

# Scheduling Hybrid Task Sets

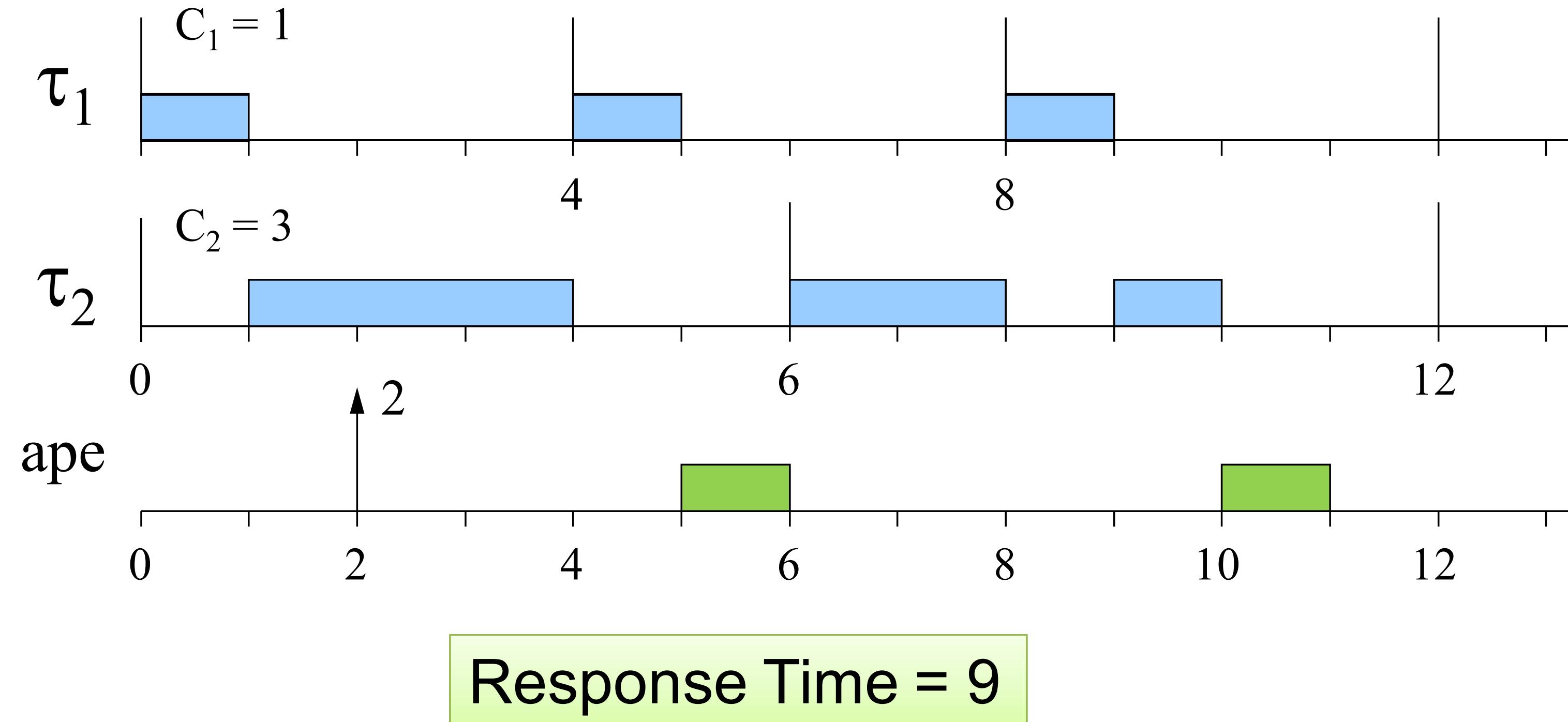
Periodic Tasks + Aperiodic Tasks

# Handling aperiodic tasks

- Aperiodic tasks are typically activated by the arrival of external events (notified by interrupts).
- From one hand, one objective of the kernel is to reduce the response time of aperiodic tasks (interrupt latency).
- On the other hand, aperiodic task execution should not jeopardize schedulability.

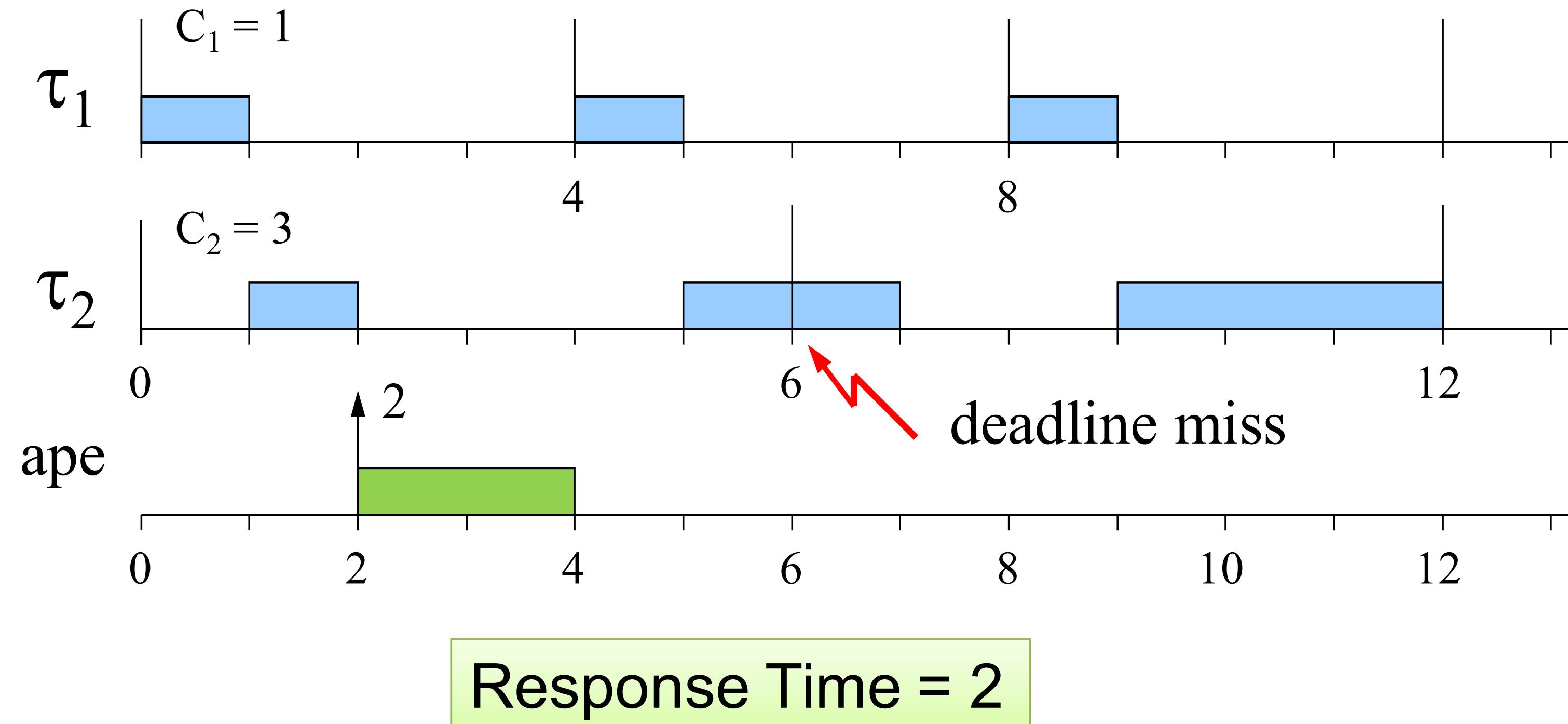
# Background service

If aperiodic jobs are scheduled in background (i.e., during idle times left by periodic tasks) their response times are too long:

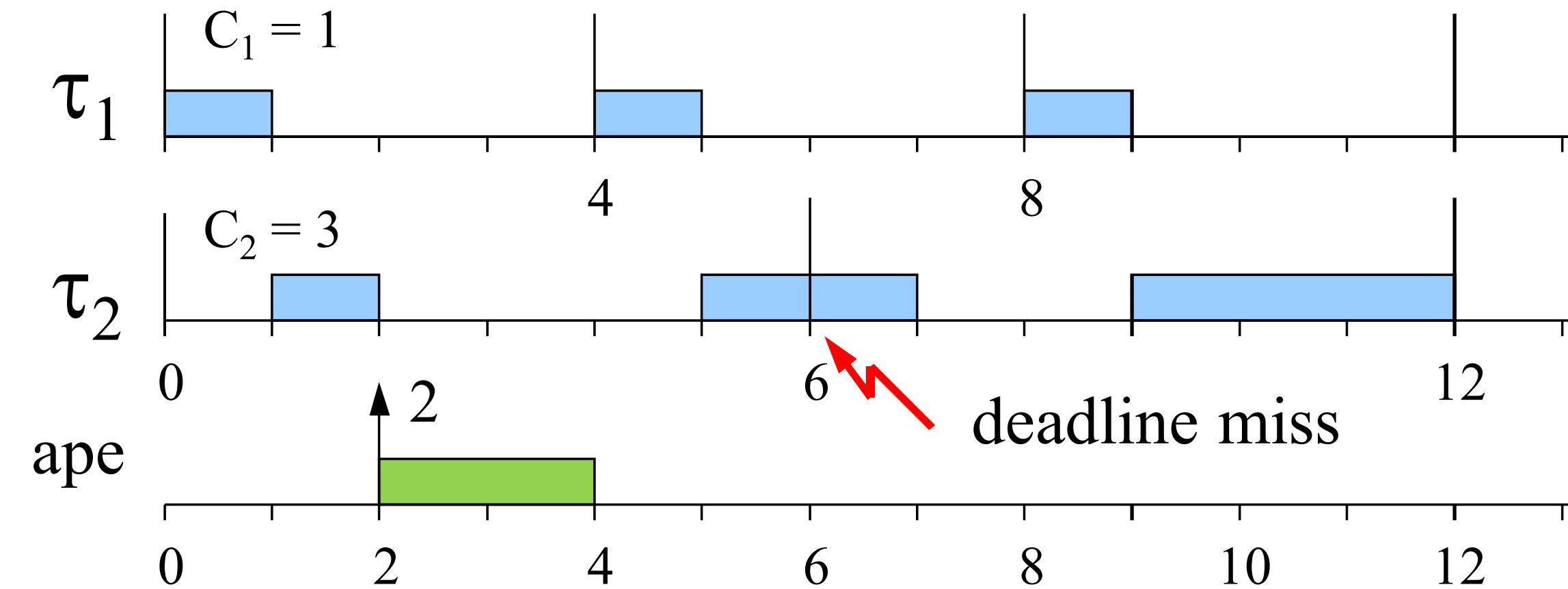


# Immediate service

On the other hand, if interrupts service routines are scheduled at the highest priority, the other tasks can miss their deadlines:



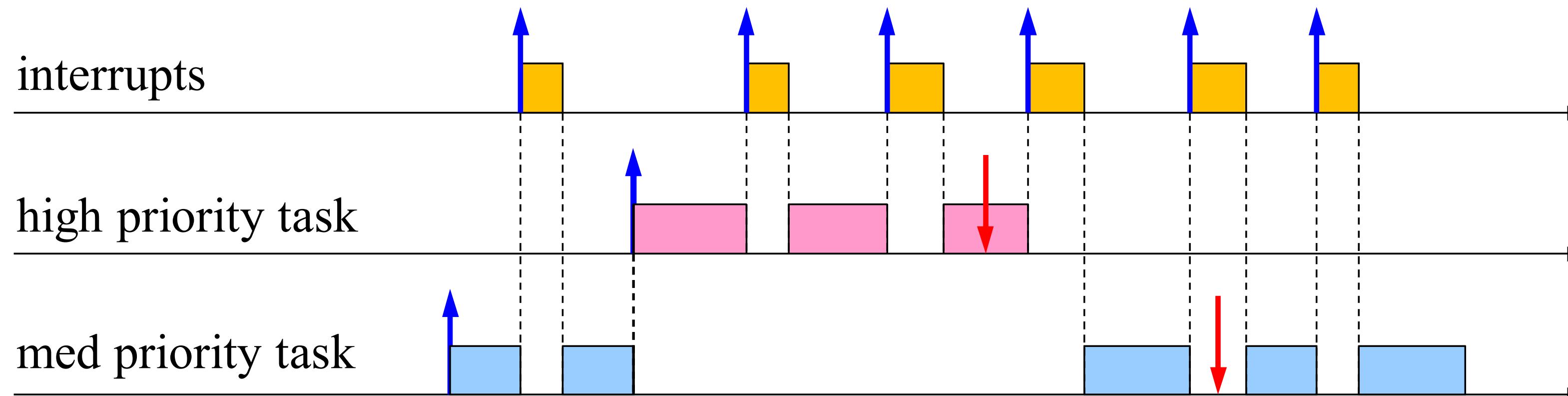
# Immediate service



A problem related to a wrong service of aperiodic tasks occurred in the **Apollo 11** computing system, during the historical mission on the Moon, in the **landing phase**.

# What happened exactly?

- During landing, sensors generated several interrupts.
- Intensive execution of drivers caused transient overload conditions.
- Task execution was delayed too much causing several timeout errors (deadline misses).



# HARD aperiodic tasks

- Aperiodic tasks with **HARD** deadlines must be guaranteed under worst-case conditions.
- Off-line guarantee is only possible if we can bound interarrival times (**sporadic tasks**).
- Hence **sporadic tasks** can be guaranteed as periodic tasks with  $C_i = \text{WCET}_i$  and  $T_i = \text{MIT}_i$ .

$$\left. \begin{array}{l} \text{WCET} = \text{Worst-Case Execution Time} \\ \text{MIT} = \text{Minimum Interarrival Time} \end{array} \right\}$$

# SOFT aperiodic tasks

- Aperiodic tasks with **SOFT** deadlines should be executed as soon as possible, but without jeopardizing HARD tasks.
- We may be interested in
  - minimizing the response time of each aperiodic request
  - performing an on-line guarantee

How can we achieve these goals?

# Aperiodic Servers

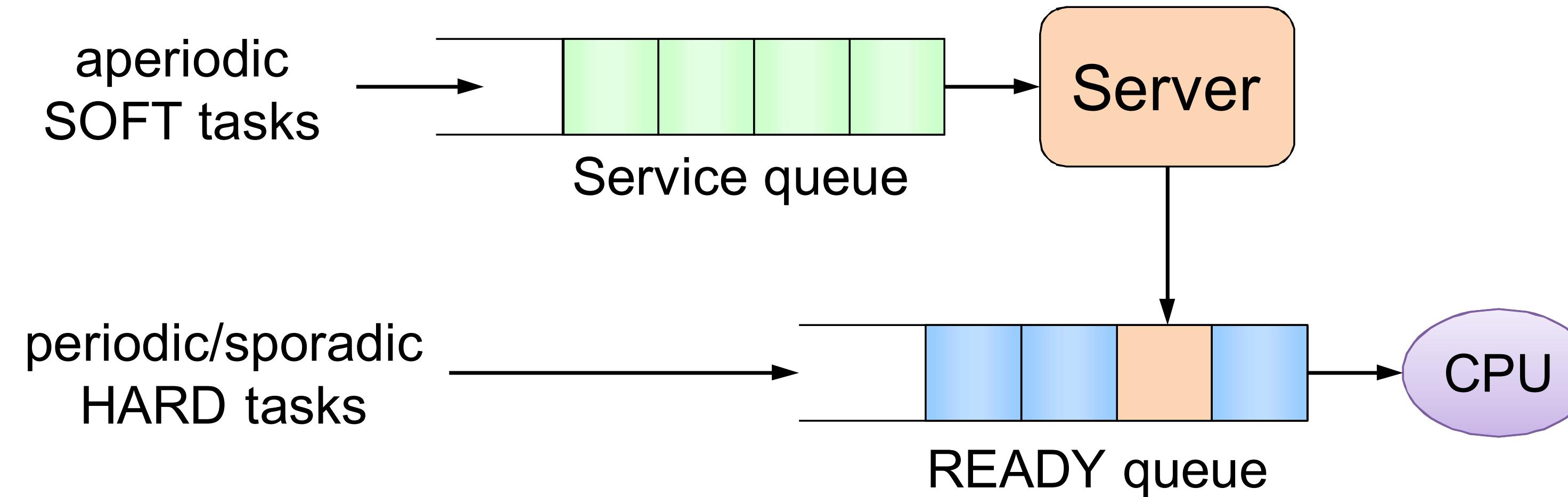
A method to limit the interference of aperiodic tasks on periodic tasks is to manage them through an **aperiodic server**.

- An **aperiodic server** can be view as a **periodic task** managing the execution of aperiodic tasks.
- It is typically described by two parameters:

$$\begin{cases} C_s & \text{capacity (or budget)} \\ T_s & \text{server period} \end{cases}$$

To preserve periodic tasks, no more than  $C_s$  units must be executed every period  $T_s$ .

# Aperiodic service queue



- The server is scheduled as any periodic task.
- Priority ties are broken in favor of the server.
- Aperiodic tasks can be selected using an arbitrary queueing discipline.

# Aperiodic Servers

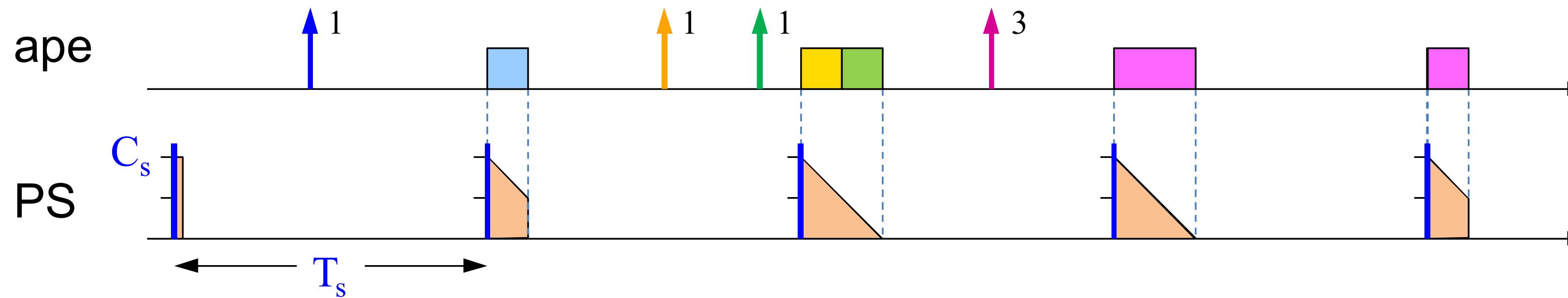
## Fixed-priority servers

- Polling Server
- Deferrable Server
- Sporadic Server
- Slack Stealer

## Dynamic-priority servers

- Dynamic Polling Server
- Dynamic Sporadic Server
- Total Bandwidth Server
- Tunable Bandwidth Server
- Constant Bandwidth Server

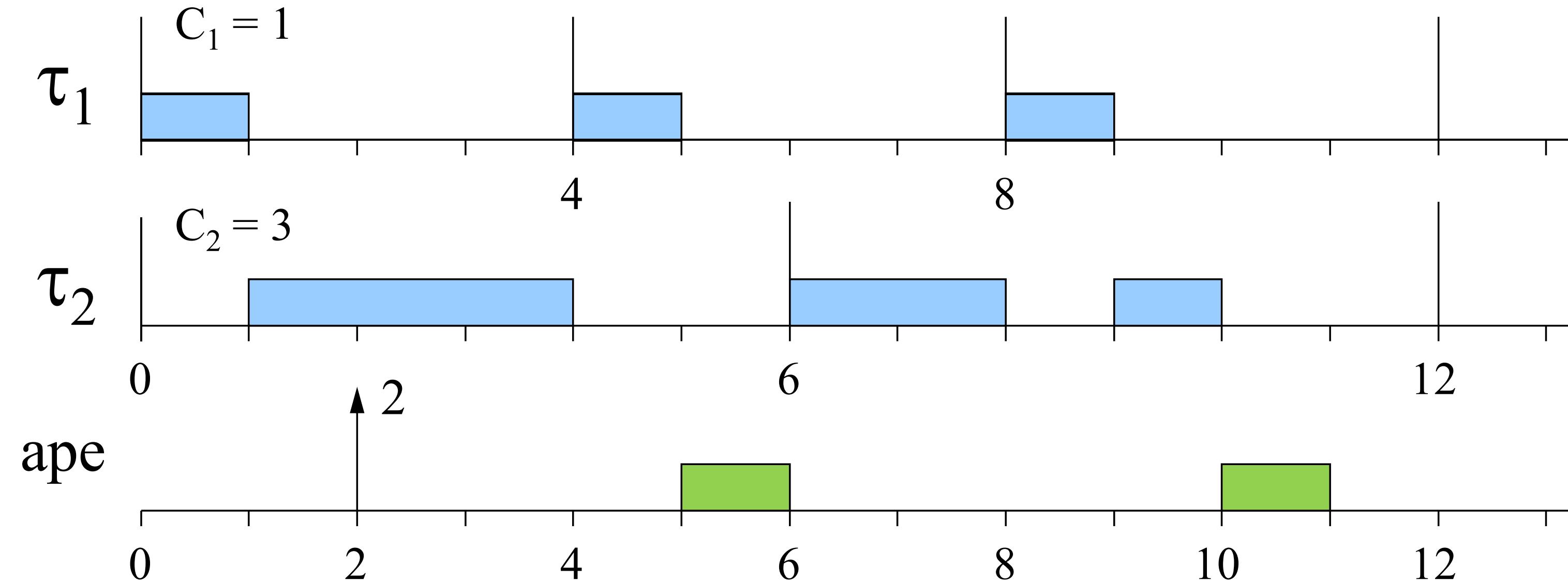
# Polling Server (PS)



- At the beginning of each period, the budget is recharged at its maximum value.
- When the server is scheduled and there are no pending jobs,  $C_s$  is discharged to zero.
- When the server is scheduled and there are pending jobs, they are served as long as  $C_s > 0$  ( $C_s$  is consumed proportionally).
- When the server budget is completely consumed ( $C_s = 0$ ) the server is suspended until the beginning of the next period.

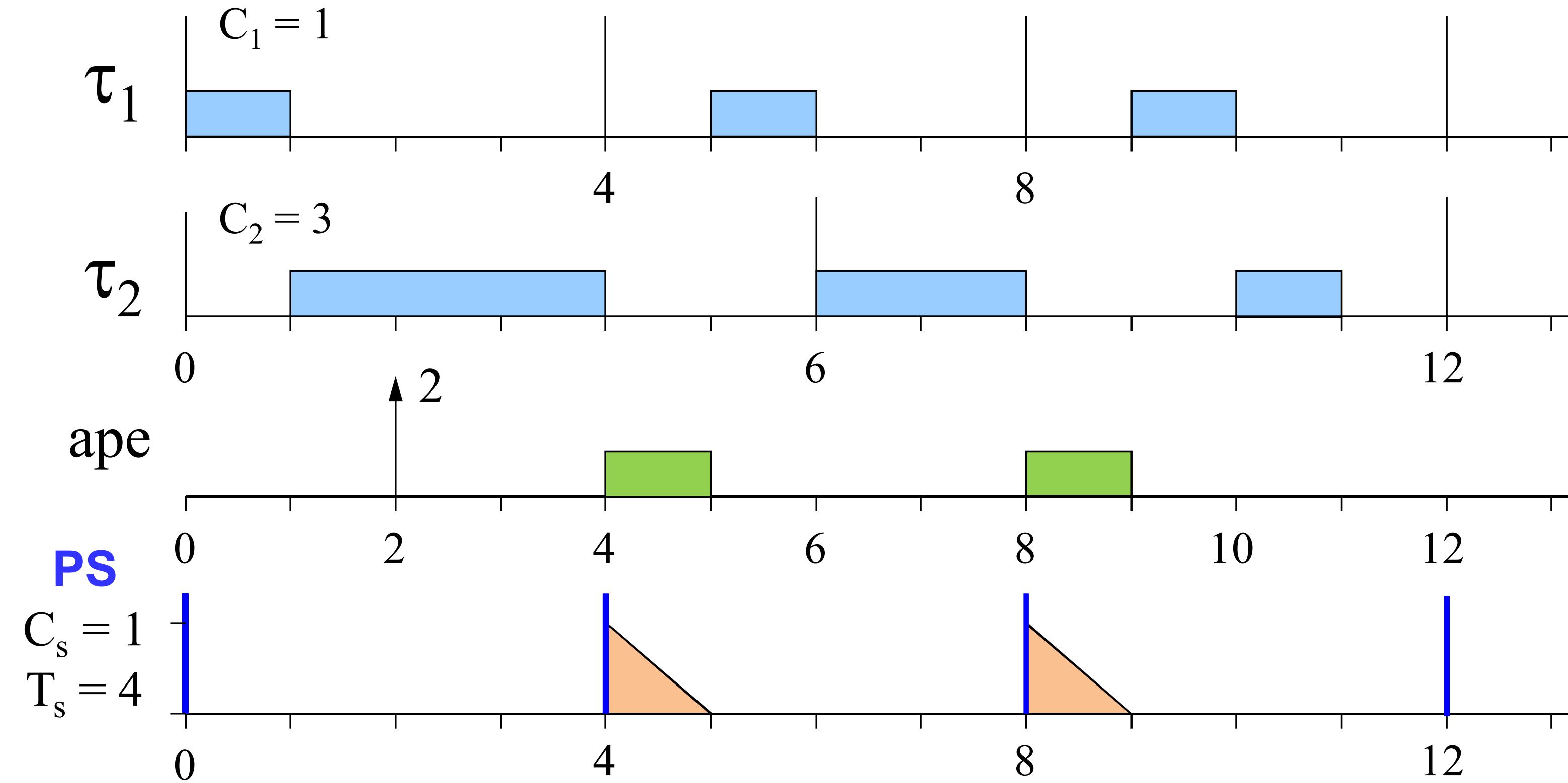
# Background service

Let's take the initial example:



Response Time = 9

# RM + Polling Server



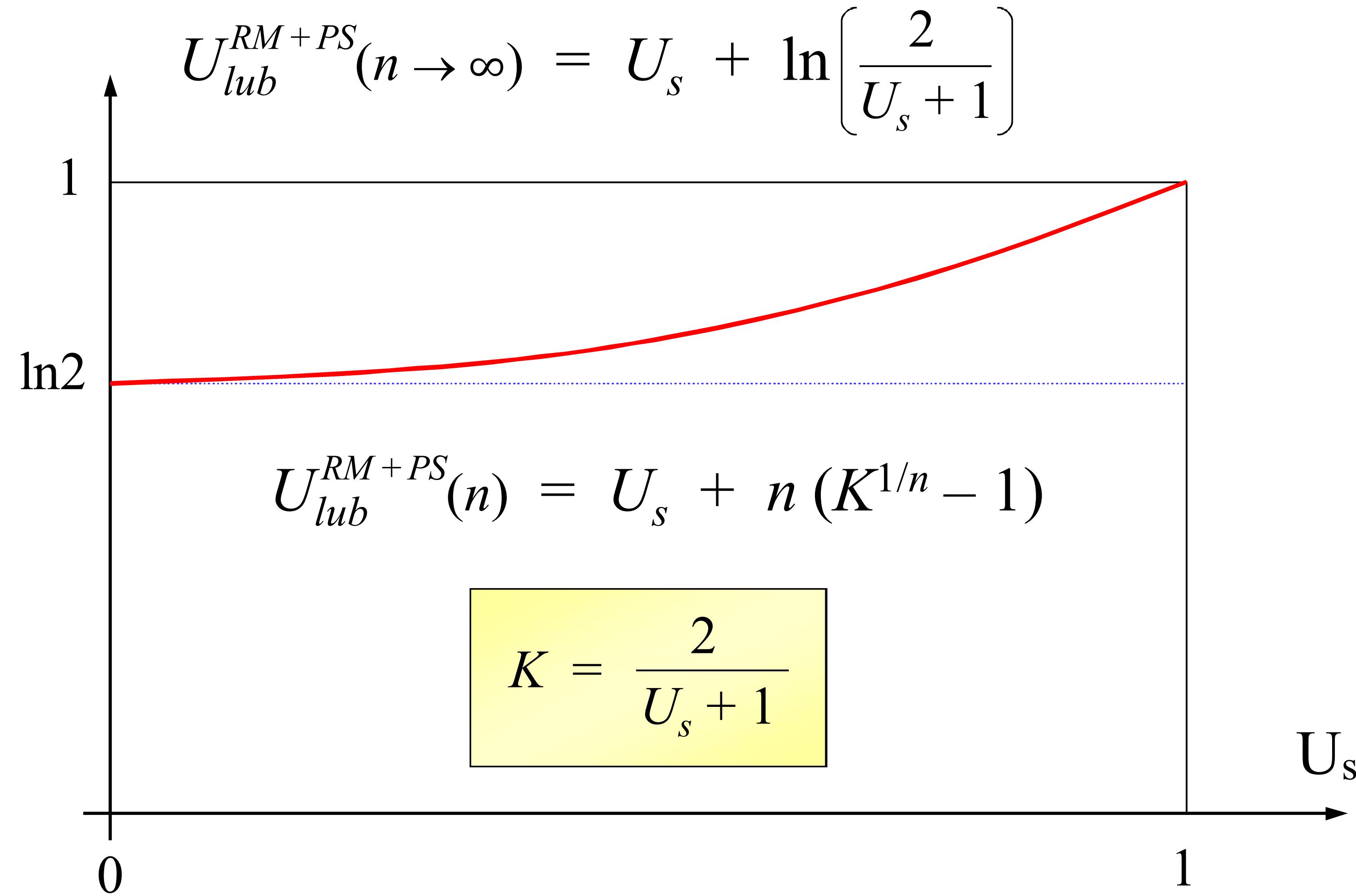
Response Time = 7

# PS properties

- In the worst-case, the PS behaves as a periodic task with utilization  $U_s = C_s/T_s$
- Aperiodic tasks execute at the highest priority if  $T_s = \min(T_1, \dots, T_n)$ .
- Liu & Layland analysis gives that:

$$U_{lub}^{RM+PS} = U_s + n \left[ \left( \frac{2}{U_s + 1} \right)^{1/n} - 1 \right]$$

# RM+PS schedulability



# Analysis with Hyperbolic Bound

A set of periodic tasks is schedulable by Rate Monotonic in the presence of a Polling Server with utilization  $U_s$  if

$$\prod_{i=1}^n (U_i + 1) \leq \frac{2}{U_s + 1}$$

Defining  $P = \prod_{i=1}^n (U_i + 1)$

the maximum server utilization that guarantees the schedulability of the periodic task set is

$$U_s^{\max} = \frac{2}{P} - 1$$

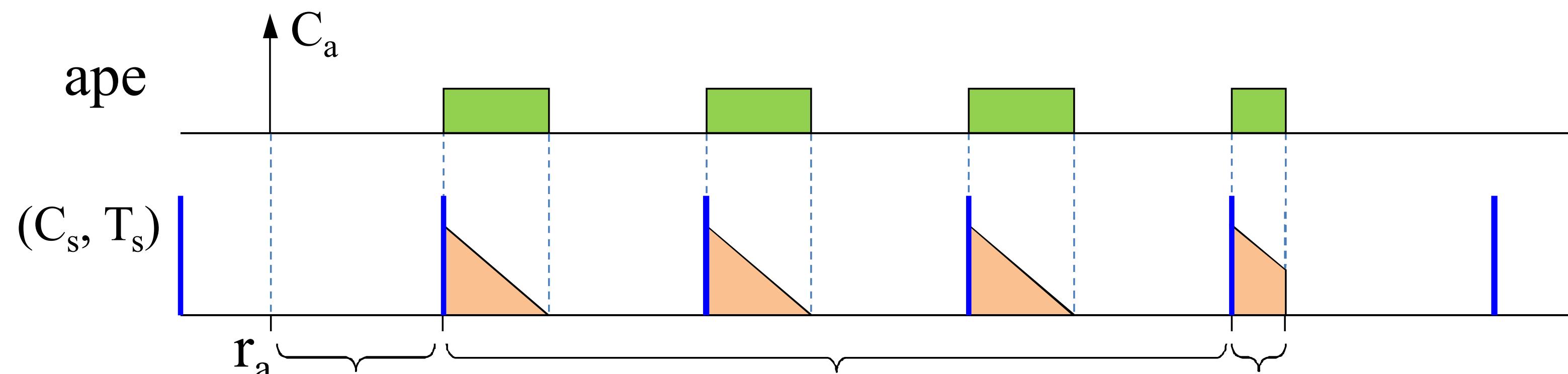
# RTA of hard periodic tasks with PS

And the **response time** of a periodic task can be computed as follows:

$$R_i^{(s)} = C_i + \underbrace{\left\lceil \frac{R_i^{(s-1)}}{T_s} \right\rceil}_{\text{Interference of the server}} C_s + \underbrace{\sum_{k=1}^{i-1} \left\lceil \frac{R_i^{(s-1)}}{T_k} \right\rceil}_{\text{Interference of the hp tasks}} C_k$$

# Response time of sporadic tasks under PS

Consider a PS running at the highest priority and an aperiodic job arriving when the server is idle:



$$\Delta_a = \left\lceil \frac{r_a}{T_s} \right\rceil T_s - r_a$$

$$F_a = \left\lceil \frac{C_a}{C_s} \right\rceil - 1$$

$$\delta_a = C_a - F_a C_s$$

$$R_a = \Delta_a + C_a + F_a(T_s - C_s)$$

# Complete RM tests

Liu & Layland test:

$$\forall i \quad \sum_{k=1}^{i-1} \frac{C_k}{T_k} + \frac{C_i + B_i}{T_i} \leq i(K^{1/i} - 1)$$

Hyperbolic test:

$$\forall i \quad \prod_{k=1}^{i-1} \left( \frac{C_k}{T_k} + 1 \right) \left( \frac{C_i + B_i}{T_i} + 1 \right) \leq K$$

Response Time:

$$R_i^{(s)} = C_i + B_i + \left\lceil \frac{R_i^{(s-1)}}{T_s} \right\rceil C_s + \sum_{k=1}^{i-1} \left\lceil \frac{R_i^{(s-1)}}{T_k} \right\rceil C_k$$

# Designing server parameters

1. Determine  $U_s^{\max}$  using  $\prod_{i=1}^n (U_i + 1) \leq K$

Defining  $P = \prod_{i=1}^n (U_i + 1)$  we have

$$\begin{cases} U_{PS}^{\max} = \frac{2}{P} - 1 \\ U_{DS}^{\max} = \frac{2-P}{2P-1} \end{cases}$$

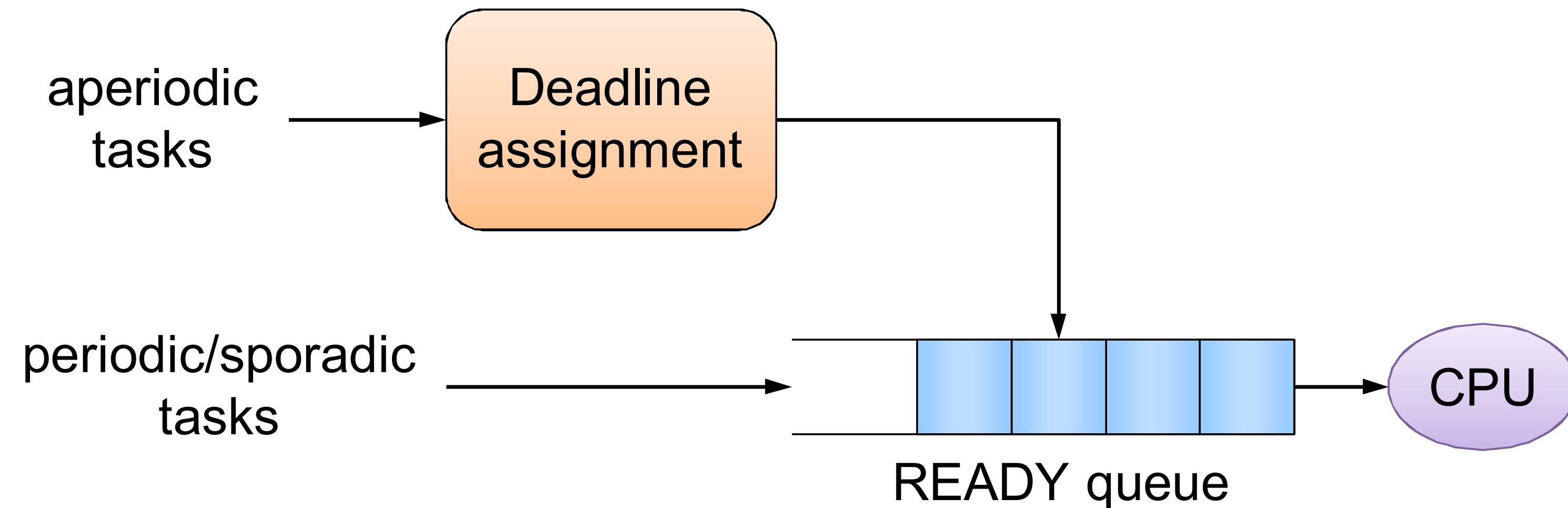
2. Set  $U_s \leq U_s^{\max}$
3. Set  $T_s = \min(T_1, \dots, T_n)$
4. Set  $C_s = U_s T_s$

# **Dynamic Priority Servers**

# Total Bandwidth Server (TBS)

- It is a dynamic priority server, used along with EDF.
- Each aperiodic request is assigned a deadline so that the server demand does not exceed a given bandwidth  $U_s$ .
- Aperiodic jobs are inserted in the ready queue and scheduled together with the HARD tasks.

# The TBS mechanism



- Deadlines ties are broken in favor of the server.
- Periodic tasks are guaranteed *if and only if*

$$U_p + U_s \leq 1$$

# Deadline assignment rule

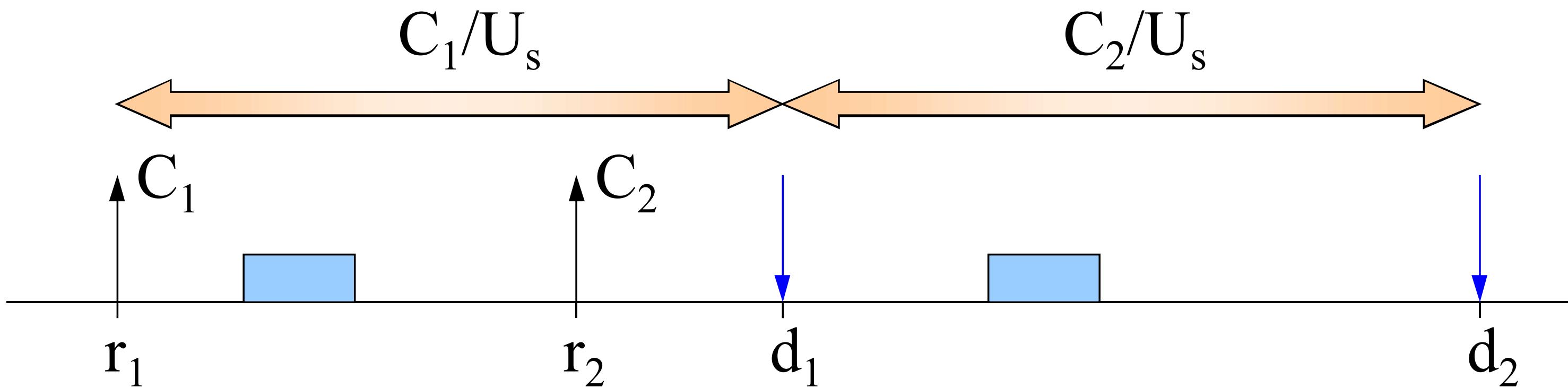
- Deadline has to be assigned not to jeopardize periodic tasks.
- A safe relative deadline is equal to the minimum period that can be assigned to a new periodic task with utilization  $U_s$ :

$$U_s = C_k / T_k \quad \rightarrow \quad T_k = d_k - r_k = C_k / U_s$$

- Hence, the absolute deadline can be set as:

$$d_k = r_k + C_k / U_s$$

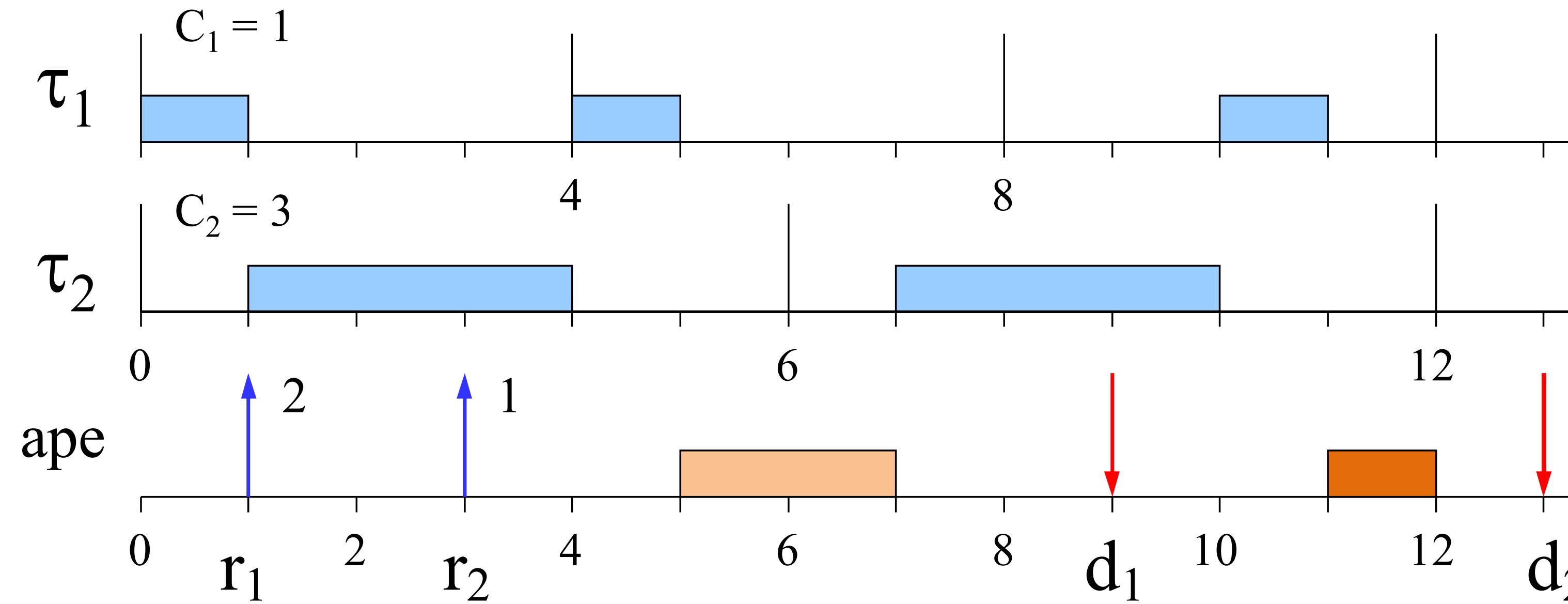
# Deadline assignment rule



- To keep track of the bandwidth assigned to previous jobs,  $d_k$  must be computed as:

$$d_k = \max(r_k, d_{k-1}) + C_k / U_s$$

# EDF + TBS schedule

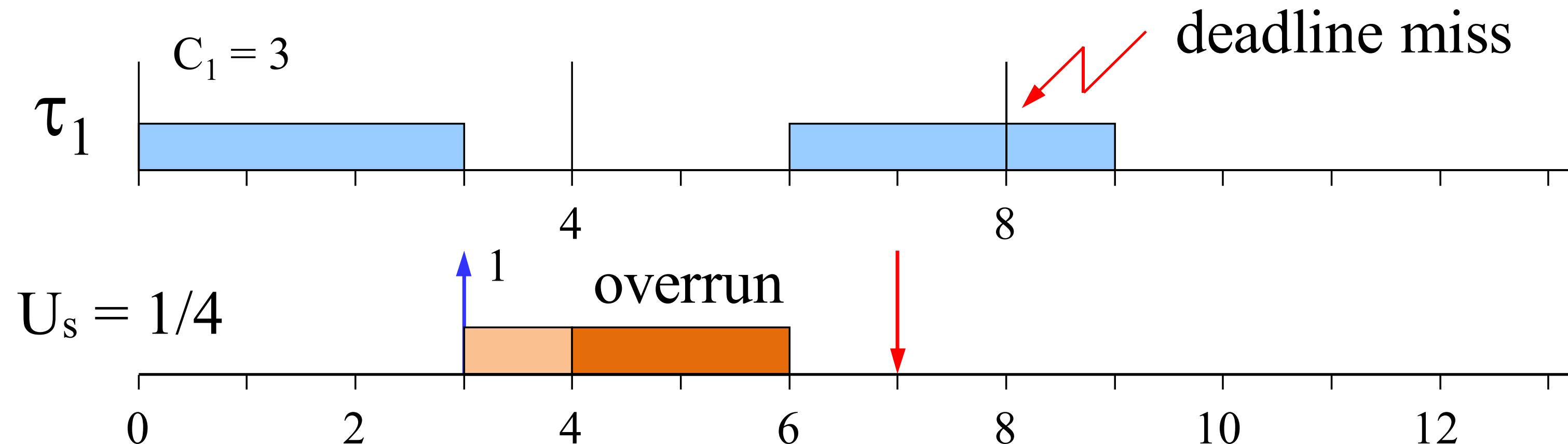


$$U_s = 1 - U_p = 1/4$$

$$\left\{ \begin{array}{l} d_1 = r_1 + C_1 / U_s = 1 + 2 \cdot 4 = 9 \\ d_2 = \max(r_2, d_1) + C_2 / U_s = 9 + 1 \cdot 4 = 13 \end{array} \right.$$

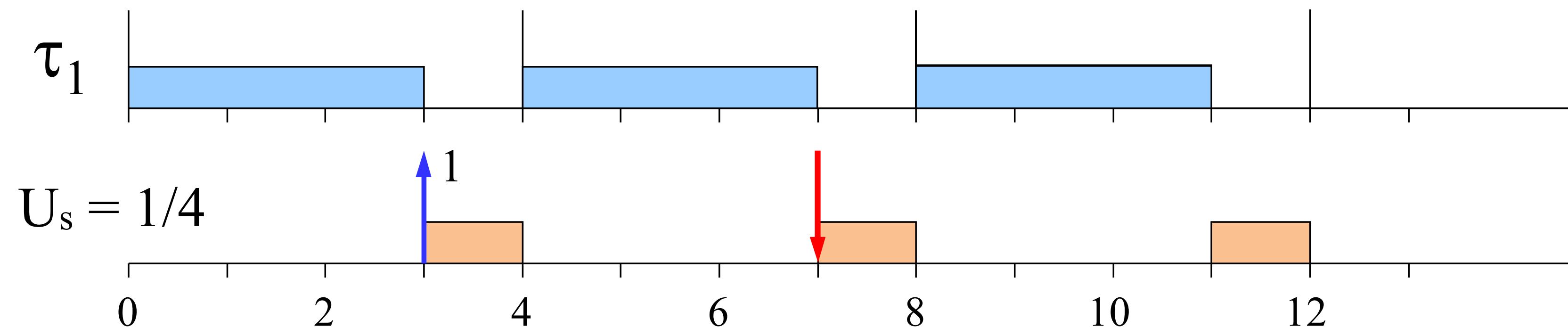
# Problems with the TBS

- Without a budget management, there is no protection against execution overruns.
- If a job executes more than expected, hard tasks could miss their deadlines.



# Overrun handling

- If a job executes more than expected (i.e., consumes its budget) it must be delayed by decreasing its priority or postponing its deadline.



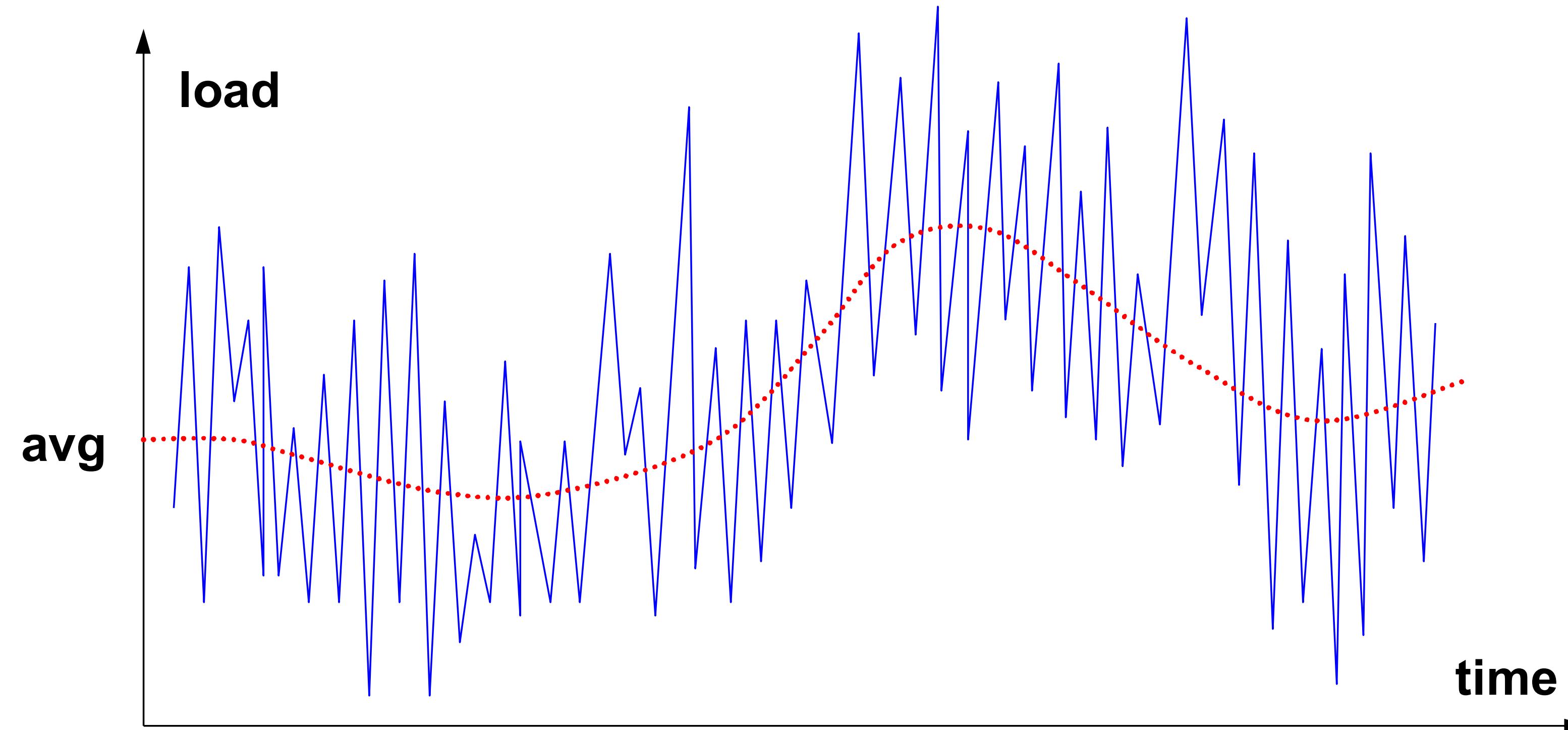
# Solution: task isolation

- In the presence of **overruns**, only the faulty task should be delayed.
- Each task  $\tau_i$  should not consume more than its declared utilization ( $U_i = C_i/T_i$ ).
- If a task executes more than expected (i.e., it **overruns**), then its priority should be decreased (or its deadline postponed).

# **Load Control Methods**

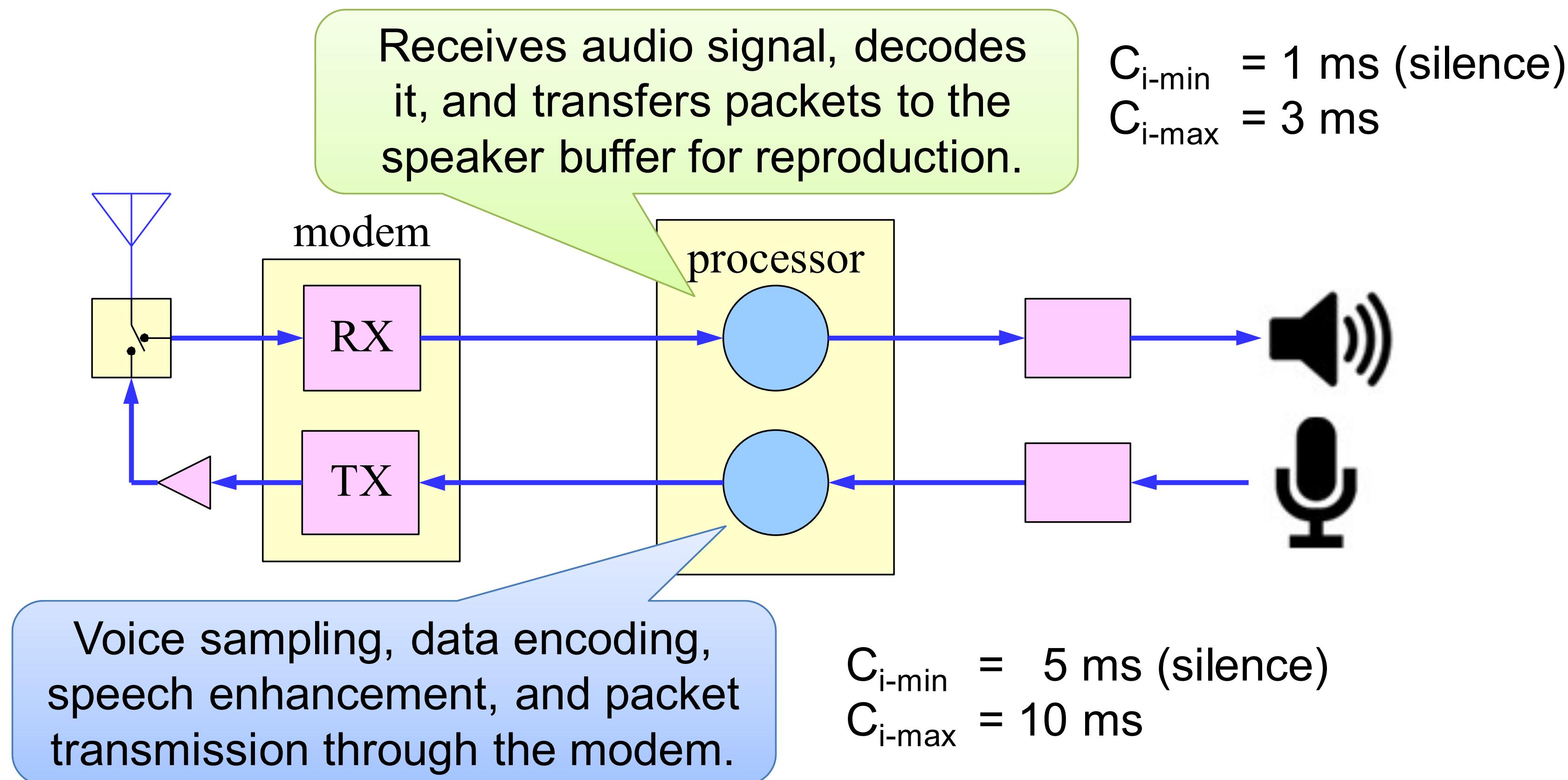
# Dynamic workload

Several applications (e.g., multimedia systems) are characterized by highly variable computational requirements:



# Example: phone call

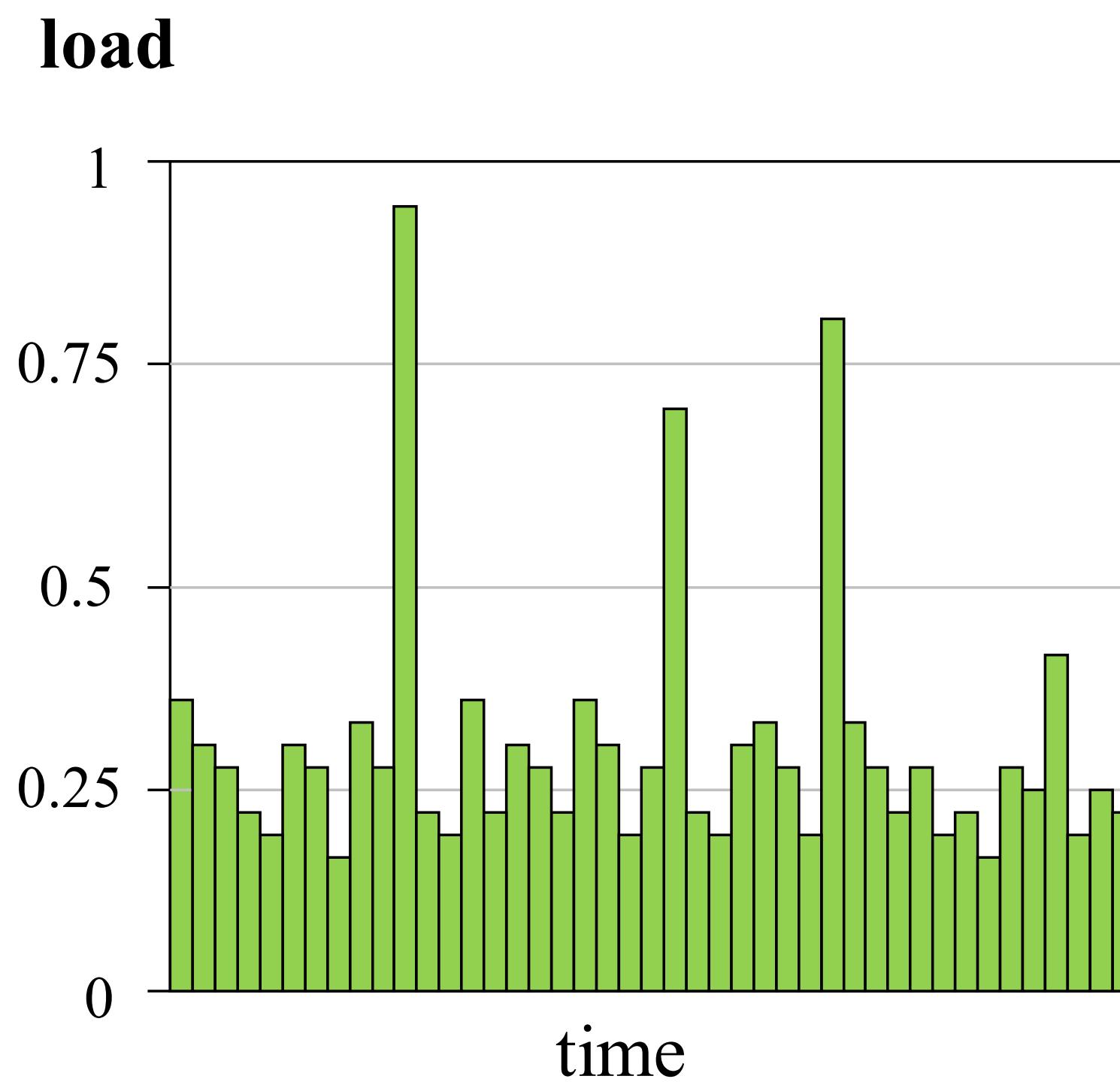
It consists at least of 2 periodic tasks, executed every 20 ms:



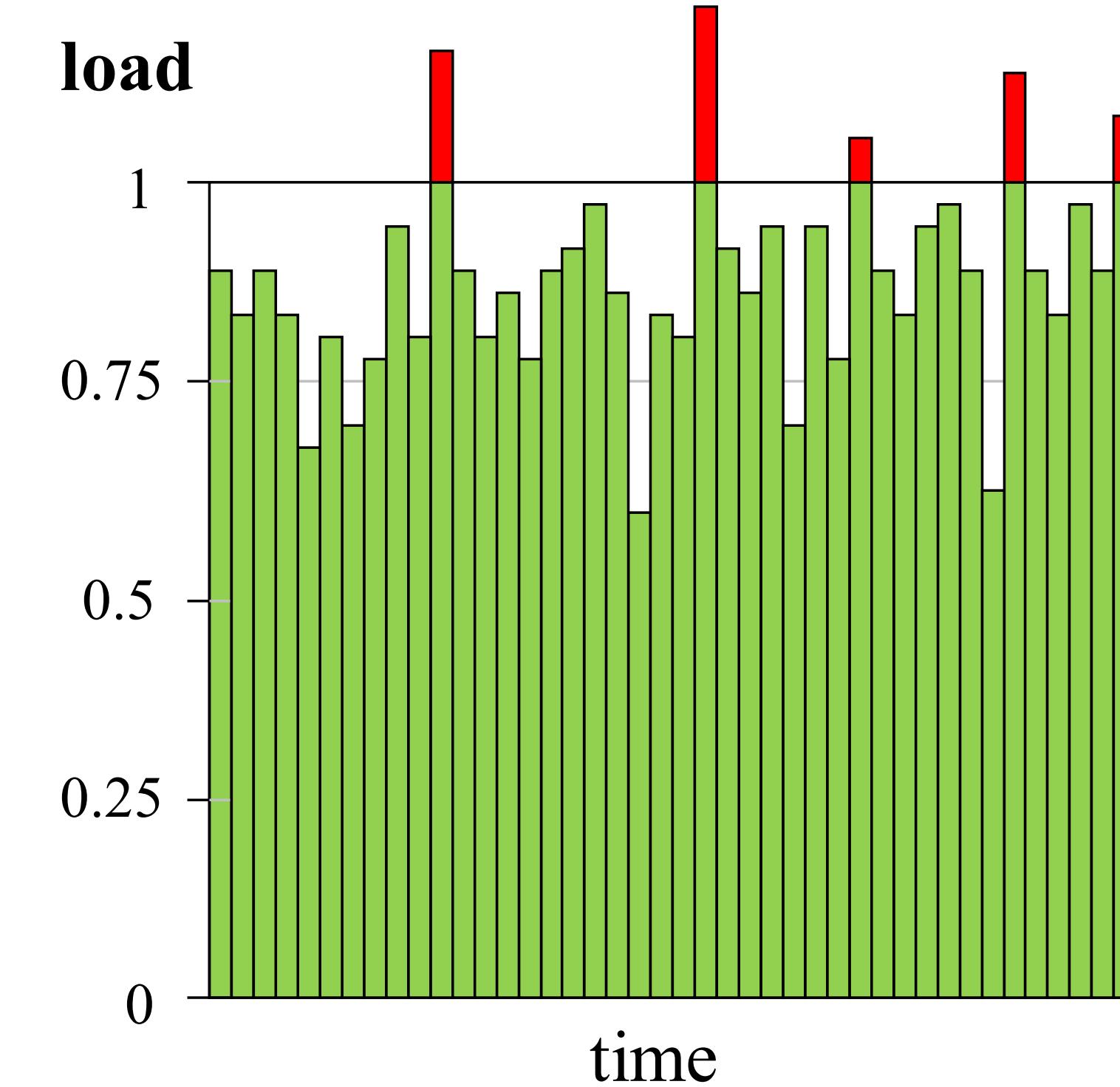
Overall CPU bandwidth: 30-65%

# Load and design assumptions

**System designed under  
worst-case assumptions**



**System designed under  
average-case assumptions**



# A matter of cost

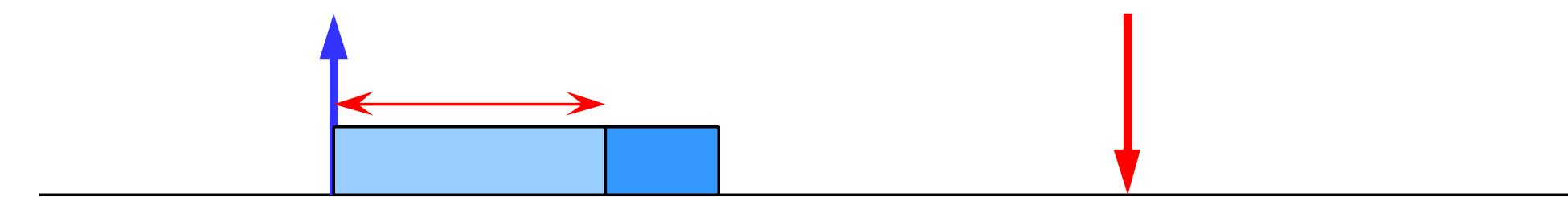
- High predictability and low efficiency means wasting resources ⇒ **high cost**
  - it can be justified only for very critical systems
- High efficiency in resource usage requires the system to:
  - handle and tolerate overloads
  - adapt for graceful degradation
  - plan for exception handling mechanisms

# Definitions

**Overrun:** Situation in which a task exceeds its expected utilization.

## Execution Overrun

The task computation time exceeds its expected value:



## Activation Overrun

The next task activation occurs before its expected time:

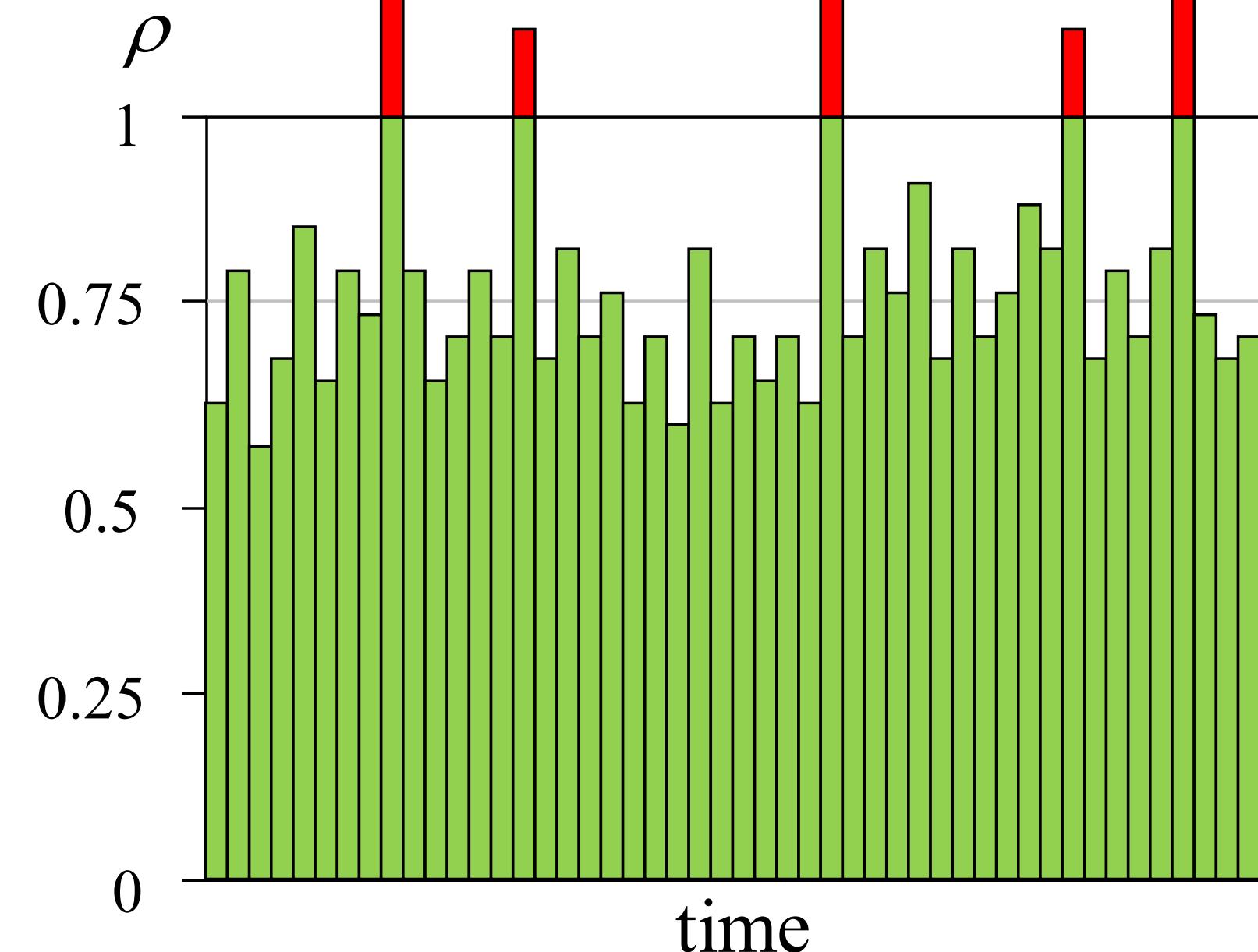


# Definitions

**Overload:** Situation in which  $\rho(t) > 1$ .

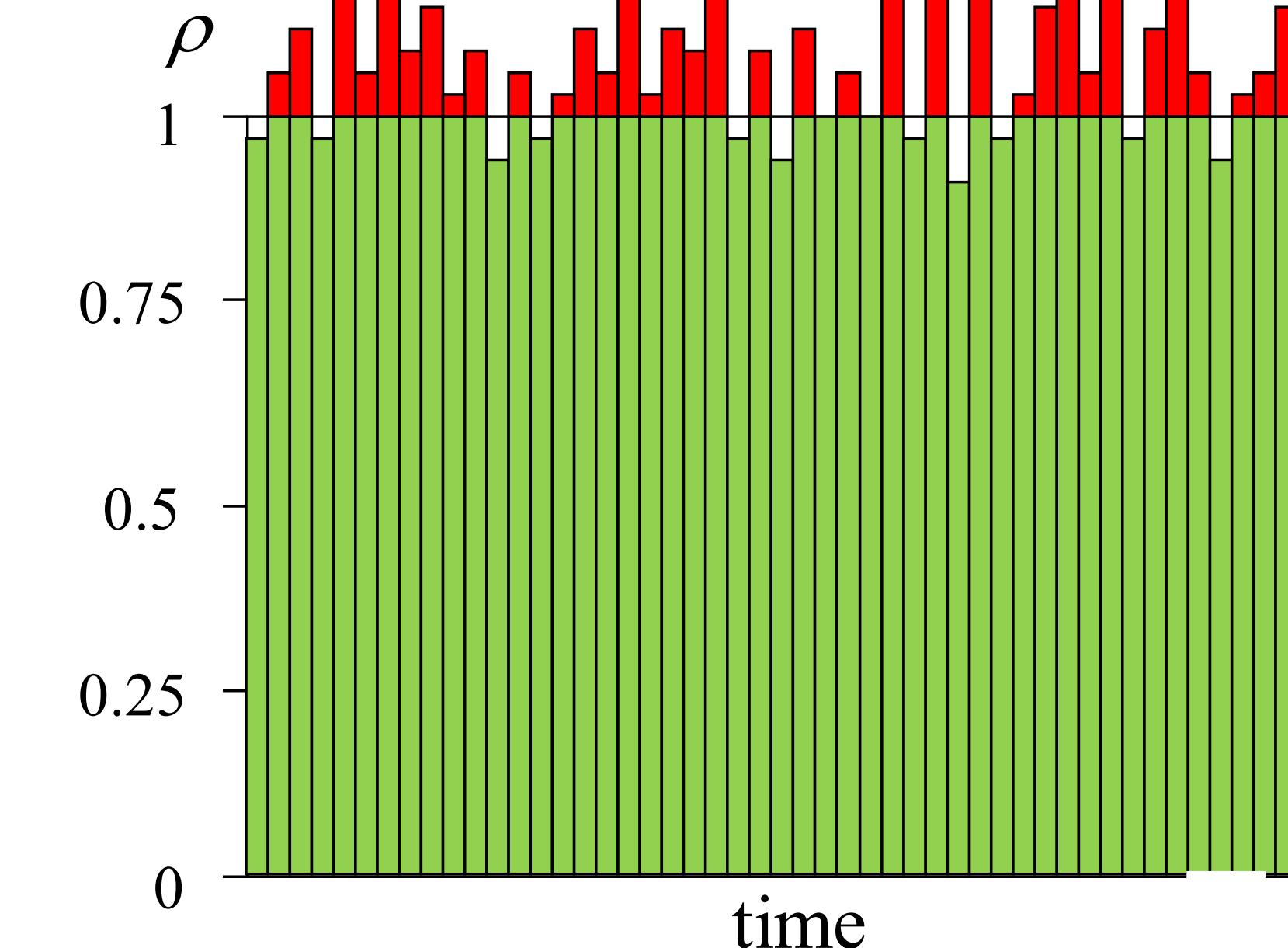
## Transient Overload

$\rho_{\max} > 1$  but  $\rho_{\text{avg}} \leq 1$



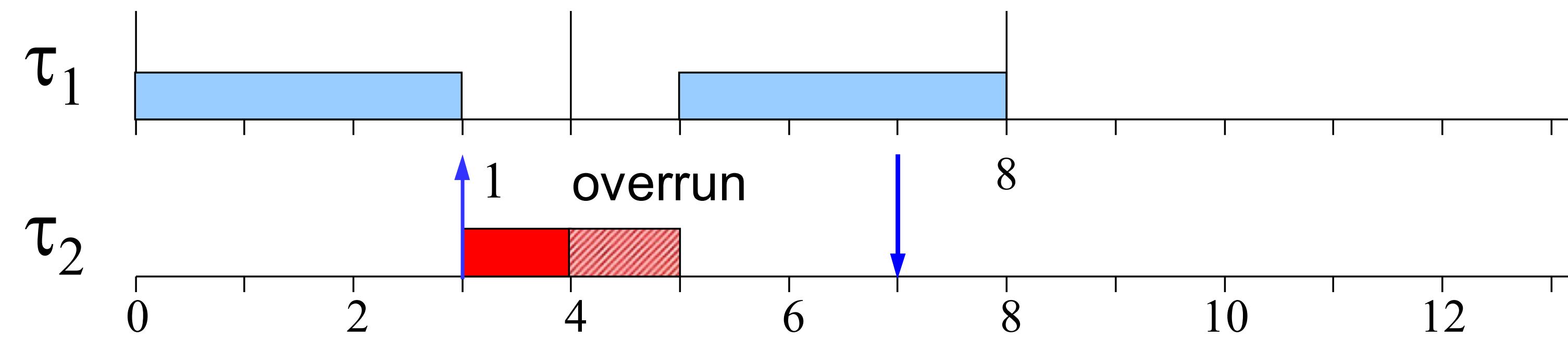
## Permanent Overload

$\rho_{\text{avg}} > 1$

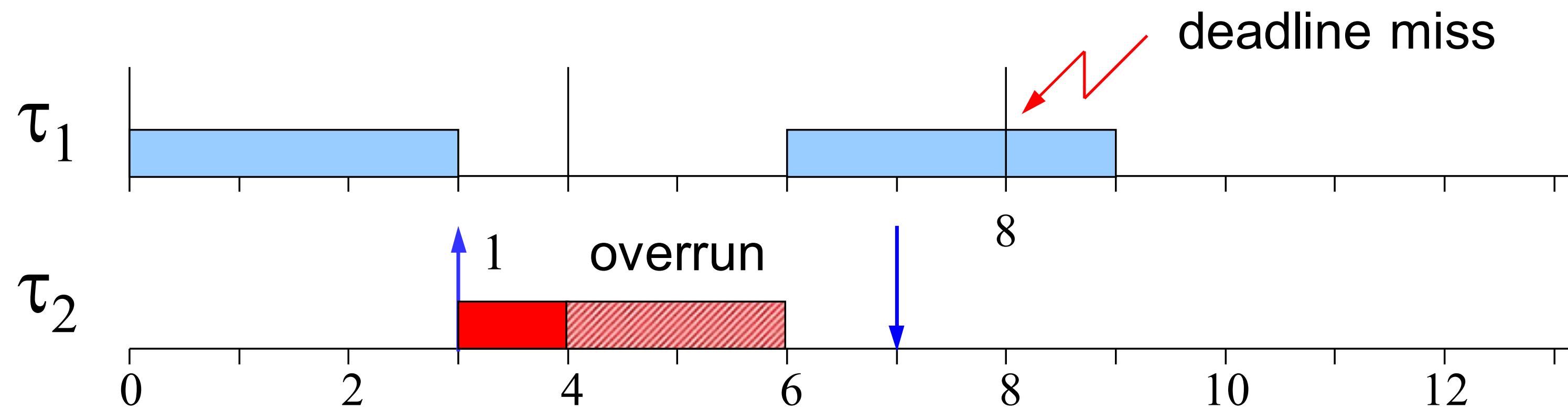


# Consequences of overruns

A task overrun may not cause an overload:

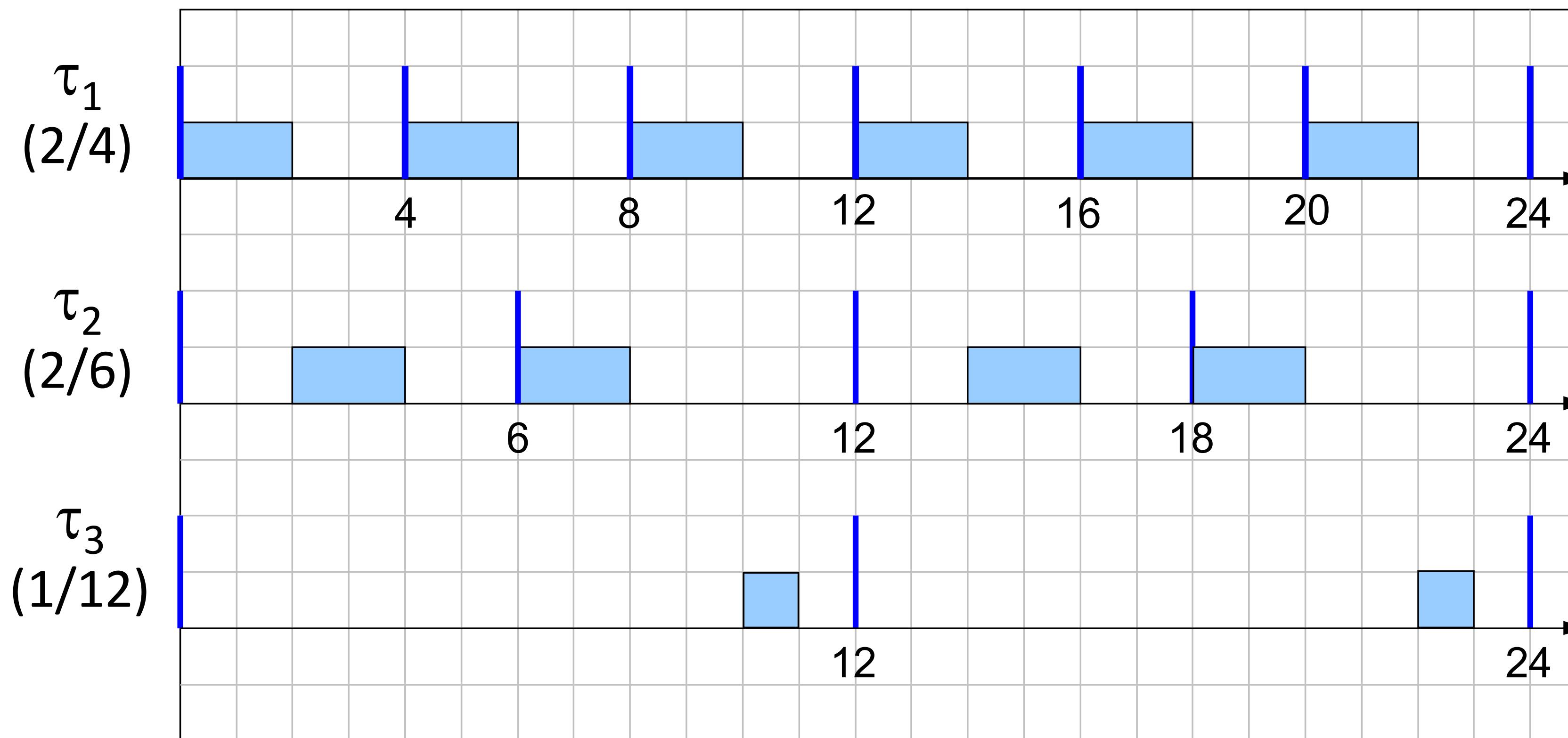


But in general may delay the execution of other tasks, causing a deadline miss:



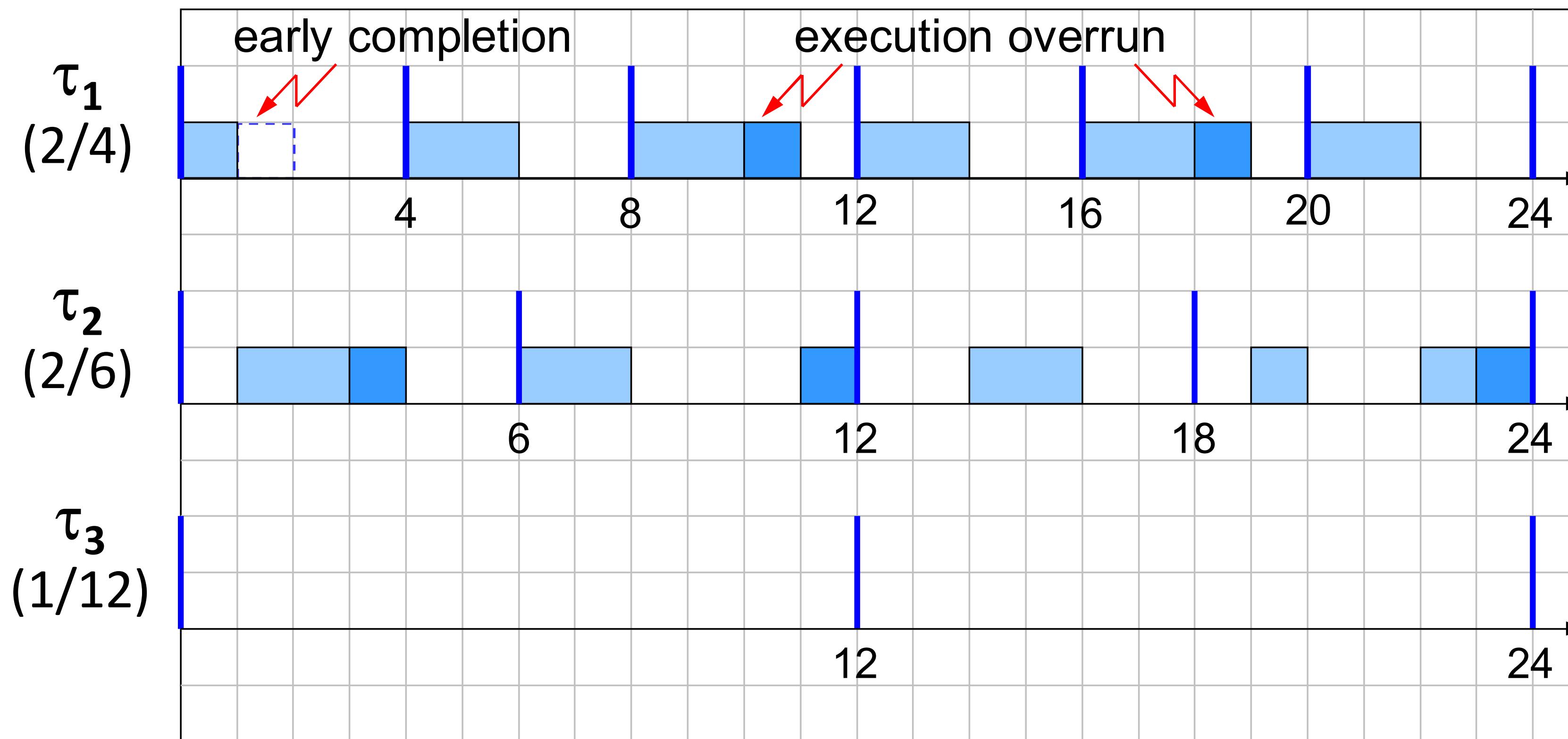
# Consequences of overruns

Consider the following applications with  $U < 1$ :

