ROBOT KINEMATICS, MODELLING AND SIMULATION

ROBOTICS 2023

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Lecture plan

No	Responsible	Contents
1	Ole Madsen	Introduction to: • the course; • robotics and robot terminology.
2	Ole Madsen	Spatial descriptions and transformation matrices
3	Ole Madsen	Orientation
4 (FIB)	Ole Madsen	Practical exercise with the on-line programming (1.5 timer/gruppe).
5	Ole Madsen	Forward Kinematics I
6	Ole Madsen	Forward Kinematics II (go though 6 DOF robot) – exercise, you go though your robot
7	Ole Madsen	Inverse kinematics I
8	Ole Madsen	Inverse kinematics II (go through 6DOF robot) – you start on your robot
9	Ole Madsen	Trajectory generation and control (joint)
10	Ole Madsen	Trajectory generation and control (cartesian)
11	Ole Madsen	Jacobian/Exam preparation

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TRAJECTORY GENERATION

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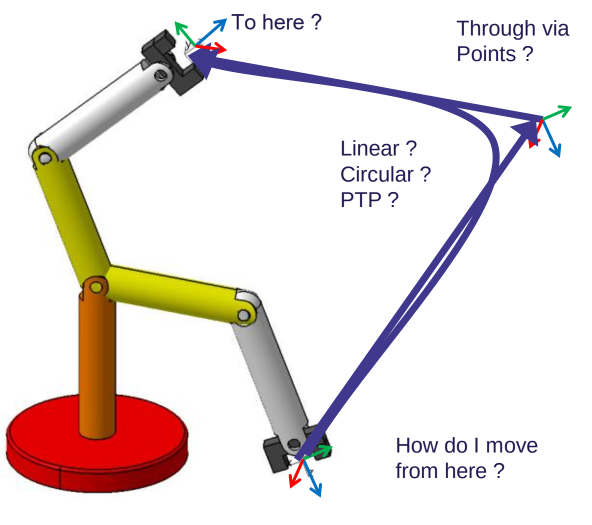
Trajectory generation



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Program

```
MODULE SVEJS
PROC main()
    MoveJ,*,v100,z50,tool0;
    ArcL\On,*,v200,sm1,wd1,wv1,fine,tool0;
    ArcL\Off,*,v100,sm1,wd1,wv1,fine,tool0;
    MoveL,*,v200,fine,tool0;
  ENDPROC
ENDMODULE
```

Controller

Control Data

Machine

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User Specified Trajectory

Location:
- Start location
- End location
- Intermediate location (via points)

Interpolation
- PTP-motion (ABB: MoveJ/FANUC: J P[0])
- Linear motion (ABB: MoveL/FANUC: L P[0])
- Circular motion (ABB: MoveC/FANUC: C P[0])

Technological Instructions
- Velocity
- Acceleration
- Tool Functions and Settings (weaving)

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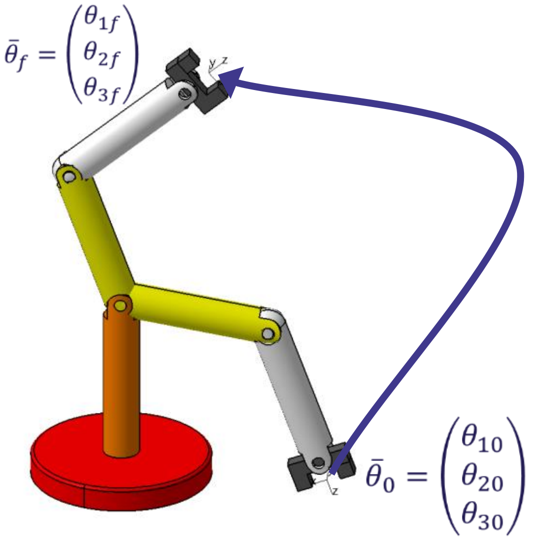
PTP-movement (MoveJ, J, Joint movement)



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Requirements to trajectory generation

- The motion of the manipulator must be smooth:
  - Continuous path
  - Continuous velocity profile
  - Sometimes continuous acceleration
- All joints reach their target location at the same time

