ROBOT KINEMATICS, MODELING AND SIMULATION

ROBOTICS 2023

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Lecture plan

No	Responsible	Contents
1	Ole Madsen	Introduction to: • the course; • robotics and robot terminology.
2	Ole Madsen	Spatial descriptions and transformation matrices
3 (FIB)	Ole Madsen + More	Practical exercise with the on-line programming (1.5 timer/gruppe).
4	Ole Madsen	Orientation
5	Ole Madsen	Forward Kinematics I
6	Ole Madsen	Forward Kinematics II (go though 6 DOF robot) – exercise, you go though your robot
7	Ole Madsen	Inverse kinematics I
8	Ole Madsen	Inverse kinematics II (go through 6DOF robot) – you start on your robot
9	Ole Madsen	Trajectory generation and control (joint)
10	Ole Madsen	Trajectory generation and control (cartesian)
11	Ole Madsen	Jacobian/Exam preparation

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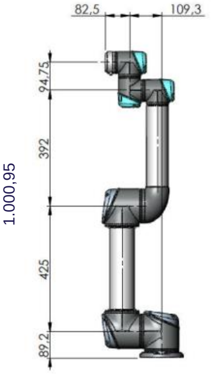
3 hints for making Forward kinematics for your robot (and one cheat)

Hints

- Use robot tool box as a first examination of the the DH parameters you have determined.
- Be fully aware about the zero location of your robot
- The dimensions of the robot in RoboDK (your physical robot) might be a bit different from your drawing.

Cheat:

- You can find the solution in RoboDK ☺



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INVERSE KINEMATICS

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Inverse kinematics

1.Set goal location of end effector
2.Calculate joint angles

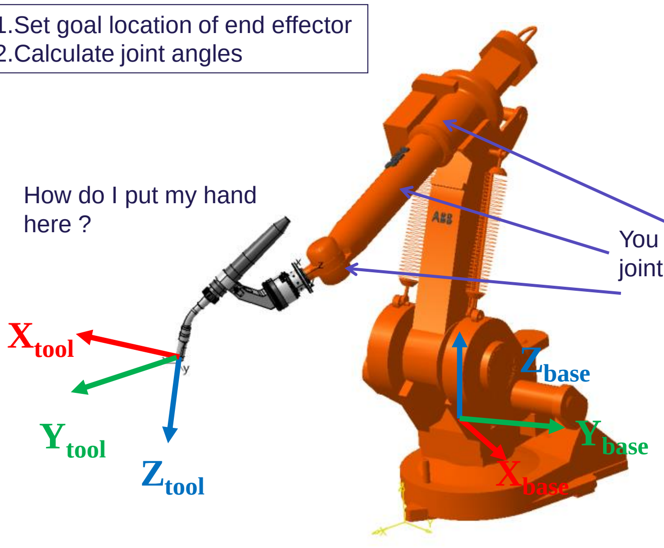

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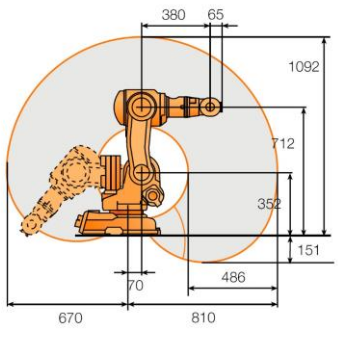
ABB IRB 140

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	70	0	θ_2
3	0	360	0	θ_3
4	-90	0	380	θ_4
5	90	0	0	θ_5
6	-90	0	0	θ_6



Forward kinematics (first three axes only). (EASY):

$$
{}^{0}_{3}T = \begin{bmatrix} c\theta_1 c\theta_2 c\theta_3 - c\theta_1 s\theta_2 s\theta_3 & -c\theta_1 c\theta_2 s\theta_3 - c\theta_1 s\theta_2 c\theta_3 & -s\theta_1 & 270c\theta_1 c\theta_2 + 75c\theta_1 \\ s\theta_1 c\theta_2 c\theta_3 - s\theta_1 s\theta_2 s\theta_3 & -s\theta_1 c\theta_2 s\theta_3 - s\theta_1 s\theta_2 c\theta_3 & c\theta_1 & 270s\theta_1 c\theta_2 + 75s\theta_1 \\ -s\theta_2 c\theta_3 - c\theta_2 s\theta_3 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 & 0 & -270s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

