Matheuristics

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Optimization Methods

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1 Generalities

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- or Math + Heuristics
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- Neighborhoods are searched by enumeration
- that is (local) brute force
- we could use MILP to do it!

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2 Usual neighborhood

Defining a neighborhood

- Back to our F2//$\sum T_j$ problem
- Restrict the search to a reasonnable amount of jobs
- Given a solution $s = (s_1, \ldots, s_n)$
- and an interval $[h..h + w - 1] \subset [1..n]$, the width is w
- let us consider the neighborhood defined by:
- all the solutions that are equal to s except on $[h..h + w - 1]$
- $\mathcal{N}_{h,w}(s) = \{\sigma / \sigma_j = s_j \; for \; j \notin [h..h + w - 1]\}$

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- There are $w!$ elements in this neighborhood
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- a best local solution
- Not using brute force, but MILP
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- and fix all the variables we know
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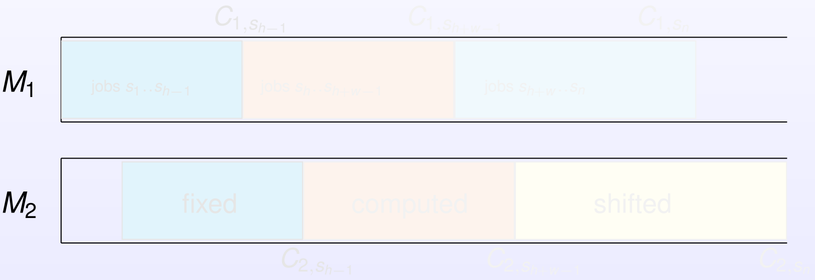

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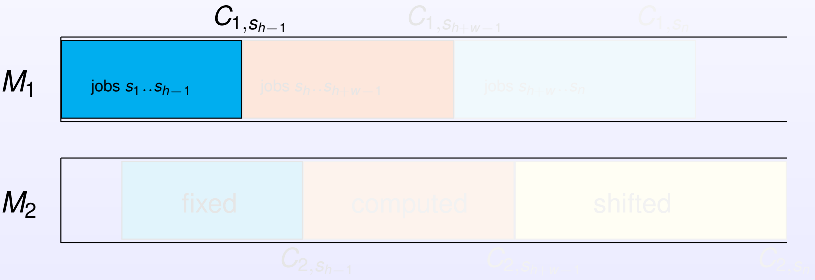

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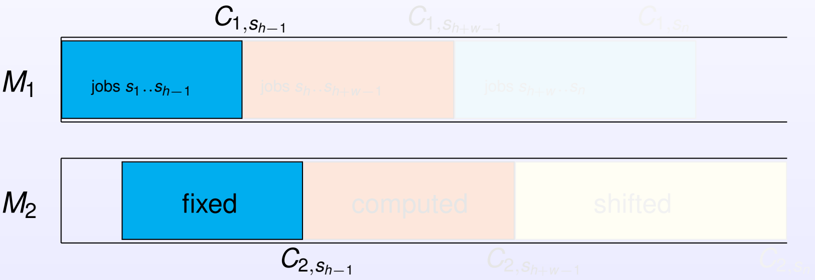

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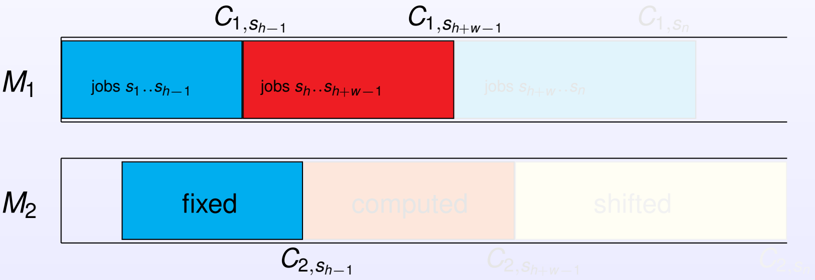

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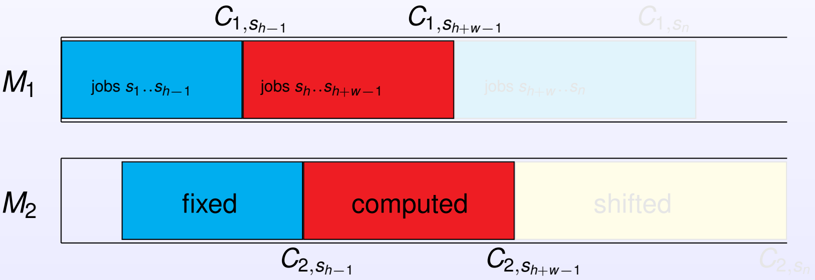

