ROBOT KINEMATICS, MODELING AND SIMULATION

ORIENTATION/ROTATIONS

ROBOTICS 2022

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Agenda

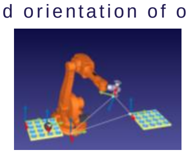
1. Follow-up from last time
2. A little about trigonometry (recap of high-school + bonus)
3. More about rotation matrices
4. Other ways to specify orientations:
   - Three angles (Euler/fixed)
   - Axis-angle representation
   - Quaternions

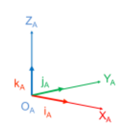
Lecture plan

No	Responsible	Contents
1	Ole Madsen	Introduction to: • the course; • robotics and robot terminology.
2	Ole Madsen	Spatial descriptions and transformation matrices
3 (FIB)	Ole Madsen + More	Practical exercise with the on-line programming (1.5 timer/gruppe).
4	Ole Madsen	Orientation
5	Ole Madsen	Forward Kinematics I
6	Ole Madsen	Forward Kinematics II (go though 6 DOF robot) – exercise, you go though your robot
7	Ole Madsen	Inverse kinematics I
8	Ole Madsen	Inverse kinematics II (go through 6DOF robot) – you start on your robot
9	Ole Madsen	Trajectory generation and control (joint)
10	Ole Madsen	Trajectory generation and control (cartesian)
11	Ole Madsen	Jacobian/Exam preparation

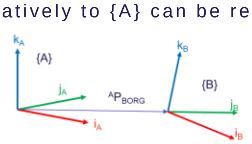
Summary from last time

- Position and orientation of objects are represented by coordinate systems.





- The location (position and orientation) of a coordinate system {B} relatively to {A} can be represented by a transformation matrix



$$ {}^A_BT = \begin{bmatrix} {}^Ai_{B,X} & {}^Aj_{B,X} & {}^Ak_{B,X} & {}^AP_{BORG,X} \\ {}^Ai_{B,Y} & {}^Aj_{B,Y} & {}^Ak_{B,Y} & {}^AP_{BORG,Y} \\ {}^Ai_{B,Z} & {}^Aj_{B,Z} & {}^Ak_{B,Z} & {}^AP_{BORG,Z} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

${}^A_BR$ *rotations matrix (3x3)*

i_B *seen from {A}*

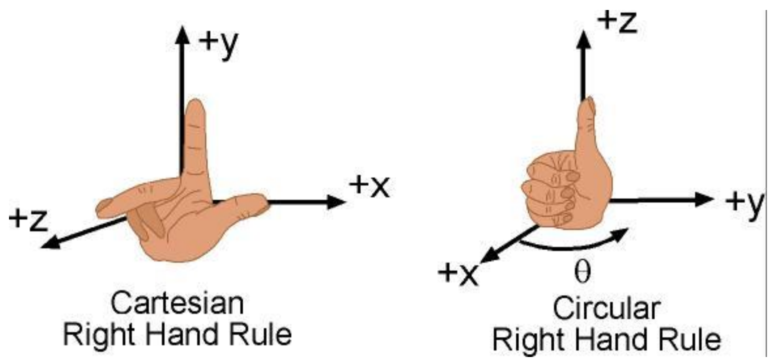
${}^AP_{BORG}$

“0 0 0 1” always

- Some important rules.

${}^AP = {}^A_BT \cdot {}^BP$	${}^B_AT = \left({}^A_BT\right)^{-1}$	${}^A_CT = {}^A_BT \cdot {}^B_CT$
${}^AV = {}^A_BR \cdot {}^BV$	${}^B_AR = \left({}^A_BR\right)^{-1} = \left({}^A_BR\right)^T$	${}^A_CR = {}^A_BR \cdot {}^B_CR$

The secret hand signs !