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Agenda

- **Joint trajectories (PTP-movements)**
  - moving one joint axes from one location to another (no via points)
  - moving from one joint axis from one location to another through via points)
  - Multi dimensional joint trajectories

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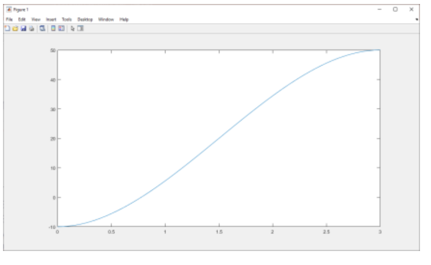
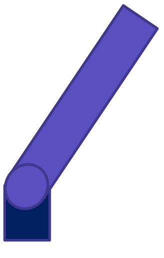
Moving one joint axes from one location to another (no via points)

- Robot with one axis
- Desired movement:
  - Start in θ_0
  - End in θ_f
  - Time for movement: t_f
  - Velocities in start and end are zero





We are looking for a function:

$\Theta(t)$

which fullfills the requirements

We will examine two options:

- Cubic polyminials
- Parabolic blend

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Cubic polynomials –no viapoints

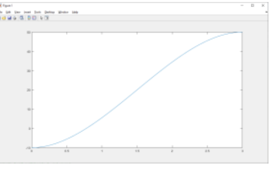
$$\theta(t) = a_0 + a_1 \cdot t + a_2 t^2 + a_3 t^3$$

$$\theta(\dot{t}) = a_1 + 2a_2 \cdot t + 3a_3 t^2$$

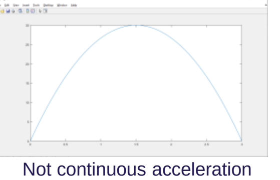
$$\theta(\ddot{t}) = 2a_2 + 6a_3 \cdot t$$

But how do we find the a's ?

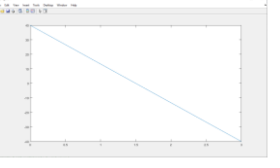
Continuous path



Continuous velocity profile



Not continuous acceleration



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Cubic polynominal –no viapoints

We know:

$$\theta(t) = a_0 + a_1 \cdot t + a_2 t^2 + a_3 t^3$$

$$\theta(\dot{t}) = a_1 + 2a_2 \cdot t + 3a_3 t^2$$

We have the following constraints:

$$\theta(0) = \theta_0$$

$$\theta(t_f) = \theta_f$$

$$\theta(\dot{0}) = 0$$

$$\theta(\dot{t}_f) = 0$$

$$\theta(0) = a_0 = \theta_0$$

$$\theta(\dot{0}) = a_1 = 0$$

$$\theta(t_f) = \theta_0 + a_2 {t_f}^2 + a_3 {t_f}^3 = \theta_f$$

$$\theta(\dot{t}_f) = 2a_2 \cdot tf + 3a_3 {t_f}^2 = 0$$

$$a_2 = \frac{3}{{t_f}^2}(\theta_f - \theta_0)$$

$$a_3 = -\frac{2}{{t_f}^3}(\theta_f - \theta_0)$$

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Cubic polynominal –no viapoints – proof (a2, a3)

$$\theta(t_f) = \theta_0 + a_2 {t_f}^2 + a_3 {t_f}^3 = \theta_f$$
$$\dot{\theta}(t_f) = 2a_2 \cdot tf + 3a_3 {t_f}^2 = 0$$

$$a_2 {t_f}^2 + a_3 {t_f}^3 = \theta_f - \theta_0$$
$$2a_2 \cdot tf + 3a_3 {t_f}^2 = 0$$

$$\begin{bmatrix} t_f^2 & t_f^3 \\ 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \theta_f - \theta_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} t_f^2 & t_f^3 \\ 2t_f & 3t_f^2 \end{bmatrix}^{-1} \begin{bmatrix} \theta_f - \theta_0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{t_f^2} & -\frac{1}{t_f} \\ -\frac{2}{t_f^2} & -\frac{1}{t_f^2} \end{bmatrix} \begin{bmatrix} \theta_f - \theta_0 \\ 0 \end{bmatrix}$$

$$a_2 = \frac{3}{{t_f}^2}(\theta_f - \theta_0)$$

$$a_3 = -\frac{2}{{t_f}^3}(\theta_f - \theta_0)$$

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Cubic polynominal –no viapoints

