ROBOT KINEMATICS, MODELLING AND SIMULATION

ROBOTICS 2022

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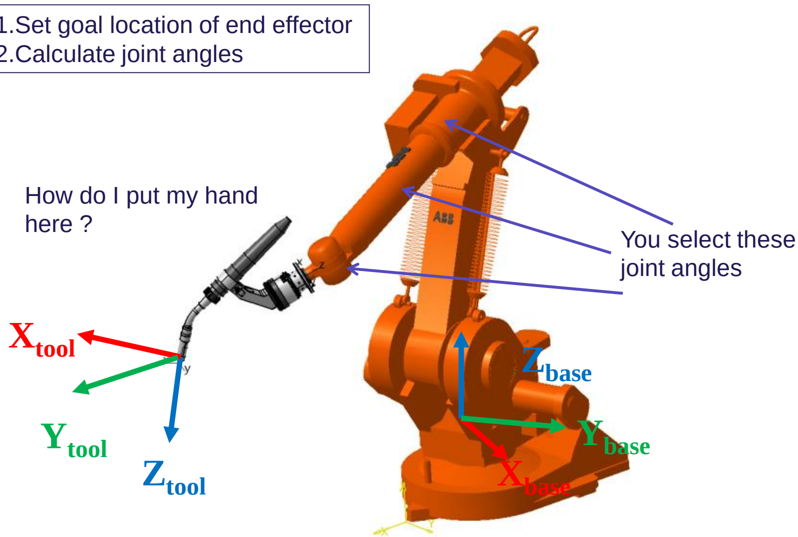
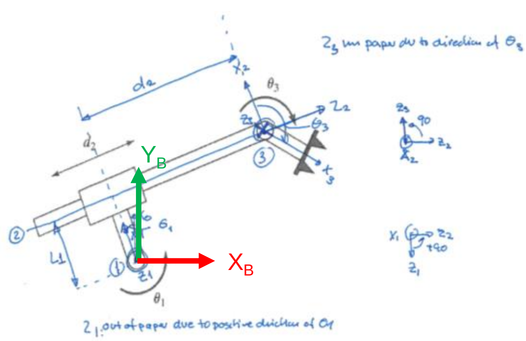
Lecture plan

No	Responsible	Contents
1	Ole Madsen	Introduction to: • the course; • robotics and robot terminology.
2	Ole Madsen	Spatial descriptions and transformation matrices
3 (FIB)	Ole Madsen + More	Practical exercise with the on-line programming (1.5 timer/gruppe).
4	Ole Madsen	Orientation
5	Ole Madsen	Forward Kinematics I
6	Ole Madsen	Forward Kinematics II (go though 6 DOF robot) – exercise, you go though your robot
7	Ole Madsen	Inverse kinematics I
8	Ole Madsen	Inverse kinematics II (go through 6DOF robot) – you start on your robot
9	Ole Madsen	Trajectory generation and control (joint)
10	Ole Madsen	Trajectory generation and control (cartesian)
11	Ole Madsen	Jacobian/Exam preparation

Inverse kinematics

1.Set goal location of end effector
2.Calculate joint angles





I	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	90	L_1	d_2	0
3	90	0	0	θ_3

$$T0\_3 := \begin{bmatrix} \cos(\theta 1-\theta 3) & \sin(\theta 1-\theta 3) & 0 & \cos(\theta 1)L1+\sin(\theta 1)d2 \\ \sin(\theta 1-\theta 3) & -\cos(\theta 1-\theta 3) & 0 & \sin(\theta 1)L1-\cos(\theta 1)d2 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Some useful equations

3. Equation:

$$C_1 \cdot cos(\theta_i) + C_2 \cdot sin(\theta_i) + C_3 = 0$$

Solution:

$$\theta_i = 2 \cdot tan^{-1}\left(\frac{C_2 \pm \sqrt{C_2^2 + C_1^2 - C_3^2}}{C_1 - C_3}\right)$$

4. Equations:

$$C_1 \cdot cos(\theta_i) + C_2 \cdot sin(\theta_i) + C_3 = 0$$
$$C_1 \cdot sin(\theta_i) - C_2 \cdot cos(\theta_i) + C_4 = 0$$

Solution:

$$\theta_i = Atan2(-C_1 C_4 - C_2 C_3, C_2 C_4 - C_1 C_3)$$

INVERSE KINEMATICS II

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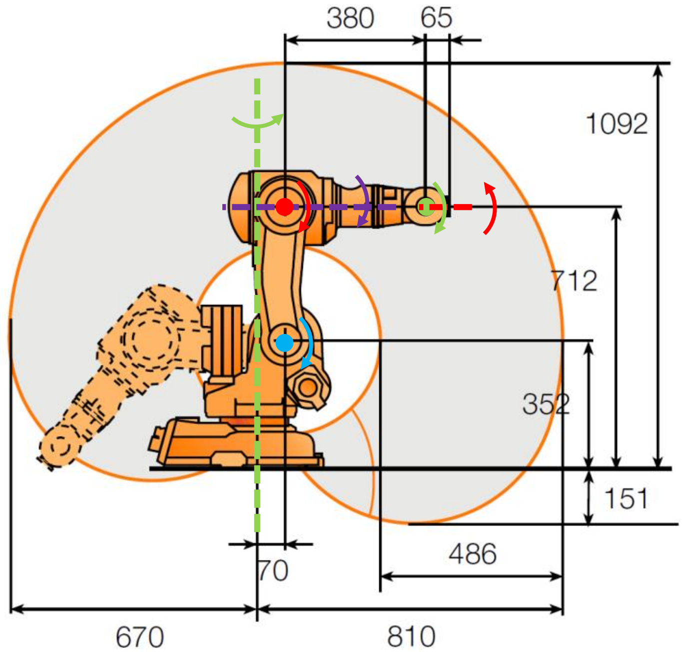


Axis (i)	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ1
2	-90	$a_1 = 70$	0	θ2-90
3	0	$a_2 = 360$	0	θ3
4	-90	0	$d_4 = 380$	θ4
5	90	0	0	θ5
6	-90	0	0	θ6

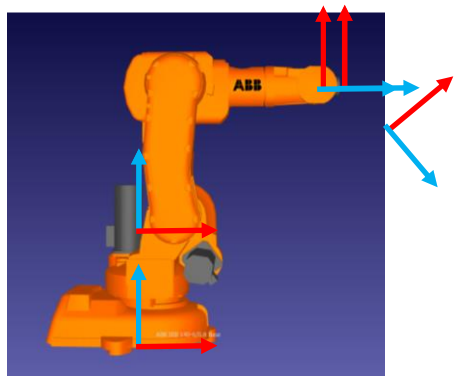
$$TB\_0 := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 352 \\ 0 & 0 & 0 & 1 \end{bmatrix}:$$

$$T6\_W := \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 65 \\ 0 & 0 & 0 & 1 \end{bmatrix}:$$

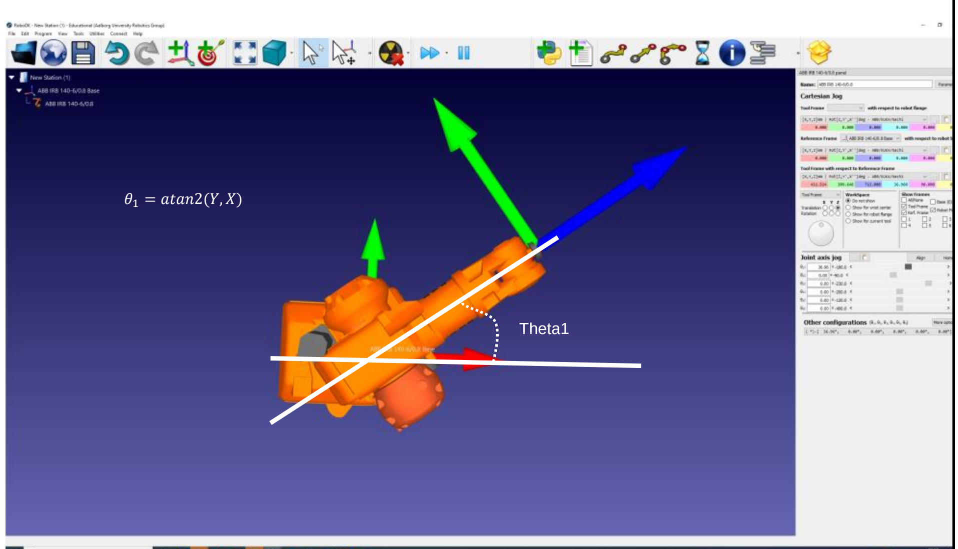
Finding $T^0_{6,target}$

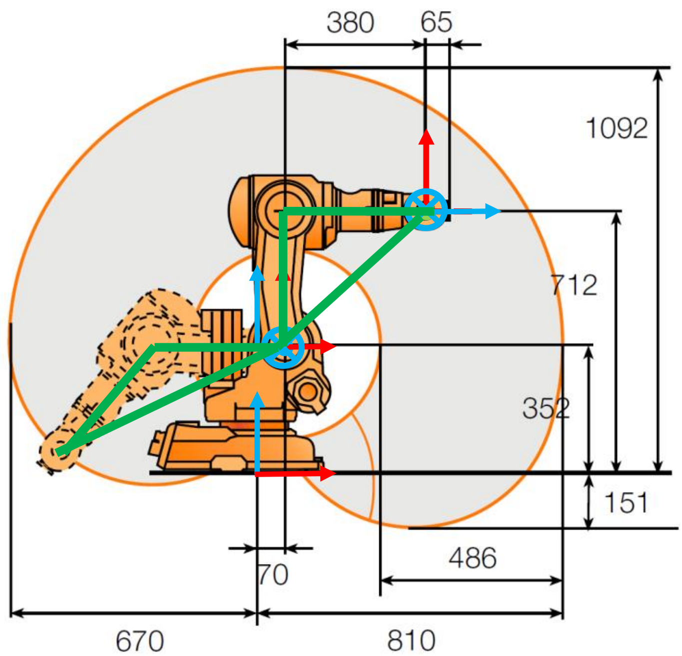
$$T^B_{T,target} = T^B_0 T^0_{6,target} T^6_W T^W_T$$