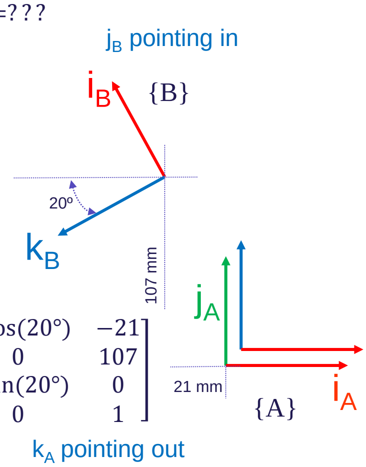


Test exercise:

Hand-in solutions for this exercise on Moodle

Find the transformation matrix: ${}^A_BT = ???$

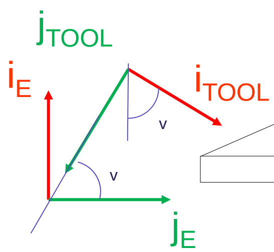
$${}^A_BT = \begin{bmatrix} -\sin(20°) & 0 & -\cos(20°) & -21 \\ \cos(20°) & 0 & -\sin(20°) & 107 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

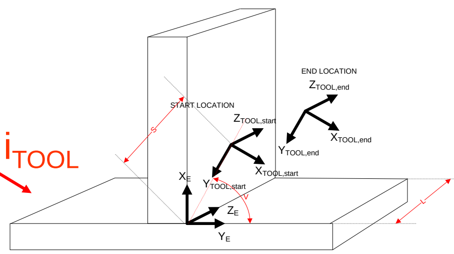


$${}^A_BT = RotXA(-90) * RotY(110)$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\sin(20°) & 0 & -\cos(20°) & -21 \\ 0 & 1 & 0 & 107 \\ \cos(20°) & 0 & -\sin(20°) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos(20) = \cos\left(20 * \frac{pi}{180} rad\right) ???$$ $\quad$ $Please\ - use\ pdf$





$$ {}_{Start}^{E}T = \begin{bmatrix} -\cos(v) & -\sin(v) & 0 & s*\sin(v) \\ \sin(v) & -\cos(v) & 0 & s*\cos(v) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ {}_{end}^{E}T = \begin{bmatrix} -\cos(v) & -\sin(v) & 0 & s*\sin(v) \\ \sin(v) & -\cos(v) & 0 & s*\cos(v) \\ 0 & 0 & 1 & L \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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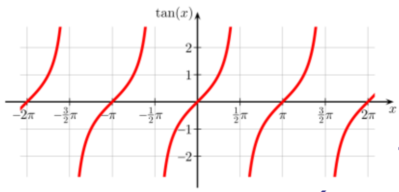
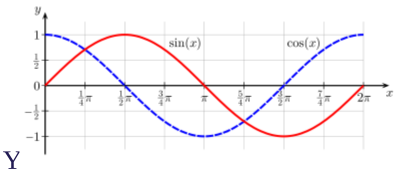
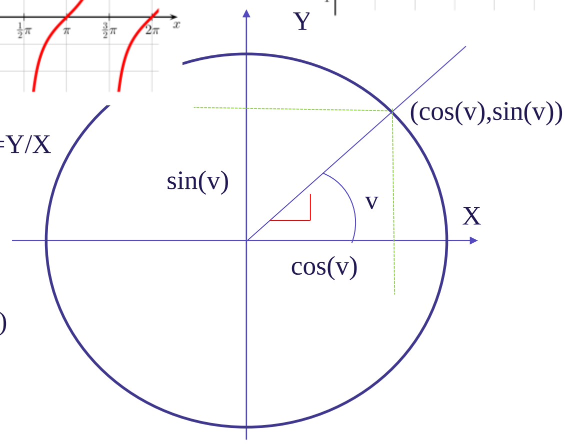




Tan(v)=sin(v)/cos(v)=Y/X

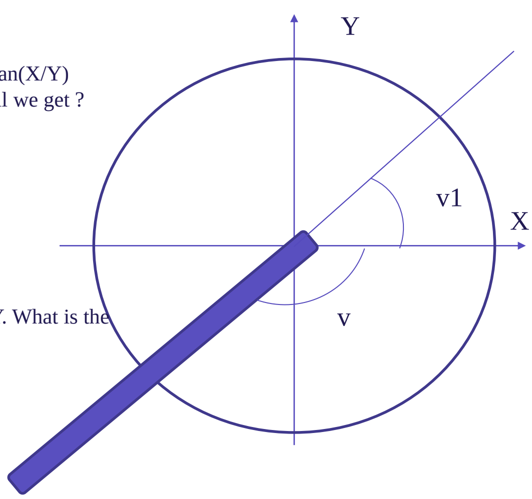


v = arctan (Y/X)

arctan(Y/X) gives
Solution between -90 and 90

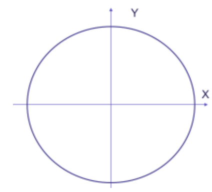
arctan(X/Y) gives
Solution between -90 and 90

If we take arctan(X/Y)
which angle will we get ?