UNKNOWN

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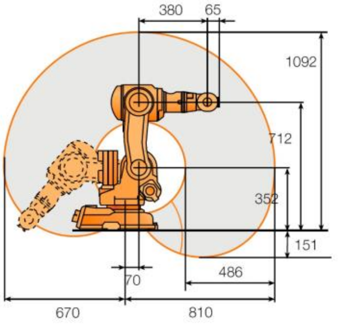
ABB IRB 140



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90	70	0	θ_2
3	0	360	0	θ_3
4	-90	0	380	θ_4
5	90	0	0	θ_5
6	-90	0	0	θ_6

Inverse kinematics (first three axes only). (DIFFICULT):

$$
{}^0_3T = \begin{bmatrix} c\theta_1 c\theta_2 c\theta_3 - c\theta_1 s\theta_2 s\theta_3 & -c\theta_1 c\theta_2 s\theta_3 - c\theta_1 s\theta_2 c\theta_3 & -s\theta_1 & 270 c\theta_1 c\theta_2 + 75 c\theta_1 \\ s\theta_1 c\theta_2 c\theta_3 - s\theta_1 s\theta_2 s\theta_3 & -s\theta_1 c\theta_2 s\theta_3 - s\theta_1 s\theta_2 c\theta_3 & c\theta_1 & 270 s\theta_1 c\theta_2 + 75 s\theta_1 \\ -s\theta_2 c\theta_3 - c\theta_2 s\theta_3 & s\theta_2 s\theta_3 - c\theta_2 c\theta_3 & 0 & -270 s\theta_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

UNKNOWNS

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Agenda

1. Solvability
2. Degree of freedom
3. Multiple solutions
4. Methods of solution:
    - Algebraic solution
    - Geometric solution

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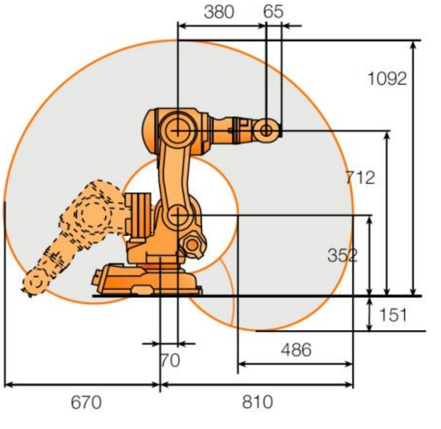
Solvability – can we reach the location ?

Reachable workspace – volume the end effector can reach

Dextrous workspace – volume end effector can reach in any orientation



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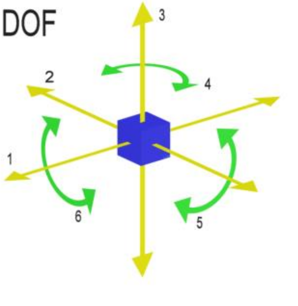
Degrees of Freedom (DOF)

From: DS/ISO 8373:Manipulating industrial robots - vocabulary

4.4
degree of freedom
DOF
one of the variables (maximum number of six) required to define the motion of a body in space

NOTE Because of possible confusion with **axes** (4.3), it is advisable not to use the term "degree of freedom" to describe the motion of the robot.



