Estimation of Cosmological Parameters in ΛCDM- and DGP-Model using Supernovae Type Ia Data

SUBMITTED BY

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Abschätzung kosmologischer Parameter im ΛCDM- and DGP-Modell anhand von Supernovae Typ Ia Daten

Bachelorarbeit

Universitätssternwarte München
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vorgelegt von

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It was a long path to achieve finally this point in my life. And it was not an easy one. But no one said, that life is going to be easy.
I was about 12 years old, when I watched the sky the very first time with my a telescope, looking out for the most beautiful planet of our solar system. Being able to admire the ring structure of Saturn opened my eyes and my heart to explore the beauty of nature, the beauty of space.

While growing up, we try, driven by curiosity and fascination, to understand the world around us. We may seek answers to “the big questions of life”.

Where do we come from ?
Why did the universe come to existence ?
How old is the universe ?
Which mechanisms in nature guide the course of all events ?
What will happen in the future ?
Which role do I play in this unimaginable big universe ?
What is the purpose of my life ?
And, where do I want to go ?

Those are not easy questions. And that is why those questions are probably the oldest ones of humanity.
Throughout my “journey of life”, I tried to find answers to those questions. Maybe, I could find some attempts to answer a few of those in this thesis. Maybe, I will never find an answer to some of these questions.

But at least, I found an answer, what the purpose of *my* life is: to discover the beauty of nature. Going on with my journey, trying to find answers. And never giving up during this adventure - no matter, how hard and how frustrating life sometimes can be. No matter, how often I have fallen or I will fall down to the harsh ground of reality. Because it is my curiosity, my ambition and my passion that makes me standing up again to turn my eyes, my gaze into the sky.

That is the most important lesson I learned in the last years, studying physics.

The fact that I made it so far is not only through reading textbooks, doing my exercise sheets or preparing for exames. Because it is not only the effort I brought up.
I am where I am thanks to the effort of several people that came into my life. I am very glad that I had the luck to get to know these people. And therefore, this is the best occasion for me to thank them.

First of all, I want to thank my teachers, who not only taught me a lot, but also did their best to answer my difficult questions and quench my thirst for knowledge, which was probably not always easy and took a lot of patience,

Pierette Al-Korey, Ulrike Blattert, Peter Bodden, Nicole Bömecke, Jörg Bühler, Reiner Büter, Rolf Heckmann, Petra Jähnigen, Thomas Schröder Klementa, Axel Knuth, Eva Krüger, Angelika Nieboer, Roland Petereit, David Stephan, Volkhard Stierhof, Stefan Usée, Selma Weiß-Tümmers, Wolf Wingenfeld, Sema Yilmaz, Mike Ziegner.

In loving memory of Simon Jacob Gallo and Gamal Tadros Mishreky

NOTATION AND CONVENTIONS

List of Books on Astrophysics, Cosmology and/or General Relativity with

(+) $\eta_{\mu\nu} = \text{diag}(-, +, +, +)$

- Scott Dodelson, Fabian Schmidt. *Modern Cosmology*
- Matthias Bartelmann. *Das kosmologische Standardmodell*

(−) $\eta_{\mu\nu} = \text{diag}(+, -, -, -)$

- Bradley W. Carroll, Dale A. Ostlie. *An Introduction to Modern Astrophysics*
- Viatcheslav Mukhanov. *Physical Foundations of Cosmology*

ABSTRACT

In 2011, Saul Perlmutter, Adam G. Riess and Brian P. Schmidt received the Nobel Prize in Physics *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*. [1]
Since then, we know that the expansion of the universe is accelerating. The cause of this accelerated expansion is still unknown, but there are some theoretical models which attempt to explain this phenomenon.

The established model in recent cosmology is the **ΛCDM-Model**, sometimes referred as the *standard model of cosmology.* In this model, the cause of the accelerated expansion is due to the so called *cosmological constant* Λ, which appears as a physical constant in Einstein's field equations of general relativity like the Newtonian gravitational constant G.

Another model, published in April 2000 by Gia Dvali, Gregory Gabadadze and Massimo Porrati, – the **DGP-Model** – proposes a modification of Einstein's field equations by introducing a fifth dimension to the four-dimensional spacetime, so that gravity behaves equivalently to Newtonian gravity on small distances, but weakens on large scales.

In this thesis, we consider measurements of type Ia Supernovae by the *Supernova Cosmology Project* [2] (dataset "Union2.1") to obtain best-fit values to the free parameters and constraints to both cosmological models.

[1] Press release: https://www.nobelprize.org/prizes/physics/2011/press-release/
[2] Supernova Cosmology Project: https://supernova.lbl.gov/Union/

CONTENTS

CHAPTER 1

INTRODUCTORY CONCEPTS OF ASTROPHYSICS

In 2011, Saul Perlmutter, Adam G. Riess and Brian P. Schmidt received the Nobel Prize in Physics *"for the discovery of the accelerating expansion of the Universe through observations of distant supernovae"*. [1]
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The established model in recent cosmology is the **ΛCDM-model**, sometimes referred as the *standard model of cosmology*. In this model, the cause of the accelerated expansion is due to the so called *cosmological constant* Λ, which appears as a physical constant in Einstein's field equations of General Relativity like the Newtonian gravitational constant G.

Another model, published in April 2000 by Gia Dvali, Gregory Gabadadze and Massimo Porrati, – the **DGP-model** – proposes a modification of Einstein's field equations by introducing a fifth dimension to the four-dimensional spacetime, so that gravity behaves equivalently to Newtonian gravity on small distances, but weakens on large scales.