A robot
Assume we know X,Y. What is the
angle v ?

Atan2



$$\operatorname{atan2}(y,x) = \begin{cases} \arctan(\frac{y}{x}) & \text{if } x > 0, \\ \arctan(\frac{y}{x}) + \pi & \text{if } x < 0 \text{ and } y \geq 0, \\ \arctan(\frac{y}{x}) - \pi & \text{if } x < 0 \text{ and } y < 0, \\ +\frac{\pi}{2} & \text{if } x = 0 \text{ and } y > 0, \\ -\frac{\pi}{2} & \text{if } x = 0 \text{ and } y < 0, \\ \text{undefined} & \text{if } x = 0 \text{ and } y = 0. \end{cases}$$

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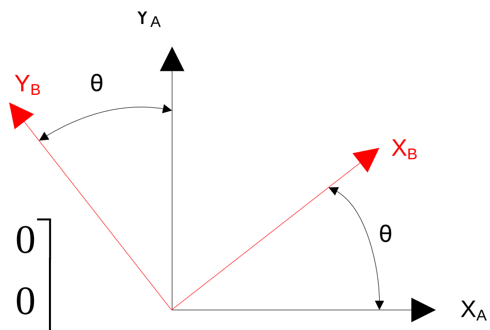
Rotation about Z



$$R_z(\theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Basic Rotations

- Rotation about X

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\theta & -S\theta \\ 0 & S\theta & C\theta \end{bmatrix}$$

- Rotation about Y

$$R_y(\theta) = \begin{bmatrix} C\theta & 0 & S\theta \\ 0 & 1 & 0 \\ -S\theta & 0 & C\theta \end{bmatrix}$$

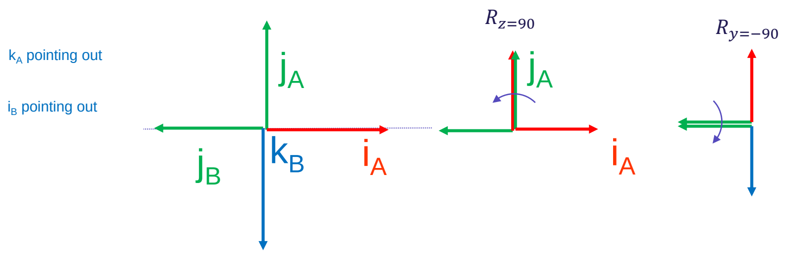
- Rotation about Z

$$R_z(\theta) = \begin{bmatrix} C\theta & -S\theta & 0 \\ S\theta & C\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A small exercise:

You come from A-> B through two rotation

Find the rotations

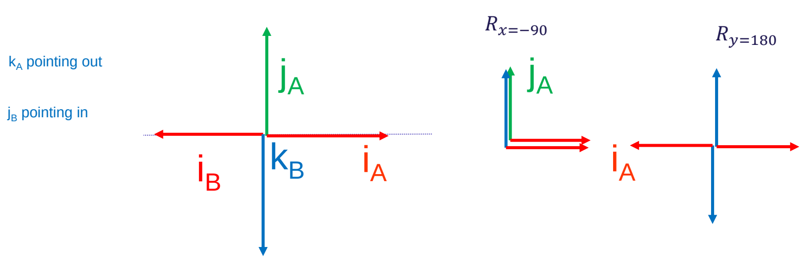


$$ {}^{A}_{B}R = R_{z=90}R_{y=-90} = \begin{bmatrix} \cos(90) & -\sin(90) & 0 \\ \sin(90) & \cos(90) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(90) & 0 & \sin(90) \\ 0 & 1 & 0 \\ -\sin(90) & 0 & \cos(90) \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix} $$

A small exercise:

You come from A-> B through two rotation

Find the rotations

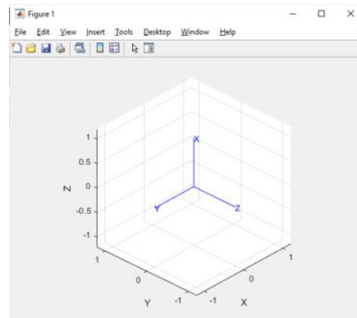


$$ {}^{A}_{B}R = R_{x=-90}R_{y=180} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(90) & -\sin(90) \\ 0 & \sin(90) & \cos(90) \end{bmatrix} \begin{bmatrix} \cos(180) & 0 & \sin(180) \\ 0 & 1 & 0 \\ -\sin(180) & 0 & \cos(180) \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & -1 & 0 \end{bmatrix} $$

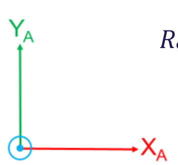
Rotz(180)*Rotx(-90) will give the same result.

