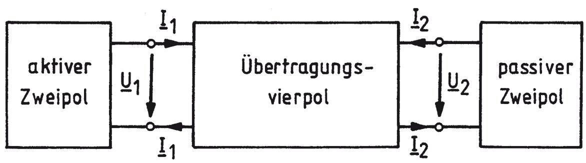
10 Vierpoltheorie

10.1 Grundlegende Zusammenhänge der Vierpoltheorie

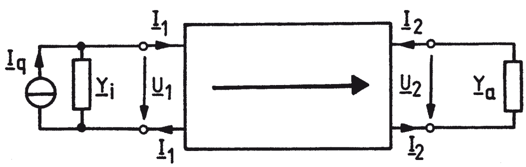
(Band 3, S.171-174)

Elektrische Schaltungen zur Übertragung von Energien oder zur Verarbeitung von Informationen sind in den meisten Fällen „Zweitore“ oder „Vierpole“, also Schaltungen mit zwei Eingangsklemmen und zwei Ausgangsklemmen.

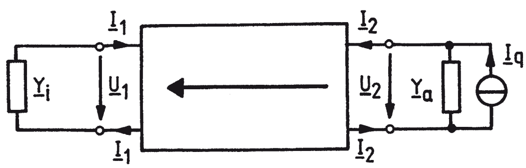
Diese Richtungsdefinitionen sind in der nachrichtentechnischen Literatur üblich:



Dem normalen Vorwärtsbetrieb ist stets eine Rückwirkung vom Ausgang zum Eingang überlagert, die auch zu Störungen bei der Signalübertragung führen kann.



Vierpolschaltung
in Vorwärtsbetrieb



Vierpolschaltung
in Rückwärtsbetrieb

10.2 Vierpolgleichungen, Vierpolparameter und Ersatzschaltungen

(Band 3, S.175-185)

Leitwertform der Vierpolgleichungen:

$$\underline{I}_1 = \underline{Y}_{11} \cdot \underline{U}_1 + \underline{Y}_{12} \cdot \underline{U}_2$$
$$\underline{I}_2 = \underline{Y}_{21} \cdot \underline{U}_1 + \underline{Y}_{22} \cdot \underline{U}_2$$

oder

$$\begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \end{pmatrix} = \begin{pmatrix} \underline{Y}_{11} & \underline{Y}_{12} \\ \underline{Y}_{21} & \underline{Y}_{22} \end{pmatrix} \cdot \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \end{pmatrix}$$

Kurzschluss-Eingangsleitwert:

$$\underline{Y}_{11} = \left(\frac{\underline{I}_1}{\underline{U}_1}\right)_{\underline{U}_2=0} = (\underline{Y}_{in})_{\underline{Y}_{a=\infty}}$$

Kurzschluss-Übertragungsleitwert rückwärts:

$$\underline{Y}_{12} = \left(\frac{\underline{I}_1}{\underline{U}_2}\right)_{\underline{U}_1=0} = (\underline{Y}_{\text{ür}})_{\underline{Y}_{i=\infty}}$$

Kurzschluss-Übertragungsleitwert vorwärts:

$$\underline{Y}_{21} = \left(\frac{\underline{I}_2}{\underline{U}_1}\right)_{\underline{U}_2=0} = (\underline{Y}_{\text{üf}})_{\underline{Y}_{a=\infty}}$$

Kurzschluss-Ausgangsleitwert:

$$\underline{Y}_{22} = \left(\frac{\underline{I}_2}{\underline{U}_2}\right)_{\underline{U}_1=0} = (\underline{Y}_{out})_{\underline{Y}_{i=\infty}}$$