$$T^0_{6,target} = (T^B_0)^{-1} T^B_{T,target} (T^W_T)^{-1} (T^6_W)^{-1}$$



Finding Theta1

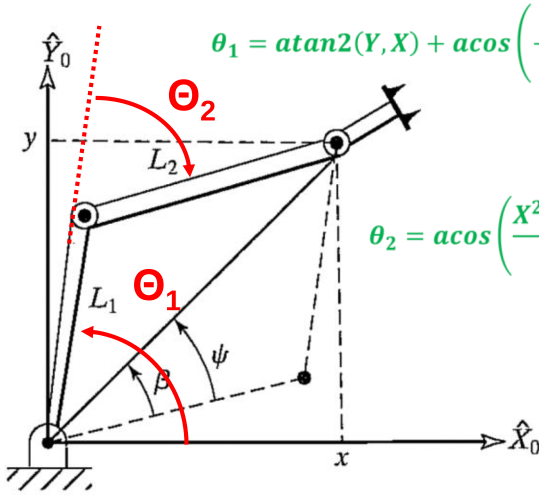




Example – Geometric solution

$$\theta_1 = atan2(Y,X) + acos\left(-\frac{L_2^2 - X^2 - Y^2 - L_1^2}{2\sqrt{X^2+Y^2}L_2}\right)$$

$$\theta_2 = acos\left(\frac{X^2+Y^2-L_1^2-L_2^2}{2L_1L_2}\right)$$

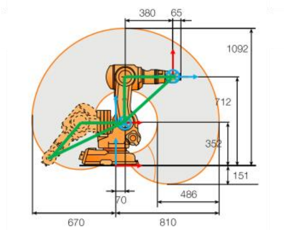


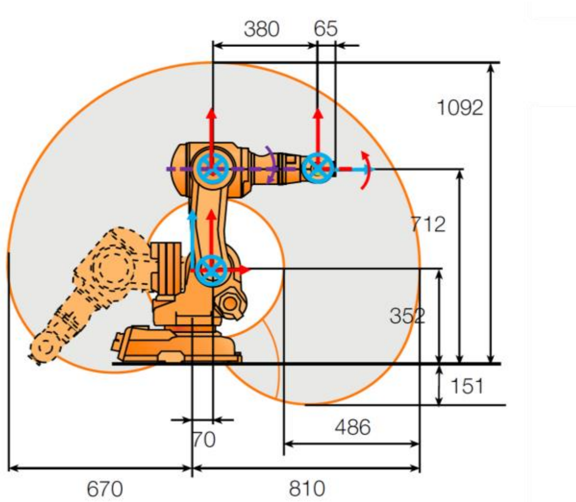
Finding theta2 og 3

```
T01=TDH(0,0,0,theta1);
T16target = inv(T01)*T06target

L3=sqrt(T16target(3,4)^2+(T16target(1,4)-70)^2);
L2=380;
L1=360;

theta2 =pi/2- (acos(-(L2^2-L1^2-L3^2)/(2*L1*L3))+atan2(T16target(3,4),T16target(1,4)-70));
theta3 = acos((L3^2-L1^2-L2^2)/(2*L1*L2))-pi/2;
```





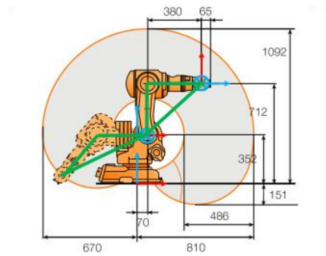
$$
{}^{3}_{6}T = \begin{bmatrix} c(\theta_4)c(\theta_5)c(\theta_6) - s(\theta_4)s(\theta_6) & -c(\theta_4)c(\theta_5)s(\theta_6) - s(\theta_4)s(\theta_6) & -c(\theta_4)s(\theta_5) & 0 \\ s(\theta_5)c(\theta_6) & -s(\theta_5)s(\theta_6) & c(\theta_5) & 380 \\ -s(\theta_4)c(\theta_5)c(\theta_6) - c(\theta_4)s(\theta_6) & s(\theta_4)c(\theta_5)s(\theta_6) - c(\theta_4)c(\theta_6) & s(\theta_5)s(\theta_4) & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Finding theta4-6

```
T12=TDH(-90*pi/180,70,0,theta2-90*pi/180);
T23=TDH(0,360,0,theta3);

T03=T01*T12*T23;

T36target = inv(T03)*T06target;
```




```
theta5 = acos(T36target(2,3));
theta4 = atan2(T36target(3,3)*sin(theta5),-T36target(1,3)*sin(theta5));
theta6 = atan2(-T36target(2,2)*sin(theta5),T36target(2,1)*sin(theta5));
```

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