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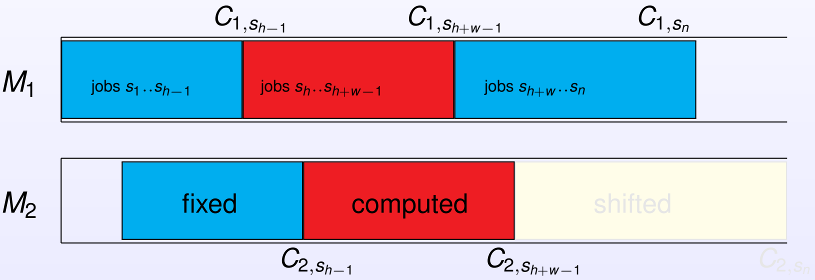

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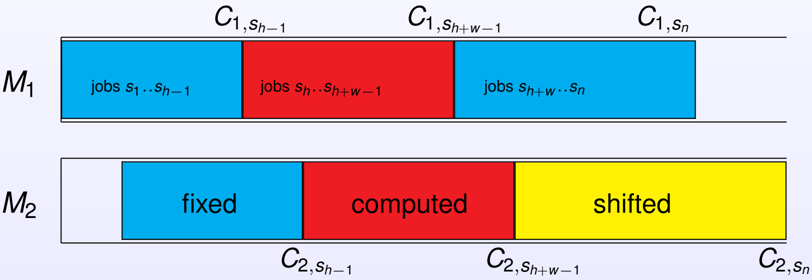

A model for the neighborhood
Variables

- Metadata: from s we compute useful values
- Variables are:
- - $X_{k,j}$ for $k \in [0..w-1]$ and $j \in [0..w-1]$.
  $X_{k,j} = 1$ if and only if the job $s[h+j]$ is in position $h+k$
- - $F1_k$, $F2_k$ and G_k for $k \in [0..w-1]$.
  They correspond to the job in position $h+k$
- - $C2_j$, T_j for the jobs in position $\in [h+w..n]$

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Constraints

- Each rank is assigned to a unique job (rows of the matrix $(X_{k,j})$):

$$\forall k \in [0..w-1] \quad \sum_{j=0}^{w-1} X_{k,j} = 1 \qquad (1)$$

- Each job has got a unique rank (columns of the matrix $(X_{k,j})$):

$$\forall j \in [0..w-1] \quad \sum_{k=0}^{w-1} X_{k,j} = 1 \qquad (2)$$

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Constraints

- Completion time on $M1$ of the job in position h (first red job)

$$F1_0 = f1_{h-1} + \sum_{j=0}^{w-1} p1_{s_{(h+j)}} X_{0,j} \qquad (3)$$

- Completion times on $M1$ of the jobs in position $h+1..h+w-1$ (other red jobs)

$$\forall k \in [1..w-1] \quad F1_k = F1_{k-1} + \sum_{j=0}^{w-1} p1_{s_{(h+j)}} X_{k,j} \qquad (4)$$

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- Completion time on *M2* of the job in position *h* (first red job)

$$F2_0 \geq f2_{h-1} + \sum_{j=0}^{w-1} p2_{s_{(h+j)}} X_{0,j} \qquad (5)$$

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$$\forall k \in [0..w-1] \quad F2_k \geq F1_k + \sum_{j=0}^{w-1} p2_{s_{(h+j)}} X_{k,j} \qquad (7)$$

- Tardinesses of the jobs in position $h..h+w-1$ (red jobs):

$$\forall k \in [0..w-1] \quad G_k \geq F2_k - \sum_{j=0}^{w-1} d_{s_{(h+j)}} X_{k,j} \qquad (8)$$

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Constraints

- Completion times on $M2$ of the jobs in position $h+w..n$ (yellow jobs):

$$\forall j \in [0..n-h-w] \quad C2_j \geq \pi^1_{h+w,h+w+j} \quad (9)$$

$$C2_j \geq F2_{w-1} + \pi^2_{h+w,h+w+j} \quad (10)$$

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A model for the neighborhood

