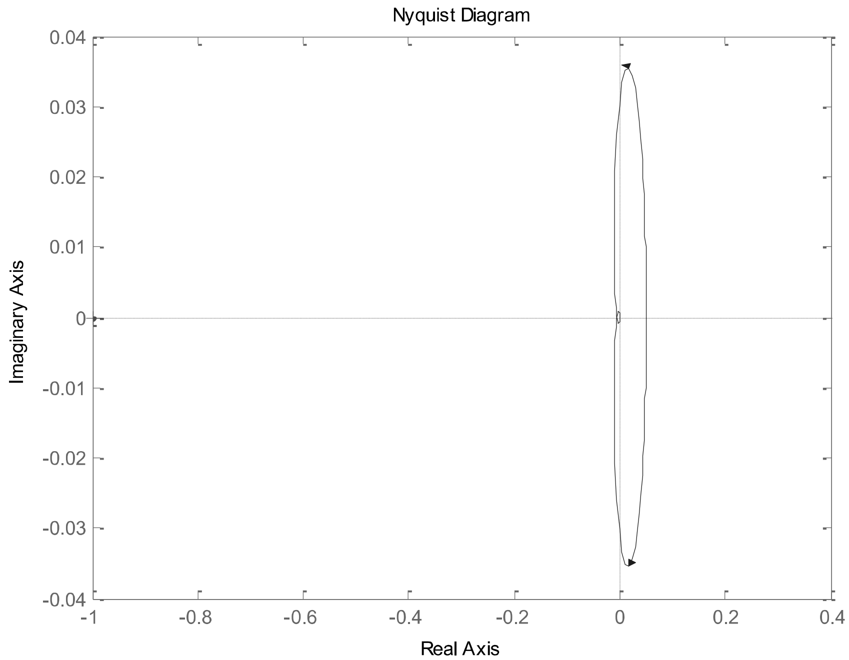

**Nyquist graph, part(b):**



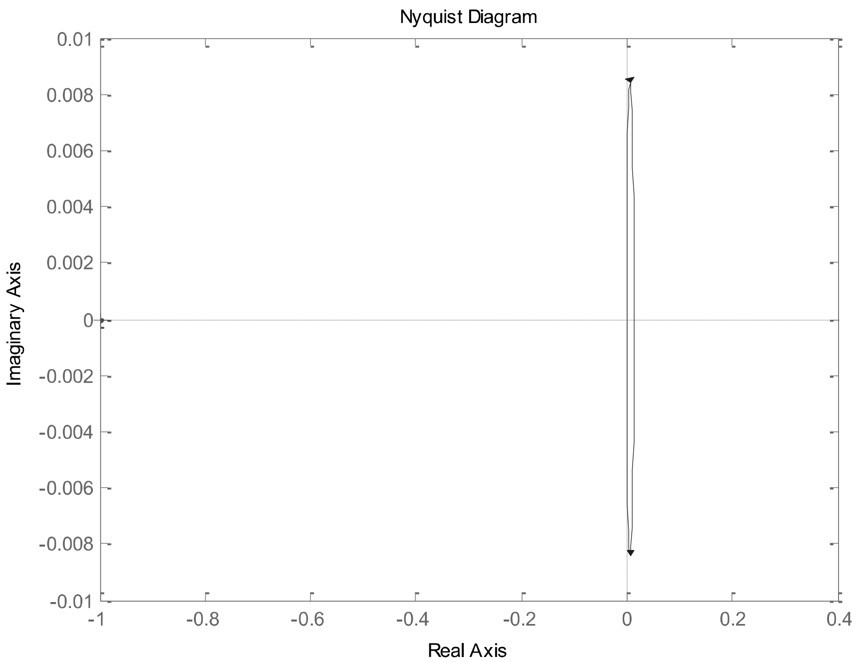

**Nyquist graph, part(c):**




**Nyquist graph, part(d):**




**Nyquist graph, part (e):**



## 10-17 (a)

$$G(s) = \frac{K}{(s+5)^2} \qquad P_\omega = 0 \qquad P = 0$$

$$\angle G(j0) = 0^o \quad (K>0) \qquad \angle G(0) = 180^o \quad (K<0) \quad |G(j0)| = \frac{K}{25}$$

$$G(j\infty) = -180^o \quad (K>0) \qquad \angle G(j\infty) = 0^o \quad (K<0) \quad |G(j\infty)| = 0$$

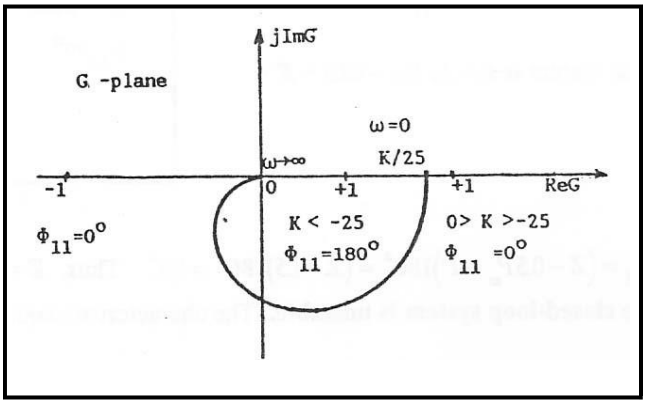
For stability, $Z = 0$.

$\Phi_{11} = -(0.5P_\omega + P)180^o = 0^o$

$0 < K < \infty \quad \Phi_{11} = 0^o$ **Stable**

$K < -25 \qquad \Phi_{11} = 180^o$ **Unstable**

$-25 < K < 0 \quad \Phi_{11} = 0^o$ **Stable**




**The system is stable for $-25 < K < \infty$ .**

## 10-17 (b)

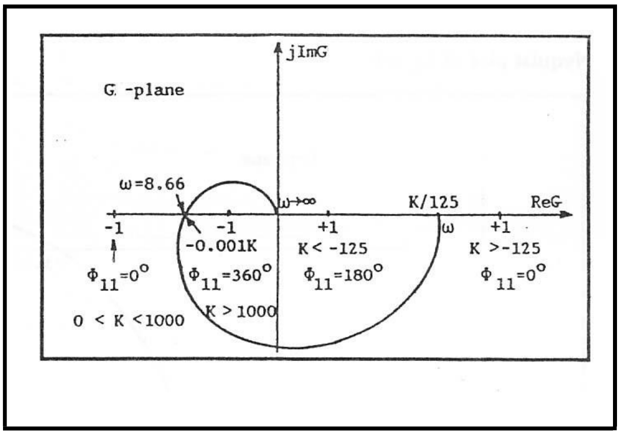
$$G(s) = \frac{K}{(s+5)^3} \qquad P_\omega = 0 \qquad P = 0$$

$$\angle G(j0) = 0^o \quad (K>0) \qquad \angle G(0) = 180^o \quad (K<0) \quad |G(j0)| = \frac{K}{125}$$

$$G(j\infty) = -270^o \quad (K>0) \qquad \angle G(j\infty) = 270^o \quad (K<0) \quad |G(j\infty)| = 0$$

For stability, $Z = 0$.

$\Phi_{11} = -\left(0.5P_{\omega} + P\right)180^{o} = 0^{o}$




| | | |
|---|---|---|
| $0 < K < 1000$ | $\Phi_{11} = 0^{o}$ | **Stable** |
| $K > 1000$ | $\Phi_{11} = 360^{o}$ | **Unstable** |
| $K < -125$ | $\Phi_{11} = 180^{o}$ | **Unstable** |
| $-125 < K < 0$ | $\Phi_{11} = 0^{o}$ | **Stable** |

**The system is stable for $-125 < K < 0$.**

## 10-17 (c)

$$G(s) = \frac{K}{(s+5)^4} \qquad P_{\omega} = P = 0$$

$$\angle G(j0) = 0^{o} \quad (K > 0) \qquad \angle G(0) = 180^{o} \quad (K < 0) \qquad \left|G(j0)\right| = \frac{K}{625}$$

$$G(j\infty) = 0^{o} \quad (K > 0) \qquad \angle G(j\infty) = 180^{o} \quad (K < 0) \qquad \left|G(j\infty)\right| = 0$$

For stability, $Z = 0$.

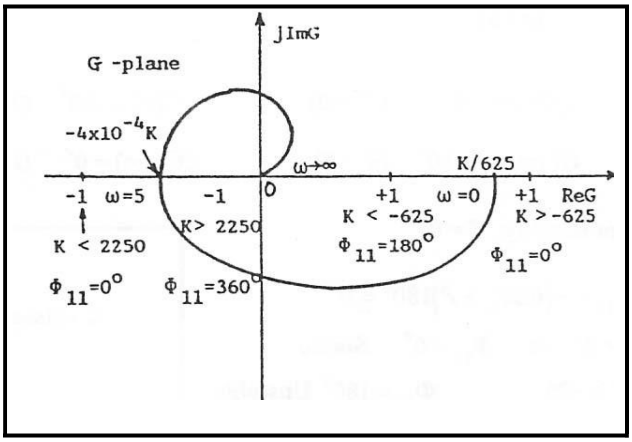
$\Phi_{11} = -(0.5P_\omega + P)180^o = 0^o$

$0 < K < 2500$ $\Phi_{11} = 0^o$ **Stable**

$K > 2500$ $\Phi_{11} = 360^o$ **Unstable**

$K < -625$ $\Phi_{11} = 180^o$ **Unstable**

$-625 < K < 0$ $\Phi_{11} = 0^o$ **Stable**




**The system is stable for –625 < *K* < 2500.**

**10-18)** The characteristic equation:

$$1 + \frac{K}{(s+1)(s^2+2s+2)} = 0$$

or

$$s^3 + 3s^2 + 4s + K + 2 = 0$$

if $K > 0$, the cross real axis at s = 0.1. ⇨ For stability $-10 < K < 10$

if K < 0, the Nyquist cross the real axis at $s = 0.5$ .So, for stability, $-2 < K < 2$

therefore, the rage of stability for the system is $-2 < K < 10$

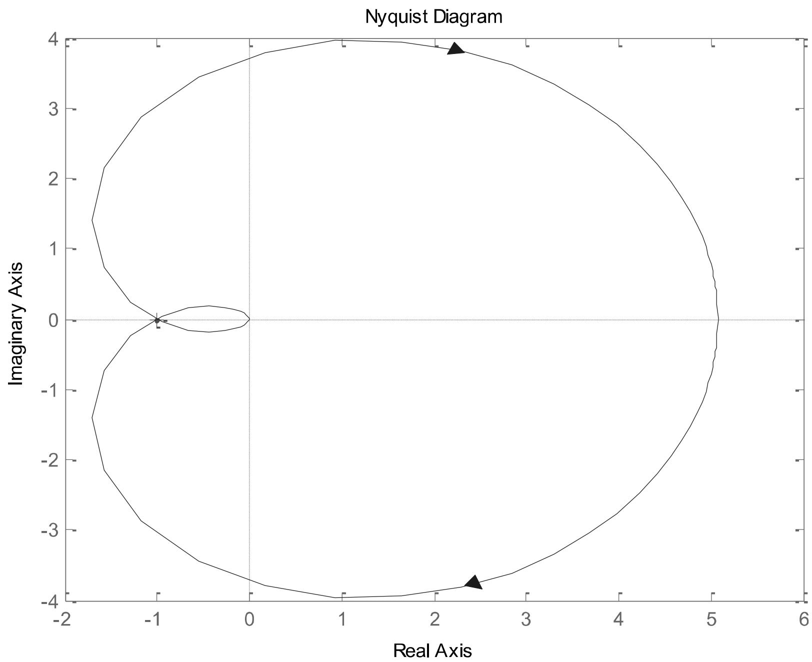
**MATLAB code:**

```
s = tf('s')

K=1
G= K/(s^2+2*s+2);
H=1/(s+1);
GH=G*H;

sisotool
```

```
K=10.15
G2= K/(s^2+2*s+2);
H2=1/(s+1);
GH2=G2*H2;
nyquist(GH2)
xlim[(-1.5,.5)]
ylim[(-1,1)]
```

After generating the feed-forward (G) and feedback (H) transfer functions in the MATLAB code, these transfer functions are imported to sisotool. Nyquist diagram is added to the results of sisotool. The overall gain of the transfer function is changed until Nyquist diagram passes through -1+0j point. Higher values of K resulted in unstable Nyquist diagram. Therefore K<10.15 determines the range of stability for the closed loop system.

**Nyquist at margin of stability:**



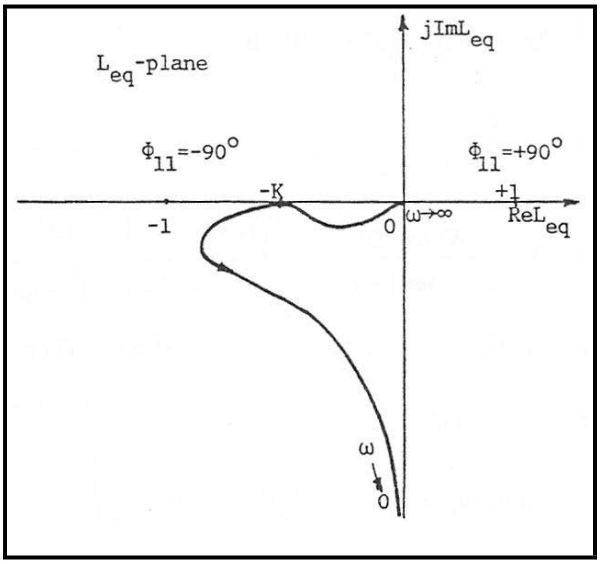

**10-19)**

$$s\left(s^3+2s^2+s+1\right)+K\left(s^2+s+1\right)=0$$

$$L_{eq}(s) = \frac{K\left(s^2+s+1\right)}{s\left(s^3+2s^2+s+1\right)} \qquad P_\omega = 1 \quad P = 0 \quad L_{eq}(j0) = \infty\angle -90^o \quad L_{eq}(j\infty) = 0\angle 180^o$$

$$L_{eq}(j\omega) = \frac{K\left[\left(1-\omega^2\right)+j\omega\right]}{\left(\omega^4-\omega^2\right)+j\omega\left(1-2\omega^2\right)} = \frac{K\left[-\left(\omega^6+\omega^4\right)-j\omega\left(\omega^4-2\omega^2+1\right)\right]}{\left(\omega^4-\omega^2\right)^2+\omega^2\left(1-2\omega^2\right)^2}$$

Setting $\mathrm{Im}\left[L_{eq}(j\omega)\right] = 0$




$$\omega^4 - 2\omega^2 + 1 = 0$$

Thus, $\omega = \pm 1$ rad/sec are the real solutions.

$$L_{eq}(j1) = -K$$

For stability,

$$\Phi_{11} = -(0.5P_\omega + P)180^o = -90^o$$

When $K = 1$ the system is marginally stable.

| | | |
|---|---|---|
| $K > 0$ | $\Phi_{11} = -90^o$ | **Stable** |
| $K < 0$ | $\Phi_{11} = +90^o$ | **Unstable** |

**Routh Tabulation**

| | | | |
|---|---|---|---|
| $s^4$ | $1$ | $K+1$ | $K$ |
| $s^3$ | $2$ | $K+1$ | |
| $s^2$ | $\frac{K+1}{2}$ | $K$ | $K>-1$ |
| $s^1$ | $\frac{K^2-2K+1}{K+1}=\frac{(K-1)^2}{K+1}$ | | |
| $s^0$ | $K$ | | $K > 0$ |

When $K$ = 1 the coefficients of the $s^1$ row are all zero. The auxiliary equation is $s^2+1=0$ The solutions are $\omega=\pm 1$ rad/sec. Thus the Nyquist plot of $L_{eq}(j\omega)$ intersects the −1 point when $K$ = 1, when $\omega=\pm 1$ rad/sec. **The system is stable for 0 < $K$ < ∞ , except at $K$ = 1.**

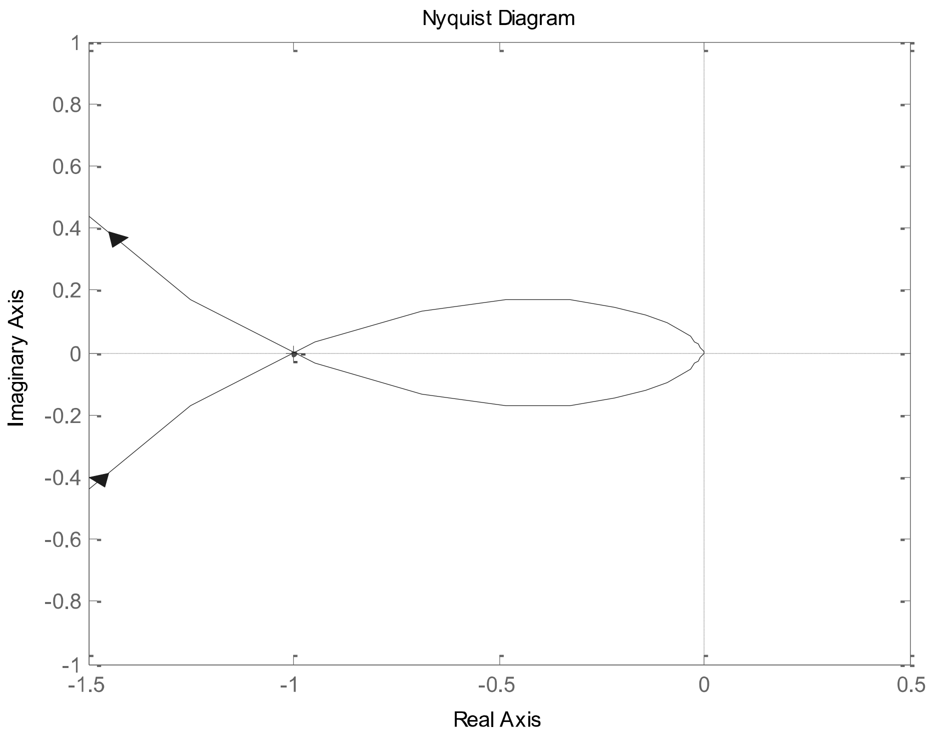
**10-20)** Solution is similar to the previous problem. Let's use Matlab as an alternative approach

**MATLAB code:**

```
s = tf('s')

figure(1);
K=8.09
num_GH= K;
den_GH=(s^3+3*s^2+3*s+1);
GH=num_GH/den_GH;
nyquist(GH)
xlim([-1.5,.5])
ylim([-1,1])

sisotool;
```

After generating the loop transfer function and analyzing Nyquist in MATLAB sisotool, it was found that for values of K higher than ~8.09, the closed loop system is unstable. Following is the Nyquist diagram at margin of stability.

**Part(a), Nyquist at margin of stability:**




**Part(b), Verification by Routh-Hurwitz criterion:**

Using Routh criterion, the coefficient table is as follows:

| | | |
|---|---|---|
| $S^3$ | 1 | 3 |
| $S^2$ | 3 | K+1 |
| $S^1$ | (8-K)/3 | 0 |
| $S^0$ | K+1 | 0 |

The system is stable if the content of the 1st column is positive:

$$(8-K)/3>0 \rightarrow K<8$$

K+1>0 → K>-1

which is consistent with the results of the Nyquist diagrams.

**10-21)**

Parabolic error constant $K_a = \lim_{s \to 0} s^2 G(s) = \lim_{s \to 0} 10\left(K_P + K_D s\right) = 10K_P = 100$ Thus $K_P = 10$

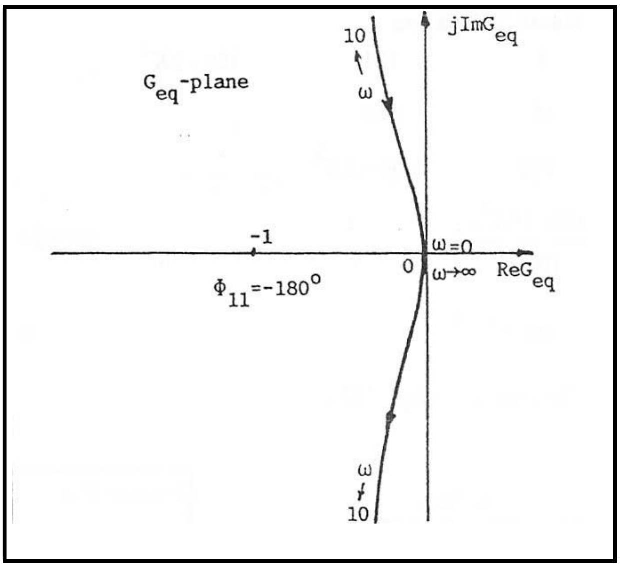
**Characteristic Equation:** $s^2 + 10K_D s + 100 = 0$

$G_{eq}(s) = \dfrac{10K_D s}{s^2 + 100}$ $P_\omega = 2$ $P = 0$

For stability,

$\Phi_{11} = -(0.5P_\omega + P)180^o = -180^o$

**The system is stable for** $0 < K_D < \infty$.



**10-22 (a)** The characteristic equation is $1+G(s)-G(s)-2[G(s)]^2 = 1-2[G(s)]^2 = 0$

$$G_{eq}(s) = -2[G(s)]^2 = \frac{-2K^2}{(s+4)^2(s+5)^2} \qquad P_\omega = 0 \qquad P = 0$$

$$G_{eq}(j\omega) = \frac{-2K^2}{\left(400-120\omega^2+\omega^4\right)+j\omega\left(360-18\omega^2\right)} = \frac{-2K^2\left[\left(400-120\omega^2+\omega^2\right)-j\omega\left(360-18\omega^2\right)\right]}{\left(400-120\omega^2+\omega^2\right)+\omega^2\left(360-18\omega^2\right)^2}$$

$G_{eq}(j0) = \dfrac{K^2}{200}\angle 180^o$ $\qquad G_{eq}(j\infty) = 0\angle 180^o$ $\qquad$ Setting $\mathrm{Im}\left[G_{eq}(j\omega)\right] = 0$

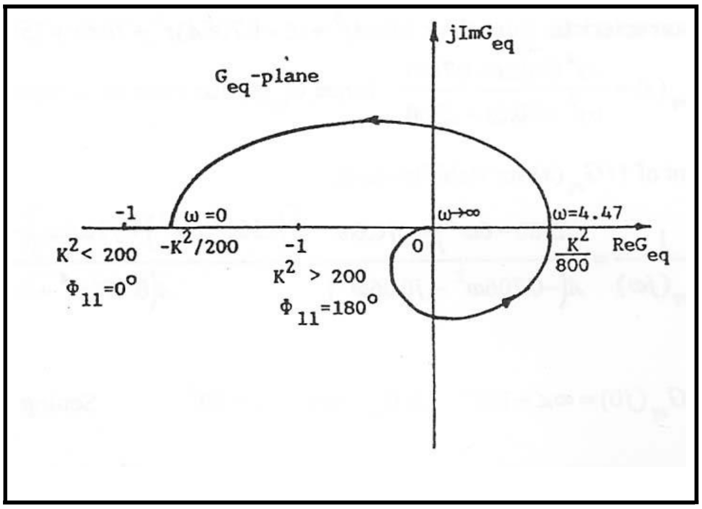
$\omega = 0$ and $\omega = \pm 4.47$ rad/sec $\qquad G_{eq}(j4.47) = \dfrac{K^2}{800}$

For stability,

$$\Phi_{11} = -\left(0.5P_\omega + P\right)180^o = 0^o$$

**The system is stable for** $K^2 < 200$

**or** $|K| < \sqrt{200}$



## 10-22 (b)

**Characteristic Equation:** $s^4 + 18s^3 + 121s^2 + 360s + 400 - 2K^2 = 0$

**Routh Tabulation**

| | | | | |
|---|---|---|---|---|
| $s^4$ | $1$ | $121$ | $400 - 2K^2$ | |
| $s^3$ | $18$ | $360$ | | |
| $s^2$ | $101$ | $400 - 2K^2$ | | |
| $s^1$ | $\dfrac{29160 - 36K^2}{101}$ | | | $29160 + 36K^2 > 0$ |
| $s^0$ | $400 - 2K^2$ | | | $K^2 < 200$ |

**Thus for stability,** $|K| < \sqrt{200}$

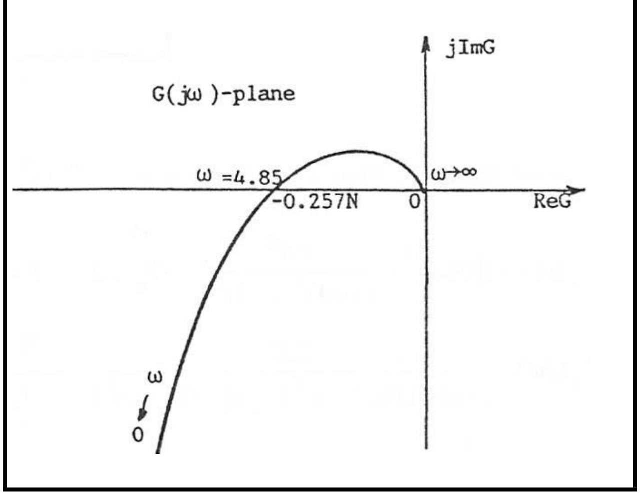
## 10-23 (a)

$$G(s) = \frac{83.33N}{s(s+2)(s+11.767)}$$

For stability, $N < 3.89$

Thus $N < 3$ since $N$ must be an integer.

Nyquist Plot



**(b)**

$$G(s)=\frac{2500}{s(0.06s+0.706)(As+100)}$$

**Characteristic Equation:** $0.06As^3+(6+0.706A)s^2+70.6s+2500=0$

$G_{eq}(s)=\dfrac{As^2(0.06s+0.706)}{6s^2+70.6s+2500}$ Since $G_{eq}(s)$ has more zeros than poles, we should sketch the Nyquist

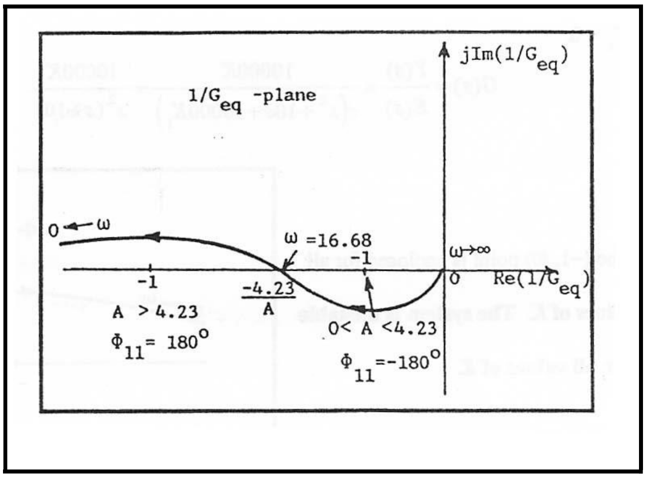
plot of $1/G_{eq}(s)$ for stability study.

$$\frac{1}{G_{eq}(j\omega)}=\frac{\left(2500-6\omega^2\right)+j70.6\omega}{A\left(-0.706\omega^2-j0.06\omega^3\right)}=\frac{\left[\left(2500-6\omega^2\right)+j70.6\omega\right]\left(-0.706\omega^2+j0.06\omega^3\right)}{A\left(0.498\omega^4+0.0036\omega^6\right)}$$

$1/G_{eq}(j0)=\infty\angle-180^o$ $1/G_{eq}(j\infty)=0\angle-90^o$ Setting $\mathrm{Im}\left[\dfrac{1}{G_{eq}(j\omega)}\right]=0$

$100.156-0.36\omega^2=0$ $\omega=\pm 16.68$ rad/sec $\dfrac{1}{G_{eq}(j16.68)}=\dfrac{-4.23}{A}$

For stability,




$$\Phi_{11}=-\left(0.5P_\omega+P\right)180^o=-180^o$$

For $A>4.23$ $\Phi_{11}=180^o$ **Unstable**

For $0<A<4.23$ $\Phi_{11}=-180^o$ **Stable**

**The system is stable for 0 < *A* < 4.23.**

**(c)**

$$G(s) = \frac{2500}{s(0.06s+0.706)(50s+K_o)}$$

**Characteristic Equation:** $s(0.06s+0.706)(50s+K_o)+2500$ **= 0**

$$G_{eq}(s) = \frac{K_o s(0.06s+0.706)}{3s^3+35.3s^2+2500} \qquad P_\omega = 0 \qquad P = 0 \qquad G_{eq}(j0) = 0\angle 90^o \qquad G_{eq}(j\infty) = 0\angle -90^o$$

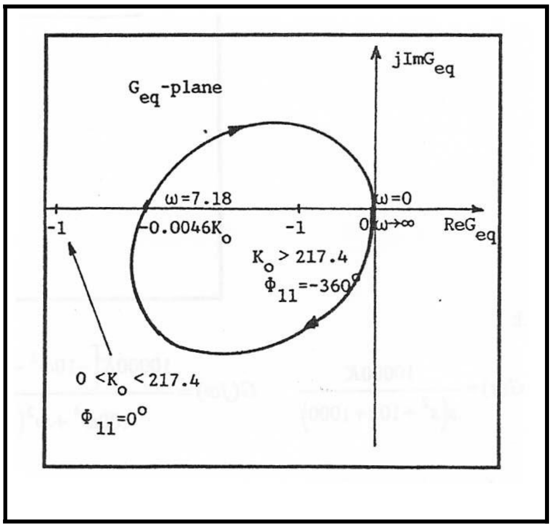
$$G_{eq}(j\omega) = \frac{K_o(-0.06\omega^3+0.706j\omega)}{\left(2500-35.3\omega^2\right)-j3\omega^3} = \frac{K_o\left(-0.06\omega^2+0.706j\omega\right)\left[\left(2500-35.5\omega^2\right)+j3\omega^3\right]}{\left(2500-35.3\omega^2\right)^2+9\omega^6}$$

Setting $\text{Im}\left[G_{eq}(j\omega)\right]=0 \quad \omega^4+138.45\omega^2-9805.55=0 \quad \omega^2=51.6 \quad \omega=\pm 7.18$ rad/sec

$$G_{eq}(j7.18) = -0.004K_o$$

For stability, $\Phi_{11} = -\left(0.5P_\omega + P\right)180^o = 0^o$

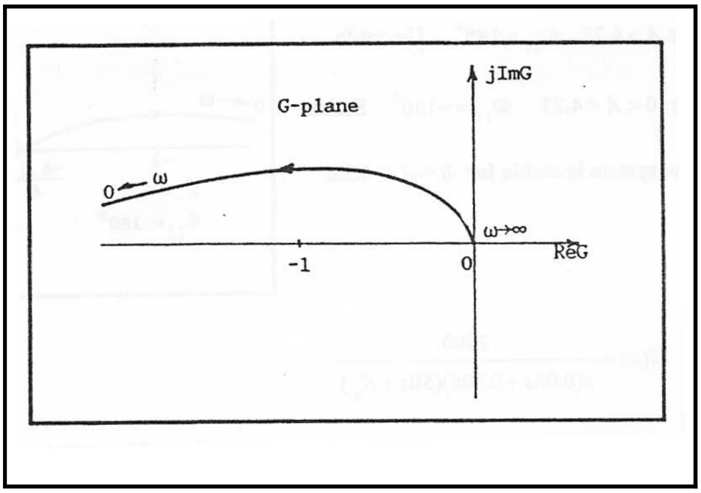
**For stability,** $0 < K_o < 217.4$



**10-24 (a)** $K_t = 0$:

$$G(s) = \frac{Y(s)}{E(s)} = \frac{10000K}{s\left(s^2 + 10s + 10000K_t\right)} = \frac{10000K}{s^2(s+10)}$$

The $(-1, j0)$ point is enclosed for all values of $K$. **The system is unstable for all values of *K*.**



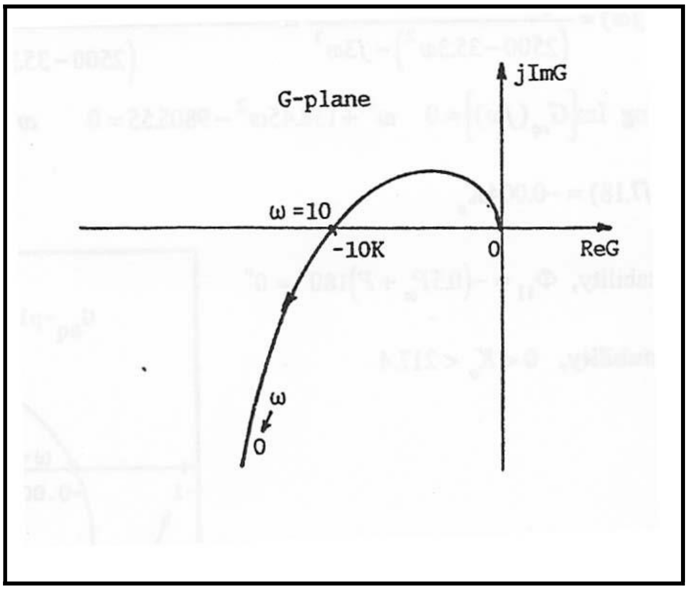

**(b)** $K_t = 0.01$:

$$G(s) = \frac{10000K}{s\left(s^2 + 10s + 100\right)} \qquad G(j\omega) = \frac{10000K\left[-10\omega^2 - j\omega\left(100 - \omega^2\right)\right]}{100\omega^4 + \omega^2\left(100 - \omega^2\right)^2}$$

