Dynamical Systems and Chaos

—

Mario Cosenza[1]

Affiliation of Author

E-mail: mcosenza@yachaytech.edu.ec

[1] *Author Website*

Contents

Preface

Compiled and expanded by Rolando S. Sánchez
Student of Professor Mario Cosenza
Yachay Tech University

This textbook originates from the lecture notes and course materials developed by Professor Mario Cosenza for his renowned course on *Dynamical Systems and Chaos*. As his student, I have undertaken the task of organizing, refining, and supplementing these invaluable resources to create this comprehensive volume.

The study of dynamical systems provides a unifying framework for understanding complexity across diverse scientific disciplines. From the microscopic interactions of molecules to the macroscopic behavior of galaxies, from the rhythmic patterns of biological oscillators to the unpredictable fluctuations of financial markets - all can be examined through the lens of nonlinear dynamics. Professor Cosenza's lectures masterfully demonstrated how simple mathematical rules can generate both orderly patterns and chaotic behavior, revealing the deep connections between determinism and unpredictability.

The core content of this book remains faithful to Professor Cosenza's original lectures, which covered:

1. Introduction to Dynamical Systems
2. Stability Analysis of Fixed Points
3. Introduction to Chaos
4. One-dimensional Maps as Models of Chaotic Systems
5. Transition to Chaos by Intermittency
6. Transition to Chaos by Quasiperiodicity
7. Strange Attractors
8. Repellers, Transient Chaos, Crises, and Control of Chaos

In preparing this text, I have:

- Organized the lecture material into a coherent textbook structure
- Added detailed derivations and worked examples
- Incorporated modern computational illustrations
- Included supplementary exercises and problems

My aim has been to preserve the intellectual depth and pedagogical clarity of Professor Cosenza's original teachings while making the material more accessible to a broader audience. This book is intended for undergraduate students across physical and mathematical sciences who wish to understand how complexity emerges from simple rules.

Rolando S. Sánchez

PART

I

Review of dynamical systems

The laws of nature and many natural phenomena can be understood through cause-effect relationships, where the evolution of a system depends on its current state and governing rules. These relationships are expressed in the universal language of mathematics—through equations, functions, and deterministic or stochastic rules. The dynamical systems framework provide a powerful framework for modeling such behavior across disciplines, from physics and biology to economics and engineering.

Whether studying the motion of planets, the spread of diseases, neural activity, or climate patterns, dynamical systems reveal how complex behavior emerges from simple interactions. By analyzing differential equations, iterative maps, or network dynamics, we uncover patterns, stability, chaos, and predictability in nature. The multidisciplinary nature of dynamical systems highlights the unity of scientific inquiry, demonstrating how mathematical principles underlie diverse real-world phenomena.

1 Dynamical Systems

Definition 1.1. A dynamical system can be described as a collection or set of time dependent state variables $\mathbf{S}$

$$\mathbf{S(t)} = \{X_1(t), X_2(t), ..., X_n(t)\}$$

that evolves with time according to a *specific known deterministic rule* $\mathbf{f}$, such that

$$S(t) \xrightarrow{\mathbf{f}} S(t+\tau)$$

This **deterministic** rule $\mathbf{f}$ corresponds to a set of operations or procedures that allows to calculate the state of a system at time $t+\tau$ from the knolwdge of its state at time t.

[1]

[1] *With some additional sidenotes*