Now you try to find the a’s realising the following movement:

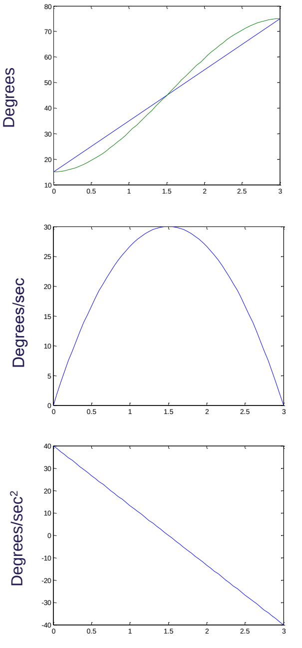
- Start in $\theta_0 = 15^\circ$
- End in $\theta_f = 75^\circ$
- Time for movement: $t_f = 3$ sec
- Velocities in start and finish are zero

Make a sketch of the position, velocity and acceleration profiles

What is the position at time 1.76 sec ?

Solution

$$\theta(t) = 15 + 20t^2 - 4.44t^3$$
$$\dot{\theta}(t) = 40 \cdot t - 13.33t^2$$
$$\ddot{\theta}(t) = 40 - 26.64 \cdot t$$



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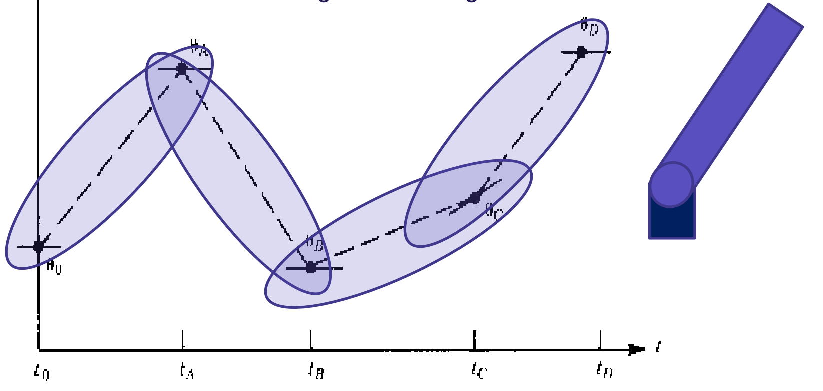
Cubic polynominal with via points

Plan segment for segment



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Cubic polynominal –with viapoints

We know:

$$\theta(t) = a_0 + a_1 \cdot t + a_2 t^2 + a_3 t^3$$

$$\dot{\theta}(t) = a_1 + 2a_2 \cdot t + 3a_3 t^2$$

We have the following constraints:

$$\theta(0) = \theta_0$$

$$\theta(t_f) = \theta_f$$

$$\dot{\theta}(0) = \dot{\theta}_0$$

$$\dot{\theta}(t_f) = \dot{\theta}_f$$

$$\theta(0) = a_0 = \theta_0$$

$$\dot{\theta}(0) = a_1 = \dot{\theta}_0$$

$$\theta(t_f) = \theta_0 + \dot{\theta}_0 t_f + a_2 t_f^2 + a_3 t_f^3 = \theta_f$$

$$\dot{\theta}(t_f) = \dot{\theta}_0 + 2a_2 \cdot tf + 3a_3 t_f^2 = \dot{\theta}_f$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$

$$a_3 = \frac{-2}{t_f^3}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0)$$

