Lecture 8, Classification, Recognition Based on Gonzales Woods, chapter 12

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Image analysis fundamental steps

image acquisition

preprocessing, enhancement

segmentation

feature extraction description

Classification, interpretation, recognition

result

This lecture

- Object vs. pixelwise classification

- Template Matching for segmentation/recognition

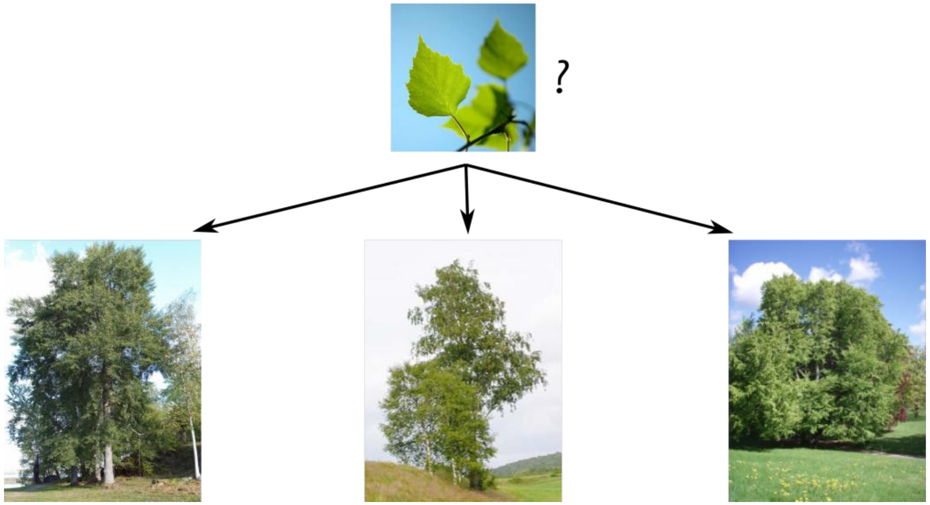
- Feature vectors and feature space, scatterplots

- Supervised classification
    - Min dist
    - Maximum likelihood
    - Decision trees
    - Neural Networks

- Unsupervised classification (clustering)
    - K-means
    - hierarchical

What is classification?

- Classification is a procedure in which individual **items** (objects/patterns/image regions/pixels) are grouped based on the similarity between the item and the description of the group.

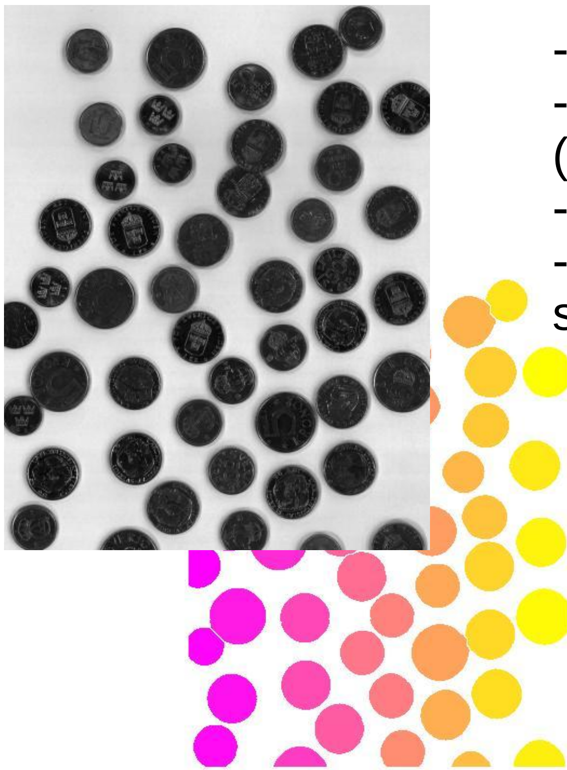


Object-wise and pixel-wise classification

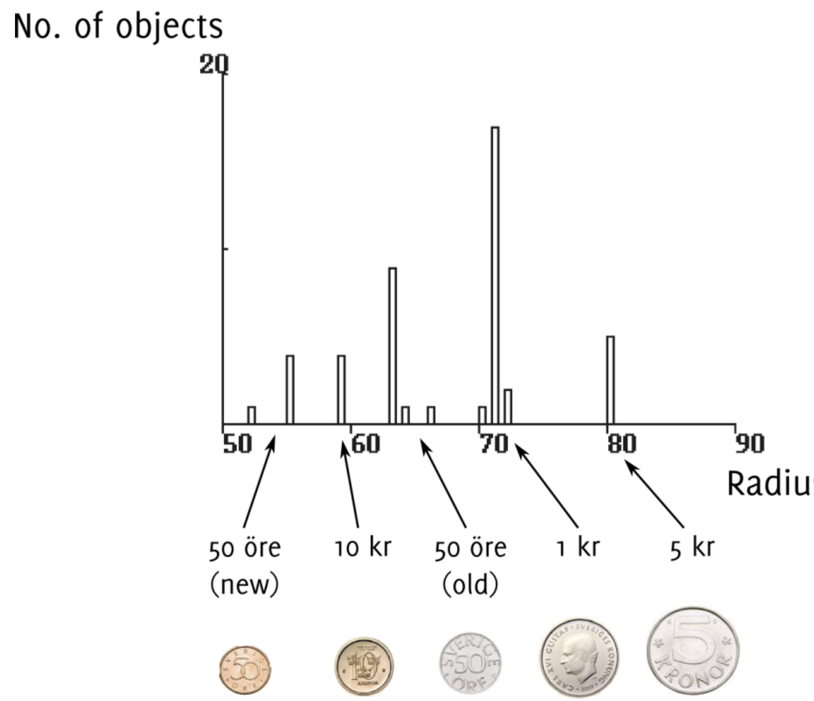
- **Object-wise classification**
  - Uses shape, size, mean intensity, mean color etc. to describe patterns.
- **Pixel-wise classification**
  - Uses intensity, color, texture, spectral information etc. (=>segmentation)

Object-wise classification

- Example



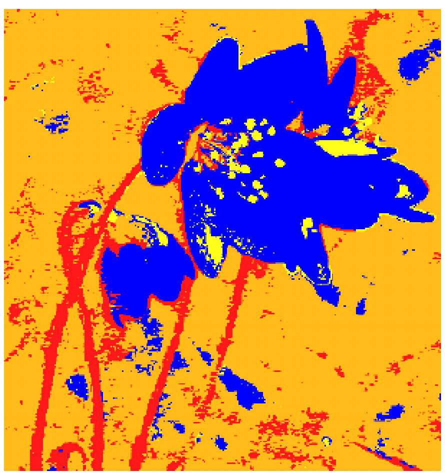
- 48 objects
- 1 feature (radius)
- 4 (5) classes
- 1-D feature space



Object-wise supervised classification

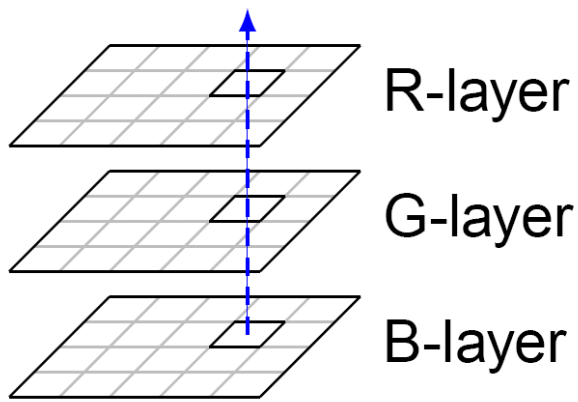
- Segment the image into regions and label them. These are the patterns to be classified.
- Extract (calculate) features for each pattern.
- Train a **classifier** on examples with known class to find **discriminant functions** in the feature space.
- For new examples decide their class using the discriminant function.

Pixel-wise classification



- Color image example.
- 256 x 256 patterns (pixels).
- 3 features (red, green and blue band).
- 4 classes (stamen, leaf, stalk and background).

$$\mathbf{x_{ij}} = [r_{ij}\, g_{ij}\, b_{ij}]^T$$

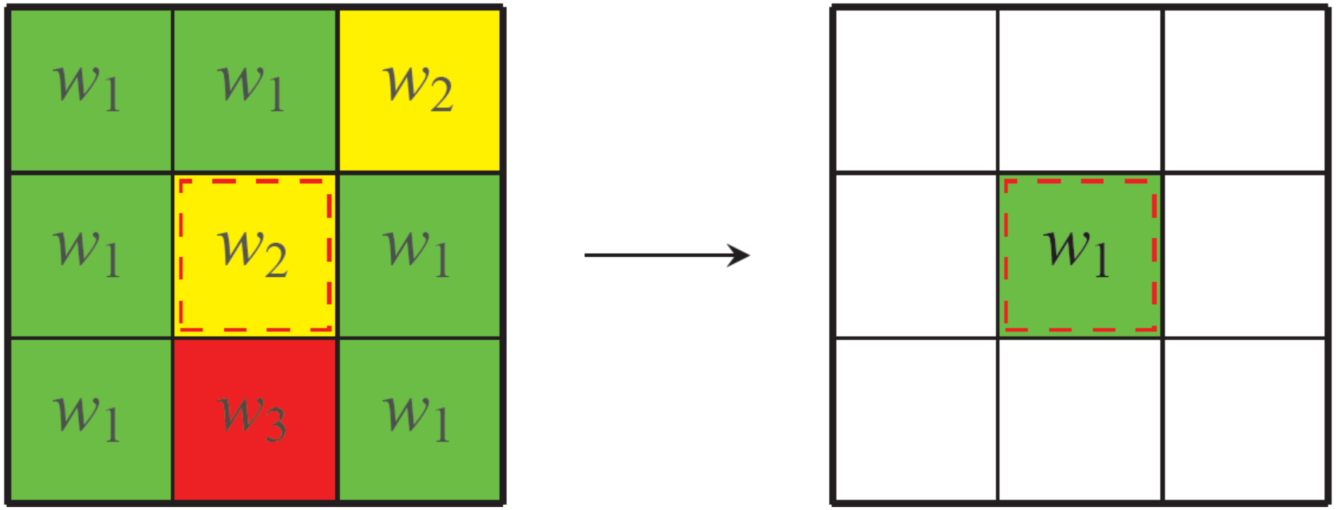


Pixel-wise classification

- The pattern is a pixel in a non-segmented image.
- Extract (calculate) features for each pattern (pixel), e.g., color, gray-level representation of texture, temporal changes.
- Train classifier.
- New samples are classified by classifier.
- Perform relaxation=majority filtering (optional).

Relaxation/majority filtering

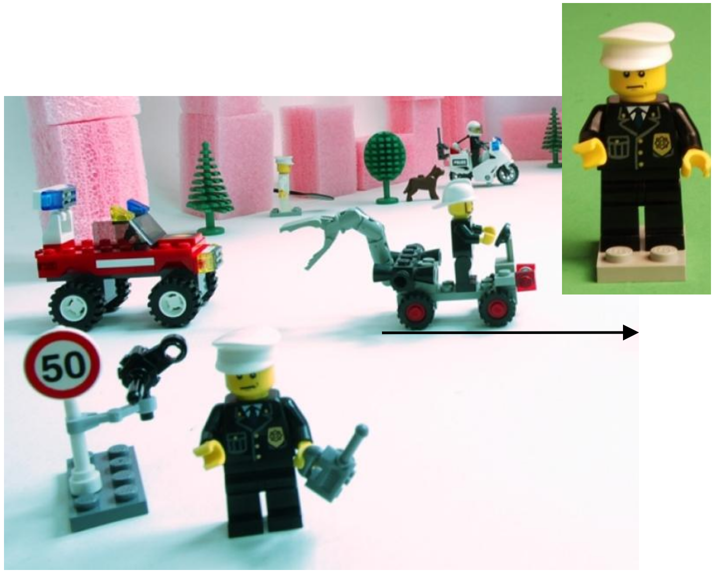
- Used in pixel-wise classification to reduce noise
- Neighborhood size determines the amount of relaxation



Matching by correlation

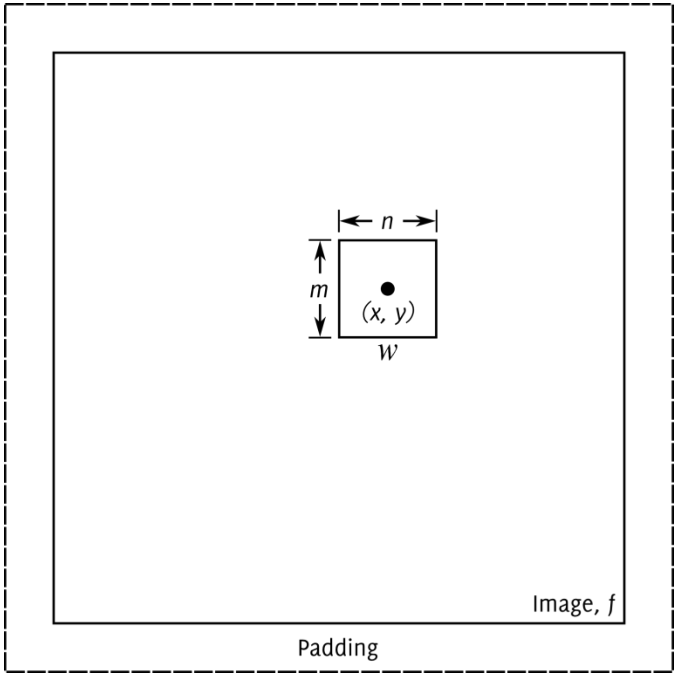
- Variant of object-wise classification.
- Locate specific objects or patterns.
- Often used for segmentation.