Before going into details about both cosmological models, let me introduce relevant concepts in physics and astronomy from which we can relate measurable physical quantities, like the brightness of stars or their redshift, to more abstract properties of the universe like its scale factor. Based on certain assumptions of a theory, it is possible to develope models that make predictions about properties of the universe, for example, how the expansion and its evolution in time influences the relation of physical quantities.

1.1 Distance measurement in Astronomy

Essential to astronomical observations and measurements is to determine the distance to objects in space like stars, star clusters, galaxies or even clusters of galaxies.
By looking at night into the sky, the only information perceived by our human eye are the brightness and some color in which objects (mostly stars) appear. How do we determine the distance to those objects ?

1.1.1 The parallax method

One method to determine the distance to an object is by using the *parallax effect*.
For this method, we assume that the observed object is almost stationary relative to earth. First, we detect the position of the measured object in the sky. After a while (for example, after a half year), the object appears at a slightly different position in the sky, since earth moved on its elliptical orbit which leads to another point of view for the observation.

[1] Press release: https://www.nobelprize.org/prizes/physics/2011/press-release/

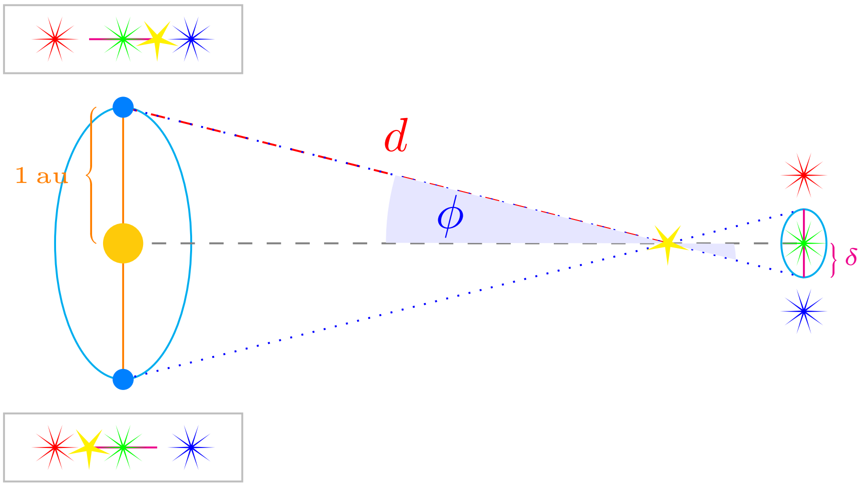

Figure 1.1: To determine the distance d between earth and the yellow star, we consider the small change of the position δ around the green star under which the yellow star appears. The difference δ relates to the angle ϕ.

By measuring the appeared difference δ of the object's position and relating this difference to the angle ϕ under which the object changed its position in the sky, we can calculate the distance d by

$$d = \frac{1\,\mathrm{au}}{\sin(\phi)} \approx \frac{1\,\mathrm{au}}{\phi}, \tag{1.1}$$

where $1\,\mathrm{au} \approx 149.6 \times 10^9\,\mathrm{m}$ ("astronomical unit"[2]) is defined as the average distance $d_{\oplus\odot}$ of earth to the sun.
The small-angle approximation can be considered justified, since the distance to earth nearest star (beyond of our solar system), Proxima-Centauri, is about $d_{\mathrm{P.C.}} \approx 4.2\,\mathrm{ly}$ and therefore $\phi_{\mathrm{P.C.}} \approx \arcsin\left(\frac{1\,\mathrm{au}}{4.2\,\mathrm{ly}}\right) \approx 3.723 \times 10^{-6} \ll 1$. The larger the distance to the measured object, the smaller the angle is.
Since the parallax method is one of the most important ways to measure the distance far away objects, we define the distance of a *parsec* ("parallax second") as the distance to an object, under which the parallax is $\phi = 1\,\mathrm{arcsec} = \frac{1}{3600}^\circ$:

$$1\,\mathrm{pc} := \frac{1\,\mathrm{au}}{\sin\left(\frac{1}{3600}^\circ\right)} \approx 3.26\,\mathrm{ly} \approx 3.086 \times 10^{16}\,\mathrm{m}. \tag{1.2}$$

This is the usual unit by which cosmological distances or parameters are expressed, for example the *Hubble constant* $H_0 \approx 67.66\,\frac{\mathrm{km}}{\mathrm{s}}\mathrm{Mpc}^{-1}$[3].

We should keep in mind that this method has of course boundaries and is practically useful for the local region of our galaxy, since the larger the distance to an observerd object is, the appeared change in position at sky gets smaller and is for objects with a distance on cosmological scale ($d \gtrsim 300\,\mathrm{Mpc}$) almost unnoticable.