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Cubic polynominal –with viapoints – proof (a2, a3)

$$\theta(t_f) = \theta_0 + \dot{\theta}_0 t_f + a_2 t_f^{\,2} + a_3 t_f^{\,3} = \theta_f$$
$$\theta(\dot{t}_f) = \dot{\theta}_0 + 2a_2 \cdot tf + 3a_3 t_f^{\,2} = \dot{\theta}_f$$

$$a_2 t_f^{\,2} + a_3 t_f^{\,3} = \theta_f - \theta_0 - \dot{\theta}_0 t_f$$
$$2a_2 \cdot tf + 3a_3 t_f^{\,2} = \dot{\theta}_f - \dot{\theta}_0$$

$$\begin{bmatrix} t_f^2 & t_f^3 \\ 2t_f & 3t_f^2 \end{bmatrix} \begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \theta_f - \theta_0 - \dot{\theta}_0 t_f \\ \dot{\theta}_f - \dot{\theta}_0 \end{bmatrix}$$

$$\begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} t_f^2 & t_f^3 \\ 2t_f & 3t_f^2 \end{bmatrix}^{-1} \begin{bmatrix} \theta_f - \theta_0 - \dot{\theta}_0 t_f \\ \dot{\theta}_f - \dot{\theta}_0 \end{bmatrix}$$

$$\begin{bmatrix} a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} \frac{3}{t_f^2} & -\frac{1}{t_f} \\ -\frac{2}{t_f^2} & \frac{1}{t_f^2} \end{bmatrix} \begin{bmatrix} \theta_f - \theta_0 - \dot{\theta}_0 t_f \\ \dot{\theta}_f - \dot{\theta}_0 \end{bmatrix}$$

$$a_2 = \frac{3}{t_f^2}\left(\theta_f - \theta_0 - \dot{\theta}_0 t_f\right) - \frac{1}{t_f}\left(\dot{\theta}_f - \dot{\theta}_0\right)$$
$$a_3 = -\frac{2}{t_f^{\,3}}\left(\theta_f - \theta_0 - \dot{\theta}_0 t_f\right) + \frac{1}{t_f^2}(\dot{\theta}_f - \dot{\theta}_0)$$

$$a_2 = \frac{3}{t_f^2}(\theta_f - \theta_0) - \frac{2}{t_f}\dot{\theta}_0 - \frac{1}{t_f}\dot{\theta}_f$$
$$a_3 = -\frac{2}{t_f^{\,3}}(\theta_f - \theta_0) + \frac{1}{t_f^2}(\dot{\theta}_f + \dot{\theta}_0)$$

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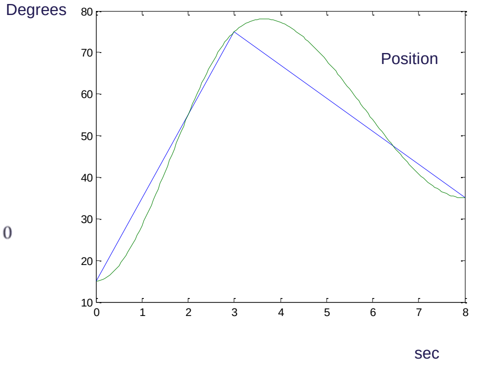
Cubic Polynomials- with via points - Example

Given:

- t_{f12} = 3 sec
- t_{f23} = 5 sec
- Θ(0) = 15 degrees
- Θ(t_{f12}) = 75 degrees
- Θ(t_{f23}) = 35 degrees
- Start and stop from rest
  - $\dot{\theta}(0) = 0; \dot{\theta}\left(t_{f12} + t_{f23}\right) = 0$
- Speed in via point:
  - $\dot{\theta}\left(t_{f12}\right) = 10 degres/sec$



$$\theta 12(t) = 15.00 + 16.66t^2 - 3.33t^3$$
$$\theta 23(t) = 75.00 + 10.00(t - t_{f12}) - 8.80(t - t_{f12})^2 - 1.04(t - t_{f12})^3$$

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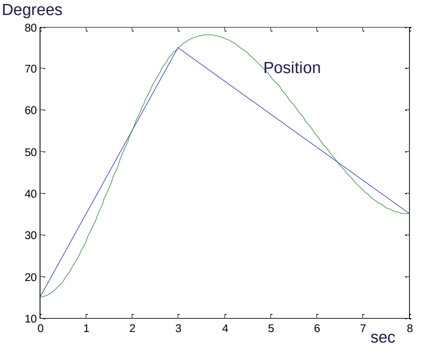
Cubic Polynomials- with via points - Example