http://vicos.fri.uni-lj.si/leegle/



Matching by correlation

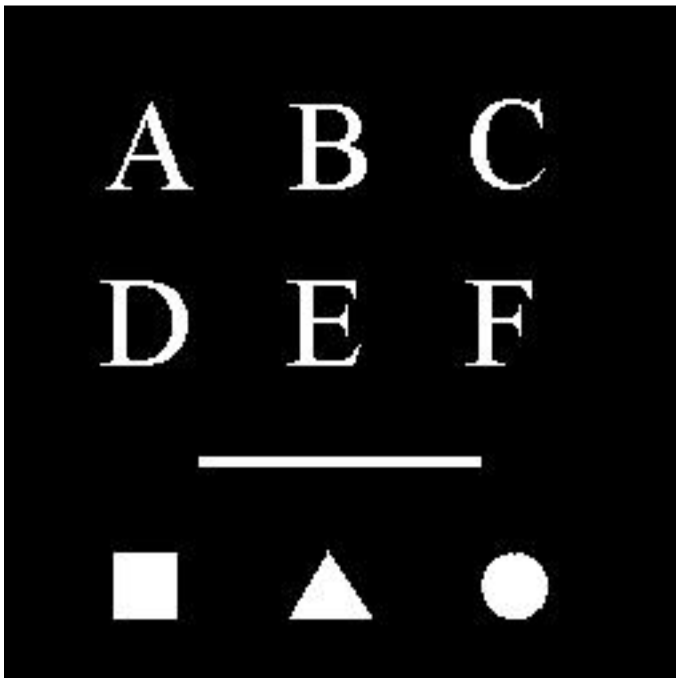
- Use correlation to match mask w(x,y) with image f(x,y).
- Slide the mask over the image and calculate correlation at each position.



$$c(x,y) = \sum_s \sum_t w(s,t) f(x+s, y+t)$$

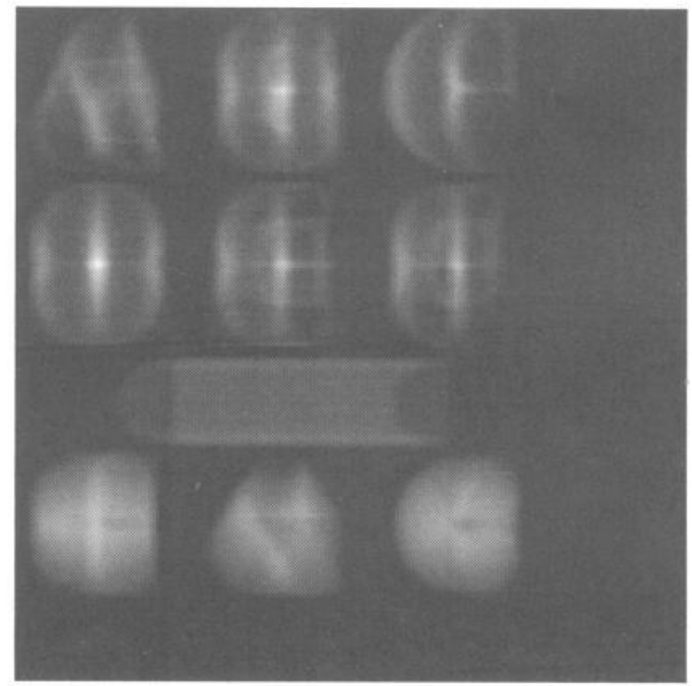
Matching by correlation

- Example



f

w



c

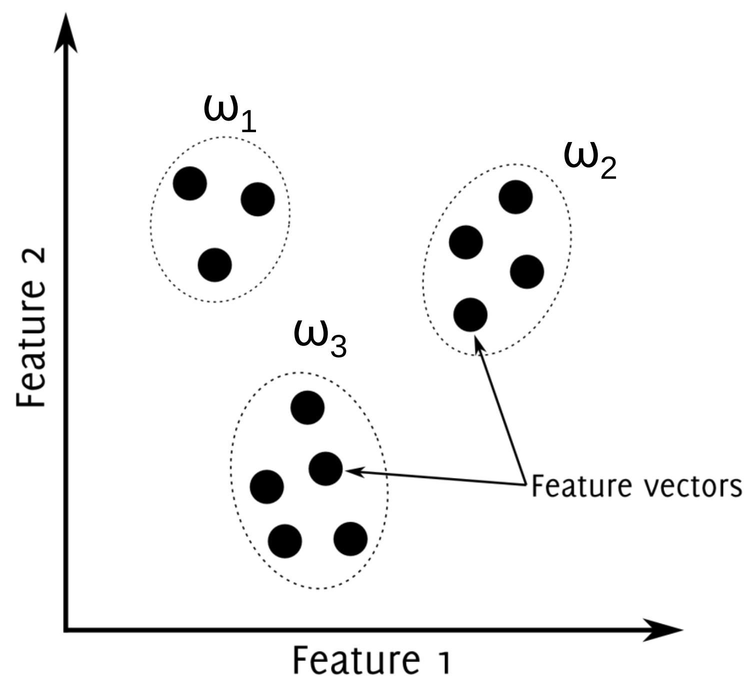
Some important concepts

- Arrangements of descriptors are often called **patterns**.
- Descriptors are often called **features**.
- The most common pattern arrangement is a **feature vector** with n dimensions.
- Patterns are placed in **classes** of objects which share common properties. A collection of W classes are denoted $\omega_{1,}\, \omega_{2,}\; \ldots.\; \omega_{W,}$

Feature vectors and feature space

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

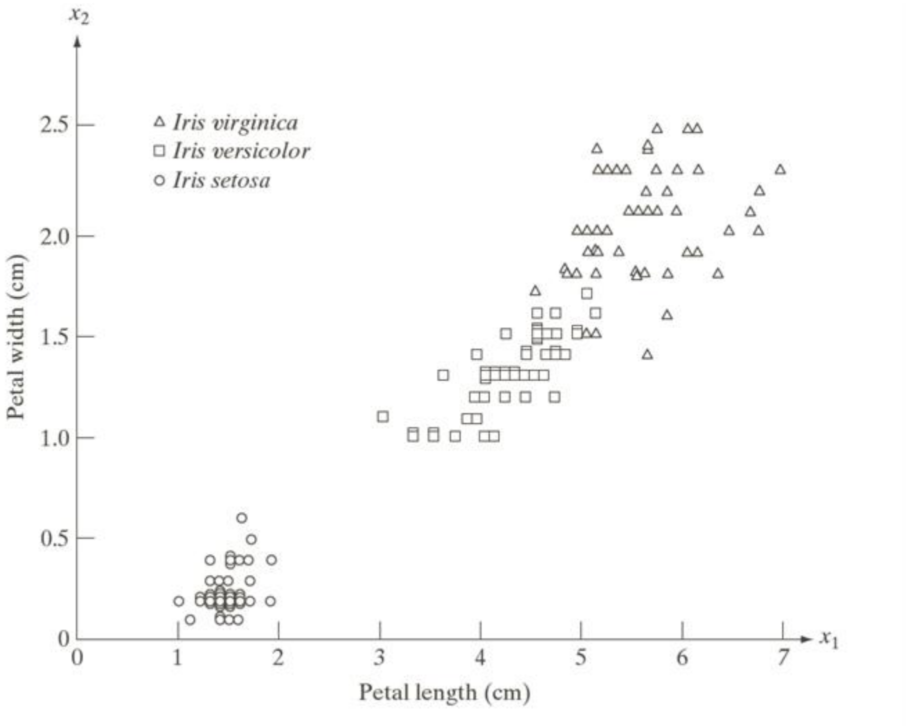
Example of a feature space (N=2)



Two features, three classes

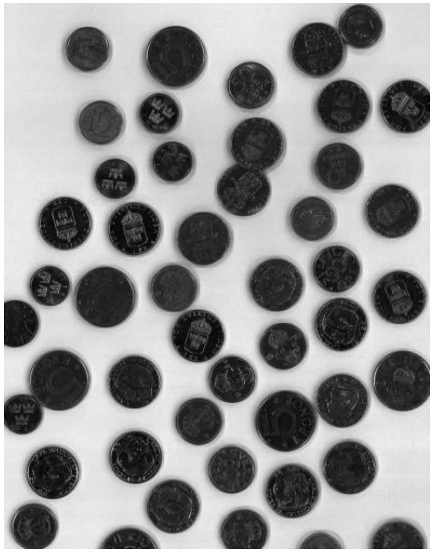
Feature vectors and feature space

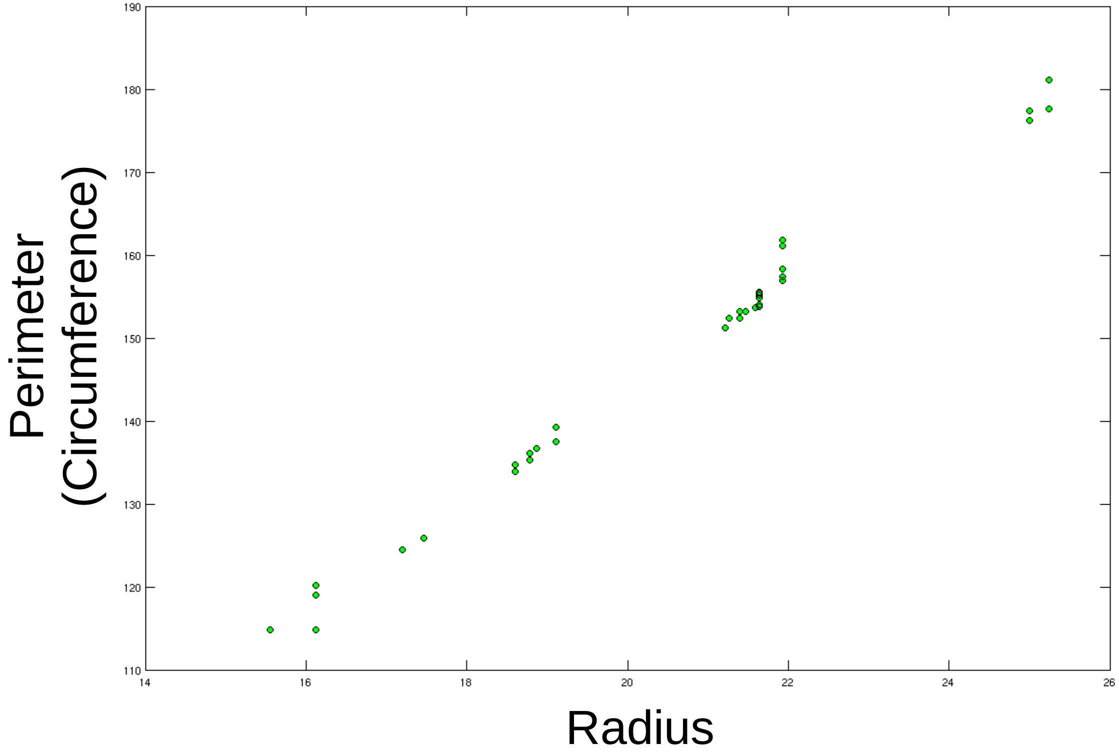
- Flower example



Scatter plots

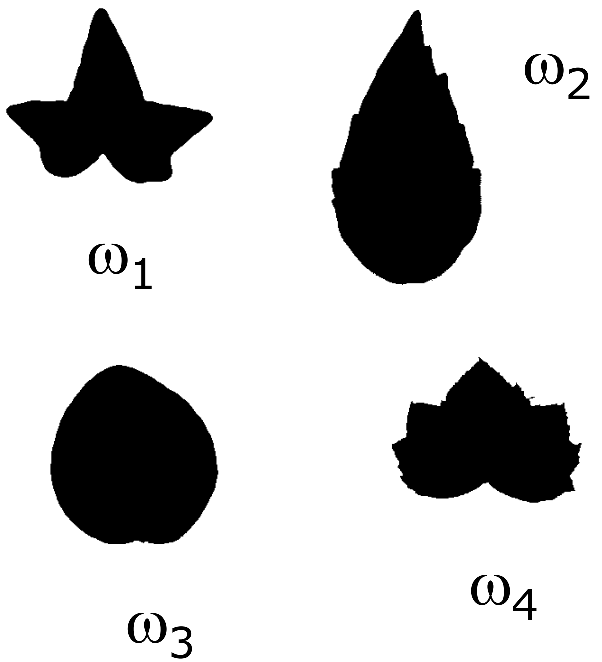
- A good way to illustrate relationships between features.