[2] Since 2012, the astronomical unit was redefined by the IAU (International Astronomical Union) to be exactly $1\,\mathrm{au} := 149\,597\,870\,700\,\mathrm{m}$, see *Resolution B2* at the XXVIII General Assembly of IAU: https://www.iau.org/static/resolutions/IAU2012_English.pdf
[3] The value of H_0 according to the Results of the *Planck Collaboration 2018*, [Col+20, Table 7]

1.1.2 The Distance Modulus

Another method to determine the distance to an object with a certain luminosity L is to measure its radiant flux F at a distance d by

$$F = \frac{L}{4\pi d^2}. \tag{1.3}$$

In general astronomical observations, it is not the radiant flux of a star that is measured. Rather, we observe *differences* of brightness or magnitude m between two objects. We define a difference in magnitude between two objects $m_1 - m_2 := \Delta m$ so that the radiant flux F_2 of object 2 is 100 times higher than the radiant flux F_1 of object 1, when their difference in magnitude is $\Delta m = 5$, so

$$\frac{F_2}{F_1} = 100 \Leftrightarrow m_1 - m_2 = \Delta m = 5. \tag{1.4}$$

This leads us to the relation between the difference of magnitude Δm and the relation of the radiant flux of two objects

$$\frac{F_2}{F_1} = 100^{\frac{m_1 - m_2}{5}} \tag{1.5}$$

and therefore with (1.3)

$$m_1 - m_2 = \frac{5}{2}\log_{10}\left(\frac{F_2}{F_1}\right) = \frac{5}{2}\log_{10}\left(\frac{L_2}{4\pi d_2^2}\frac{4\pi d_1^2}{L_1}\right) = \frac{5}{2}\log_{10}\left(\frac{L_2}{L_1}\right) + 5\log_{10}\left(\frac{d_1}{d_2}\right). \tag{1.6}$$

So, to calculate the distance to one of both objects, for example object 2, would require us to know their *relative* magnitudes m_1 and m_2, their luminosities L_1 and L_2 and the distance d_1 to object 1.
To eliminate the need of knowing five quantities, we can define an *absolute* magnitude M as the magnitude an object would have at a distance of $d = 10\,\mathrm{pc}$. If we consider only one object, we therefore can calculate the distance to this object by

$$m - M = \underbrace{\frac{5}{2}\log_{10}\left(\frac{L}{L}\right)}_{=0} + 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right) = 5\log_{10}\left(\frac{d}{10\,\mathrm{pc}}\right). \tag{1.7}$$

We call this equation the *distance modulus*.
The benefit of the distance modulus is that we only have to know two quantities, m and M, to calculate the distance to an object.
Unfortunately, we can not measure the absolute magnitude M of an object that easily (since we can not fly to far away stars and measure the observed magnitude or radiant flux at a distance of $10\,\mathrm{pc}$ to them). That is why we rely on (1.6). But at least, we could reduce the amount of unknown variables by calibrating the relation (1.6) with an object which luminosity and distance is known.

One could choose to calibrate equation (1.6) by measuring the properties of the nearest star to earth – our sun. Given the magnitude $m_\odot$, the luminosity $L_\odot$ and the distance to sun $d_{♁\odot} = 1\,\mathrm{au}$, we obtain

$$m = m_\odot - \frac{5}{2}\log_{10}\left(\frac{L}{L_\odot}\right) + 5\log_{10}\left(\frac{d}{1\,\mathrm{au}}\right). \tag{1.8}$$

By this calibration, we can determine the distance to an observed object, if we could know its luminosity L and measure its relative magnitude m. Generally, the luminosity of objects like stars could varying arbitrarly. To obtain a reliable distance measurement, we are looking for objects which luminosity can be predicted very precisely – so called *standard candles*.

1.1.3 Possible candidates for Standard Candles

Generally, there are two established candidates for standard candles in astronomy. Since we deal in this thesis with data of type Ia supernovae, the focus lies on the second paragraph of this subsection. For the sake of completeness, however, the cepheids as possible standard candles should not be unmentioned – also to emphasize, why we rely on supernovae of type Ia to determinine cosmological distances.

1.1.3.1 Cepheids

There are several types and classes of cepheids, but they all have in common that those stars obey a certain, periodic relation of luminosity and time.

Without going into details[4], the periodicity of luminosity is caused by fluctuations of temperature dependent opacity in the stars photosphere due to transitions between single- and dual-ionized Helium inside the star.
It is important to identify cepheids of the same type – cepheids that share the same physical properties, like their metalicity or the same periodicity pattern in their luminosity, to ensure that the same physical process is occuring in all cepheids of a certain type. From then on, it is possible to determine the luminosity of all cepheids of the same type by observing one cepheid, measuring its relative magnitude m and its distance d by the parallax method.

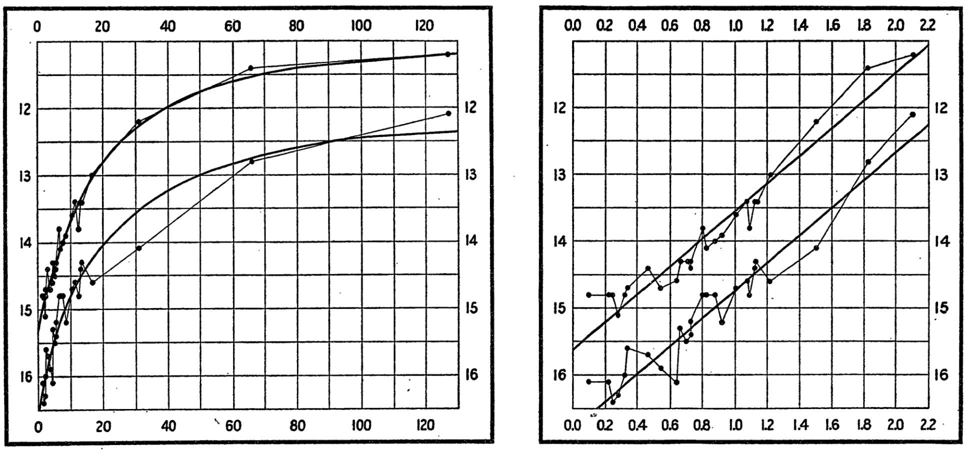

Fig. 1. Fig. 2.