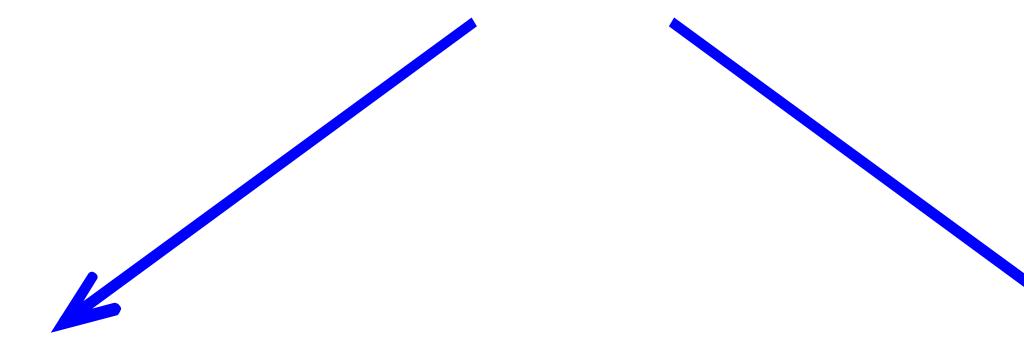


# Consequences of overruns

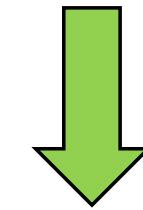
$U < 1$  but sporadic overruns can prevent  $\tau_3$  to run:



# Load control methods



**Overrun handling**



Resource Reservation

**Overload handling**



{  
  Reactive vs. Proactive  
  Local vs. Global

# Resource Reservation Methods

# Providing temporal isolation

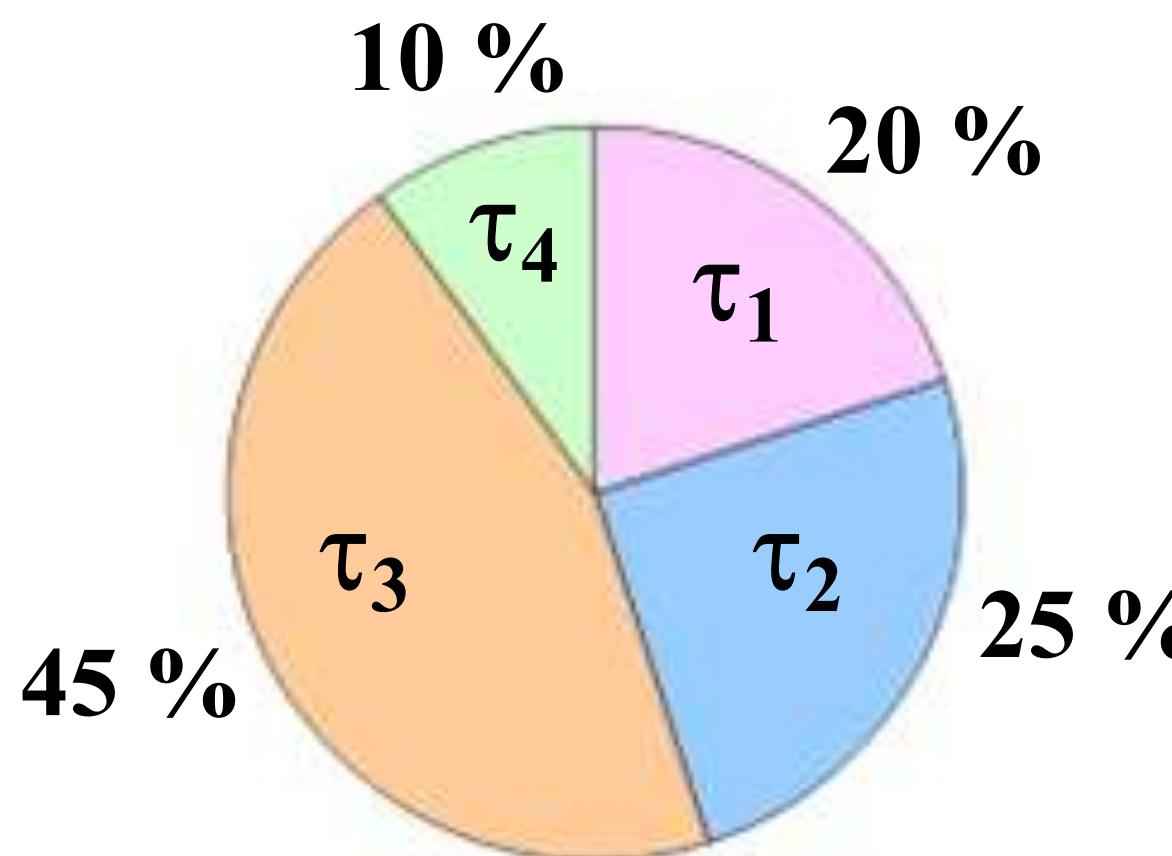
Transient overloads can be efficiently handled through **Resource Reservation**: an operating system mechanism to bound the resource consumption of tasks in order to limit the interference caused to other tasks.

To implement Resource Reservation we have to:

1. reserve a fraction of the processor bandwidth to the tasks we want to keep under load control;
2. prevent the such tasks to use more bandwidth than the reserved fraction.

# Resource Reservation

## Resource partition



Each task receives a bandwidth  $\alpha_i$  and behaves as it were executing alone on a slower processor of speed  $\alpha_i$

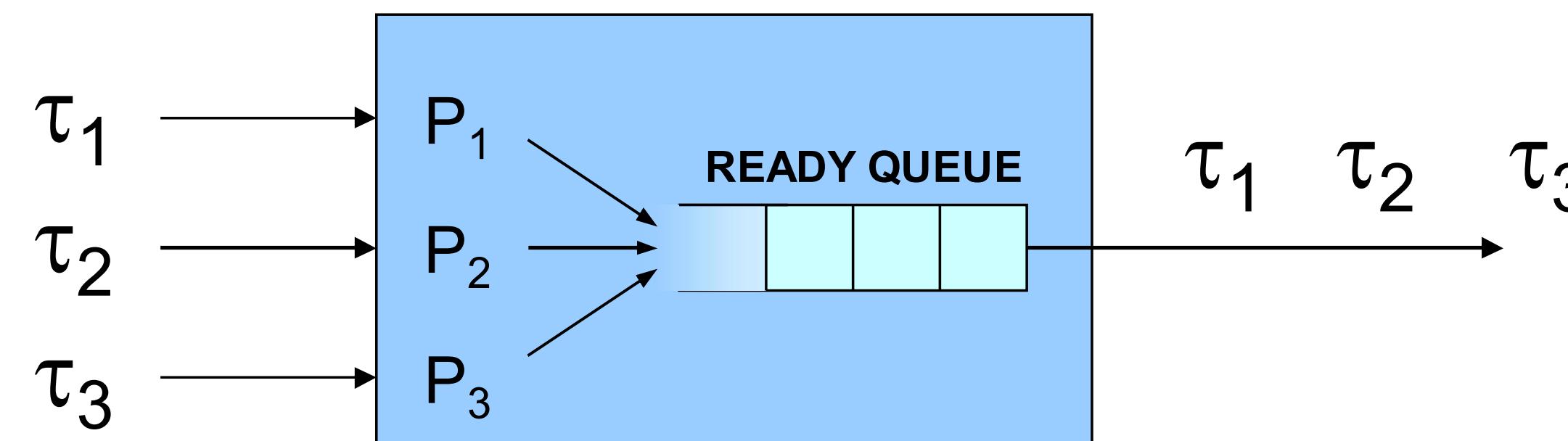
## Resource enforcement



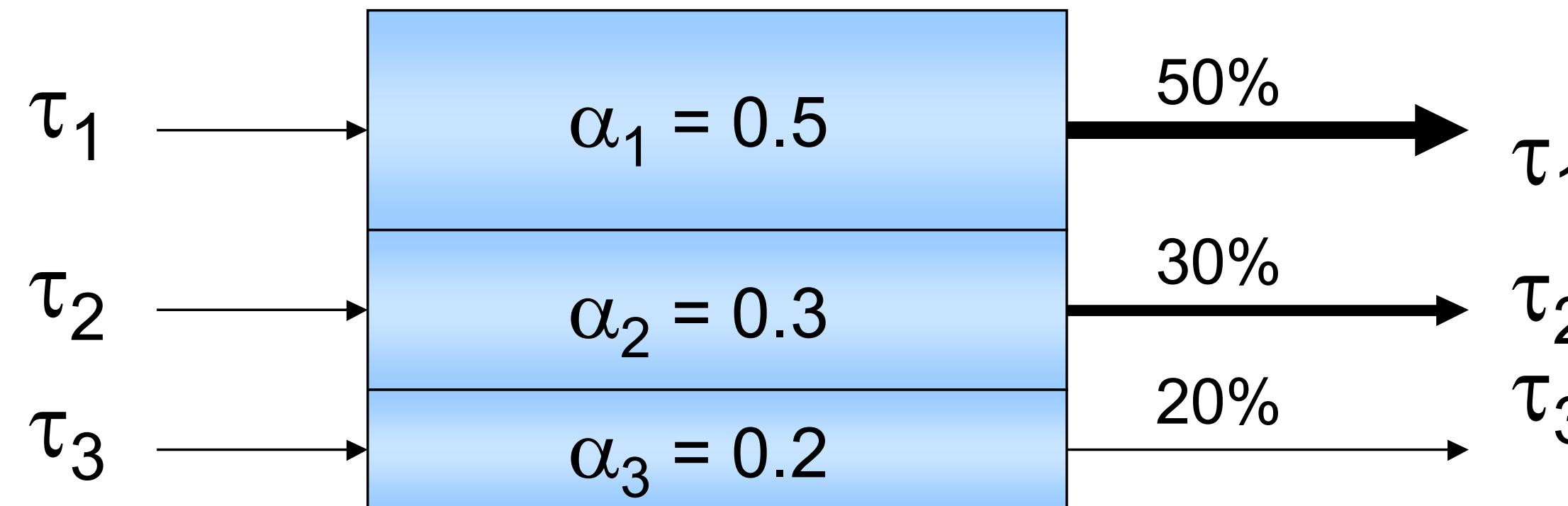
- A mechanism that prevents a task to consume more than its reserved amount.
- If a task executes more, it is delayed, preserving the resource for other tasks.

# Priorities vs. Reservations

Prioritized Access

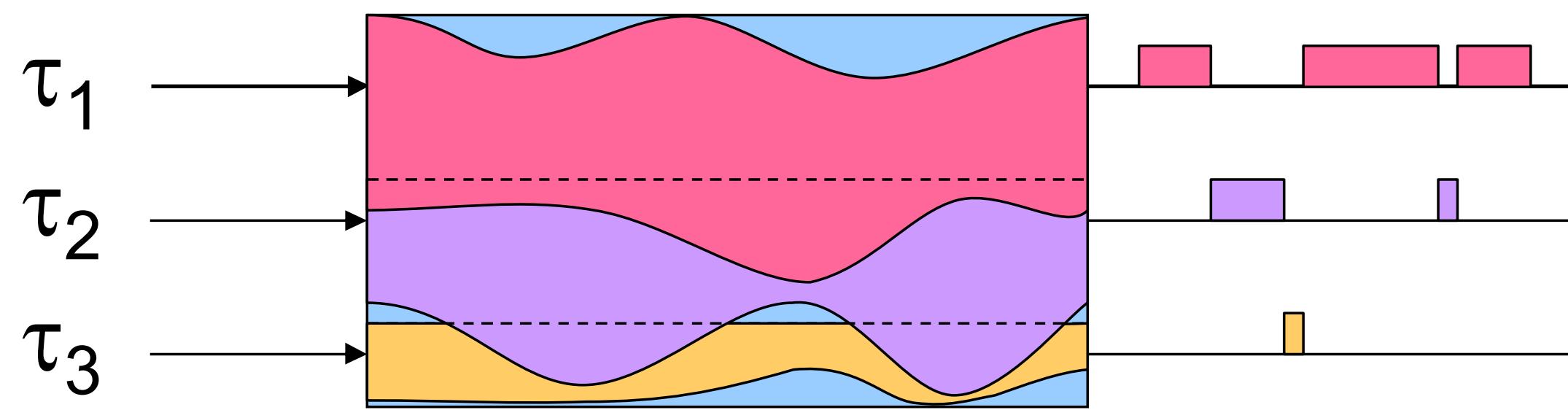


Resource Reservation

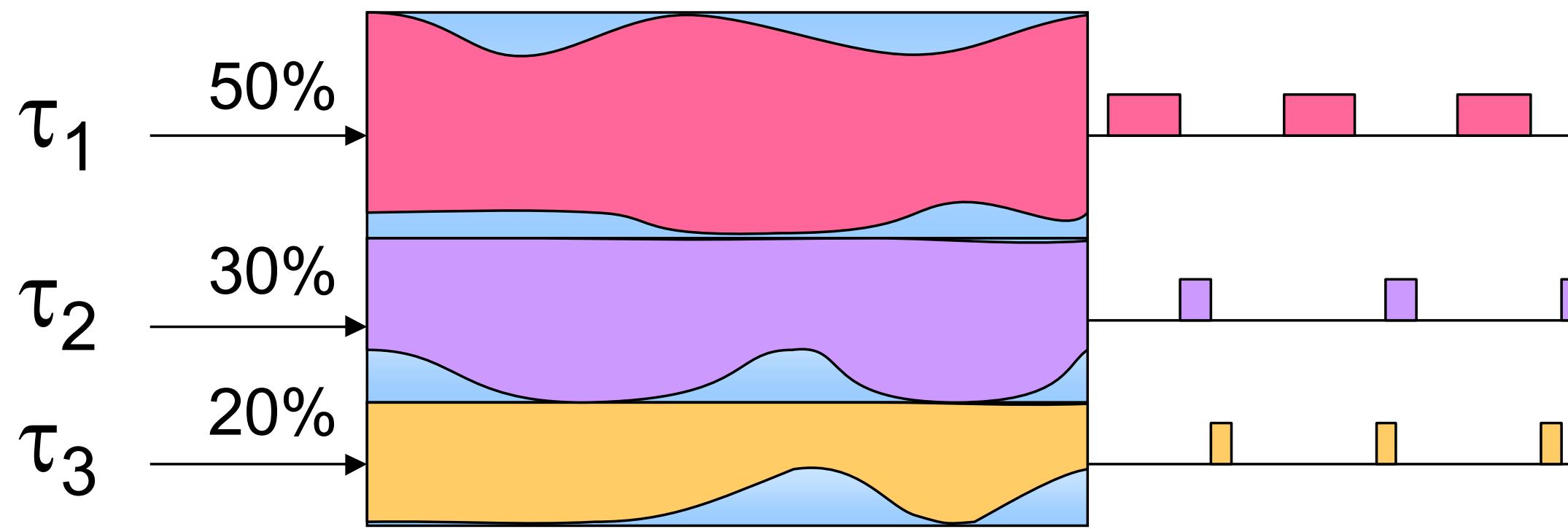


# Priorities vs. Reservations

Prioritized  
Access



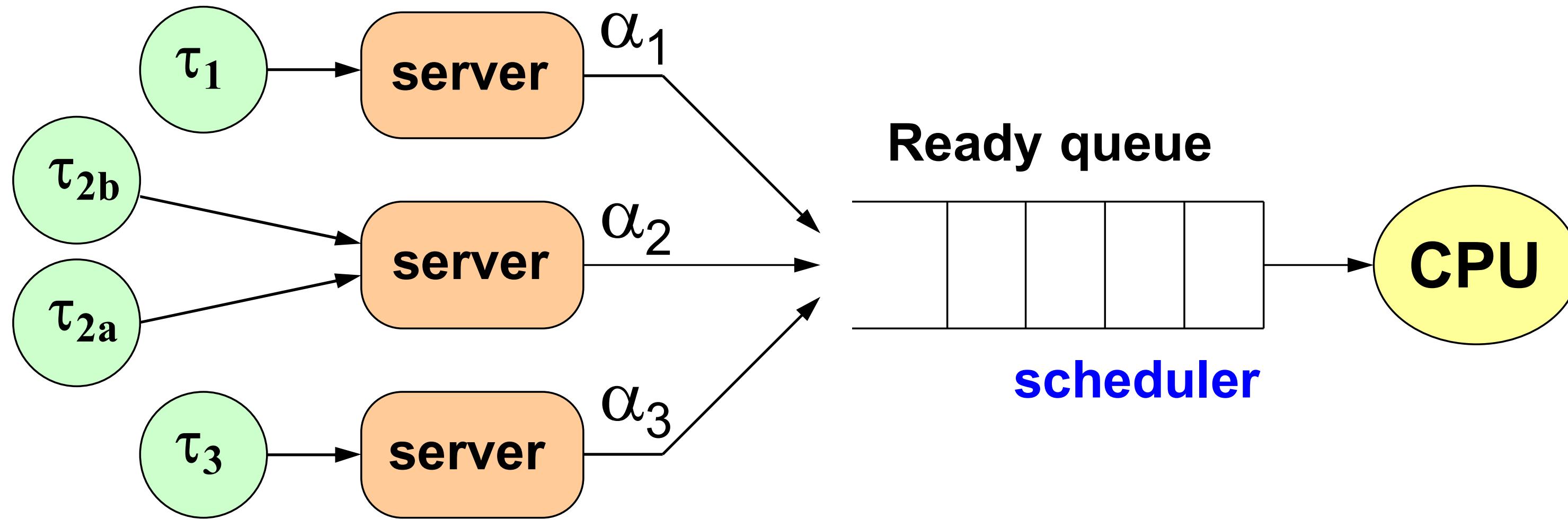
Resource  
Reservation



# Benefits of Resource Reservation

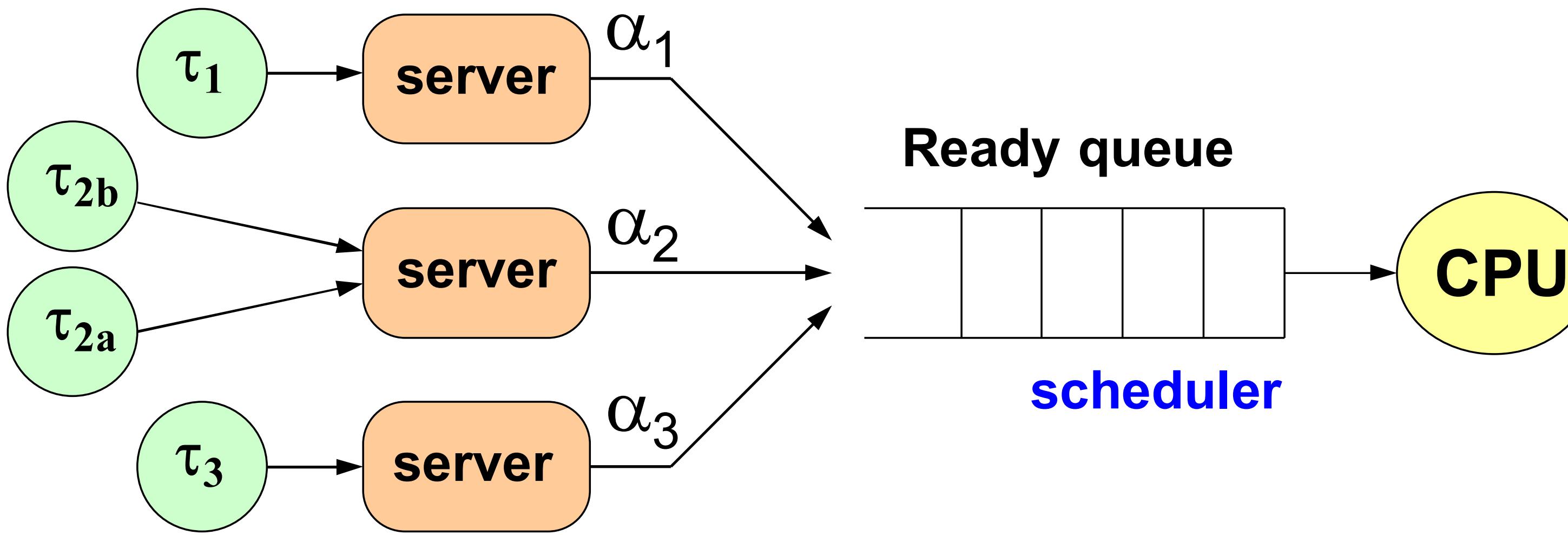
1. Resource allocation is easier than priority mapping.
2. It provides temporal isolation: overruns occurring in a reservation do not affect other tasks.
  - Important for modularity and scalability
3. Simpler schedulability analysis:
  - Response times only depends on the application demand and the amount of reserved resource.

# Implementing RR



	scheduler	server
Fixed priorities	<b>RM/DM</b>	<b>Sporadic Server</b>
Dynamic priorities	<b>EDF</b>	<b>CBS</b>

# Analysis under RR



If a processor is partitioned into  $n$  reservations, we must have that:

$$\sum_{i=1}^n \alpha_i \leq U_{\text{lub}}^A$$

where  $A$  is the adopted scheduling algorithm.

# Types of reservations

**HARD:** when the budget is exhausted, the server is blocked until the next budget replenishment.

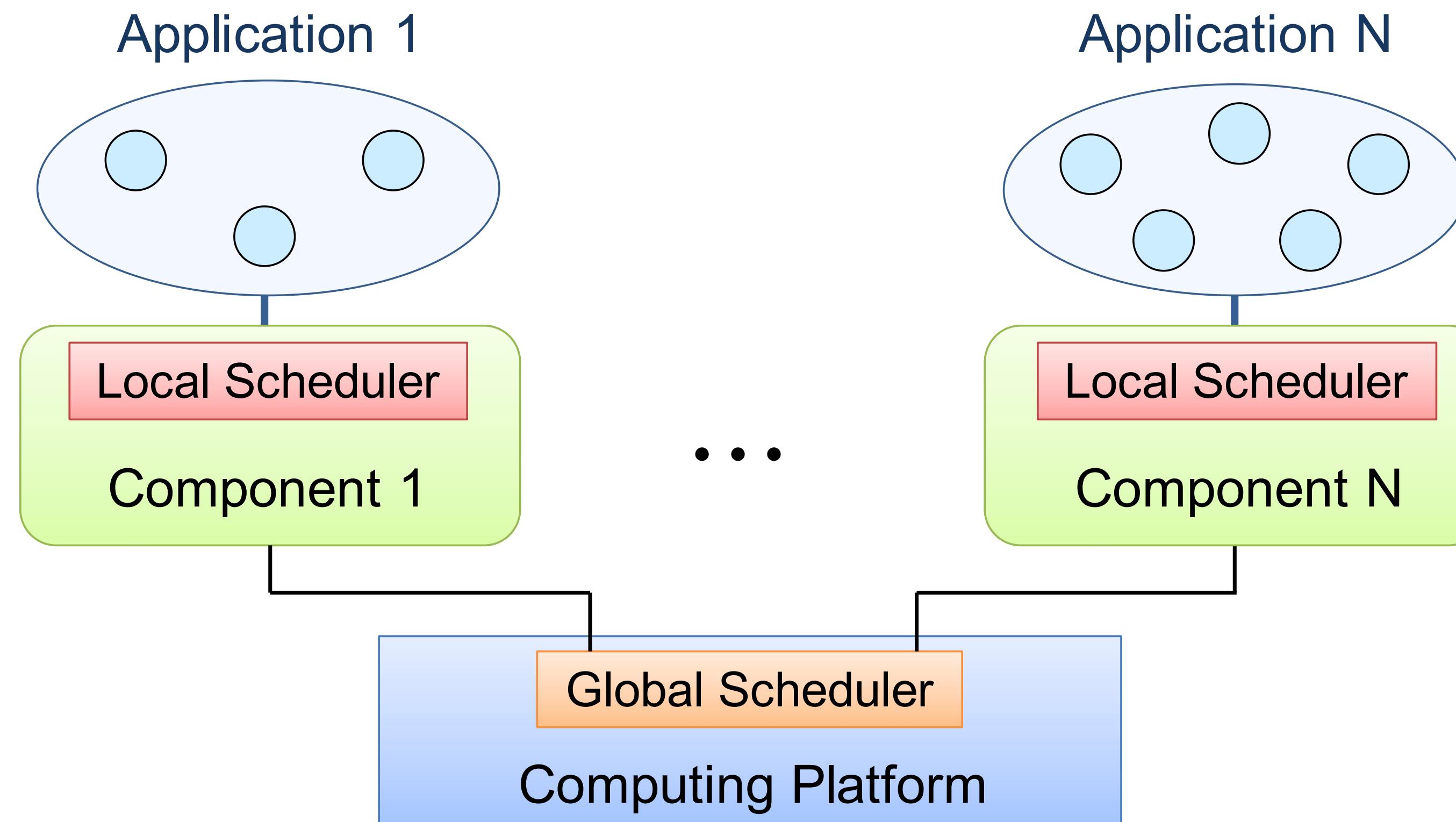
**SOFT:** when the budget is exhausted, the server remains active but at a lower priority level.