Setting $\mathrm{Im}\left[G(j\omega)\right] = 0 \quad \omega^2 = 100$

$\omega = \pm 10 \text{ rad/sec} \quad G(j10) = -10K$

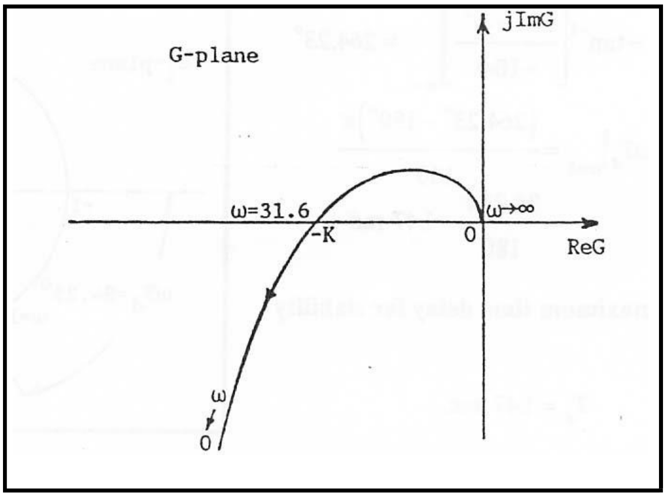
**The system is stable for $0 < K < 0.1$**



**(c)** $K_t = 0.1$:

$$G(s) = \frac{10000K}{s\left(s^2 + 10s + 1000\right)} \qquad G(j\omega) = \frac{10000K\left[-10\omega^2 - j\omega\left(1000 - \omega^2\right)\right]}{100\omega^4 + \omega^2\left(1000 - \omega^2\right)^2}$$

Setting $\text{Im}\left[G(j\omega)\right] = 0 \quad \omega^2 = 100 \qquad \omega = \pm 31.6$ rad/sec $\qquad G(j31.6) = -K$




**For stability, 0 < *K* < 1**

**10-25)** The characteristic equation for *K* = 10 is:

$$s^3 + 10s^2 + 10{,}000K_t s + 100{,}000 = 0$$

$$G_{eq}(s) = \frac{10{,}000K_t s}{s^3 + 10s^2 + 100{,}000} \qquad P_\omega = 0 \qquad P = 2$$

$$G_{eq}(j\omega) = \frac{10{,}000K_t j\omega}{100{,}000 - 10\omega^2 - j\omega^3} = \frac{10{,}000K_t\left[-\omega^4 + j\omega\left(10{,}000 - 10\omega^2\right)\right]}{\left(10{,}000 - 10\omega^2\right)^2 + \omega^6}$$

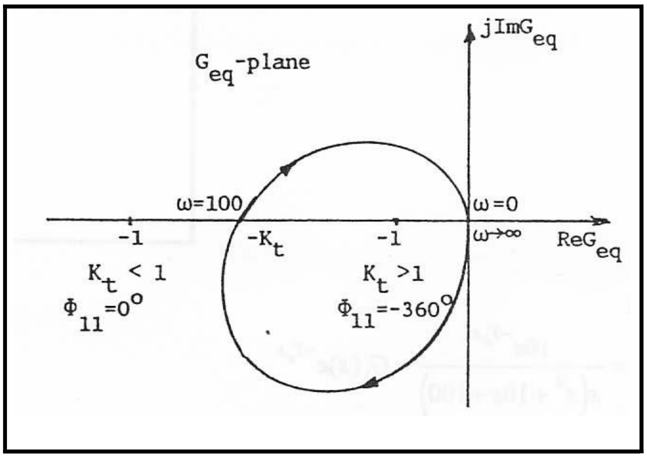
Setting $\text{Im}\left[G_{eq}(j\omega)\right] = 0$

$\omega = 0, \quad \omega^2 = 10{,}000$

$\omega = \pm 100 \text{ rad/sec} \qquad G_{eq}(j100) = -K_t$

For stability,

$\Phi_{11} = -(0.5P_\omega + P)180^o = -360^o$




**The system is stable for** $K_t > 0.$

**10-26)**

$$\frac{Y(s)}{X(s)} = \frac{KK_f}{Js^2 + (a + KK_f)s + KK_f}$$

a) $K_f = 0 \Rightarrow \frac{Y(s)}{X(s)} = \frac{K}{Js^2+as+K} = \frac{K}{s^2+s+K}$

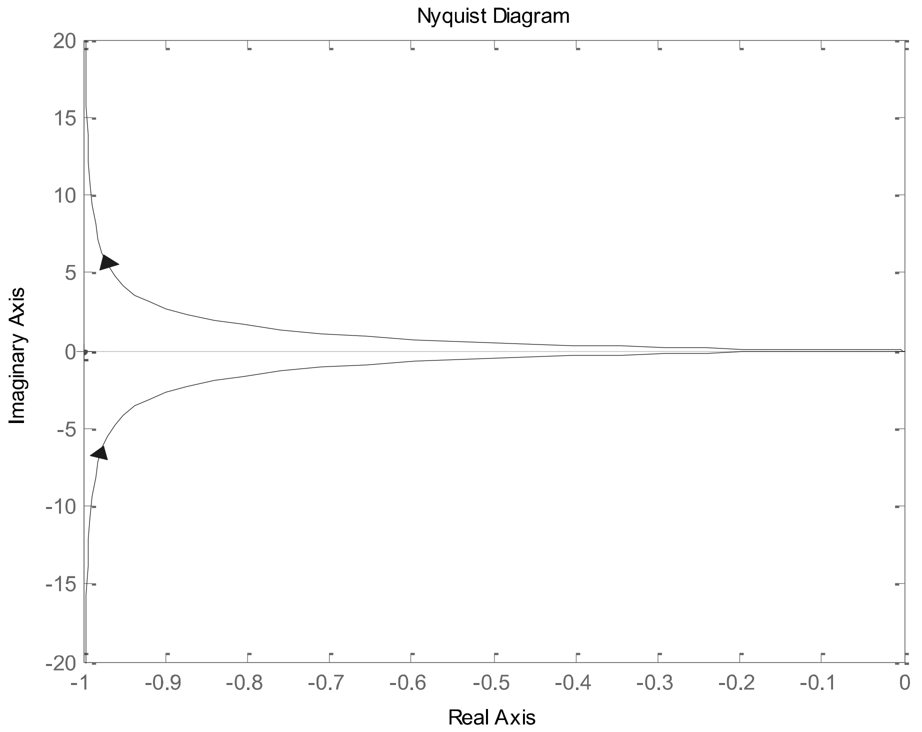
b) $K_f = 0.1 \Rightarrow \frac{Y(s)}{X(s)} = \frac{0.1K}{Js^2+(a+0.1K)s+0.1K} = \frac{0.1K}{s^2+(1+0.1K)s+0.1K}$

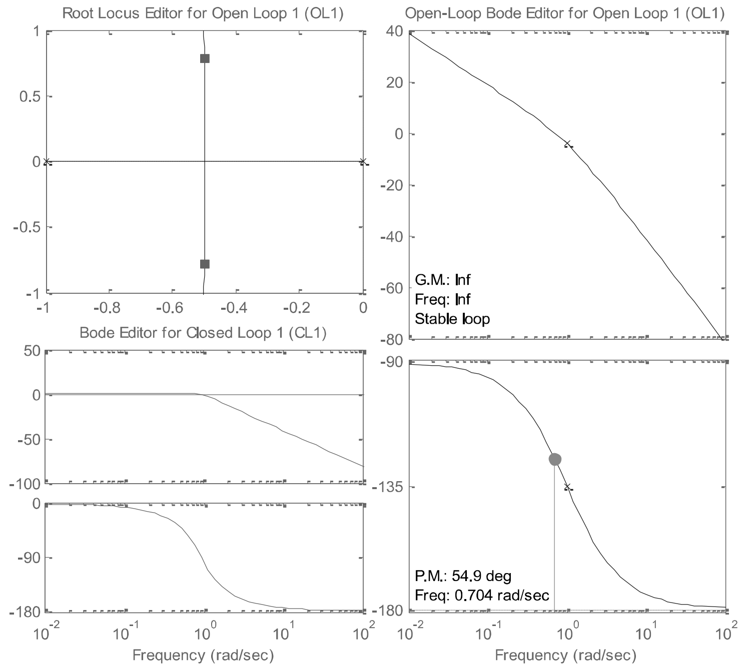
c) $K_f = 0.2 \Rightarrow \frac{Y(s)}{X(s)} = \frac{0.2K}{s^2+(-1+0.2K)s+0.2K}$

**MATLAB code:**

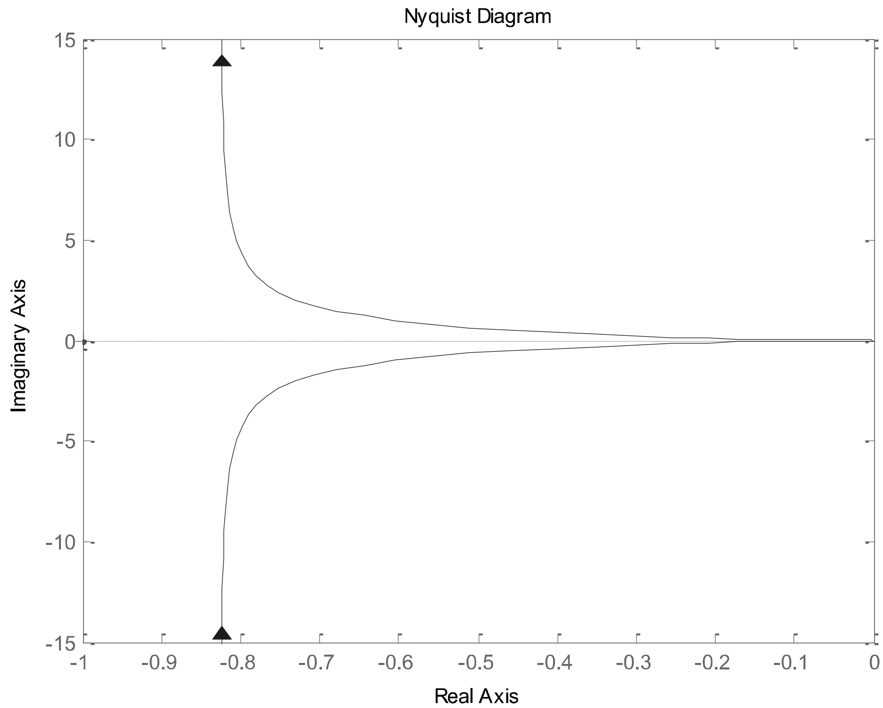
```
s = tf('s')
figure(1);
J=1;
B=1;
K=1;
Kf=0
G1= K/(J*s+B);
CL1=G1/(1+G1*Kf);
H2 = 1;
G1G2 = CL1/s;
L_TF=G1G2*H2;
nyquist(L_TF)

sisotool
```

**Part (a),** `Kf=0`**:** by plotting the Nyquist diagram in sisotool and varying the gain, it was observed that all values of gain (K) will result in a stable system. Location of poles in root locus diagram of the second figure will also verify that.

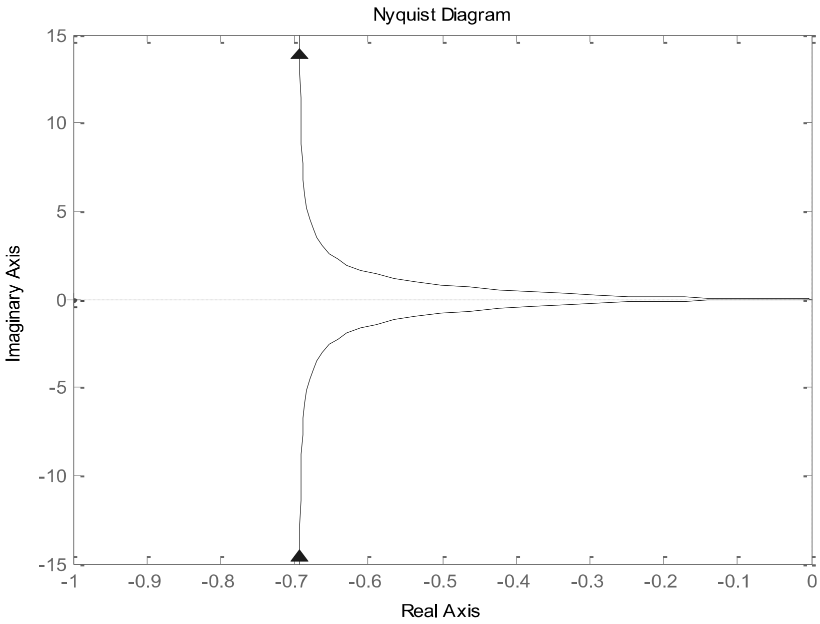





**Part (b),** `Kf=0.1`**:** The result and approach is similar to part (a), a sample of Nyquist diagram is presented for his case as follows:



**Part (c),** `Kf=0.2`**:** The result and approach is similar to part (a), a sample of Nyquist diagram is presented for his case as follows:



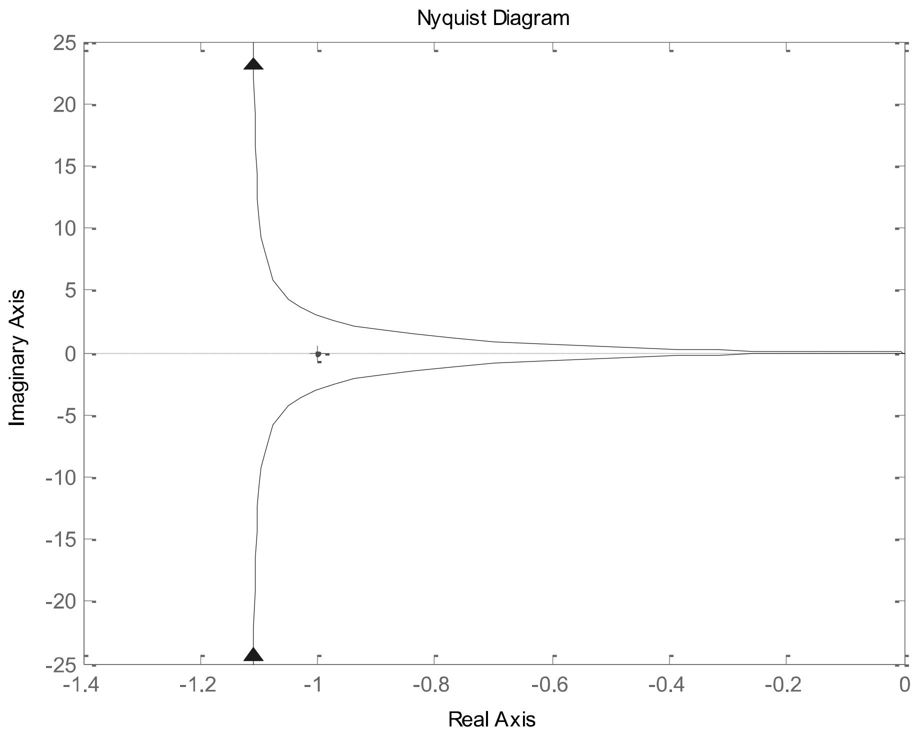

**10-27)**

$$\frac{Y(s)}{X(s)} = \frac{10K_f}{s^2 + (1 + 10K_f)s + 10K_f}$$

**MATLAB code:**

```
s = tf('s')
figure(1);
J=1;
B=1;
K=10;
Kf=0.2
G1= K/(J*s+B);
CL1=G1/(1+G1*Kf);
H2 = 1;
G1G2 = CL1/s;
L_TF=G1G2*H2;
nyquist(L_TF)
```

After assigning K=10, different values of Kf has been used in the range of $0.01<K<10^4$. The Nyquist diagrams shows the stability of the closed loop system for all $0<K<\infty$. A sample of Nyquist diagram is plotted as follows:




**10-28)** a) K > 2 ⇨ system is stable

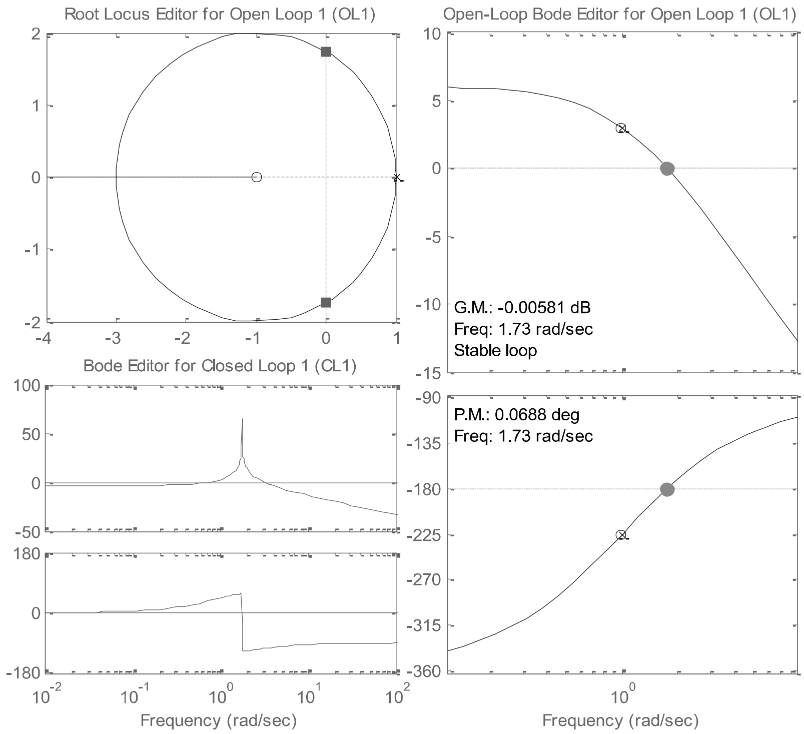
b) 0 < K < 1 and -2 < K < 0 ⇨ -2 < K < 1 ⇨system is stable

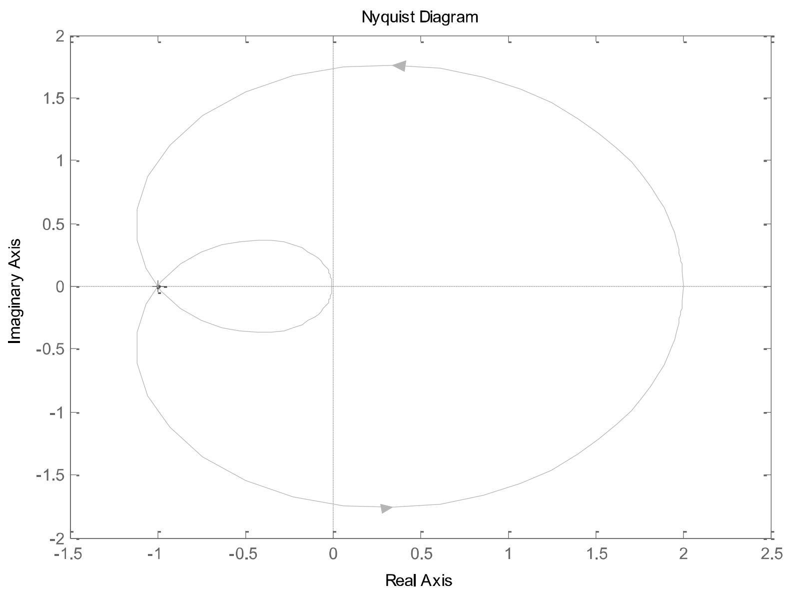
**MATLAB code:**

```
s = tf('s')
%a)
figure(1);
K=1
num_GH_a= K*(s+1);
den_GH_a=(s-1)^2;
GH_a=num_GH_a/den_GH_a;
nyquist(GH_a)
%b)
figure(2);
K=1
num_GH_b= K*(s-1);
den_GH_b=(s+1)^2;
GH_b=num_GH_b/den_GH_b;
nyquist(GH_b)
```

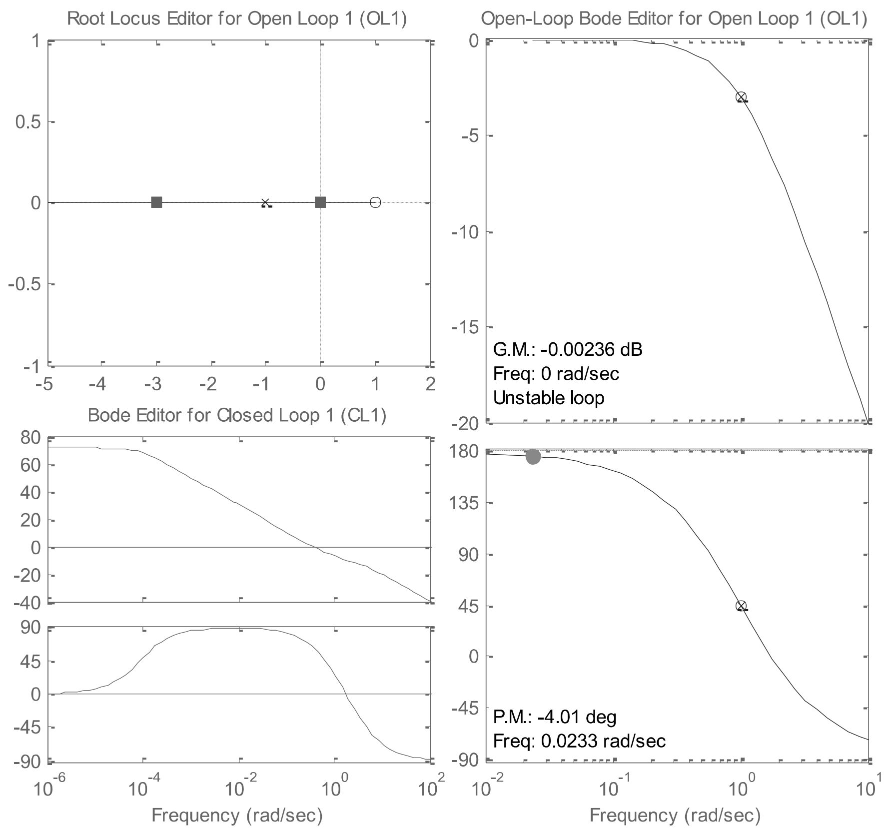
```
sisotool
```

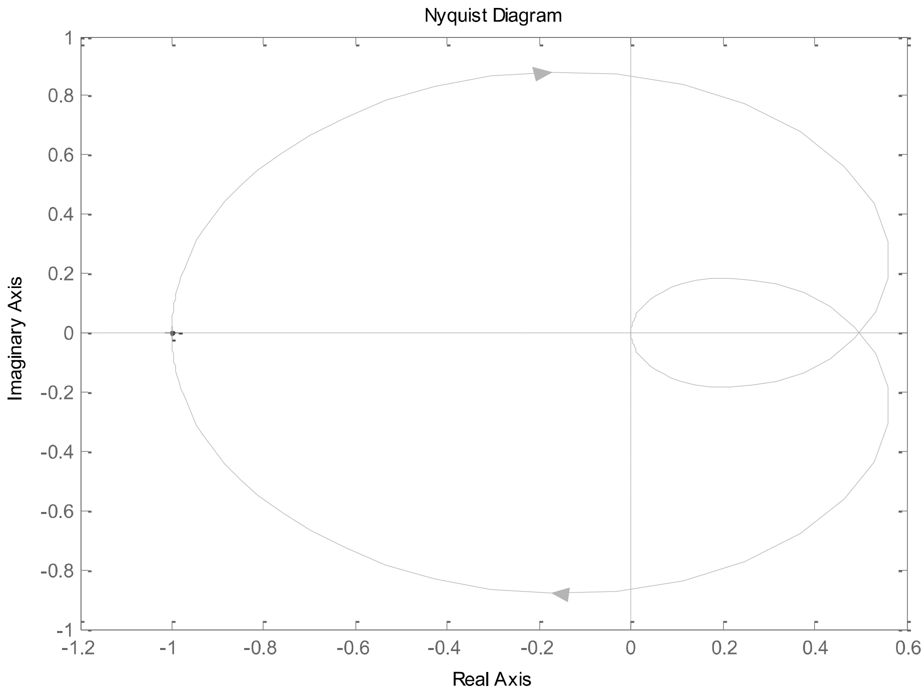
**Part(a):** Using MATLAB sisotool, the transfer function gain can be iteratively changed in order to obtain different phase margins. By changing the gain so that PM=0 (margin of stability), K>~2 resulted in stable Nyquist diagram for part(a). Following two figures illustrate the sisotool and Nyquist results at margin of stability for part (a).






**Part(b):** Similar methodology applied as in part (a). K<1 results in closed loop stability. Following are sisotool and Nyquist results at margin of stability (K=1):






**10-29) (a)**

Let $G(s) = G_1(s)e^{-T_d s}$ Then $G_1(s) = \dfrac{100}{s\left(s^2 + 10s + 100\right)}$

Let $\left|\dfrac{100}{-10\omega^2 + j\omega\left(100 - \omega^2\right)}\right| = 1$ or $\dfrac{100}{\left[100\omega^4 + \omega^2\left(100 - \omega^2\right)^2\right]^{1/2}} = 1$

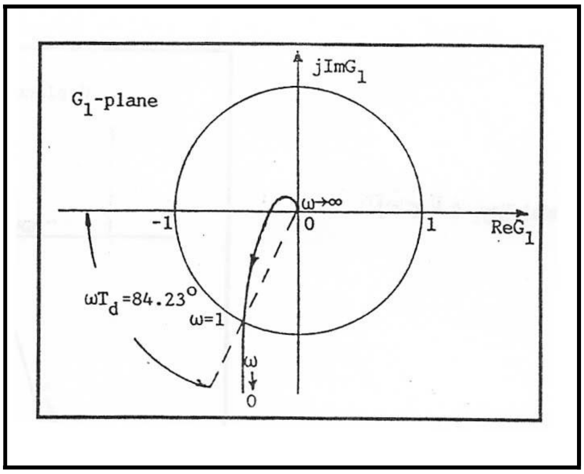
Thus $100\omega^4 + \omega^2\left(100 - \omega^2\right)^2 = 10,000$ $\omega^6 - 100\omega^4 + 10,000\omega^2 - 10,000 = 0$

The real solution for $\omega$ are $\omega = \pm 1$ rad/sec.

$$\angle G_1(j1) = -\tan^{-1}\left[\frac{100-\omega^2}{-10\omega}\right]\Bigg|_{\omega=1} = 264.23^o$$

Equating $\omega T_d\big|_{\omega=1} = \dfrac{\left(264.23^o - 180^o\right)\pi}{180}$

$$= \frac{84.23\pi}{180} = 1.47 \text{ rad}$$




**Thus the maximum time delay for stability is**

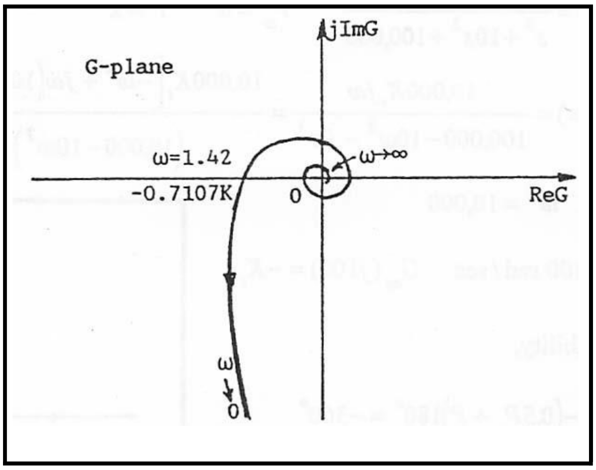
$T_d = 1.47$ **sec.**

**(b)** $T_d = 1$ sec.

$$G(s) = \frac{100Ke^{-s}}{s\left(s^2+10s+100\right)} \qquad G(j\omega) = \frac{100Ke^{-j\omega}}{-10\omega^2 + j\omega\left(100-\omega^2\right)}$$

At the intersect on the negative real axis, $\omega = 1.42$ rad/sec.

$G(j1.42) = -0.7107K.$




**The system is stable for**

**0 < *K* < 1.407**

**10-30 (a)** ***K* = 0.1**

$$G(s) = \frac{10e^{-T_d s}}{s\left(s^2+10s+100\right)} = G_1(s)e^{-T_d s}$$

Let $\left|\dfrac{10}{-10\omega^2 + j\omega\left(100-\omega^2\right)}\right| = 1$ or $\dfrac{10}{\left[100\omega^4+\omega^2\left(100-\omega^2\right)^2\right]^{1/2}} = 1$

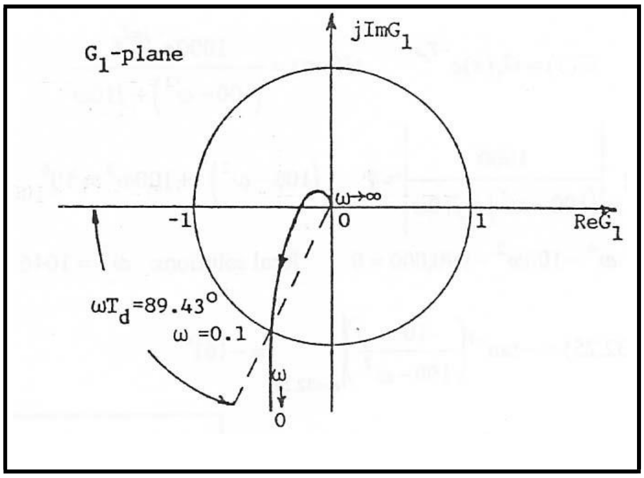
Thus $\omega^6 - 100\omega^4 + 10{,}000\omega^2 - 100 = 0$ The real solutions for $\omega$ is $\omega = \pm 0.1$ rad/sec.

$$\angle G_1(j0.1) = -\tan^{-1}\left[\frac{100-\omega^2}{-10\omega}\right]\Bigg|_{\omega=0.1} = 269.43^o$$

Equate $\omega T_d\big|_{\omega=0.1} = \dfrac{\left(269.43^o - 180^o\right)\pi}{180^o} = 1.56$ rad We have $T_d = 15.6$ sec.

We have the maximum time delay

for stability is 15.6 sec.




