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# Machine Learning

## Answer Sheet for Homework 5

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November 23, 2015

### Problem 1

The hard-margin support vector machine is with  $d + 1$  variables. For soft-margin support vector machine, there are  $N$  more variables  $\xi_n$ ,  $1 \leq n \leq N$ .

So soft-margin support vector machine is a quadratic programming problem with  $N + d + 1$  variables.

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### Problem 2

I wrote a `Q02.py` to help me get the answer. By using Python package `cvxopt`, with

$$\mathbf{z} = \begin{bmatrix} 1 & -2 \\ 4 & -5 \\ 4 & -1 \\ 5 & -2 \\ 7 & -7 \\ 7 & 1 \\ 7 & 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ +1 \\ +1 \\ +1 \\ +1 \end{bmatrix} \quad (1)$$

and

$$\mathbf{Q} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \mathbf{p} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad (2)$$

$$\mathbf{A}^T = \begin{bmatrix} -1 & -1 & 2 \\ -1 & -4 & 5 \\ -1 & -4 & 1 \\ 1 & 5 & -2 \\ 1 & 7 & -7 \\ 1 & 7 & 1 \\ 1 & 7 & 1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \quad (3)$$

To use this package, I gave `solvers.qp(Q, p, -AT, -c)` and got

$$b = -9, \mathbf{w} = [2, 0] \quad (4)$$

So the hyperplane is

$$2z_1 - 9 = 0 \Rightarrow z_1 = 4.5 \quad (5)$$

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### Problem 3

I wrote a `Q03.py` to help me get the answer. By using Python package `cvxopt`, with

$$\mathbf{Q} = \begin{bmatrix} 4 & 1 & 1 & 0 & -1 & -1 & -1 \\ 1 & 4 & 0 & -1 & -9 & -1 & -1 \\ 1 & 0 & 4 & -1 & -1 & -9 & -1 \\ 0 & -1 & -1 & 4 & 1 & 1 & 9 \\ -1 & -9 & -1 & 1 & 25 & 9 & 1 \\ -1 & -1 & -9 & 1 & 9 & 25 & 1 \\ -1 & -1 & -1 & 9 & 1 & 1 & 25 \end{bmatrix}, \mathbf{p} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}, \quad (6)$$

$$-\mathbf{A}^T = \begin{bmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (7)$$

with

$$\mathbf{G} = \begin{bmatrix} -1 & -1 & -1 & 1 & 1 & 1 & 1 \end{bmatrix} \text{ and } h = 0 \quad (8)$$

and To use this package, I gave `solvers.qp(Q, p, -AT, c, G, h)` and got

$$\alpha = [4.32 \times 10^{-9} \approx 0, 0.704, 0.704, 0.889, 0.259, 0.259, 5.27 \times 10^{-10} \approx 0] \quad (9)$$

where `cvxopt` needs conditions

$$-\mathbf{A}^T \boldsymbol{\alpha} \preceq \mathbf{c} \text{ and } \mathbf{G} \boldsymbol{\alpha} = h \quad (10)$$

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## Problem 4

I wrote a `Q04.py` to help me get the answer. By using python package `sympy` and

$$\mathbf{w} = \sum_{n=1}^N \alpha_n y_n K \left( \mathbf{x}_n, \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) + b \quad (11)$$

$$b = y_s - \sum_{n=1}^N \alpha_n y_n K(\mathbf{x}_n, \mathbf{x}_s) \quad (12)$$

we have

$$\mathbf{w} = \frac{1}{9} (8x_1^2 - 16x_1 + 6x_2^2 - 15) \quad (13)$$

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## Problem 5

Since kernel function  $K(\mathbf{x}, \mathbf{x}') = (1 + \mathbf{x}^T \mathbf{x}')^2$  is different from  $\mathbf{z} = (\phi(\mathbf{x}), \phi(\mathbf{x}'))$ , the curves should be different in the  $\mathcal{X}$  space.

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## Problem 6

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## Problem 7

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**Problem 8**

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**Problem 9**

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**Problem 10**

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**Problem 11**

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**Problem 15**

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### Problem 16



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### Problem 17



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### Problem 18



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### Problem 19



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### Problem 20



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## Reference

- [1] Lecture Notes by Hsuan-Tien LIN, Department of Computer Science and Information Engineering, National Taiwan University, Taipei 106, Taiwan.