ROBOT KINEMATICS, MODELLING AND SIMULATION

ROBOTICS 2023

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Lecture plan

No	Responsible	Contents
1	Ole Madsen	Introduction to: • the course; • robotics and robot terminology.
2	Ole Madsen	Spatial descriptions and transformation matrices
3	Ole Madsen	Orientation
4 (FIB)	Ole Madsen	Practical exercise with the on-line programming (1.5 timer/gruppe).
5	Ole Madsen	Forward Kinematics I
6	Ole Madsen	Forward Kinematics II (go though 6 DOF robot) – exercise, you go though your robot
7	Ole Madsen	Inverse kinematics I
8	Ole Madsen	Inverse kinematics II (go through 6DOF robot) – you start on your robot
9	Ole Madsen	Trajectory generation and control (joint)
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11	Ole Madsen	Jacobian/Exam preparation

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JACOBIANS INTRO

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Kinematic relations

$$\theta = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix} \qquad \xrightarrow{X=FK(\theta)} \qquad \xleftarrow{\theta = IK(X)} \qquad X = \begin{bmatrix} x \\ y \\ z \\ R_x \\ R_y \\ R_z \end{bmatrix}$$

Joint Space

Cartesian Space

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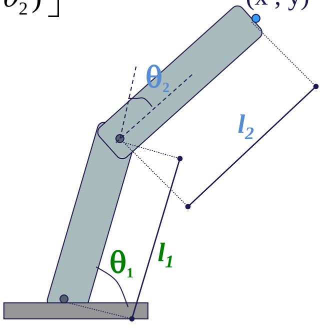
Example – 2 DOF planar robot

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{bmatrix}$$



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Velocity relations

$$\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} \qquad \dot{X} = J(\theta)\dot{\theta} \qquad \dot{\theta} = J^{-1}(\theta)\dot{X} \qquad \dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Joint Space

Cartesian Space

JACOBIAN matrix J(θ)

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Jacobian

From forward kinematics:

$$X = h(\theta)$$

$$X = \begin{bmatrix} x \\ y \\ z \\ \phi \\ \theta \\ \varphi \end{bmatrix} = h\left(\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \\ \theta_5 \\ \theta_6 \end{bmatrix}\right) = \begin{bmatrix} h_1(\theta_1, \theta_2, \cdots, \theta_6) \\ h_2(\theta_1, \theta_2, \cdots, \theta_6) \\ h_3(\theta_1, \theta_2, \cdots, \theta_6) \\ h_4(\theta_1, \theta_2, \cdots, \theta_6) \\ h_5(\theta_1, \theta_2, \cdots, \theta_6) \\ h_6(\theta_1, \theta_2, \cdots, \theta_6) \end{bmatrix}$$

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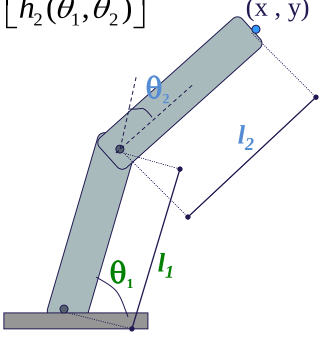
Example – 2 DOF planar robot

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1 + \theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1 + \theta_2) \end{bmatrix} = \begin{bmatrix} h_1(\theta_1, \theta_2) \\ h_2(\theta_1, \theta_2) \end{bmatrix}$$



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Jacobian Matrix

Forward kinematics

$$X_{6\times1} = h(\theta_{n\times1})$$

$$\dot{X} = \frac{d}{dt}h(\theta_{nx1}) = \frac{dh(\theta)}{d\theta}\frac{d\theta}{dt} = \frac{dh(\theta)}{d\theta}\dot{\theta}$$

$$\begin{bmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\\ \omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \boxed{\left[\frac{dh(\theta)}{d\theta}\right]_{6xn}}\begin{bmatrix}\dot{\theta}_1\\ \dot{\theta}_2\\ \dot{\theta}_3\\ \dot{\theta}_4\\ \dot{\theta}_5\\ \dot{\theta}_6\end{bmatrix}$$

$$\boxed{J = \frac{dh(\theta)}{d\theta}}$$

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Jacobian Matrix

Jacobian is a function of θ, it is not a constant!