**10-30 (b)** $T_d = 0.1$ sec.

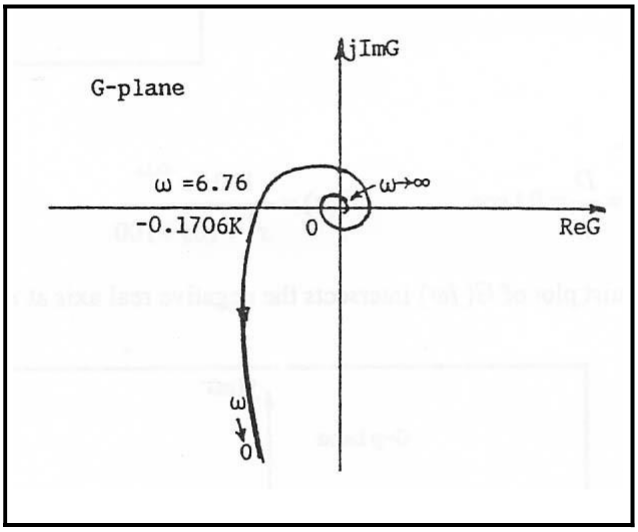
$$G(s) = \frac{100Ke^{-0.1s}}{s\left(s^2+10s+100\right)} \qquad G(j\omega) = \frac{100Ke^{-0.1j\omega}}{-10\omega^2 + j\omega\left(100-\omega^2\right)}$$

At the intersect on the negative real axis,

$\omega = 6.76$ rad/sec. $G(j6.76) = -0.1706K$

**The system is stable for**

**0 < *K* < 5.86**

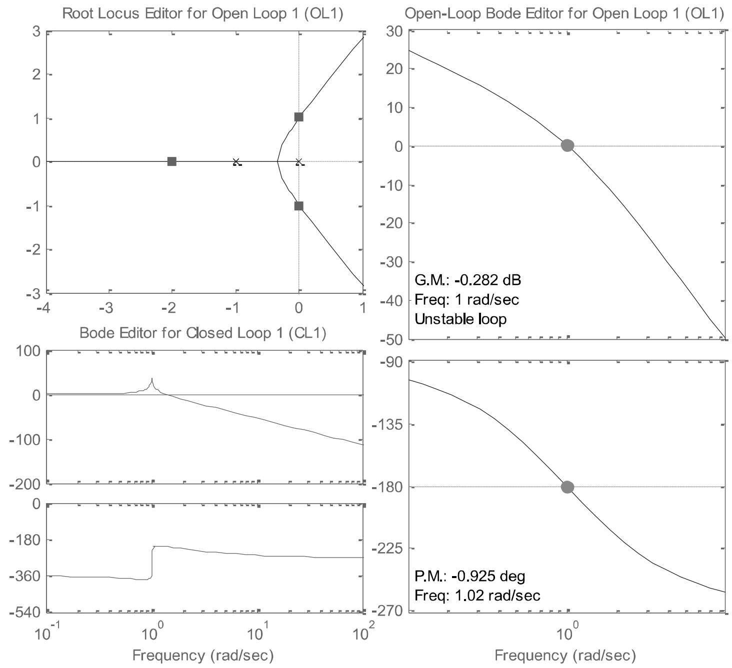



**10-31)**

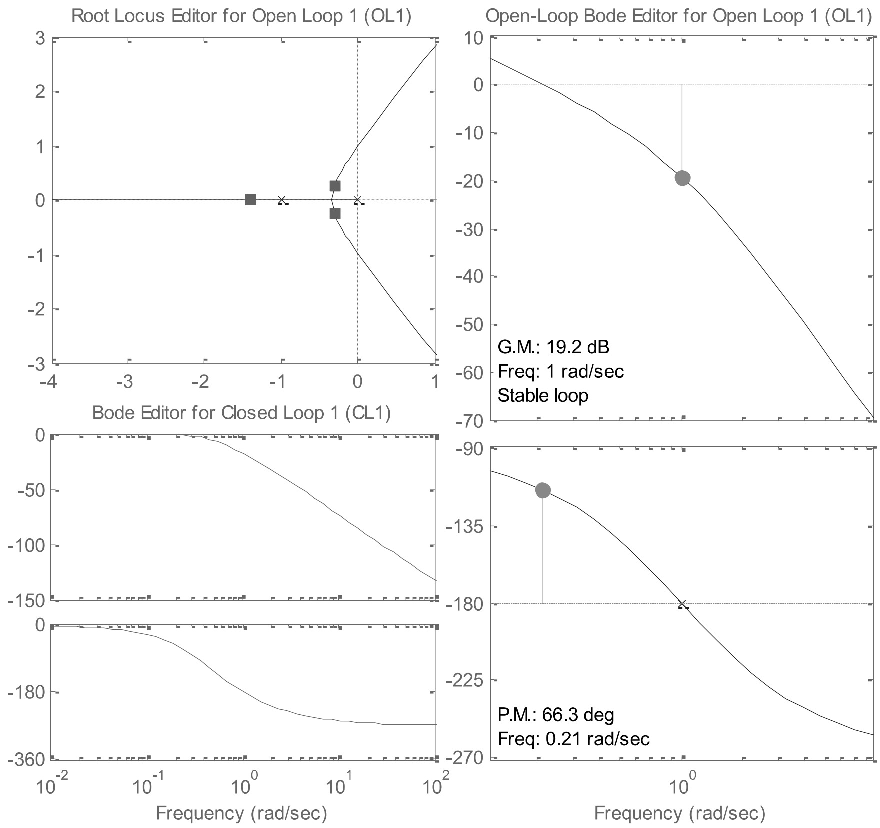
**MATLAB code:**

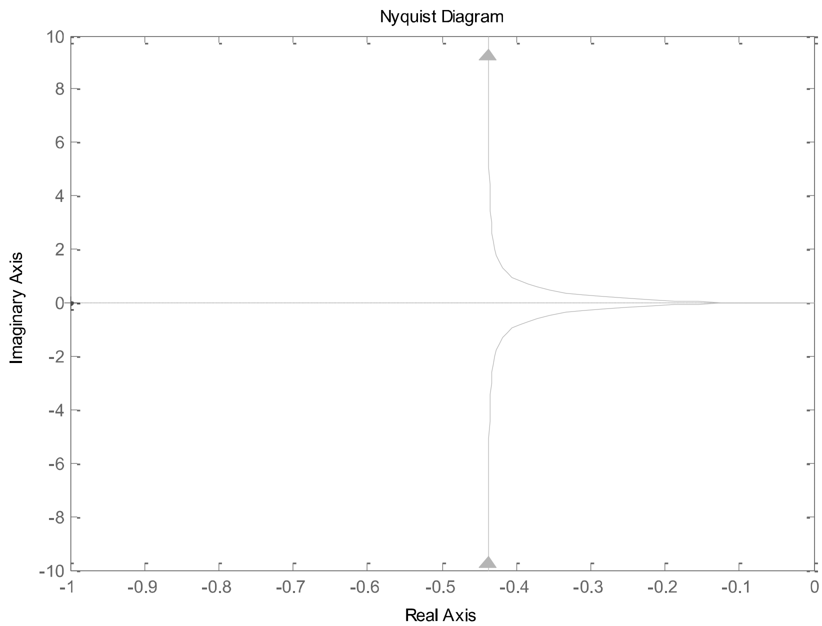
```
s = tf('s')
%a)

figure(1);
K=1
num_GH_a= K;
den_GH_a=s*(s+1)*(s+1);
GH_a=num_GH_a/den_GH_a;
nyquist(GH_a)

%b)
figure(2);
K=20
num_GH_b= K;
den_GH_b=s*(s+1)*(s+1);
GH_b=num_GH_b/den_GH_b;
nyquist(GH_b)

sisotool;
```

By using sisotool and importing the loop transfer function, different values of K has been tested which resulted in a stable system when K<2, and unstable system for K>2. Following diagrams correspond to margin of stability:

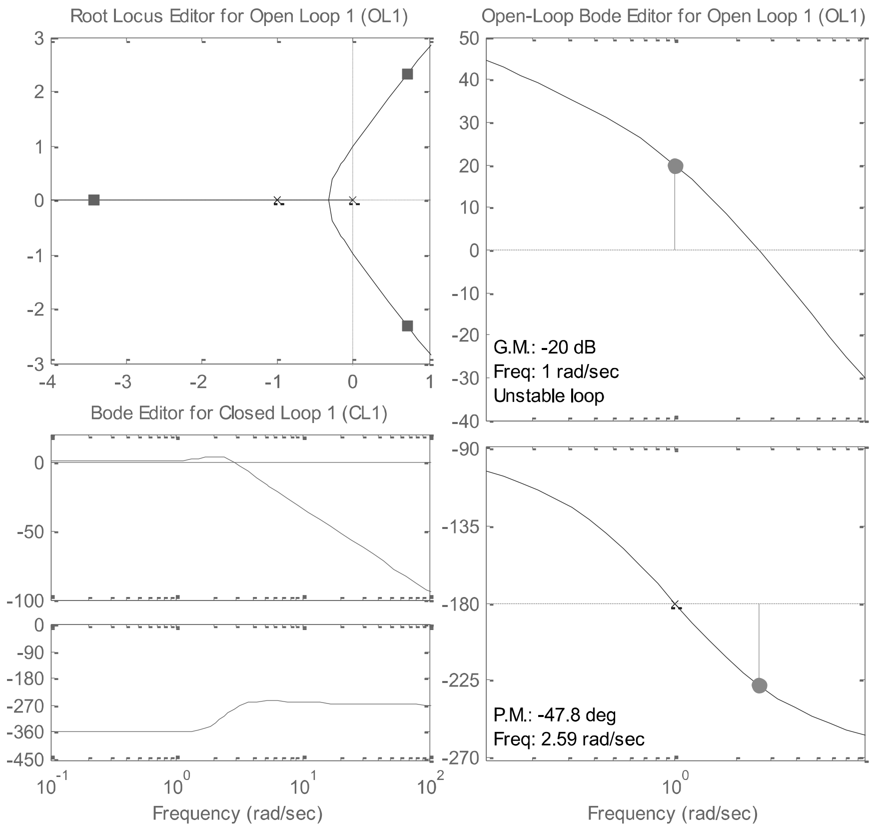


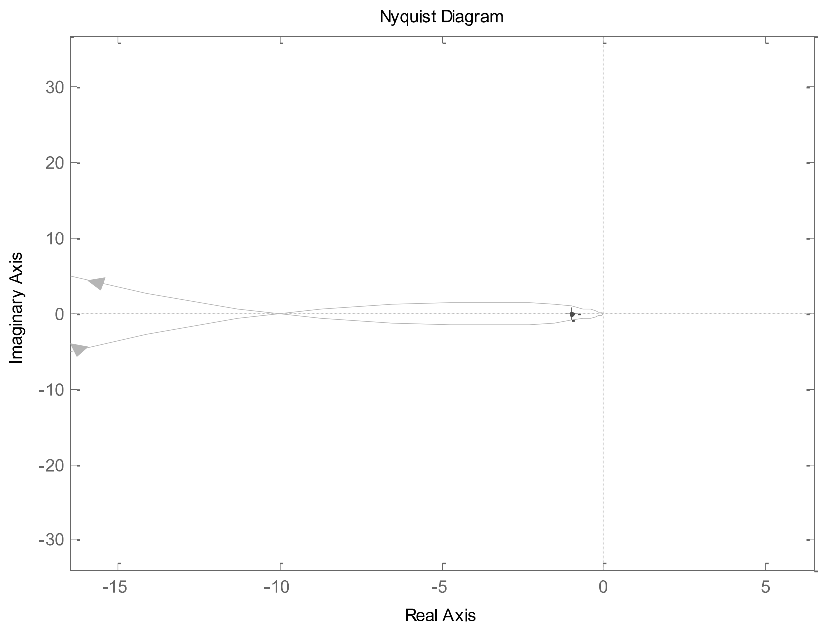

**Part(a):** small K resulted in stable system as shown below for K=0.219:




**Part(b):** Large K resulted in unstable system as shown below for K=20:


Root Locus Editor for Open Loop 1 (OL1)
Open-Loop Bode Editor for Open Loop 1 (OL1)
G.M.: -20 dB
Freq: 1 rad/sec
Unstable loop
Bode Editor for Closed Loop 1 (CL1)
P.M.: -47.8 deg
Freq: 2.59 rad/sec
Frequency (rad/sec)
Frequency (rad/sec)


Nyquist Diagram
Imaginary Axis
Real Axis

The system is stable for small value of K, since there is no encirclement of the s = -1

The system is unstable for large value of K, since the locus encirclement the $s = -1$ twice in CCW; which means two poles are in the right half s-plane.

**10-32)** **(a)** The transfer function (gain) for the sensor-amplifier combination is 10 V/0.1 in = 100 V/in. The velocity of flow of the solution is

$$v = \frac{10\ \text{in}^3/\text{sec}}{0.1\ \text{in}} = 100\ \text{in/sec}$$

The time delay between the valve and the sensor is $T_d = D/v$ sec. The loop transfer function is

$$G(s) = \frac{100Ke^{-T_d s}}{s^2 + 10s + 100}$$

**(b)** $K = 10$:

$$G(s) = G_1(s)e^{-T_d s} \qquad G(j\omega) = \frac{1000e^{-j\omega T_d}}{\left(100-\omega^2\right)+j10\omega}$$

Setting $\left|\dfrac{1000}{\left(100-\omega^2\right)+j10\omega}\right| = 1 \qquad \left(100-\omega^2\right)^2 + 100\omega^2 = 10^6$

Thus, $\omega^4 - 100\omega^2 - 990{,}000 = 0$ Real solutions: $\omega^2 = 1046.2$ $\omega = 32.35$ rad/sec

$$\angle G_1(j32.25) = -\tan^{-1}\left(\frac{10\omega}{100-\omega^2}\right)\Bigg|_{\omega=32.25} = -161^o$$

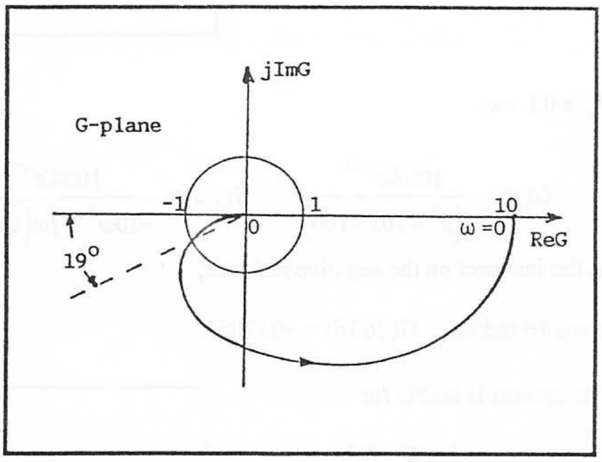
Thus,

$$32.35T_d = \frac{19^o \pi}{180^o} = 0.33 \text{ rad}$$

Thus,

$$T_d = 0.0103 \text{ sec}$$

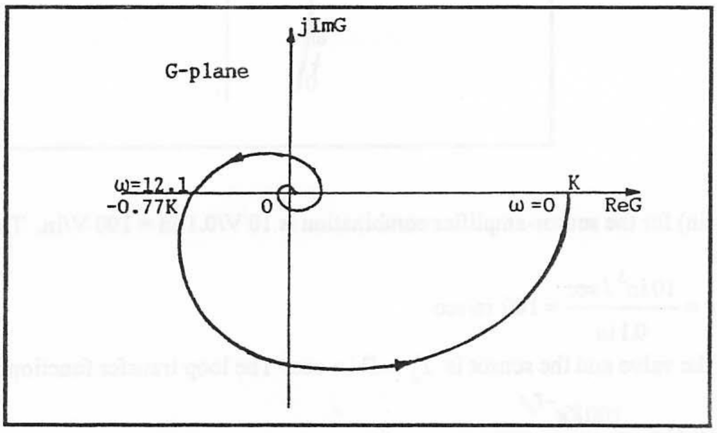
Maximum $D = vT_d = 100 \times 0.0103 = 1.03$ in




**(c)** $D = 10$ **in.**

$$T_d = \frac{D}{v} = 0.1 \text{ sec} \qquad G(s) = \frac{100Ke^{-0.1s}}{s^2+10s+100}$$

The Nyquist plot of $G(j\omega)$ intersects the negative real axis at $\omega = 12.1$ rad/sec.



**10-33)**

**(a)** The transfer function (gain) for the sensor-amplifier combination is 1 V/0.1 in = 10 V/in. The velocity of flow of the solutions is

$$v = \frac{10\ \text{in}^3/\text{sec}}{0.1\ \text{in}} = 100\ \text{in}/\text{sec}$$

The time delay between the valve and sensor is $T_d = D/v$ sec. The loop transfer function is

$$G(s) = \frac{10Ke^{-T_d s}}{s^2 + 10s + 100}$$

**(b)** ***K*** **= 10:**

$$G(s) = G_1(s)e^{-T_d s} \qquad G(j\omega) = \frac{100e^{-j\omega T_d}}{\left(100-\omega^2\right) + j10\omega}$$

Setting $\left|\dfrac{100}{\left(100-\omega^2\right)+j10\omega}\right| = 1 \qquad \left(100-\omega^2\right)^2 + 100\omega^2 = 10,000$

Thus, $\omega^4 - 100\omega^2 = 0$ Real solutions: $\omega = 0, \quad \omega = \pm 10\ \text{rad}/\text{sec}$

$$\angle G_1(j10) = -\tan^{-1}\left(\frac{10\omega}{100-\omega^2}\right)\bigg|_{\omega=10} = -90^o$$

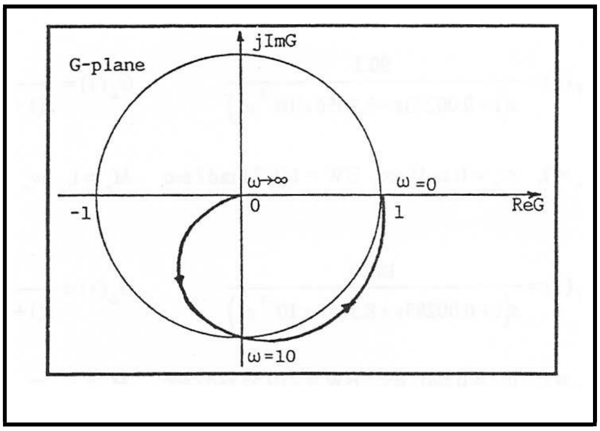
Thus,

$$10T_d = \frac{90^o \pi}{180^o} = \frac{\pi}{2}\ \text{rad}$$

Thus,

$$T_d = \frac{\pi}{20} = 0.157\ \text{sec}$$

Maximum $D = vT_d = 100 \times 0.157 = 15.7$ in

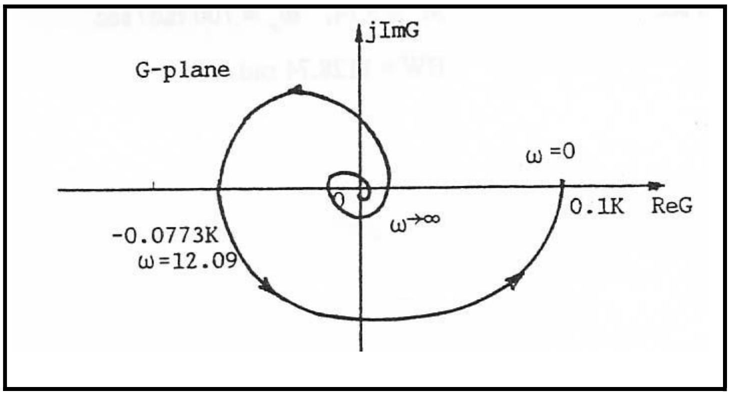


**(c)** ***D*** **= 10 in.**

$$T_d = \frac{D}{v} = \frac{10}{100} = 0.1 \text{ sec} \qquad G(s) = \frac{10Ke^{-0.1s}}{s^2 + 10s + 100}$$

The Nyquist plot of $G(j\omega)$ intersects the negative real axis at $\omega = 12.09$ rad/sec. $G(j) = -0.0773K$

For stability, the maximum value of *K* is 12.94 .




**10-34)**

The system (GH) has zero poles in the right of s plane: P=0.

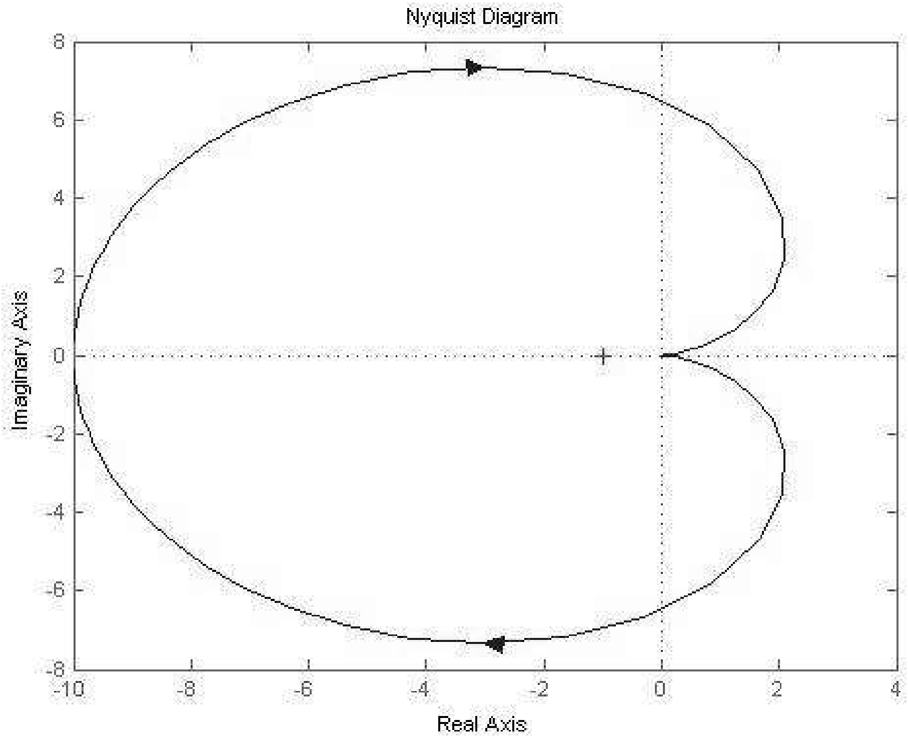
According to Nyquist criteria (Z=N+P), to ensure the stability which means the number of right poles of 1+ GH=0 should be zero (Z=0), we need N clockwise encirclements of Nyquist diagram about -1+0j point. That is N=-P or in other words, we need P counter-clockwise encirclement about -1+0j. In this case, we need 0 CCW encirclements.

**10-34(a)** According to Nyquist diagrams, this happens when K<-1. The three Nyquist diagrams are plotted with K=-10, K=-1, K=10 as examples:

**MATLAB code:**

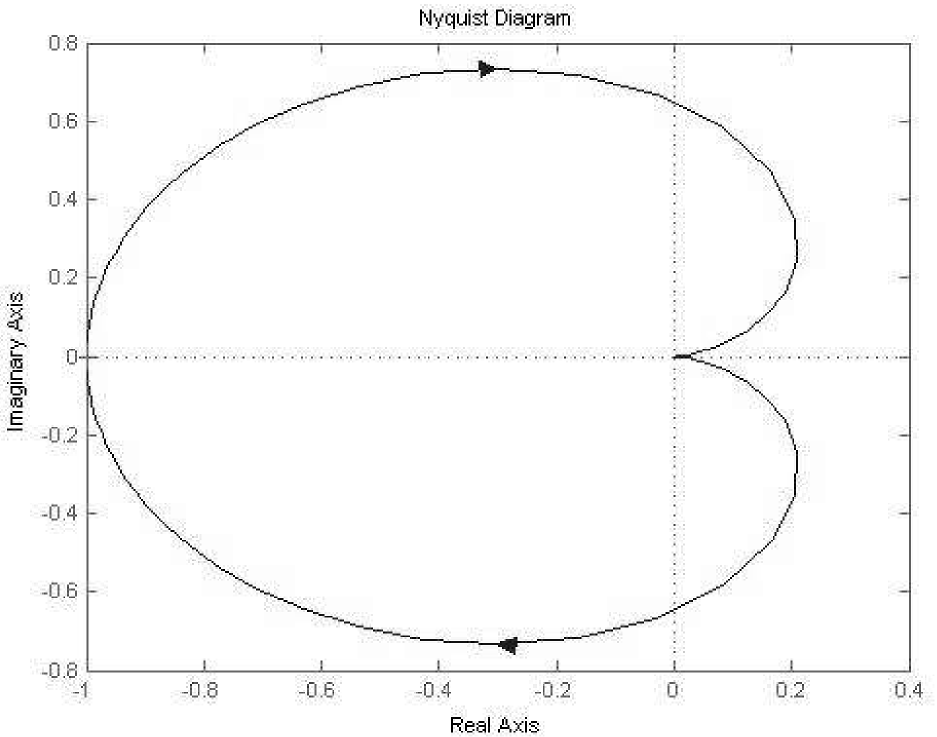
```
s = tf('s')
%a)
figure(1);
K=-10
num_G_a = K ;
```

```
den_G_a =(s+1);
num_H_a = (s+2);
den_H_a = (s^2+2*s+2);
G_a=num_G_a/den_G_a;
H_a = num_H_a/den_H_a;
OL_a = G_a*H_a
nyquist(OL_a)
```



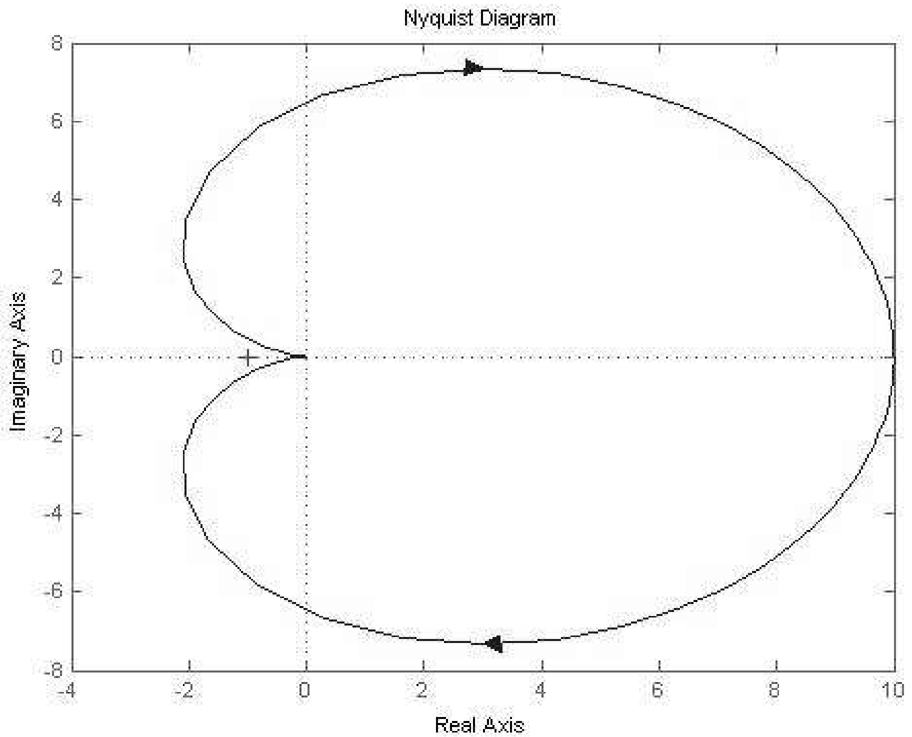

**Case 1) Nyquist graph, K=-10:** margin of stability K<-1 unstable

```
figure(2);
K=-1
num_G_a = K ;
den_G_a =(s+1);
num_H_a = (s+2);
den_H_a = (s^2+2*s+2);
G_a=num_G_a/den_G_a;
H_a = num_H_a/den_H_a;
%CL_a = G_a/(1 + G_a*H_a);
OL_a = G_a*H_a
nyquist(OL_a)
```




**Case 2) Nyquist graph, K=-1:** marginally unstable

```
figure(3);
K=10
num_G_a = K ;
den_G_a =(s+1);
num_H_a = (s+2);
den_H_a = (s^2+2*s+2);
G_a=num_G_a/den_G_a;
H_a = num_H_a/den_H_a;
%CL_a = G_a/(1 + G_a*H_a);
OL_a = G_a*H_a
nyquist(OL_a)
```




**Case 3) Nyquist graph, K=10:** stable case, -1<K no CCW encirclement about -1+0j point

**10-34 (b)**

For K<-1 (unstable), there will be 1 real pole in the right hand side of s-plane for the closed loop system, by running the following code.

```
K=-10
num_G_a = K ;
den_G_a =(s+1);
num_H_a = (s+2);
den_H_a = (s^2+2*s+2);
G_a=num_G_a/den_G_a;
H_a = num_H_a/den_H_a;
OL_a = G_a*H_a
nyquist(OL_a)
CL=1/(1+OL_a)
pole(CL)

K =

   -10


Transfer function:
     -10 s - 20
---------------------
s^3 + 3 s^2 + 4 s + 2


Transfer function:
s^3 + 3 s^2 + 4 s + 2
----------------------
s^3 + 3 s^2 - 6 s - 18


ans =

    2.4495
   -3.0000
   -2.4495
```

For K=-1 (marginally unstable), there will be 2 negative complex conjugate poles and a pole at zero for the closed loop system, by running the following code.

```
K=-1
num_G_a = K ;
den_G_a =(s+1);
num_H_a = (s+2);
den_H_a = (s^2+2*s+2);
```

```
G_a=num_G_a/den_G_a;
H_a = num_H_a/den_H_a;
OL_a = G_a*H_a
nyquist(OL_a)
CL=1/(1+OL_a)
pole(CL)
```

```
K =

    -1



Transfer function:
      -s - 2
---------------------
s^3 + 3 s^2 + 4 s + 2



Transfer function:
s^3 + 3 s^2 + 4 s + 2
---------------------
  s^3 + 3 s^2 + 3 s



ans =

        0
  -1.5000 + 0.8660i
  -1.5000 - 0.8660i
```

**Note: you may also wish to use MATLAB** `sisotool`.
**See alternative solution to 10-38.**

**10-34(c) The Characteristic Equation is: $s^3 + 3s^2 + (4+K)s + 2+2K$**

Using Routh criterions, the coefficient table is as follows:

| | | |
|---|---|---|
| $s^3$ | $1$ | $4+K$ |
| $s^2$ | $3$ | $2K+2$ |
| $s^1$ | $K+10$ | $0$ |
| $s^0$ | $2K+2$ | $0$ |

The system is stable if the content of the 1st column is positive:

$$K+10>0 \rightarrow K>-10$$

$$2K+2>0 \rightarrow K>-1$$

which is consistent with the results of the Nyquist diagrams. For $K>-1$ system is **stable**.