W. Weißgerber, *Elektrotechnik für Ingenieure – Formelsammlung*,
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Widerstandsform der Vierpolgleichungen

$$\underline{U}_1 = \underline{Z}_{11}\cdot\underline{I}_1 + \underline{Z}_{12}\cdot\underline{I}_2$$
$$\underline{U}_2 = \underline{Z}_{21}\cdot\underline{I}_1 + \underline{Z}_{22}\cdot\underline{I}_2$$

oder

$$\begin{pmatrix}\underline{U}_1\\ \underline{U}_2\end{pmatrix} = \begin{pmatrix}\underline{Z}_{11} & \underline{Z}_{12}\\ \underline{Z}_{21} & \underline{Z}_{22}\end{pmatrix}\cdot\begin{pmatrix}\underline{I}_1\\ \underline{I}_2\end{pmatrix}$$

Leerlauf-Eingangswiderstand:

$$R_{11} = \underline{Z}_{11} = \left(\frac{\underline{U}_1}{\underline{I}_1}\right)_{\underline{I}_2=0} = (\underline{Z}_{in})_{\underline{Y}_{a=0}}$$

Leerlauf-Übertragungswiderstand rückwärts:

$$R_{12} = \underline{Z}_{12} = \left(\frac{\underline{U}_1}{\underline{I}_2}\right)_{\underline{I}_1=0} = (\underline{Z}_{ür})_{\underline{Y}_{i=0}}$$

Leerlauf-Übertragungswiderstand vorwärts:

$$R_{21} = \underline{Z}_{21} = \left(\frac{\underline{U}_2}{\underline{I}_1}\right)_{\underline{I}_2=0} = (\underline{Z}_{üf})_{\underline{Y}_{a=0}}$$

Leerlauf-Ausgangswiderstand:

$$R_{22} = \underline{Z}_{22} = \left(\frac{\underline{U}_2}{\underline{I}_2}\right)_{\underline{I}_1=0} = (\underline{Z}_{out})_{\underline{Y}_{i=0}}$$

Reihen-Parallel-Form der Vierpolgleichungen

$$\underline{U}_1 = \underline{H}_{11}\cdot\underline{I}_1 + \underline{H}_{12}\cdot\underline{U}_2$$
$$\underline{I}_2 = \underline{H}_{21}\cdot\underline{I}_1 + \underline{H}_{22}\cdot\underline{U}_2$$

oder

$$\begin{pmatrix}\underline{U}_1\\ \underline{I}_2\end{pmatrix} = \begin{pmatrix}\underline{H}_{11} & \underline{H}_{12}\\ \underline{H}_{21} & \underline{H}_{22}\end{pmatrix}\cdot\begin{pmatrix}\underline{I}_1\\ \underline{U}_2\end{pmatrix}$$

Kurzschluss-Eingangswiderstand:

$$\underline{H}_{11} = \left(\frac{\underline{U}_1}{\underline{I}_1}\right)_{\underline{U}_2=0} = (\underline{Z}_{in})_{\underline{Y}_{a=\infty}}$$

Leerlauf-Spannungsrückwirkung:

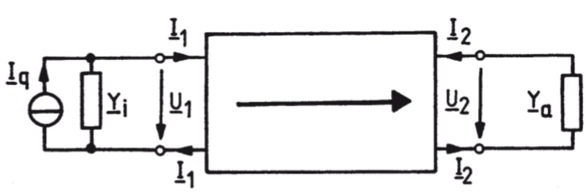
$$\underline{H}_{12} = \left(\frac{\underline{U}_1}{\underline{U}_2}\right)_{\underline{I}_1=0} = (\underline{V}_{ur})_{\underline{Y}_{i=0}}$$

Kurzschluss-Stromübersetzung vorwärts:

$$\underline{H}_{21} = \left(\frac{\underline{I}_2}{\underline{I}_1}\right)_{\underline{U}_2=0} = (\underline{V}_{if})_{\underline{Y}_{a=\infty}}$$

Leerlauf-Ausgangsleitwert:

$$\underline{H}_{22} = \left(\frac{\underline{I}_2}{\underline{U}_2}\right)_{\underline{I}_1=0} = (\underline{Y}_{out})_{\underline{Y}_{i=0}}$$