Figure 1.2: First observed direct, logarithmic periodicity between time (x-axis in days) and magnitude (y-axis) of 25 stars in the Small Magellanic Cloud by Henrietta Leavitt, published 1912. Source: [LP12].

[4] For further information and more detailed explanation how stellar pulsation and its underlying κ-mechanism can be calculated, I recommend chapter 14 "*Stellar Pulsation*" in [CO07]

However, in order to use cepheids for distance determination, they must first be observed and resolved. Even with our most powerful telescopes that we have today, resolution of cepheids is only possible in our local, galactic neighborhood, for example in the Large Magellanic Cloud or the Andromeda Galaxy. Therefore, the boundary to resolute cepheids in other galaxies is currently about $\sim 30\,\text{Mpc}$ ([Bar19, p. 47] and [Eng13, p. 3]).
For larger scales, we need much brighter standard candles.

1.1.3.2 Supernovae of Type Ia

In general, supernovae are abrupt bursts of luminosity from massive stars, often accompanied by explosive thermonuclear reactions. While supernovae of type Ib, Ic and II occur due to an imbalance between the star's gravity, which causes its nucleus to collapse, and radiation emitted by nuclear fusion inside the star, which pushes against its photosphere, supernovae of type Ia are caused by a white dwarf, accreting a companion star and therefore increasing in mass.
If the white dwarf reaches due to accreting a companion star a mass limit over $\sim 1.4M_\odot$ (where $M_\odot \approx 1.98 \times 10^{30}\,\text{kg}$ is the mass of the sun, [Bar19, p. 48]), which is called the *Chandrasekhar limit* M_{Ch}, the nucleus of the white dwarf beocmes unstable, since the degeneracy pressure of the electrons (Pauli exclusion principle) can not resist the gravitational forces any more.
Inside the white dwarf's nucleus, a fusion of carbon and oxygen is triggered, which leads to further nuclear processes and an amount of energy release up to $\sim 10^{44}\,\text{J}$ ([Mag17, p. 295], [Spa17, p. 321]). This makes supernovae of type Ia one of the most energetic – and therefore also luminous – phenomena known in nature. Supernovae of type Ia are almost as bright as their host galaxy. Since they are not only extremly luminous, but the process of star collapse is triggered after crossing a fixed limit of mass (the Chandrasekhar limit), this makes them also very unique and therefore good candidates for standard candles.
We can distinguish between supernovae of type Ia from other supernovae types by observing and analizing their spectrum.

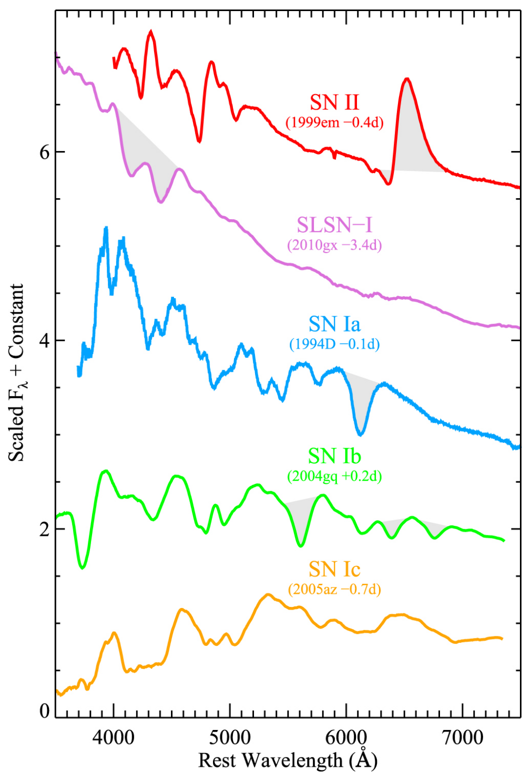

Figure 1.3: Key features in the spectra of different supernovae types are shaded in grey. While supernovae of type II have significant hydrogen lines, supernovae of type Ia and Ib are lacking of hydrogen lines. In type Ia supernovae, the Si II line (at $\sim 6150\,\text{Å}$) is very significant, while it is weak in type Ib supernovae. In type Ib supernovae, the helium lines seems to be very strong. Supernovae of type Ic do have non of the mentioned features.
Source: [Qui+18, Figure 1]

Yet there are small problems that occur here.
First, the mechanism which leads to the supernovae explosion is somewhat controversial, since there are two possible scenarios: the "single-degenerate"-model, in which the companion star is a star of the main sequence or a giant star, and the "double-degenerate"-model, in which the companion star is also a white dwarf.

In the "single-degenerate"-model, the accretion must not be too slow, since the hydrogen-rich material of the companion star could be burnt at the same rate as it is accreted, which results in no growth of mass for the white dwarf. On the other hand, the accretion must not be too high, since the accretion might stop due to the companion star's loss of mass at a high rate or being engulfed by the accreted material ([Mag17, p. 308]).

In the "double-degenrate"-model, it is assumed that one of both white dwarfs reaches the Chandrasekhar limit if they get too close ([Bar19, p. 48]), but there are also other possible scenarios that could occur (see [Mag17, p. 308/309]).
This could lead to slight variations in the luminosity behavior.

