Estimating Skill Prices in a Model of Continuous Task Choice

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Supervisor: JProf. Michael Böhm, Ph.D.

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Daniel Lüke
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1 Introduction

The economic literature of recent years has identified significant changes in demand on the labor markets in Europe and the US, particularly during the 1980s and 90s. The concept of a skill-biased technological change resulting in higher returns to skills is well established and widely recognized to be a driver behind these changes in labor demand. It builds on the idea that developments in production technology increase the demand for more skilled workers (**acemoglu2011skills**). This increase in returns to skills, in particular returns to education, has been accompanied by shifts in the composition of employment, often referred to as polarization, which describes the simultaneous growth in both, high skill - high paying and low skill - low paying occupations, leading to relative decline in medium skill - medium paying occupations. An overview of these labor market trends is presented, e.g., in Acemoglu and Autor (**acemoglu2011skills**).

The most widely discussed source of this polarization is the ”routinization hypothesis”, introduced by Autor et al. (**autor2003skill**). This approach on skill-biased technological change is based on the idea of routine tasks being easier to either automate or offshore, resulting in a technology driven decrease in demand for workers executing such tasks. In this framework, the emerging computer technology generally complements high skilled workers who tend to execute non-routine tasks while substituting for workers involved in routine tasks. These two mechanisms lead to an increase in relative demand for workers that have a comparative advantage in the execution of non-routine tasks and thus are favorable for relatively high educated workers (**autor2003skill**). At the opposite end of the spectrum, the increase in service occupation employment is accountable for the relative growth in employment in low skilled and low payed occupations (**autor2010inequality**). Building on this, Goos and Manning (**goos2007lousy**) present evidence that both, high and low skill occupations are likely to involve non-routine tasks, while medium skilled occupations appear to consist of more routine tasks The authors can thus show that the ”routinization hypothesis” is consistent with the observed polarization of labor demand. A comprehensive review of related literature is presented in, e.g., **green2015has**. Studies focusing on more recent years reveal, in contrast to the polarization hypothesis, little to no employment growth of high skilled occupations (see, e.g., **beaudry2016great**). Analyzing relative employment data, Deming (**deming2017growing**) presents evidence of differences in the growth rates of employment in different high skilled and cognitive occupations. Within this group of occupations, he shows that those which are in the the fields of science, technology, engineering and mathematics (STEM) experienced a relative decline in employment. At the same time, management, teaching, and nursing are identified as fastest growing occupations within the high-skilled category. This motivates the conjectured connection that it is the content of interpersonal interactions which determines the occupational growth. Building on this, Deming (**deming2017growing**) provides evidence on the importance of social skill on labor market outcomes.

Deming (**deming2017growing**) hypothesizes that one potential driver of this occupational growth in favor of interpersonal interaction intensive occupations might be that this kind of interaction shows to be hard to automate. Autor (**autor2015there**) argues that generally those skill and tasks that cannot be substituted by automation are complemented by it. These findings motivates a more detailed perception of occupations. In understanding these changes in growth of occupational employment, a more nuanced view on the relative importance of different tasks contained within occupations might be helpful, instead of broadly categorizing occupations to be, e.g., either cognitive or manual.

This approach is being pursued by a line of literature, that views occupations as bundles of tasks. For example, Firpo et al. (**firpo2011occupational**) employ a five dimensional measurement for task contents of occupations which captures the effects of technological changes on occupational wages. In an application this task content measure has explanatory value for changes in the distribution of occupational wages. Böhm (**bohm2019price**) presents a propensity method with discrete task choice which can be utilized in the estimation of changes in task prices. Using this method, he identifies changes of task prices that are consistent with routine-biased technological change. Yamaguchi (**yamaguchi2018changes**) builds on a task choice model that distinguishes between cognitive and motor tasks. He finds that differences in skill endowments between men and women, in combination with changes in returns to task specific skills that are driven by changes in technology, can explain changes in the gender wage gap.

Following this line of literature, the model presented in this thesis understands occupations as bundles of tasks. Within the framework of this model, workers optimally choose from a continuity of task combinations under consideration of the resulting realized wage as well as the amenities associated with the choice of tasks. Following Böhm et al. (**bohm2019occupation**), I rely on a distinction between realized wages and the wage paid per constant unit of skill (*skill prices*) for the identification of skill specific changes in task prices which the authors argue to be direct reflections of shifts in labor demand. A worker's productivity in executing each of these tasks is determined by her endowment of the respective task specific skill.

Utilizing a base period which is characterized by an absence of changes in relative skill prices, the theoretical framework discussed in this thesis can be useful in the identification of changes in relative prices of task-specific skills in panel data. Building on the identification strategy derived in this thesis, I employ a simulation study to validate this finding. The results of a Monte Carlo estimation show that the method presented in this thesis is, indeed, able to identify the simulated changes in relative skill prices.

The remainder of this thesis is structured as follows. Section 2 introduces the model of continuous task choice. Beginning with a utility maximization problem, I present the identification strategy used to arrive at an estimable representation of changes in relative skill prices. Starting from a very general setting, I illustrate these findings by means of a model with two different tasks. Subsequently, section 3 presents an application of the identification strategy in the context of a simulation study. This proof of concept helps to examine previously established results in a Monte Carlo estimation of changes in relative task specific skill prices. A discussion and concluding remarks are provided in the final section 4 of this thesis.

2 A Model of Continuous Task Choice

This section presents a model of continuous task choice which can be employed to analyze changes in task specific skill prices. In this framework, workers maximize utility by allocating their working time between tasks. In doing so, workers are facing a trade-off between realized wages determined through their choice and associated amenities that depend on the workers preferences regarding their choice of tasks. This trade-off is reflected in a utility function defined as a composition of two parts: the pecuniary part, particularly realized wages and a non-pecuniary part, i.e. the amenities associated with the executed tasks. The model I develop is related to the one presented in Böhm (**bohm2019price**), where workers maximize their utility by choosing the task that offers the highest sum of pecuniary and non-pecuniary rewards. However, I deviate from this approach with discrete task choice in that I allow a continuous combination of tasks.

As a starting point, I first introduce the utility maximization problem each worker faces. In general, workers experience positive utility from wages which is accompanied by a dislike towards strong specialization in executed tasks as well as deviations from the preferred choice of tasks. In section 2.2, I present the solution of this choice problem by means of a two task version of this model. Subsequently, I derive the identification of changes in relative skill prices, based on the two task version of the model in section 2.3. Section 2.4 discusses how in an empirical application this identification strategy can be employed to estimate such changes in skill prices.

2.1 The Utility Maximization Problem

A worker's realized wage is modeled as a function of the task-specific wages that would be obtained if a worker spent her entire work-time on one particular task (*potential wages*), and the fraction of work-time that she spends on each task, i.e. her *task choice*.

The amenity valuation associated with each task choice is determined by, first, an exogenous preference for a certain allocation of working time to tasks. In particular, agents experience negative utility from deviations from their preferred allocation. Second, amenities are modeled to capture a general dislike towards strong task-specialization. Notice that both these properties translate into non positive utility. With this conceptualization of the amenities at hand, it appears to be more intuitive to think of this non-pecuniary part of the utility function in terms of a penalty. Therefore, I will use the notion *penalty term* in the further course of this thesis.

Since the goal of this analysis is an investigation of relative quantities, I use a notation in logarithmic values when describing my model. Above described properties are translated into a task choice model as follows. In each period $t \in \{0, ..., T\}$, all agents $i \in \{1, ..., N\}$ individually maximize their utility $u_{i,t}$ by choosing the fractions of their working time spent on each of J tasks. These task choices are captured in $\lambda_{i,j,t},$ with $0 \leq \lambda_{i,j,t} \leq 1,$ and $j \in \{1, ..., J\},$ s.t. $\sum_{j=1}^{J} \lambda_{i,j,t} = 1$. This approach allows to represent occupations as continuous compositions of tasks.