**10-35) (a)** $M_r = 2.06,\quad \omega_r = 9.33 \text{ rad/sec},\ \text{BW} = 15.2 \text{ rad/sec}$

**(b)**

$$M_r = \frac{1}{2\zeta\sqrt{1-\zeta^2}} = 2.06 \qquad \zeta^4 - \zeta^2 + 0.0589 = 0$$ The solution for $\zeta < 0.707$ is $\zeta = 0.25$.

$$\omega_r\sqrt{1-2\zeta^2} = 9.33 \text{ rad/sec} \quad \text{Thus} \quad \omega_n = \frac{9.33}{0.9354} = 9.974 \text{ rad/sec}$$

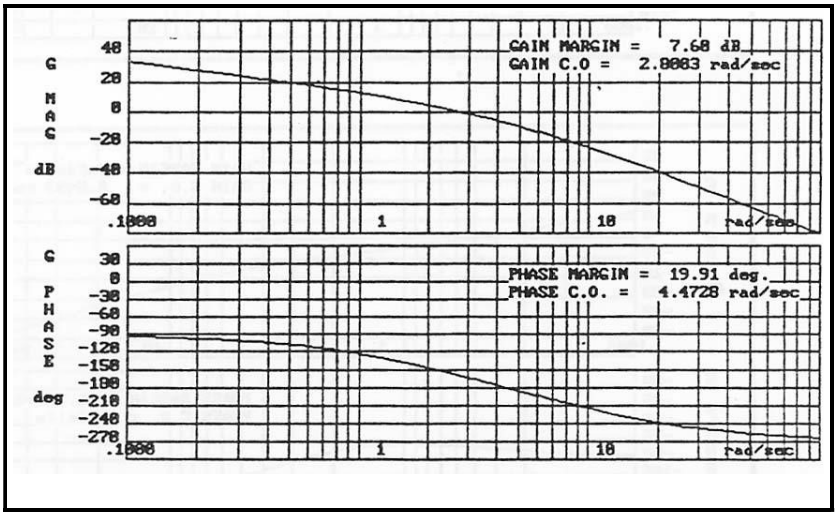
$$G_L(s) = \frac{\omega_n^2}{s(s+2\zeta\omega_n)} = \frac{99.48}{s(s+4.987)} = \frac{19.94}{s(1+0.2005s)}$$

**BW = 15.21 rad/sec**

**10-36)** Assuming a unity feedback loop ($H=1$), $G(s)H(s)=G(s)$

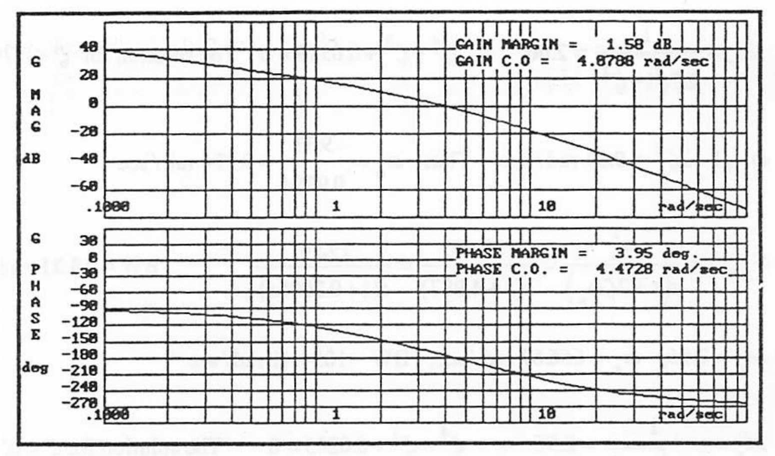
**(a)**

$$G(s)=\frac{5}{s(1+0.5s)(1+0.1s)}$$



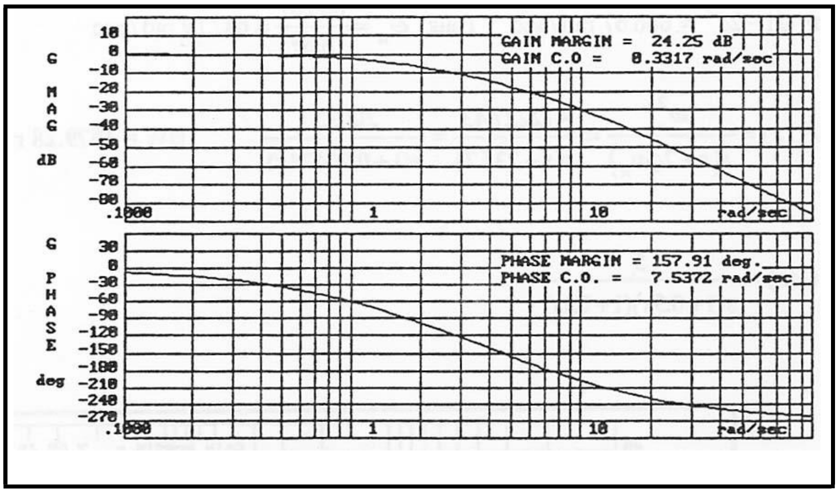
**10-36 (b)**

$$G(s) = \frac{10}{s(1+0.5s)(1+0.1s)}$$



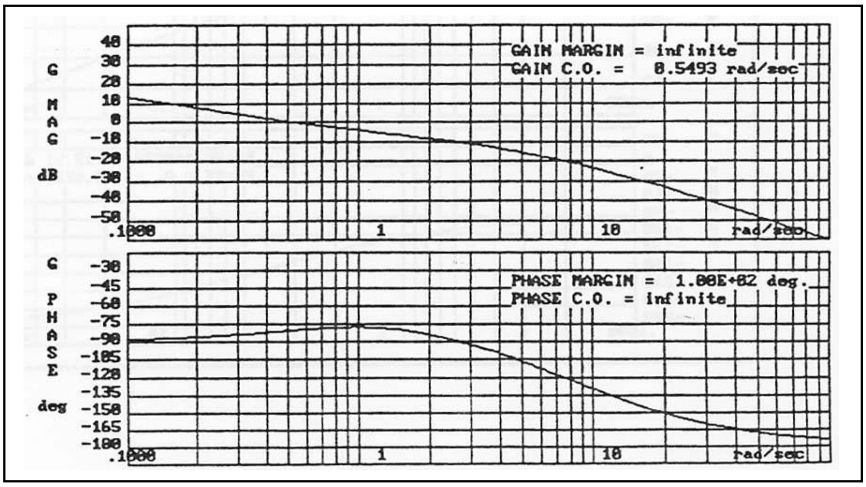

**(c)**

$$G(s) = \frac{500}{(s+1.2)(s+4)(s+10)}$$



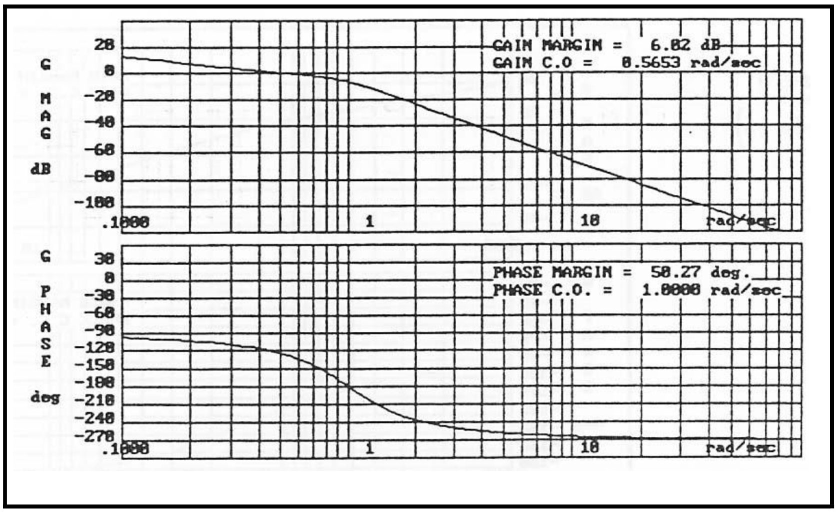
**(d)**

$$G(s)=\frac{10(s+1)}{s(s+2)(s+10)}$$



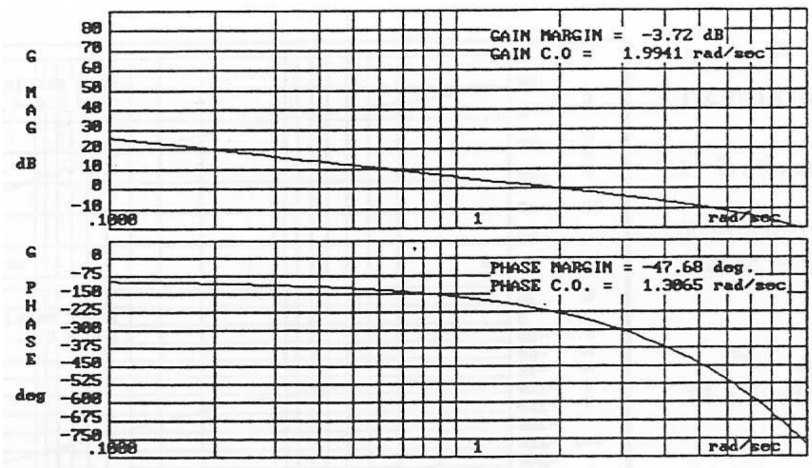

**10-36 (e)**

$$G(s)=\frac{0.5}{s\left(s^2+s+1\right)}$$



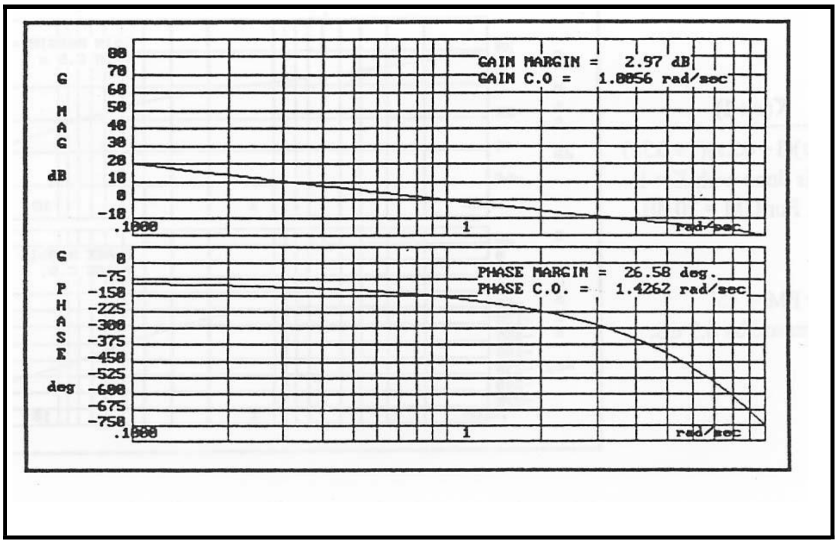
**(f)**

$$G(s) = \frac{100e^{-s}}{s\left(s^2 + 10s + 50\right)}$$



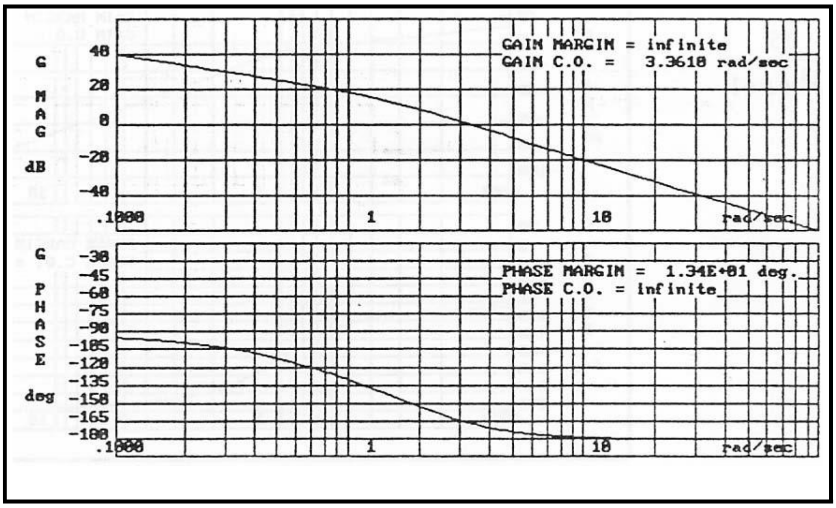
**(g)**

$$G(s) = \frac{100e^{-s}}{s\left(s^2 + 10s + 100\right)}$$




**10-36 (h)**

$$G(s) = \frac{10(s+5)}{s\left(s^2 + 5s + 5\right)}$$

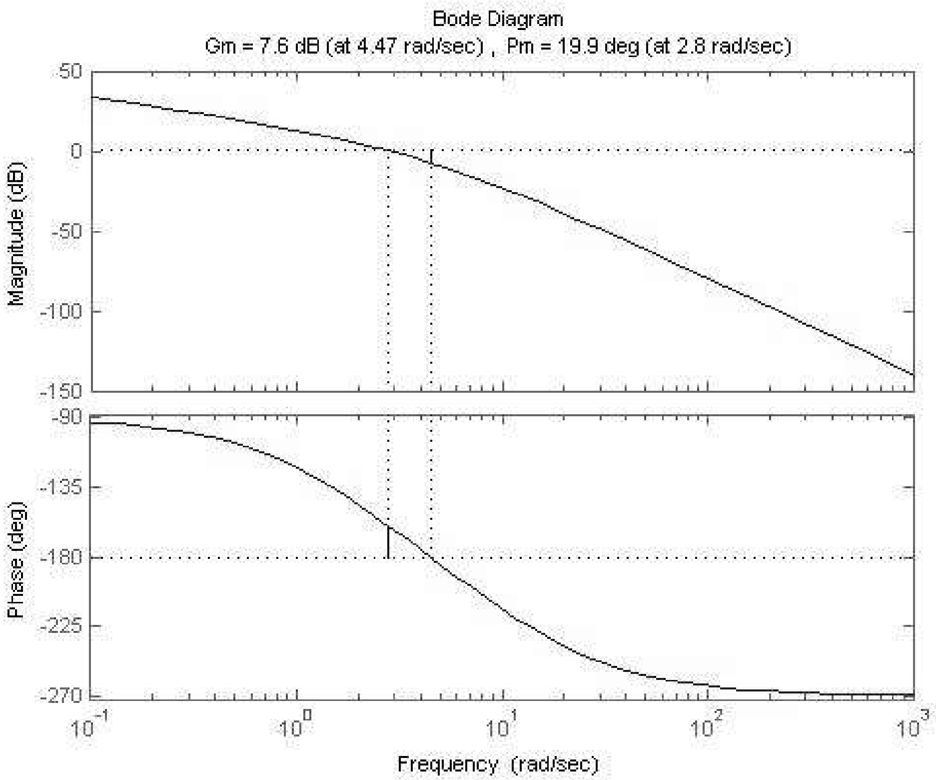



**(a)**

**MATLAB code:**

```
s = tf('s')
num_G_a= 5;
den_G_a=s*(0.5*s+1)*(0.1*s+1);
G_a=num_G_a/den_G_a
margin(G_a)
```

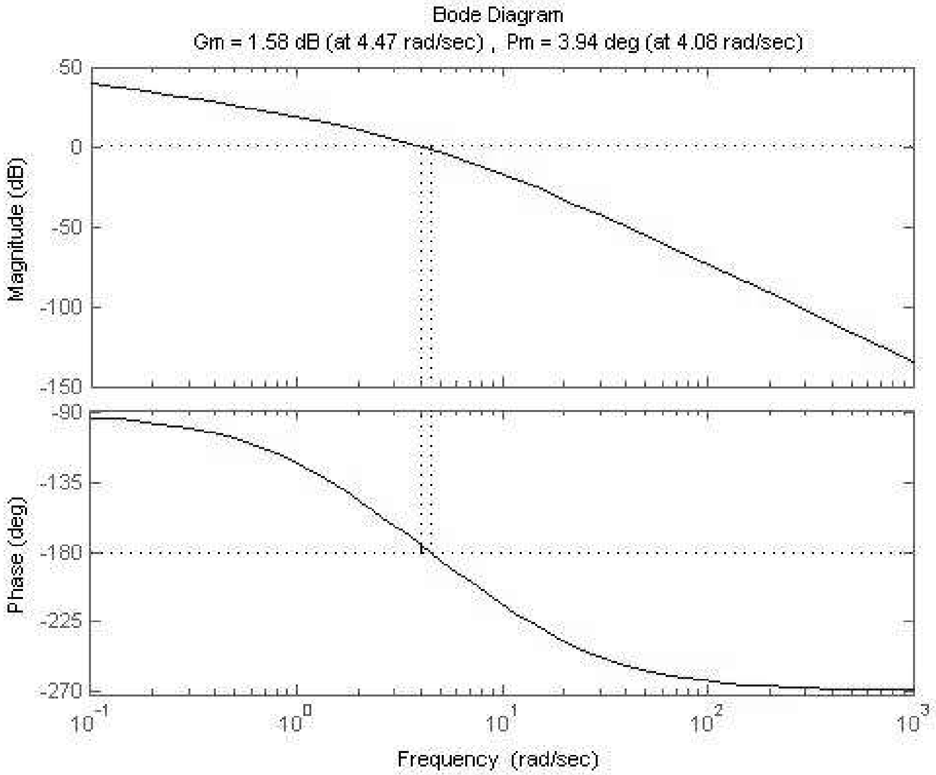
**Bode diagram:**




**(b)**

**MATLAB code:**

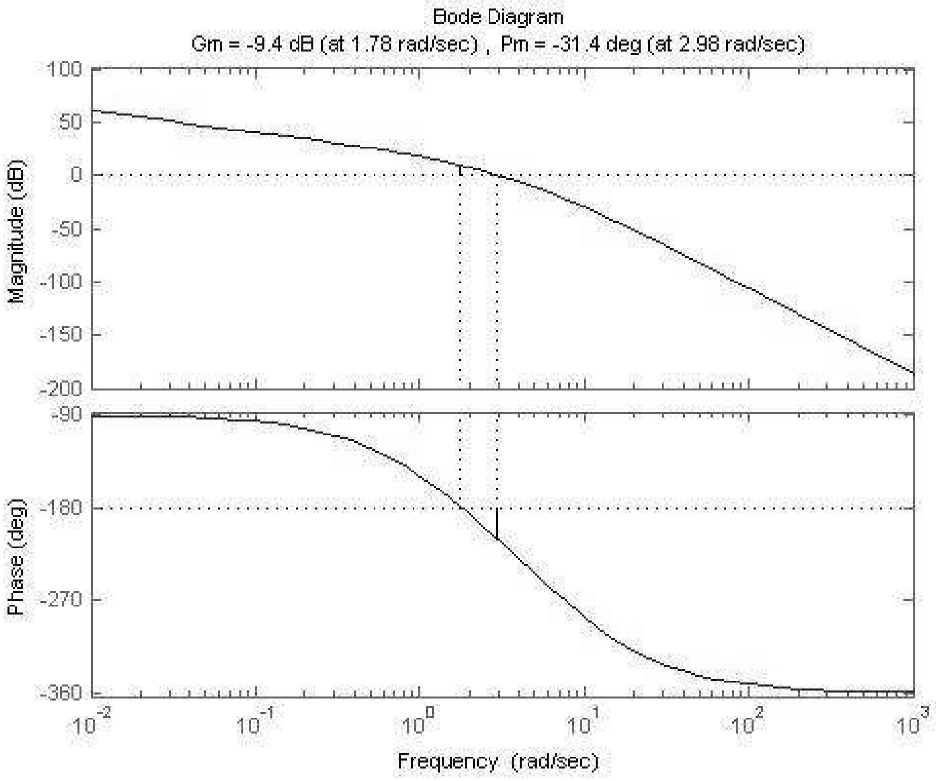
```
s = tf('s')
num_G_a= 10;
den_G_a=s*(1+0.5*s)*(1+0.1*s);
G_a=num_G_a/den_G_a
margin(G_a)
```




**(c)**

**MATLAB code:**

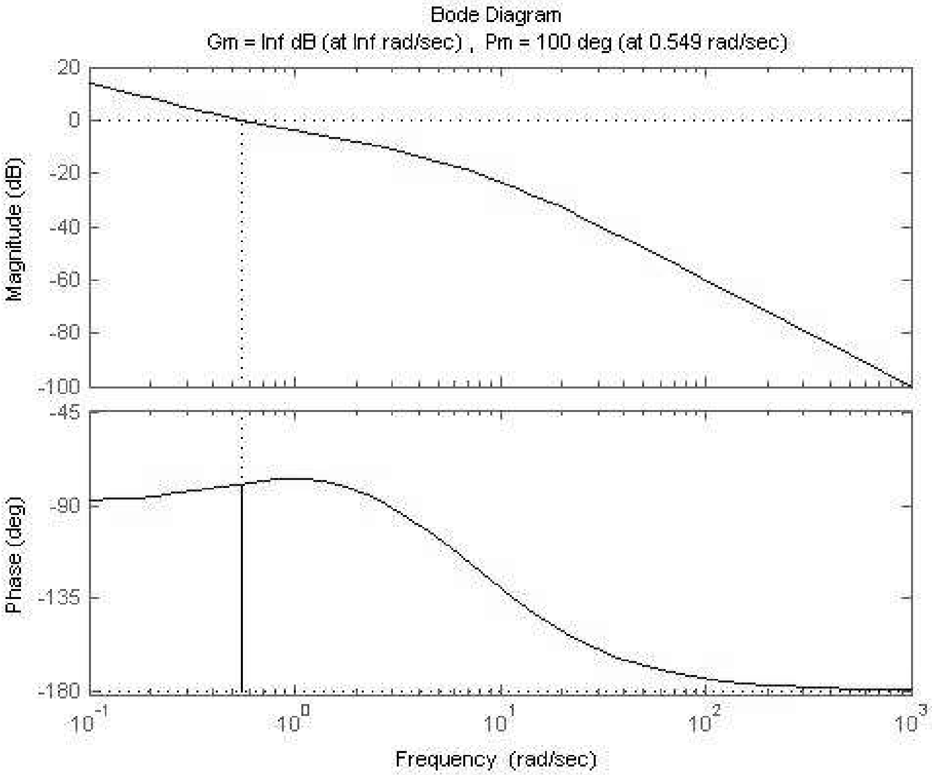
```
s = tf('s')
num_G_a= 500;
den_G_a=s*(s+1.2)*(s+4)*(s+10);
G_a=num_G_a/den_G_a
margin(G_a)
```




**(d)**

**MATLAB code:**

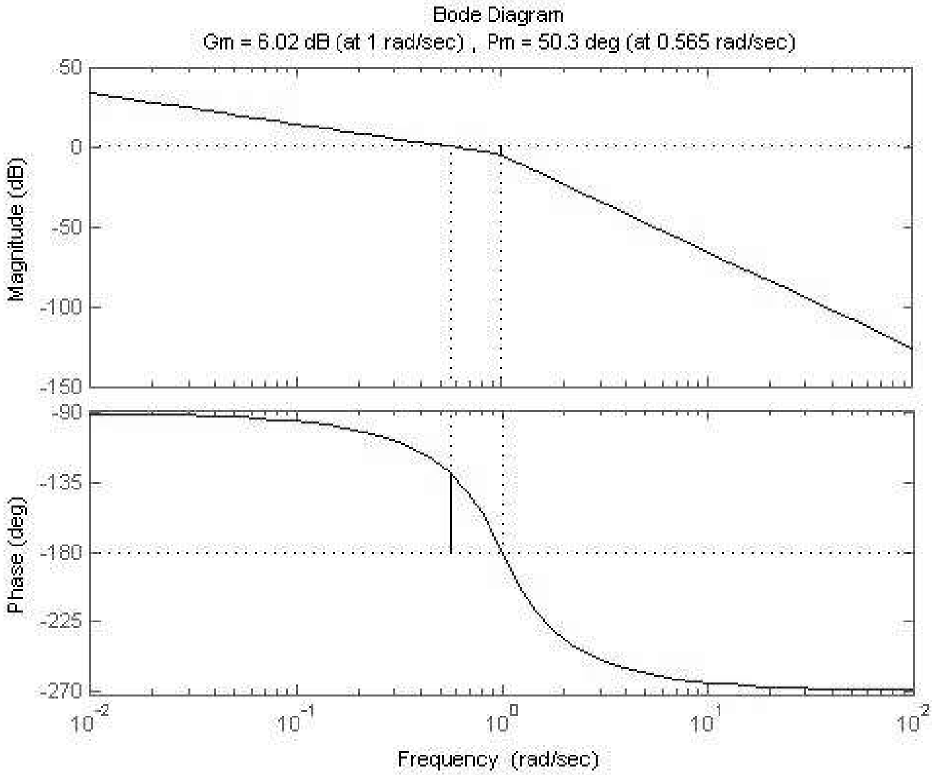
```
s = tf('s')
num_G_a= 10*(s+1);
den_G_a=s*(s+2)*(s+10);
G_a=num_G_a/den_G_a
margin(G_a)
```




**(e)**

**MATLAB code:**

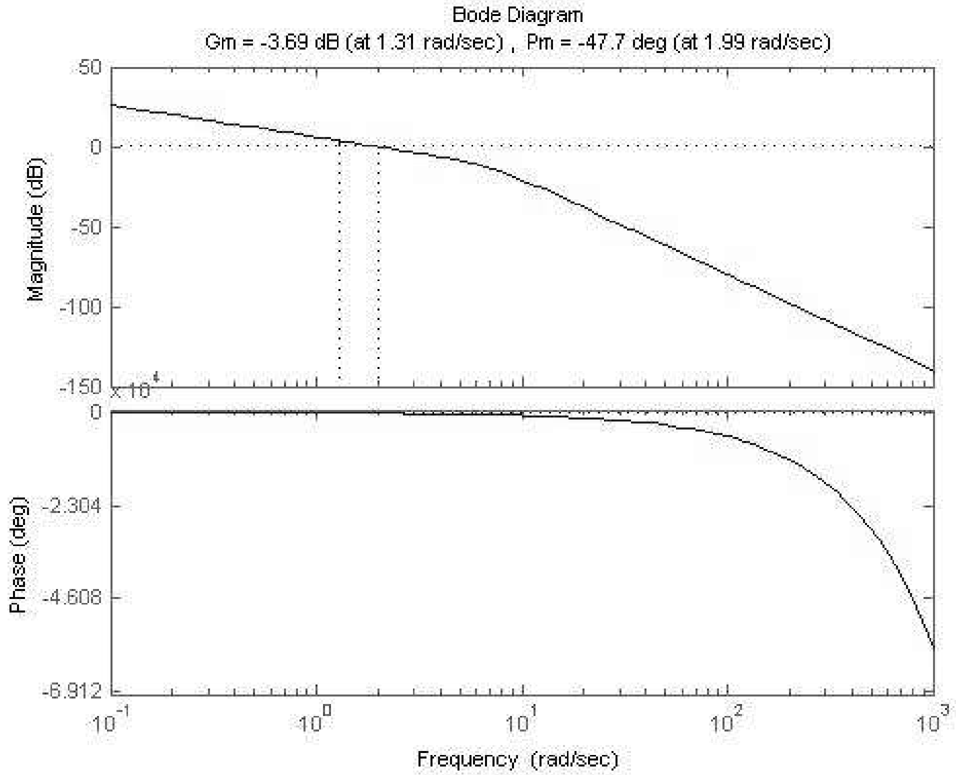
```
s = tf('s')
num_G_a= 0.5;
den_G_a=s*(s^2+s+1);
G_a=num_G_a/den_G_a
margin(G_a)
```




**(f)**

**MATLAB code:**

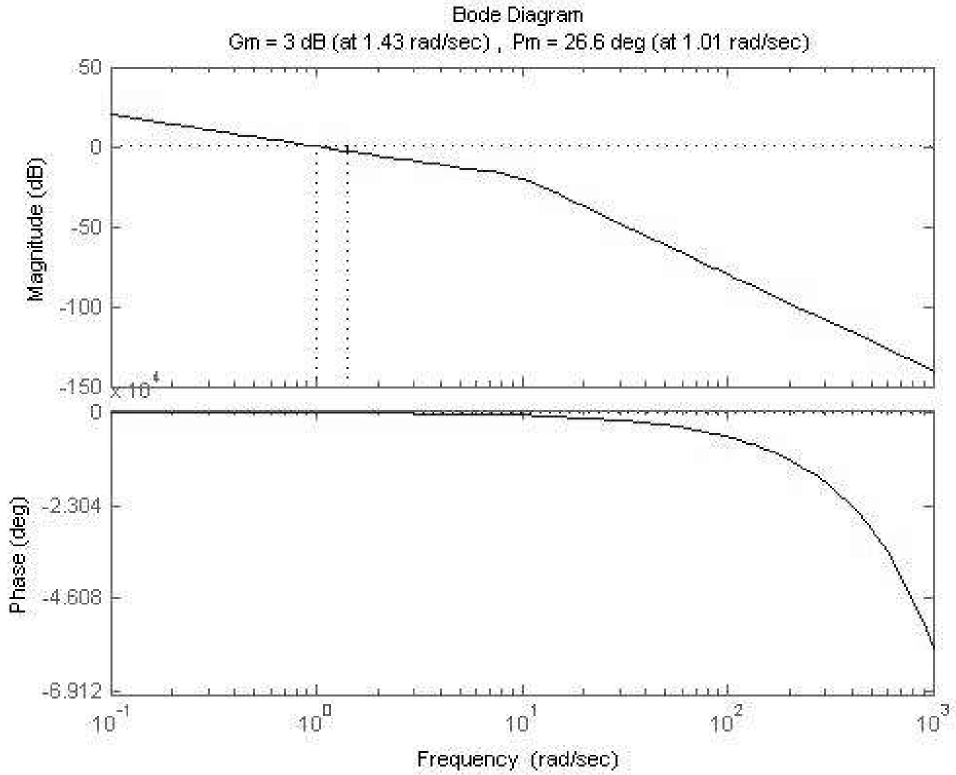
```
s = tf('s')
num_G_a= 100*exp(-s);
den_G_a=s*(s^2+10*s+50);
G_a=num_G_a/den_G_a
margin(G_a)
```




**(g)**

**MATLAB code:**

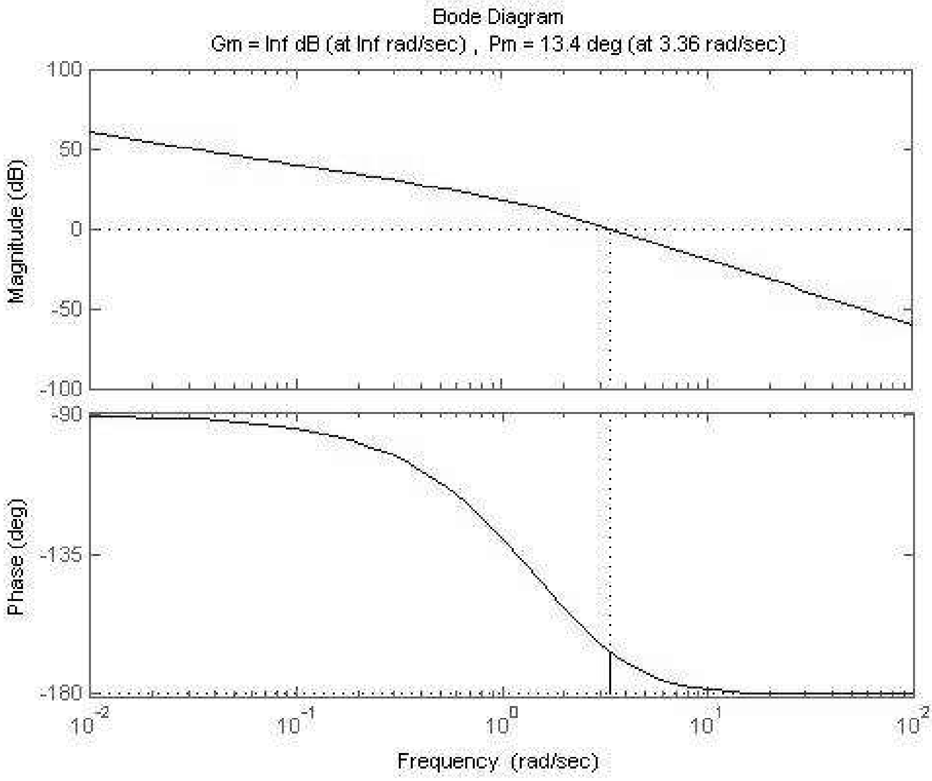
```
s = tf('s')
num_G_a= 100*exp(-s);
den_G_a=s*(s^2+10*s+100);
G_a=num_G_a/den_G_a
margin(G_a)
```




**(h)**

**MATLAB code:**

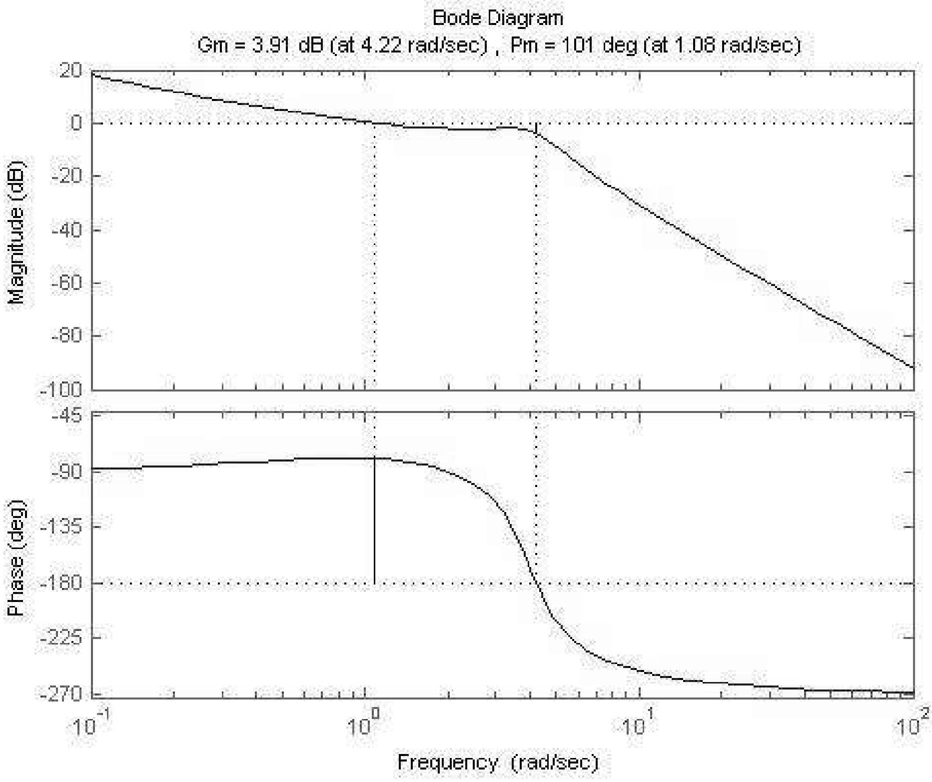
```
s = tf('s')
num_G_a= 10*(s+5);
den_G_a=s*(s^2+5*s+5);
G_a=num_G_a/den_G_a
margin(G_a)
```


Bode Diagram
Gm = Inf dB (at Inf rad/sec) , Pm = 13.4 deg (at 3.36 rad/sec)
Magnitude (dB)
100
50
0
-50
-100
Phase (deg)
-90
-135
-180
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
Frequency (rad/sec)

**10-37)**

**MATLAB code:**

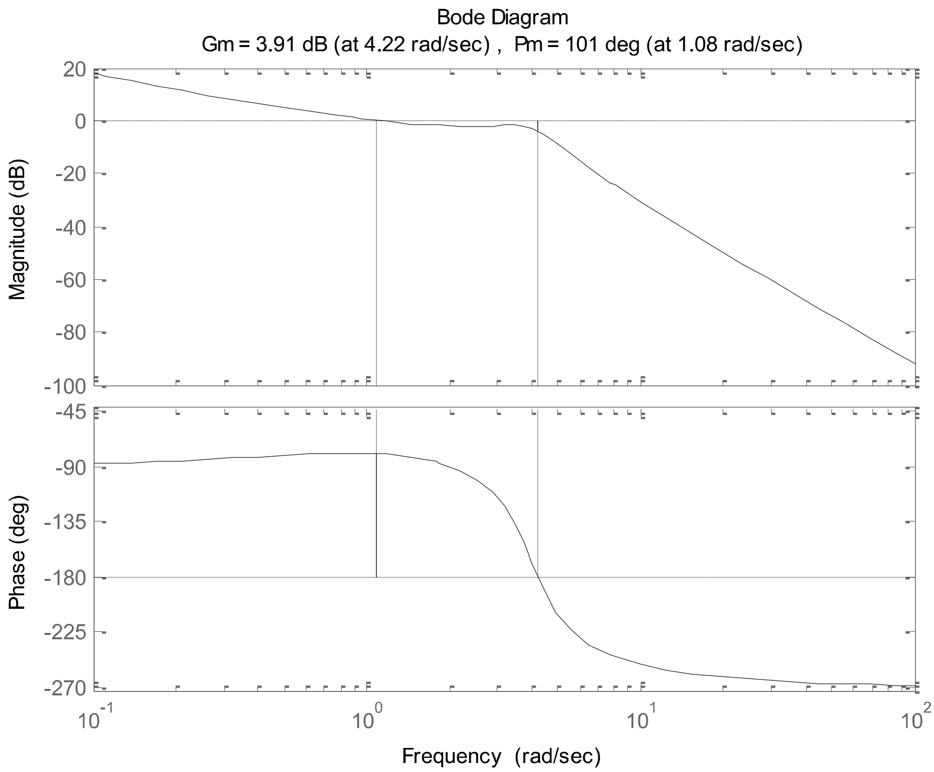
```
s = tf('s')
num_GH_a= 25*(s+1);
den_GH_a=s*(s+2)*(s^2+2*s+16);
GH_a=num_GH_a/den_GH_a
margin(GH_a)
```