Actuation DOFs:
- The joint axes

End-effector DOF:
- The location of the end-effector in space (Max 6)

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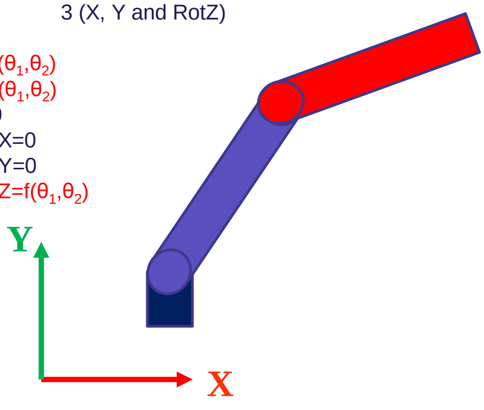
Example 1:

Number of actuation DOF:
2 (θ_1,θ_2)
Number of end-effector DOF:
3 (X, Y and RotZ)

X=f(θ_1,θ_2)
Y=f(θ_1,θ_2)
Z=0
RotX=0
RotY=0
RotZ=f(θ_1,θ_2)



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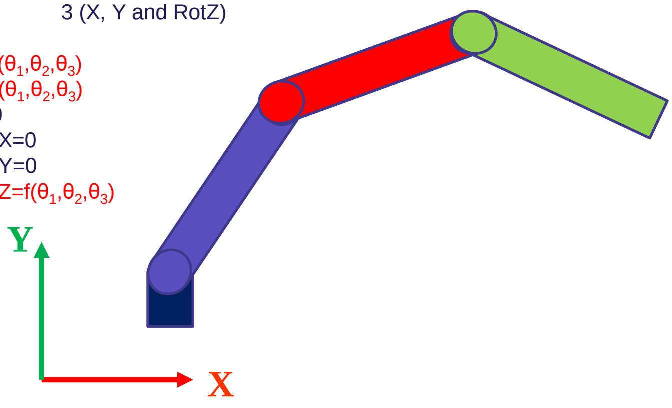
Example 2

Number of actuation DOF:
3 (θ_1,θ_2,θ_3)
Number of end-effector DOF:
3 (X, Y and RotZ)

X=f(θ_1,θ_2,θ_3)
Y=f(θ_1,θ_2,θ_3)
Z=0
RotX=0
RotY=0
RotZ=f(θ_1,θ_2,θ_3)



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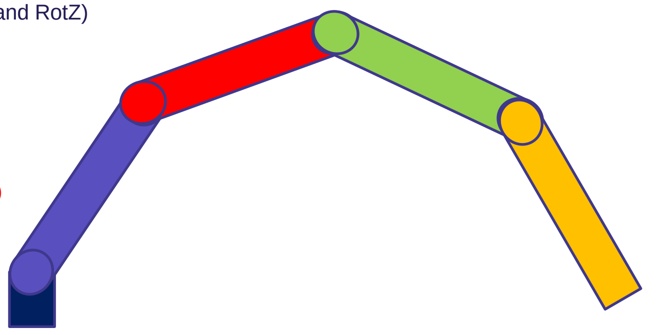
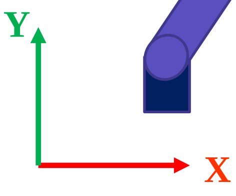
Example 3

Number of actuation DOF:
4 (θ_1,θ_2,θ_3,θ_4)
Number of end-effector DOF:
3 (X, Y and RotZ)

$X=f(\theta_1,\theta_2,\theta_3,\theta_4)$
$Y=f(\theta_1,\theta_2,\theta_3,\theta_4)$
Z=0
RotX=0
RotY=0
$RotZ=f(\theta_1,\theta_2,\theta_3,\theta_4)$





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Actuator and end-effector DOF of these robots?

Manipulate objects in
X, Y, Z and RotZ

4 joint axis

Manipulate objects in
X, Y, Z, RotX, RotY, RotZ

6 joint axis

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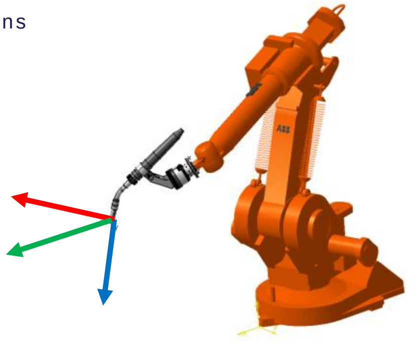
Solvability – for 6 End-effector DOF - is there a solution and how many ?