Parallel-Reihen-Form der Vierpolgleichungen

$$\underline{I}_1 = \underline{C}_{11} \cdot \underline{U}_1 + \underline{C}_{12} \cdot \underline{I}_2$$
$$\underline{U}_2 = \underline{C}_{21} \cdot \underline{U}_1 + \underline{C}_{22} \cdot \underline{I}_2$$

oder

$$\begin{pmatrix} \underline{I}_1 \\ \underline{U}_2 \end{pmatrix} = \begin{pmatrix} \underline{C}_{11} & \underline{C}_{12} \\ \underline{C}_{21} & \underline{C}_{22} \end{pmatrix} \cdot \begin{pmatrix} \underline{U}_1 \\ \underline{I}_2 \end{pmatrix}$$

Leerlauf-Eingangsleitwert:

$$\underline{C}_{11} = \left(\frac{\underline{I}_1}{\underline{U}_1}\right)_{\underline{I}_2=0} = (\underline{Y}_{in})_{\underline{Y}_{a=0}}$$

Kurzschluss-Stromrückwirkung:

$$\underline{C}_{12} = \left(\frac{\underline{I}_1}{\underline{I}_2}\right)_{\underline{U}_1=0} = (\underline{V}_{ir})_{\underline{Y}_{i=\infty}}$$

Leerlauf-Spannungsübersetzung vorwärts:

$$\underline{C}_{21} = \left(\frac{\underline{U}_2}{\underline{U}_1}\right)_{\underline{I}_2=0} = (\underline{V}_{uf})_{\underline{Y}_{a=0}}$$

Kurzschluss-Ausgangswiderstand:

$$\underline{C}_{22} = \left(\frac{\underline{U}_2}{\underline{I}_2}\right)_{\underline{U}_1=0} = (\underline{Z}_{out})_{\underline{Y}_{i=\infty}}$$

Kettenform der Vierpolgleichungen

$$\underline{U}_1 = \underline{A}_{11} \cdot \underline{U}_2 + \underline{A}_{12} \cdot (-\underline{I}_2)$$
$$\underline{I}_1 = \underline{A}_{21} \cdot \underline{U}_2 + \underline{A}_{22} \cdot (-\underline{I}_2)$$

oder

$$\begin{pmatrix} \underline{U}_1 \\ \underline{I}_1 \end{pmatrix} = \begin{pmatrix} \underline{A}_{11} & \underline{A}_{12} \\ \underline{A}_{21} & \underline{A}_{22} \end{pmatrix} \cdot \begin{pmatrix} \underline{U}_2 \\ -\underline{I}_2 \end{pmatrix}$$

reziproke Leerlauf-Spannungsübersetzung vorwärts:

$$\underline{A}_{11} = \left(\frac{\underline{U}_1}{\underline{U}_2}\right)_{\underline{I}_2=0} = \left(\frac{1}{\underline{V}_{uf}}\right)_{\underline{Y}_{a=0}}$$

negativer reziproker Kurzschluss-Übertragungsleitwert vorwärts:

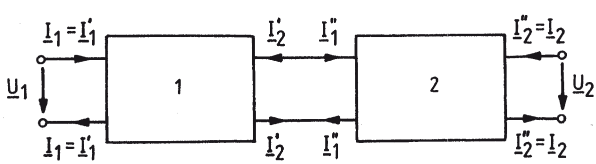
$$\underline{A}_{12} = \left(\frac{\underline{U}_1}{-\underline{I}_2}\right)_{\underline{U}_2=0} = \left(\frac{1}{-\underline{Y}_{üf}}\right)_{\underline{Y}_{a=\infty}}$$

reziproker Leerlauf-Übertragungswiderstand vorwärts:

$$\underline{A}_{21} = \left(\frac{\underline{I}_1}{\underline{U}_2}\right)_{\underline{I}_2=0} = \left(\frac{1}{\underline{Z}_{üf}}\right)_{\underline{Y}_{a=0}}$$

negative reziproke Kurzschluss-Stromübersetzung vorwärts:

$$\underline{A}_{22} = \left(\frac{\underline{I}_1}{-\underline{I}_2}\right)_{\underline{U}_2=0} = \left(\frac{1}{-\underline{V}_{if}}\right)_{\underline{Y}_{a=\infty}}$$