In robotic toolbox:

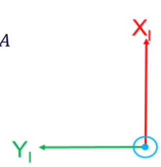
```
syms theta1 theta2

R=rotx(theta1)*rotz(theta2)

theta1 =90*pi/180;
theta2 = 90*pi/180;


R=rotx(theta1)*rotz(theta2)

trplot(R);
```



Post/pre multiplication



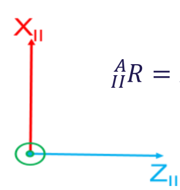
$R_{90,ZA}$



Given a coordinate-system {A}

Rotate 90 degrees about Z_A

$R_{90,XI}$

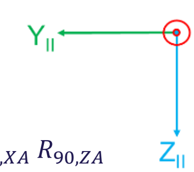


$${}^{A}_{II}R = R_{90,ZA}\, R_{90,XI}$$

Rotate 90 degrees about X_I (the new X)

Rotate 90 degrees about X_A (the original X)

$R_{90,XA}$



$${}^{A}_{II}R = R_{90,XA}\, R_{90,ZA}$$

Some terminology

Extrinsic rotations:

- are elemental rotations that occur about the axes of the fixed coordinate system.

Intrinsic rotations:

- elemental rotations that occur about the axes of the rotating coordinate system, which changes its orientation after each elemental rotation.

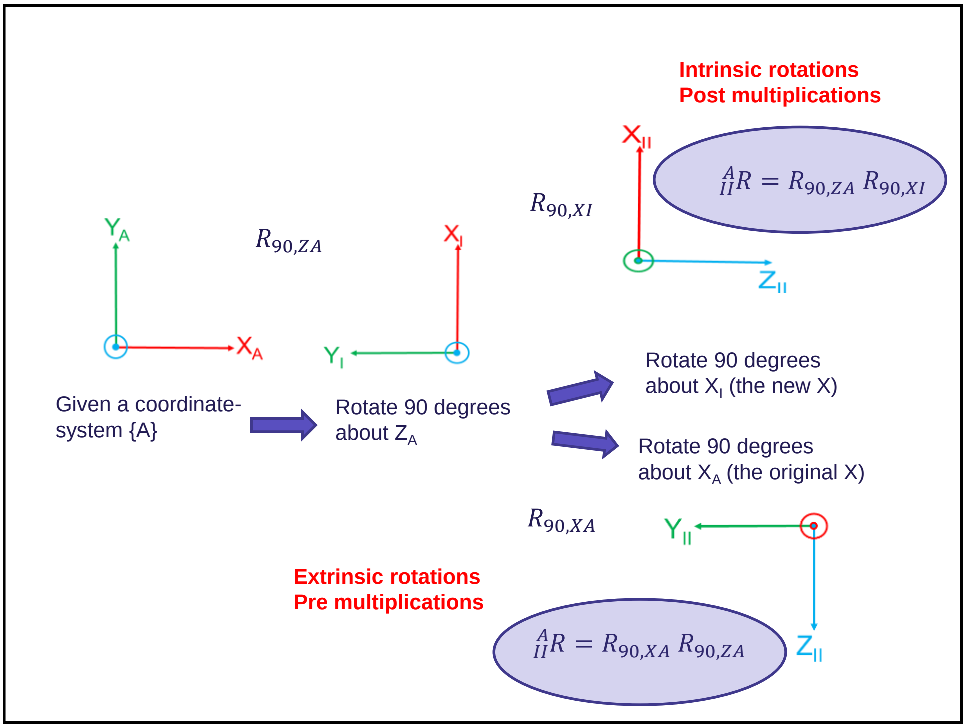


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ABB: Orientation specified by quaternions

UR: Orientation specified by the axis-angle representation

KUKA, FANUC: Orientation specified by three angles

ZYX intrinsic rotation (Euler)