Realized wages of a worker result as the sum of potential task specific wages weighted by task choices. This definition of realized wage is captured in equation (1), where $\boldsymbol{\lambda}_{i,t}$ and $\boldsymbol{w}_{i,t}$ denote vectors of length J that contain the chosen work time fraction and the potential wage for each task, respectively. Alternatively, realized wages can be thought of as a linear combination of potential wages.

$$w(\boldsymbol{\lambda}_{i,t}, \boldsymbol{w}_{i,t}) = \sum_{j=1}^{J} \lambda_{i,j,t} w_{i,j,t} \tag{1}$$

Following above description of negative amenities associated with task choices, the penalty term is defined as a function of task choices and preferred time allocations. Furthermore, this penalty incorporates the assumed dislike towards specialization. These properties are captured by the function presented in equation (2).

$$\Phi(\boldsymbol{b}_i, \boldsymbol{\lambda}_{i,t}, \phi, \theta) = \theta \sum_{j=1}^{J} |b_{i,j} - \lambda_{i,j,t}|^{\phi} \tag{2}$$

In this functional form of the penalty term, $\boldsymbol{b}_{i,j}$, with $0 \leq b_{i,j} \leq 1 \; \forall \; j \in \{1, ..., J\}$ captures the individually preferred time allocation on tasks. This preference is assumed to be time invariant. From equation (2) one can see that the penalty is minimized at a value of zero by setting task choices exactly equal to the preferred allocation, i.e., $\lambda_{i,j,t} = b_{i,j} \;\; \forall \;\; j \in \{1, ..., J\}$. Ceteris paribus, increasing the difference between a worker's preferred time allocation and her respective task choice in associated with an increase of the penalty, hence decreasing utility.
Parameter $\theta \in \mathbb{R}_+$ works as a scaling factor, allowing to adjust relative relevance of the amenity and wage terms for the utility function. This weighting parameter is limited to positive values since switching signs of this function results in a loss of its previously described desired properties. All else equal, an increase in penalty weight θ entails higher penalties from a deviation from the preferred time allocation. As seen from the absence of subscripts, θ is modeled to be constant across individuals and time.
Finally, the penalty term's exponent ϕ determines the dislike towards strong task specializations. Since the base of this exponent by construction is between zero and one (i.e., $|b_{i,j} - \lambda_{i,j,t}| \in [0, 1]$), a larger exponent ϕ, ceteris paribus, decreases penalties from deviations. ϕ is required to be weakly positive in order to ensure continuity of the first order condition of the resulting maximization problem (see section 2.2 below). Similar to the penalty weight parameter, ϕ is assumed to be time invariant and identical across individuals as well as tasks.
Equation (3) below presents both components, realized wage and penalty term, combined to the utility function. Overall, the functional form of the penalty term is modeled to be time invariant, which will be important for the estimation strategy of changes in task prices in section 2.4.

$$\begin{aligned} u_{i,t} &= w(\boldsymbol{\lambda}_{i,t}, \boldsymbol{w}_{i,t}) - \Phi(\boldsymbol{b}_i, \boldsymbol{\lambda}_{i,t}, \phi, \theta) \\ &= \sum_{j=1}^{J} \lambda_{i,j,t} w_{i,j,t} - \theta \sum_{j=1}^{J} |b_{i,j} - \lambda_{i,j,t}|^{\phi} \end{aligned} \tag{3}$$

Each worker is assumed to act utility maximizing by setting task choices. Solving this optimization problem provides optimal task choices $\boldsymbol{\lambda}^*_{i,t}$, which are defined as follows.

$$\boldsymbol{\lambda}^*_{i,t} \equiv \arg\max_{\boldsymbol{\lambda}_{i,t}} u_{i,t}(\boldsymbol{b}_i, \boldsymbol{\lambda}_{i,t}, \phi, \theta) = \arg\max_{\boldsymbol{\lambda}_{i,t}} w(\boldsymbol{\lambda}_{i,t}, \boldsymbol{w}_{i,t}) - \Phi(\boldsymbol{b}_i, \boldsymbol{\lambda}_{i,t}, \phi, \theta). \tag{4}$$

2.2 Implications in a Model with J=2 Tasks

Up to this point the model introduced in this thesis captures the general setting of a continuous choice between J tasks. With the complexity of occupations in the real world in mind, a subdivision of each occupation into a multitude of tasks would be conceivable. The desired number of dimensions of this decomposition depends on the specific context of the model and can be specified accordingly. In this subsection, I consider a very tractable version of this model with $J = 2$ tasks. This facilitates solving the model and results in accessible interpretations. The insights gained from this specification, however, are transferable to a higher dimensional task differentiation.

A distinction between $J = 2$ tasks could, for instance, be used to analyze a setting similar to the one presented in Yamaguchi (**yamaguchi2018changes**) who distinguishes between "cognitive" and manual (specifically, "motor") tasks. Similarly, Beaudry et al. (**beaudry2016great**) build their arguments on a model that considers two distinct tasks: "cognitive" and "routine".

A specification of the model with $J = 2$ tasks allows to restate the utility function in equation (3) in a more concise way, using that both, $\lambda_{i,j,t}$ and $b_{i,j}$ sum to one over all tasks. Furthermore, I normalize $w_{i,j=1,t}$ in every period $t \in \{1, ..., T\}$, so that potential wages in task two yield a quantity that is relative to the potential wage in task one. This does not impact the model's informative value regarding the relative task specific wages and will facilitate the identification thereof. Consequently, the utility function of a case with $J = 2$ tasks results as following expression, the derivation of which can be found in appendix A.1.

$$u_{i,t} = \lambda_{i,t}\tilde{w}_{i,t} - 2\theta|b_i - \lambda_{i,t}|^{\phi}, \tag{5}$$

where

$$\lambda_{i,t} \equiv \lambda_{i,j=2,t}$$

$$b_i \equiv b_{i,j=2}$$

$$\tilde{w}_{i,t} \equiv w_{i,j=2,t} - w_{i,j=1,t}.$$

Drawing on the utility maximization problem, which is presented in a general version in equation (4), workers maximize utility by optimally setting choice parameter $\lambda_{i,t}$. Thus, the first order condition presented in equation (6) must be satisfied. This results in the piecewise defined optimal task choice presented in equation (7)[1]. For a derivation of the first and second order

[1]Note that $\tilde{w}_{i,t} < 0$, if and only if the potential wage in task 2 is smaller then the potential wage in task 1. This case is accompanied by a shift in task choices towards task 1, which will result in $b_i > \lambda_{i,t}$ (this corresponds to the second case in equation (7)). Analogously, it can be seen that $\tilde{w}_{i,t} > 0 \iff b_i < \lambda_{i,t}$. The exponent in equation (7), therefore, is by construction always over a strictly positive base and thereby defined for any real number of $\tilde{w}_{i,t}$.

conditions of this optimization problem, see appendix A.2. A detailed derivation of optimal task choices $\lambda^*_{i,t}$ can be found in appendix A.3.

$$\frac{\partial u_{i,t}(b_i, \lambda_{i,t}, \phi, \theta)}{\partial \lambda_{i,t}} \stackrel{!}{=} 0 \tag{6}$$

$$\lambda^*_{i,t} = \begin{cases} b_i - (\frac{-\tilde{w}_{i,t}}{2\phi\theta})^{\frac{1}{\phi-1}}, & \text{if } b_i \geq \lambda_{i,t} \\ b_i + (\frac{\tilde{w}_{i,t}}{2\phi\theta})^{\frac{1}{\phi-1}}, & \text{if } b_i < \lambda_{i,t} \end{cases} \tag{7}$$

From this representation of optimal task choices one can see that the penalty exponent ϕ is required to be weakly positive in order to provide continuity of optimal choices.[2]. Apart from this, constraining $\phi > 1$ ensures strict concavity of the utility function which is the sufficient condition for a global maximum of $u_{i,t}$ at $\lambda^*_{i,t}$, given its domain of definition, $\lambda^*_{i,t} \in [0, 1]$ (see appendix A.2 for a derivation of this result).