Constraint of improvement and objective function

- We want to improve the current solution:

$$\sum_{j=1}^{h-1} g_j + \sum_{k=0}^{w-1} G_k + \sum_{j=0}^{n-h-w} T_j \leq tbar(s) - 1 \qquad (12)$$

- Objective function:

$$Minimize \quad \sum_{k=0}^{w-1} G_k + \sum_{j=0}^{n-h-w} T_j$$

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Running this model

Function One-Pass(w,step)

1: *improved* $\leftarrow$ **false**
2: **for** h **from** 0 **while**$(h + w \leq n)$ **by** *step* **do**
3:     Run the model
4:     **if** (there is a solution *news*) **then**
5:         $s \leftarrow$ *news*
6:         *improved* $\leftarrow$ **true**
7:     **end if**
8: **end for**
9: **return** *improved*

- w: width of the interval
- *step*: pace for the progression of the interval
- line 4: if there is a solution, then it is better than s
- s is modified by side effect

Running this model

In the function One-pass, the model is lauched $\left\lfloor \dfrac{n-w}{step} \right\rfloor$ times.

The function One-Pass is used while the solution is improved.

A fixed point is reached.

Function Several-Passes(w,step)

1: $s \leftarrow$ *initial solution*
2: **while** *One-Pass(w,step)* **do**
3: Pass
4: **end while**
5: **return** s

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s ← initial solution
while One-Pass(w,step) do
    Pass
end while
return s
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3 Other neighborhoods

- The Shake model
- The Swap model
- The extended swap model
- The EBSR and EFSR models
- The Random model

A second neighborhood

- In the local search, we considered swapping consecutive jobs
- Now we can allow jobs to change their positions in a limited way
- for each job, new-rank $\in$ [old-rank$-\delta$.. old-rank$+\delta$]
- This will be called the Shake model

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Defining a second neighborhood

$s()$

	0	...	δ									$n-1$
0	X	X	X	□	□	□	□	□	□	□	□	□
...	X	X	X	X	□	□	□	□	□	□	□	□
δ	X	X	X	X	X	□	□	□	□	□	□	□
	□	X	X	X	X	X	□	□	□	□	□	□
	□	□	X	X	X	X	X	□	□	□	□	□
	□	□	□	X	X	X	X	X	□	□	□	□
	□	□	□	□	X	X	X	X	X	□	□	□
	□	□	□	□	□	X	X	X	X	X	□	□
	□	□	□	□	□	□	X	X	X	X	X	□
	□	□	□	□	□	□	□	X	X	X	X	X
	□	□	□	□	□	□	□	□	X	X	X	X
$n-1$	□	□	□	□	□	□	□	□	□	X	X	X

The X correspond to possible moves

thus to binary variables.

The ranks of the jobs are recomputed.

Defining a second neighborhood

$s()$

	0	$\cdots$	h	$\cdots$	$h+\delta$					$h+w$		$n-1$
0	□	□	□	□	□	□	□	□	□	□	□	□
$\cdots$	□	□	□	□	□	□	□	□	□	□	□	□
h	□	□	X	X	X	□	□	□	□	□	□	□
$\cdots$	□	□	X	X	X	X	□	□	□	□	□	□
$h+\delta$	□	□	X	X	X	X	X	□	□	□	□	□
	□	□	□	X	X	X	X	X	□	□	□	□
	□	□	□	□	X	X	X	X	X	□	□	□
	□	□	□	□	□	X	X	X	X	□	□	□
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$h+w$	□	□	□	□	□	□	□	□	□	□	□	□
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$n-1$	□	□	□	□	□	□	□	□	□	□	□	□

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The parameters of the method are:

- δ
- w: width of the interval
- *step*

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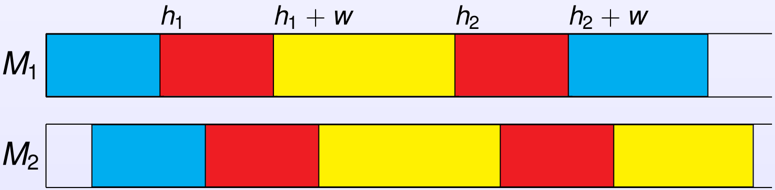


