Iterated Local Search

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Otimization Methods



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Content

- Previously on Optmization Methods
- 2 The initial solution
- 3 Experimentation
- Mathematical modeling

Local search for the $F2/\overline{T}$

- These moves can be used with every problem the solution of which is a permutation
- Move 1: consecutive swap k and k+1
- Move 2: any swap k_1 and k_2 : 2-opt
- Move 3: rotation (k_1, k_2, k_3) :
- Move 4: Extraction and Backward Shift Reinsertion::
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Algorithm of exploring a neighborhood

Local search

```
function LocalSearch(s)
```

Require: s is a solution of the problem

Ensure: The neighborhood of *s* is explored and *bestneighbor* is returned. A boolean value *Improved* is also returned.

- 1: $currentvalue \leftarrow f(s)$
- 2: bestvalue ← currentvalue
- 3: bestneighbor ← s
- 4: **for** *t* **in** neighborhood(*s*) **do**
- 5: **if** (f(t) < bestvalue) **then**
- 6: $bestvalue \leftarrow f(t)$
- 7: $bestneighbor \leftarrow t$
- 8: end if
- 9: end for
- 10: *Improved* ← (*bestvalue* < *currentvalue*)
- 11: return (Improved, bestneighbor)

Algorithm of the Iterated Local Search

Iterated Local Search

function IteratedLS(s)

Require: s is a solution of the problem

Ensure: The neighborhood of *s* is explored as long as an improvement is proved. Then the current best solution is returned.

1: *Improved* ← **true**

2: while (Improved) do

3: $(Improved, bestneighbor) \leftarrow LocalSearch(s)$

4: end while

5: return bestneighbor



- Any solution
- $\{1, 2, \dots, n\}$ or a random one
- A constructive method
- Earliest Due Date first method
- Jobs are sorted in the increasing order of the d_i's
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Algorithm Earliest Due Date

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function EDD(s)

Require: The list of the d_i 's

Ensure: A permutation corresponding to EDD is returned.

1: Pair the d_i 's with $j \# [(d_1, 1), (d_2, 2), \dots, (d_n, n)]$

2: Sort the pairs in increasing order of the first elements

3: return The sequence of the second elements

Remark: EDD is optimal for the 1//*Lmax* problem

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Algorithm NEH for the $F2/\overline{T}$

NEH

```
function NEH(s)
Require: The list of the (p_{1,i}, p_{2,i}, d_i)'s
Ensure: A permutation corresponding to NEH is returned.
 1: s \leftarrow \text{Sort the jobs in an initial order } \# \text{ e.g. EDD or} \dots?
 2: neh \leftarrow [s(1)]
 3: for k from 2 to n do # neh is a list of k-1 elements
 4:
           (mini, nextneh) \leftarrow (+\infty, [])
          for j from 1 to k do
 5:
 6:
                \sigma \leftarrow \text{Insert job } s(k) \text{ in position } j \text{ in } neh
                if (\overline{T}(\sigma) < mini) then
 7:
                      (mini, nextneh) \leftarrow (\overline{T}(\sigma), \sigma)
 8:
 9:
                end if
10:
     end for
     neh ← nextneh
11:
12: end for
13: return neh
```



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- The cardinality of the neighborhood
- Most of the times, nothing much
- Practical efficiency is the target
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Raw results

See the spreadsheet.

14 / 22

Conclusion

What can we conclude from all these numbers?

- Is the consec. swap nbh included in 2opt-nbh?
- Is the 2-opt nbh included in the 3-opt nbh?
- Consec swap nbh included in 2-opt, EBSR and EFSF
- ullet If $Nbh_1 \subset Nbh_2$, is LS_2 better than LS_1 ?
- If Nbh₁ ⊂ Nbh₂ is t// S₂ better than t// S₂?

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Thinking further

Look at this case:

	$p_{1,j}$	$p_{2,j}$	dj
1	10	1	12
2	10	1	13
3	1	10	14
4	1	10	22

1 before 2 and 3 before 4

Domination rule: if $p_{1,j_1} = p_{1,j_2}$, $p_{2,j_1} = p_{2,j_2}$ and $d_{j_1} \le d_{j_2}$ then j_1 can be scheduled before j_2

Swapping j_2 before j_1 cannot increase the criterion. We can restrict our research to the set of the solutions such that j_1 is before j_2 .

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18 / 22

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- Write the constraints
- and the objective function
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- indicating whether j_1 is before j_2

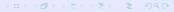
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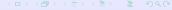


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Optim. Meth.

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Constraints

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$$\forall (j_1, j_2), 1 \leq j_1 < j_2 \leq n, \ C_{1,j_1} + p_{1,j_1} \leq C_{1,j_2} \text{ if } Z_{j_1,j_2} = 0$$

$$C_{1,j_2} + p_{1,j_2} \leq C_{1,j_1} \text{ if } Z_{j_1,j_2} = 1$$

$$\forall (j_1, j_2), \qquad C_{1,j_1} + p_{1,j_1} \le C_{1,j_2} + HV \times Z_{j_1,j_2}$$

$$1 \le j_1 < j_2 \le n, C_{1,j_2} + p_{1,j_2} \le C_{1,j_1} + HV \times (1 - Z_{j_1,j_2})$$
(2)

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 (4)

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Constraints and objective function

$$\forall j, 1 \leq j \leq n, \quad C_{1,j} + p_{1,j} \leq C_{2,j}$$
 (5)
 $T_j \geq C_{2,j} - d_j$ (6)

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Minimize
$$\sum_{j=1}^{n} \overline{T}_{j}$$

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Objective function

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