## Note that

- a HARD reservation  $(Q_s, P_s)$  guarantees at most a budget  $Q_s$  every period  $P_s$
- a SOFT reservation  $(Q_s, P_s)$  guarantees at least a budget  $Q_s$  every period  $P_s$

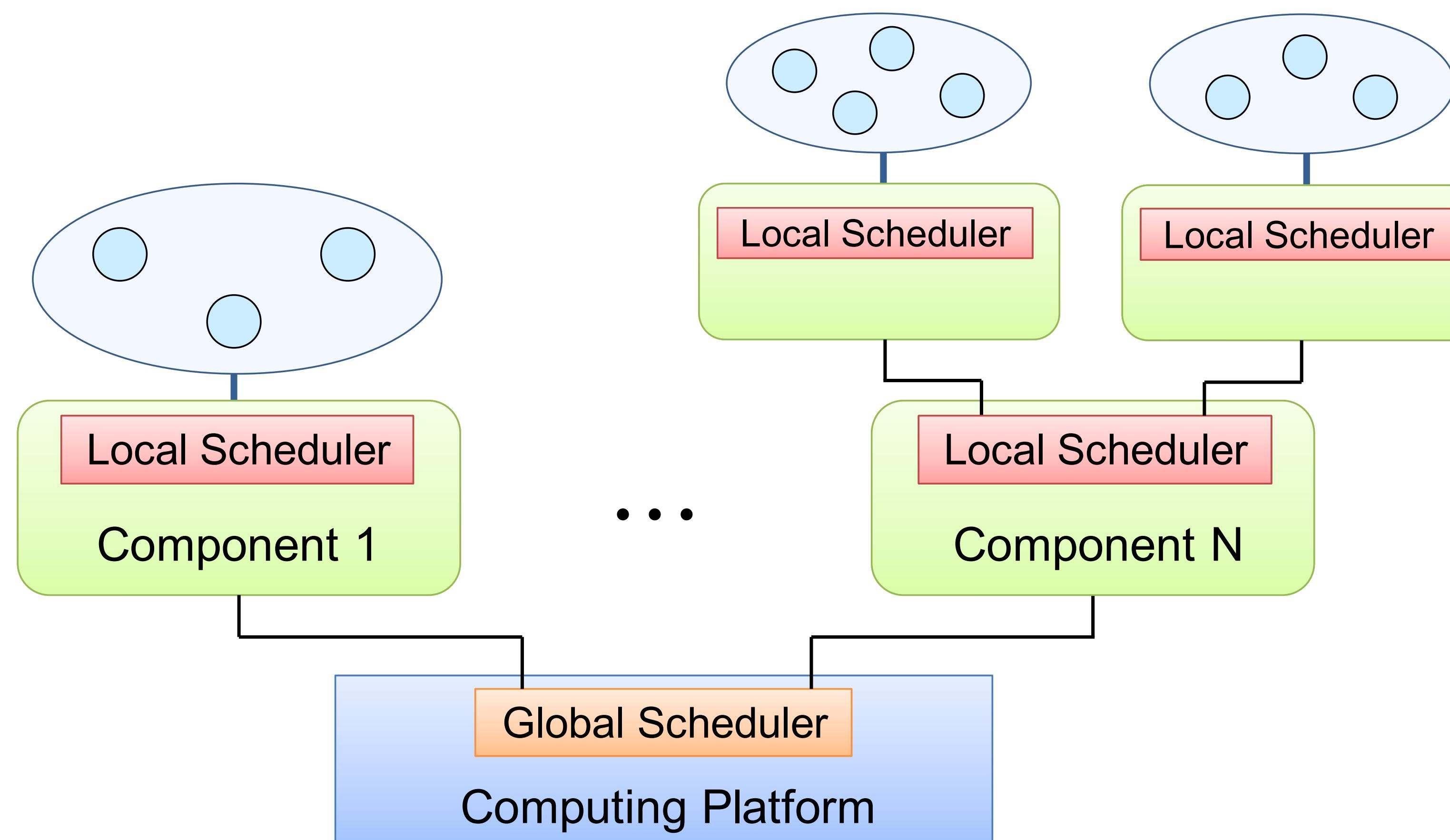
# Hierarchical scheduling

Resource reservation can be used to develop hierarchical systems, where each component is implemented within a reservation:



# Hierarchical scheduling

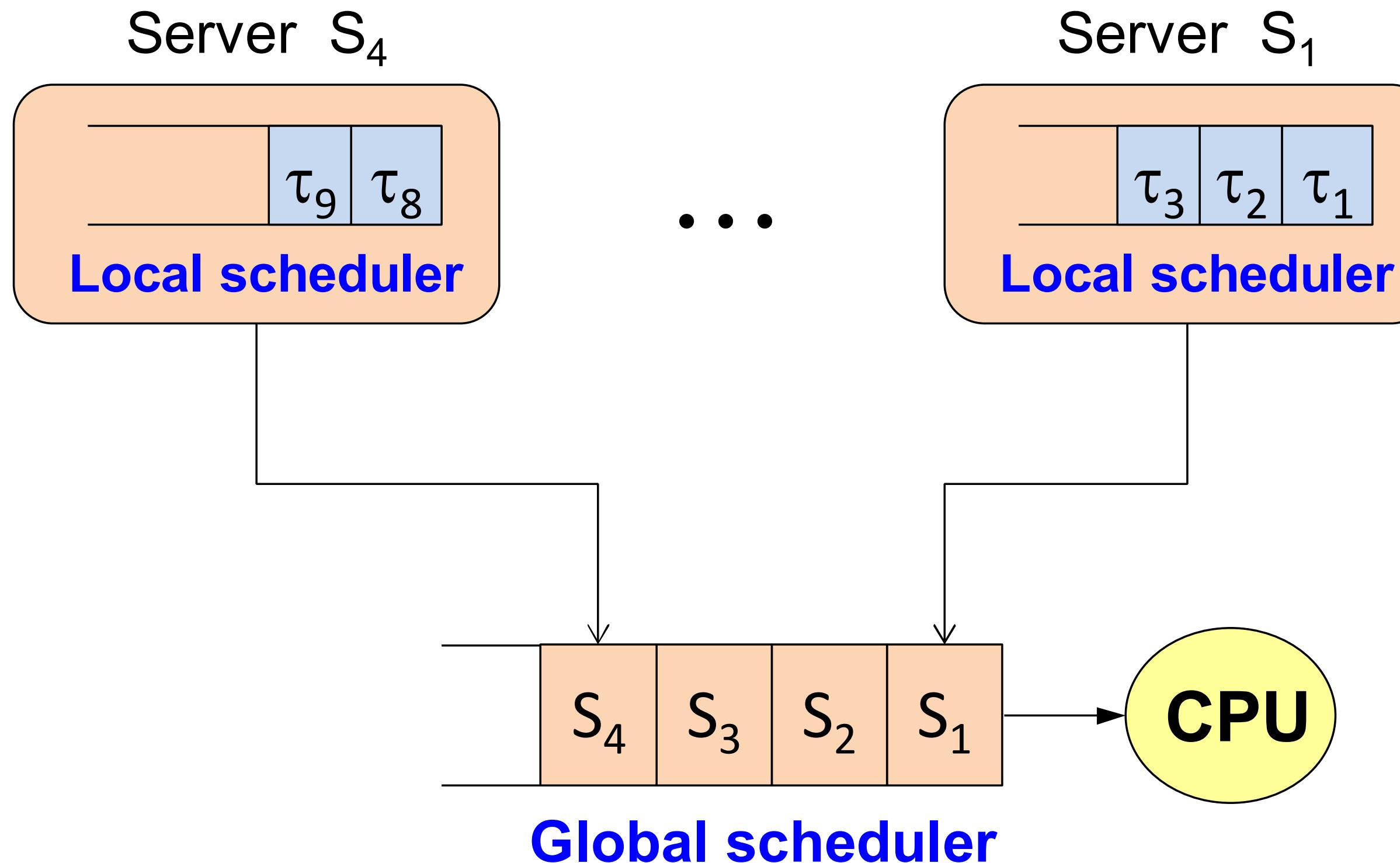
In general, a component can also be divided in other sub-components



# Hierarchical scheduling

**Global scheduler:** the one managing the system ready queue

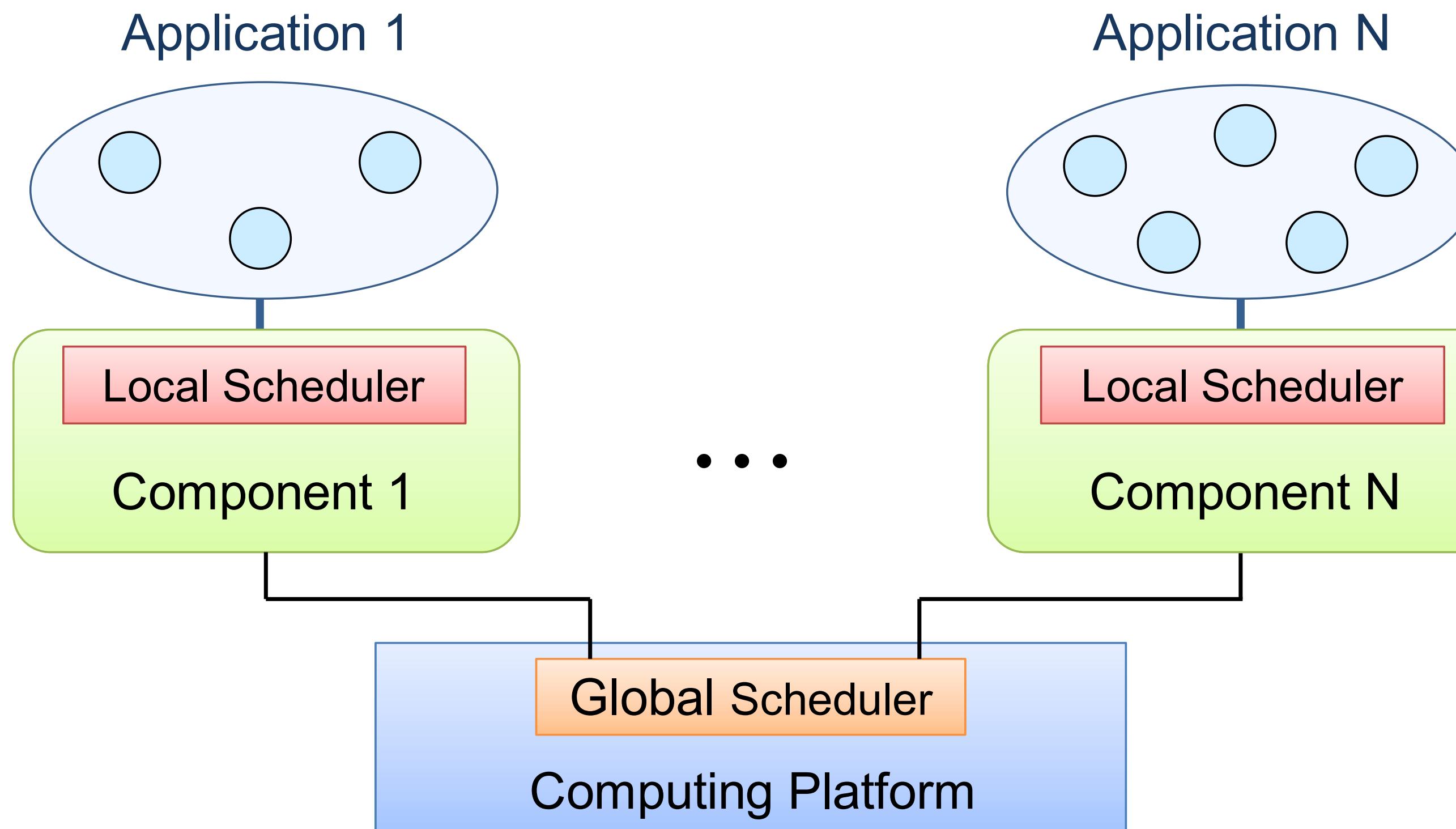
**Local schedulers:** the ones managing the servers queues



# Hierarchical Analysis

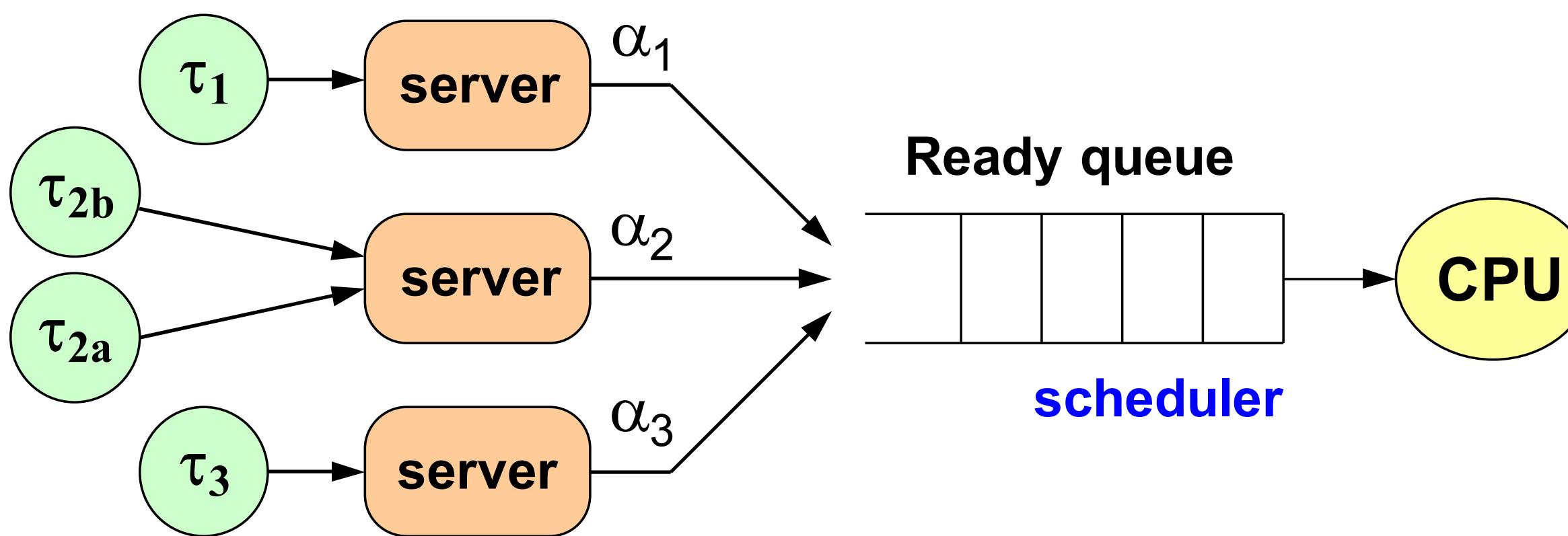
**Global analysis:** Servers must be schedulable by the global scheduler running on the physical platform.

**Local analysis:** Applications must be scheduled by the local schedulers running on the components.



# Global Analysis

**Global analysis:** Servers must be schedulable by the global scheduler running on the physical platform.



If a processor is partitioned into  $n$  reservations, we must have that:

$$\sum_{i=1}^n \alpha_i \leq U_{\text{lub}}^A$$

where  $A$  is the adopted scheduling algorithm.

# Local Analysis under RR

**Processor  
Demand  
Criterion**  
(under EDF)

$$\forall t \in D \quad dbf(t) \leq t$$

The diagram shows two red-bordered boxes at the bottom. The left box contains "Timing behavior of the application". The right box contains "processing time available in  $[0, t]$ ". Two red arrows point from these boxes up to the red circles containing  $dbf(t)$  and  $t$  respectively in the main equation.

**Workload  
Analysis**  
(under fixed  
priorities):

$$\forall i = 1, \dots, n \quad \exists t \in (0, D_i] : W_i(t) \leq t$$

The diagram shows two red-bordered boxes at the bottom. The left box contains " $\forall i = 1, \dots, n$ ". The right box contains " $\exists t \in (0, D_i] : W_i(t) \leq t$ ". Two red arrows point from these boxes up to the red circles containing  $W_i(t)$  and  $t$  respectively in the main equation.

# Local Analysis under RR

Under EDF, the analysis of an application within a reservation is done through the Processor Demand Criterion:

$$\forall t > 0, \quad dbf(t) \leq t$$

Under Fixed Priority Systems (FPS), the analysis is done through the Workload Analysis:

$$\forall i = 1, \dots, n \quad \exists \ t \in (0, D_i] : W_i(t) \leq t$$

The difference is that in an interval of length  $t$  the processor is only partially available.

# Local Analysis under RR

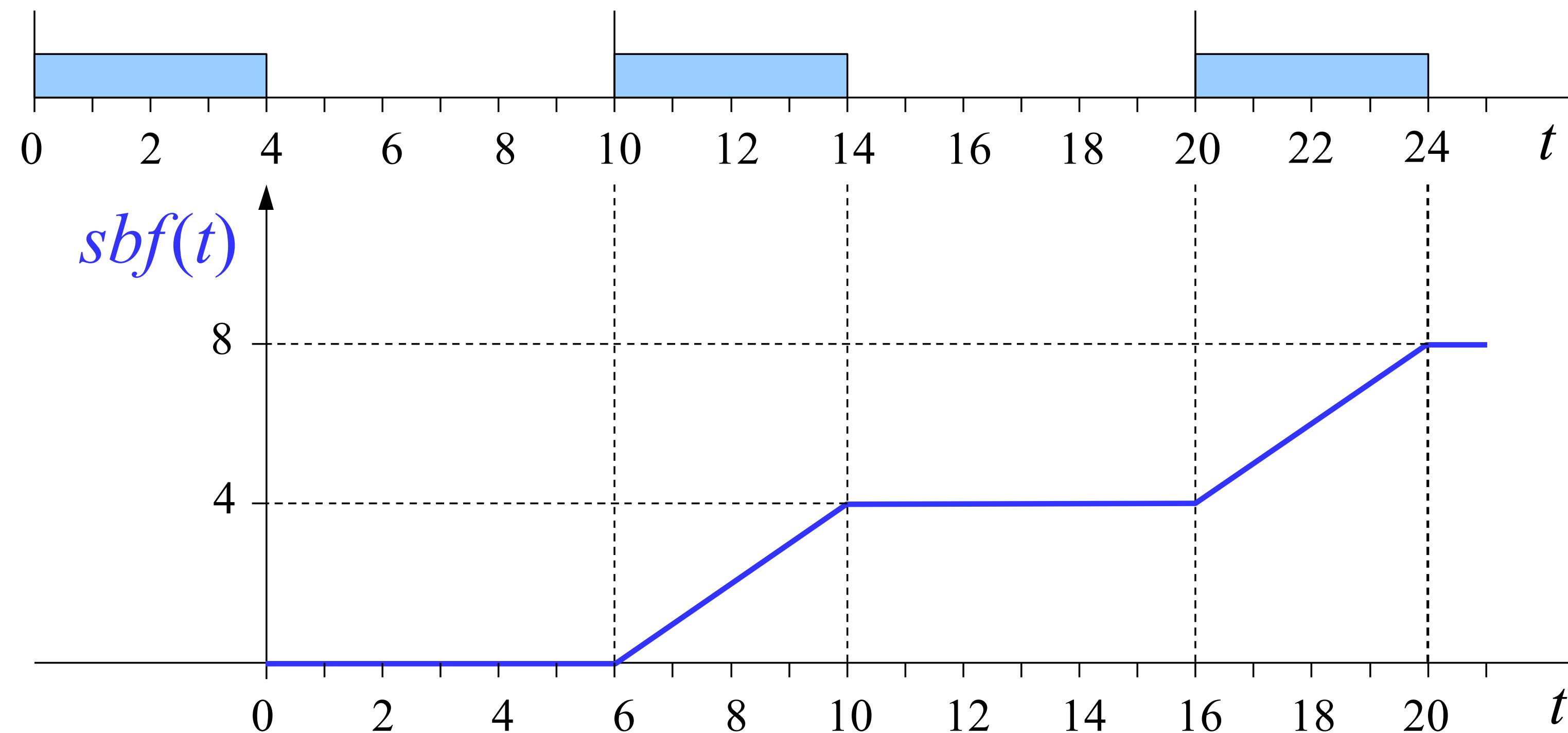
To describe the time available in a reservation, we need to identify, for any interval  $[0, t]$ , the minimum time allocated in the worst-case situation.

***Supply bound function sbf(t):***

*minimum amount of time available in reservation  $R_k$  in every time interval of length  $t$ .*

# Example: Static time partition

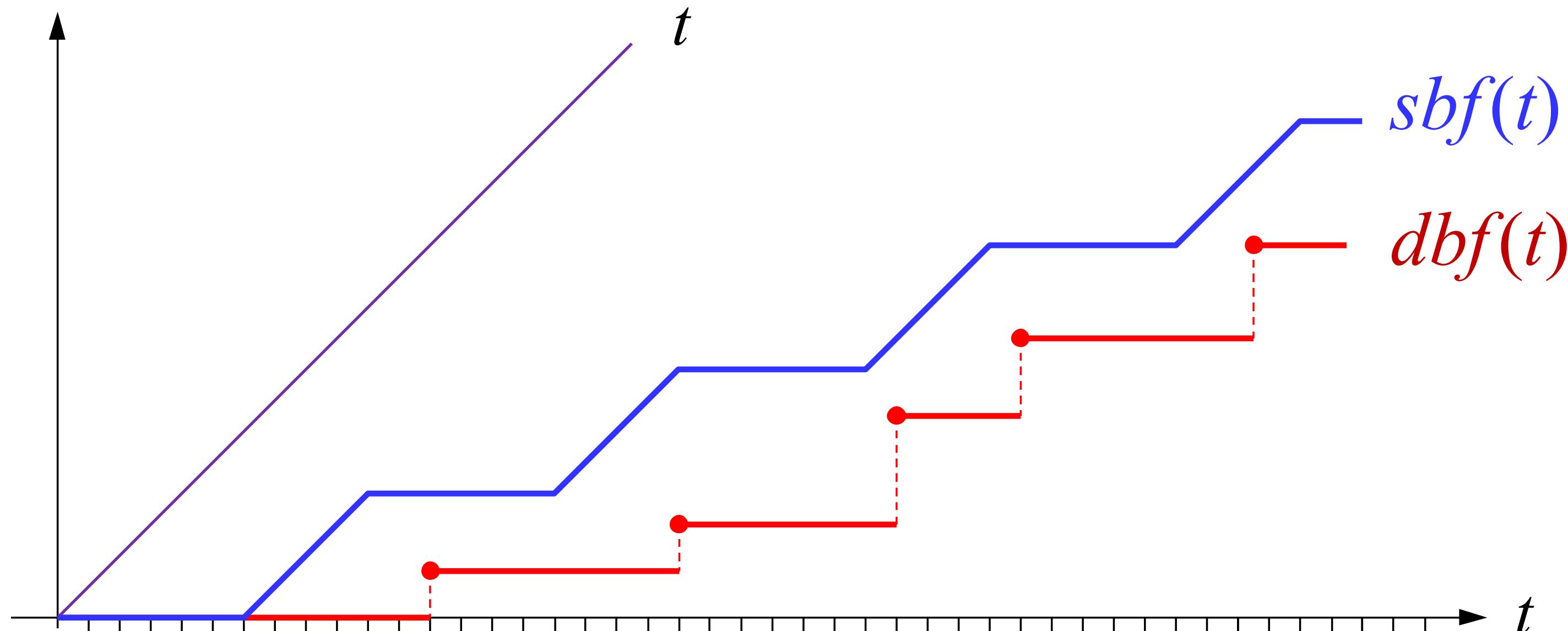
Example of reservation providing 4 units of time every 10  
(bandwidth = 0.4):



# Analysis under RR

Hence the Processor Demand Criterion can be reformulated as follows:

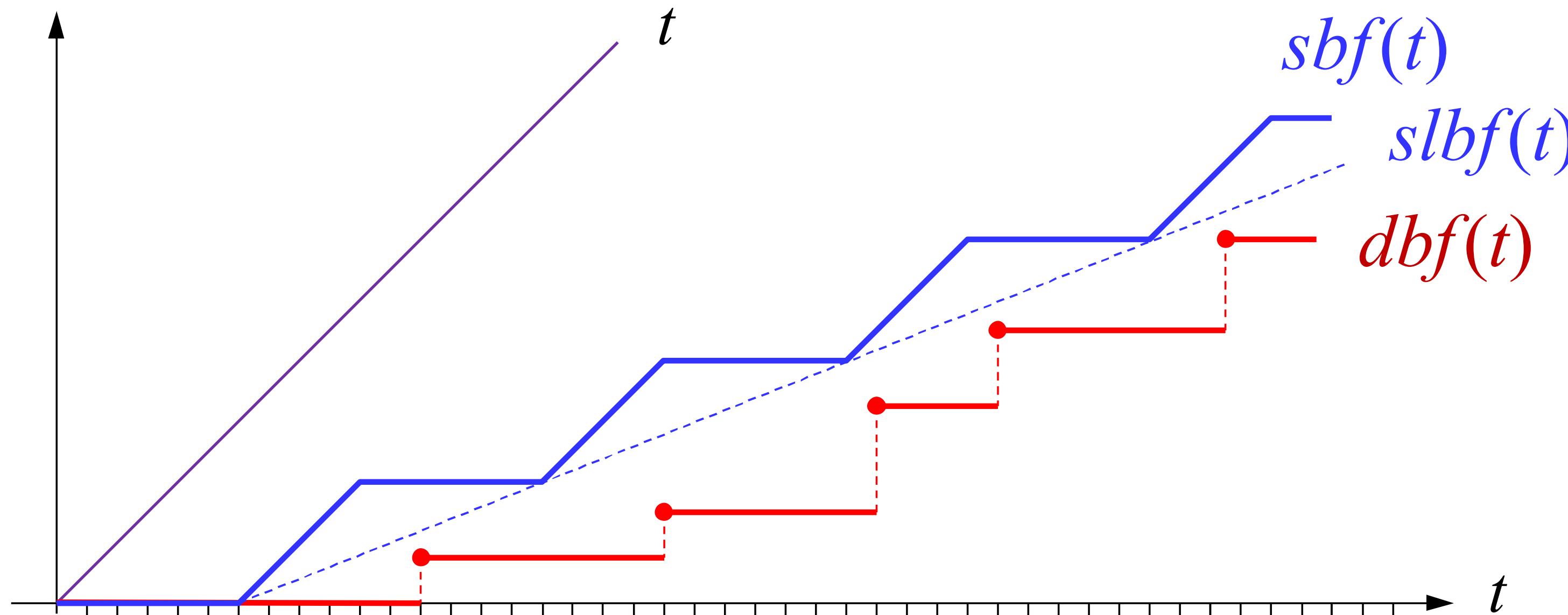
$$\forall t > 0, \quad dbf(t) \leq sbf(t)$$



# Local Analysis under RR

A simpler sufficient test, can be done by replacing  $sbf(t)$  with a lower bound, called supply lower-bound function  $slbf(t)$ :

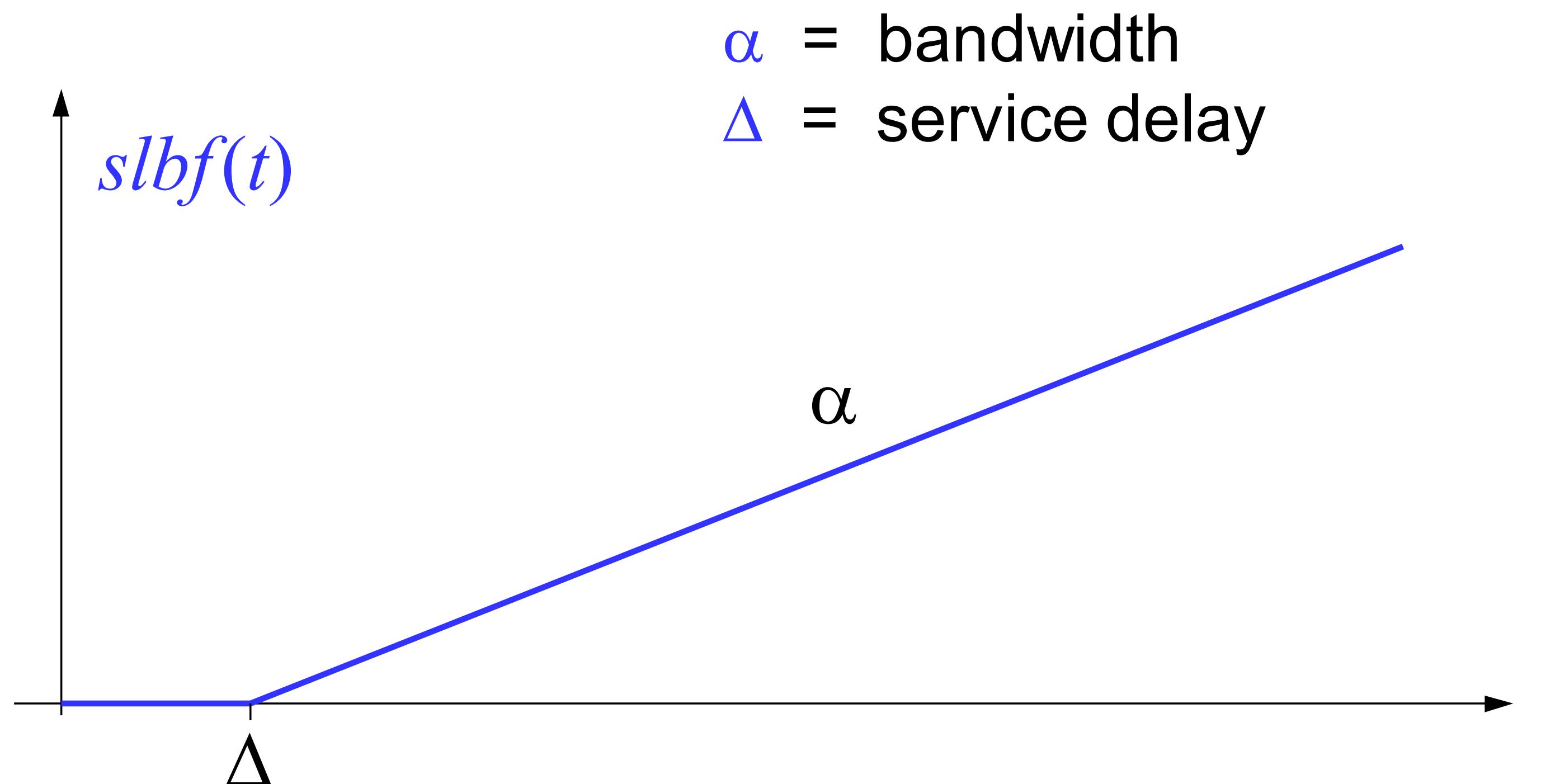
$$\forall t > 0, \quad dbf(t) \leq slbf(t)$$