- Start with a frame {B} coincident with a known reference frame {A}:
  - rotate {B} k_B (Z) by an angle α
  - rotate about j_B (Y) by an angle β
  - rotate about i_B (X) by an angle γ

$$
\begin{aligned}
{}^{A}_{B}R_{Z'Y'X'}(\alpha,\beta,\gamma) &= R_Z(\alpha)\cdot R_Y(\beta)\cdot R_X(\gamma) \\
&= \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix} \\
&= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}
\end{aligned}
$$

Inverse ZYX intrinsic rotations

- I know: $${}^{A}_{B}R_{Z'Y'X'} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

- what are the Euler Angles:

$$\beta = A\tan 2(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2})$$

$$\alpha = A\tan 2(\frac{r_{21}}{c\beta}, \frac{r_{11}}{c\beta})$$

$$\gamma = A\tan 2(\frac{r_{32}}{c\beta}, \frac{r_{33}}{c\beta})$$

$$\beta = 90$$
$$\alpha = 0.0$$
$$\gamma = A\tan 2(r_{12}, r_{22})$$

$$\beta = -90$$
$$\alpha = 0.0$$
$$\gamma = -A\tan 2(r_{12}, r_{22})$$

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$$\sqrt{(c\alpha c\beta)^2 + (s\alpha c\beta)^2} = \sqrt{(c\alpha^2 + s\alpha^2)\, c\beta^2} = c\beta$$

- what are the Euler Angles:

$$\beta = A\tan 2(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2})$$
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$$\beta = 90$$
$$\alpha = 0.0$$
$$\gamma = A\tan 2(r_{12}, r_{22})$$

$$\beta = -90$$
$$\alpha = 0.0$$
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Inverse ZYX intrinsic rotations

- I know: $${}^{A}_{B}R_{Z'Y'X'} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} = \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

- what are the Euler Angles:

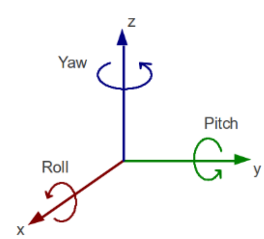
$$\beta = A\tan 2(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2})$$
$$\alpha = A\tan 2(\frac{r_{21}}{c\beta}, \frac{r_{11}}{c\beta})$$
$$\gamma = A\tan 2(\frac{r_{32}}{c\beta}, \frac{r_{33}}{c\beta})$$

$$\beta = 90$$
$$\alpha = 0.0$$
$$\gamma = A\tan 2(r_{12}, r_{22})$$

$$\beta = -90$$
$$\alpha = 0.0$$
$$\gamma = -A\tan 2(r_{12}, r_{22})$$

Extrinsic XYZ rotation (Craig call these: roll, pitch, yaw)

- Start with a frame {B} coincident with a known reference frame {A}:
  - rotate {B} i_A (X) by an angle γ (Yaw)
  - rotate about j_A (Y) by an angle β (pitch)
  - rotate about k_A (Z) by an angle α (Roll)



$${}^{A}_{B}R_{XYZ}(\alpha,\beta,\gamma) = R_Z(\alpha)\cdot R_Y(\beta)\cdot R_X(\gamma)$$
$$= \begin{bmatrix} c\alpha & -s\alpha & 0 \\ s\alpha & c\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} c\beta & 0 & s\beta \\ 0 & 1 & 0 \\ -s\beta & 0 & c\beta \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\gamma & -s\gamma \\ 0 & s\gamma & c\gamma \end{bmatrix}$$
$$= \begin{bmatrix} c\alpha c\beta & c\alpha s\beta s\gamma - s\alpha c\gamma & c\alpha s\beta c\gamma + s\alpha s\gamma \\ s\alpha c\beta & s\alpha s\beta s\gamma + c\alpha c\gamma & s\alpha s\beta c\gamma - c\alpha s\gamma \\ -s\beta & c\beta s\gamma & c\beta c\gamma \end{bmatrix}$$

Inverse Extrinsic (Fixed angel) XYZ rotation (roll, pitch, yaw)

- I know: $^{A}_{B}R_{XYZ} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{22} & r_{23} & r_{33} \end{bmatrix}$