Other variations in luminosity behavior could occur due the amount of ^{56}Ni in the thermonuclear process, which decay influences the peak of the supernovae type Ia light curve ([Mag17, p. 295]). But those variations can be compensated very well since a relation between the lumninosity's decline and its peak is found so that it is possible to "normalize" or "stretch" the light curves so they obey a uniform distribution ([PS03, p. 4], [Phi93]).
The fact that the light curves of type Ia supernovae are distributed exactly the same way after some "normalization" or "stretching" shows us, that the same physical process underlies all light curves of type Ia supernovae, even if they seem to be stretched, not only due to the relation of luminosity's decline and luminosity peak. We will mention later, why this property of supernovae type Ia light curves verify the expansion of the universe.

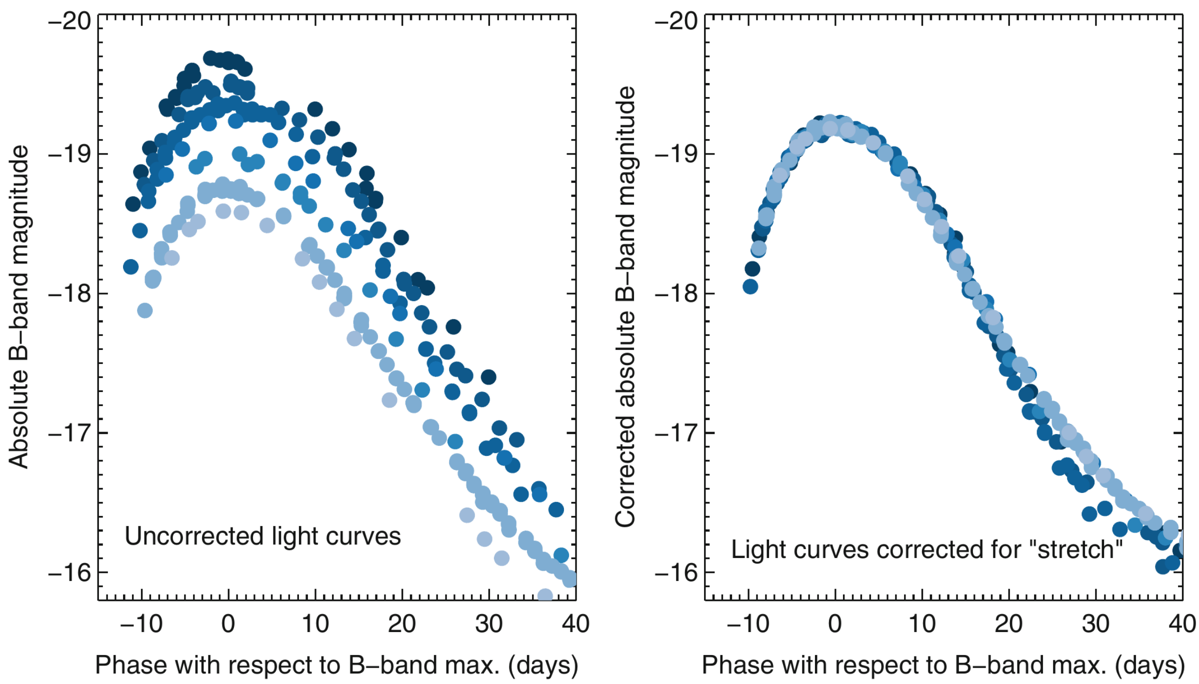

Figure 1.4: Uncorrected vs. corrected sample of supernovae type Ia light curves.
Source: [Mag17, p. 298, Figure 2]

Despite the mentioned slight variations in the luminosity spectrum, supernovae of type Ia can be considered as the best candidates for standard candles at the current state of research, not only because their light curves are very homogeneous, but they are also much brighter than cepheids and therefore also at large distances visible, which is essential for cosmological research.

1.1.4 Redshift and Hubble's Observation

The most important tool to determine distances on a cosmological scale is by observing the redshift of extragalactic objects. The redshift is a shift in the observed spectrum of light emmited by a source with wavelength λ_e to an observed wavelength λ_o and defined as

$$z := \frac{\lambda_o - \lambda_e}{\lambda_e} = \frac{\lambda_o}{\lambda_e} - 1. \tag{1.9}$$

The term "red" in "redshift" might be misleading, since z could take also values < 0 and therefore $\lambda_o < \lambda_e$, which implicates a shift of the light's spectrum to the blue.
This term has been established, because observations show that the spectrum of most galaxies and extragalactic objects is shifted towards red.

In the end of the 1920s, Edwin Hubble observed a certain relation between the measured redshift of the galaxies and extragalactic objects and their distance to us. If one assumes the redshift due to (relativistic) doppler effect[5], one could calculate the galaxys velocity component towards the direction of observation by

$$z = \gamma\left(1 + \frac{v}{c}\right) = \sqrt{\frac{c+v}{c-v}} - 1. \tag{1.10}$$

An observed shift of the spectrum to the red would imply, that the observed object moves away from the observer, and a shift of the spectrum to the blue implies a motion towards the observer.
Edwin Hubble applied a linear relation (see figure 1.6) between the measured distance d to the observed objects and the calculated velocity v due to the doppler effect

$$v = H_0 d, \tag{1.11}$$

which is known as the *Hubble law*. The proportionality constant H_0 has the dimension of an inverse time. It is one of the most important parameters in cosmology and its precise determination a challenge of active research (often called "Hubble tension", [Val+21]).

For small velocities ($v \ll c$), we can approximate (1.10) (first order taylor series) as $z \approx \frac{v}{c}$, which leads with (1.11) to

$$d \approx \frac{c}{H_0} z. \tag{1.12}$$

We will see later that this relation is actually a first order approximation between the *luminosity distance* d_L and the redshift z.