Another way to understand the implications of choices of the penalty exponent ϕ on the solution to this maximization problem is in terms of the optimal task choice. Allowing for $0 \leq \phi \leq 1$ results in a weakly convex utility function with a kink point at $\lambda_{i,t} = b_i$. In this case, it can be seen from the first derivative of the utility function (appendix A.2) that workers will either stick with their preferred time allocation b_i regardless of relative wages or changes thereof, and alternatively they will fully focus on one single task as soon as its relative wage is high enough resulting, in corner solutions. In such a model specification at most three optimal task choices are feasible: $\lambda^*_{i,t} \in \{0, b_i, 1\}$. While the first case results in a trivial version of the model without inter temporal task adjustments, the second case results in a model where each worker either sticks with her preferred task split b_i or fully focuses on the higher paid task. Both cases are not desirable for the analyses in this thesis, so that it is throughout this work assumed that $\phi > 1$.

Furthermore, notice that the solution presented above accommodates the special case of $\phi = 2$ in which, first, the penalty term no longer needs to be defined piecewise, and second, optimal task choices are linear in changes in relative potential wages. I will fall back to this case in the context of the simulation study that is presented at a later stage in this thesis in section 3.

2.3 Identifying Changes in Skill Prices

Previous subsection presents the solution to the maximization problem workers are facing. Following this, I now show how the changes in skill prices in discrete time changes can be identified in this framework. Detailed derivations of the results presented throughout this section can be found in appendix A.4. Starting from the utility function in the case of $J = 2$ different tasks, presented in equation (5), changes in time are captured in the total derivative thereof with respect to time. Using the product and chain rules, this yields the expression in equation (8).

For the total derivative with respect to time, I make use of a more detailed notation where, instead of using subscripts to denote time dependence, such variables are written as explicit functions of time. Relative potential wages ($\tilde{w}_{i,t}$), thus are expressed as $\tilde{w}_i(t)$. Consequently,

[2]Considering equation (2), one can easily see that $\lim_{|b_i - \lambda_{i,t}| \to 0} \Phi(\theta, \lambda_{i,t}, b_i, \phi) = \infty$, $\forall \phi < 0$ and $\theta > 0$

optimal task choices are expressed as an indirect function of time, i.e. $\lambda^*_{i,t} \equiv \lambda^*_i(\tilde{w}_i(t))$. The utility function follows analogously.

$$\frac{d}{dt}u_i(\lambda^*_i(\tilde{w}_i(t)),\tilde{w}_i(t)) = \frac{\partial u_i(\lambda^*_i(\tilde{w}_i(t)),\tilde{w}_i(t))}{\partial \lambda^*_i(\tilde{w}_i(t))}\frac{\partial \lambda^*_i(\tilde{w}_i(t))}{\partial \tilde{w}_i(t)}\frac{d\tilde{w}_i(t)}{dt} + \frac{\partial u_i(\lambda^*_i(\tilde{w}_i(t)),\tilde{w}_i(t))}{\partial \tilde{w}_i(t)}\frac{d\tilde{w}_i}{dt} \tag{8}$$

By application of the envelope theorem, the indirect effect of changes relative wages via a change in optimal task choices λ^*_i equates to zero so that only the direct effect of changes in relative wages needs to be considered[3]. The total derivative of utility with respect to time, therefore, can be restated as follows.

$$\frac{d}{dt}u_i(\lambda^*_i(\tilde{w}_i(t)),\tilde{w}_i(t)) = \lambda^*(\tilde{w}_i(t))\frac{d\tilde{w}_i(t)}{dt} \tag{9}$$

For the problem addressed in this thesis, however, changes in discrete time need to be identified. Therefore, equation (9) is integrated from $t-1$ to t. Throughout the remainder of this thesis, I use Δ as notation for discrete changes of time-dependent variables. Hence, I formulate the integral over marginal changes in equation (9) from $t-1$ to t as follows.

$$\Delta u_{i,t} = \int_{t-1}^{t}\left(\frac{d}{d\tau}u_i\big(\lambda^*_i(\tilde{w}_i(\tau))\big)\right)d\tau = \int_{\tilde{w}_{i,t-1}}^{\tilde{w}_{i,t}}\lambda^*_i(\tilde{w}_{i,\tau})d\tilde{w}_{i,\tau} \tag{10}$$

Where in the last part of expression (10) integration by substitution is used to restate the integral in the second term. By substituting $\tilde{w}_t(\tau)$ with $\tilde{w}_{i,\tau}$, I again rely on the abbreviated notation of time dependence, which I will stick with for the remainder of this work.

Applying a linear interpolation of task choices, presented in equation (11), leads to the intermediate result which is shown in equation (12). For the identification of the model, this approximation of $\lambda^*_{i,\tau}$ allows to arrive at discrete changes in utility as a function of discrete changes in potential wages. However, against the background of an empirical application of this result, a different interpretation is conceivable: For an unknown functional form of optimal task choices, a linear interpolation between two observable points in time offers a approximation of the interim task choices.

$$\lambda^*_i(\tilde{w}_{i,\tau}) \approx \lambda^* + \frac{\lambda^*(\tilde{w}_{i,t}) - \lambda^*(\tilde{w}_{i,t-1})}{\tilde{w}_{i,t} - \tilde{w}_{i,t-1}}(\tilde{w}_{i,\tau} - \tilde{w}_{i,t-1}) \tag{11}$$

$$\Delta u_{i,t} = \bar{\lambda}^*_{i,t}\Delta\tilde{w}_{i,t} \tag{12}$$

Where $\bar{\lambda}^*_{i,t} \equiv \frac{\lambda^*_{i,t-1}+\lambda^*_{i,t}}{2}$ denotes the mean task choice of periods $t-1$ and t. Notice that the linear interpolation of optimal task choices is not an approximation in case the penalty function is quadratic (i.e., penalty exponent $\phi = 2$). It can be seen from equation (7) that a quadratic penalty term will result in optimal task choices $\lambda^*_{i,t}$ that are linear in relative potential wages.

[3]As seen from equation (6), the partial derivative of the utility function with respect to the task choice parameter must be zero by the first order condition of the utility maximization problem.