- what are the Euler Angles:

$$\beta = A\tan 2(-r_{31}, \sqrt{r_{11}^2 + r_{21}^2})$$

$$\alpha = A\tan 2(\frac{r_{21}}{c\beta}, \frac{r_{11}}{c\beta})$$

$$\gamma = A\tan 2(\frac{r_{32}}{c\beta}, \frac{r_{33}}{c\beta})$$

$$\beta = 90$$
$$\alpha = 0.0$$
$$\gamma = A\tan 2(r_{12}, r_{22})$$

$$\beta = -90$$
$$\alpha = 0.0$$
$$\gamma = -A\tan 2(r_{12}, r_{22})$$

There are many intrinsic/extrinsic rotations

12 intrinsic rotations (Craig: Euler rotations):
- XYZ
- XZY
- YZX
- XZX
- YXY
- YZY
- ZXZ
- ZYZ

12 extrinsic rotations (Craig: fixed axis rotations):
- XYZ
- XZY
- YZX
- XZX
- YXY
- YZY
- ZXZ
- ZYZ

HOW ABOUT YOUR ROBOT ?

In robotic toolbox:

```
%Euler ZYZ:
R=rotz(0.1)*roty(0.2)*rotz(0.3);

R1= eul2r(0.1,0.2,0.3);
EULER= tr2eul(R1);


%Roll, pitch, yaw (Corke uses
intrinsic rotatons)
R2=rotx(0.1)*roty(0.2)*rotz(0.3);

R3=rpy2r(0.1,0.2,0.3)
RPY = tr2rpy(R3);
```

```
Command Window
R =

    0.9021   -0.3836    0.1977
    0.3875    0.9216    0.0198
   -0.1898    0.0587    0.9801


R1 =

    0.9021   -0.3836    0.1977
    0.3875    0.9216    0.0198
   -0.1898    0.0587    0.9801


EULER =

    0.1000    0.2000    0.3000


R2 =

    0.9363   -0.2896    0.1987
    0.3130    0.9447   -0.0978
   -0.1593    0.1538    0.9752


R3 =

    0.9363   -0.2896    0.1987
    0.3130    0.9447   -0.0978
   -0.1593    0.1538    0.9752


RPY =

    0.1000    0.2000    0.3000

>>
```

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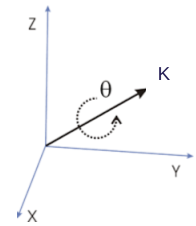
Equivalent angle-axis

Start with a frame {B} coincident with a known reference frame {A}.

Then rotate {B} about the vector K by an angle θ according to the right-hand rule.



$$ {}^{A}_{B}R_{K}(\theta) = \begin{bmatrix} k_x k_x (1-c\theta)+c\theta & k_x k_y (1-c\theta)-k_z s\theta & k_x k_z (1-c\theta)+k_y s\theta \\ k_x k_y (1-c\theta)+k_z s\theta & k_y k_y (1-c\theta)+c\theta & k_y k_z (1-c\theta)-k_x s\theta \\ k_x k_z (1-c\theta)-k_y s\theta & k_y k_z (1-c\theta)+k_x s\theta & k_z k_z (1-c\theta)+c\theta \end{bmatrix} $$

Inverse Angle-axis representation

- I know:

$$ {}^{A}_{B}R_{\overline{K},\theta} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{22} & r_{23} & r_{33} \end{bmatrix} $$

- what are K and theta:

$$ \theta = A\cos\left(\frac{r_{11}+r_{22}+r_{33}-1}{2}\right) $$

$$ \overline{K} = \frac{1}{2\sin\theta} \begin{bmatrix} r_{32}-r_{23} \\ r_{13}-r_{31} \\ r_{21}-r_{12} \end{bmatrix} $$

Quaternions

Start with a frame {B} coincident with a known reference frame {A}. Then rotate {B} about the vector K by an angle θ according to the right-hand rule. The quaternions are:

$$\varepsilon_1 = k_x \sin\frac{\theta}{2}$$
$$\varepsilon_2 = k_y \sin\frac{\theta}{2}$$
$$\varepsilon_3 = k_z \sin\frac{\theta}{2}$$
$$\varepsilon_4 = \cos\frac{\theta}{2}$$

In robotic toolbox:

```
%Euler ZYZ:
R=rotx(0.1)*roty(0.2)*rotz(0.3)

[theta,v] =tr2angvec(R)
R1=angvec2tr(theta,v)

%quaternions
R2=rotx(0.1)*roty(0.2)*rotz(0.3)

q=Quaternion(R2)
R3=q.R
```