# Supply lower-bound function

A supply bound function has the following form:

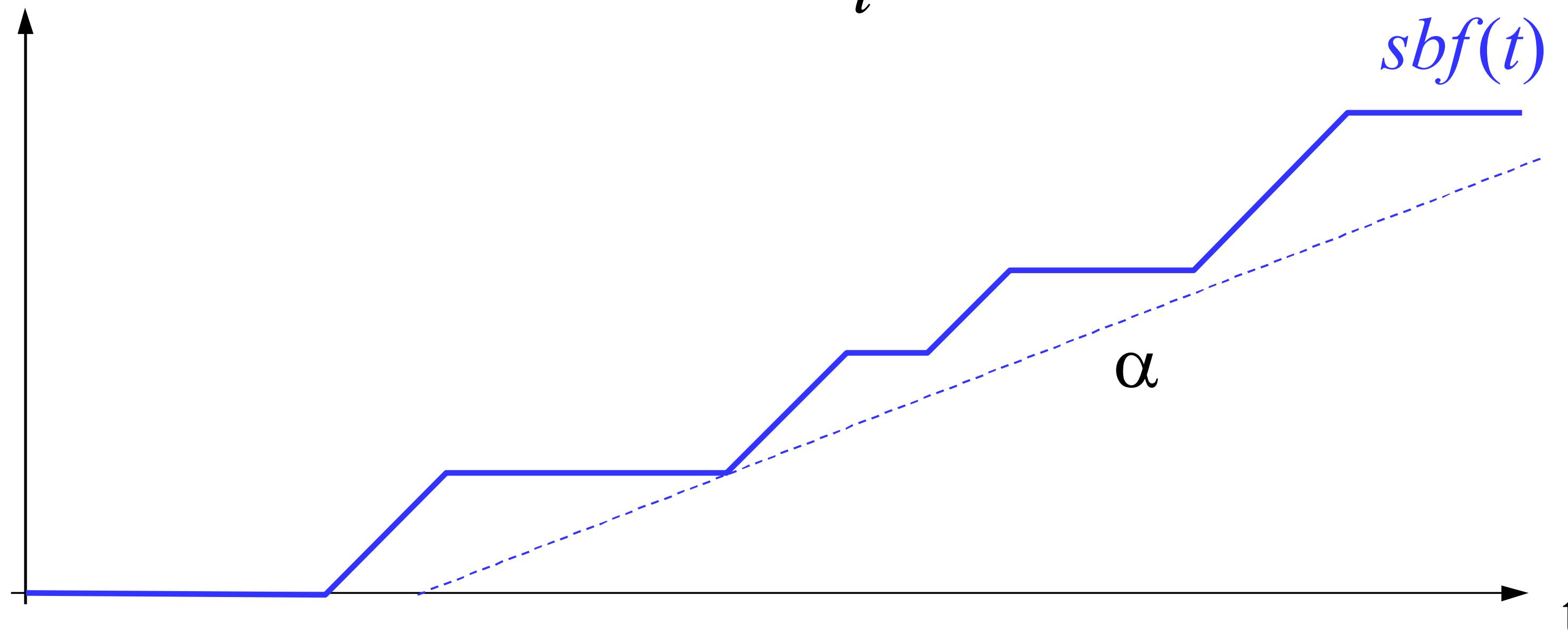
$$slbf(t) = \max \{0, \alpha(t - \Delta)\}$$



# Deriving $\alpha$ and $\Delta$

Given a generic supply function  $sbf(t)$ , the bandwidth  $\alpha$  is the equivalent slope computed for long intervals:

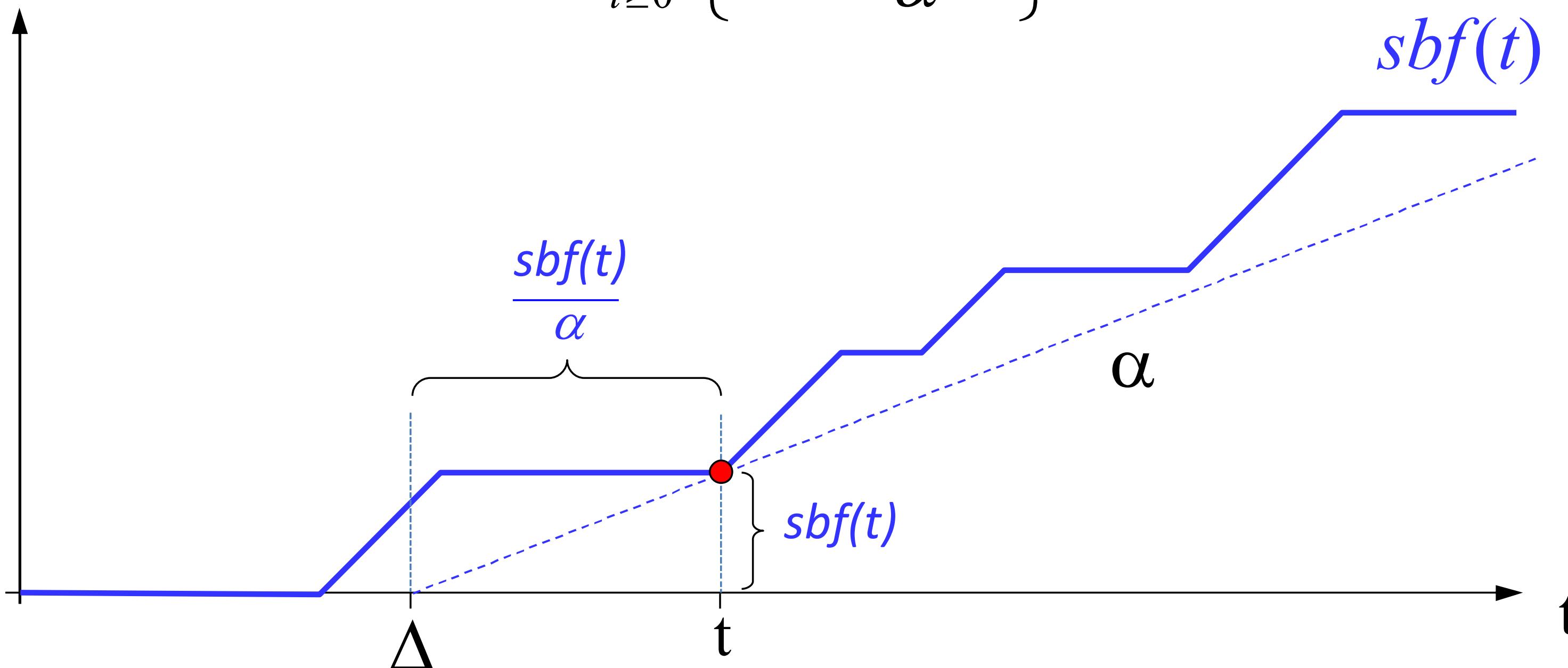
$$\alpha = \lim_{t \rightarrow \infty} \frac{sbf(t)}{t}$$



# Deriving $\alpha$ and $\Delta$

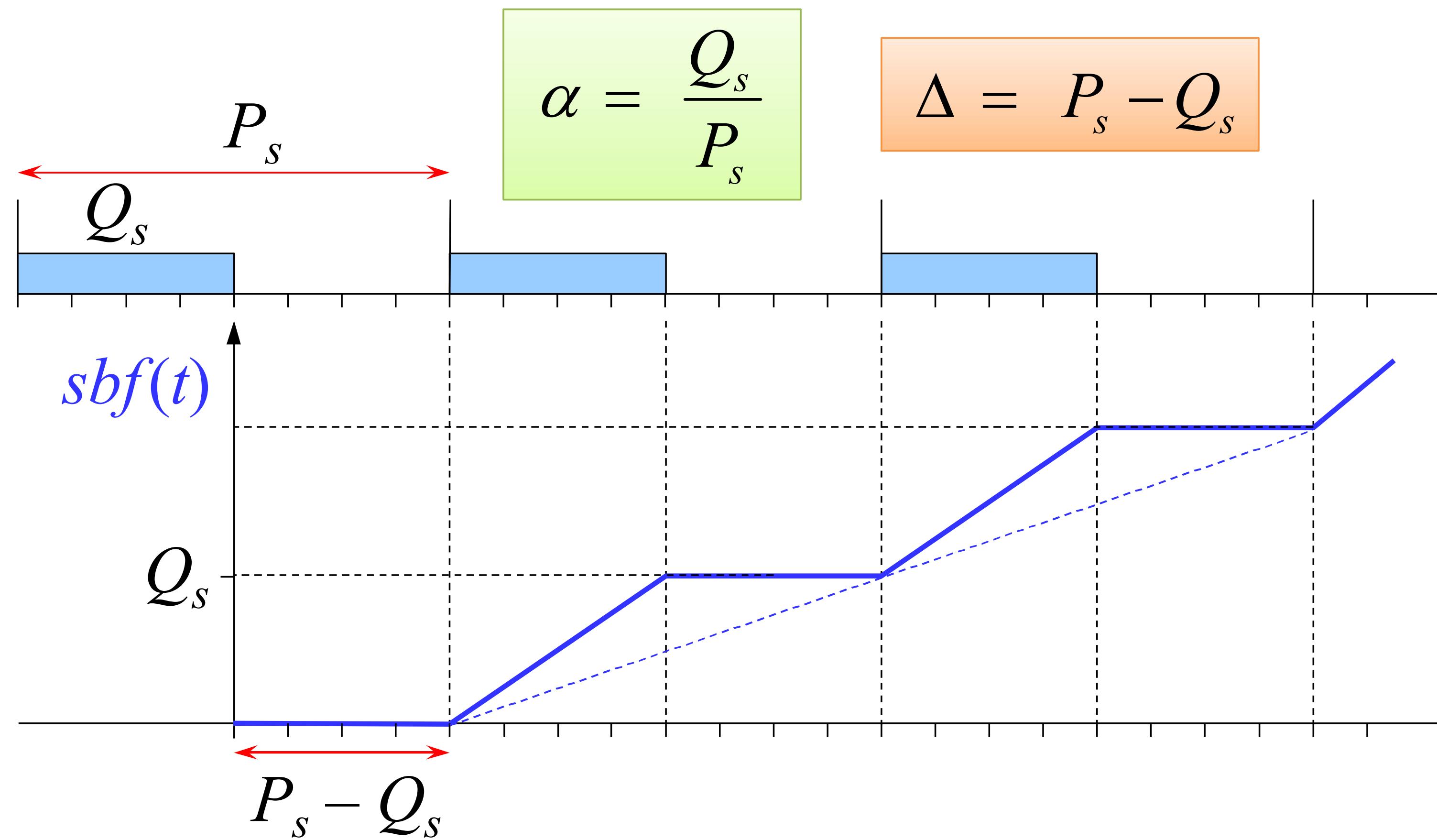
While the delay  $\Delta$  is the highest intersection with the time axis of the line of slope  $\alpha$  touching the  $sbf(t)$ :

$$\Delta = \sup_{t \geq 0} \left\{ t - \frac{sbf(t)}{\alpha} \right\}$$



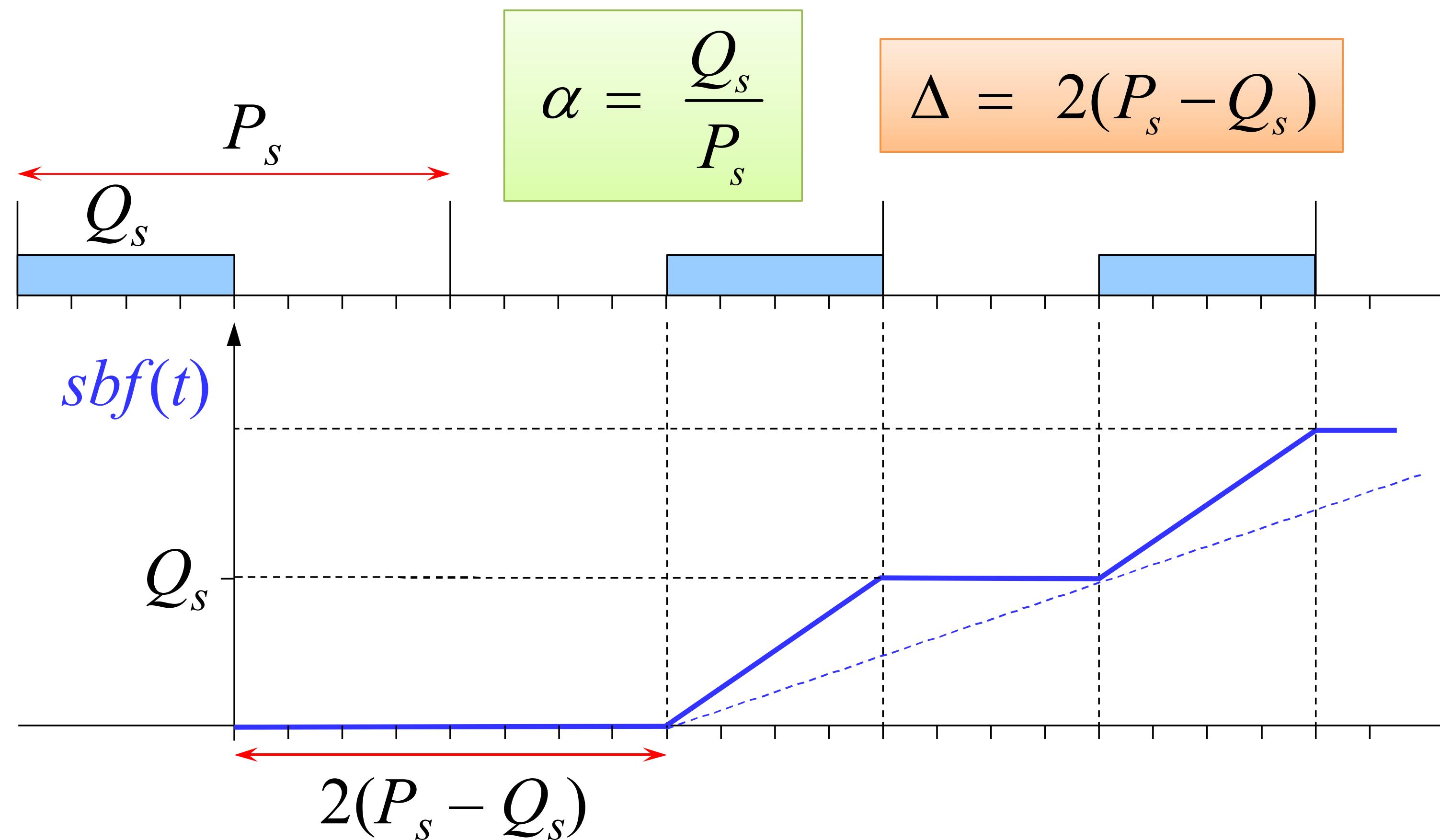
# Example: Periodic Server

For a periodic server with budget  $Q_s$  and period  $P_s$  running at the highest priority, we have:



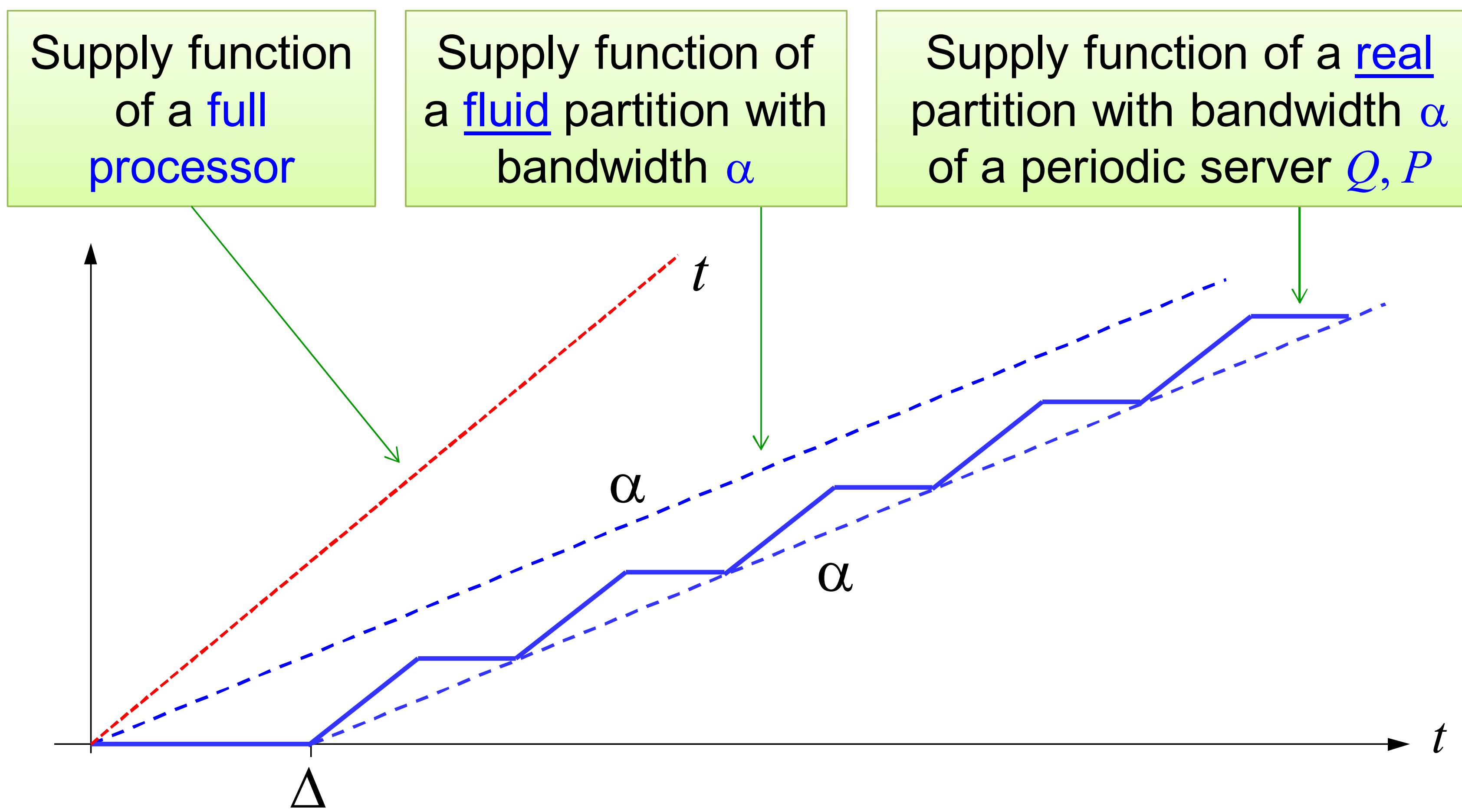
# Example: Periodic Server

For a periodic server with budget  $Q_s$  and period  $P_s$  running at unknown priority, we have:



# Observation

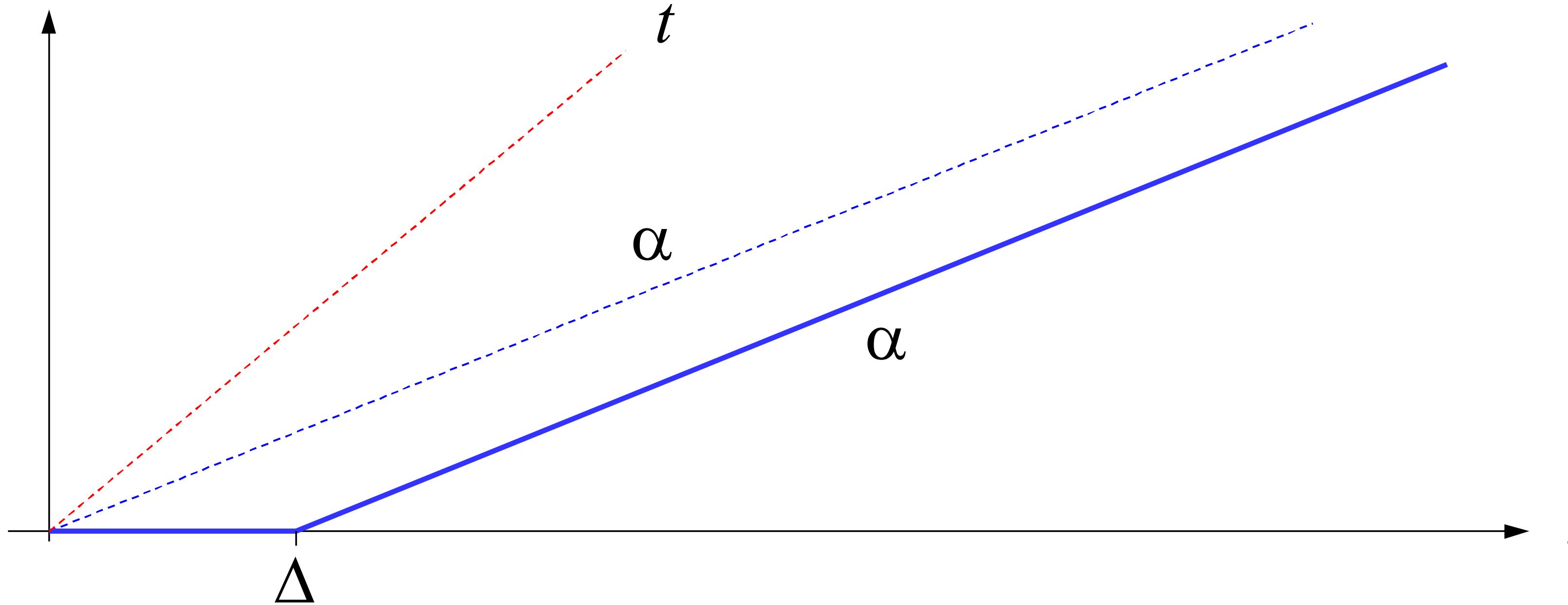
It is worth comparing the following supply functions:



# Observation

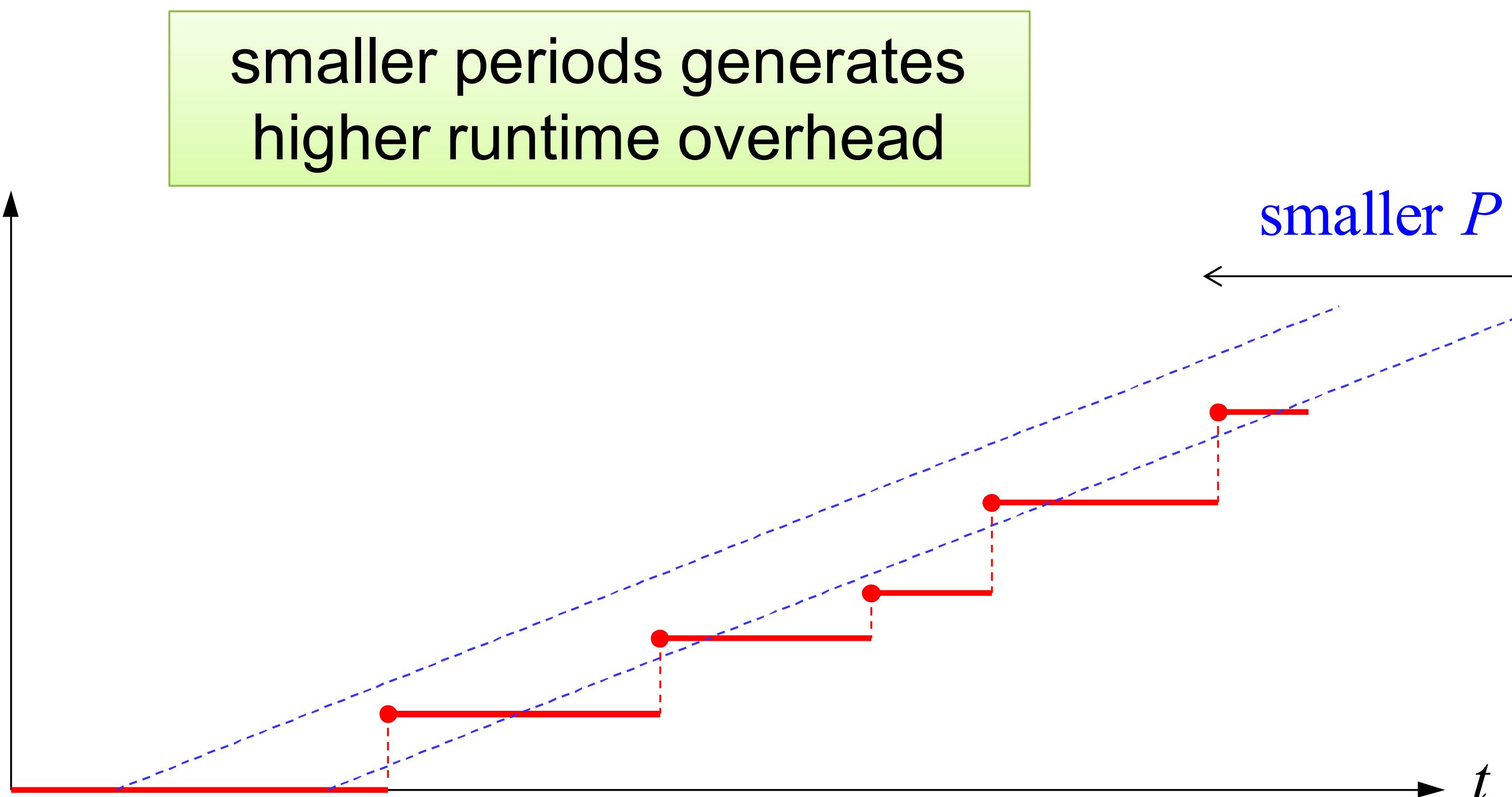
In a periodic server with bandwidth  $\alpha$ , we have that:

$$\left\{ \begin{array}{l} Q = \alpha P \\ \Delta = 2(P - Q) = 2P(1 - \alpha) \end{array} \right. \rightarrow \text{The delay } \Delta \text{ is proportional to the server period } P$$



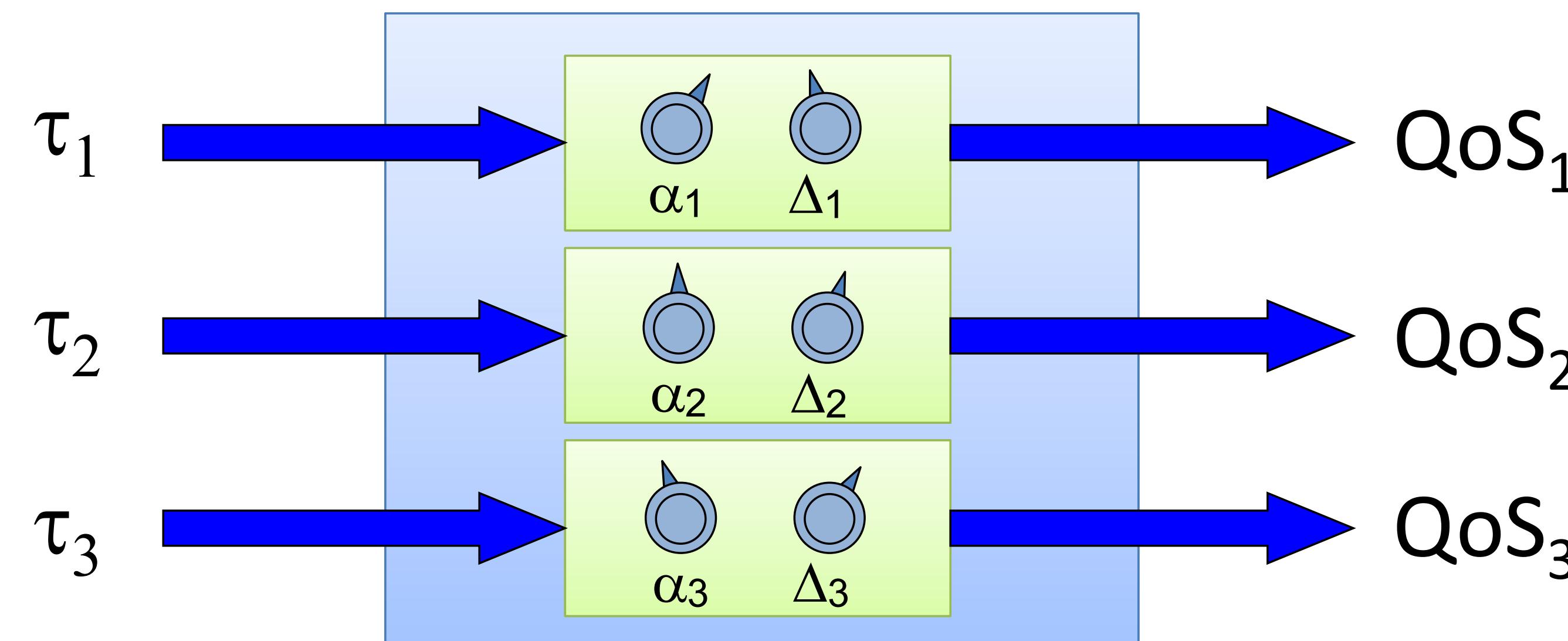
# Observation

Note that, for a given bandwidth  $\alpha$ , reducing  $P$  reduces the delay  $\Delta$  and improves schedulability, tending to a fluid reservation, **but ...**