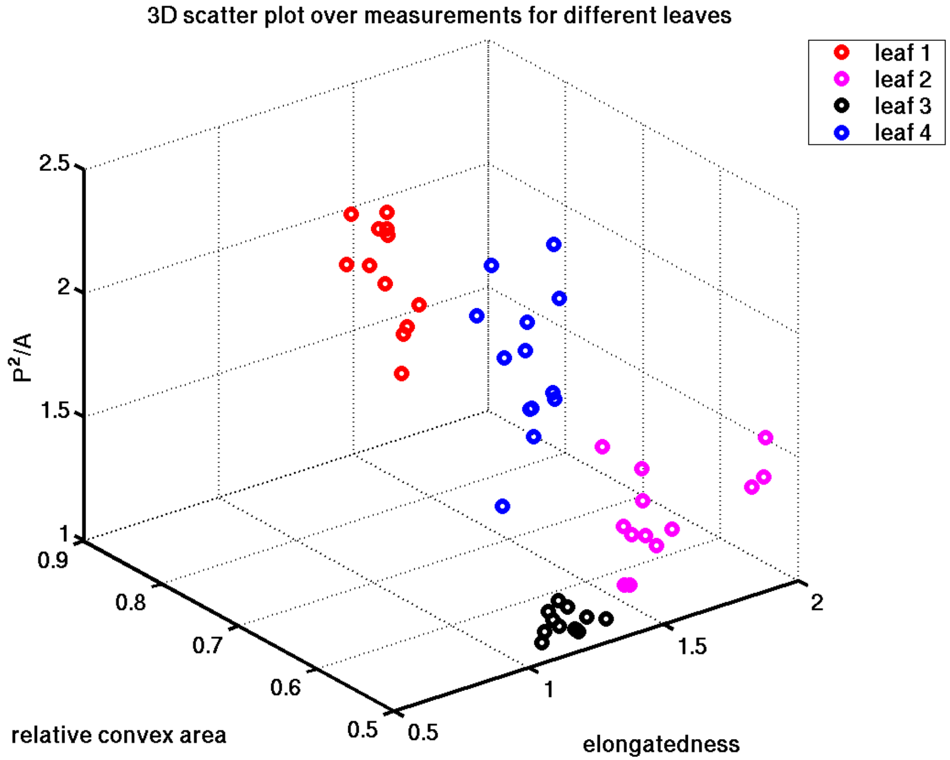




Scatter plots

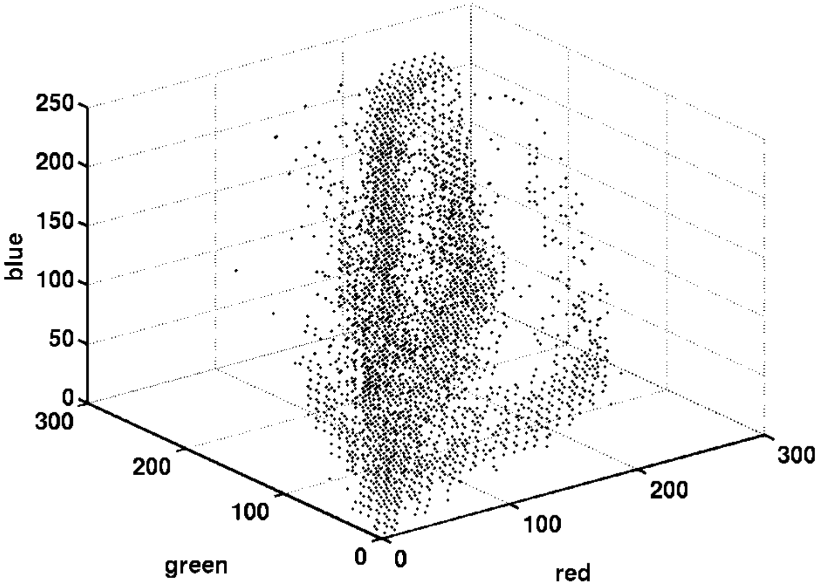
- Example of 3-dimensional plot





Scatter plots

- Example: RGB color image.

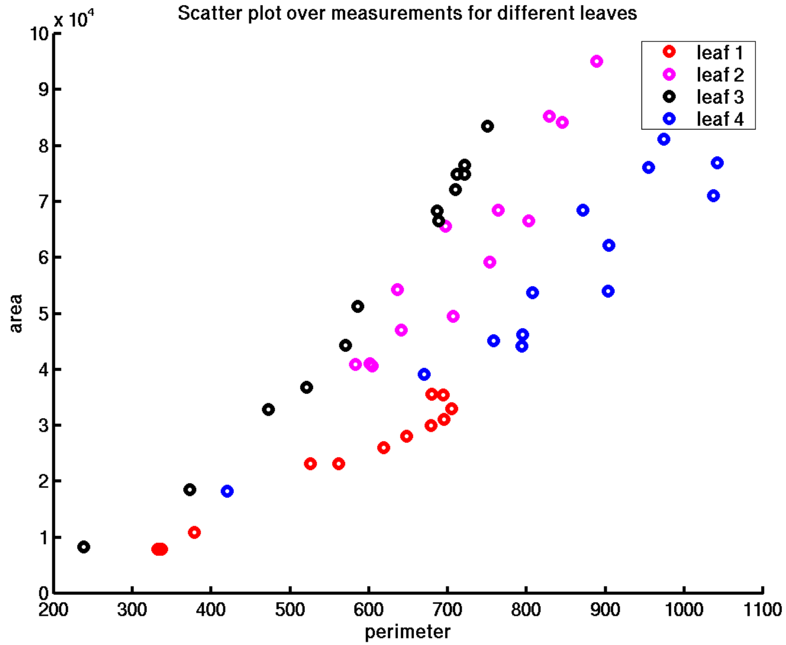


Feature selection

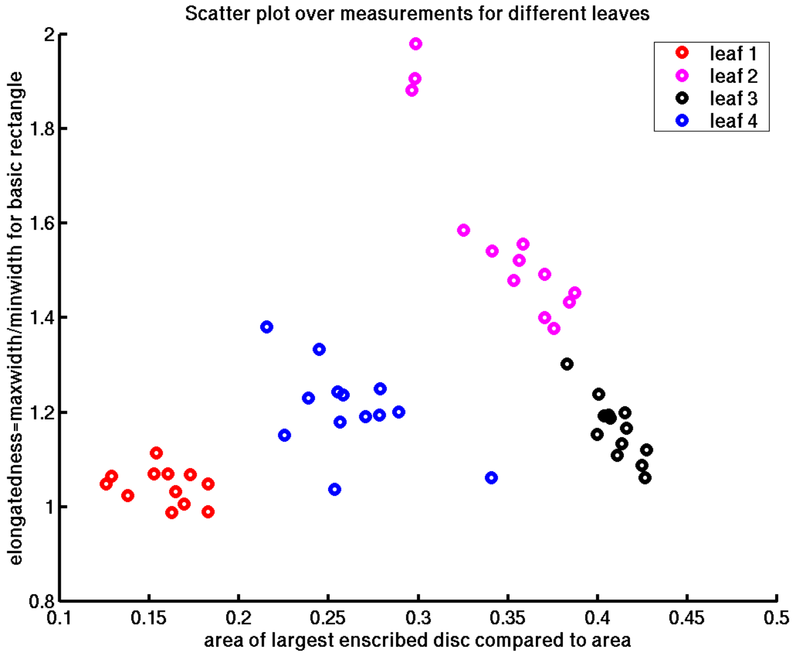
- The goal in feature selection (which is a prerequisite for ALL kinds of classification) is to find a limited set of features that can discriminate between the classes.
- Adding features without “verification” will often NOT improve the result.

Feature selection

- Some examples



Limited separation between classes



Good separation between classes

Train and classify=supervised classification

- **Training**
  - Find rules and discriminant functions that separate the different classes in the feature space using known examples.
- **Classification**
  - Take a new unknown example and put it into the correct class using the discriminant functions.

Supervised classification->
First apply knowledge, then classify

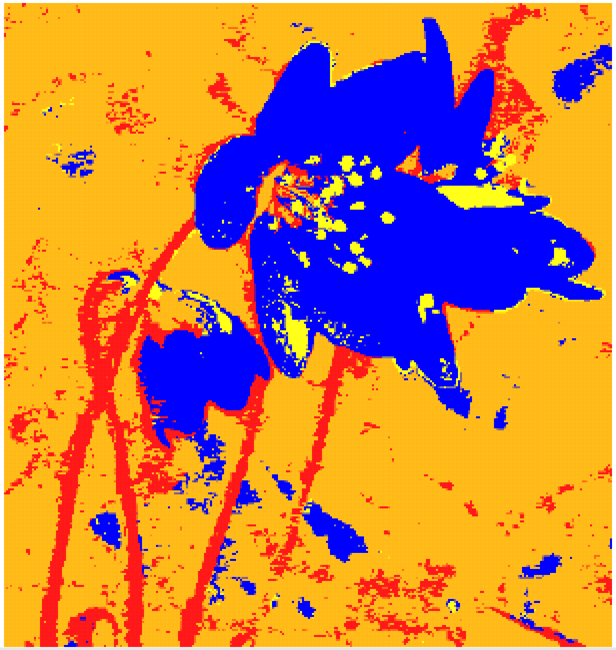
Train and classify

- Regions in the image are used as training examples (pixel-wise classification).

Original image

Training areas

Classification



Discriminant functions

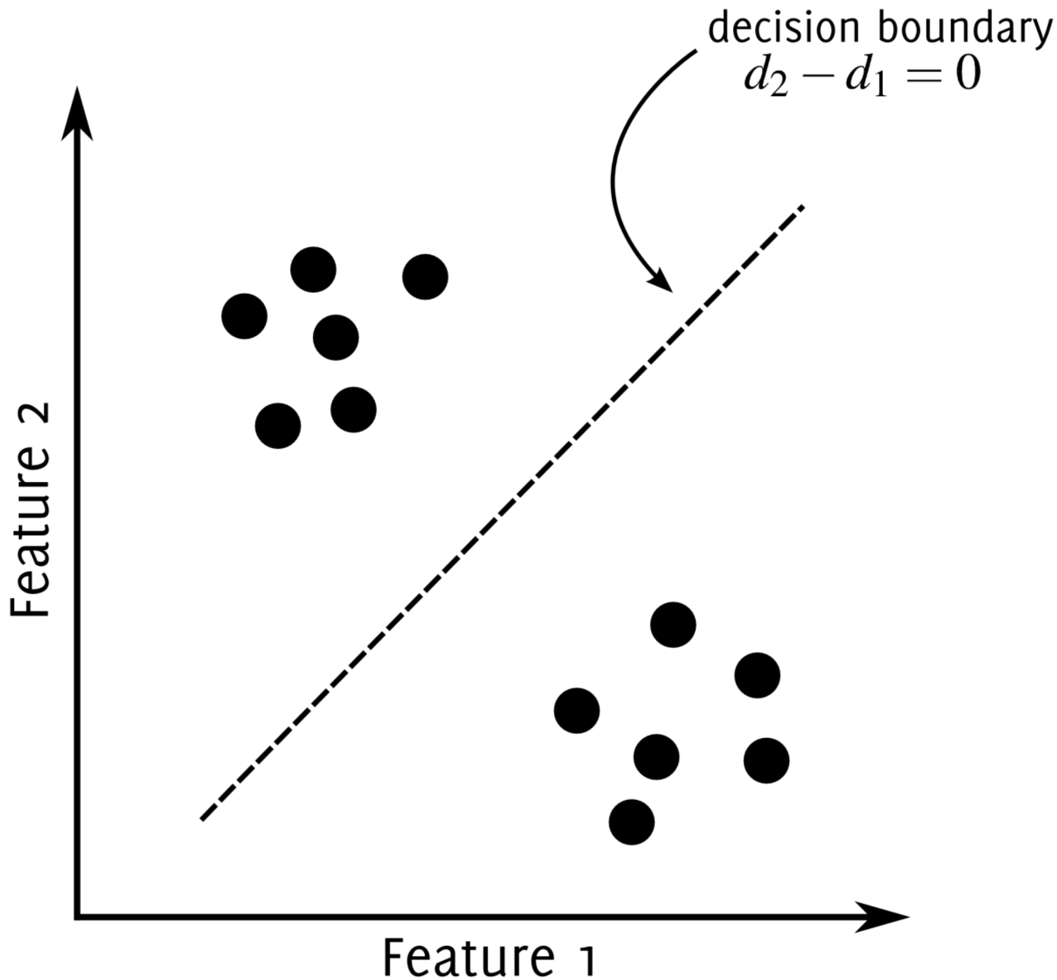
- A **discriminant function** for a class is a function that will yield larger values than functions for other classes if the pattern belongs to the class.

$$d_i(\mathbf{x}) > d_j(\mathbf{x}) \quad j = 1, 2, ..., W; \quad j \neq i$$

- For W pattern classes, we have W discriminant functions.
- The **decision boundary** between class i and j

$$d_i(\mathbf{x}) - d_j(\mathbf{x}) = 0$$

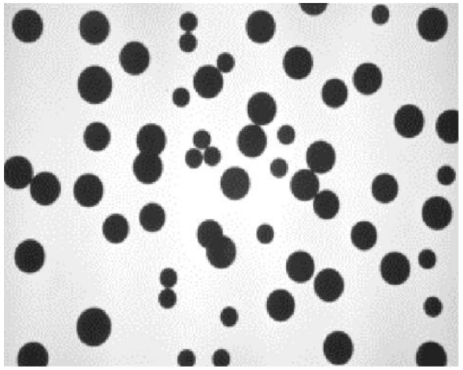
Decision boundary



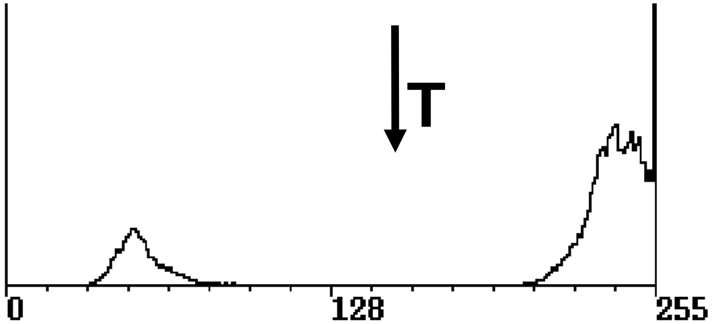
Example:Thresholding (simple classification -1D)

- Classify image into foreground and background.