The functional form of optimal task choices is not an approximation but exactly correct in this case.
Similar to the model presented in Böhm et al. (**bohm2019occupation**), the key identifying assumption is the divisibility of potential wages into wage paid per constant unit of skill, i.e. skill prices ($\pi_{j,t}$), and skill endowments ($s_{i,j,t}$) In log notation, this assumption allows for a decomposition as shown in equation (13) for each task $j \in \{1, ..., J\}$.

$$w_{i,j,t} = \pi_{j,t} + s_{i,j,t} \tag{13}$$

This implies that changes in relative potential wages can be decomposed into changes in relative skill endowments and changes in relative skill prices, i.e. $\Delta\tilde{w}_{i,t} = \Delta\tilde{\pi}_i + \Delta\tilde{s}_{i,t}$. Similarly, changes in utility can be split into wage changes and changes in the penalty term, as shown in the general version of the utility function presented in equation (3), i.e. $\Delta u_{i,t} = \Delta w_{i,t} - \Delta\Phi_{i,t}$. Making use of both decompositions, equation (12) can be restated to an expression of wage changes:

$$\Delta w_{i,t} = \bar{\lambda}^*_{i,t}\Delta\tilde{\pi}_t + \bar{\lambda}^*_{i,t}\Delta\tilde{s}_{i,t} + \left[\Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1})\right] \tag{14}$$

Considering equation (14) changes in observable realized wages can be partitioned into three subcomponents: First, changes in skill prices ($\bar{\lambda}^*_{i,t}\Delta\tilde{\pi}_t$), which is what this section aims to identify. Second, changes in skill endowments ($\bar{\lambda}^*_{i,t}\Delta\tilde{s}_{i,t}$) that result from some skill accumulation process, and third, changes in the penalty term ($\Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1})$). Identifying the relation between changes in realized wages and changes in skill prices, therefore, requires the determining both, the second and third summands of equation (14) in a first step. In the following section I hence discuss the strategy used to distinguish price changes from these other influences on realized wages.

2.4 Estimation of the Model

Equation (14) shows that determining changes in relative skill prices requires to distinguish them from changes in both, skill endowments as well as in the penalty term. In a estimation with optimal task choices as explanatory variables, however, price changes would not be distinguishable from changes in skill endowments and, moreover, requires controlling for changes in the penalty term. A method to isolate changes in skill prices is presented in Böhm et al. (**bohm2019occupation**). In a similar setup, the authors distinguish between effects of skill accumulation and changes in skill prices relying on a base period $t = 0, ..., T_{base}$ in which they set $\Delta\pi_t = 0$. In the article, the authors argue that this can either be seen as an assumption or a normalization. In the context of this thesis, it is straight forward to think of it as a normalization for two reasons. First, the model introduced in this work by construction cannot provide insights on absolute levels of prices, but relative prices instead. It is, therefore, intuitive to interpret changes in skill prices relative to a base period. Second, in order to answer the research question of this thesis on changes in the relative importance of different task specific skills, one inherently has to consider changes in task prices in comparison to some earlier period.

In addition to skill accumulation, the base period can also be used to estimate changes in the penalty term. In this model, penalty terms are a direct function of optimal task choices, only. In the absence of changes in skill prices, it is therefore possible to estimate $\Delta\Phi_{i,t} = \Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1})$ and treat it as confounding variable in subsequent periods[4]. Exploiting the base period to estimate both, skill accumulation and changes in the penalty term comes with the downside that these estimated effects cannot be distinguished. However, this is not of relevance to the specific question of this thesis, which is focused on the identification of skill prices. Furthermore, there are other, more sophisticated approaches to work out the skill accumulation process in panel data. These I will discuss in more detail in the section 4.

The estimation results for the contributions of skill accumulation and changes in the penalty term can be used to adjust wage changes in subsequent periods for these effects. Particularly, the assumptions that both, the skill accumulation process as well the functional form of the penalty term are time invariant allows to transfer the estimated results from the base period to the subsequent period. Finally, an estimation of changes in skill prices can be obtained by regressing the residual wage change, i.e. observed change in realized wages adjusted for changes in skill endowments and penalty term, on average task choices.

3 Simulation Study

In the previous section I show how the model presented in this thesis can be useful in the identification of changes in skill prices in panel data. Following this identification strategy, changes in the prices of skills can be determined on the basis of the realized wages and task choices. In this section, I provide an exemplary application of this identification in the format of a simulation study[5].

As part of this simulation study I, first, generate a representation of a panel data set that. The data generating process endogenously provides optimal task choices and according wages for each simulation period on the basis of potential task specific wages. In a second step, I rely on this simulated data in an estimation of skill prices following above described identification strategy.

3.1 Data Generating Process

In order to produce a representation of a panel data set, the data generating process provides data on three points in time so that two inter temporal changes can be calculated. The first thereof serves as base period and the second as estimation period. Following the discussion of the model in section 2.2, this simulation is based on two different tasks. With the goal of simulating a utility maximization process in mind, both wages and penalty term need to be simulated (see equation 3). Therefore, I first describe the data generating process for potential task

[4] see Böhm et al. **bohm2019occupation** who present similar, more general results on a model with "non-pecuniary benefits" with unknown functional form.

[5] Program codes of this simulation study are openly available on GitHub: github.com/DaLueke/social_skill_prices

specific wages. Second, I present the parameterization of the penalty term.

The wage function depends on task specific potential wages, which are assumed to result as sum of respective skill prices and individual skill endowments (see section 2.3, particularly equation (13)). Prices of each skill in each period are exogenous in the model. In this simulation study specifically, they are modeled to be constant during the base period and changing during the estimation period. In order to facilitate the evaluation of estimation results, the data generating process used in this simulation study is specified to have a deterministic change in relative prices.

Skill endowments at the first observed point in time are randomly drawn from a normal distribution. In subsequent periods, skills endowments experience an accumulation that is modeled as a learning by doing process. In particular, workers gain relative skill in that task, which they execute to a relatively large extent. The stronger a worker focuses on one single task, the larger is the gain in relative skill herein. This results in a skill accumulation that depends on task choices, hence can be estimated using these task choices. It therefore meets the conditions for being estimated using the method described in section 2.4.

In accordance to the normalization of potential wages introduced in section 2.2, potential wages of task one are normalized to zero so that potential wages of task two need to be understood as relative sizes. Specifically, a skill endowment for task two that is positive indicates that a worker has a comparative advantage in doing this task and vice versa for negative skill endowments. Analogously, a relative skill price that is larger than zero implies a higher compensation for executing task two.

The penalty term is a function of individual time allocation preference, captured in parameter b_i. For this simulation study this parameter is generally assumed to be uniformly distributed within some borders $\{b_{low}, b_{high}\}$ with $b_{low}, b_{high} \in [0, 1]$ and $b_{low} \leq b_{high}$. The resulting interval is centered at the equal split of work time on both tasks, so that neither task is systematically preferred to the other. Additionally, it is assumed that workers dislike strong specialization (section 2.1). To do justice to this assumption, the limits of the parameter for preferred allocations should, therefore, not be chosen in the extremes. In the data generated process that is used for the simulation study, the borders of individual time allocation preferences are set to $b_{low} = 0.3$ and $b_{high} = 0.7$.

For this simulation study, I use a quadratic specification of the penalty function, i.e. the penalty exponent is set to $\phi = 2$. This specification meets the assumed properties of the penalty function. At the same time, it results optimal task choices that are a linear function in relative potential wages, which makes the linear interpolation of inter temporal changes in task choices an exact solution. The penalty weight parameter τ is set to a value of $\tau = 15$ in this data generating process. This weight is chosen so that neither realized wages nor the penalty dominates the utility function. While the first case could result in a large number of corner solutions where every worker focuses on the task with higher potential wage, the second case would produce a dataset in which workers mostly stick with their preferred task choice so that potential wages, and more importantly changes thereof, impact task choice very little.

Based on both, potential wages and the penalty term, the optimal task choice parameter $\lambda^*_{i,t}$ can be obtained in each period by maximizing the individual utility (i.e., realized wage lessened by the penalty term). Furthermore, resulting realized wages can be calculated using this optimal task choice. Utilizing both, task choices and resulting wages, allows to estimate the change in relative skill prices on which the data generating process is based. In the next section, I show how changes in skill prices can be estimated relying on the identification strategy discussed in section 2.4 and using data generated by the process described in this section.