- Six joints (Actuator DOF):
  - A final number of solutions (if pose inside dextrous workspace)
- Fewer than six joints:
  - limited in the poses an end-effector poses can attain (hence there may not be any solution)
  - Under-actuated/over constrained
- More than six joints:
  - Often infinite number of solutions
  - Redundant robot/under constrained



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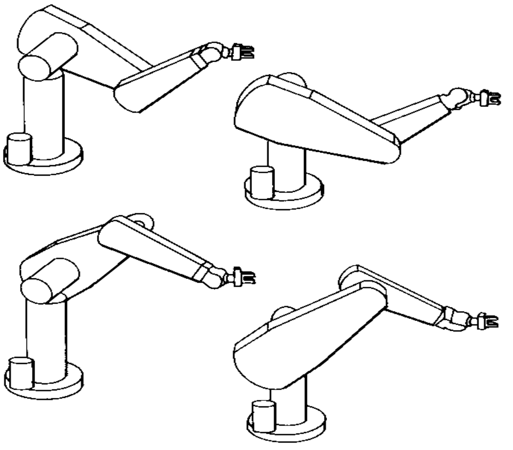
Multiple solutions



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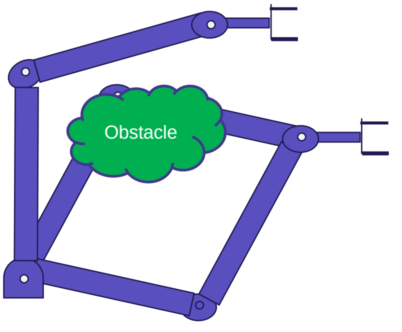
Multiple solutions



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Multiple solutions

Number of solution dependents on kinematic structure

For a robot with six axes:

a_i	Number of solutions
$a_1 = a_3 = a_5 = 0$	≤ 4
$a_3 = a_5 = 0$	≤ 8
All $a_i \neq 0$	≤ 16

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Agenda

1. Solvability
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    - Geometric solution

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Solving inverse kinematics

Inverse kinematics solution

- Closed form solution
    - Algebraic solution
        - Solving kinematic transformation equations
    - Geometric solution
        - Decompose spatial arm geometry to plane cases to solve
- Numerical solution

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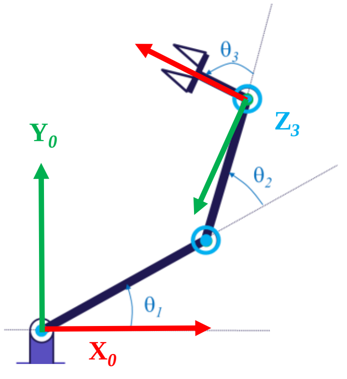
Example – Algebraic solution – End-effector DOF



Manipulate objects in
X, Y and RotZ (3 DOF)

$$ {}^0_3T(X,Y,Rot_Z) = \begin{bmatrix} \cos(Rot_Z) & -\sin(Rot_Z) & 0 & X \\ \sin(Rot_Z) & \cos(Rot_Z) & 0 & Y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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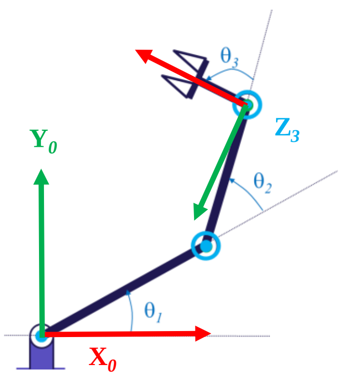
Example – Algebraic solution – setup DH-parameters



i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	0	L1	0	θ_2
3	0	L2	0	θ_3

$$ {}^0_3T(\theta_1,\theta_2,\theta_3) = \begin{bmatrix} \cos(\theta_1+\theta_2+\theta_3) & -\sin(\theta_1+\theta_2+\theta_3) & 0 & l_1\cos(\theta_1)+l_2\cos(\theta_1+\theta_2) \\ \sin(\theta_1+\theta_2+\theta_3) & \cos(\theta_1+\theta_2+\theta_3) & 0 & l_1\sin(\theta_1)+l_2\sin(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