**10-38) MATLAB code:**

```
s = tf('s')
num_G_a= 25*(s+1);
den_G_a=s*(s+2)*(s^2+2*s+16);
G_a=num_G_a/den_G_a
margin(G_a)
```

**Bode diagram:** PM=101 deg, GM=3.91 dB @ 4.22 rad/sec



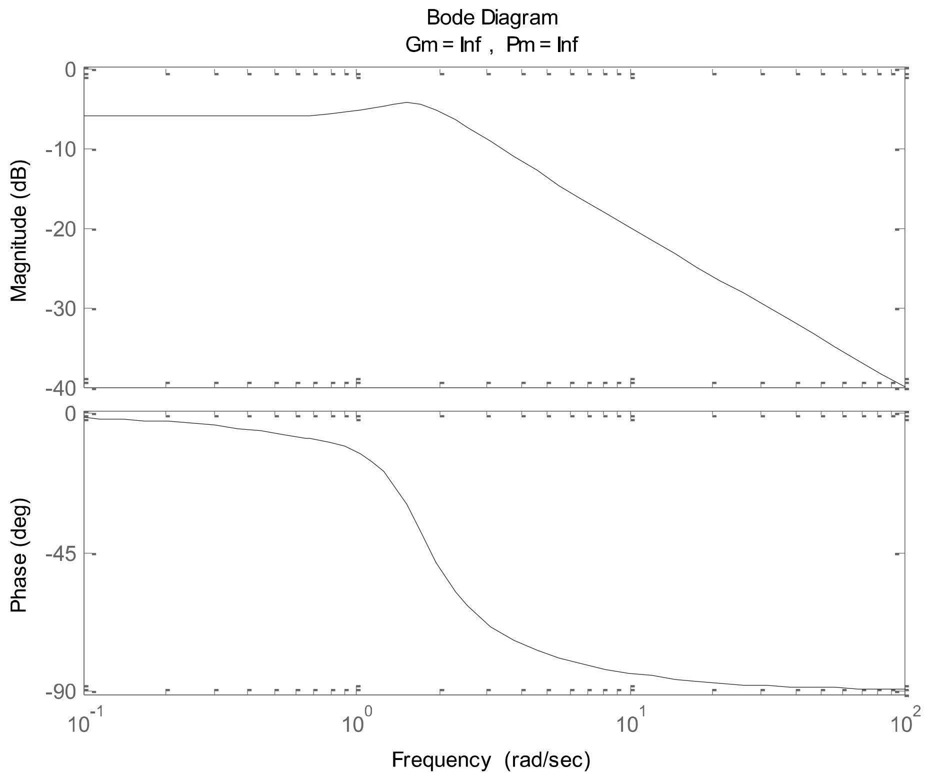

**10-38 Alternative solution**

**MATLAB code:**

```
s = tf('s')

%a)
figure(1);
num_G_a = 1 ;
den_G_a =(s+1);
num_H_a = (s+2);
den_H_a = (s^2+2*s+2);
```

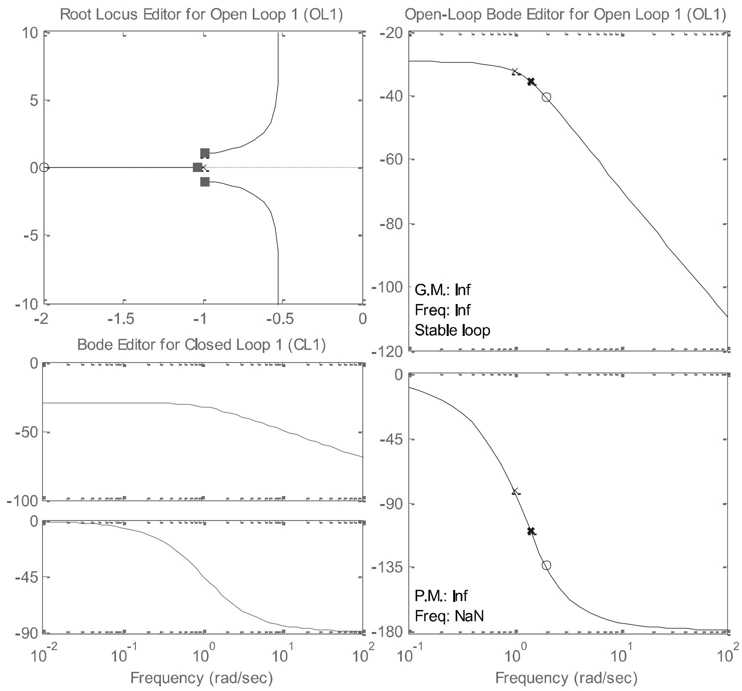
```
G_a=num_G_a/den_G_a;
H_a = num_H_a/den_H_a;
CL_a = G_a/(1 + G_a*H_a);
margin(CL_a)

sisotool
```

**Bode diagram:** for k=1

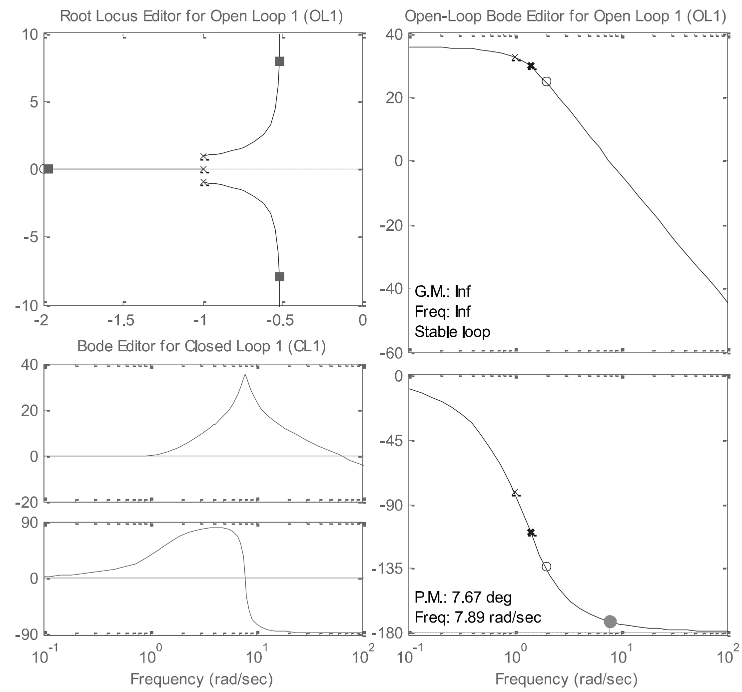



Using MATLAB sisotool, the transfer function gain can be iteratively changed in order to obtain different phase margins. By changing the gain K between very small and very big numbers, it was found that the closed loop system are stable (positive PM) ***<u>for every positive K in this system</u>***.

**K=0.034**


Root Locus Editor for Open Loop 1 (OL1)
Open-Loop Bode Editor for Open Loop 1 (OL1)
G.M.: Inf
Freq: Inf
Stable loop
Bode Editor for Closed Loop 1 (CL1)
P.M.: Inf
Freq: NaN
Frequency (rad/sec)
Frequency (rad/sec)

**K=59.9**

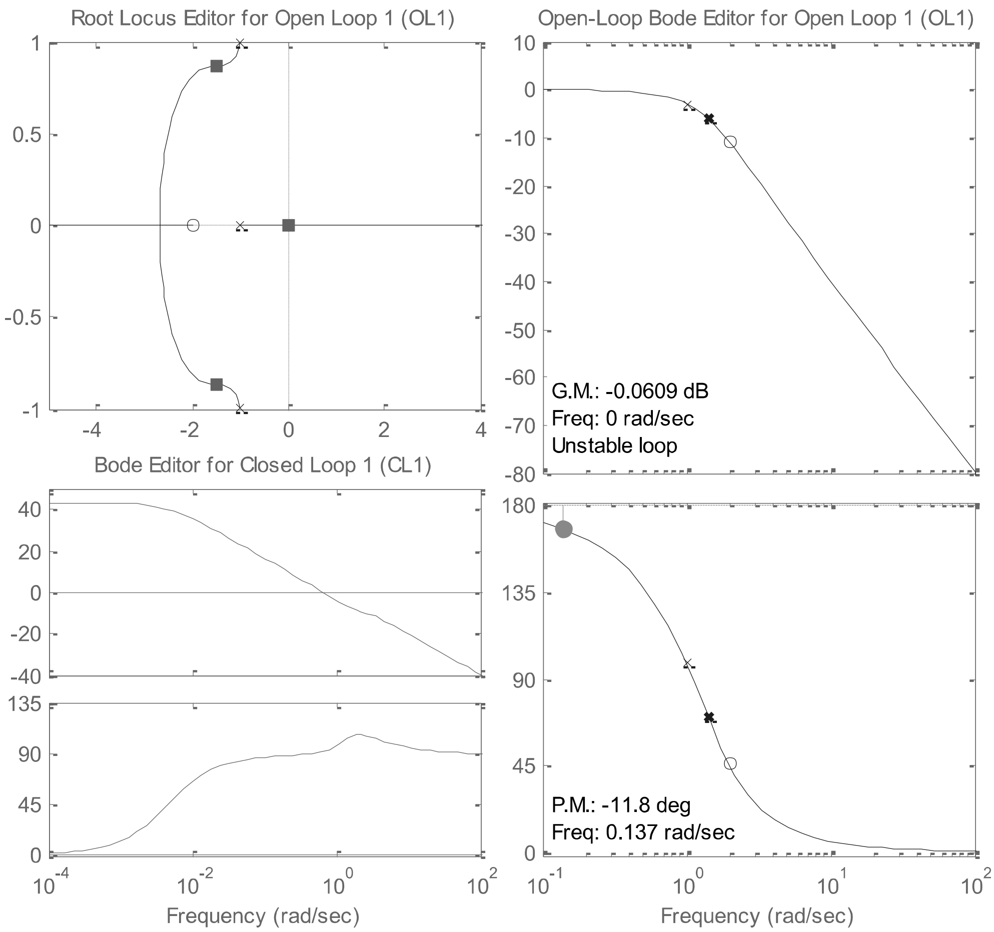

Root Locus Editor for Open Loop 1 (OL1)
Open-Loop Bode Editor for Open Loop 1 (OL1)
G.M.: Inf
Freq: Inf
Stable loop
Bode Editor for Closed Loop 1 (CL1)
P.M.: 7.67 deg
Freq: 7.89 rad/sec
Frequency (rad/sec)
Frequency (rad/sec)

In order to test the negative range of K, -1 was multiplied to the loop transfer function through the following code, and sisotool was used again.

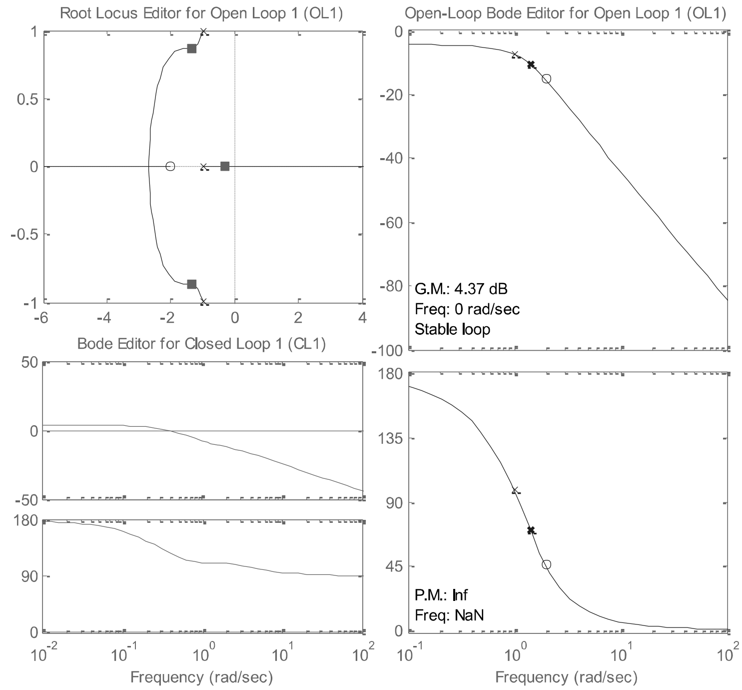
```
figure(1);
num_G_a = -1 ;
den_G_a =(s+1);
num_H_a = (s+2);
den_H_a = (s^2+2*s+2);
G_a=num_G_a/den_G_a;
H_a = num_H_a/den_H_a;
CL_a = G_a/(1 + G_a*H_a);
margin(CL_a)

sisotool
```

at K=-1, margin of stability is observed as PM~=0:

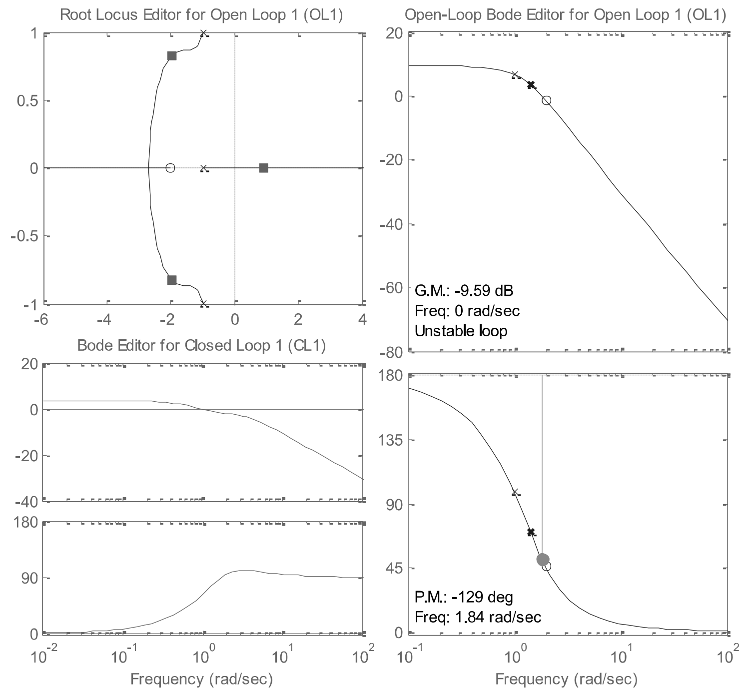
**K= -1**




The system is stable for K>-1 as follows: **K= -0.6**




And the system is unstable for K< -1: **K= -3**

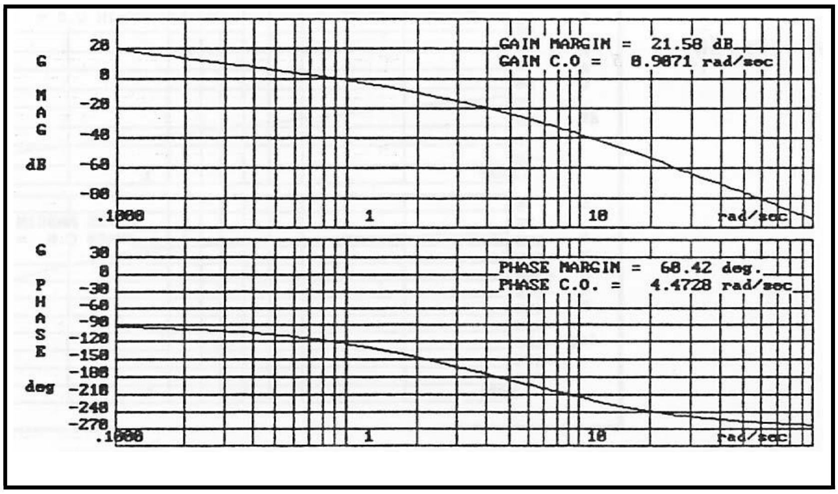



*Combining the individual ranges for K, the system will be stable in the range of K> -1

**10-39 See sample MATLAB code in Part e. The MATLAB codes are identical to problem 10-36.**

**(a)**

$$G(s)=\frac{K}{s(1+0.1s)(1+0.5s)}$$




The Bode plot is done with *K* = 1.

GM = 21.58 dB For GM = 20 dB,

*K* must be reduced by –1.58 dB.

Thus *K* = 0.8337

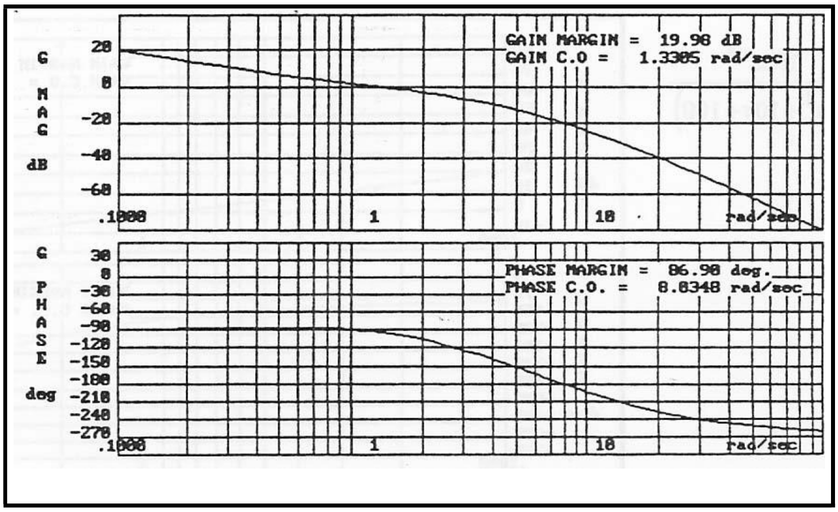
PM = $60.42^o$. For PM = $45^o$

*K* should be increased by 5.6 dB.

Or, *K* = 1.91

**(b)**

$$G(s)=\frac{K(s+1)}{s(1+0.1s)(1+0.2s)(1+0.5s)}$$ The Bode plot is done with *K* = 1.




GM = 19.98 dB. For GM = 20 dB,

$K \cong 1.$

PM = $86.9^o$. For PM = $45^o$

*K* should be increased by 8.9 dB.

Or, *K* = 2.79.

**8-39 (c) See the top plot**

$$G(s) = \frac{K}{(s+3)^3}$$

The Bode plot is done with $K$ = 1.

GM = 46.69 dB

PM = infinity.

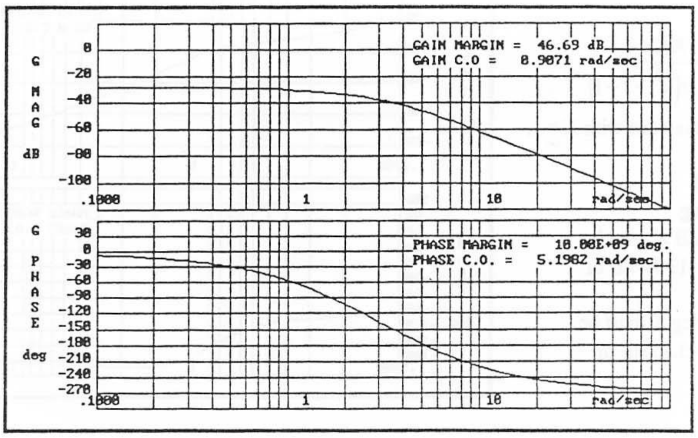
For GM = 20 dB $K$ can be

increased by 26.69 dB or $K$ = 21.6.

For PM = 45 deg. $K$ can be

increased by 28.71 dB, or

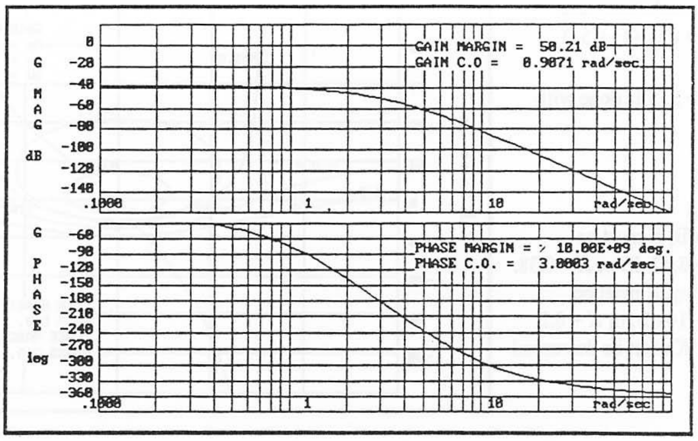
$K$ = 27.26.




**(d) See the middle plot**

$$G(s) = \frac{K}{(s+3)^4}$$

The Bode plot is done with $K$ = 1.




GM = 50.21 dB

PM = infinity.

For GM = 20 dB $K$ can be

increased by 30.21 dB or $K$ = 32.4

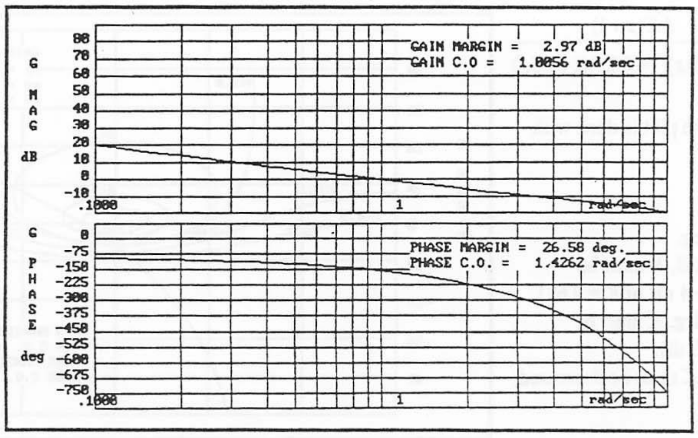
For PM = 45 deg. $K$ can be

increased by 38.24 dB, or

$K$ = 81.66

**(e) See the bottom plot**

The Bode plot is done with $K$ = 1.



$$G(s) = \frac{Ke^{-s}}{s\left(1+0.1s+0.01s^2\right)}$$

GM=2.97 dB; PM = 26.58 deg

For GM = 20 dB *K* must be

decreased by –17.03 dB or

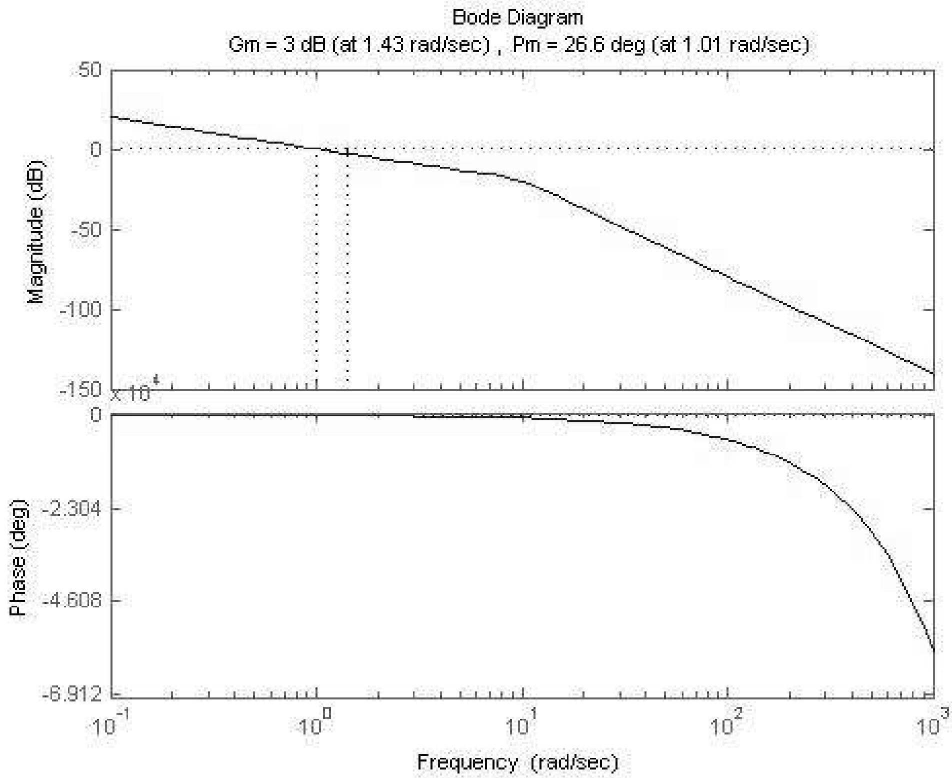
*K* = 0.141.

For PM = 45 deg. *K* must be

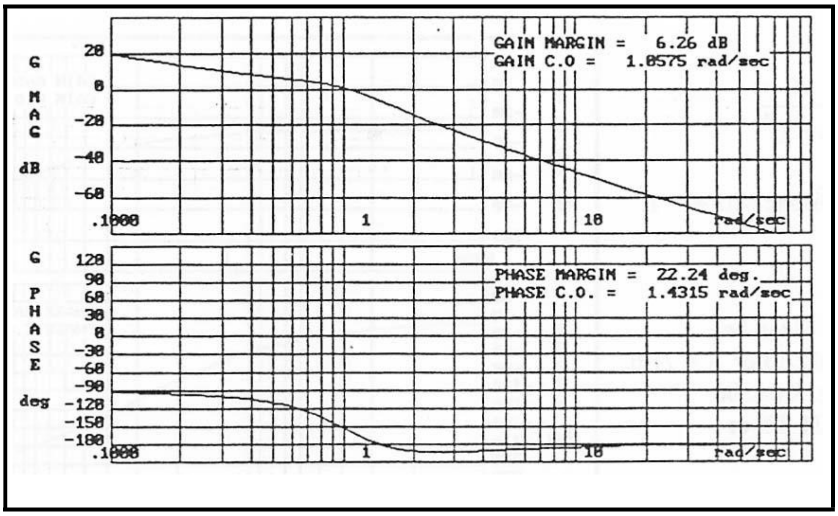
decreased by –2.92 dB or *K* = 0.71.

**MATLAB code:**

```
s = tf('s')
num_G_a= exp(-s);
den_G_a=s*(0.01*s^2+0.1*s+1);
G_a=num_G_a/den_G_a
margin(G_a)
```



## 10-39 (f)




$$G(s) = \frac{K(1+0.5s)}{s\left(s^2+s+1\right)}$$

The Bode plot is done with $K = 1$.

GM = 6.26 dB

PM = 22.24 deg

For GM = 20 dB $K$ must be decreased by –13.74 dB or

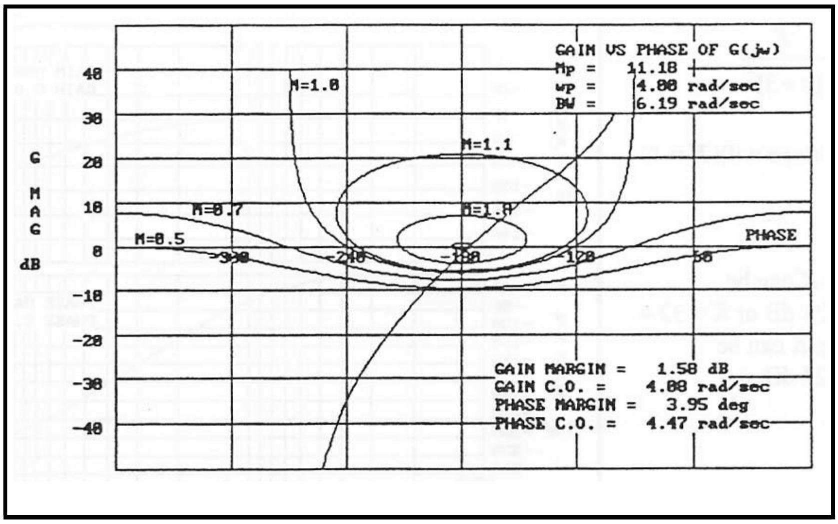
$K = 0.2055$.

For PM = 45 deg $K$ must be decreased by –3.55 dB or

$K = 0.665$.

## 10-40 (a)

$$G(s) = \frac{10K}{s(1+0.1s)(1+0.5s)}$$




The gain-phase plot is done with

$K = 1$.

GM = 1.58 dB

PM = 3.95 deg.

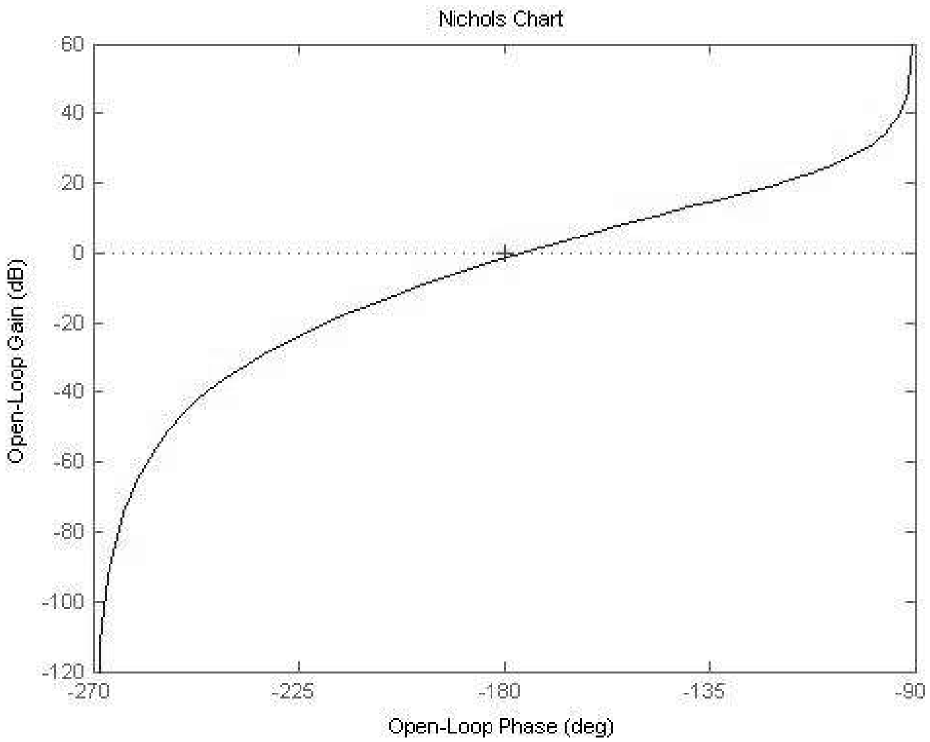
For GM = 10 dB, $K$ must be decreased by –8.42 dB or $K = 0.38$.

For PM = 45 deg, $K$ must be decreased by –14 dB, or $K = 0.2$.

For $M_r = 1.2$, $K$ must be decreased to 0.16.

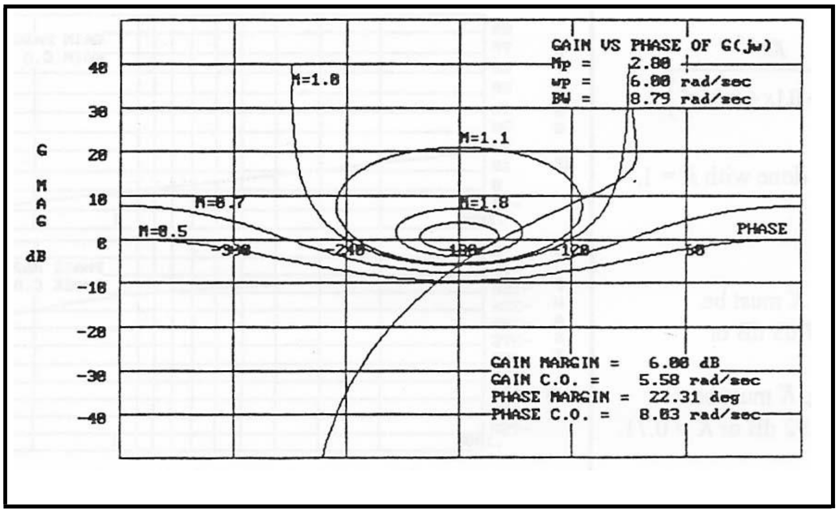
**Sample MATLAB code:**

```
s = tf('s')
num_G_a= 10;
den_G_a=s*(1+0.1*s)*(0.5*s+1);
G_a=num_G_a/den_G_a
nichols(G_a)
```




**(b)**

$$G(s) = \frac{5K(s+1)}{s(1+0.1s)(1+0.2s)(1+0.5s)}$$




The Gain-phase plot is done with

*K* = 1.

GM = 6 dB

PM = 22.31 deg.

For GM = 10 dB, *K* must be decreased by –4 dB or *K* = 0.631.