A representation of the simulated data can be seen in figure 1. Particularly, the figure shows the combinations of optimal task choices $\lambda^*_{i,t=0}$ and realized wages $w_{i,t=0}$ for all $N = 100$ simulated workers in one of the Monte Carlo iterations. From the graphic it can be seen that task choices (horizontal axis) are distributed around the equal split, with no worker engaging in an extreme focus on either one of the tasks. The histogram of realized relative wages (vertical axis) shows an increased frequency at the center of the distribution around a relative wage of zero. Both, relatively high and low realized relative wages are less frequent.

In the data generating process, the relative skill price of task two is fixed at 0.05 which implies that the skill price of task two is 5% higher than the price of task one. This setting should result positive relative realized wages for workers that spend the majority of their working time on task two. Indeed, workers who have higher $\lambda^*_{i,t}$ tend to earn relatively high wages. This impression is supported by the position of a regression line through the point cloud (black dotted line in figure 1).

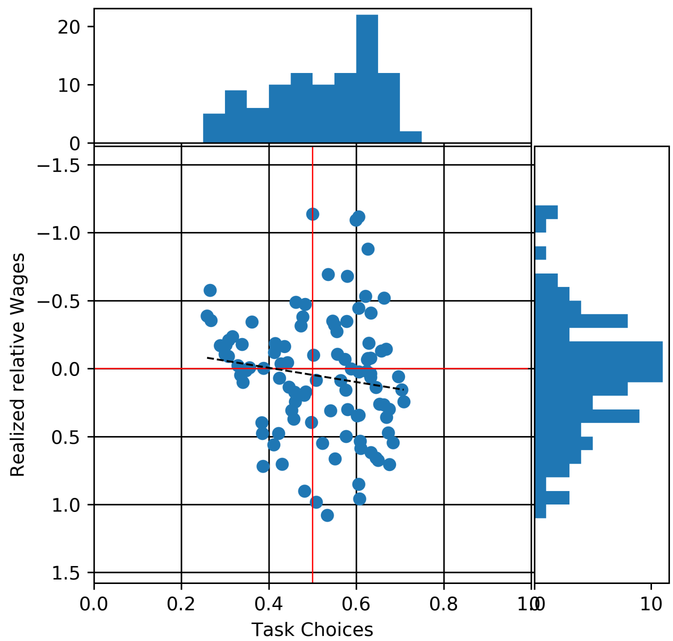

Figure 1: Simulated data resulting from a data generating process that is specified as described in section 3.1. This figure illustrates $N = 100$ combinations of realized wages and optimal task choices in a scatter plot with histograms of the respective variables along each axis. The dashed line results from an OLS fit.

3.2 Monte Carlo Estimation of Skill Prices

This section presents the results of a Monte Carlo estimation of changes in skill prices and serves as a proof of concept of the model I present in section 2 of this thesis, particularly of the estimation strategy discussed in section 2.4. For this purpose it relies on data generated in the process described in previous section.

The estimation of skill prices consists of three steps: First, I make use of a base period to estimate the effects of skill accumulation and changes in the penalty term in the absence of changes in skill prices. Second, observed wage changes of a subsequent period are adjusted for these previously estimated effects. Third, I estimate changes in skill prices based on these residual wage changes. Each of these steps is repeated $M = 1000$ times in a Monte Carlo estimation, each time considering data on $N = 100$ simulated individuals.

In the particular model discussed in this work both, changes in skills and changes in the penalty term from $t-1$ to t, are functions of task choices $\lambda^*_{i,t-1}$ and $\lambda^*_{i,t}$. This can be seen from equation (14). Specifically, skill accumulation results as a function of the mean task choice of both periods, $\bar{\lambda}^*_{i,t}$. The change in penalty term, however, results as a polynomial of degree ϕ of these optimal task choices (see appendix B.1 for a derivation of this result). In the particular setting of the data generating process that is used in this simulation study, it is assumed that $\phi = 2$. In this case, changes in the penalty term can be shown to be equivalent to the expression in equation (15) below as shown in appendix B.1. Therefore, both $\lambda^*_{i,t-1}$ and $\lambda^*_{i,t}$ as well as their squared values need to be included to capture changes in skill endowments and penalty terms.

$$\Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1}) = \lambda^{*2}_{i,t-1} + \lambda^{*2}_{i,t} + 2b_i(\lambda^*_{i,t-1} + \lambda^*_{i,t}) \tag{15}$$

In an estimation of skill accumulation and changes in the penalty term, however, not all of these regressors can be included due to their strong multicollinearity. Not only will each task choice be correlated with its square. In addition, task choices of subsequent periods will be correlated. This is for two reasons: First, optimal task choices (equation (7)) are a function of potential wages. Potential wages, in term, are modeled as the sum of log skills and log skill prices. By the assumption of skill accumulation as a learning by doing process, potential wages will correlate between periods and, thereby, optimal task choices will be correlated as well. Second, optimal task choices are a function of the allocation preferences b_i which are modeled to be time-invariant. As seen from figure 2, all task choice related regressors are, indeed, almost perfectly correlated.

In order to address potential multicollinearity, only task choices of the later period are included as regressors. As seen from the estimation result in table 1, this estimation specification appears to fit the simulation data well with an adjusted R^2 of 0.997. Each iteration of the Monte Carlo estimation is based on $N = 100$ simulated observations. While this number is small enough to keep the computational load low, the resulting estimates have small standard deviations and high significance ($p < 0.001$ for both estimates).

In each of the $M = 1000$ iterations of this Monte Carlo estimation, changes in skills and penalty

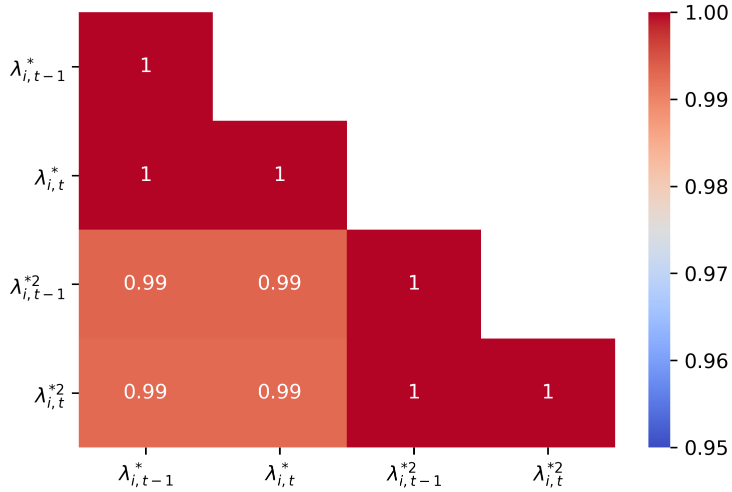

Figure 2: Heatmap of correlation coefficients of task choices in periods $t-1$ and t of the base period as well as their squared values. It should be noted that the scale is compressed and only covers values between 0.95 and 1.0 to make the small differences visible.

Dep. Variable:	y	**R-squared (uncentered):**	0.997
Model:	OLS	**Adj. R-squared (uncentered):**	0.997
Method:	Least Squares	**F-statistic:**	1.721e+04
Date:	Wed, 22 Apr 2020	**Prob (F-statistic):**	1.62e-125
Time:	21:11:52	**Log-Likelihood:**	423.85
No. Observations:	100	**AIC:**	-843.7
Df Residuals:	98	**BIC:**	-838.5
Df Model:	2		

	coef	**std err**	**t**	**P> |t|**	**[0.025**	**0.975]**
$\lambda^*_{i,t}$	-0.4803	0.004	-128.958	0.000	-0.488	-0.473
$\lambda^{*2}_{i,t}$	0.9633	0.006	150.591	0.000	0.951	0.976

Omnibus:	36.470	**Durbin-Watson:**	2.010
Prob(Omnibus):	0.000	**Jarque-Bera (JB):**	111.225
Skew:	1.222	**Prob(JB):**	7.05e-25
Kurtosis:	7.552	**Cond. No.**	12.9

Table 1: Results of an OLS estimation of wage changes in the base period on both, $\lambda^*_{i,t}$ and $\lambda^{*2}_{i,t}$. This estimation result is from one of the $M = 1000$ simulated datasets used in the Monte Carlo estimation. Regressors do not include a constant because the true model intersects the point of origin dur to the normalization of $w_{i,j=1,t} = 0$.

term are estimated using this regression model. After that, wage changes of the subsequent time period are adjusted for the product of the later period's task choices and the estimated coefficient of the base period. Resulting residual wage changes are the basis of the estimation of changes in skill prices.