Now you try to find the a's realising the following movement:

- t_{f12} = 3 sec
- t_{f23} = 5 sec
- $\Theta(0)$ = 15°
- $\Theta(t_{f12})$ = 75°
- $\Theta(t_{f12}+t_{f23})=35$
- Start and stop from rest
  $\dot{\theta}(0)= \dot{\theta}(t_{f12}+t_{f23})=0$
- Velocity in via point:
  $\dot{\theta}(t_{f12})=10$ °/sec



$$\theta 12(t) = 15.00 + 16.66t^2 - 3.33t^3$$

$$\theta 23(t) = 75.00 + 10.00(t - t_{f12}) - 8.80(t - t_{f12})^2 - 1.04(t - t_{f12})^3$$

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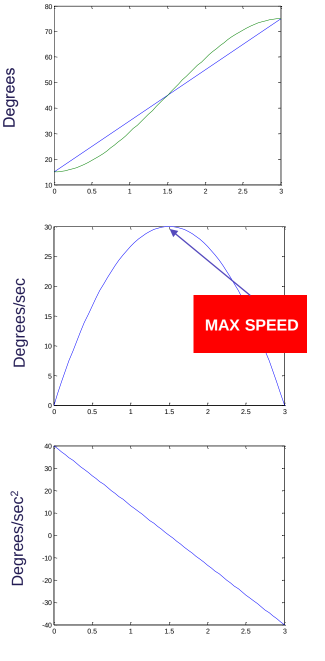
Cubic polynominal –no viapoints

Now you try to find the a's realising the following movement:

- Start in θ_0 = 15°
- End in θ_f = 75°
- Time for movement: t_f = 3 sec
- Velocities in start and finish are zero



Solution

$$\theta(t) = 15 + 20t^2 - 4.44t^3$$
$$\dot{\theta}(t) = 40 \cdot t - 13.33t^2$$
$$\ddot{\theta}(t) = 40 - 26.64 \cdot t$$

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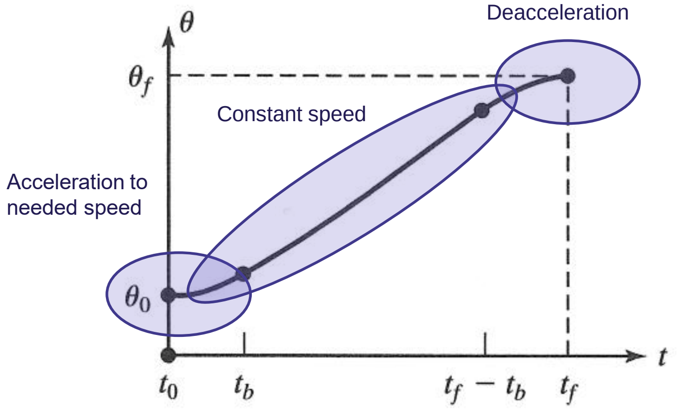
Liniar function with parabolic blend



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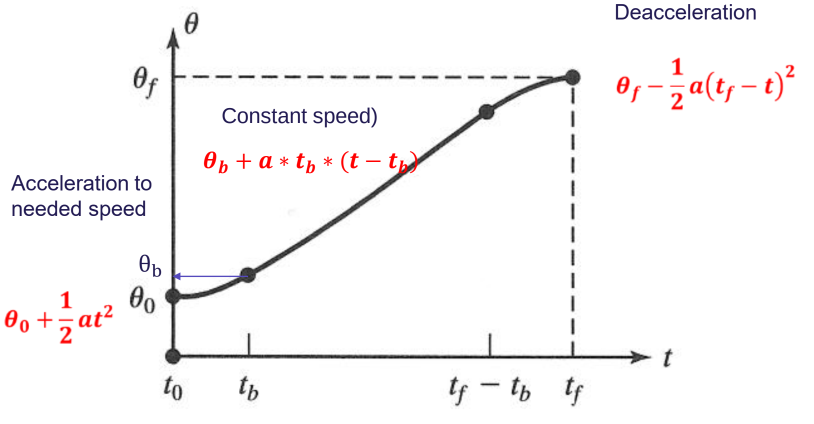
Liniar function with parabolic blend