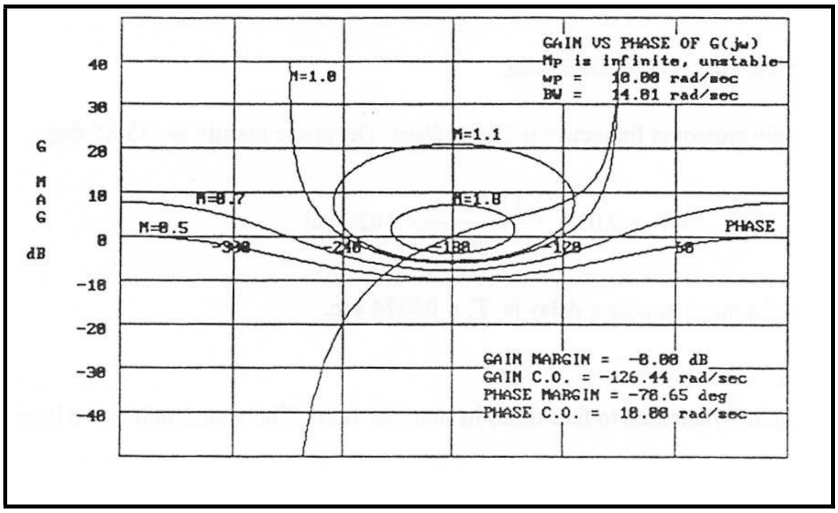
For PM = 45 deg, *K* must be

decrease by –5 dB.

For $M_r = 1.2$, *K* must be decreased to 0.48.

## 10-40 (c)

$$G(s) = \frac{10K}{s\left(1+0.1s+0.01s^2\right)}$$




The gain-phase plot is done for

*K* = 1.

GM = 0 dB $\quad M_r = \infty$

PM = 0 deg

For GM = 10 dB, *K* must be decreased by –10 dB or *K* = 0.316.

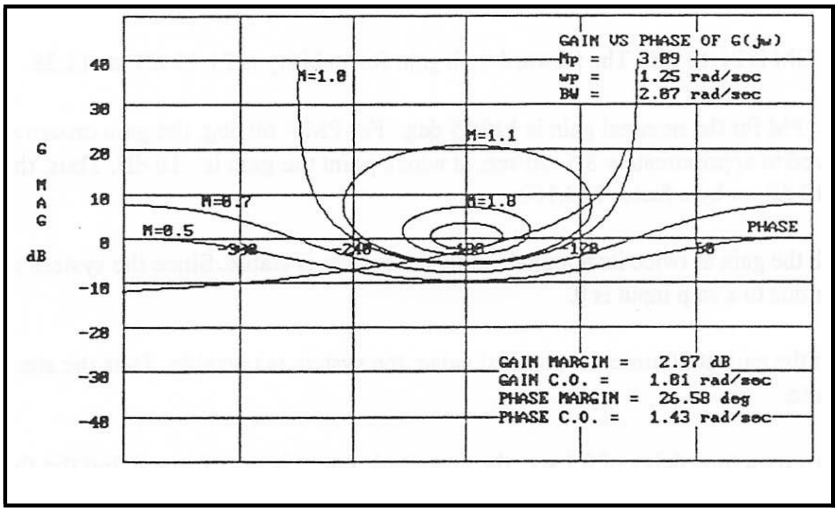
For PM = 45 deg, *K* must be decreased by –5.3 dB, or

*K* = 0.543.

For $M_r = 1.2, K$ must be decreased to 0.2213.

## (d)

$$G(s) = \frac{Ke^{-s}}{s\left(1 + 0.1s + 0.01s^2\right)}$$




The gain-phase plot is done for

$K = 1$.

GM = 2.97 dB $\quad M_r = 3.09$

PM = 26.58 deg

For GM = 10 dB, $K$ must be decreased by –7.03 dB, $K = 0.445$.

For PM = 45 deg, $K$ must be decreased by –2.92 dB, or

$K = 0.71$.

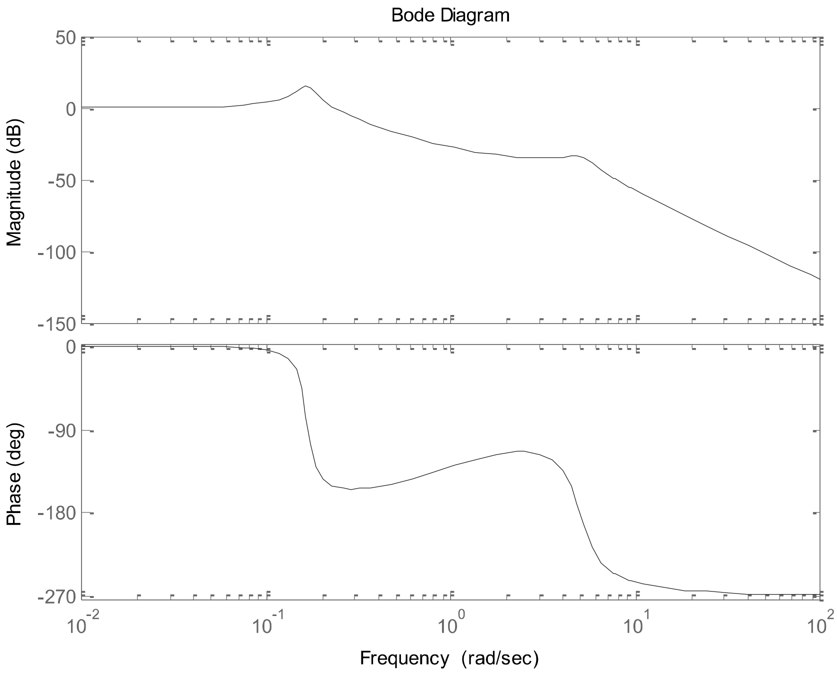
For $M_r = 1.2,\ K = 0.61.$

**10-41**

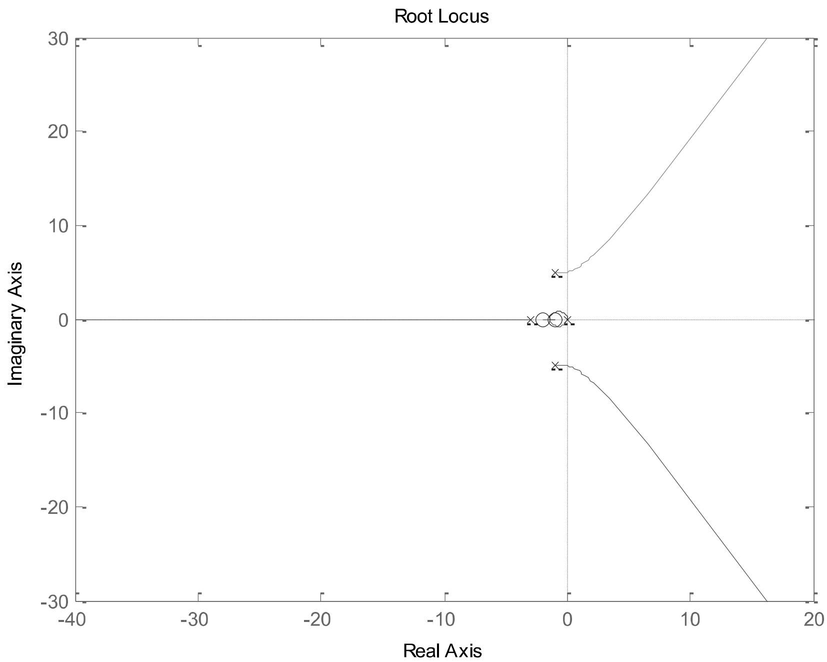
**MATLAB code:**

```
s = tf('s')
%a)
num_GH_a= 1*(s+1)*(s+2);
den_GH_a=s^2*(s+3)*(s^2+2*s+25);
GH_a=num_GH_a/den_GH_a;
CL_a = GH_a/(1+GH_a)
figure(1);
bode(CL_a)

%b)
figure(2);
rlocus(GH_a)

%c)
num_GH_c= 53*(s+1)*(s+2);
den_GH_c=s^2*(s+3)*(s^2+2*s+25);
GH_c=num_GH_c/den_GH_c;
figure(3);
nyquist(GH_c)
xlim([-2 1])
ylim([-1.5 1.5])

%d)
num_GH_d= (s+1)*(s+2);
den_GH_d=s^2*(s+3)*(s^2+2*s+25);
GH_d=num_GH_d/den_GH_d;
CL_d = GH_d/(1+GH_d)
figure(4);
margin(CL_d)

sisotool
```

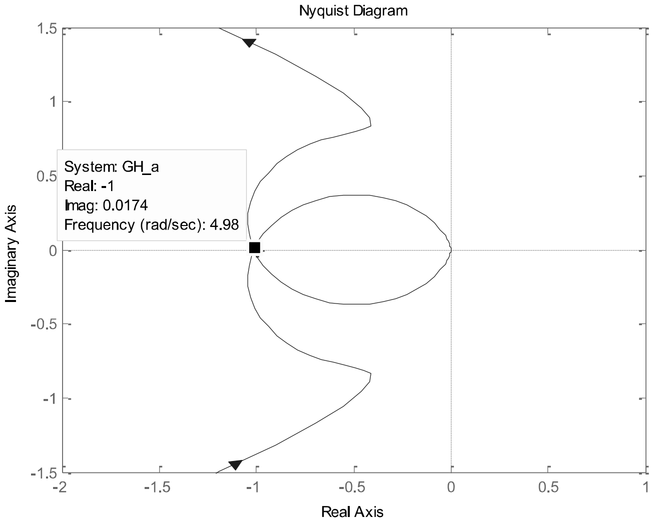
**Part (a), Bode diagram:**




**Part (b), Root locus diagram:**



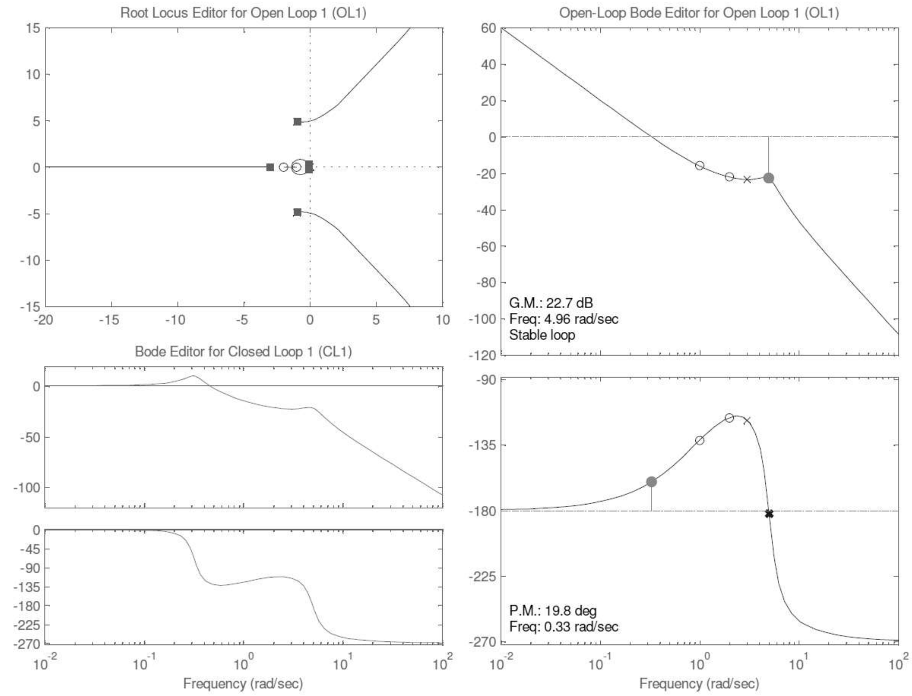

**Part (c), Gain and frequency that instability occurs:** Gain=53, Freq = 4.98 rad/sec, as seen in the data point in the figure:




**Part (d), Gain and frequency that instability occurs:**

Gain=0.127 at PM =20 deg:

By running sisotool command in MATLAB, the transfer functions are imported and the gain is iteratively changed until the phase margin of PM=20 deg is achieved. The corresponding Gain is K= 0.127.




**Part (e):** The corresponding gain margin is GM = 22.7 dB is seen in the figure

**10-42** **(a)** Gain crossover frequency = 2.09 rad/sec PM = 115.85 deg

Phase crossover frequency = 20.31 rad/sec GM = 21.13 dB

**(b)** Gain crossover frequency = 6.63 rad/sec PM = 72.08 deg

Phase crossover frequency = 20.31 rad/sec GM = 15.11 dB

**(c)** Gain crossover frequency = 19.1 rad/sec PM = 4.07 deg

Phase crossover frequency = 20.31 rad/sec GM = 1.13 dB

**(d)** For GM = 40 dB, reduce gain by (40 – 21.13) dB = 18.7 dB, or gain = 0.116 $\times$ nominal value.

**(e)** For PM = 45 deg, the magntude curve reads –10 dB. This means that the loop gain can be increased by 10 dB from the nominal value. Or gain = 3.16 $\times$ nominal value.

**(f)** The system is type 1, since the slope of $|G(j\omega)|$ is –20 dB/decade as $\omega \to 0$.

**(g)** GM = 12.7 dB. PM = 109.85 deg.

**(h)** The gain crossover frequency is 2.09 rad/sec. The phase margin is 115.85 deg.

Set

$$\omega T_d = 2.09 T_d = \frac{115.85^o \pi}{180^o} = 2.022 \text{ rad}$$

Thus, the maximum time delay is $T_d = 0.9674$ sec.

**10-43 (a)** The gain is increased to four times its nominal value. The magnitude curve is raised by 12.04 dB.

Gain crossover frequency = 10 rad/sec PM = 46 deg

Phase crossover frequency = 20.31 rad/sec GM = 9.09 dB

**(b)** The GM that corresponds to the nominal gain is 21.13 dB. To change the GM to 20 dB we need to increase the gain by 1.13 dB, or 1.139 times the nominal gain.

**(c)** The GM is 21.13 dB. The forward-path gain for stability is 21.13 dB, or 11.39.

**(d)** The PM for the nominal gain is 115.85 deg. For PM = 60 deg, the gain crossover frequency must be moved to approximately 8.5 rad/sec, at which point the gain is –10 dB. Thus, the gain must be increased by 10 dB, or by a factor of 3.162.

**(e)** With the gain at twice its nominal value, the system is stable. Since the system is type 1, the steady-state error due to a step input is 0.

**(f)** With the gain at 20 times its nominal value, the system is unstable. Thus the steady-state error would be infinite.

**(g)** With a pure time delay of 0.1 sec, the magnitude curve is not changed, but the the phase curve is subject to a negative phase of $-0.1\omega$ rad. The PM is

$$\text{PM} = 115.85 - 0.1 \times \text{gain crossover frequency} = 115.85 - 0.209 = 115.64 \text{ deg}$$

The new phase crossover frequency is approximately 9 rad/sec, where the original phase curve is reduced by –0.9 rad or –51.5 deg. The magnitude of the gain curve at this frequency is –10 dB. Thus, the gain margin is 10 dB.

**(h)** When the gain is set at 10 times its nominal value, the magnitude curve is raised by 20 dB. The new gain crossover frequency is approximately 17 rad/sec. The phase at this frequency is –30 deg. Thus, setting

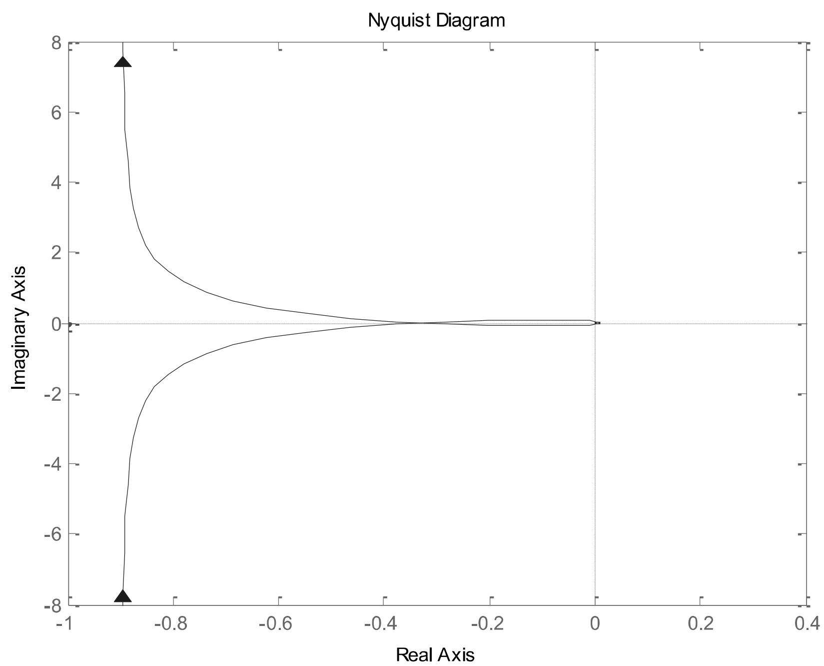
$$\omega T_d = 17 T_d = \frac{30^o \pi}{180^o} = 0.5236 \qquad \text{Thus} \qquad T_d = 0.0308 \text{ sec.}$$

**10-44**

**MATLAB code:**

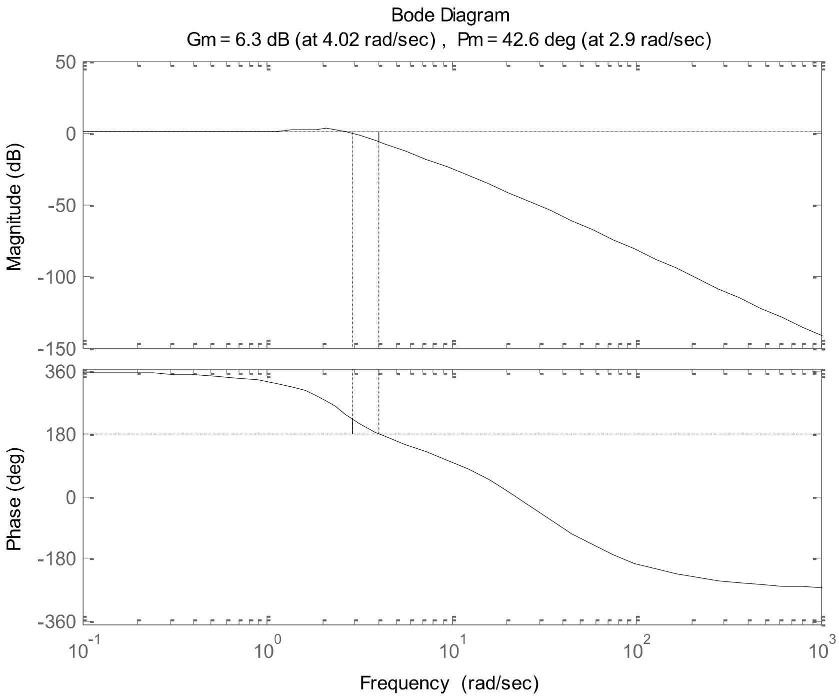
```
s = tf('s')
%using pade cammand for PADE approximation of exponential term
num_G_a= pade((80*exp(-0.1*s)),2);
den_G_a=s*(s+4)*(s+10);
G_a=num_G_a/den_G_a;
CL_a = G_a/(1+G_a)
OL_a = G_a*1;

%(a)
figure(1)
nyquist(OL_a)

%(b) and (c)
figure(2);
margin(CL_a)
```

**Part (a), Nyquist diagram:**



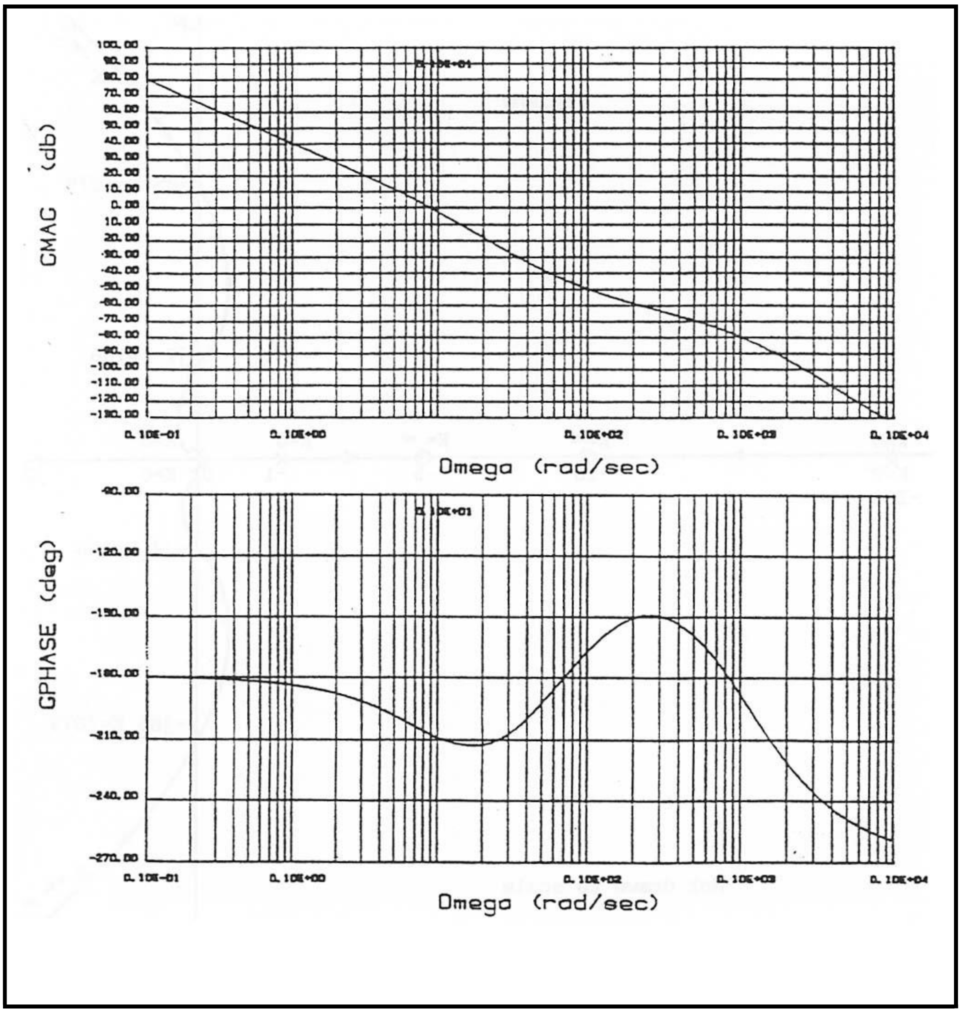

**Part (b) and (c), Bode diagram:**

Using Margin command, the gain and phase margins are obtained as GM = 6.3 dB, PM = 42.6 deg:

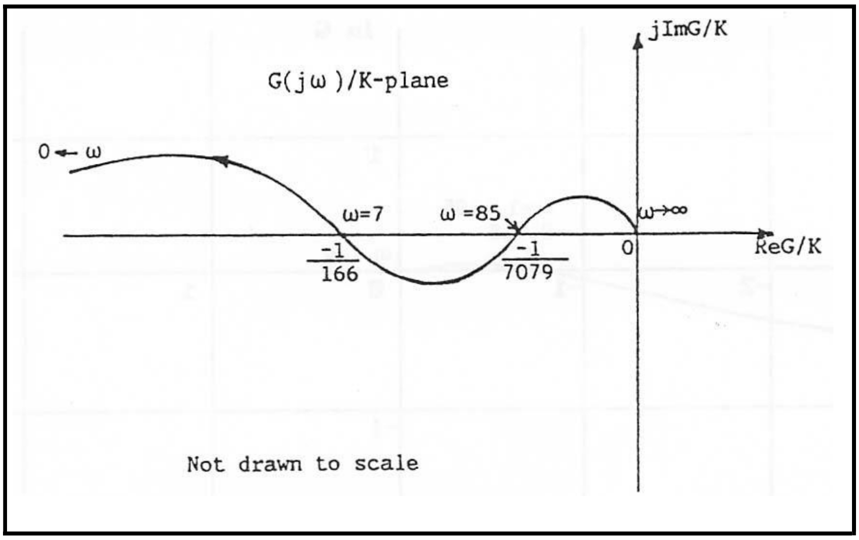

Bode Diagram
Gm = 6.3 dB (at 4.02 rad/sec) , Pm = 42.6 deg (at 2.9 rad/sec)
Magnitude (dB)
50
0
-50
-100
-150
Phase (deg)
360
180
0
-180
-360
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$
Frequency (rad/sec)

**10-45 (a) Bode Plot:**

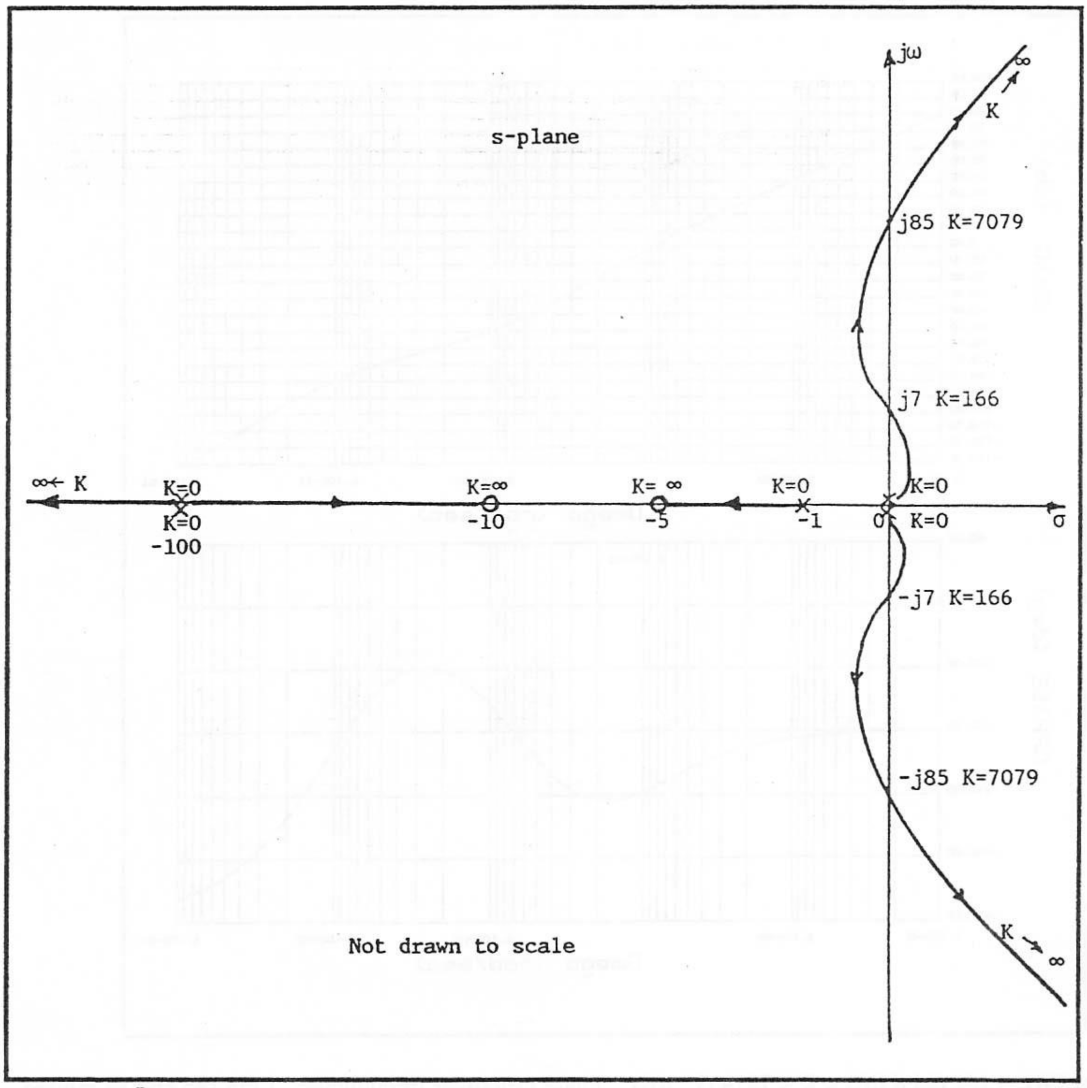
**For stability: 166 (44.4 dB) < *K* < 7079 (77 dB)** Phase crossover frequencies: 7 rad/sec and 85 rad/sec



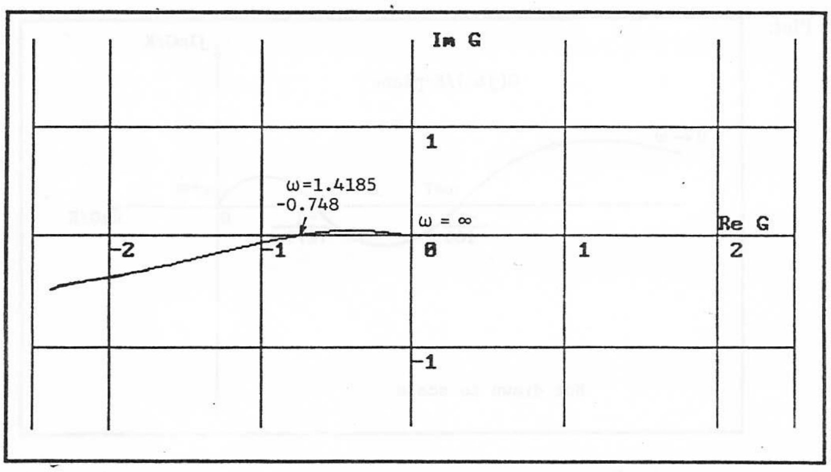
**Nyquist Plot:**



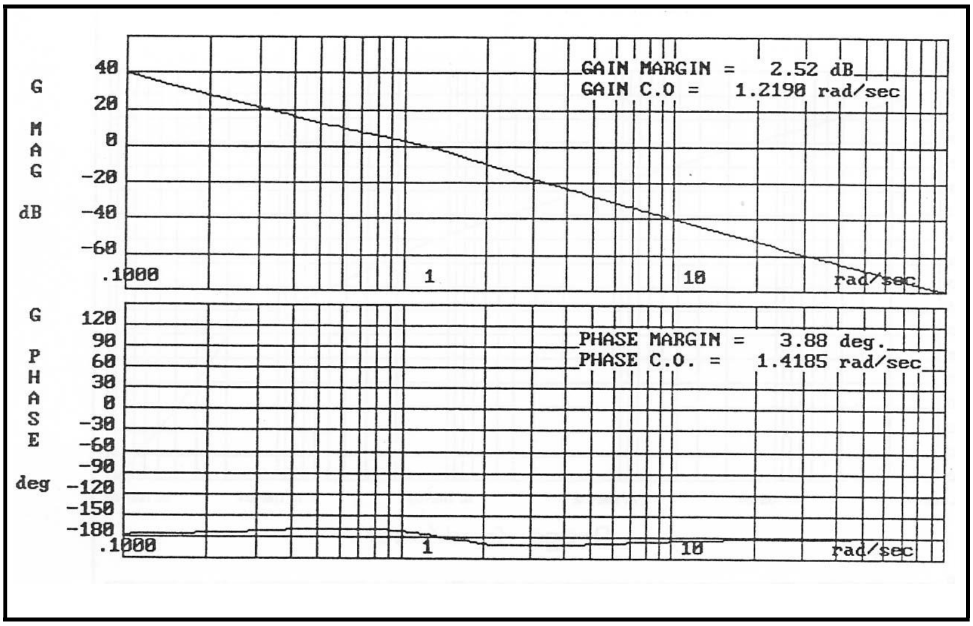
**10-45 (b)** **Root Loci.**



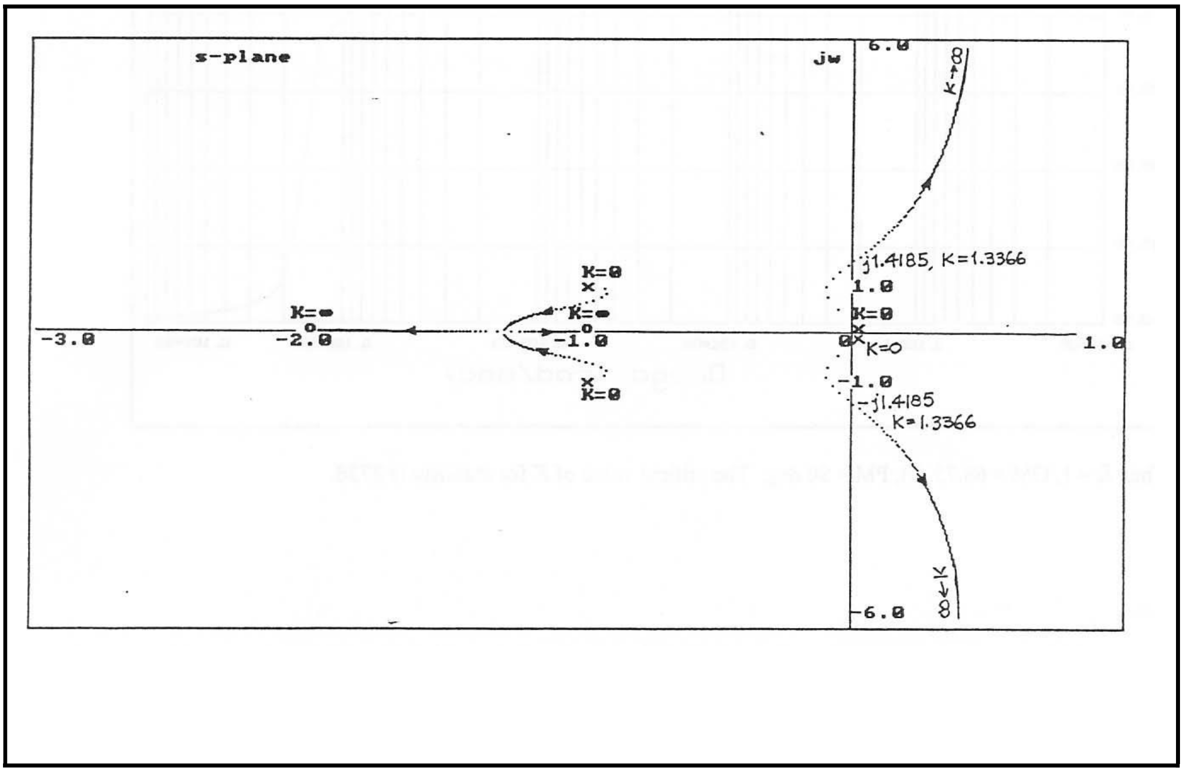
**10-46 (a) Nyquist Plot**




**Bode Plot**



**Root Locus**

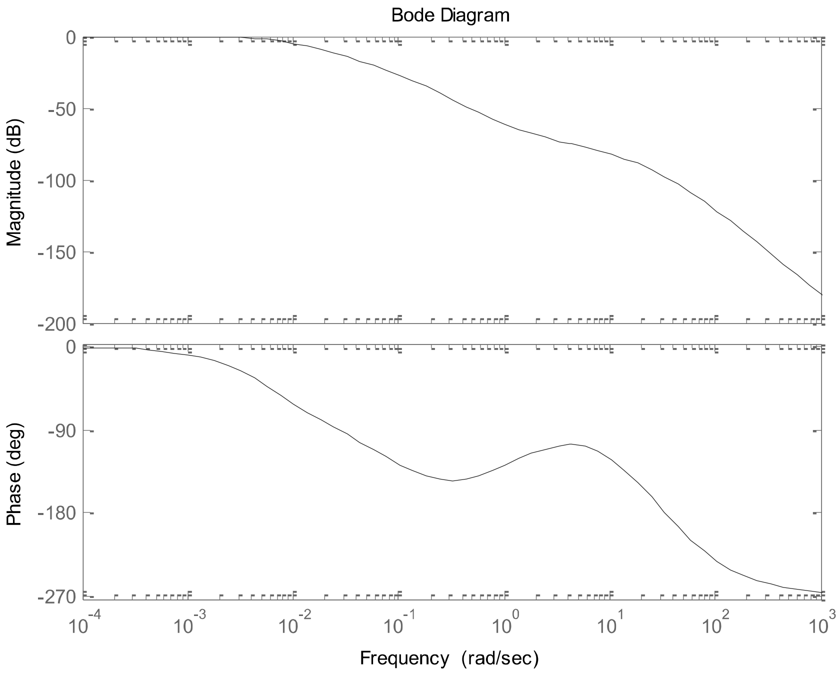

s-plane
jw
6.0
K→∞
j1.4185, K=1.3366
1.0
K=0
K=0
K=∞
K=∞
K=0
-3.0
-2.0
-1.0
0
K=0
1.0
-1.0
K=0
-j1.4185
K=1.3366
K→∞
-6.0