Figure 3 presents a histogram of the estimation results for $M = 1000$ Monte Carlo estimation

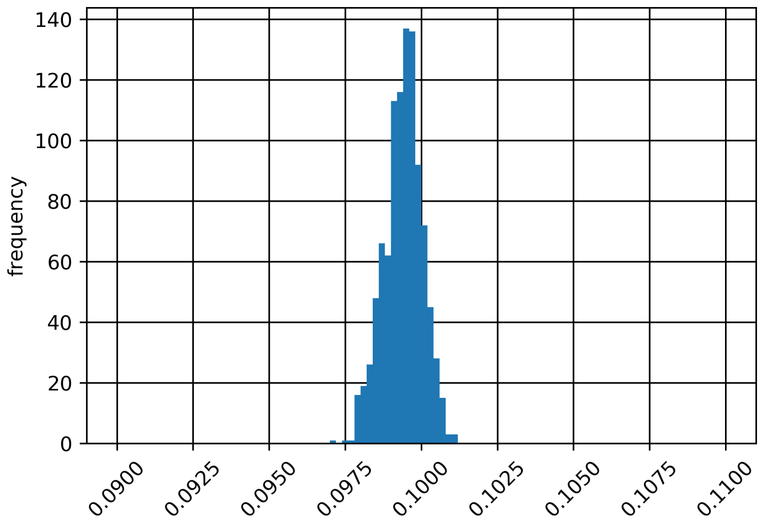

Figure 3: Histogram of the result of $M = 1000$ Monte Carlo estimations of changes in relative skill prices. The true price changed used in the data generating process is 0.1.

in a model with true changes in relative prices of 0.1. While the peak of the distribution of estimation results is slightly below this true change in relative prices at approximately 0.0995, the true value of 0.1 is well covered by this distribution.

4 Conclusions

This thesis presents a method to estimate changes in task specific skill prices. In a theoretical framework of continuous task choice, workers behave utility maximizing by optimally choosing in what tasks they engage under consideration of resulting wages and accompanying occupational amenities. The identification strategy presented in this thesis can be used to infer on changes in relative skill prices from observable wage changes in panel data. As a proof of concept, I test this theoretical result in a simulation study. Herein, simulated changes in skill prices are estimated accurately in a Monte Carlo estimation. This result contributes to the research on changes in wage structure and changing importance of occupational tasks.

Applying the developed method to actual observation data could provide valuable insights into changes in labor demand at a task level. These insights could help to better understand the larger shifts in labor demand that are summarized in the concept of job polarization. Such an application to panel data would, furthermore, provide the opportunity to refine the estimation method. In section 2.3 I show that wage changes result as a composition of changes in task specific skill prices as well as skill accumulation and changes in amenities. The estimation of both, skill accumulation as well as changes in amenities could be refined using observable characteristics of a panel data set. Particularly, Böhm et al. (**bohm2019occupation**) make use of a saturated skill model to estimate changes thereof as a result of occupational choices and observable characteristics controlling the speed of skill acquisition of a worker. In their online appendix, Böhm et al. (**bohm2019occupation**) furthermore provide a method to estimate changes in the relative amenity value of occupational choices between periods and in comparison to a reference occupation. In a setting where the personal characteristics of workers allow an indication of the amenities experienced from their task choice, the authors present a method that enables an estimation of changes in amenities using workers' task choice and characteristics.

With this model of continuous task choice at hand, an arbitrarily differentiated examination of changes in relevance of different occupational tasks for labor market outcomes in the sense of employment and wage growth is possible. In contrast to the discrete division into routine versus non-routine or cognitive versus manual tasks that is usually the basis in similar studies, the model allows for a continuous scale. This can potentially increase the precision of estimations of task prices by attributing wages to occupational tasks more accurately. Additionally, this framework allows to consider a larger number of occupational tasks and according skills in a comprehensive model.

A Modeling Continuous Task Choice: Derivations

A.1 Deriving the utility function with $J = 2$ Tasks

Point of departure is equation (3) where $J = 2$:

$$
\begin{aligned}
u_{i,t} &= \sum_{j=1}^{2} \lambda_{i,j,t} w_{i,j,t} - \theta \sum_{j=1}^{2} |b_{i,j} - \lambda_{i,j,t}|^{\phi} \\
&= \lambda_{i,j=1,t} w_{i,j=1,t} + \lambda_{i,j=2,t} w_{i,j=2,t} - \theta \left(|b_{i,j=1} - \lambda_{i,j=1,t}|^{\phi} + |b_{i,j=2} - \lambda_{i,j=2,t}|^{\phi} \right)
\end{aligned}
$$

With $\sum_{j=1}^{J} \lambda_{i,j,t} = 1$ and $\sum_{j=1}^{J} b_{i,j} = 1$ I can substitute $\lambda_{i,j=1,t}$ by $(1 - \lambda_{i,j=2,t})$ and $b_{i,j=1}$ by $(1 - b_{i,j=2})$ this can be rearranged to:

$$
u_{i,t} = w_{i,j=1,t} + \lambda_{i,t} \tilde{w}_{i,t} - 2\theta |b_i - \lambda_{i,t}|^{\phi}
$$

where

$$
\begin{aligned}
\lambda_{i,t} &\equiv \lambda_{i,j=2,t}, \\
\tilde{w}_{i,t} &\equiv w_{i,j=2,t} - w_{i,j=1,t}, \text{ and} \\
b_i &\equiv_{i,j=2} .
\end{aligned}
$$

Finally, normalizing $w_{i,j=1,t} = 0$, we arrive at equation 5:

$$
u_{i,t} = \lambda_{i,t} \tilde{w}_{i,t} - 2\theta |b_i - \lambda_{i,t}|^{\phi} \tag{5}
$$

A.2 First and Second Order Conditions of the Utility Maximization Problem

Starting from the utility function in equation (5), this section presents the first and second derivative of the utility function $u_{i,t}$ w.r.t. choice parameter $\lambda_{i,t}$. Based on these results, the first and second order conditions for a global maximum of the utility function can be identified.

Starting from the utility function in a setting with $J = 2$ tasks.

$$
u_{i,t} = \lambda_{i,t} \tilde{w}_{i,t} - 2\theta |b_i - \lambda_{i,t}|^{\phi} \tag{5}
$$

Calculate the first derivative of utility w.r.t. task choice parameter $\lambda_{i,t}$ by application of the chain rule.

$$\frac{\partial u_{i,t}}{\partial \lambda_{i,t}} = \tilde{w}_{i,t} - 2\theta\phi|b_i - \lambda_{i,t}|^{\phi-1} \frac{\partial}{\partial \lambda_{i,t}}[|b_i - \lambda_{i,t}|^{\phi}]$$

Applying the derivation rules for absolute value functions:

$$= \tilde{w}_{i,t} + 2\theta\phi|b_i - \lambda_{i,t}|^{\phi-1} \frac{b_i - \lambda_{i,t}}{|b_i - \lambda_{i,t}|}$$

Finally, after some rearrangements:

$$= \tilde{w}_{i,t} + 2\theta\phi \frac{|b_i - \lambda_{i,t}|^{\phi}}{b_i - \lambda_{i,t}} \tag{A.1}$$

Notice that this derivative is not defined at $b_i = \lambda_{i,t}$.