$$J=\left(\frac{dh(\theta)}{d\theta}\right)_{6xn} = \begin{bmatrix}\frac{\partial h_1}{\partial\theta_1} & \frac{\partial h_1}{\partial\theta_2} & \cdots & \frac{\partial h_1}{\partial\theta_n}\\ \frac{\partial h_2}{\partial\theta_1} & \frac{\partial h_2}{\partial\theta_2} & \cdots & \frac{\partial h_2}{\partial\theta_2}\\ \vdots & \vdots & \ddots & \vdots\\ \frac{\partial h_n}{\partial\theta_1} & \frac{\partial h_n}{\partial\theta_2} & \cdots & \frac{\partial h_n}{\partial\theta_n}\end{bmatrix}$$

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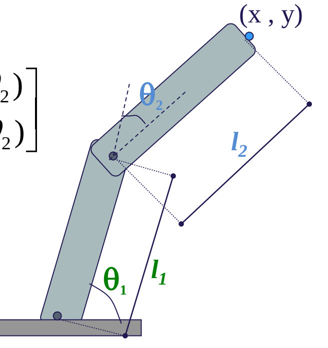
Example – 2 DOF planar robot

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) \\ l_1 \sin\theta_1 + l_2 \sin(\theta_1+\theta_2) \end{bmatrix} = \begin{bmatrix} h_1(\theta_1,\theta_2) \\ h_2(\theta_1,\theta_2) \end{bmatrix}$$



Given l_1, l_2, ***Find: Jacobian***

$$J = \begin{bmatrix} \frac{\partial h_1}{\partial \theta_1} & \frac{\partial h_1}{\partial \theta_2} \\ \frac{\partial h_2}{\partial \theta_1} & \frac{\partial h_2}{\partial \theta_2} \end{bmatrix} = \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) & -l_2 \sin(\theta_1+\theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) \end{bmatrix}$$

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Velocity relations

$$\dot{\theta} = \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \\ \dot{\theta}_4 \\ \dot{\theta}_5 \\ \dot{\theta}_6 \end{bmatrix} \qquad \xrightarrow{\dot{X} = J(\theta)\dot{\theta}} \qquad \xleftarrow[\dot{\theta} = J^{-1}(\theta)\dot{X}]{} \qquad \dot{X} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

Joint Space

Cartesian Space

What if det(J) = 0 ??

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Singularities

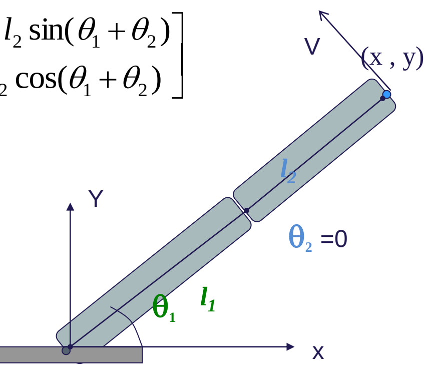
Singular points are such values of θ that cause the determinant of the Jacobian to be zero

$$\det[J(\theta)] = 0$$

Example – 2 DOF planar robot

Find the singularity configuration of the 2-DOF planar robot arm

$$J = \begin{bmatrix} -l_1 \sin\theta_1 - l_2 \sin(\theta_1+\theta_2) & -l_2 \sin(\theta_1+\theta_2) \\ l_1 \cos\theta_1 + l_2 \cos(\theta_1+\theta_2) & l_2 \cos(\theta_1+\theta_2) \end{bmatrix}$$

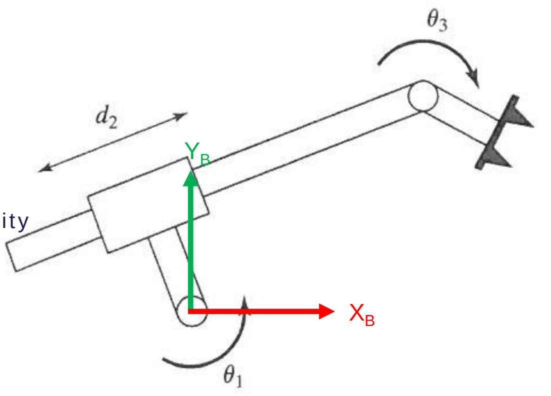


Det(J)=0

$$\theta_2 = 0$$

Exercise

For the robot to the left
- Determine the Jacobian:
  1. Determine the DH-parameters
  2. Setup an equation describing the forward kinematics
  3. Determine the positioning capability
  4. Differentiate



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