Running the Swap model

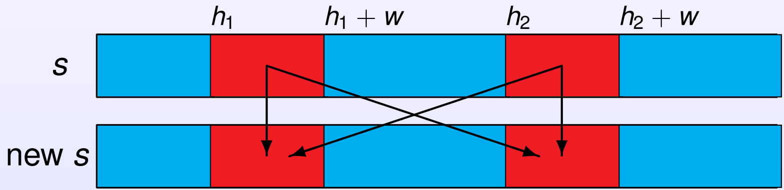
Function One-Pass(w,step)

improved ← **false**
for h_1 **from** 0 **while**$(h_1 + 2w \leq n)$ **by** $step_1$ **do**
    for h_2 **from** $h_1 + w$ **while**$(h_2 + w \leq n)$ **by** $step_2$ **do**
        Run the model
        if (there is a solution *news*) **then**
            $s \leftarrow news$
            improved ← **true**
        end if
    end for
end for
return *improved*

- w: width of the interval
- $step_1, step_2$: pace for the progression of h_1 and h_2
- the model is launched $\approx \frac{(n-2w)^2}{2 step_1 step_2}$ times

A variant of the Swap model

- In the Swap model, the (blue) jobs with the rank $[1..h_1 - 1]$, $[h_1 + w..h_2 - 1]$ and $[h_2 + w..n]$ do not change
- Only the (red) jobs with the ranks within the intervals $[h_1..h_1 + w - 1]$ and $[h_2..h_2 + w - 1]$ are recomputed.



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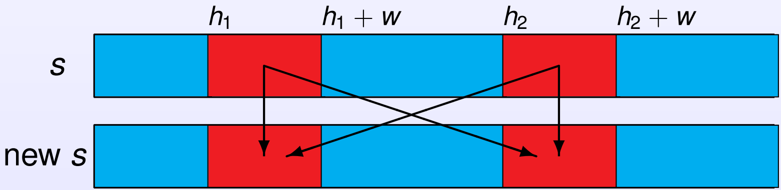

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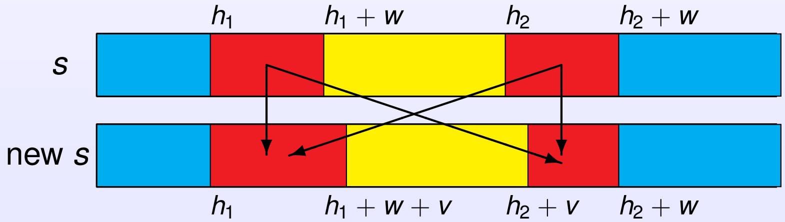
- The (red) jobs with the ranks within the intervals $[h_1..h_1 + w - 1] \cup [h_2..h_2 + w - 1]$ are recomputed
- within the intervals $[h_1..h_1 + w + v - 1] \cup [h_2 + v..h_2 + w - 1]$ where v is a small positive or negative variable.



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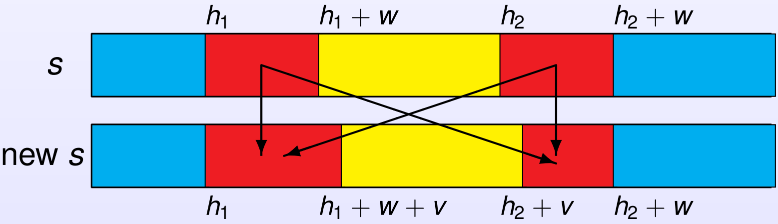

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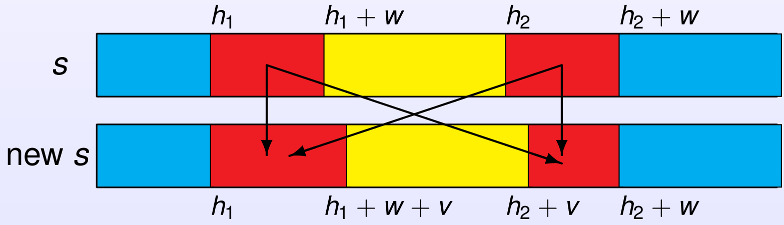

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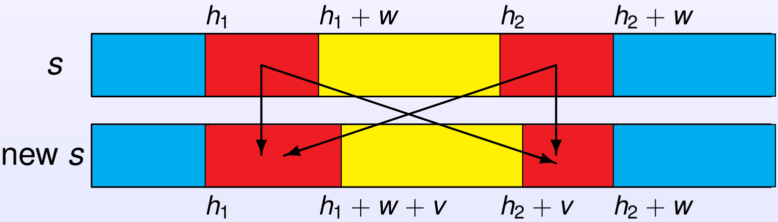
- The subsequence of the (yellow) jobs in the interval $[h_1 + w..h_2 - 1]$ is shifted of v positions
- leftward or rightward, depending on the sign of v
- HV's are to be used in the model to enable this possibility (the price of the free v)