[5] Some raised objections against the interpretation that the observed redshift is a result of the doppler effect, but claimed that it is caused by energy loss of the photons traveling through space ("tired light"-hypothesis). We will address later, why this hypothesis can not be hold anymore in the light of tremendous evidence for the expansion model.

The correct relation between the luminosity distance d_L and the redshift z will play an essential role when estimating parameters of cosmological models.

From Hubble's law follows that the further the distance to a galaxy (or other cosmological object) is, the higher the redshift and therefore, the faster it seems to move apart from us. This observation was one of the milestones in the history of cosmology and was the first indicator, that our Universe is truly expanding.

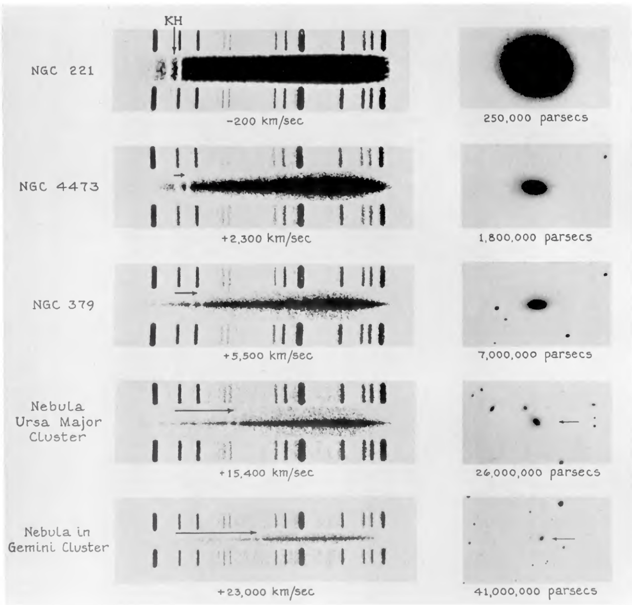

Figure 1.5: Observed redshift of H and K lines of calcium, shifted to the red, by Milton L. Humason.
Source: [Hum36, Figure "Red-shifts in the Spectra of Extra-galactic Nebulae"]

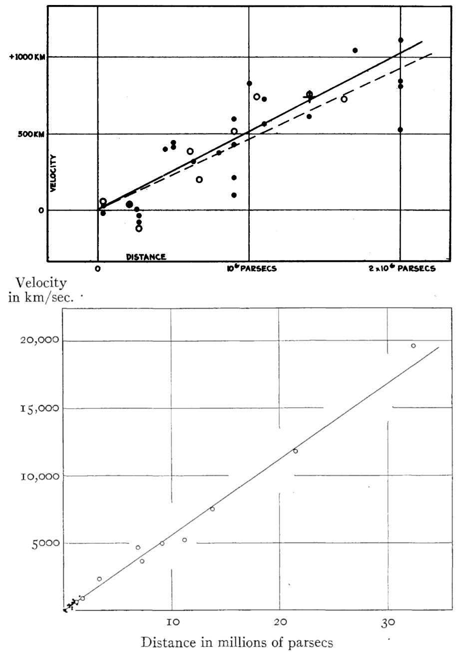

Figure 1.6: Linear regression between the distance of observed objects and the calculated velocity due to redshift.
Source: [Hub29] and [HH31, Figure 5]

1.2 The Theory of Gravity and Spacetime

How can one speak about the nature on large scales without mentioning the force that dominates in this regime ? Beginning with Newton's ideas about gravity, the modern formulation about gravity is described in the framework of Einstein's General Theory of Relativity, in which gravity is, strictly speaking, not anymore a force in the Newtonian sense, but rather a property of the four dimensional spacetime that interacts with matter.

For a deep understanding of General Relativity, it is required to have knowledge on the mathematics of differential geometry.
Despite the fact that, as an undergraduate student, I do not have this knowledge (yet), this would be beyond the scope of this bachelor thesis.
But to motivate the basic equations of the ΛCDM-Model, which are derived from Einstein's field equations under certain assumptions that we formulate in the next chapter, I would like to mention the concept of a *metric* and give brief view on Einstein's field equations.

1.2.1 The Metric of Spacetime

Generally speaking, a metric is a function that takes two points in space and returns a distance. For example, let $\mathbb{E}^2$ be the Euclidean, two dimensional space, then

$$d(\cdot,\cdot):\mathbb{E}^2\times\mathbb{E}^2\to\mathbb{R},(\boldsymbol{p}_1,\boldsymbol{p}_2)\mapsto d(\boldsymbol{p}_1,\boldsymbol{p}_2):=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2} \tag{1.13}$$

would be a function that gives us the distance between two points $\boldsymbol{p}_1=(x_1,y_1)^{\mathrm{T}}$ and $\boldsymbol{p}_2=(x_2,y_2)^{\mathrm{T}}$. This distance function d would return the pythagorean distance in cartesian coordinates that we are familiar with.
Let us define $x:=x_1-x_2$, $y:=y_2-y_2$ and $s:=d(\boldsymbol{p}_1,\boldsymbol{p}_2)$ so we could write for the (infinitesimal) distance

$$\mathrm{d}s^2=\mathrm{d}x^2+\mathrm{d}y^2\,. \tag{1.14}$$

Now, let us switch to polar coordinates so that $\boldsymbol{p}_1:=(r_1,\phi_1)^{\mathrm{T}}$ and $\boldsymbol{p}_2:=(r_2,\phi_2)^{\mathrm{T}}$. With the given distance function d, we would obtain

$$d(\boldsymbol{p}_1,\boldsymbol{p}_2)=\sqrt{(r_1-r_2)^2+(\phi_1-\phi_2)^2}. \tag{1.15}$$