**10-47**

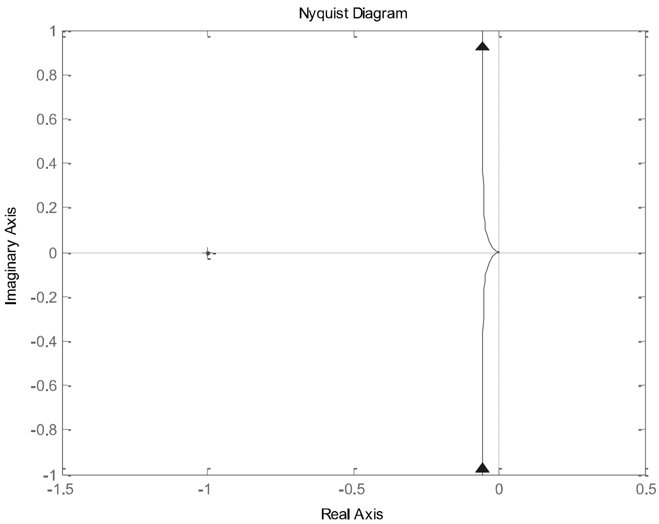
**MATLAB code:**

```
%a)
k=1
num_GH= k*(s+1)*(s+5);
den_GH=s*(s+0.1)*(s+8)*(s+20)*(s+50);
GH=num_GH/den_GH;
CL = GH/(1+GH)
figure(1);
bode(CL)
figure(2);
OL = GH;
nyquist(GH)
xlim([-1.5 0.5]);
ylim([-1 1]);

sisotool
```

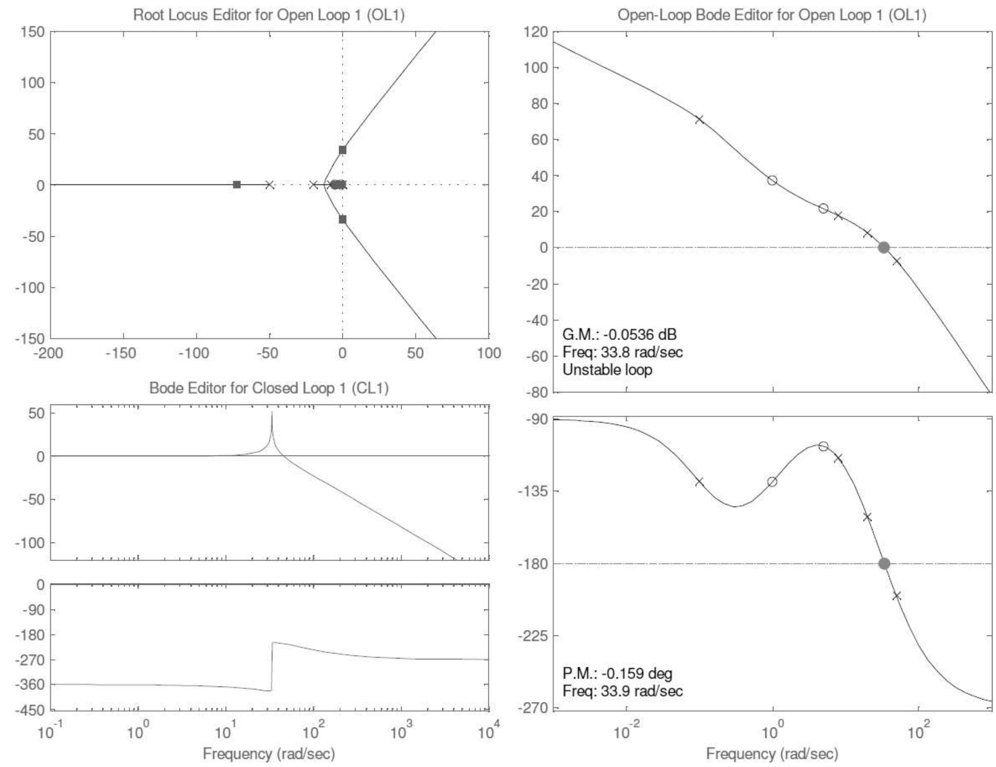
**Part (a), Bode diagram:**




**Part (a), Nyquist diagram:**

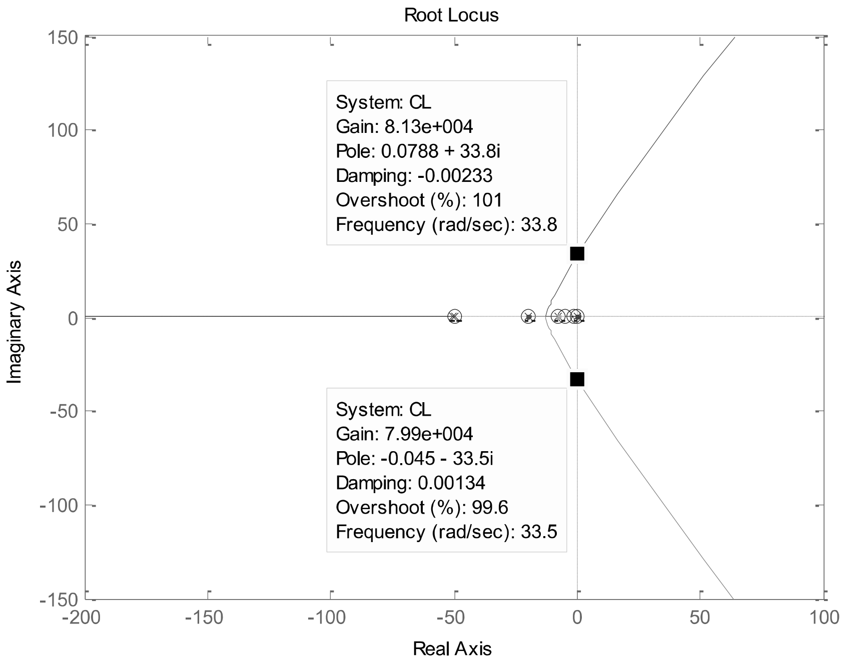



**Part (a), range of K for stability:** By running sisotool command in MATLAB, the transfer functions are imported and the gain is iteratively changed until the phase margin of PM=0 deg is achieved (where K = ) which is the margin of stability. The stable rang for K is K>8.16x$10^4$:

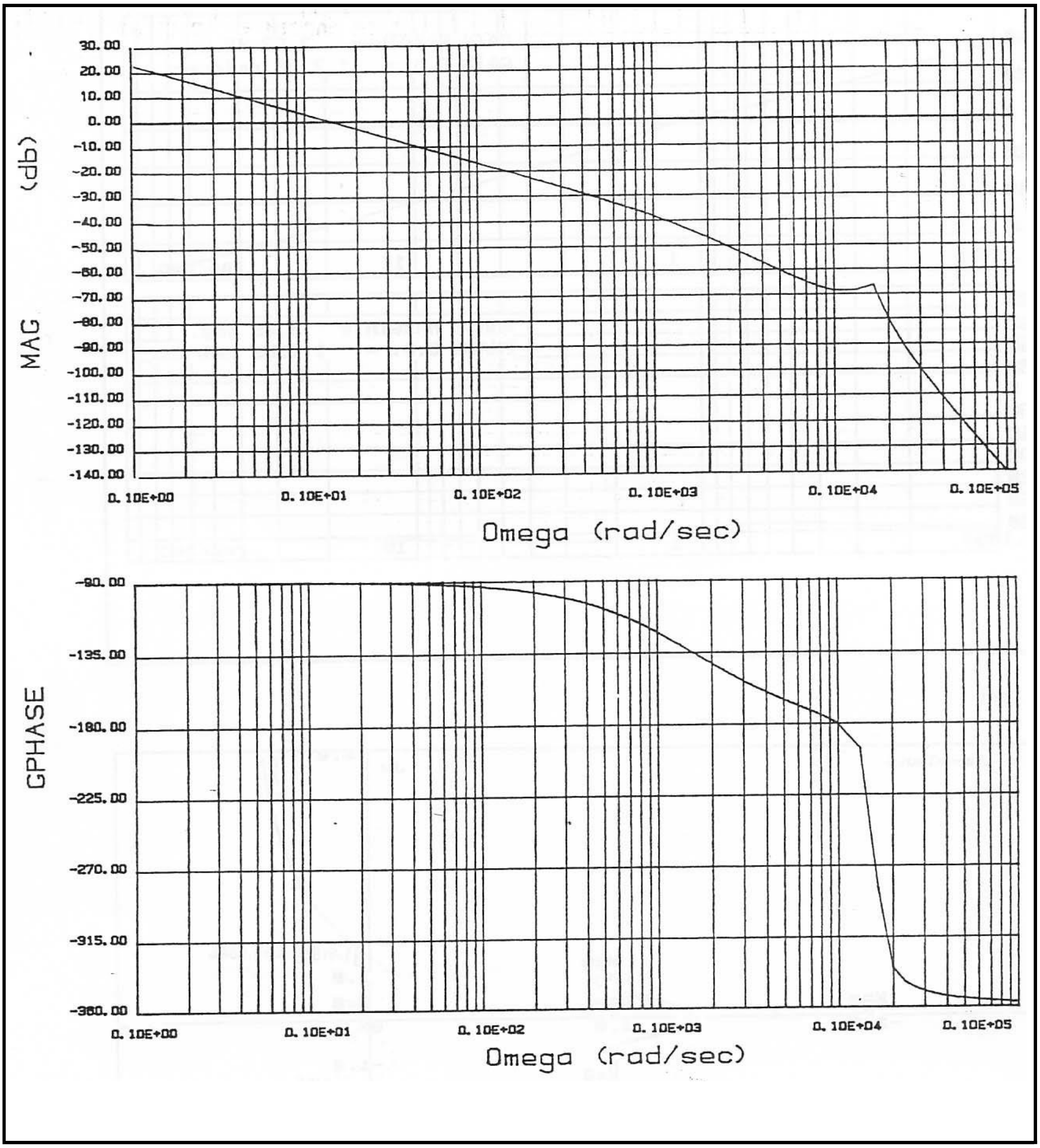



**Part (b), Root-locus diagram,** *K* and $\omega$ at the points where the root loci cross the $j\omega$-axis**:**

As can be seen in the figure at K=8.13x$10^4$ and $\omega$=33.8 rad/sec, the poles cross the j$\omega$ axis. Both of these values are consistent with the results of part(a) from sisotool.


Root Locus
System: CL
Gain: 8.13e+004
Pole: 0.0788 + 33.8i
Damping: -0.00233
Overshoot (%): 101
Frequency (rad/sec): 33.8
System: CL
Gain: 7.99e+004
Pole: -0.045 - 33.5i
Damping: 0.00134
Overshoot (%): 99.6
Frequency (rad/sec): 33.5
Imaginary Axis
Real Axis
150
100
50
0
-50
-100
-150
-200
-150
-100
-50
0
50
100

**10-48** **Bode Diagram**




When $K = 1$, GM = 68.75 dB, PM = 90 deg. The critical value of $K$ for stability is 2738.

**10-49 (a) Forward-path transfer function:**

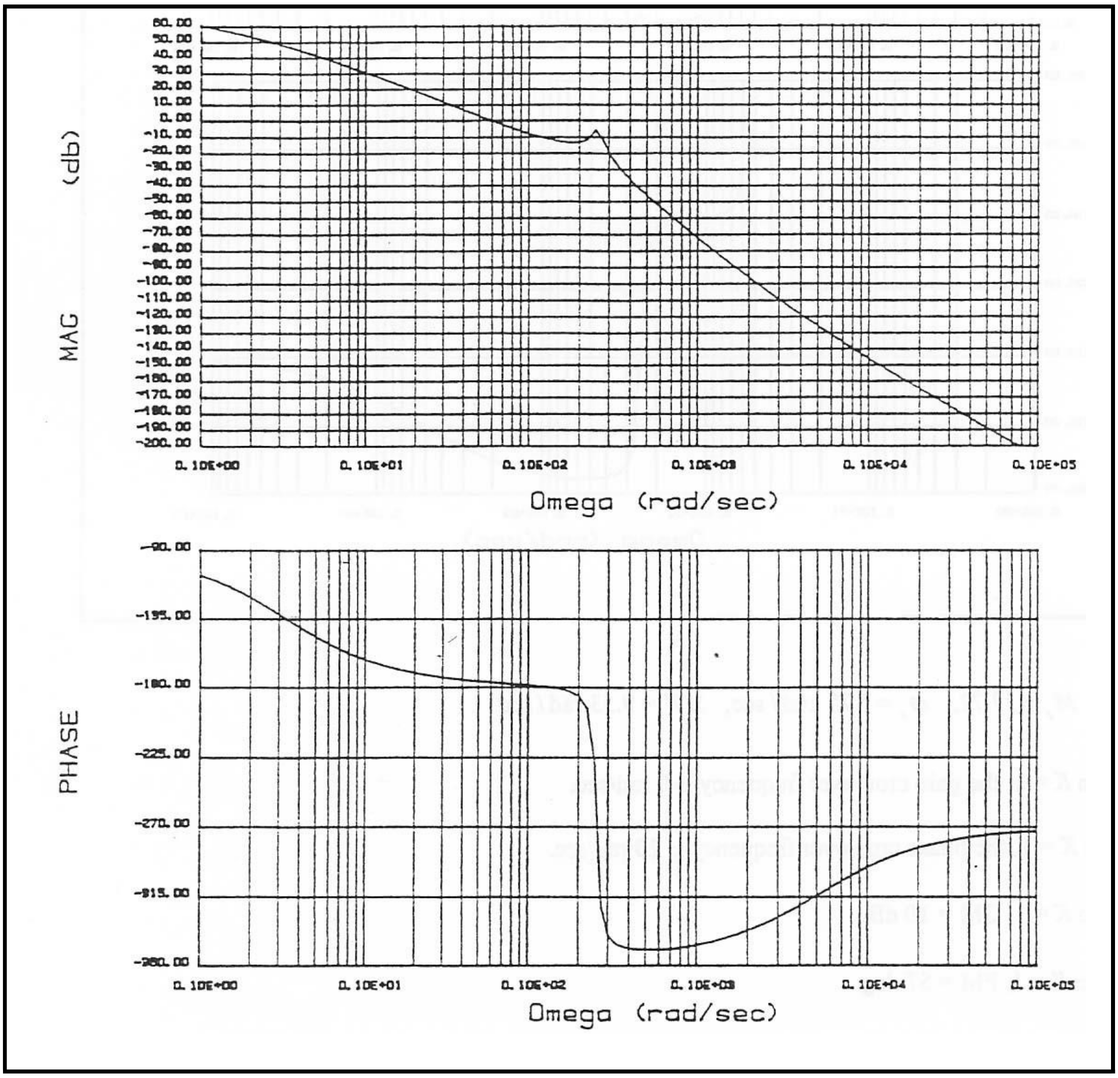
$$G(s) = \frac{\Theta_L(s)}{E(s)} = K_a G_p(s) = \frac{K_a K_i (Bs + K)}{\Delta_o}$$

where

$$\Delta_o = 0.12s\left(s + 0.0325\right)\left(s^2 + 2.5675s + 6667\right)$$
$$= s\left(0.12s^3 + 0.312s^2 + 80.05s + 26\right)$$

$$G(s) = \frac{43.33(s + 500)}{s\left(s^3 + 2.6s^2 + 667.12s + 216.67\right)}$$

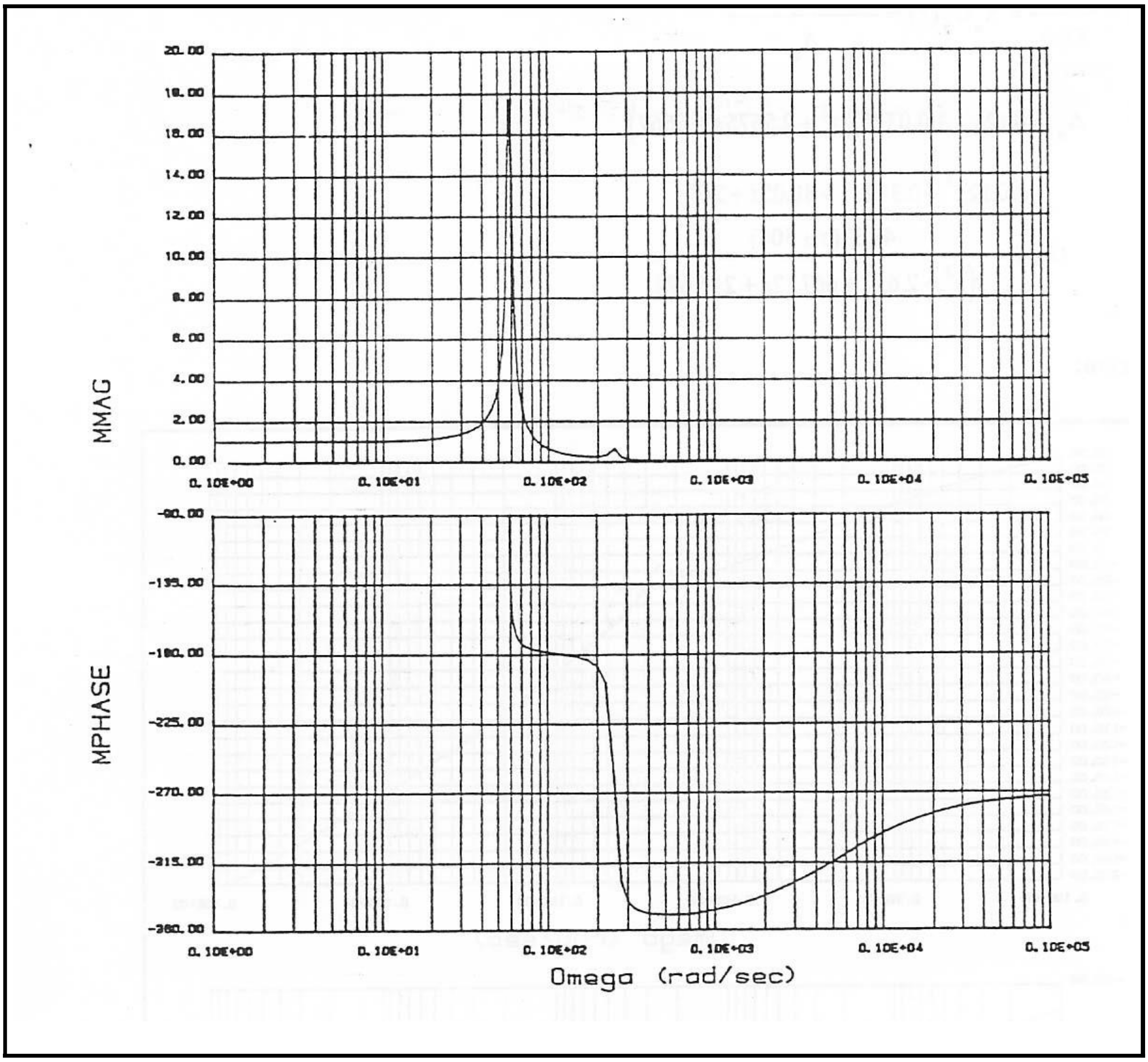
**(b) Bode Diagram:**




Gain crossover frequency = 5.85 rad/sec PM = 2.65 deg.

Phase crossover frequency = 11.81 rad/sec GM = 10.51 dB

**10-49 (c) Closed-loop Frequency Response:**




$M_r = 17.72, \quad \omega_r = 5.75 \text{ rad/sec}, \quad \text{BW} = 9.53 \text{ rad/sec}$

**10-50**

**(a)** $G(s)H(s) = \frac{Kmgd}{L\left(\frac{J}{r^2}+m\right)s^2}$

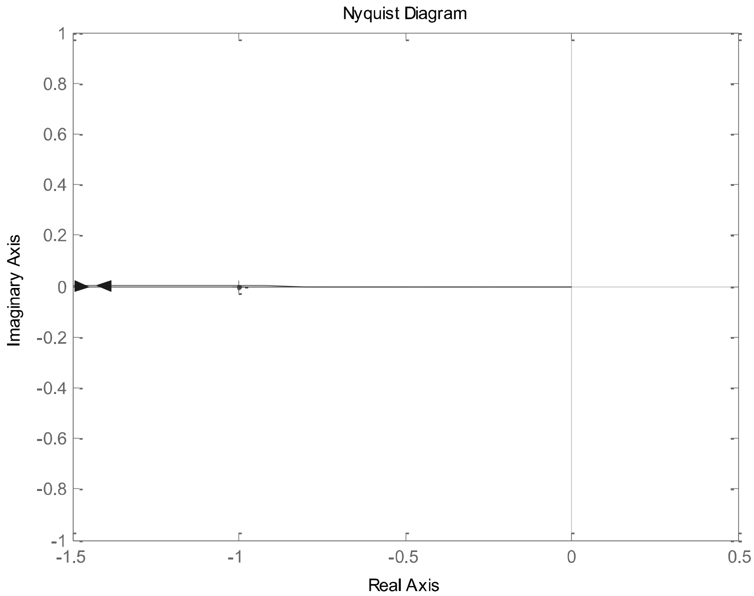
**(b)** $\frac{G(s)H(s)}{1+G(s)H(s)} = \frac{Kmgd}{L\left(\frac{J}{r^2}+m\right)s^2+Kmgd}$

**(c) to (e)**

**MATLAB code:**

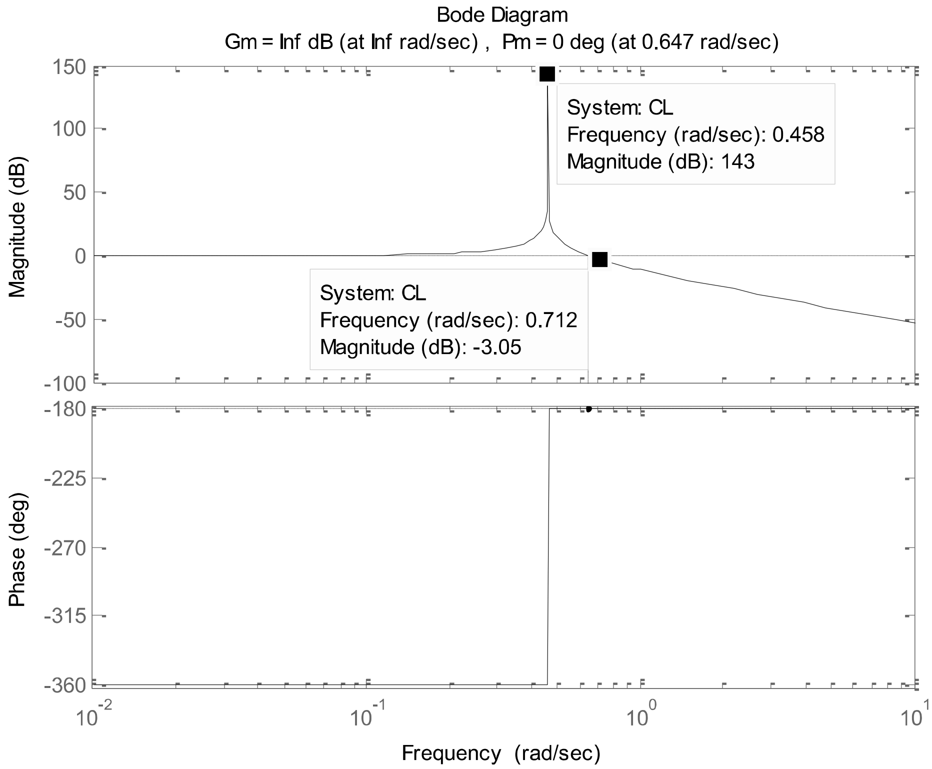
```
s = tf('s')

m = 0.11;
r = 0.015;
d = 0.03;
g = 9.8;
L = 1.0;
J = 9.99*10^-6

K=1
num_GH= K*m*g*d;
den_GH=L*(J/r^2+m)*s^2;
GH=num_GH/den_GH;
CL = GH/(1+GH)

%c)
figure(1);
nyquist(GH)
xlim([-1.5 0.5]);
ylim([-1 1]);

%d)
figure(2);
margin(CL)
```

**Part (c):** since the system is a double integrator ($1/s^2$), the phase is always -180 deg, and the system is always marginally stable for **any K**, leading to a complicated control problem.




**Part (d),** Bode diagram:

As explained in section (c), since the system is always marginally stable, GM=inf and PM = 0, as can be seen by MATLAB MARGIN command, resulting in the following figure:




**Part (e),** $M_r$,= 143 dB, $\omega_r$ = 0.458 rad/sec, and BW= 0.712 rad/sec as can be seen in the data points in the above figure.

**10-51 (a)** When $K$ = 1, the gain crossover frequency is 8 rad/sec.

**(b)** When $K$ = 1, the phase crossover frequency is 20 rad/sec.

**(c)** When $K$ = 1, GM = 10 dB.

**(d)** When $K$ = 1, PM = 57 deg.

**(e)** When $K$ = 1, $M_r = 1.2$.

**(f)** When $K$ = 1, $\omega_r = 3$ rad/sec.

**(g)** When $K$ = 1, BW = 15 rad/sec.

**(h)** When $K$ = –10 dB (0.316), GM = 20 dB

**(i)** When $K$ = 10 dB (3.16), the system is marginally stable. The frequency of oscillation is 20 rad/sec.

**(j)** The system is type 1, since the gain-phase plot of $G(j\omega)$ approaches infinity at –90 deg. Thus, the steady-state error due to a unit-step input is zero.

**10-52** When $K$ = 5 dB, the gain-phase plot of $G(j\omega)$ is raised by 5 dB.

**(a)** The gain crossover frequency is ~10 rad/sec.

**(b)** The phase crossover frequency is ~20 rad/sec.

**(c)** GM = 5 dB.

**(d)** PM = ~34.5 deg.

**(e)** When $K$ = 5, $M_r$= ~ 2 (smallest circle tangent to an ***M*** circle).

**(f)** $\omega_r = 15$ rad/sec

**(g)** BW = 30 rad/sec

**(h)** When $K$ = –30 dB, the GM is 40 dB (shift the graph of $K$=1, 30 dbs down).

When ***K* = 10** dB, the gain-phase plot of $G(j\omega)$ is raised by 10 dB.

**(a)** The gain crossover frequency is 20 rad/sec.

**(b)** The phase crossover frequency is 20 rad/sec.

**(c)** GM = 0 dB.

**(d)** PM = 0 deg.

**(e)** When $K$ = 10, $M_r$= ~ 1.1 (smallest circle tangent to an ***M*** circle).

**(f)** $\omega_r = 5$ rad/sec

**(g)** BW = ~40 rad/sec

**(h)** When $K$ = –30 dB, the GM is 40 dB (shift the graph of $K$=1, 30 dbs down).

**10-53**

Since the function has exponential term, PADE command has been used to obtain the transfer function.