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Example – Algebraic solution – Solve equations

Desired location   Forward kinematics

$$ {}^0_3T(X,Y,Rot_Z) = {}^0_3T(\theta_1,\theta_2,\theta_3) $$

$$\begin{bmatrix} c(Rot_Z) & -s(Rot_Z) & 0 & X \\ s(Rot_Z) & c(Rot_Z) & 0 & Y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} c(\theta_1+\theta_2+\theta_3) & -s(\theta_1+\theta_2+\theta_3) & 0 & l_1c(\theta_1)+l_2c(\theta_1+\theta_2) \\ s(\theta_1+\theta_2+\theta_3) & c(\theta_1+\theta_2+\theta_3) & 0 & l_1s(\theta_1)+l_2s(\theta_1+\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X = l_1c(\theta_1)+l_2c(\theta_1+\theta_2)$$
$$Y = l_1s(\theta_1)+l_2s(\theta_1+\theta_2)$$
$$Rot_Z = \theta_1+\theta_2+\theta_3$$

Some useful equations

1. Cosinus/sinus rules:

$$\cos^2(x)+\sin^2(x)=1$$

Sum:

$$\sin(x+y)=\sin(x)\cos(y)+\cos(x)\sin(y)$$
$$\cos(x+y)=\cos(x)\cos(y)-\sin(x)\sin(y)$$

Subtraction

$$\sin(x-y)=\sin(x)\cos(y)-\cos(x)\sin(y)$$
$$\cos(x-y)=\cos(x)\cos(y)+\sin(x)\sin(y)$$

2. Atan2:

$$\operatorname{atan2}(y,x)=\begin{cases} \arctan\left(\frac{y}{x}\right) & x>0 \\ \arctan\left(\frac{y}{x}\right)+\pi & y\ge 0, x<0 \\ \arctan\left(\frac{y}{x}\right)-\pi & y<0, x<0 \\ +\frac{\pi}{2} & y>0, x=0 \\ -\frac{\pi}{2} & y<0, x=0 \\ \text{undefined} & y=0, x=0 \end{cases}$$

Some useful equations

3. Equation:

$$C_1 \cdot cos(\theta_i) + C_2 \cdot sin(\theta_i) + C_3 = 0$$

Solution:

$$\theta_i = 2 \cdot tan^{-1}\left(\frac{C_2 \pm \sqrt{C_2^2 + C_1^2 - C_3^2}}{C_1 - C_3}\right)$$

4. Equations:

$$C_1 \cdot cos(\theta_i) + C_2 \cdot sin(\theta_i) + C_3 = 0$$
$$C_1 \cdot sin(\theta_i) - C_2 \cdot cos(\theta_i) + C_4 = 0$$

Solution:

$$\theta_i = Atan2(-C_1 C_4 - C_2 C_3, C_2 C_4 - C_1 C_3)$$

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Example – Algebraic solution – Finding theta2

$$X = l_1 c(\theta_1) + l_2 c(\theta_1 + \theta_2)$$
$$Y = l_1 s(\theta_1) + l_2 s(\theta_1 + \theta_2)$$
$$Rot_z = \theta_1 + \theta_2 + \theta_3$$

$$X^2 + Y^2 = (l_1 c(\theta_1) + l_2\, c(\theta_1 + \theta_2))^2 + (l_1 s(\theta_1) + l_2\, s(\theta_1 + \theta_2))^2$$

$$= \left(l_1 c(\theta_1)\right)^2 + (l_2\, c(\theta_1 + \theta_2))^2 + 2 l_1 l_2\, c(\theta_1 + \theta_2) c(\theta_1) + \left(l_1 s(\theta_1)\right)^2 + (l_2\, s(\theta_1 + \theta_2))^2 + 2\, l_1 l_2\, s(\theta_1 + \theta_2) s(\theta_1)$$

$$= l_1^2 (c(\theta_1)^2 + s(\theta_1)^2) + l_2^2 (c(\theta_1 + \theta_2)^2 + s(\theta_1 + \theta_2)^2) + 2 l_1 l_2 (c(\theta_1 + \theta_2) c(\theta_1) + s(\theta_1 + \theta_2) s(\theta_1))$$

$$= l_1^2 + l_2^2 + 2 l_1 l_2 c(\theta_2)$$

$$\theta_2 = \arccos\left(\frac{X^2 + Y^2 - l_1^2 - l_2^2}{2 l_1 l_2}\right)$$

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Example – Algebraic solution – Finding theta1