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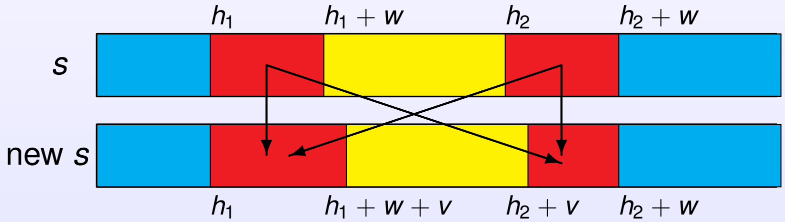

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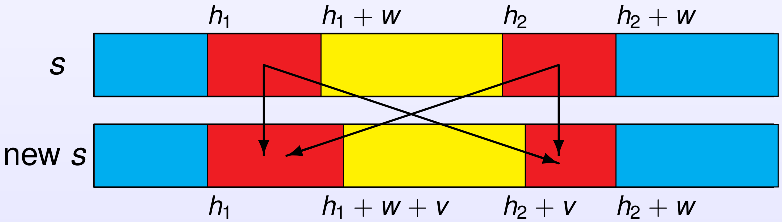

The EBSR model
Extraction and Backward Shifted Reinsertion

- The subsequence of the (red) jobs in the interval $[h_2 - w..h_2 - 1]$ gives place to the intervals $[h_1..h_1 + v - 1]$ and $[h_2 + v - w..h_2 - 1]$
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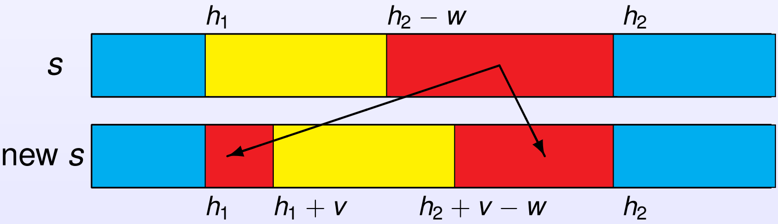

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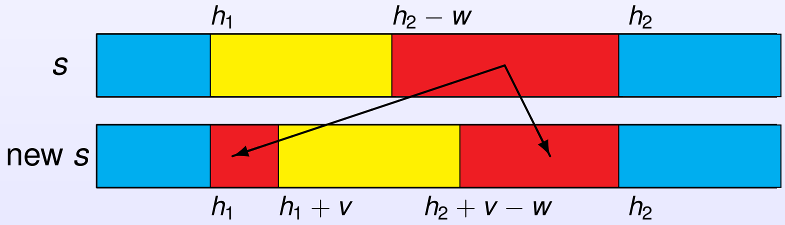

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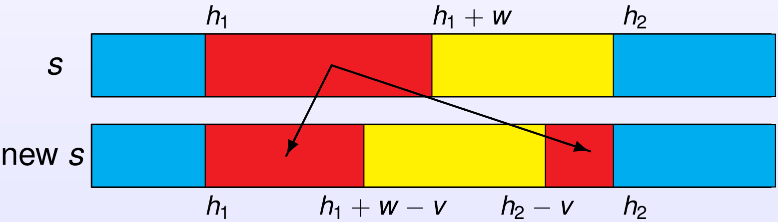

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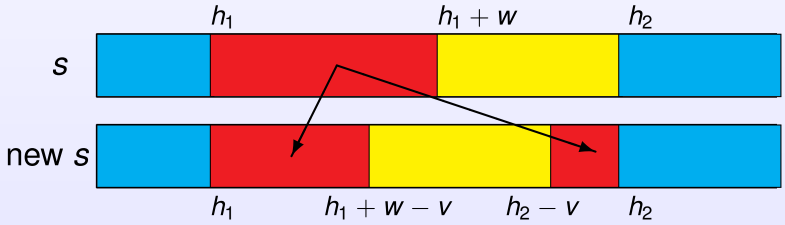

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- We choose w jobs
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4 Conclusion

Various Matheuristic methods

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2. The Shake method
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