

Report HW1

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1 Exercise 1

Gaussians:

N°	Mean	Variance	Probability
1	-2	2	0.15
2	4	1	0.25
3	10	3	0.35
4	15	2	0.25

Let X be the random variable representing the drawn number from one of the four Gaussians. Let Y be the random variable representing the Gaussian from which the number is drawn.

$$\begin{aligned} E[X] &= E[X|Y=1]P(Y=1) + E[X|Y=2]P(Y=2) + E[X|Y=3]P(Y=3) + E[X|Y=4]P(Y=4) \\ &= -2 \cdot 0.15 + 4 \cdot 0.25 + 10 \cdot 0.35 + 15 \cdot 0.25 \\ &= -0.3 + 1 + 3.5 + 3.75 \\ &= 7.95 \end{aligned}$$

$$\begin{aligned} \text{Var}[X] &= E[\text{Var}[X|Y]] + \text{Var}[E[X|Y]] \\ &= \sigma_1^2 \cdot P(Y=1) + \sigma_2^2 \cdot P(Y=2) + \sigma_3^2 \cdot P(Y=3) + \sigma_4^2 \cdot P(Y=4) \\ &\quad + E[E[X|Y]^2] - E[X]^2 \\ &= 2 \cdot 0.15 + 1 \cdot 0.25 + 3 \cdot 0.35 + 2 \cdot 0.25 \\ &\quad + 0.15 \cdot (-2)^2 + 0.25 \cdot 4^2 + 0.35 \cdot 10^2 + 0.25 \cdot 15^2 \\ &\quad - 7.95^2 \\ &= 0.3 + 0.25 + 1.05 + 0.5 \\ &\quad + 0.6 + 4 + 35 + 56.25 \\ &\quad - 63.2025 \\ &= 2.1 + 95.85 - 63.2025 \\ &= 34.7475 \end{aligned}$$