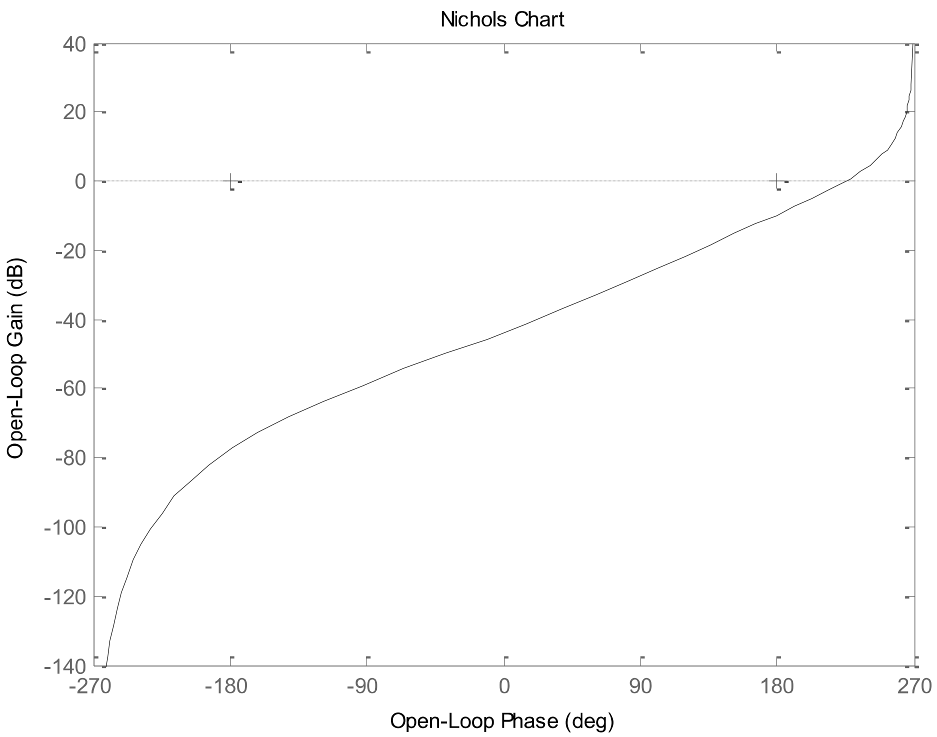
$$G(s)H(s) = \frac{80e^{-0.1s}}{s(s+4)(s+10)}$$

**MATLAB code:**

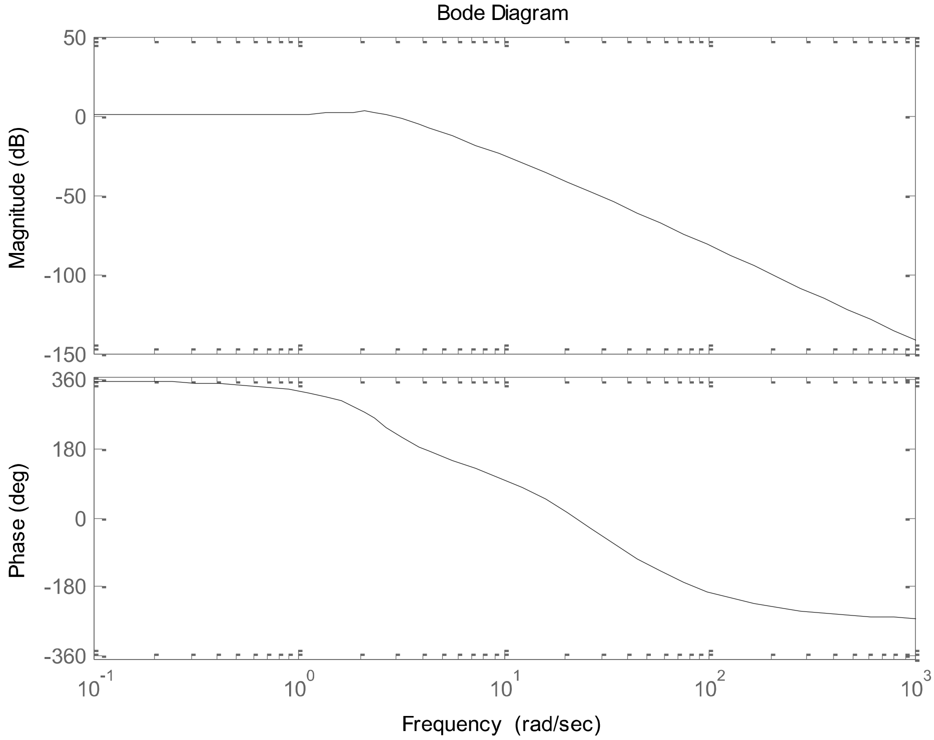
```
s = tf('s')
%a)
```

```
num_GH= pade(80*exp(-0.1*s),2);
den_GH=s*(s+4)*(s+10);
GH=num_GH/den_GH;
CL = GH/(1+GH)
BW = bandwidth(CL)
bode(CL)

%b)
figure(2);
nichols(GH)
```

**Part(a), Nicholas diagram:**




**Part(b), Bode diagram:**




**10- 54) Note:** $G_{CL} = \dfrac{G}{1+G}$

To draw the Bode and polar plots use the closed loop transfer function, $G_{CL}$, and find BW. Use $G$ to obtain the gain-phase plots and $Gm$ and $Pm$. Use the Bode plot to graphically obtain $M_r$.

**Sample MATLAB code:**

```
s = tf('s')
%a)
num_G= 1+0.1*s;
den_G=s*(s+1)*(0.01*s+1);
G=num_G/den_G
figure(1)
nyquist(G)
figure(2)
margin(G)
GCL = G/(1+G)
BW = bandwidth(GCL)
figure(3)
bode(GCL)
```

Transfer function:

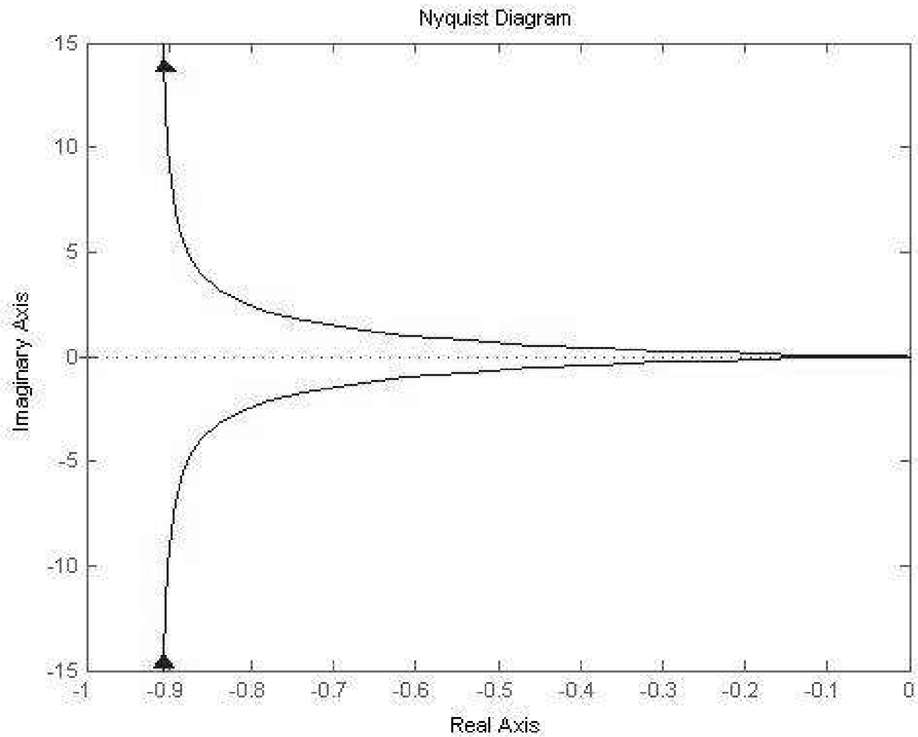
```
      0.1 s + 1
-----------------------
0.01 s^3 + 1.01 s^2 + s
```

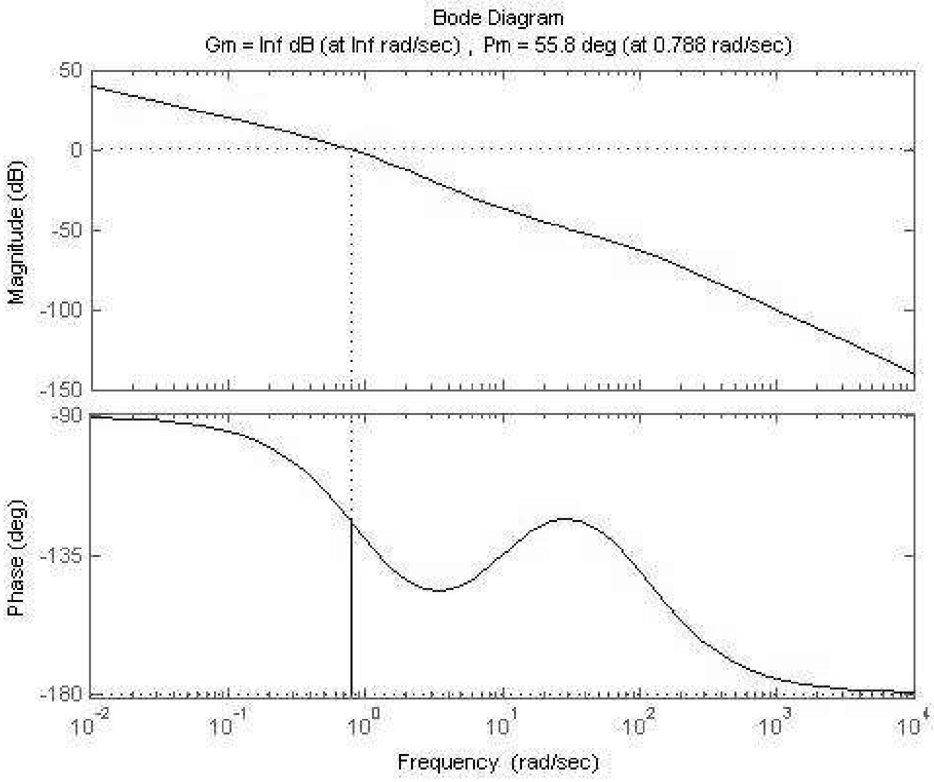
Transfer function:

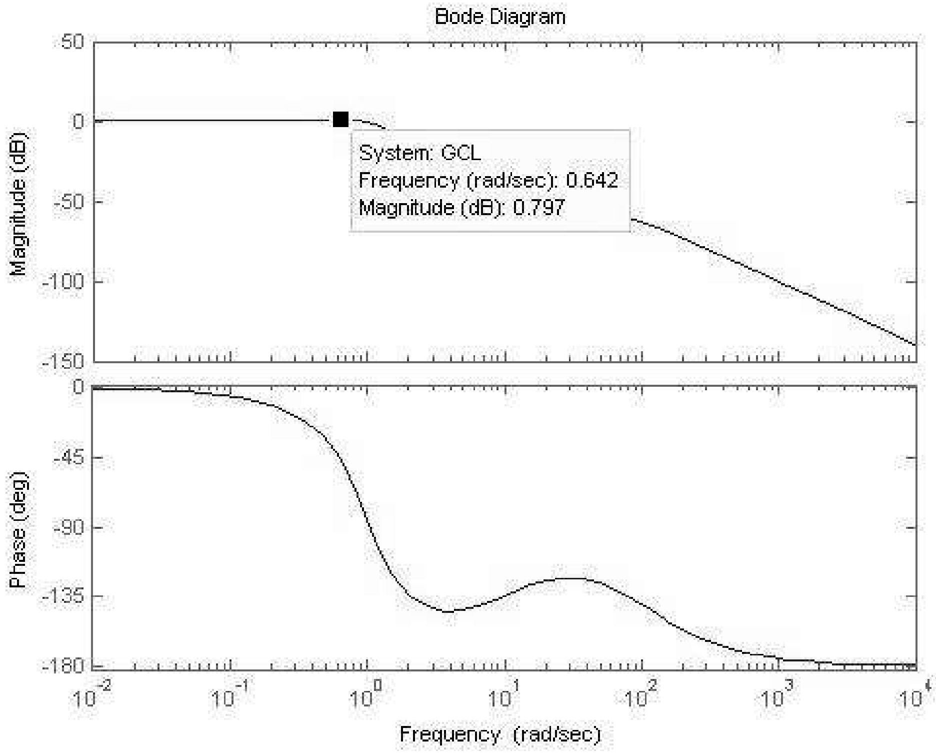
```
           0.001 s^4 + 0.111 s^3 + 1.11 s^2 + s
---------------------------------------------------------------
0.0001 s^6 + 0.0202 s^5 + 1.041 s^4 + 2.131 s^3 + 2.11 s^2 + s
```

BW =

1.2235


Nyquist Diagram
Imaginary Axis
Real Axis
15
10
5
0
-5
-10
-15
-1
-0.9
-0.8
-0.7
-0.6
-0.5
-0.4
-0.3
-0.2
-0.1
0


Bode Diagram
Gm = Inf dB (at Inf rad/sec) , Pm = 55.8 deg (at 0.788 rad/sec)
Magnitude (dB)
50
0
-50
-100
-150
Phase (deg)
-90
-135
-180
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
$10^{3}$
$10^{4}$
Frequency (rad/sec)




**10-55) (a)** The phase margin with $K = 1$ and $T_d = 0$ sec is approximately 57 deg. For a PM of 40 deg, the time delay produces a phase lag of –17 deg. The gain crossover frequency is 8 rad/sec.

Thus,

$$\omega T_d = 17^o = \frac{17^o \pi}{180^o} = 0.2967 \text{ rad / sec} \qquad \text{Thus} \quad \omega = 8 \text{ rad / sec}$$

$$T_d = \frac{0.2967}{8} = 0.0371 \text{ sec}$$

**(b)** With $K = 1$, for marginal stability, the time delay must produce a phase lag of –57 deg.

Thus, at $\omega = 8$ rad/sec,

$$\omega T_d = 57^o = \frac{57^o \pi}{180^o} = 0.9948 \text{ rad} \qquad T_d = \frac{0.9948}{8} = 0.1244 \text{ sec}$$

**10-56 (a)** The phase margin with *K* = 5 dB and $T_d = 0$ is approximately 34.5 deg. For a PM of 30 deg, the time delay must produce a phase lag of –4.5 deg. The gain crossover frequency is 10 rad/sec. Thus,

$$\omega T_d = 4.5^o = \frac{4.5^o \pi}{180^o} = 0.0785 \text{ rad} \qquad \text{Thus} \qquad T_d = \frac{0.0785}{10} = 0.00785 \text{ sec}$$

**(b)** With *K* = 5 dB, for marginal stability, the time delay must produce a phase lag of –34.5 deg.

Thus at $\omega = 10$ rad/sec,

$$\omega T_d = 34.5^o = \frac{34.5^o \pi}{180^o} = 0.602 \text{ rad} \qquad \text{Thus} \qquad T_d = \frac{0.602}{10} = 0.0602 \text{ sec}$$

**10-57)** For a GM of 5 dB, the time delay must produce a phase lag of –34.5 deg at $\omega = 10$ rad/sec. Thus,

$$\omega T_d = 34.5^o = \frac{34.5^o \pi}{180^o} = 0.602 \text{ rad} \qquad \text{Thus} \qquad T_d = \frac{0.602}{10} = 0.0602 \text{ sec}$$

**10-58 (a) Forward-path Transfer Function:**

$$G(s) = \frac{Y(s)}{E(s)} = \frac{e^{-2s}}{(1+10s)(1+25s)}$$

From the Bode diagram, phase crossover frequency = 0.21 rad/sec GM = 21.55 dB gain crossover frequency = 0 rad/sec PM = infinite

**(b)**

$$G(s) = \frac{1}{(1+10s)(1+25s)(1+2s+2s^2)}$$

From the Bode diagram, phase crossover frequency = 0.26 rad/sec GM = 25 dB

gain crossover frequency = 0 rad/sec PM = infinite

**(c)**

$$G(s) = \frac{1-s}{(1+s)(1+10s)(1+2s)}$$

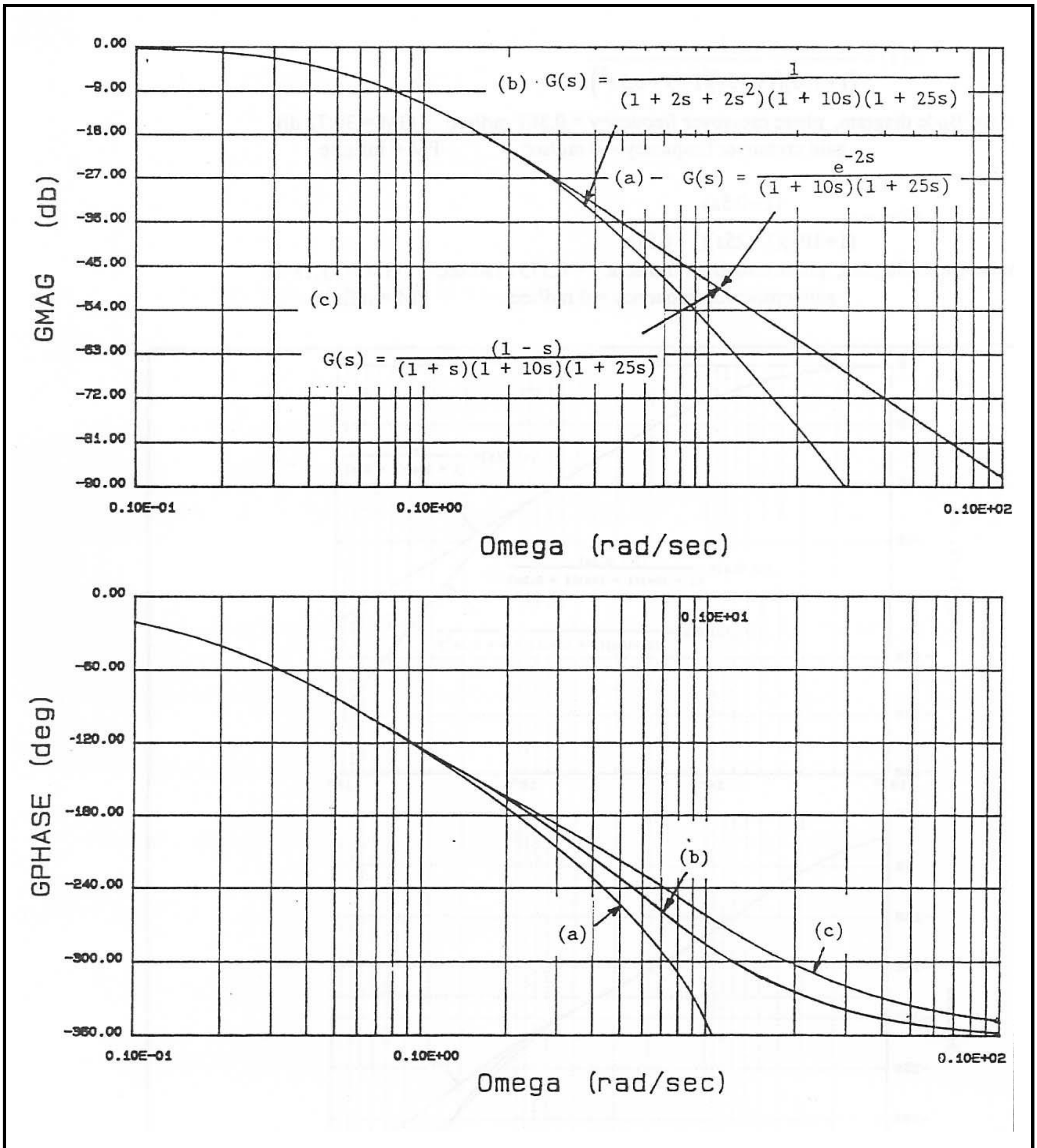
From the Bode diagram, phase crossover frequency = 0.26 rad/sec GM = 25.44 dB

gain crossover frequency = 0 rad/sec PM = infinite

**Sample MATLAB code**

```
s = tf('s')
%a)
num_G=exp(-2*s);
den_G=(10*s+1)*(25*s+1);
G=num_G/den_G
figure(1)
margin(G)
```

**10-58 (continued)** **Bode diagrams for all three parts.**



**10-59 (a)** **Forward-path Transfer Function:**

$$G(s) = \frac{e^{-s}}{(1+10s)(1+25s)}$$

From the Bode diagram, phase crossover frequency = 0.37 rad/sec GM = 31.08 dB

gain crossover frequency = 0 rad/sec PM = infinite

**(b)**

$$G(s) = \frac{1}{(1+10s)(1+25s)(1+s+0.5s^2)}$$

From the Bode diagram, phase crossover frequency = 0.367 rad/sec GM = 30.72 dB

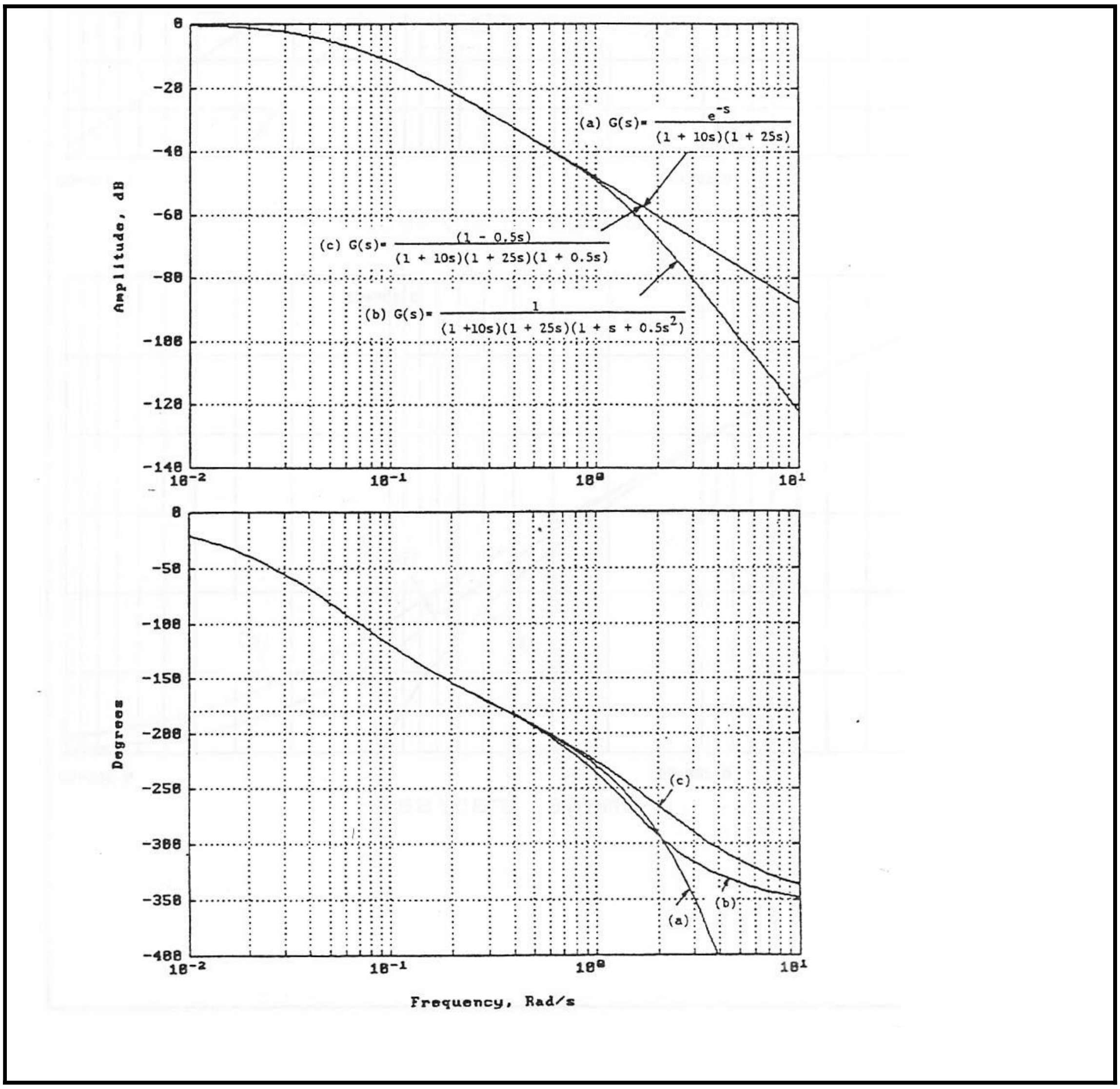
gain crossover frequency = 0 rad/sec PM = infinite

**(c)**

$$G(s) = \frac{(1-0.5s)}{(1+10s)(1+25s)(1+0.5s)}$$

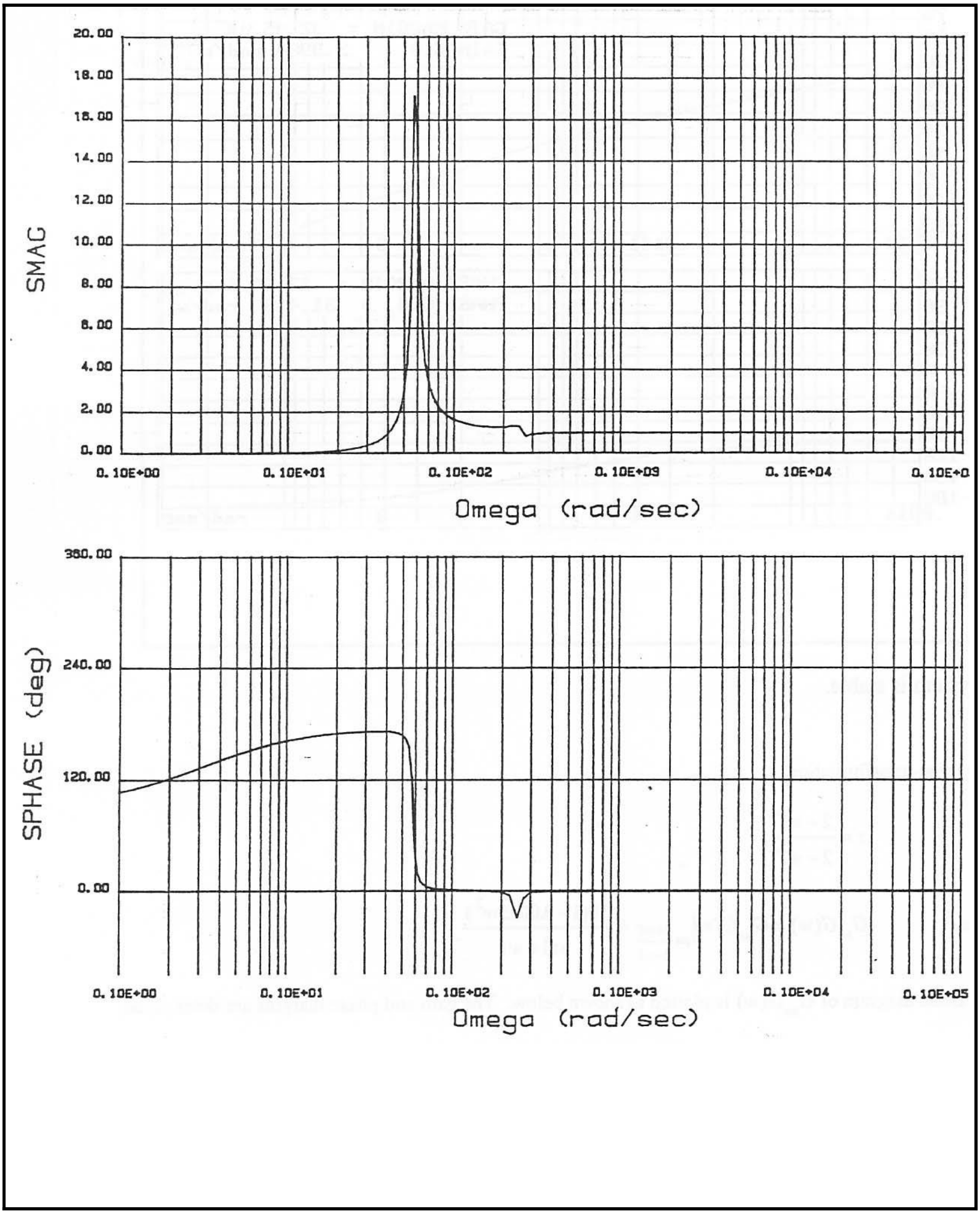
From the Bode diagram, phase crossover frequency = 0.3731 rad/sec GM = 31.18 dB

gain crossover frequency = 0 rad/sec PM = infinite




**Plots 10-59 (a-c)**

**10-60** **Sensitivity Plot:**



$$\left|S_G^M\right|_{\max} = 17.15 \qquad \omega_{\max} = 5.75 \text{ rad/sec}$$

**10-61)**

**(a)** $G(s)H(s) = \frac{K(1.151s+0.1774)}{s^3+0.739s^2+0.921s}$

**(b)** $\frac{G(s)H(s)}{1+G(s)H(s)} = \frac{K(1.151s+0.1774)}{s^3+0.739s^2+(0.921+1.151K)s+0.1774K}$

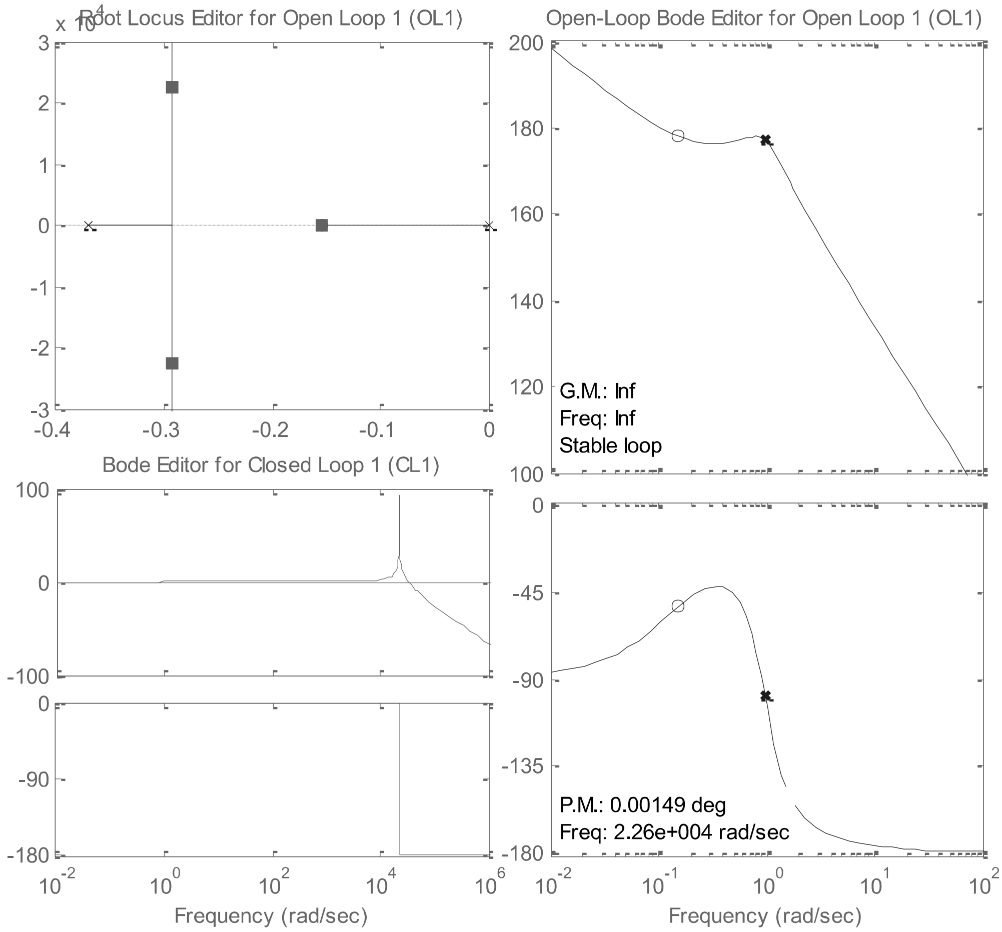
**(c)&(d)**

**MATLAB code:**

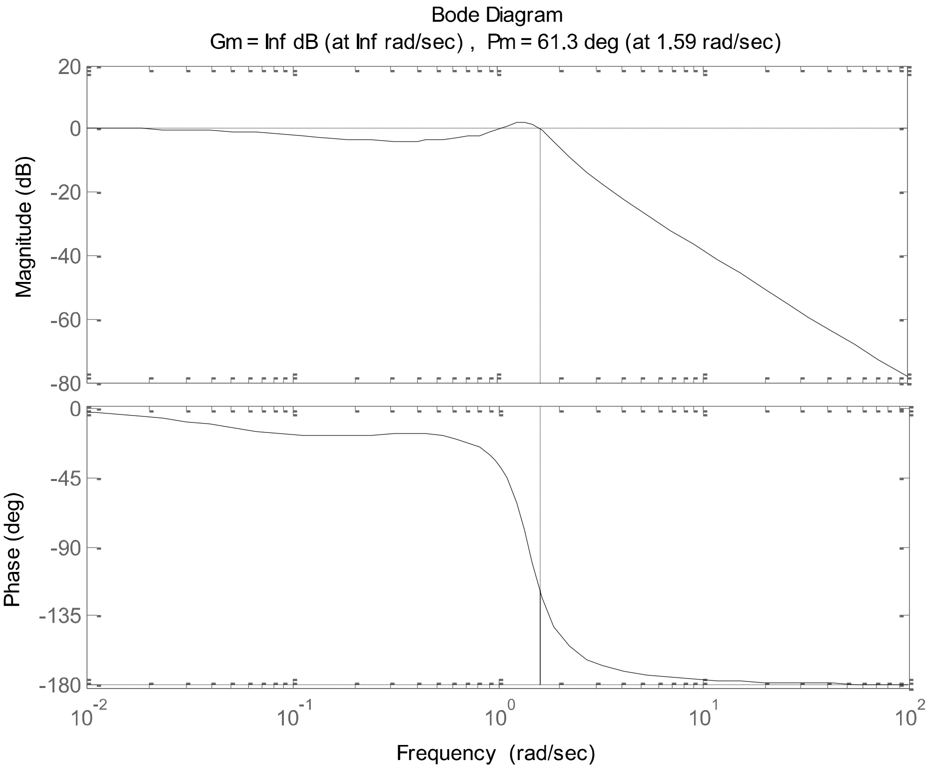
```
s = tf('s')
%c)
K = 1
num_GH= K*(1.151*s+0.1774);
den_GH=(s^3+0.739*s^2+0.921*s);
GH=num_GH/den_GH;
CL = GH/(1+GH)
sisotool
%(d)
figure(1)
margin(CL)
```

**Part (c), range of K for stability:**

Sisotool Result shows that by changing K between 0 and inf., all the roots of closed loop system remain in the left hand side plane and PM remains positive. Therefore, the system is stable for all positive K.




**Part (d), Bode, GM & PM for K=1:**


Bode Diagram
Gm = Inf dB (at Inf rad/sec) , Pm = 61.3 deg (at 1.59 rad/sec)
Magnitude (dB)
20
0
-20
-40
-60
-80
Phase (deg)
0
-45
-90
-135
-180
$10^{-2}$
$10^{-1}$
$10^{0}$
$10^{1}$
$10^{2}$
Frequency (rad/sec)