$X = l_1 c(\theta_1) + l_2 c(\theta_1 + \theta_2)$

$Y = l_1 s(\theta_1) + l_2 s(\theta_1 + \theta_2)$

$Rot_z = \theta_1 + \theta_2 + \theta_3$

$\theta_2 = \arccos(\frac{X^2 + Y^2 - l_1^2 - l_2^2}{2 l_1 l_2})$

$X = l_1 c(\theta_1) + l_2 c(\theta_1 + \theta_2) = l_1 c(\theta_1) + l_2\, c(\theta_1) c(\theta_2) - l_2\, s(\theta_1) s(\theta_2)$
$= (l_1 + l_2 c(\theta_2)) c(\theta_1) - l_2\, s(\theta_2) s(\theta_1) = C_1 c(\theta_1) - C_2 s(\theta_1)$

$Y = l_1 s(\theta_1) + l_2 s(\theta_1 + \theta_2) = l_1 s(\theta_1) + l_2\, c(\theta_1) s(\theta_2) + l_2\, s(\theta_1) c(\theta_2)$
$= (l_1 + l_2 c(\theta_2)) s(\theta_1) + l_2 s(\theta_2) c(\theta_1) = C_1 s(\theta_1) + C_2 c(\theta_1)$

$0 = C_1 s(\theta_1) + C_2 c(\theta_1) + C_3$

$0 = C_1 c(\theta_1) - C_2 s(\theta_1) + C_4$

Where

$C_1 = (l_1 + l_2 c(\theta_2)); C_2 = l_2 s(\theta_2); C_3 = -Y; C_4 = -X$

$\theta_1 = Atan2(-C_1 C_4 - C_2\, C_3, C_2 C_4 - C_1\, C_3)$

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Example – Algebraic solution – Finding theta3

$X = l_1 c(\theta_1) + l_2 c(\theta_1 + \theta_2)$

$Y = l_1 s(\theta_1) + l_2 s(\theta_1 + \theta_2)$

$Rot_z = \theta_1 + \theta_2 + \theta_3$

$\theta_2 = \arccos(\frac{X^2 + Y^2 - l_1^2 - l_2^2}{l_1 l_2})$

$\theta_1 = Atan2(-C_1 C_4 - C_2\, C_3, C_2 C_4 - C_1\, C_3)$

Where

$C_1 = (l_1 + l_2 c(\theta_2)); C_2 = l_2 s(\theta_2); C_3 = -Y; C_4 = -X$

$\theta_3 = Rot_z - \theta_1 - \theta_2$

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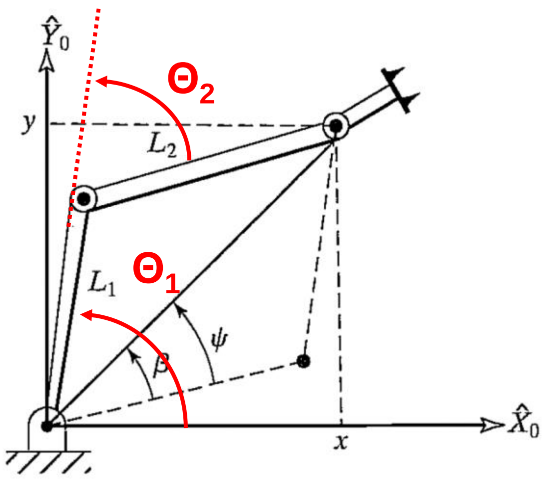
Example – Geometric solution