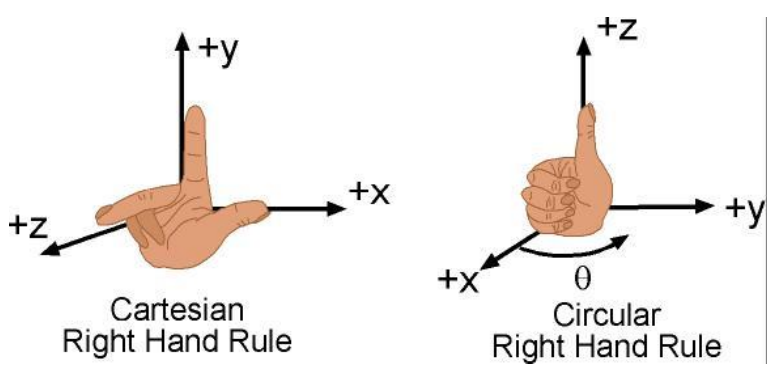
```
R =

    0.9363   -0.2896    0.1987
    0.3130    0.9447   -0.0978
   -0.1593    0.1538    0.9752


theta =

    0.3816


v =

    0.3379    0.4807    0.8092


R1 =

    0.9363   -0.2896    0.1987         0
    0.3130    0.9447   -0.0978         0
   -0.1593    0.1538    0.9752         0
         0         0         0    1.0000


R2 =

    0.9363   -0.2896    0.1987
    0.3130    0.9447   -0.0978
   -0.1593    0.1538    0.9752


q =

0.98186 < 0.064071, 0.091158, 0.15344 >


R3 =

    0.9363   -0.2896    0.1987
    0.3130    0.9447   -0.0978
   -0.1593    0.1538    0.9752
```

Summery of Rotations

We saw that a rotation can be represented by a rotation matrix
Matrix has 9 variables

Rotation matrix can be parameterized in different manners:
- — Roll, pitch and yaw angles
- — Euler Angles
- — Angle-axis representation
- — quaternions

The secret hand signs !



Test exercise

- Start with a frame {B} coincident with a known reference frame {A}. Give the rotation matrix which accomplishes the following rotations of {B}:
  1. rotate about j_A (Y_A) by an angle 30 degrees
  2. rotate about i_A (X_A) by an angle 45 degrees

- A frame {B} is located as follows: initially coincident with a frame {A}. We rotate {B} about Z_B by θ degrees and then we rotate the resulting frame about X_B by ϕ degrees

Hand-in solutions for this exercise on Moodle

Exercise 1

- Try to explain the differences between three angle rotation, angle/axis rotation and quaternions to each others.

- Compute the rotation matrices which correspond to:
  1. Extrinsic (fixed axis rotation) rotation about XYZ (Craigs roll, pitch, yaw):
     Roll(X)=30;Pitch(Y)=50,Yaw(Z)=-20
  2. Intrinsic rotation about ZYX
     X=20;Y=-15,Z=30
  3. Angle/axis representation
     theta = 21.8583 degrees
     kx = 0.3379
     ky = 0.4808
     kz = 0.8093
  4. Quaternion:
     e1=0.064071;
     e2=0.091158;
     e3=0.15344;
     e4=0.98186;

Exercise 2

- Given the rotation matrix R below:

$$R = \begin{bmatrix} 0.9752 & -0.0370 & 0.2184 \\ 0.0978 & 0.9564 & -0.2751 \\ -0.1987 & 0.2896 & 0.9363 \end{bmatrix}$$

- Compute the corresponding:

  1. Extrinsic (fixed axis rotation) XYZ angles (Craigs roll, pitch, yaw)
  2. Angle/axis representation
  3. Quaternions

Exercise 3

2.27 [15] Referring to Fig. 2.25, give the value of ${}^{A}_{B}T$.

2.30 [15] Referring to Fig. 2.25, give the value of ${}^{C}_{A}T$.

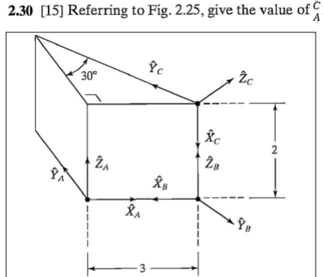


FIGURE 2.25: Frames at the corners of a wedge.

Exercise 4

- Determine which representation your robot is using (for now use RoboDK).
- Setup the equations used by the robot
- Test the equations.

- Determine the coordinate systems that are involved in your project (e.g on a sketch of your robotcell)

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