Continuing from equation (A.1), now calculate the second derivative of the utility function w.r.t. task choice parameter $\lambda_{i,t}$ by application of the division rule.

$$\frac{\partial^2 u_{i,t}}{\partial \lambda_{i,t}^2} = 2\theta\phi \left(\frac{\frac{\partial}{\partial \lambda_{i,t}}[|b_i - \lambda_{t,i}|^{\phi}](b_i - \lambda_{i,t}) - |b_i - \lambda_{i,t}|^{\phi} \frac{\partial}{\partial \lambda_{i,t}}[b_i - \lambda_{i,t}]}{(b_i - \lambda_{i,t})^2} \right)$$

By application of the chain rule and derivation rules for absolute value functions:

$$= 2\theta\phi \left(\frac{-\phi|b_i - \lambda_{i,t}|^{\phi} + |b_i - \lambda_{i,t}|^{\phi}}{(b_i - \lambda_{i,t})^2} \right)$$

Rearranging:

$$= 2\theta\phi(1 - \phi) \frac{|b_i - \lambda_{i,t}|^{\phi}}{(b_i - \lambda_{i,t})^2} \tag{A.2}$$

From equation (A.2) it can be seen immediately that the second derivative of the utility function w.r.t. task choices $\lambda_{i,t}$ is (strictly) positive for penalty exponents ϕ (strictly) larger than one, resulting in a (strictly) concave utility function. Analogously, penalty exponents (strictly) smaller than one will result in a (strictly) convex utility function.
Similarly to the first derivative, the second derivative is not defined at $b_i = \lambda_{i,t}$.

A.3 Derivation of the optimal task choice

We start from the first order condition of the utility maximization:

$$\frac{\partial u_{i,t}(b_i, \lambda_{i,t}, \phi, \theta, \tilde{w}_{i,t})}{\partial \lambda_{i,t}} = \tilde{w}_{i,t} + 2\phi\theta \frac{(|b_i - \lambda_{i,t}|)^{\phi}}{b_i - \lambda_{i,t}} \stackrel{!}{=} 0 \tag{A.1}$$

Defining $x \equiv b_i - \lambda_{i,t}$, this can be restated as follows.

$$\begin{aligned} & \tilde{w}_t + 2\phi\theta \frac{(|x|)^{\phi}}{x} \stackrel{!}{=} 0 \\ \Leftrightarrow \qquad & \frac{(|x|)^{\phi}}{x} = -\frac{\tilde{w}_t}{2\phi\theta} \end{aligned}$$

The absolute value function for any real number x is defined as follows.

$$|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$$

Using this definition, two cases can be distinguished when solving for the optimal task choice $\lambda^*_{i,t}$.

Case (1): $x \geq 0$ (i.e., $b_i \geq \lambda_{i,t}$).

$$\begin{aligned} & \frac{x^{\phi}}{x} = -\frac{\tilde{w}_{i,t}}{2\phi\theta} \\ \Leftrightarrow \quad & x = \left(-\frac{\tilde{w}_{i,t}}{2\phi\theta}\right)^{\frac{1}{\phi-1}}, \text{with } x = b_i - \lambda_{i,t} \\ \Leftrightarrow & \lambda^*_{i,t} = b_i - \left(-\frac{\tilde{w}_{i,t}}{2\phi\theta}\right)^{\frac{1}{\phi-1}} \end{aligned}$$

Case (2): $x < 0$ (i.e., $b_i < \lambda_{i,t}$).

$$\begin{aligned} & \frac{(-x)^{\phi}}{x} = -\frac{\tilde{w}_{i,t}}{2\phi\theta} \\ \Leftrightarrow & \frac{(-x)^{\phi}}{-x} = \frac{\tilde{w}_{i,t}}{2\phi\theta} \\ \Leftrightarrow \quad & -x = \left(\frac{\tilde{w}_{i,t}}{2\phi\theta}\right)^{\frac{1}{\phi-1}}, \text{with } x = b_i - \lambda_{i,t} \\ \Leftrightarrow \quad & \lambda^*_{i,t} = b_i + \left(\frac{\tilde{w}_{i,t}}{2\phi\theta}\right)^{\frac{1}{\phi-1}} \end{aligned}$$

Which can be rewritten to the expression in equation (7):

$$\lambda_{i,t}^{*} = \begin{cases} b_i - (\frac{-\tilde{w}_{i,t}}{2\phi\theta})^{\frac{1}{\phi-1}}, & \text{if } b_i \geq \lambda_{i,t} \\ b_i + (\frac{\tilde{w}_{i,t}}{2\phi\theta})^{\frac{1}{\phi-1}}, & \text{if } b_i < \lambda_{i,t} \end{cases} \tag{7}$$

A.4 Identifying Changes in Skill Prices

This appendix provides a more detailed derivation of the identification of skill prices that is presented in section 2.3. Just like in that section, I start from the total derivative of the utility function with respect to time.

$$\begin{aligned} \frac{d}{dt}u_i(\lambda_i^*(\tilde{w}_i(t)), \tilde{w}_i(t)) = & \underbrace{\frac{\partial u_i(\lambda_i^*(\tilde{w}_i(t)), \tilde{w}_i(t))}{\partial \lambda_i^*(\tilde{w}_i(t))} \frac{\partial \lambda_i^*(\tilde{w}_i(t))}{\partial \tilde{w}_i(t)} \frac{d\tilde{w}_i(t)}{dt}}_{(1)} \\ & + \underbrace{\frac{\partial u_i(\lambda_i^*(\tilde{w}_i(t)), \tilde{w}_i(t))}{\partial \tilde{w}_i(t)} \frac{d\tilde{w}_i(t)}{dt}}_{(2)} \end{aligned} \tag{8}$$

By the first order condition, part (1) of above equation equals zero for utility maximizing task choices $\lambda_{i,t}^*$. This can be throught of as an application of the envelope theorem, where $u_{i,t}$is the objective function of a parameterized optimization problem and $\lambda_{i,t}^*(\tilde{w}_{i,t})$ is the parameterized optimizer. As parameter $\tilde{w}_{i,t}$ changes, changes in the optimizer $\lambda_{i,t}^*(\tilde{w}_{i,t})$ of the objective function do not contribute to the change in the objective function. The partial derivation of $u_{i,t}$ w.r.t. $\tilde{w}_{i,t}$ in part (2) is very easily calculated for the particular realized wage function that results for $J = 2$ tasks:

$$\begin{aligned} \frac{\partial u_i(\lambda_i^*(\tilde{w}_i(t)), \tilde{w}_i(t)}{\tilde{w}_i(t)} \frac{d\tilde{w}_i(t)}{dt} = & \frac{\partial}{\partial \tilde{w}_i(t)} \left(\lambda_i^* \tilde{w}_i(t) - 2\theta|b_i - \lambda_i^*|^{\phi} \right) \frac{d\tilde{w}_i(t)}{dt} \\ = & \lambda_i^* \frac{d\tilde{w}_i(t)}{dt} \end{aligned}$$

By substituting this transformation of part (2) into the total differential (equation (8)), we obtain equation (9):

$$\frac{d}{dt}u_i(\lambda_i^*(\tilde{w}_i(t)), \tilde{w}_i(t)) = \lambda^*(\tilde{w}_i(t)) \frac{d\tilde{w}_i(t)}{dt} \tag{9}$$