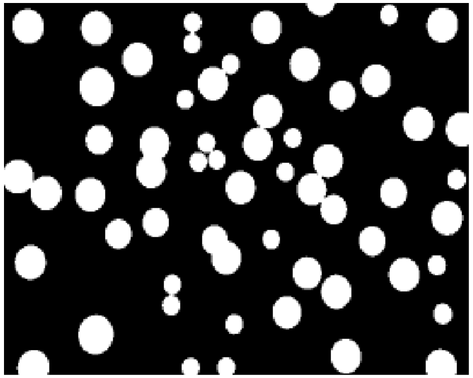
$f(x,y)$



$$g(x,y) = \begin{cases} 1 & \text{if } f(x,y) \geq T; \\ 0 & \text{if } f(x,y) < T. \end{cases}$$



Object Background

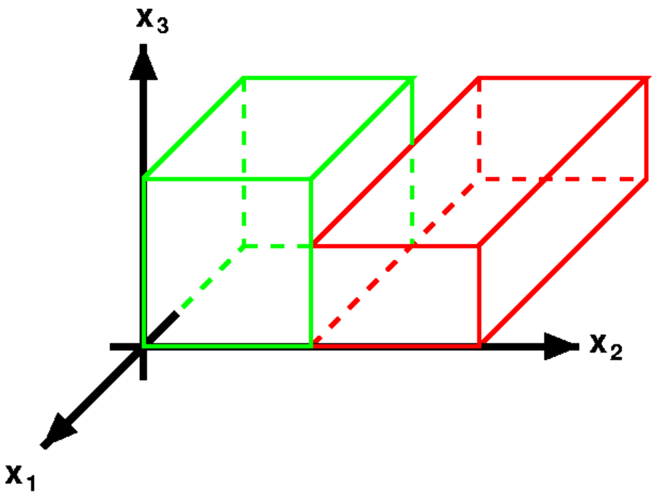


$g(x,y)$

Result: binary image

Example:Box classification

- Intervals for each class and feature
- All objects with feature vectors within the same box belong to the same class
- Generalized thresholding
  - Multispectral thresholding

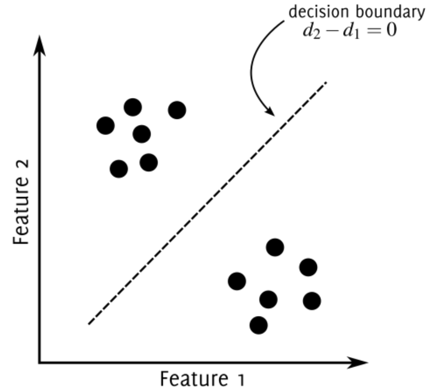


Bayesian classifiers

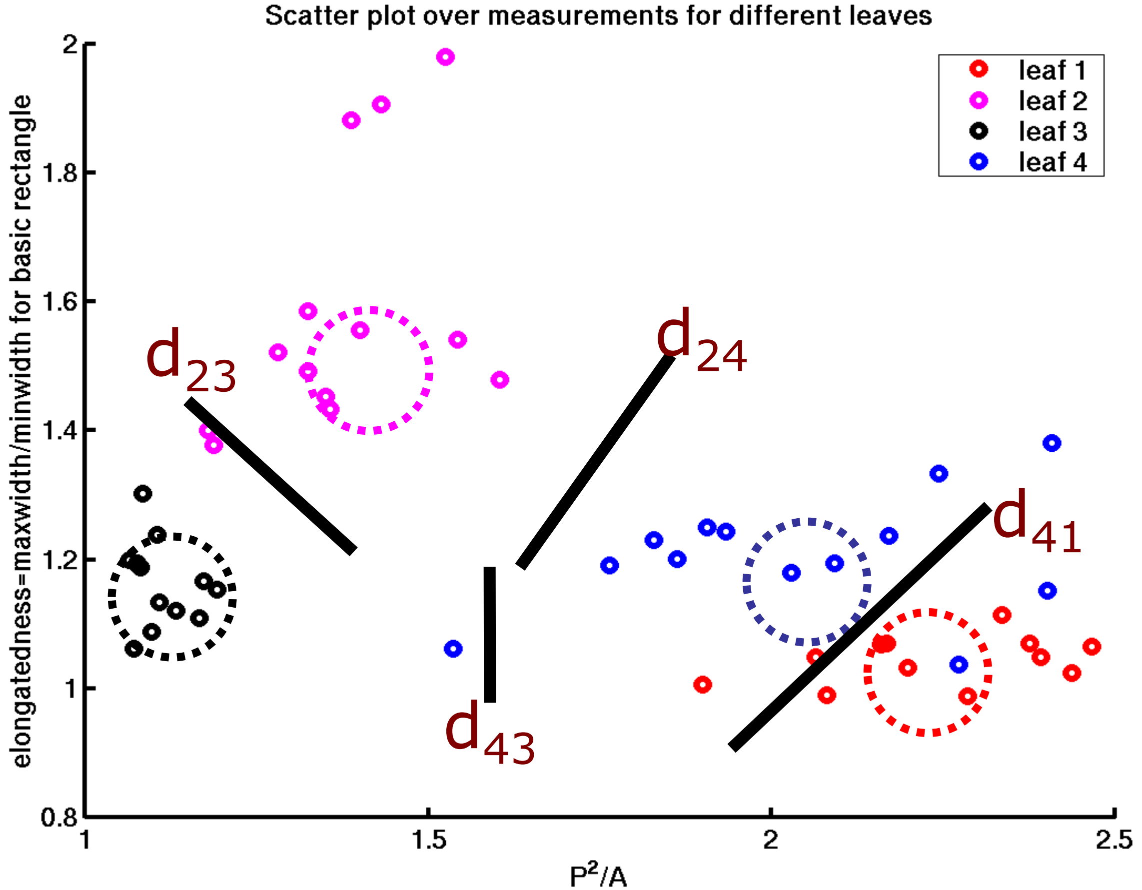
- Bayes 1702-1761
- Includes *a priori* knowledge of class probability
- Cost of errors
- Combination gives an optimum statistical classifier (in theory), minimizes total average loss
- Assumptions to simplify classifier
  - Minimum distance (MD) classifier
  - Maximum likelihood (ML) classifier

Minimum distance classifier

- Each class is represented by its mean vector
- Training is done using the objects/pixels of known class and calculate the mean of the feature vectors for the objects within each class
- New objects are classified by finding the closest mean vector



Minimum distance classifier leaves: elongatedness – P^2/A



Limitation of minimum distance classifier

Minimum distance classifier is suitable to use when:

Distance between means is **large** compared to randomness of each class with respect to its mean

Optimum performance:
distribution forms spherical hypercloud in nD pattern space

Maximum likelihood classifier

- Classify according to the greatest probability (taking variance and covariance into consideration)
- Assume that the distribution within each class is Gaussian
- The distribution within each class can be described by a mean vector and a covariance matrix

Variance & covariance

- Variance: spread or randomness for class&feature
- Covariance: influence/dependency between different features
- Described by *covariance matrix*

	feature 1	feature 2	feature 3
feature 1	1	1 & 2	1 & 3
feature 2	1 & 2	2	2 & 3
feature 3	1 & 3	2 & 3	3

Computation of covariance

- Features: x_1, x_2, x_3, ...
- Feature vector for object i: $x_{1,i}$, $x_{2,i}$, $x_{3,i}$, ...
- Mean for each feature (and class): x_{mean_1} x_{mean_2} x_{mean_3} ...

$$\mathrm{cov}\left(x_i, x_j\right) = \frac{1}{n-1} \sum_{k=1}^{n} \left(x_{i,k} - x_{mean_i}\right) \cdot \left(x_{j,k} - x_{mean_j}\right)$$

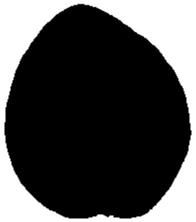
Mean vector for each class

Computed from training data

$$\begin{array}{l} \text{feature 1} \\ \text{feature 2} \\ \text{feature 3} \end{array} \quad \begin{pmatrix} 1.0441 \\ 2.2383 \\ 0.7008 \end{pmatrix} \quad \begin{pmatrix} 1.5855 \\ 1.3763 \\ 0.5275 \end{pmatrix} \quad \begin{pmatrix} 1.1654 \\ 1.3763 \\ 0.5275 \end{pmatrix} \quad \begin{pmatrix} 1.2073 \\ 2.0342 \\ 0.5756 \end{pmatrix}$$

ω_1

ω_2



ω_3

ω_4

Covariance matrix for the leaves

$$C_{\omega_1} = \begin{pmatrix} 0.0014 & 0.0061 & 0.0011 \\ 0.0061 & 0.0299 & 0.0052 \\ 0.0011 & 0.0052 & 0.0010 \end{pmatrix}$$

$$C_{\omega_3} = \begin{pmatrix} 0.0042 & 0.0026 & 0.0002 \\ 0.0026 & 0.0018 & 0.0001 \\ 0.0002 & 0.0001 & 0.0000 \end{pmatrix}$$

$$C_{\omega_2} = \begin{pmatrix} 0.0409 & 0.0244 & 0.0034 \\ 0.0244 & 0.0163 & 0.0022 \\ 0.0034 & 0.0022 & 0.0003 \end{pmatrix}$$

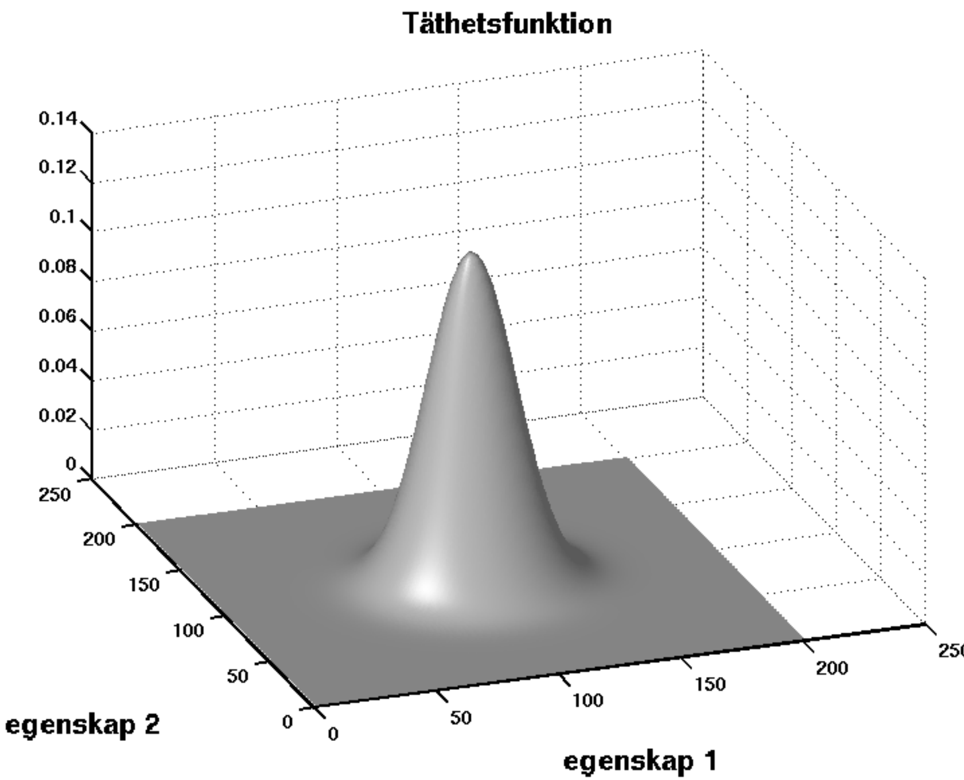
$$C_{\omega_4} = \begin{pmatrix} 0.0088 & 0.0232 & 0.0015 \\ 0.0232 & 0.0680 & 0.0042 \\ 0.0015 & 0.0042 & 0.0003 \end{pmatrix}$$

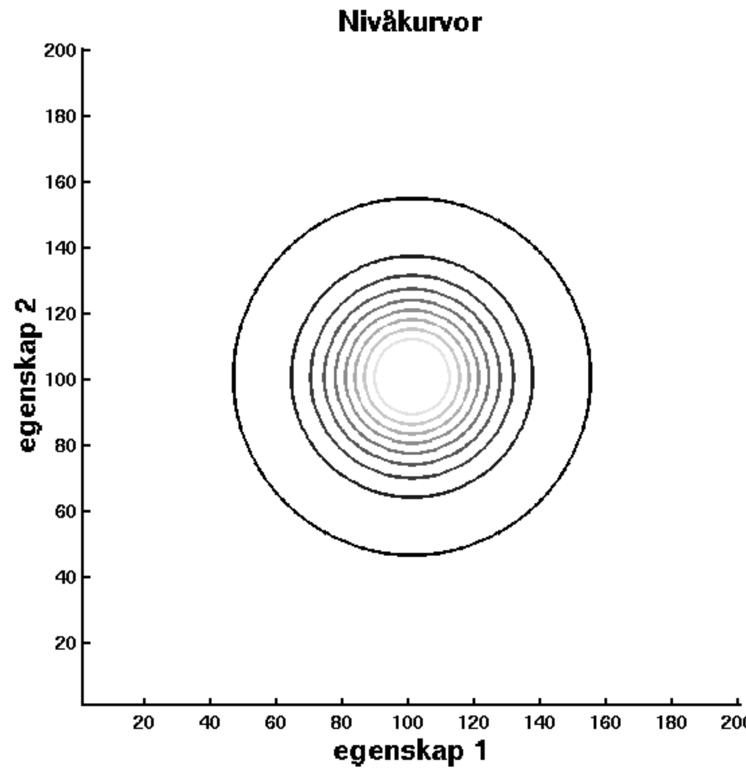
x_1=elongatedness
x_2=P^2/A
x_3=relative convex area

$$C_{\omega_i} = \begin{pmatrix} \operatorname{cov}(x_1, x_1) & \operatorname{cov}(x_1, x_2) & \operatorname{cov}(x_1, x_3) \\ \operatorname{cov}(x_1, x_2) & \operatorname{cov}(x_2, x_2) & \operatorname{cov}(x_2, x_3) \\ \operatorname{cov}(x_1, x_3) & \operatorname{cov}(x_2, x_3) & \operatorname{cov}(x_3, x_3) \end{pmatrix}$$