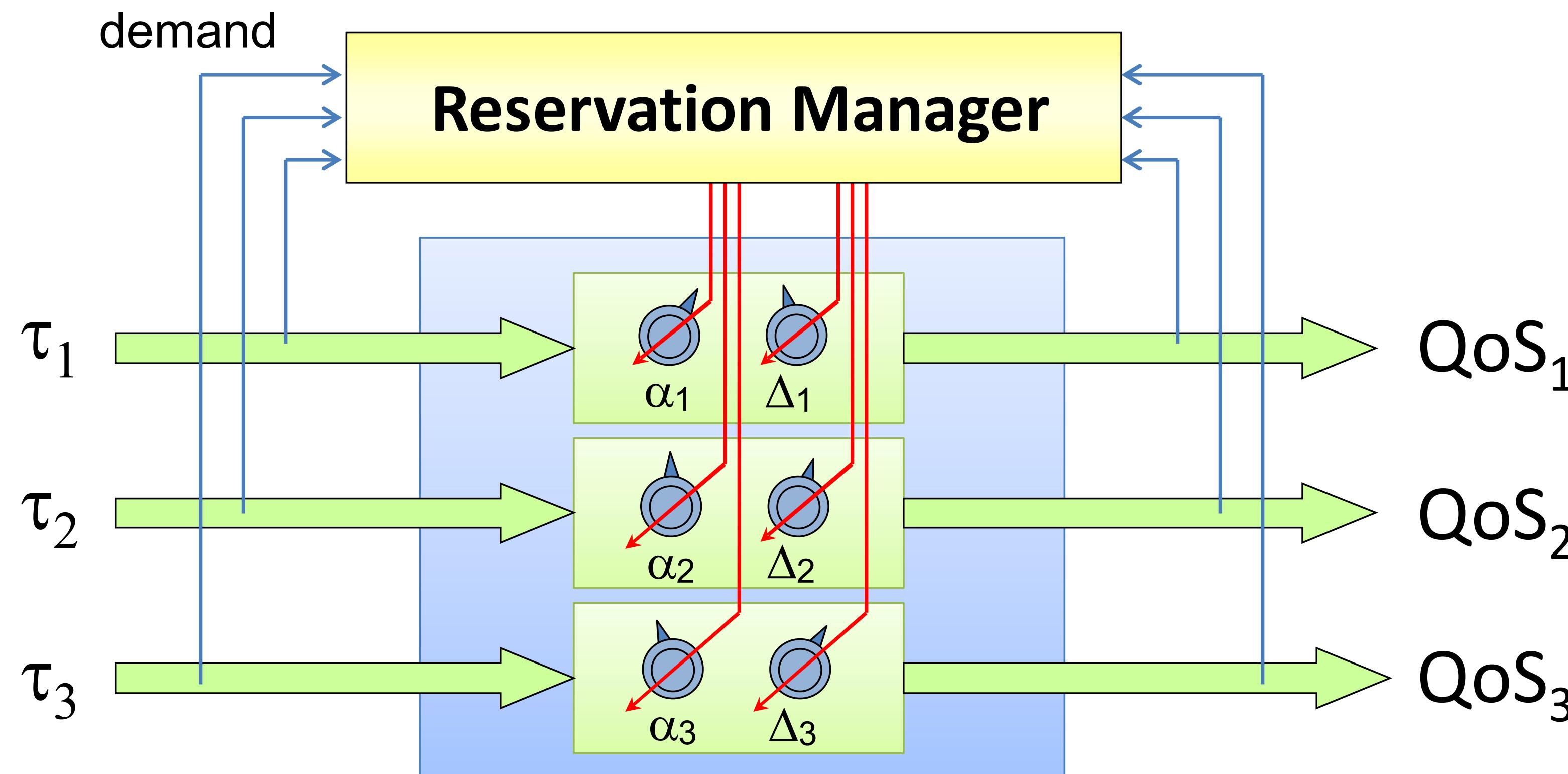
# Reservation interface

Note that the  $(\alpha, \Delta)$  parameters offer an alternative interface, which is independent of the implementation mechanism (static partitions or Q-P server):



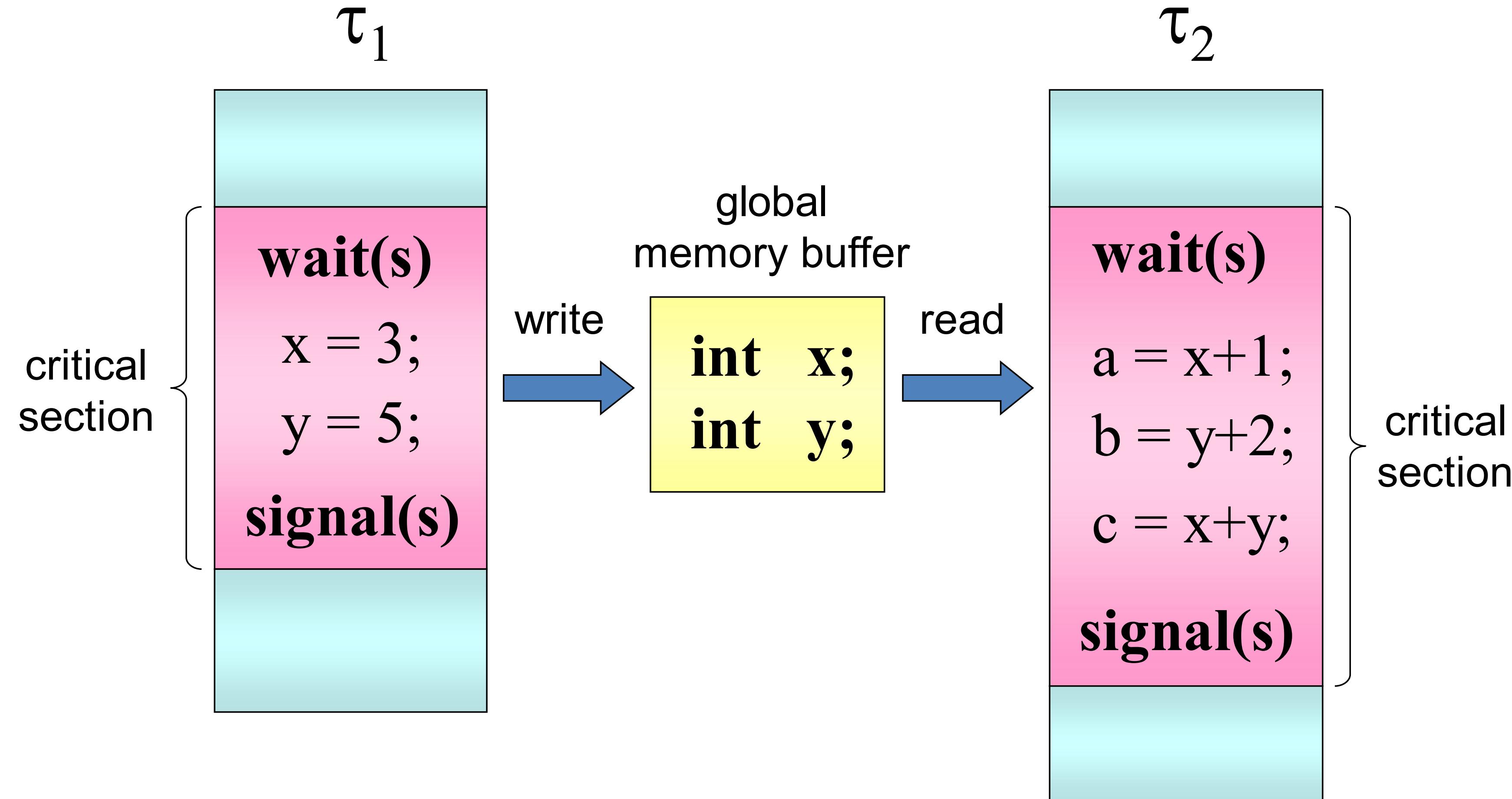
# Adaptive QoS Management

Reservation parameters can be changed at run time by a **Reservation Manager**:



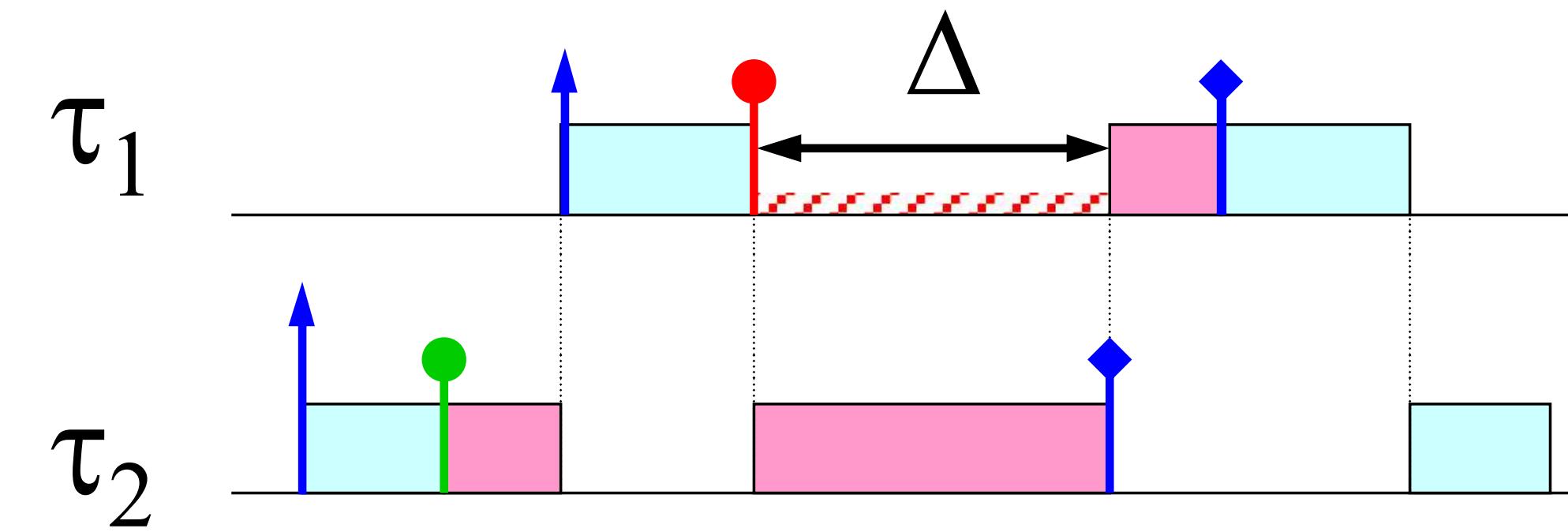
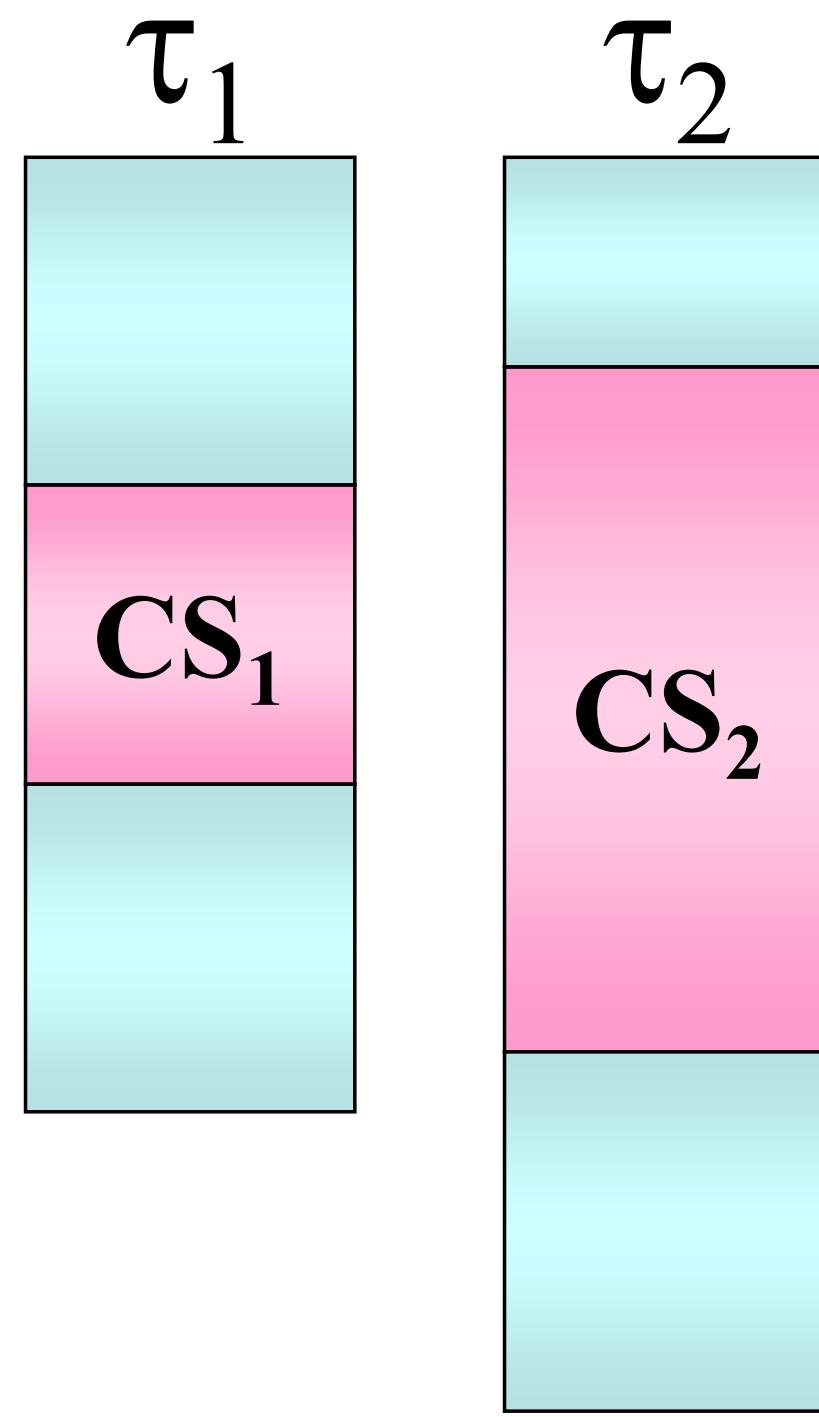
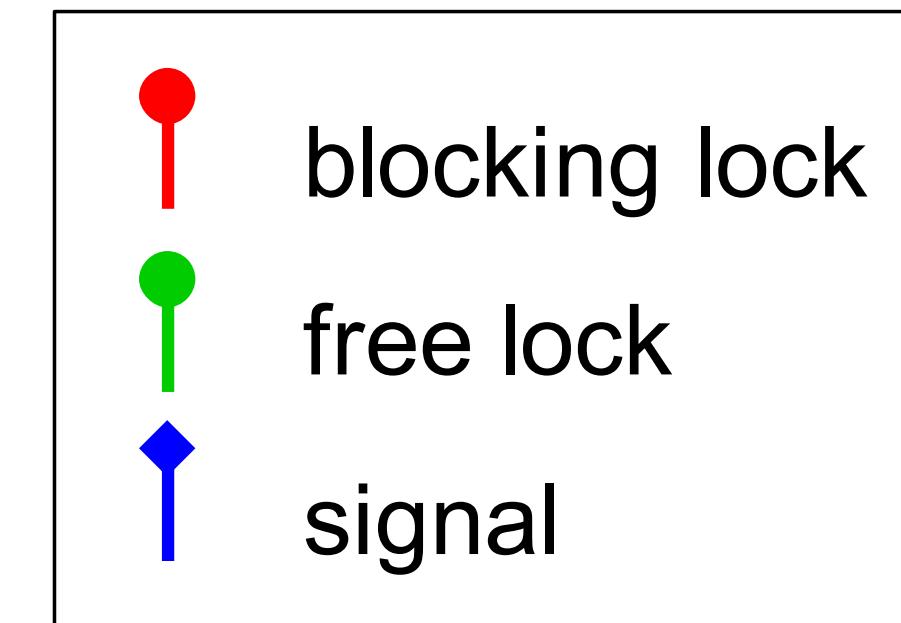
# **Handling Shared Resources**

# Critical sections



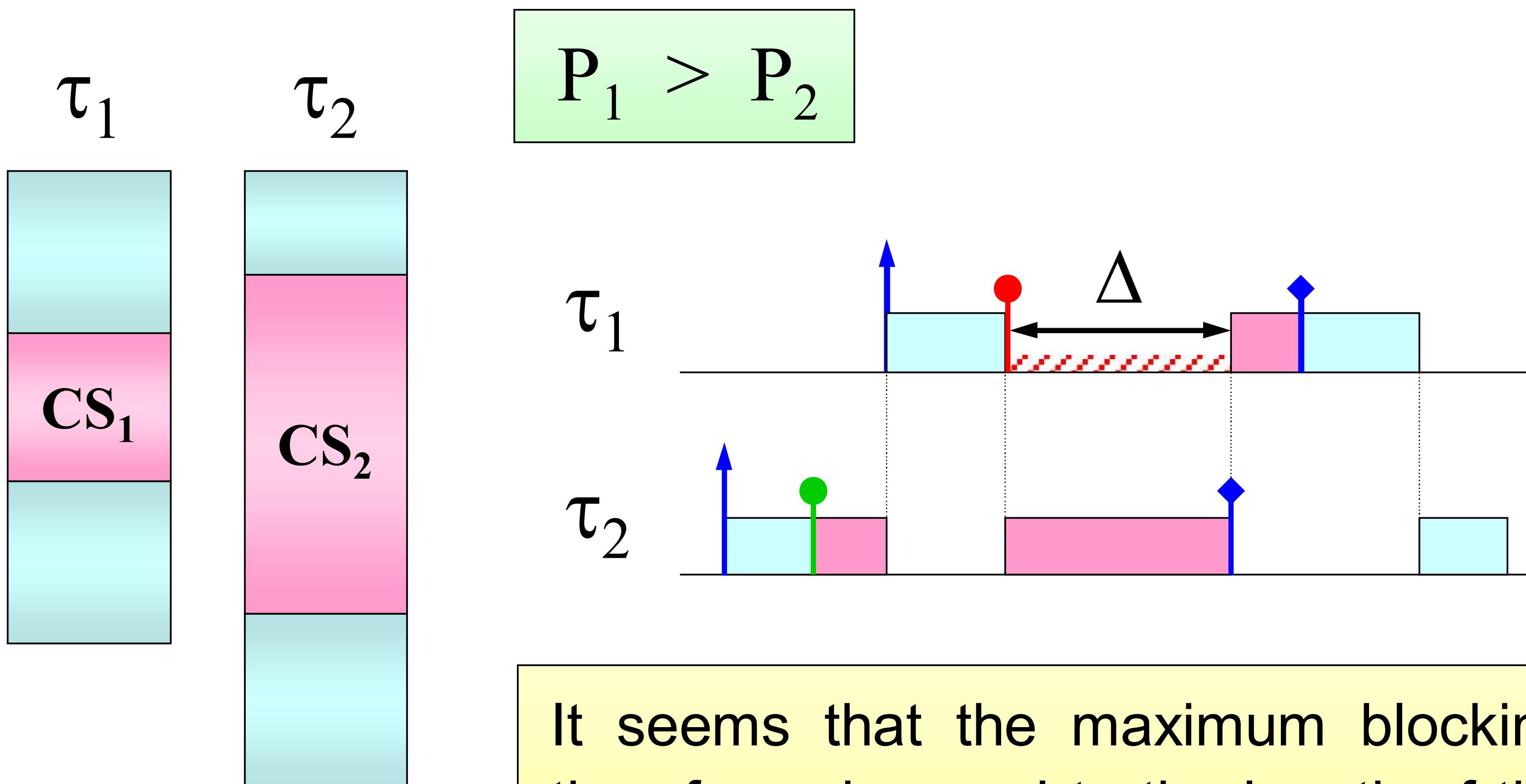
# Blocking on a semaphore

$$P_1 > P_2$$



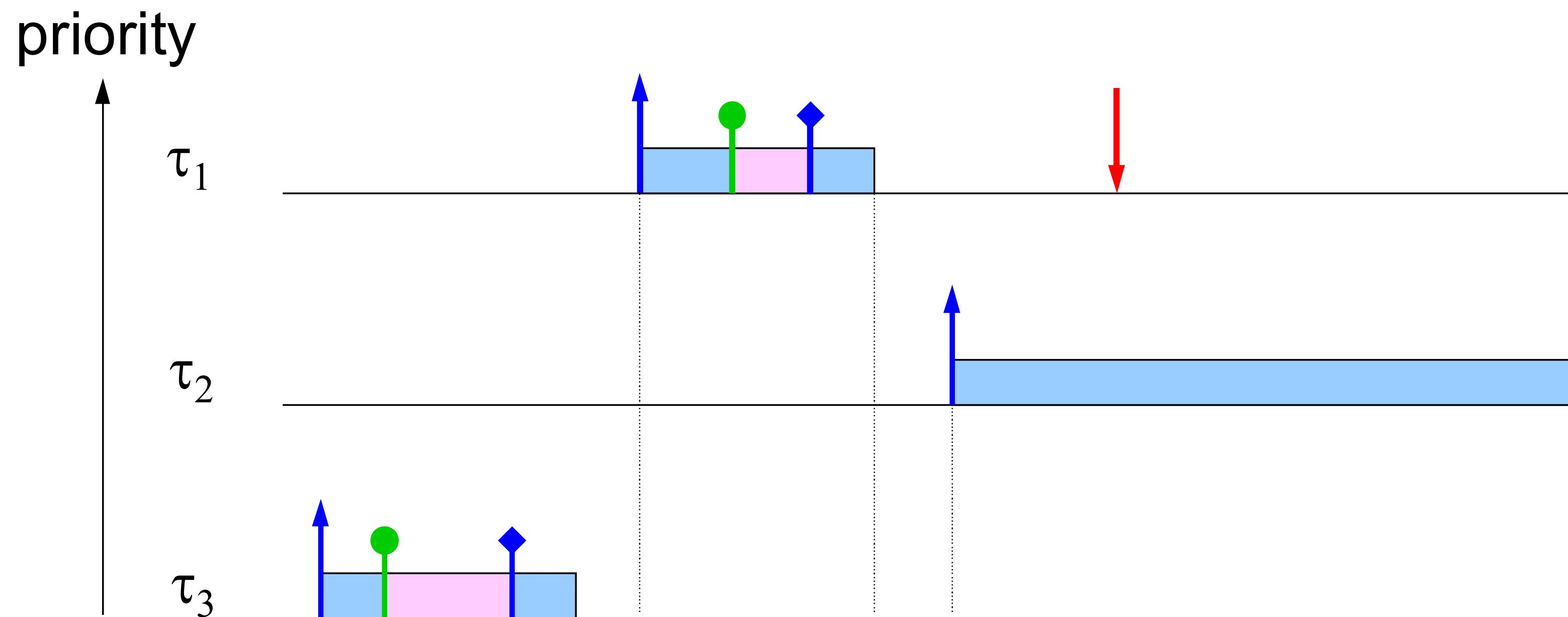
So the blocking time of  $\tau_1$  depends on  
the length of the critical section of  $\tau_2$

# How long is blocking time?

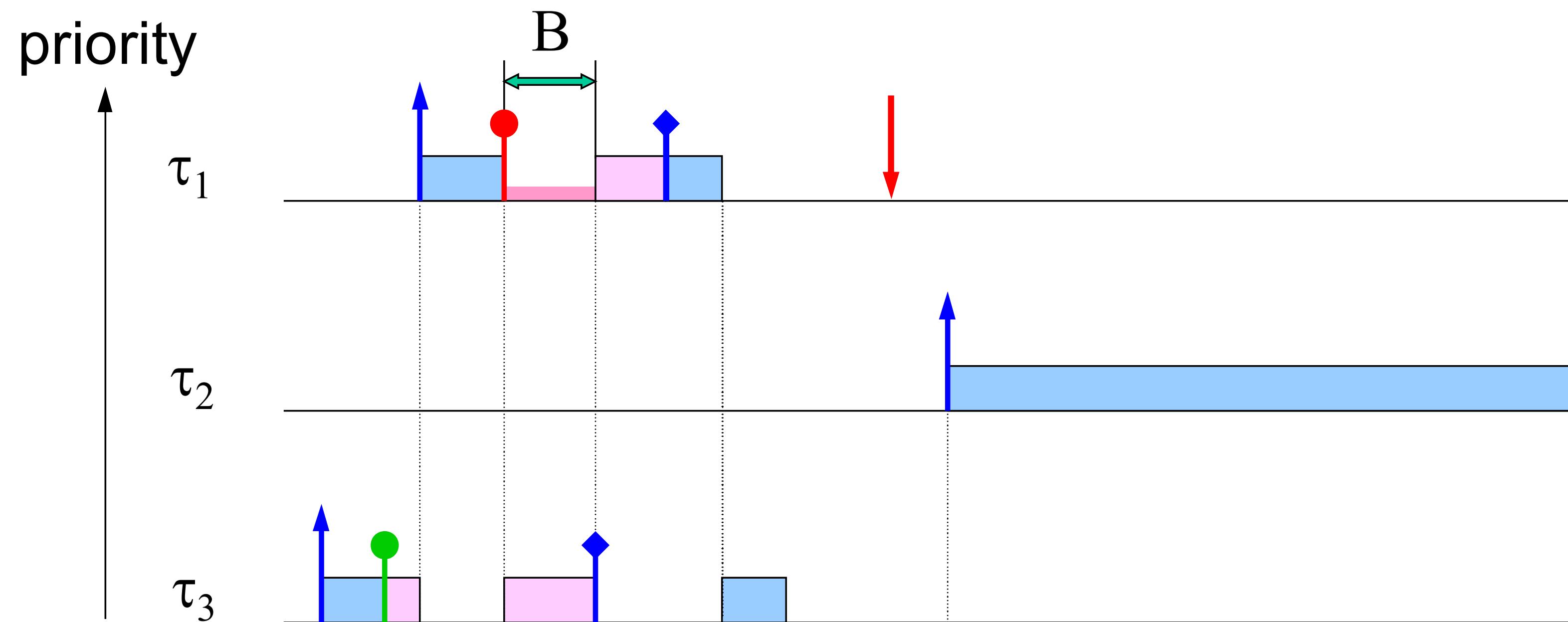


It seems that the maximum blocking time for  $\tau_1$  is equal to the length of the critical section ( $CS_2$ ) of  $\tau_2$ , but ...

