Affixing frames to joints

1. Identify the joint axes and draw lines along them.

For step 2 through 5 below consider two of these neighbouring lines (at axes i-1 and i):

2. Identify the common perpendicular between them, or point of intersection. At the point of intersection, or at the point where the common perpendicular meets the i-1th axis, assign the link frame origin.
3. Assign Z_{i-1} pointing along the direction of axis i-1.
4. Assign X_{i-1} pointing along the common perpendicular, or if the axes intersect, assign X_{i-1} to be normal to the plane containing the two axes.
5. Assign Y_{i-1} to complete a right-hand coordinate system.
6. Assign {0} to match {1} when the joint variable is zero. For {N} choose an origin location and X_N direction freely, but generally so as to cause as many link parameters as possible to become zero.

Link transformation

$$
{}^{i-1}_{i}T = {}^{i-1}_{R}T \cdot {}^{R}_{Q}T \cdot {}^{Q}_{P}T \cdot {}^{P}_{i}T
$$

$$
= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_{i-1} & -s\alpha_{i-1} & 0 \\ 0 & s\alpha_{i-1} & c\alpha_{i-1} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & a_{i-1} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i c\alpha_{i-1} & c\theta_i c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i s\alpha_{i-1} & c\theta_i s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

Denavit Hartenberg Parameters

α_{i-1} = the angle between Z_{i-1} to Z_i measured about X_{i-1}

a_{i-1} = the distance from Z_{i-1} to Z_i measured along X_{i-1}

d_i = the distance from X_{i-1} to X_i measured along Z_i

θ_i = the angle between X_{i-1} to X_i measured about Z_i

i	α_i	a_{i-1}	d_i	θ_i
1				
2				
3