Density function

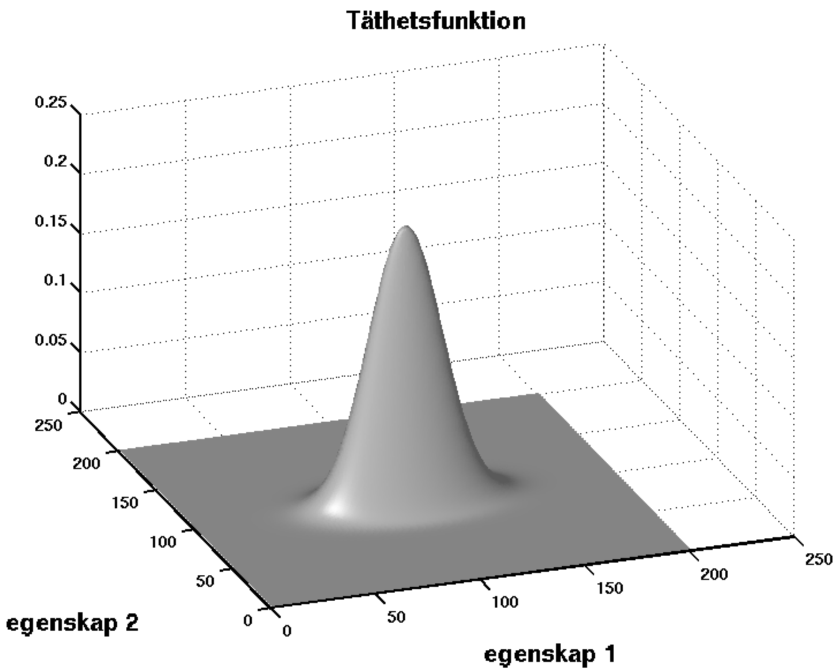
Equal variance

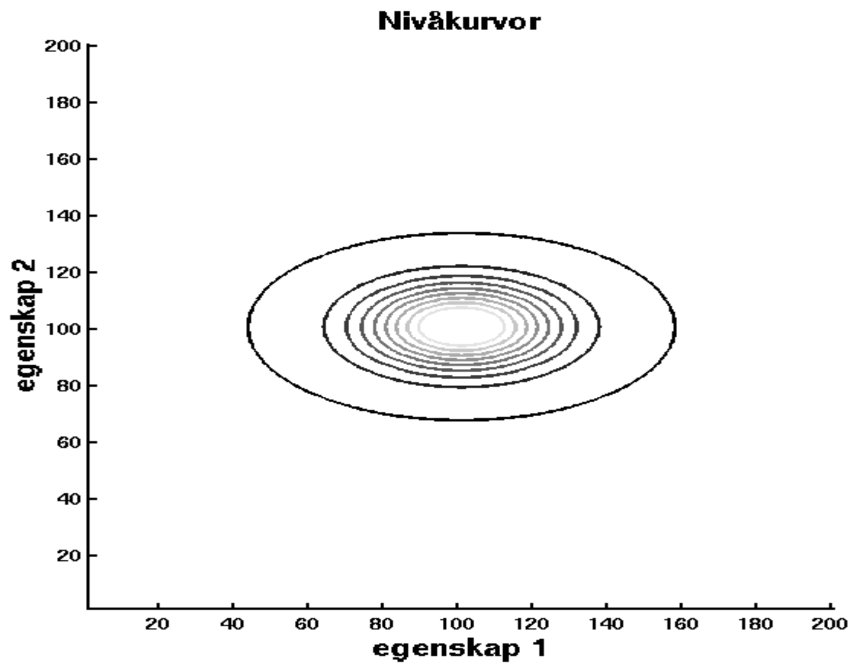




Density function

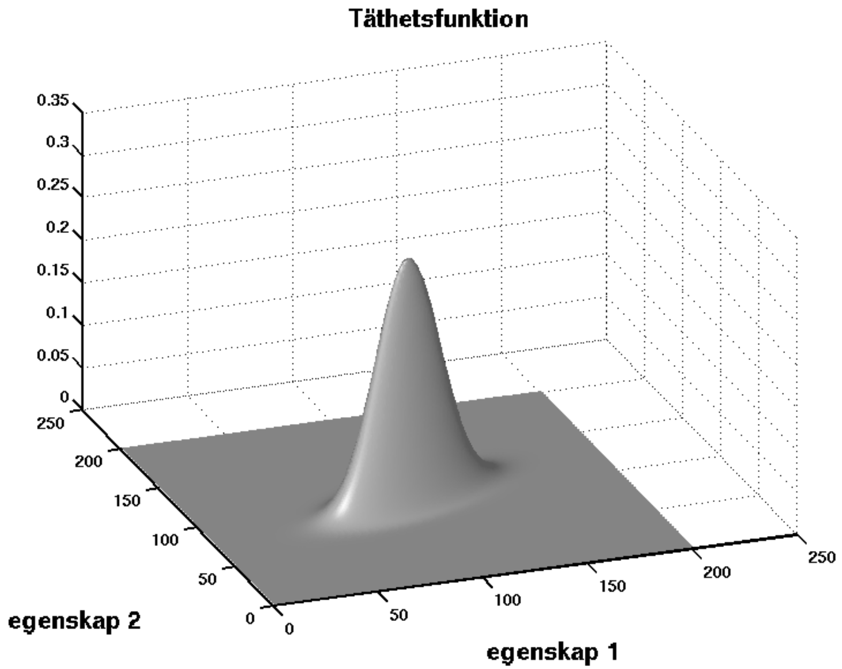
Different variance

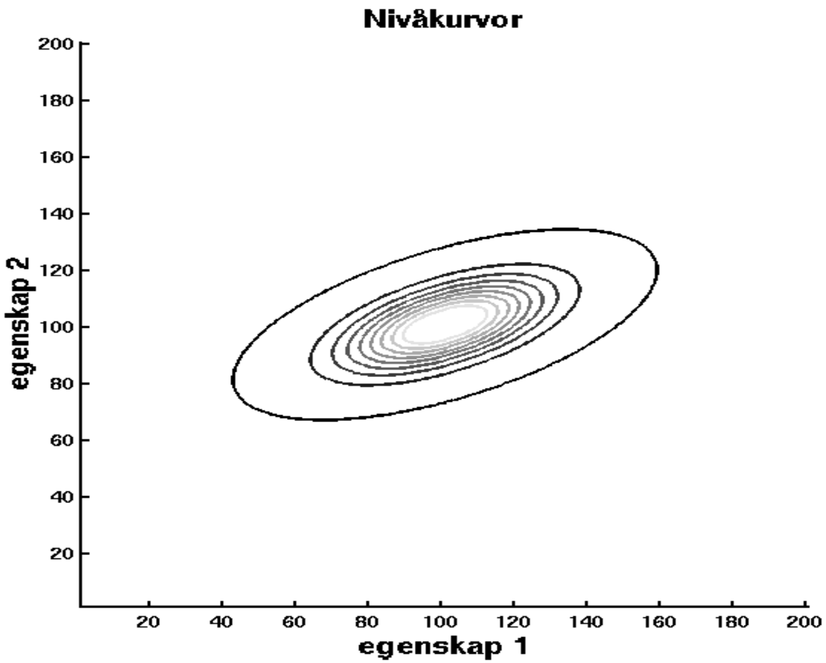




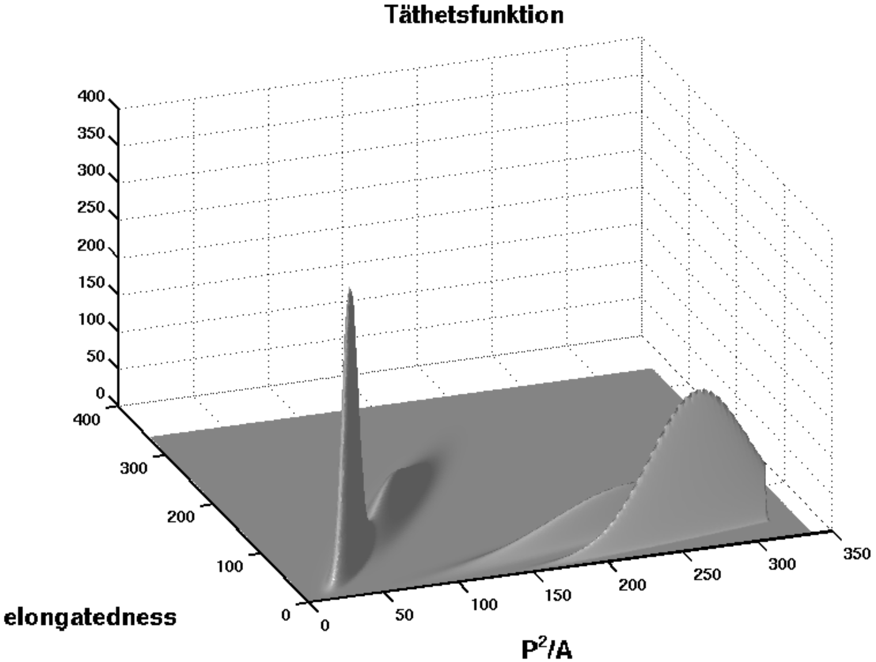
Density function

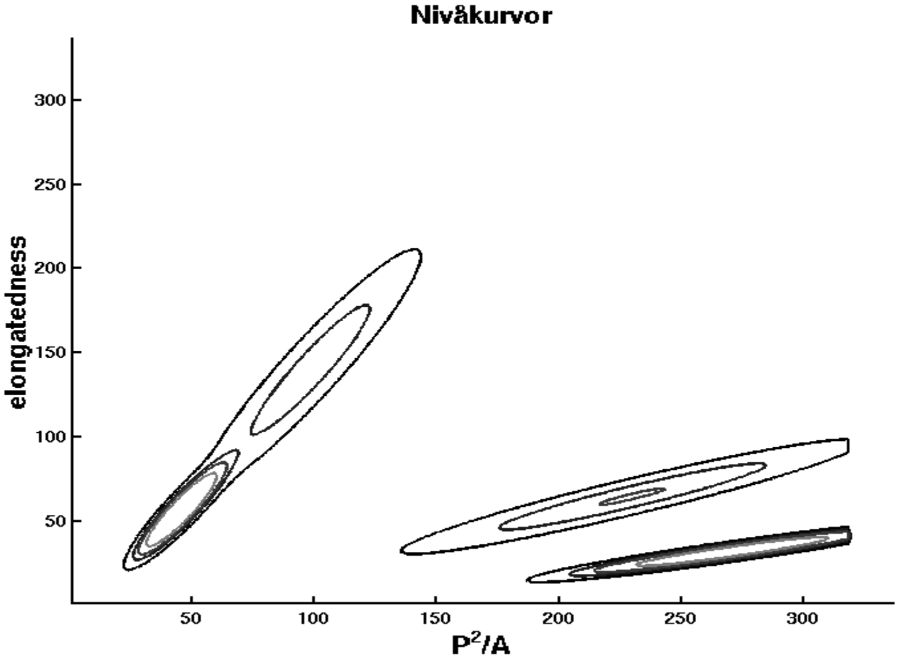
Covariance≠0





Likelihood
Leaves: elongatedness – P^2/A





Assumptions on covariance matrix

- Case 1 (**MinDist**)
  - independent features → no covariance
  - equal variance

$$d_j(\mathbf{x}) = \mathbf{x}^T \mathbf{m}_j - \frac{1}{2}\mathbf{m}_j \mathbf{m}_j$$

- Case 3 (**EqualCovar**)
  - same covariance for all classes

Case 2 (**UnCorrelated**)

independent features → no covariance

different variance for different features

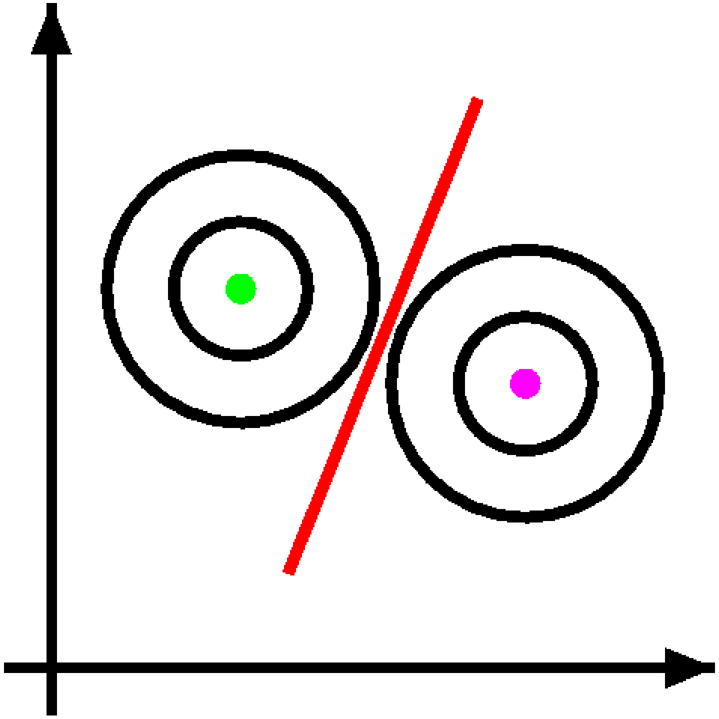
Case 4 (**General**)

different covariance matrices for alla classes

$$d_j(\mathbf{x}) = \ln P(\omega_j) + \mathbf{x}^T \mathbf{C}^{-1} \mathbf{m}_j - \mathbf{m}_j^T \mathbf{C}^{-1} \mathbf{m}_j$$