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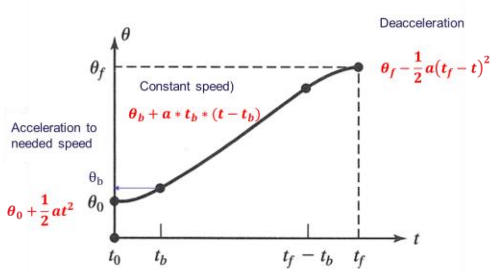
Liniar function with parabolic blend

Assume the following is given:

- Θ_0, θ_f, t_f and a

Compute t_b and θ_b :

$$t_b = \frac{t_f}{2} - \frac{\sqrt{a^2 t_f^2 - 4a\left(\theta_f - \theta_0\right)}}{2a} \qquad \theta_b = \frac{1}{2} a t_b^2 + \theta_0$$



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Liniar function with parabolic blend

$$\dot{\theta}(t_b) = a \cdot t_b$$

$$\dot{\theta}(t_b) = \frac{\theta_f - 2\theta_b + \theta_0}{t_f - 2t_b}$$

$$\theta_b = \frac{1}{2} a {t_b}^2 + \theta_0$$

$$\frac{\theta_f - 2(\frac{1}{2} a {t_b}^2 + \theta_0) + \theta_0}{t_f - 2t_b} = a \cdot t_b$$

$$\theta_f - a{t_b}^2 - \theta_0 = a \cdot t_b (t_f - 2t_b)$$

$$a{t_b}^2 - a t_f t_b + \theta_f - \theta_0 = 0$$

$$t_b = \frac{t_f}{2} - \frac{\sqrt{a^2 t_f^2 - 4a\left(\theta_f - \theta_0\right)}}{2a} \qquad a >= \frac{4\left(\theta_f - \theta_0\right)}{t_f^2}$$

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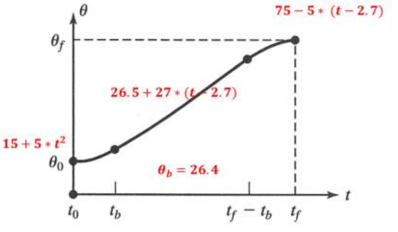
Liniar function with parabolic blend

Now you try:

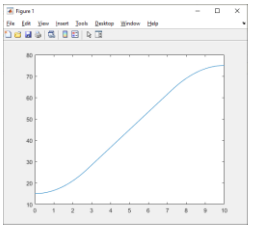
Given

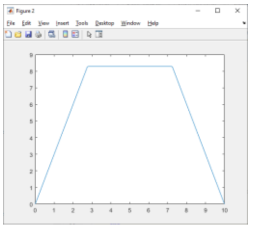
- $\Theta(0) = 15$
- $\Theta(t) = 75$
- $a = 3$ degree/s^2
- $t_f = 10$ s

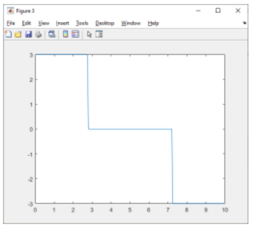
$t_b = 2.7$
$\theta_b = 26.5$



Design the parabolic blend and sketch the position, velocity and acceleraction curves.







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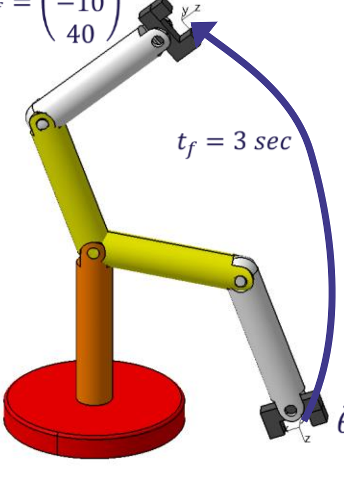
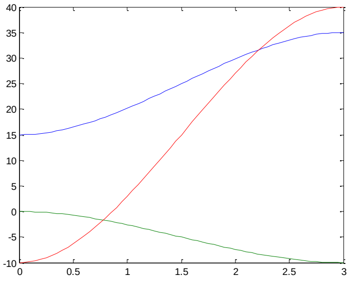
Cubic Polynomials – multi dimensional trajectories