But this is *not* the same distance as in cartesian coordinates. The distance that would correspond to the same distance as in cartesian coordinates (1.14), is

$$\mathrm{d}s^2=\mathrm{d}r^2+r^2\,\mathrm{d}\phi^2 \tag{1.16}$$

with $r:=r_1-r_2$, $\phi:=\phi_1-\phi_2$.
We have to introduce the *metric tensor* (in most applications of physics a 3×3- or 4×4-matrix) that garantuees as the invariance of the distance function d under coordinate transformation. We define

$$g_{ij}\overset{\text{cartestian}}{=}\begin{pmatrix}1&0\\0&1\end{pmatrix}\overset{\text{polar}}{=}\begin{pmatrix}1&0\\0&r\end{pmatrix},\quad x^1\overset{\text{cartesian}}{=}x\overset{\text{polar}}{=}r,\quad x^2\overset{\text{cartesian}}{=}y\overset{\text{polar}}{=}\phi, \tag{1.17}$$

so that we can express the invariant distance between $\boldsymbol{p}_1$ and $\boldsymbol{p}_2$ through

$$\mathrm{d}s^2=\sum_{i=1}^{2}\sum_{j=1}^{2}g_{ij}\,\mathrm{d}x^i\,\mathrm{d}x^j=g_{11}\left(\mathrm{d}x^1\right)^2+\underbrace{g_{12}}_{=0}\mathrm{d}x^1\,\mathrm{d}x^2+\underbrace{g_{21}}_{=0}\mathrm{d}x^2\,\mathrm{d}x^1+g_{22}\left(\mathrm{d}x^2\right)^2 \tag{1.18}$$

$$=g_{11}\left(\mathrm{d}x^1\right)^2+g_{22}\left(\mathrm{d}x^2\right)^2\overset{\text{cartesian}}{=}\mathrm{d}x^2+\mathrm{d}y^2\overset{\text{polar}}{=}\mathrm{d}r^2+r^2\,\mathrm{d}\phi^2\,. \tag{1.19}$$

In the framework of relativity, we express the distance (called “world line”) between to *events* $\boldsymbol{p}_1:=(ct_1,x_1,y_1,z_1)^{\mathrm{T}}$ and $\boldsymbol{p}_2:=(ct_2,x_2,y_2,z_2)^{\mathrm{T}}$ in four dimensional spacetime as

$$\mathrm{d}s^2=g_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu \tag{1.20}$$

with the implicit sum over double occuring indices $\mu,\nu\in\{0,1,2,3\}$. In flat Minkowski-Spacetime of Special Relativity for example, we have

$$g_{\mu\nu}=\eta_{\mu\nu}:=\mathrm{diag}(-1,1,1,1) \tag{1.21}$$

and therefore for distances in spacetime

$$\mathrm{d}s^2=\eta_{\mu\nu}\,\mathrm{d}x^\mu\,\mathrm{d}x^\nu=-(c\,\mathrm{d}t)^2+\mathrm{d}x^2+\mathrm{d}y^2+\mathrm{d}z^2\,. \tag{1.22}$$

1.2.2 Einstein's field equations

Similar to the derivation of the Euler-Lagrange equations in classical mechanics (or classical field theory) by formulating an action $S[\boldsymbol{q}(t)]$ (or $S[\phi(x)]$) and find the path $\boldsymbol{q}(t)$ (or field $\phi(x)$) that extremizes the action ($\delta S = 0$), Einstein's field equations can be derived[6] from the so called Einstein-Hilbert action given by

$$S_{\mathrm{EH}}[g_{\mu\nu}] = \int \mathrm{d}^4x \sqrt{-\det(g_{\mu\nu})} \left[\frac{c^4}{16\pi G}(R - 2\Lambda) + \mathcal{L}_{\mathrm{M}} \right], \tag{1.23}$$

where d^4x is the four dimensional spacetime-volume element, $g_{\mu\nu}$ the metric tensor, c the speed of light (in vacuum), G the Newtonian gravitational constant, R the Ricci scalar, Λ the cosmological constant and $\mathcal{L}_{\mathrm{M}}$ the lagrange density of matter fields.
With the action principle, the variation $\delta S[g_{\mu\nu}]$ of the Einstein-Hilbert action with respect to the metric leads to Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{1.24}$$

where $R_{\mu\nu}$ is the Ricci curvature tensor and $T_{\mu\nu}$ the energy-momentum tensor.
Without getting too much into details about the single components of this equation, we keep in mind that the left hand side of (1.24) describes how spacetime behaves, while the right hand side of (1.24), how matter behaves.

1.2.2.1 The Cosmological Constant Λ

Originally, Einstein formulated his field equations without the cosmological constant Λ. Since Einstein believed in a static universe and found that his equations can not hold for a static universe (it would have collapsed due to the gravity of matter), he introduced the Λ term that acts repulsive towards the attraction of gravity, so that a static universe as he proposed would be possible.

[6] For a proper and detailed derivation, I recommend subsection 4.3 "*Lagrangian Formulation*" in [Car19, p. 159]

CHAPTER 2

THE STANDARD MODEL OF COSMOLOGY

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like “Huardest gefburn”? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

2.1 The Cosmological Principle

2.1.1 Homogenity

2.1.2 Isotropy

2.2 Friedmann equations

2.2.1 FLRW-Metric

2.2.2 Cosmological Parameters

2.3 Evidence

CHAPTER 3
THE DVALI-GABADADZE-PORRATI-MODEL

Hello, here is some text without a meaning. This text should show what a printed text will look like at this place. If you read this text, you will get no information. Really? Is there no information? Is there a difference between this text and some nonsense like "Huardest gefburn"? Kjift – not at all! A blind text like this gives you information about the selected font, how the letters are written and an impression of the look. This text should contain all letters of the alphabet and it should be written in of the original language. There is no need for special content, but the length of words should match the language.