Case 1

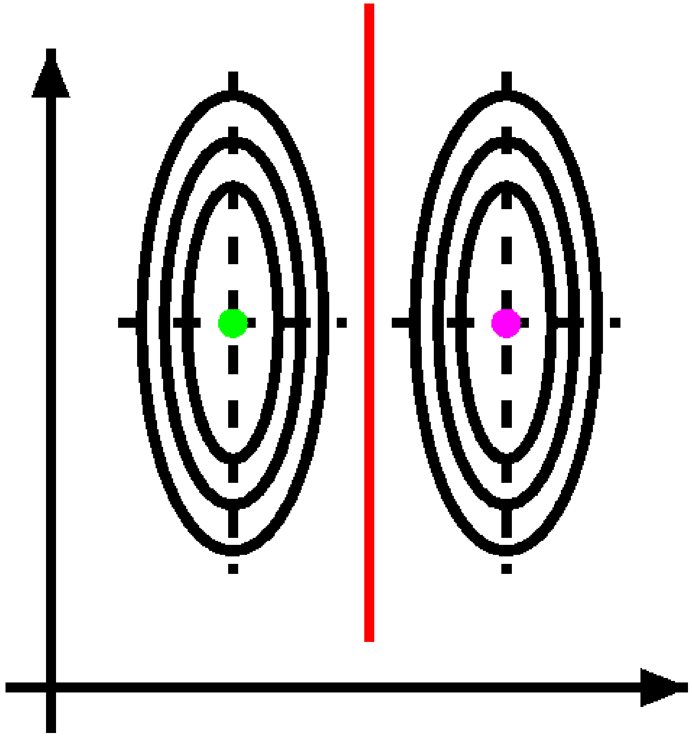
Minimum distance classifier:
- independent features
- equal variance



Case 2

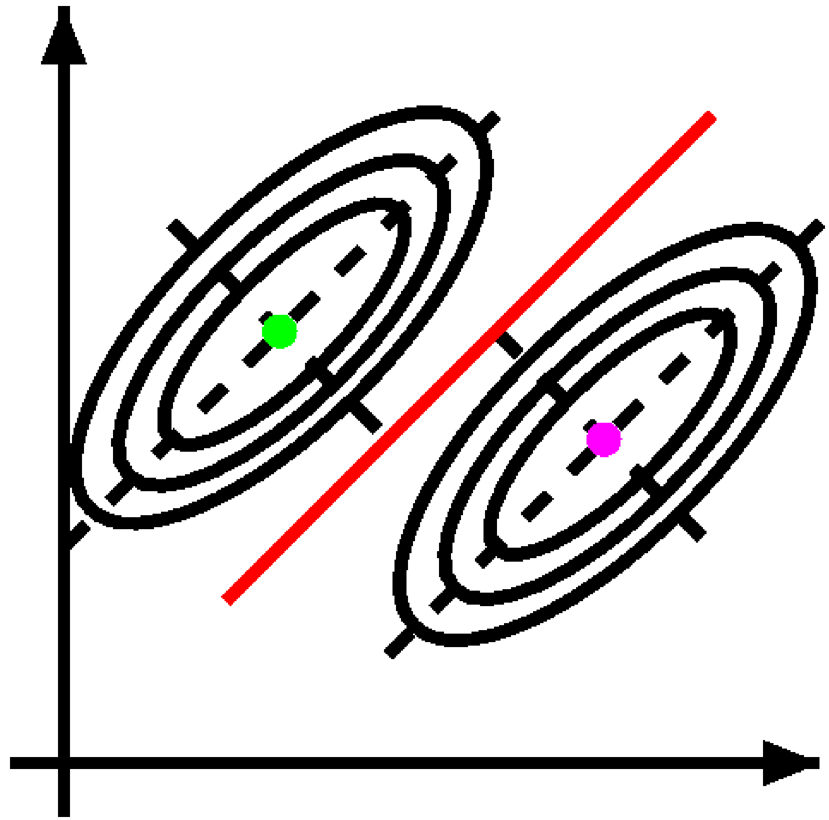
Uncorrelated features:

- independent features
- different variance for different features

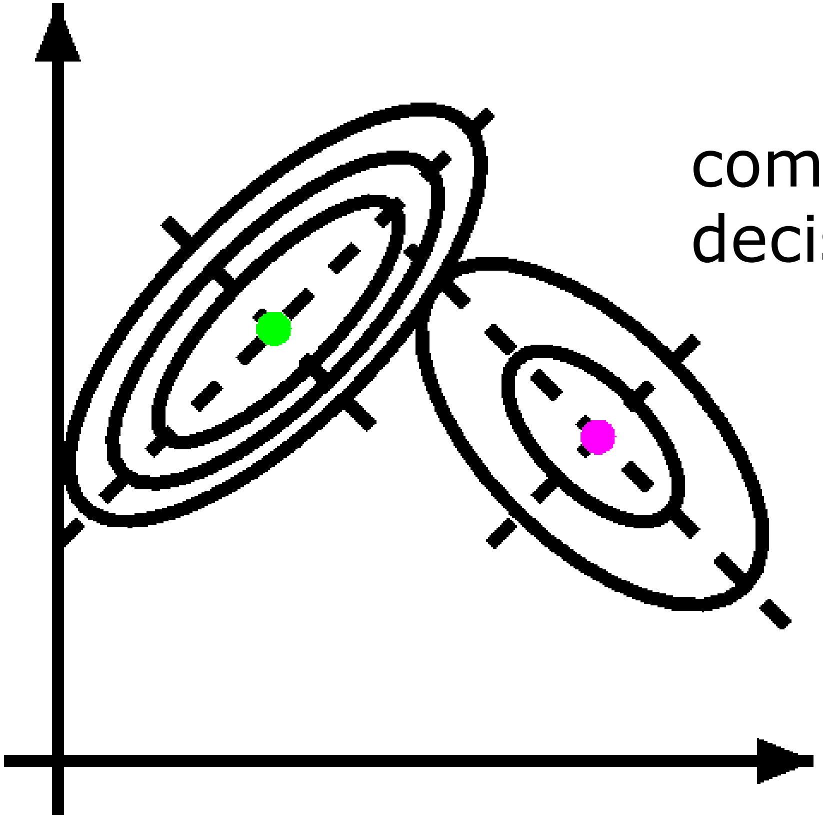


Case 3

equal covariance for all classes



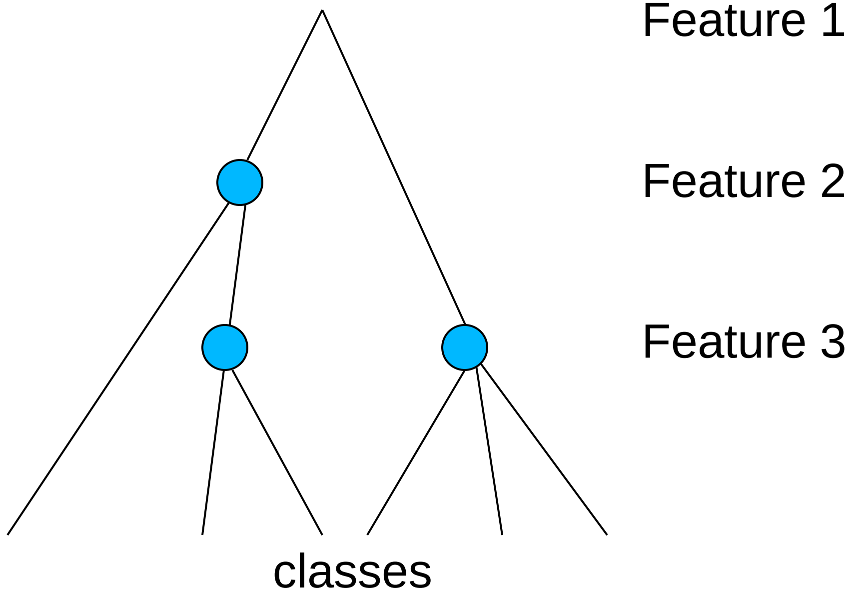
Case 4 (general)



complicated
decision boundaries

Decision Tree

- Divide samples into classes by ”thresholding” one feature at a time
- Training algorithms/automtic tree constructing algorithms exist

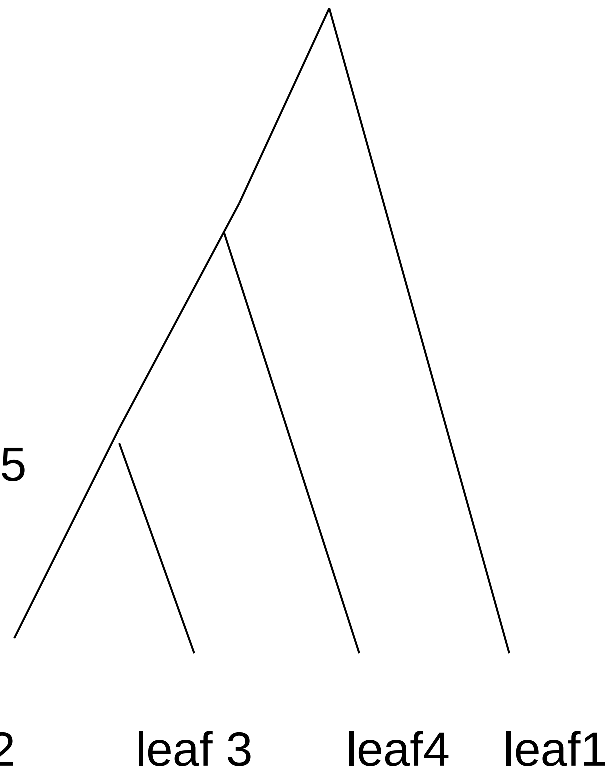


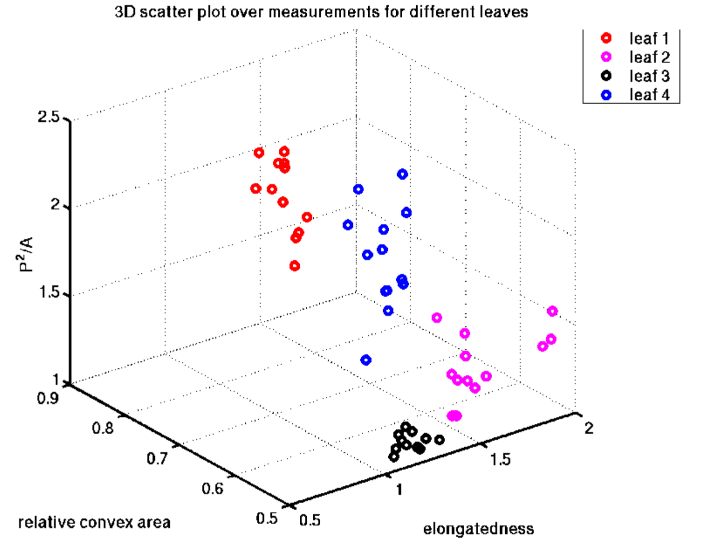
Decision tree example

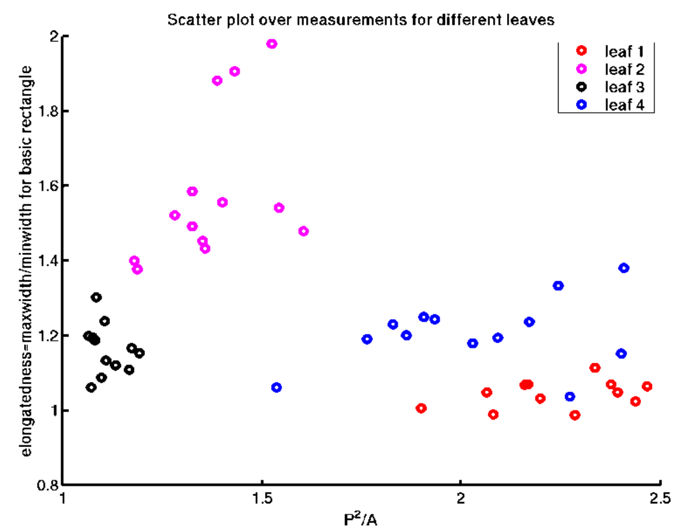
F3 <0.6, F3>0.6

F2<1.5, F2>1.5

F1<1.25,F1>1.25







Artificial Neural Networks (ANNs)

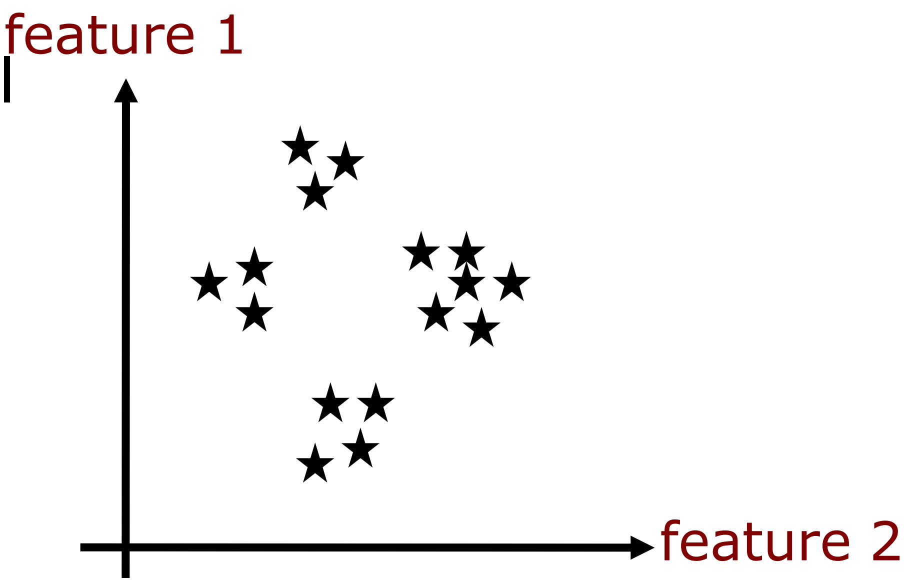
- Create a classifier by adaptive development of coefficients for decisions found via training.
- Do not assume a normal (Gaussian) probability distribution.
- Simulate the association of neurons in the brain.
- Can draw decision borders in feature space that are more complicated than hyper quadratics.
- Require careful training

About trained (supervised) systems

- The features should be based on their ability to separate the classes
- Addition of new features may lead to decreased performance
- The training data should be much larger than the number of features
- Linearly dependent features should be avoided