$$\bar{\theta}_f = \begin{pmatrix} 35 \\ -10 \\ 40 \end{pmatrix}$$



$t_f = 3\ sec$

$$\bar{\theta}_0 = \begin{pmatrix} 15 \\ 0 \\ -10 \end{pmatrix}$$



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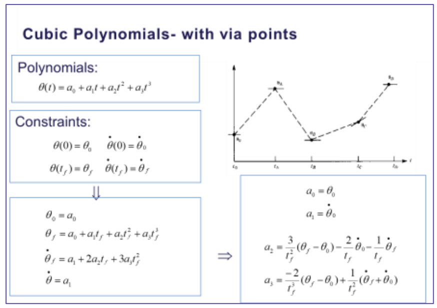
PTP-movement (MoveJ, J, Joint movement)

1. Plan the movement e.g: if we use cubic polynominals: find the a's.



2. Interpolate find the location for a given t ∈ [0;t_f]:

$$\theta_1(t) = a_{01} + a_{11} \cdot t + a_{21}t^2 + a_{31}t^3$$
$$\theta_2(t) = a_{02} + a_{12} \cdot t + a_{22}t^2 + a_{32}t^3$$
$$\theta_3(t) = a_{0Z} + a_{13} \cdot t + a_{23}t^2 + a_{33}t^3$$

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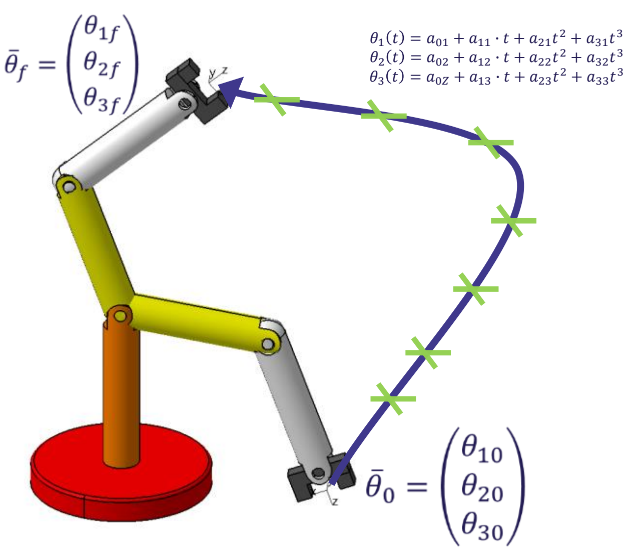
PTP-movement (MoveJ, J, Joint movement)



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Exercises

1. A single-axis robot with a rotary joint is motionless at θ_0 = -5°. It is desired to move the joint in a smooth manner to θ_f = 80° in 4 sec.
    a) Find the coefficients of a cubic which accomplishes this motion.
    b) Draw the graphs of position velocity and acceleration.
    c) Test results in Matlab
2. It is desired that a single-axis robot is moving as follows: start in rest from: θ_1 = 5.0 degrees, go via the point θ_2 = 15.0 degrees, to the ending position at θ_3 = 40.0 degrees. Assume that the duration of each segment is 1.0 second and that the velocity in the via point is 15.5 degrees/sec.
    a) Find the coefficients of the cubics which accomplish this motion.
    b) Where is the robot after 0.876 sec ?
    c) Plot the graphs of position, velocity and acceleration.
    d) Test results in Matlab

Exercises

3. A single-axis robot with a rotary joint is motionless at θ_0 = -50°. It is desired to move the joint in a smooth manner to θ_f = 100° in 10 sec.
    a) Find the equations of the segments in a a parabolic blend which accomplishes this motion. Assume the acceleration is 10 mm/sec
    b) Plot the graphs of position, velocity and acceleration
    c) Test results in Matlab
4. Take the motion from question 3.
    a) Find the equations of the segments in a parabolic blend which accomplishes this motion. This time assume that tb should be 3 sec.
    b) Test results in Matlab

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