Integrating equation (9) from $t-1$ to t:

$$\int_{t-1}^{t} \left(\frac{\partial u_i(\lambda_i^*(\tilde{w}_i(\tau)), \tilde{w}_i(\tau)}{\tilde{w}_i(\tau)} \frac{d\tilde{w}_i(\tau)}{dt} \right) d\tau = \int_{t-1}^{t} \left(\lambda^*(\tilde{w}_i(t)) \frac{d\tilde{w}_i(t)}{dt} \right) d\tau$$

I define Δ to denote the discrete change in a variable between two periods. Specifically, in case of the utility $\Delta u_{i,t}$ denotes the discrete change in utility from period $t-1$ to t, i.e.:

$$\begin{aligned}\Delta u_{i,t} &\equiv \int_{t-1}^{t} \left(\frac{\partial u_i(\lambda_i^*(\tilde{w}_i(\tau)), \tilde{w}_i(\tau)}{\tilde{w}_i(\tau)} \frac{d\tilde{w}_i(\tau)}{dt} \right) d\tau \\ &= \int_{t-1}^{t} \left(\lambda^*(\tilde{w}_i(t)) \frac{d\tilde{w}_i(t)}{dt} \right) d\tau\end{aligned}$$

Rewriting this by application of integration by substitution with $\tilde{w}_i(t) = \tilde{w}_{i,t}$ and relying on the abridged notation with t as subscript again:

$$\Delta u_{i,t} = \int_{\tilde{w}_{i,t-1}}^{\tilde{w}_{i,t}} \lambda_{i,t}^* d\tilde{w}_{i,\tau} \tag{10}$$

I now apply the following linear approximation of the task choice parameter:

$$\begin{aligned}\frac{\lambda_{i,\tau}^* - \lambda_{i,t-1}^*}{\tilde{w}_{i,\tau} - \tilde{w}_{i,t-1}} &\approx \frac{\lambda_{i,t}^* - \lambda_{i,t-1}^*}{\tilde{w}_{i,t} - \tilde{w}_{i,t-1}} \\ \Leftrightarrow \lambda_{i,\tau}^* &\approx \lambda_{i,t-1}^* + \frac{\lambda_{i,t}^* - \lambda_{i,t-1}^*}{\tilde{w}_{i,t} - \tilde{w}_{i,t-1}} (\tilde{w}_{i,\tau} - \tilde{w}_{i,t-1})\end{aligned} \tag{11}$$

Now substituting equation (11) into equation (10):

$$\Delta u_{i,t} = \int_{w_{i,t-1}}^{w_{i,t}} \left(\lambda_{i,t-1}^* + \frac{\lambda_{i,t}^* - \lambda_{i,t-1}^*}{\tilde{w}_{i,t} - \tilde{w}_{i,t-1}} (\tilde{w}_{i,\tau} - \tilde{w}_{i,t-1}) \right) dw_{i,\tau}$$

Calculate the closed interval by evaluating the antiderivative:

$$= \left[\lambda_{i,t-1}^* \tilde{w}_{i,\tau} \frac{\lambda_{i,t}^* - \lambda_{i,t-1}^*}{\tilde{w}_{i,t} - \tilde{w}_{i,t-1}} (\frac{1}{2} \tilde{w}_{i,\tau}^2 - \tilde{w}_{i,t-1} \tilde{w}_{i,\tau}) \right]_{\tilde{w}_{i,t-1}}^{\tilde{w}_{i,t}}$$

Evaluating the closed interval:

$$= \lambda_{i,t-1}^* \Delta \tilde{w}_{i,t} + \frac{\lambda_{i,t}^* - \lambda_{i,t-1}^*}{\tilde{w}_{i,t} - \tilde{w}_{i,t-1}} (\frac{1}{2} \tilde{w}_{i,t}^2 - \tilde{w}_{i,t-1} \tilde{w}_{i,t} + \frac{1}{2} \tilde{w}_{i,t-1}^2)$$

Rearranging and making use of the binomial expansion:

$$= \lambda_{i,t-1}^* \Delta \tilde{w}_{i,t} + \frac{1}{2} \frac{\lambda_{i,t}^* - \lambda_{i,t-1}^*}{\tilde{w}_{i,t} - \tilde{w}_{i,t-1}} (\tilde{w}_{i,t} - \tilde{w}_{i,t-1})^2$$

Which can be rearranged to:

$$= \bar{\lambda}_{i,t}^* \Delta \tilde{w}_{i,t} \tag{12}$$

B Simulation Study: Derivations

B.1 Deriviation of Changes in the Penalty Term

As part of the estimation of changes in skill prices, changes in the penalty term need to be estimated. These changes in the penalty term are a funtion of optimal task choices $\lambda^*_{i,\tau},\ \tau \in \{t-1,t\}$. In this appendix I show that changes in the penalty term result as a polynomial with penalty exponent of degree ϕ.
From equation (14) we have that wage changes come as a composition of three parts:

$$\Delta w_{i,t} = \bar{\lambda}^*_{i,t}\Delta\tilde{\pi}_t + \bar{\lambda}^*_{i,t}\Delta\tilde{s}_{i,t} + \left[\Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1})\right] \tag{14}$$

In the following I show that the part related to changes in the penalty term (i.e., $\left[\Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1})\right]$) can be restated as a polynomial of optimal task choice parameters $\lambda^*_{i,\tau},\ \tau \in \{t-1,t\}$ of degree ϕ.

Starting from

$$\Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1}) \tag{B.1}$$

with

$$\Phi_i(\lambda^*_{i,\tau}) = \theta|b_i - \lambda^*_{i,\tau}|^\phi,\ \text{for } \tau \in \{t-1,t\}$$

With regard to the absolute value function, two cases have to be considered:

$$|b_i - \lambda^*_{i,\tau}| = \begin{cases} b_i - \lambda^*_{i,\tau}, & \text{if } b_i - \lambda^*_{i,\tau} \geq 0 \\ -(b_i - \lambda^*_{i,\tau}), & \text{if } b_i - \lambda^*_{i,\tau} < 0 \end{cases}$$

Consider first case: $b_i - \lambda^*_{i,\tau} \geq 0$. In this case equation B.1 can be written as follows:

$$\Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1}) = (b_i - \lambda^*_{i,t})^\phi - (b_i - \lambda^*_{i,t-1})^\phi$$

Each of the two binomials on the right hand side can be rewritten as bivariate polynomials by application of the binomial expansion:

$$(b_i - \lambda^*_{i,\tau})^\phi = \sum_{k=0}^{\phi} \binom{\phi}{k} b_i^{\phi-k}(-\lambda_{i,\tau})^k,\ \text{for } \tau \in \{t-1,t\}$$

It is straightforward to see that in the second case ($b_i - \lambda^*_{i,\tau} < 0$) the result in analogous with opposite sign.
Thus, changes in the penalty term can be expressed as the differencee between two polynomials

of $\lambda_{i,t-1}$ and $\lambda_{i,t}$, respectively. The degree of the resulting polynomial is equal to the penalty exponent ϕ.

In the simulation study, it is assumed that $\phi = 2$. The changes in the penalty function in this particular case, therefore, result as follows:

$$\begin{aligned}\Phi_i(\lambda^*_{i,t}) - \Phi_i(\lambda^*_{i,t-1}) &= (b_i - \lambda_{i,t})^2 - (b_i - \lambda_{i,t-1})^2 \\ &= \lambda^{*2}_{i,t-1} + \lambda^{*2}_{i,t} + 2b_i(\lambda^*_{i,t-1} + \lambda^*_{i,t})\end{aligned} \tag{15}$$