Unsupervised classification: cluster analysis

- Divide feature space into clusters based on the mutual similarity of the subset elements
- Explorative analysis
- After clustering :
  - Compare with reference data
  - Identify the classes



Unsupervised classification: First classify, then apply knowledge

Unsupervised methods (clustering)

- *k*-means
  - Top down approach (divisive)
  - Predetermined number of clusters
  - Tries to find natural centers in the data
  - Result difficult to illustrate for more than 3 dimensions

Hierarchical

Most often bottom up approach (agglomerative)

Merges patterns until all are one class

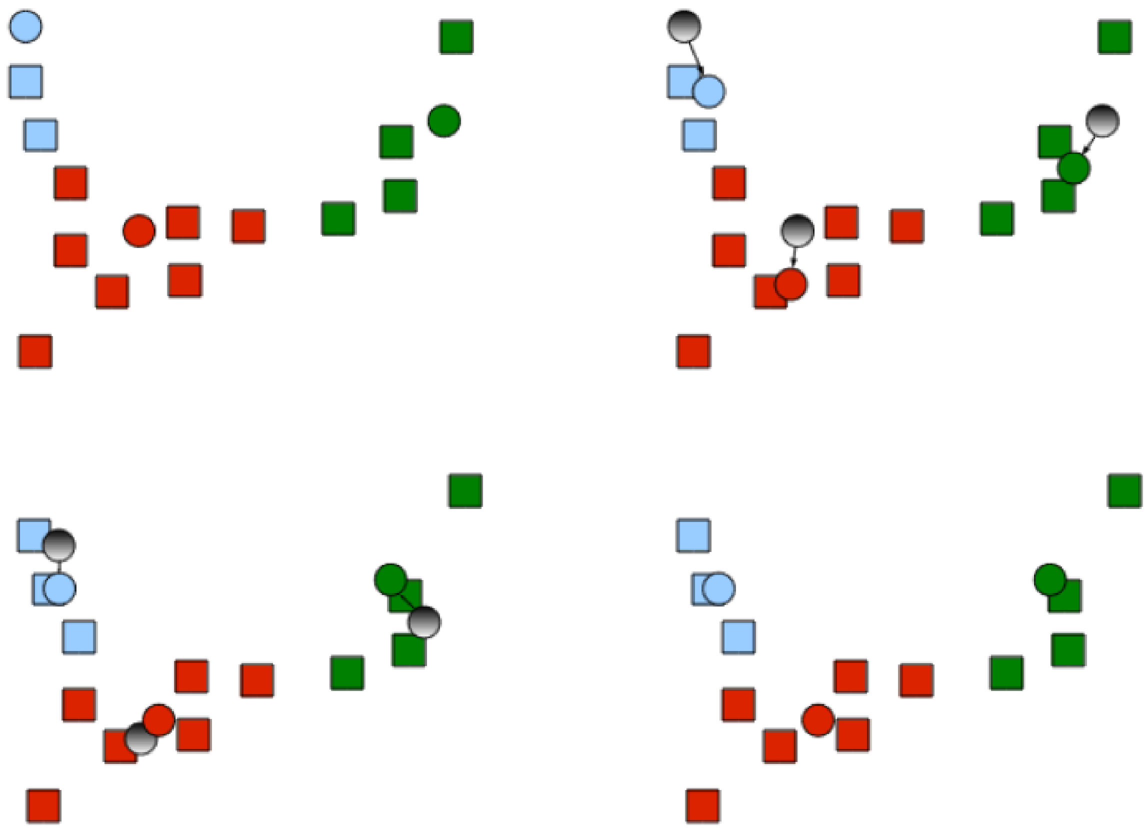
The user decides which clusters are natural

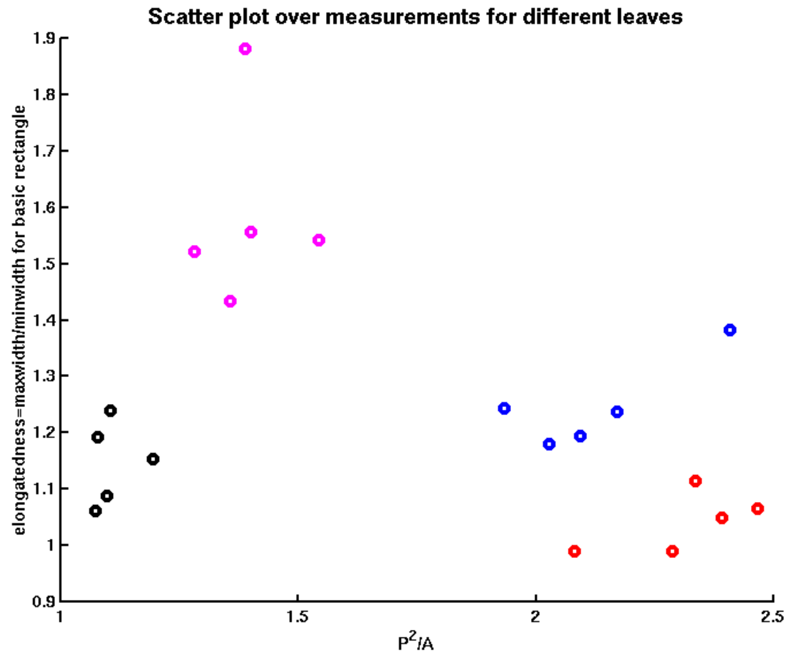
Illustrates results through a dendrogram

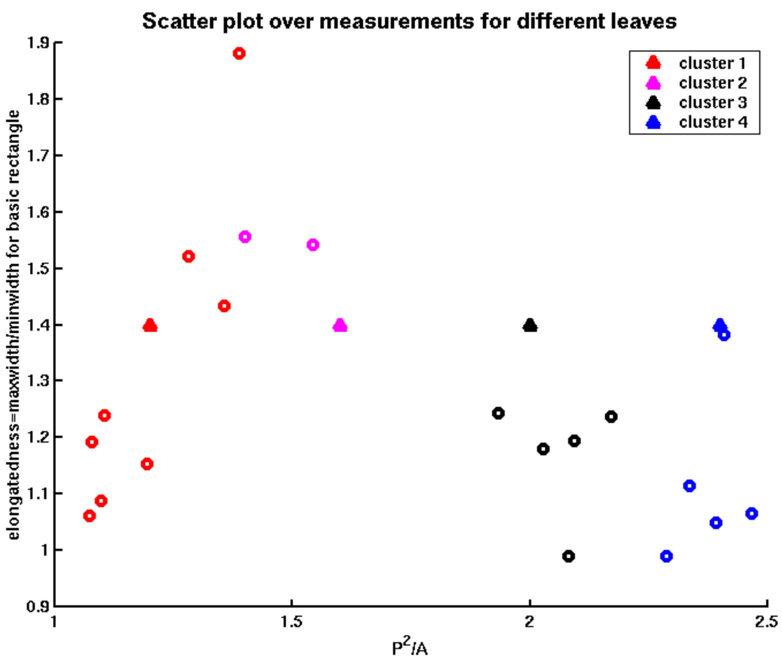
k-means clustering

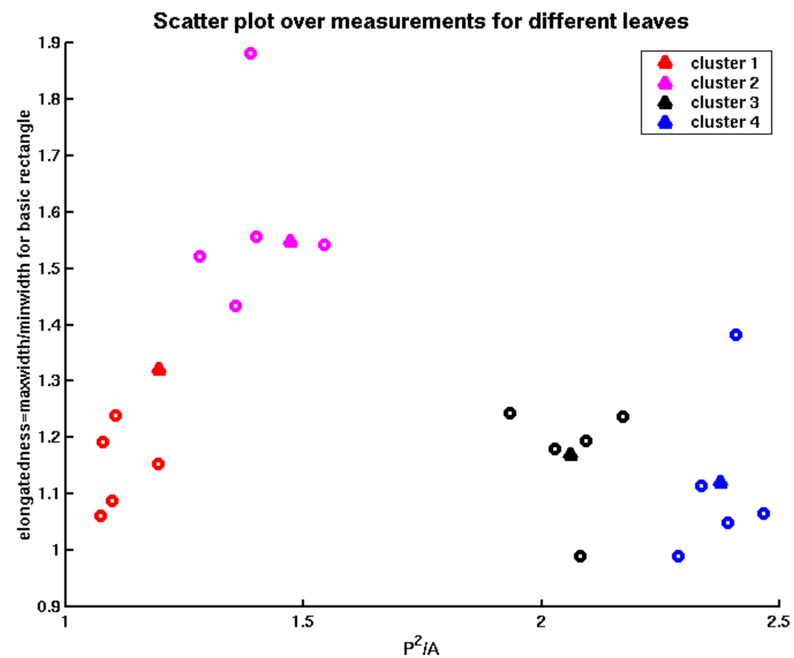
- number of clusters = k
- initialize k starting points (randomly or according to some distribution)
- assign each object to the closest cluster and recompute the centre for that cluster
- move objects between the clusters in order to
  - minimize the variance within each cluster
  - maximize the variance between clusters

k-means clustering




Scatter plot over measurements for different leaves
elongatedness=maxwidth/minwidth for basic rectangle
P^2/A
0.9
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
1
1.5
2
2.5


Scatter plot over measurements for different leaves
cluster 1
cluster 2
cluster 3
cluster 4
elongatedness=maxwidth/minwidth for basic rectangle
P^2/A


Scatter plot over measurements for different leaves
cluster 1
cluster 2
cluster 3
cluster 4
elongatedness=maxwidth/minwidth for basic rectangle
P^2/A

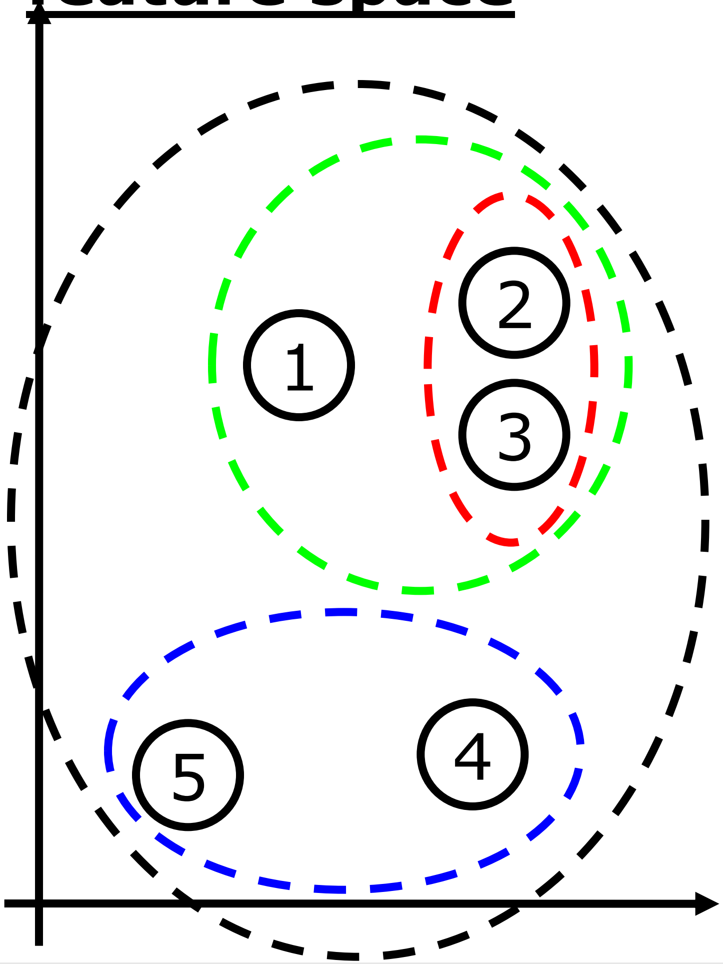
Hierarchical clustering

construct clustering tree (dendrogram)

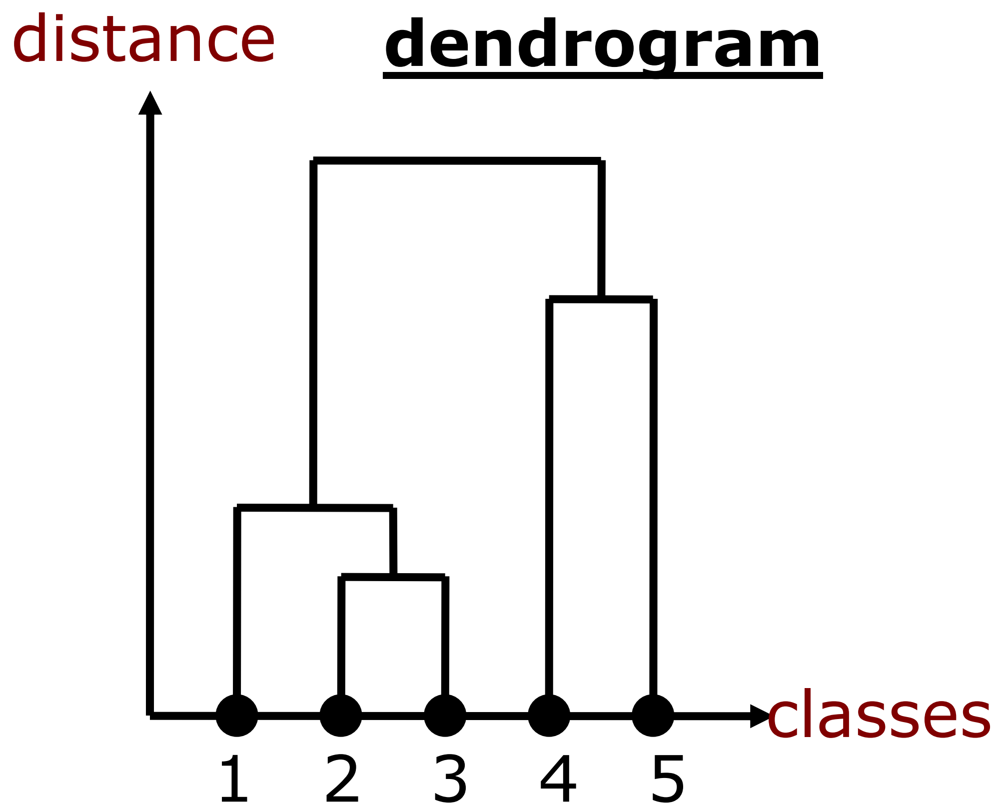
- start with each object/pixel as its own class
- merge the classes that are closest according to some distance measure
- continue until only one class is achieved
- decide the number of classes based on the distances in the tree