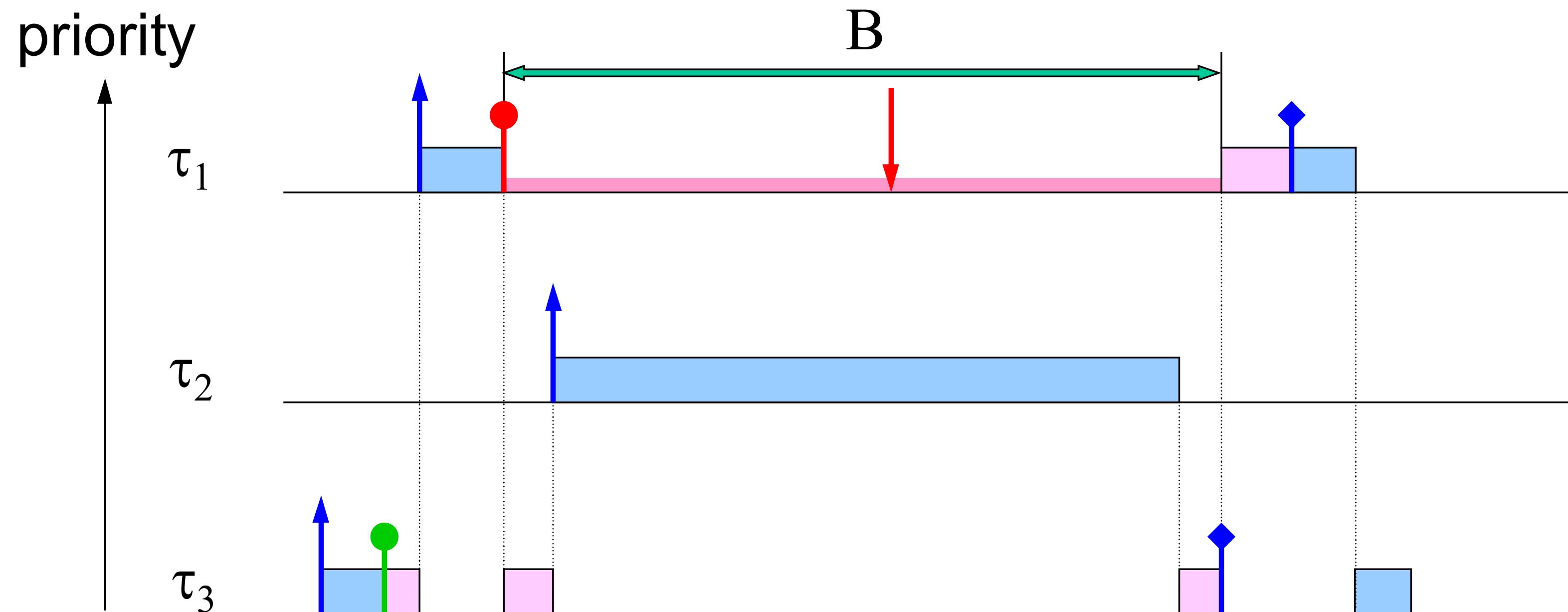
# Schedule with no conflicts



# Conflict on a critical section



# Conflict on a critical section



# Priority Inversion

A high priority task is blocked by a lower priority task  $a$  for an unbounded interval of time

## Solution

Introduce a [concurrency control protocol](#) for accessing critical sections.

# Protocol key aspects.

**Access Rule:** Decides whether to block and when.

**Progress Rule:** Decides how to execute inside a critical section.

**Release Rule:** Decides how to order the pending requests of the blocked tasks.

## Other aspects

**Analysis:** estimates the worst-case blocking times.

**Implementation:** finds the simplest way to encode the protocol rules.

# Resource Access Protocols

- Classical semaphores (**No protocol**)
- Non Preemptive Protocol (**NPP**)
- Highest Locker Priority (**HLP**)
- Priority Inheritance Protocol (**PIP**)
- Priority Ceiling Protocol (**PCP**)
- Stack Resource Policy (**SRP**)

# Non Preemptive Protocol

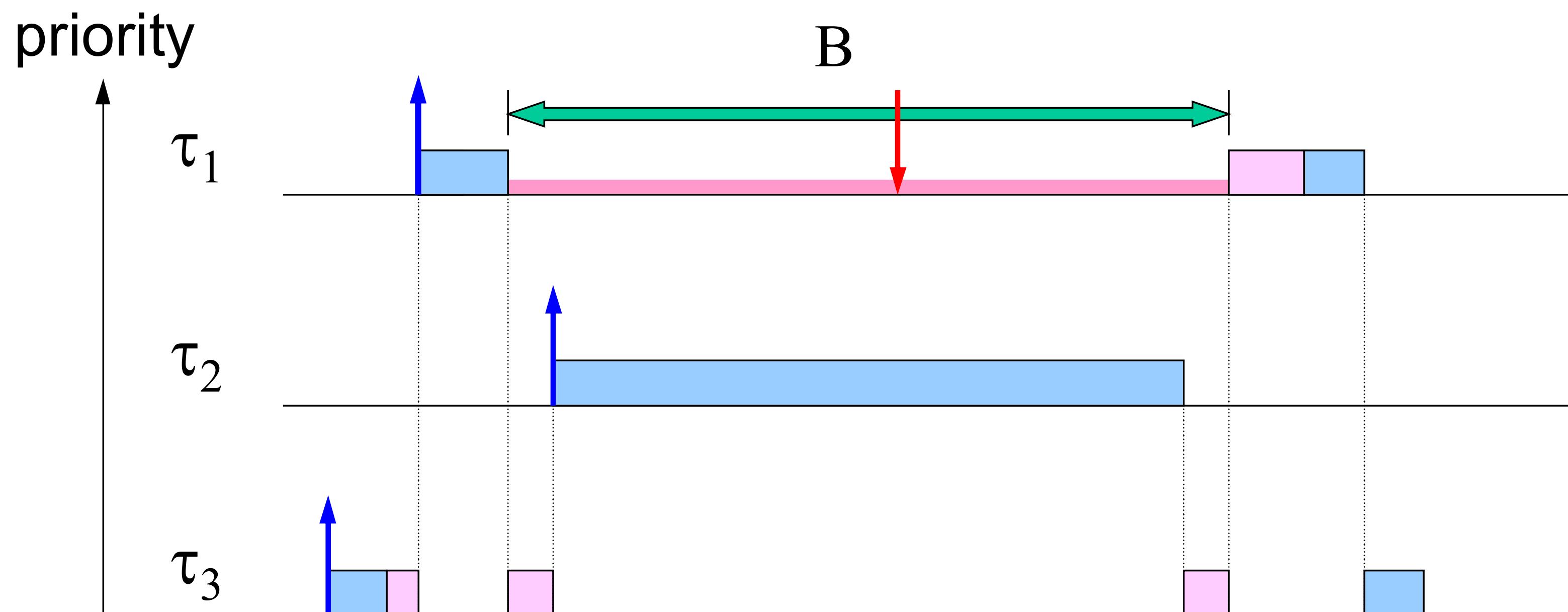
**Access Rule:** A task never blocks at the entrance of a critical section, but at its activation time.

**Progress Rule:** Disable preemption when executing inside a critical section.

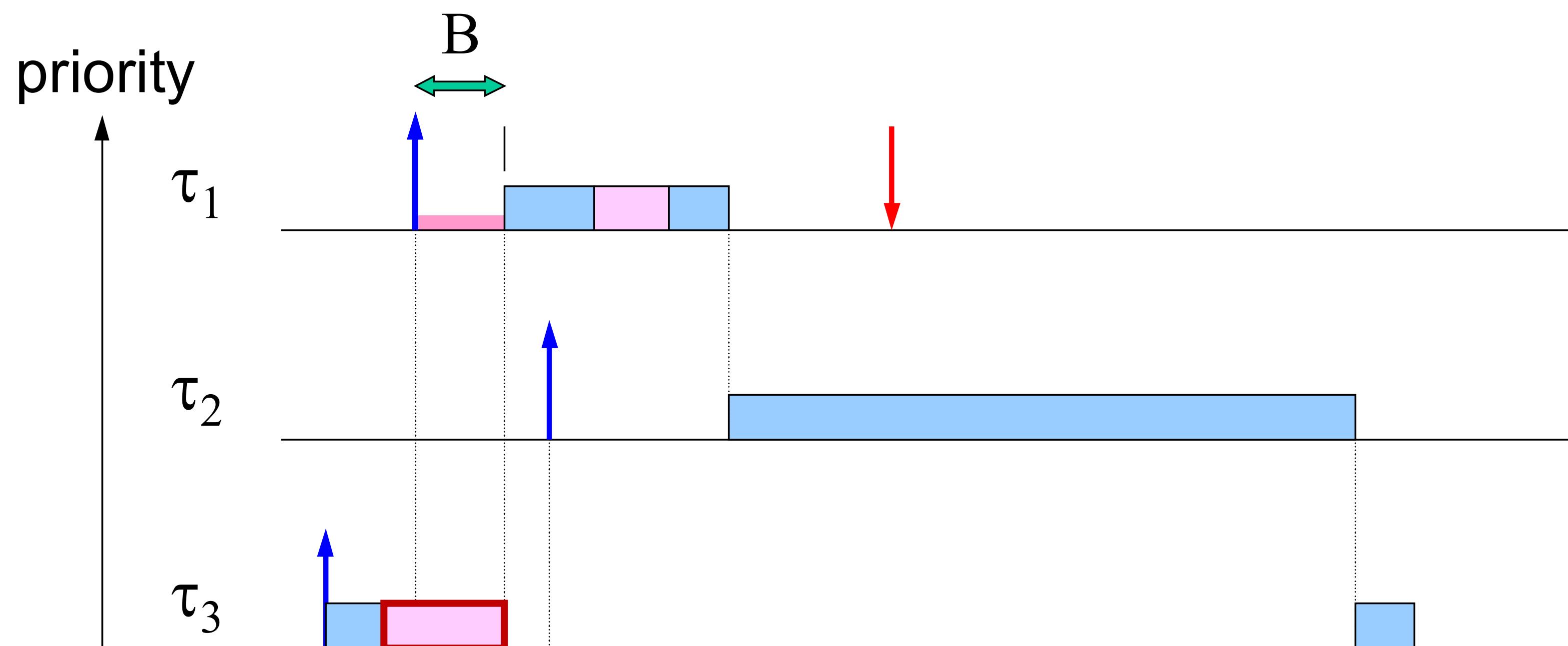
**Release Rule:** At exit, enable preemption so that the resource is assigned to the pending task with the highest priority.

# Conflict on a critical section

(using classical semaphores)



# NPP: example



# NPP: implementation notes

Each task  $\tau_i$  must be assigned two priorities:

- a **nominal priority**  $P_i$  (fixed) assigned by the application developer;
- a **dynamic priority**  $p_i$  (initialized to  $P_i$ ) used to schedule the task and affected by the protocol.

Then, the protocol can be implemented by changing the behavior of the **wait** and **signal** primitives:

**wait(s):**  $p_i = \max(P_1, \dots, P_n)$

**signal(s):**  $p_i = P_i$

# NPP: pro & cons

**ADVANTAGES:** simplicity and efficiency.

- Semaphores queues are not needed, because tasks never block on a `wait(s)`.
- Each task can block at most on a single critical section.
- It prevents deadlocks and allows stack sharing.
- It is transparent to the programmer.

**PROBLEMS:**

1. Tasks may block even if they do not use resources.
2. Since tasks are blocked at activation, blocking could be unnecessary (pessimistic assumption).

# Priority Ceiling Protocol

**Access Rule:** A task can access a resource only if it passes the **PCP access test**.

**Progress Rule:** Inside resource R, a task executes with the highest priority of the tasks blocked on R.

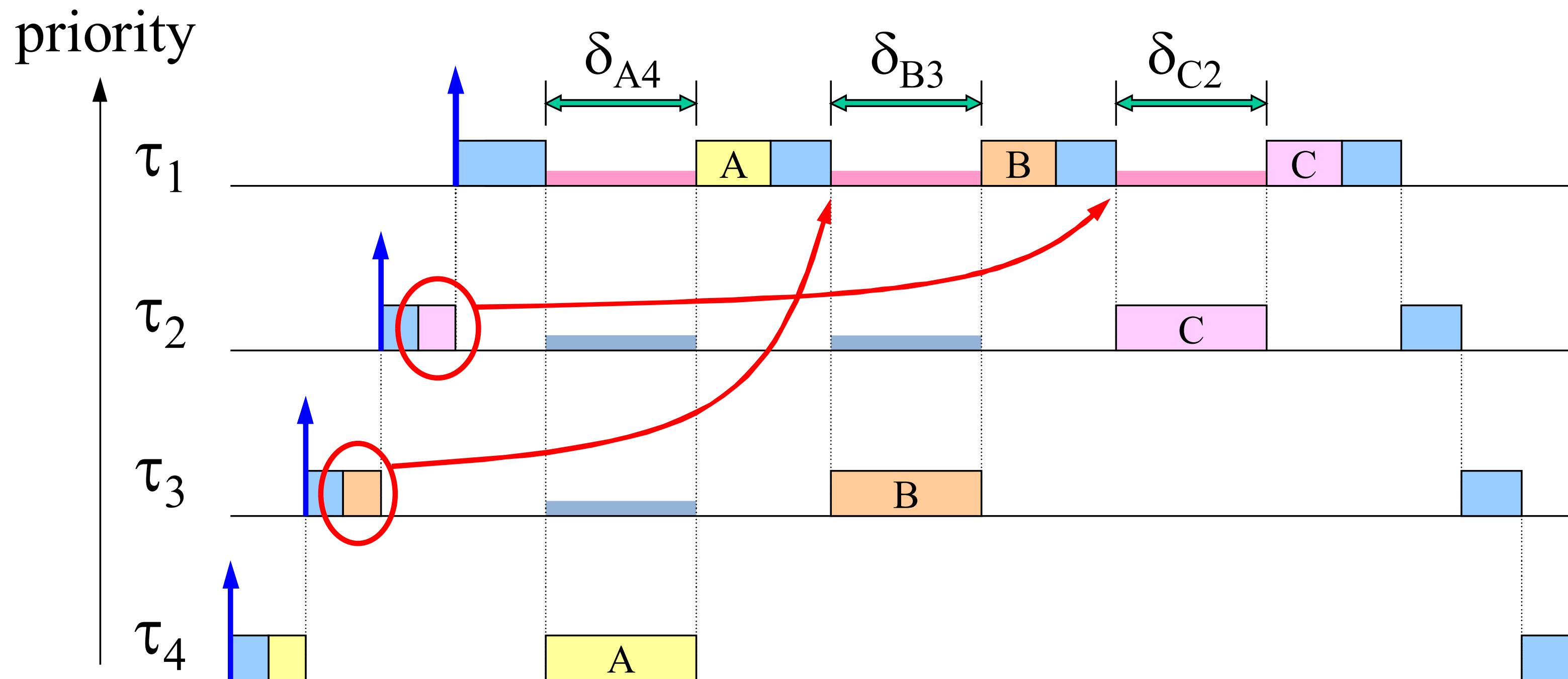
**Release Rule:** At exit, the dynamic priority of the task is reset to its nominal priority  $P_i$ .

**NOTE:** **PCP** can be viewed as **PIP + access test**

## Reference

L. Sha, R. Rajkumar, and J. P. Lehoczky, "Priority inheritance protocols: An approach to real-time synchronization", *IEEE Transactions on Computers*, 39(9), September 1990.

# Avoiding chained blocking



To avoid multiple blocking of  $\tau_1$  we must prevent  $\tau_3$  and  $\tau_2$  to enter their critical sections (even if they are free), because a low priority task ( $\tau_4$ ) is holding a resource used by  $\tau_1$ .

# Resource Ceilings

To keep track of resource usage by high-priority tasks, each resource is assigned a **resource ceiling**:

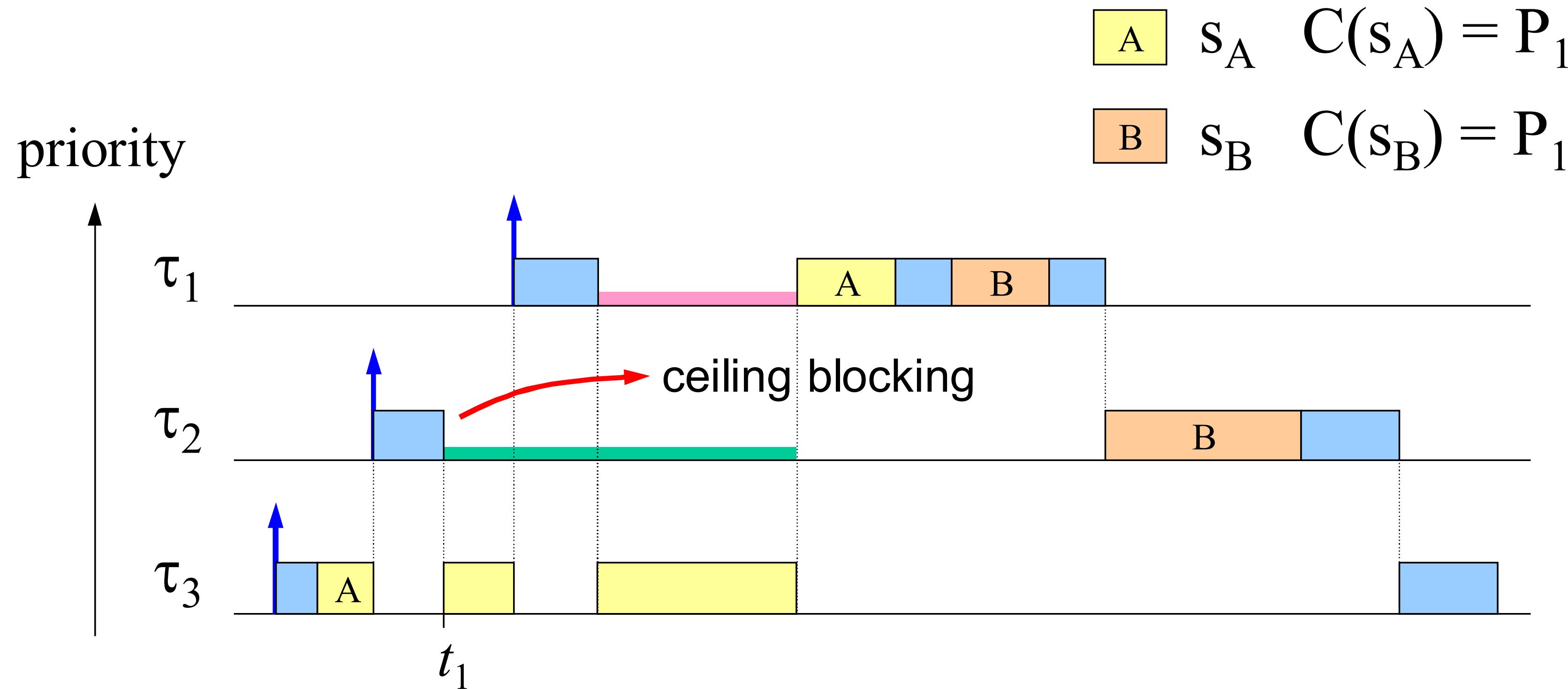
$$C(s_k) = \max \{ P_i \mid \tau_i \text{ uses } s_k \}$$

Then a task  $\tau_i$  can enter a critical section only if its priority is higher than the maximum ceiling of the locked semaphores:

**PCP access test**

$$P_i > \max \{ C(s_k) : s_k \text{ locked by tasks } \neq \tau_i \}$$

# PCP: example



$t_1$ :  $\tau_2$  is blocked by the PCP, since  $P_2 < C(s_A)$

# PCP: properties

## Theorem 1

Under PCP, a task can block at most on a single critical section.

## Theorem 2

PCP prevents chained blocking.

## Theorem 3

PCP prevents deadlocks.

# PCP: pro & cons

## ADVANTAGES:

- It limits blocking to the length of a single critical section.
- It prevents deadlocks when using nested critical sections.

## PROBLEMS:

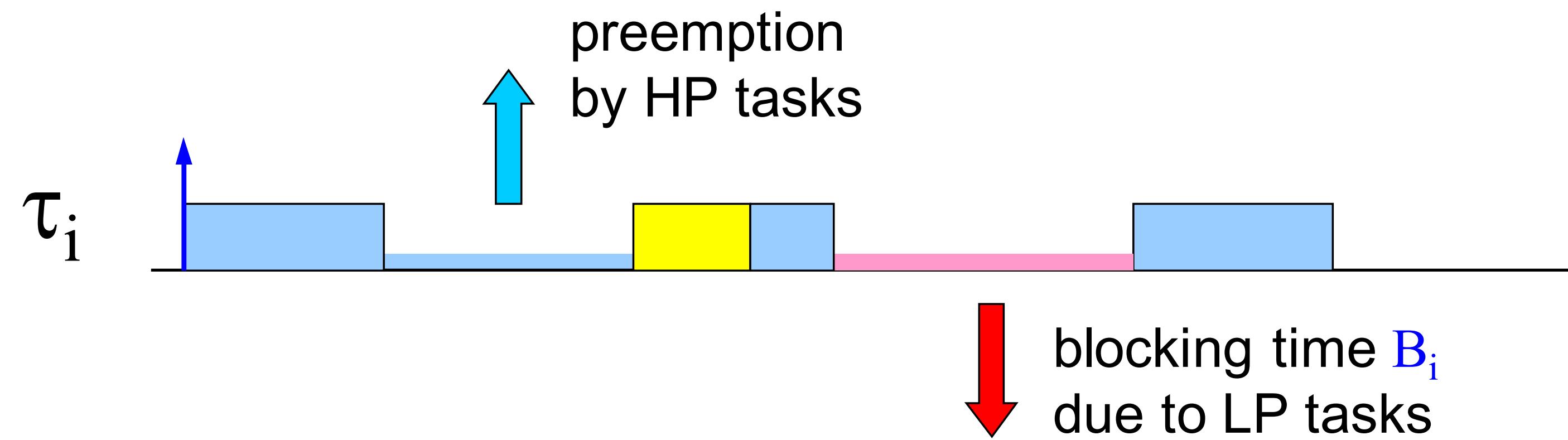
- It is complex to implement (like PIP).
- It can create unnecessary blocking (it is pessimistic like HLP).
- It is not transparent to the programmer: resource ceilings must be specified in the source code.

# Analysis under shared resources

1. Select a **scheduling algorithm** to manage tasks and a **protocol** for accessing shared resources.
2. Compute the maximum **blocking time  $B_i$**  for each task.
3. Perform the **guarantee test** including the blocking terms.

# Analysis under RM

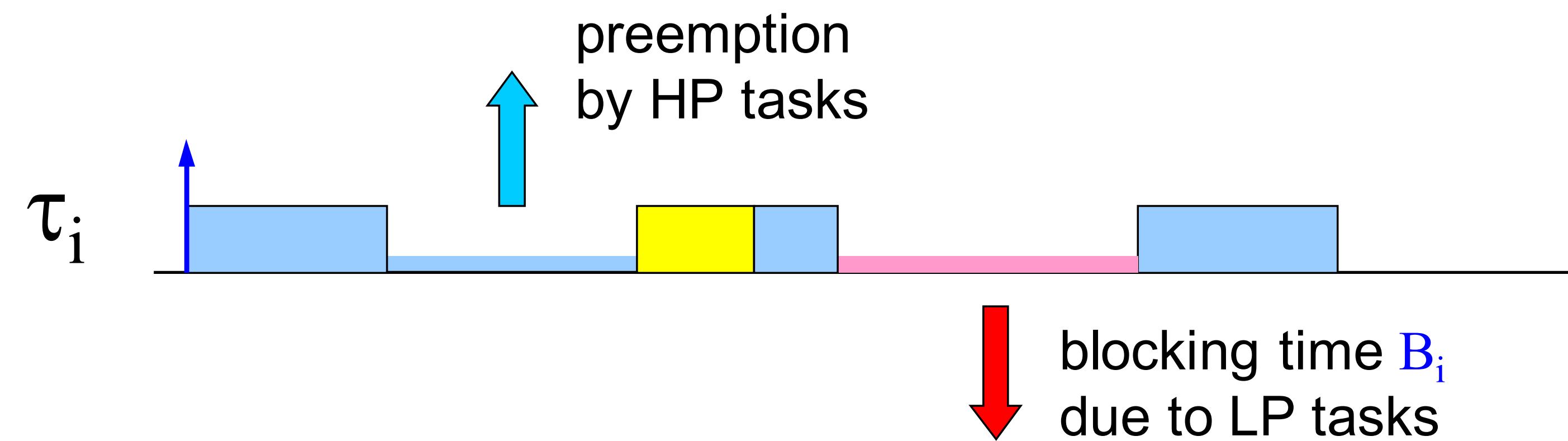
Once we identify the **maximum blocking time  $B_i$**  for each task, we can extend the LL schedulability test as follows:



$$\forall i \quad \sum_{k=1}^{i-1} \frac{C_k}{T_k} + \frac{C_i + B_i}{T_i} \leq i(2^{1/i} - 1)$$

# Hyperbolic Bound

Similarly, we can extend the HB schedulability test as follows:



$$\forall i \quad \prod_{k=1}^{i-1} \left( \frac{C_k}{T_k} + 1 \right) \left( \frac{C_i + B_i}{T_i} + 1 \right) \leq 2$$

# Response Time Analysis

$$R_i = C_i + B_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i}{T_k} \right\rceil C_k$$

**Iterative solution:**

$$\begin{cases} R_i^0 = C_i + B_i \\ R_i^{(s)} = C_i + B_i + \sum_{k=1}^{i-1} \left\lceil \frac{R_i^{(s-1)}}{T_k} \right\rceil C_k \end{cases}$$

iterate while  
 $R_i^s > R_i^{(s-1)}$

# Resource Sharing under EDF

The protocols analyzed so far have been originally developed for fixed priority scheduling schemes. However:

- **NPP** can also be used under EDF
- **PIP** has been extended under EDF by Spuri (1997).
- **PCP** has been extended under EDF by Chen-Lin (1990)
- In 1990, Baker proposed a new access protocol, called **Stack Resource Policy (SRP)** that works both under fixed priorities and EDF.

# Resource ceiling for EDF

Each resource  $R_k$  is assigned a dynamic ceiling equal to the highest preemption level of the tasks that may block on  $R_k$ :

$$C_R(n_k) = \max \{0, \pi_i : n(R_k) < \mu_i(R_k)\}$$

**NOTE:**  $C_R(n_k)$  increases only when a resource is locked  
 $C_R(n_k)$  decreases only when a resource is released

# Schedulability analysis under EDF

When  $D_i = T_i$

A task set is schedulable if

$$\forall i \quad \sum_{k=1}^{i-1} \frac{C_k}{T_k} + \frac{C_i + B_i}{T_i} \leq 1$$

$B_i$  can be computed as under PCP and refers to the length of longest critical section that can block  $\tau_i$ .

# Schedulability analysis under EDF

When  $D_i \leq T_i$

A task set is schedulable if  $U < 1$  and  $\forall L \in \mathcal{D}$

$$B(L) + \sum_{k=1}^n \left\lfloor \frac{L + T_k - D_k}{T_k} \right\rfloor C_k \leq L$$

where:  $B(L) = \max \{ \delta_{jh} \mid (D_j > L) \text{ and } (D_h \leq L) \}$

$$\mathcal{D} = \{ d_k \mid d_k \leq \max(D_{max}, \min(H, L^*)) \}$$

$$\begin{cases} D_{max} = \max(D_1, \dots, D_n) \\ H = lcm(T_1, \dots, T_n) \end{cases} \quad L^* = \frac{\sum_{i=1}^n (T_i - D_i) U_i}{1 - U}$$

# Comparison

Prot	prio.	pessim.	$\alpha_i$	block. instant	trans- parent	dlock avoid.	stack sharing	impl
<b>NPP</b>	any	high	1	on arrival	YES	YES	YES	easy
<b>HLP</b>	fixed	medium	1	on arrival	NO	YES	YES	easy
<b>PIP</b>	fixed	low	$\min(n_i, m_i)$	on access	YES	NO	NO	hard
<b>PCP</b>	fixed	low	1	on access	NO	YES	NO	hard
<b>SRP</b>	any	medium	1	on arrival	NO	YES	YES	easy