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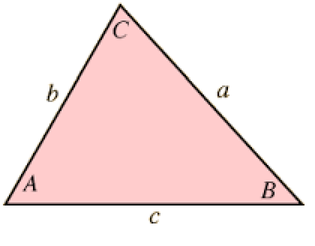
Some useful equations

5. Low of cosines:

$$c^2 = a^2 + b^2 - 2ab\cos C$$



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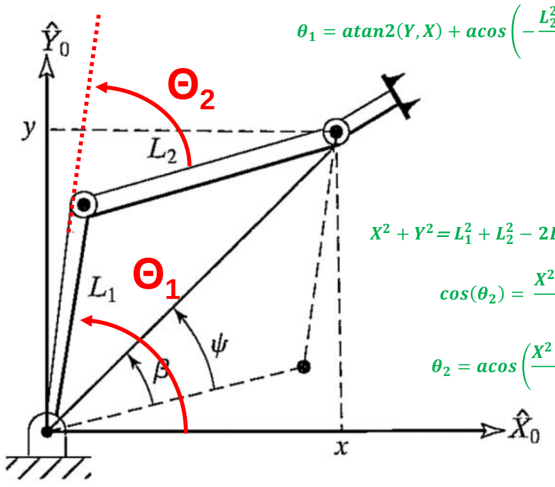
Example – Geometric solution



$$\theta_1 = atan2(Y,X) + acos\left(-\frac{L_2^2 - X^2 - Y^2 - L_1^2}{2\sqrt{X^2+Y^2}L_2}\right)$$

$$X^2 + Y^2 = L_1^2 + L_2^2 - 2L_1L_2cos(180 - \theta_2)$$

$$cos(\theta_2) = \frac{X^2 + Y^2 - L_1^2 - L_2^2}{2L_1L_2}$$

$$\theta_2 = acos\left(\frac{X^2 + Y^2 - L_1^2 - L_2^2}{2L_1L_2}\right)$$

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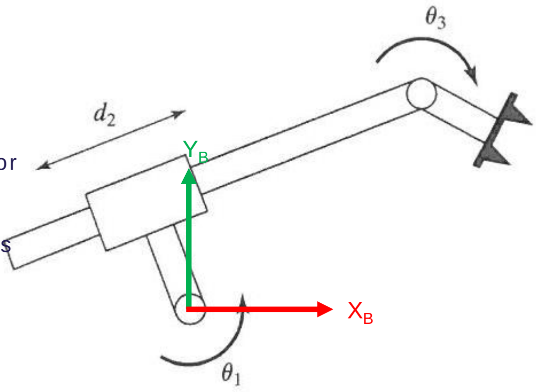
Exercise I

For the robot to the left

- Determine the DH-parameters
- Setup an equation describing the forward kinematics
- Determine the end-effector DOF.
- Establish equations for making inverse kinematics
- Test your algorithms in robotic tool box



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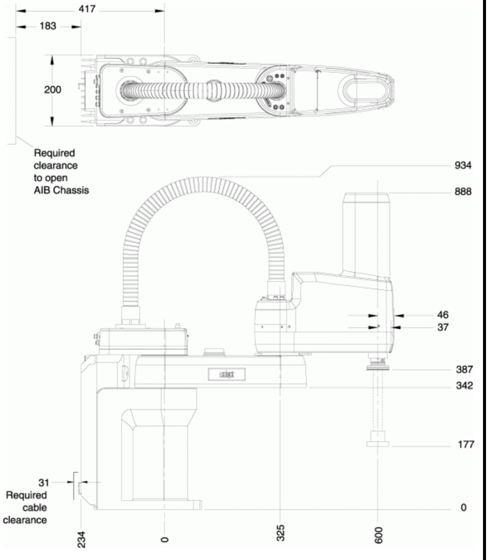
Exercise II

For the SCARA robot (ADEPT COBRA s600):

- Determine the DH-parameters
- Setup an equation describing the forward kinematics
- Determine the end-effector DOF
- Establish equations for making inverse kinematics
- Test your algorithms in robotic tool box



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