Machine Learning

Answer Sheet for Homework 5

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Problem 1

The hard-margin support vector machine is with d+1 variables. For soft-margin support vector machine, there are N more variables ξ_n , $1 \le n \le N$.

So soft-margin support vector machine is a quadratic programming problem with N+d+1 variables.

Problem 2

I wrote a Q02.py to help me get the answer. By using Python package cvxopt, with

$$\mathbf{z} = \begin{bmatrix} 1 & -2 \\ 4 & -5 \\ 4 & -1 \\ 5 & -2 \\ 7 & -7 \\ 7 & 1 \\ 7 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ +1 \\ +1 \\ +1 \\ +1 \end{bmatrix}$$
(1)

and

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \tag{2}$$

$$\mathbf{A}^{T} = \begin{bmatrix} -1 & -1 & 2 \\ -1 & -4 & 5 \\ -1 & -4 & 1 \\ 1 & 5 & -2 \\ 1 & 7 & -7 \\ 1 & 7 & 1 \\ 1 & 7 & 1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$
(3)

To use this package, I gave solvers.qp $(\mathbf{Q}, \mathbf{p}, -\mathbf{A}^T, -\mathbf{c})$ and got

$$b = -9, \mathbf{w} = [2, 0] \tag{4}$$

So the hyperplane is

$$2z_1 - 9 = 0 \Rightarrow z_1 = 4.5 \tag{5}$$

Problem 3

I wrote a Q03.py to help me get the answer. By using Python package cvxopt, with

$$\mathbf{Q} = \begin{bmatrix}
4 & 1 & 1 & 0 & -1 & -1 & -1 \\
1 & 4 & 0 & -1 & -9 & -1 & -1 \\
1 & 0 & 4 & -1 & -1 & -9 & -1 \\
0 & -1 & -1 & 4 & 1 & 1 & 9 \\
-1 & -9 & -1 & 1 & 25 & 9 & 1 \\
-1 & -1 & -9 & 1 & 9 & 25 & 1 \\
-1 & -1 & -1 & 9 & 1 & 1 & 25
\end{bmatrix}, \mathbf{p} = \begin{bmatrix}
-1 \\
-1 \\
-1 \\
-1 \\
-1 \\
-1
\end{bmatrix},$$

$$(6)$$

$$\mathbf{A}^{T} = \begin{bmatrix}
-1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1
\end{bmatrix}, \mathbf{c} = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}$$

$$(7)$$

$$-\mathbf{A}^{T} = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$
(7)

with

$$\mathbf{G} = [-1 \ -1 \ -1 \ 1 \ 1 \ 1 \ 1] \text{ and } h = 0$$
 (8)

and To use this package, I gave solvers.qp $(\mathbf{Q},\mathbf{p},-\mathbf{A}^T,\mathbf{c},\mathbf{G},h)$ and got

$$\alpha = \left[4.32 \times 10^{-9} \approx 0, 0.704, 0.704, 0.889, 0.259, 0.259, 5.27 \times 10^{-10} \approx 0\right] \tag{9}$$

where cvxopt needs conditions

$$-\mathbf{A}^T \boldsymbol{\alpha} \leq \mathbf{c} \text{ and } \mathbf{G} \boldsymbol{\alpha} = h \tag{10}$$

Problem 4

I wrote a Q04.py to help me get the answer. By using python package sympy and

$$\mathbf{w} = \sum_{n=1}^{N} \alpha_n y_n K\left(\mathbf{x}_n, (x_1, x_2)\right) + b \tag{11}$$

$$b = y_s - \sum_{n=1}^{N} \alpha_n y_n K(\mathbf{x}_n, \mathbf{x}_s)$$
(12)

we have

$$\mathbf{w} = \frac{1}{9} \left(8x_1^2 - 16x_1 + 6x_2^2 - 15 \right) \tag{13}$$

Problem 5

Since kernel function $K(\mathbf{x}, \mathbf{x}') = (1 + \mathbf{x}^T \mathbf{x}')^2$ is different from $\mathbf{z} = (\phi(\mathbf{x}), \phi(\mathbf{x}))$, the curves should be different in the \mathcal{X} space.

Problem 6

Problem 7

Problem 8	
Problem 9	
Problem 10	
Problem 11	
Problem 12	
Problem 13	
Problem 14	
Problem 15	

Problem 16	
Problem 17	
Problem 18	
Problem 19	
Problem 20	

Reference

[1] Lecture Notes by Hsuan-Tien LIN, Department of Computer Science and Information Engineering, National Taiwan University, Taipei 106, Taiwan.