CHAPTER 4

CONCLUSION

APPENDIX A

THE FIRST APPENDIX

Here is the first appendix.

APPENDIX B

THE SECOND APPENDIX

Here comes the second appendix.

BIBLIOGRAPHY

[Col+20] Planck Collaboration et al. *"Planck 2018 results: I. Overview and the cosmological legacy of Planck"*. en. In: *Astronomy & Astrophysics* 641 (Sept. 2020), A1. ISSN: 0004-6361, 1432-0746. DOI: 10.1051/0004-6361/201833880. URL: https://www.aanda.org/10.1051/0004-6361/201833880.

[CO07] Bradley W. Carroll and Dale A. Ostlie. *An Introduction to Modern Astrophysics*. Ed. by San Francisco: Pearson Addison-Wesley. 2nd (International). 2007. ISBN: 978-0-321-44284-0.

[LP12] Henrietta S. Leavitt and Edward C. Pickering. *Periods of 25 variable stars in the Small Magellanic Cloud*. Mar. 1912. URL: https://ui.adsabs.harvard.edu/link_gateway/1912HarCi.173....1L/ADS_PDF.

[Bar19] Matthias Bartelmann. *Das kosmologische Standardmodell*. Springer Verlag, 2019. ISBN: 978-3-662-59626-5. DOI: 10.1007/978-3-662-59627-2.

[Eng13] Philipp Engelmann. *"Cepheid Stars as standard candles for distance measurements"*. In: Sept. 2013. URL: https://www.haus-der-astronomie.de/3440685/04Engelmann.pdf.

[Mag17] Kate Maguire. *"Type Ia Supernovae"*. In: ed. by Athem W. Alsabti and Paul Murdin. Springer International Publishing, 2017, pp. 293–316. ISBN: 978-3-319-21845-8. DOI: 10.1007/978-3-319-21846-5_36. URL: https://link.springer.com/content/pdf/10.1007/978-3-319-21846-5_36.pdf.

[Spa17] Karl-Heinz Spatschek. *Astrophysik. Eine Einführung in Theorie und Grundlagen*. 2nd ed. Springer Spektrum Berlin, Heidelberg, Nov. 2017. ISBN: 978-3-662-55466-1. DOI: https://doi.org/10.1007/978-3-662-55467-8.

[Qui+18] Robert M. Quimby et al. *"Spectra of Hydrogen-poor Superluminous Supernovae from the Palomar Transient Factory"*. In: *The Astrophysical Journal* 855.1 (Feb. 2018), p. 2. DOI: 10.3847/1538-4357/aaac2f. URL: https://dx.doi.org/10.3847/1538-4357/aaac2f.

[PS03] Saul Perlmutter and Brian P. Schmidt. *Supernovae and Gamma-Ray Bursters*. Mar. 2003. Chap. Measuring Cosmology with Supernovae. DOI: 10.1007/3-540-45863-8_11. URL: https://arxiv.org/pdf/astro-ph/0303428.pdf.

[Phi93] M. M. Phillips. *"The Absolute Magnitudes of Type IA Supernovae"*. In: *The Astrophysical Journal* 413 (Aug. 1993). DOI: 10.1086/186970. URL: https://articles.adsabs.harvard.edu/pdf/1993ApJ...413L.105P.

[Val+21] Eleonora Di Valentino et al. *"In the realm of the Hubble tension—a review of solutions"*. In: *Classical and Quantum Gravity* 38.15 (July 2021), p. 153001. DOI: 10.1088/1361-6382/ac086d. URL: https://dx.doi.org/10.1088/1361-6382/ac086d.

[Hum36] Milton L. Humason. *"The Apparent Radial Velocities of 100 Extra-Galactic Nebulae"*. In: *Astrophysical Journal* 83 (Jan. 1936), p. 10. DOI: 10.1086/143696. URL: https://ui.adsabs.harvard.edu/abs/1936ApJ....83...10H.

[Hub29] Edwin Hubble. *"A Relation between Distance and Radial Velocity among Extra-Galactic Nebulae"*. In: *Proceedings of the National Academy of Science* 15.3 (Mar. 1929), pp. 168–173. DOI: 10.1073/pnas.15.3.168. URL: https://ui.adsabs.harvard.edu/abs/1929PNAS...15..168H.

[HH31] Edwin Hubble and Milton L. Humason. *"The Velocity-Distance Relation among Extra-Galactic Nebulae"*. In: *Astrophysical Journal* 74 (July 1931), p. 43. DOI: 10.1086/143323. URL: https://ui.adsabs.harvard.edu/abs/1931ApJ....74...43H.

[Car19] Sean M. Carroll. *Spacetime and Geometry: An Introduction to General Relativity*. Cambridge University Press, Aug. 2019. ISBN: 978-1-108-48839-6. DOI: 10.1017/9781108770385.

[Sch09] Bernard Schutz. *A First Course in General Relativity*. 2nd ed. Cambridge University Press, 2009. ISBN: 978-0-521-88705-2. DOI: 10.1017/CBO9780511984181.

DECLARATION OF AUTHORSHIP

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