Simple dendrogram

feature space

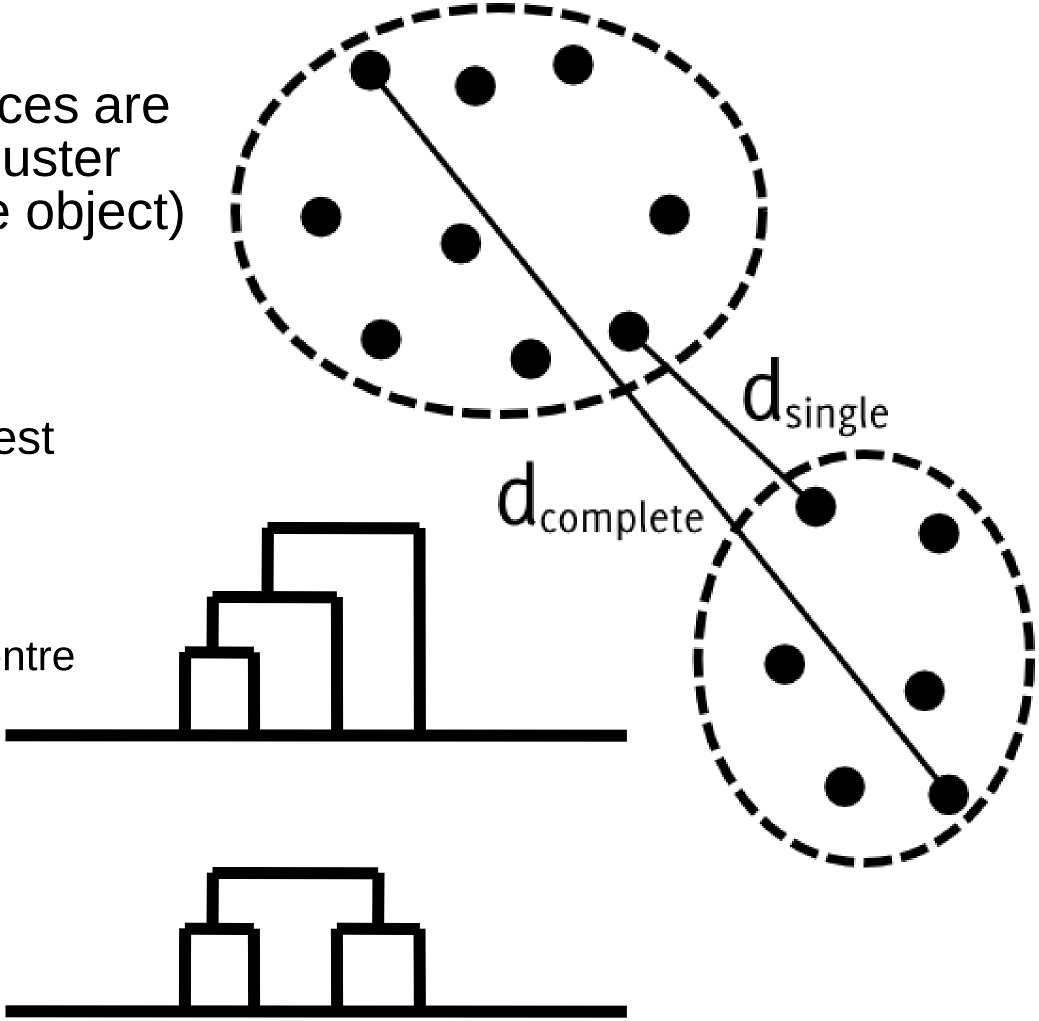


dendrogram

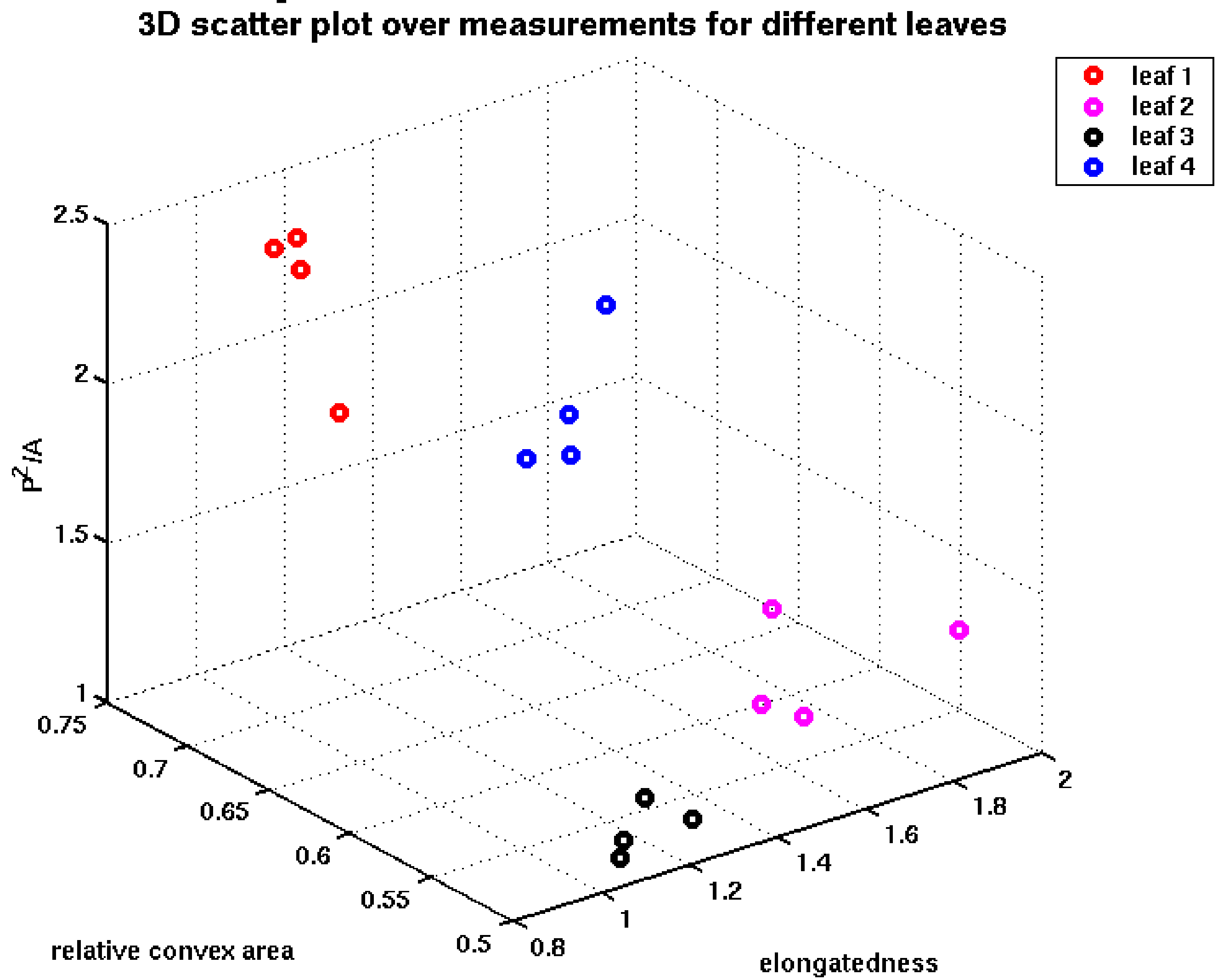


Linking rules

- Linking rules (how distances are measured when the cluster contain more than one object)
  - single linkage (nearest neighbour)
    - shortest distance
  - complete linkage (furthest neighbour)
    - longest distance
  - mean linkage
    - distance to cluster centre

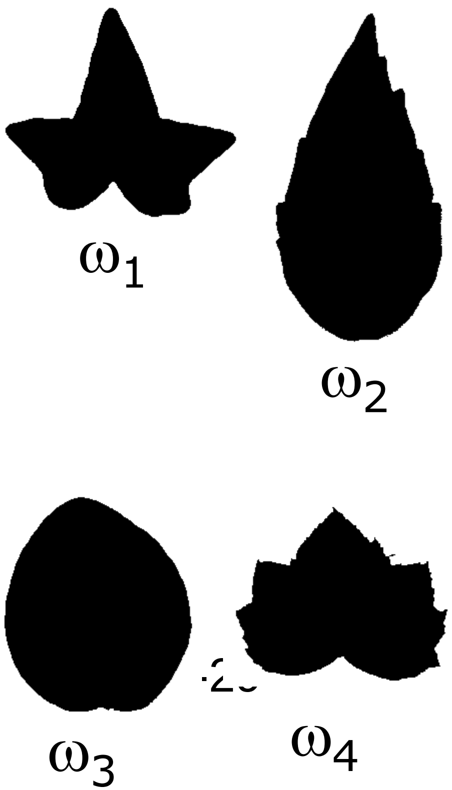


scatterplot for used features

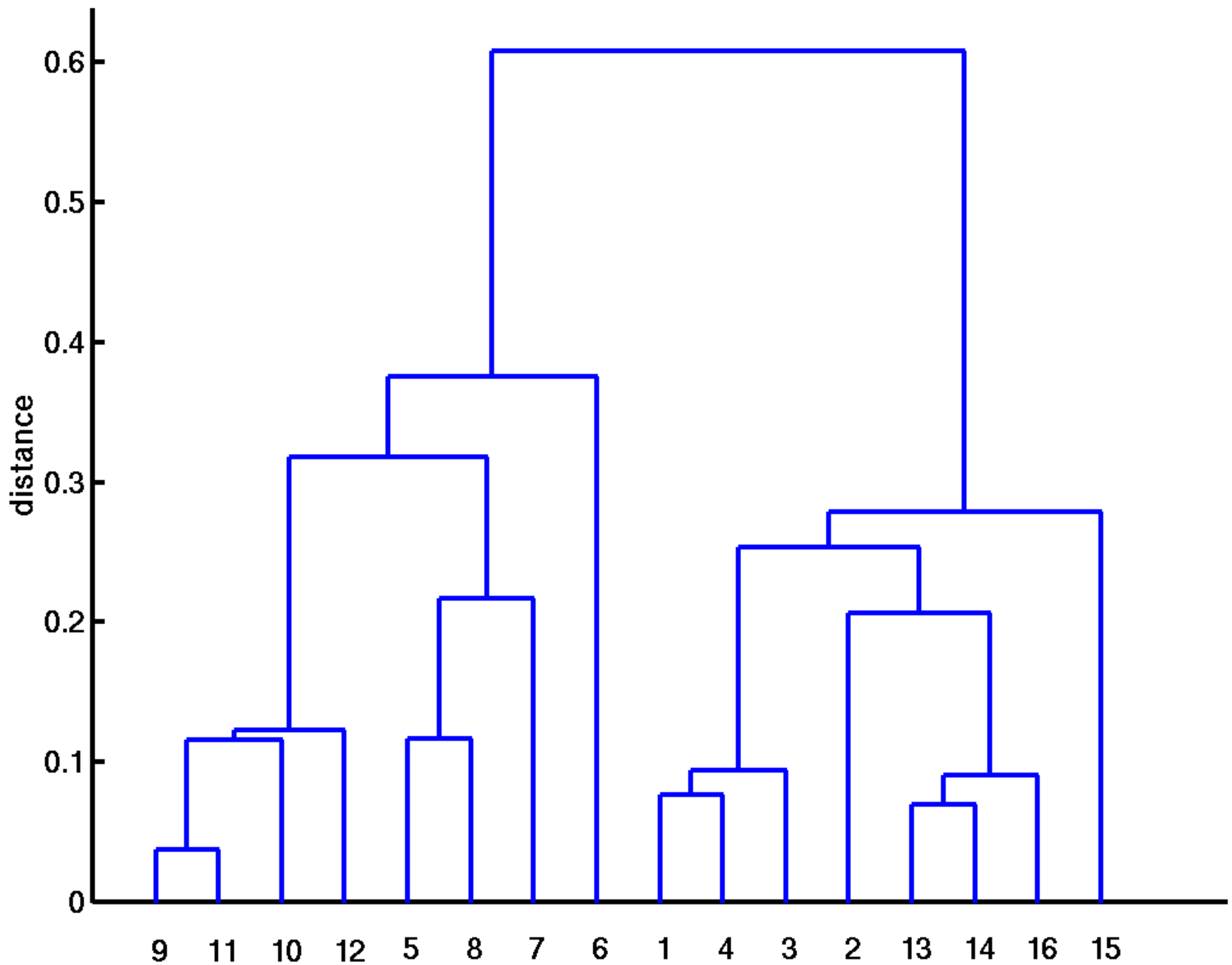


labels:

- ω_1= 1, 2, 3, 4
- ω_2= 5, 6, 7, 8
- ω_3=9,10,11,12
- ω_4=13,14,15,16



Dendrogram over leaves using Euclidean distance and single linkage

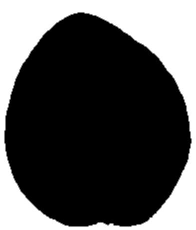


label:

- ω_1= 1, 2, 3, 4
- ω_2= 5, 6, 7, 8
- ω_3=9,10,11,12
- ω_4=13,14,15,16

ω_1

ω_2



ω_3

ω_4

Dendrogram over leaves using Euclidean distance and complete linkage