Definition der $\underline{A}$-Parameter mittels Kettenschaltung

Umrechnung der Vierpolparameter von einer Form in eine andere

(Y)	$\underline{Y}_{11}$	$\underline{Y}_{12}$	$\frac{\underline{Z}_{22}}{\det\underline{Z}}$	$\frac{-\underline{Z}_{12}}{\det\underline{Z}}$	$\frac{1}{\underline{H}_{11}}$	$\frac{-\underline{H}_{12}}{\underline{H}_{11}}$	$\frac{\det\underline{C}}{\underline{C}_{22}}$	$\frac{\underline{C}_{12}}{\underline{C}_{22}}$	$\frac{\underline{A}_{22}}{\underline{A}_{12}}$	$\frac{-\det\underline{A}}{\underline{A}_{12}}$
	$\underline{Y}_{21}$	$\underline{Y}_{22}$	$\frac{-\underline{Z}_{21}}{\det\underline{Z}}$	$\frac{\underline{Z}_{11}}{\det\underline{Z}}$	$\frac{\underline{H}_{21}}{\underline{H}_{11}}$	$\frac{\det\underline{H}}{\underline{H}_{11}}$	$\frac{-\underline{C}_{21}}{\underline{C}_{22}}$	$\frac{1}{\underline{C}_{22}}$	$\frac{-1}{\underline{A}_{12}}$	$\frac{\underline{A}_{11}}{\underline{A}_{12}}$
(Z)	$\frac{\underline{Y}_{22}}{\det\underline{Y}}$	$\frac{-\underline{Y}_{12}}{\det\underline{Y}}$	$\underline{Z}_{11}$	$\underline{Z}_{12}$	$\frac{\det\underline{H}}{\underline{H}_{22}}$	$\frac{\underline{H}_{12}}{\underline{H}_{22}}$	$\frac{1}{\underline{C}_{11}}$	$\frac{-\underline{C}_{12}}{\underline{C}_{11}}$	$\frac{\underline{A}_{11}}{\underline{A}_{21}}$	$\frac{\det\underline{A}}{\underline{A}_{21}}$
	$\frac{-\underline{Y}_{21}}{\det\underline{Y}}$	$\frac{\underline{Y}_{11}}{\det\underline{Y}}$	$\underline{Z}_{21}$	$\underline{Z}_{22}$	$\frac{-\underline{H}_{21}}{\underline{H}_{22}}$	$\frac{1}{\underline{H}_{22}}$	$\frac{\underline{C}_{21}}{\underline{C}_{11}}$	$\frac{\det\underline{C}}{\underline{C}_{11}}$	$\frac{1}{\underline{A}_{21}}$	$\frac{\underline{A}_{22}}{\underline{A}_{21}}$
(H)	$\frac{1}{\underline{Y}_{11}}$	$\frac{-\underline{Y}_{12}}{\underline{Y}_{11}}$	$\frac{\det\underline{Z}}{\underline{Z}_{22}}$	$\frac{\underline{Z}_{12}}{\underline{Z}_{22}}$	$\underline{H}_{11}$	$\underline{H}_{12}$	$\frac{\underline{C}_{22}}{\det\underline{C}}$	$\frac{-\underline{C}_{12}}{\det\underline{C}}$	$\frac{\underline{A}_{12}}{\underline{A}_{22}}$	$\frac{\det\underline{A}}{\underline{A}_{22}}$
	$\frac{\underline{Y}_{21}}{\underline{Y}_{11}}$	$\frac{\det\underline{Y}}{\underline{Y}_{11}}$	$\frac{-\underline{Z}_{21}}{\underline{Z}_{22}}$	$\frac{1}{\underline{Z}_{22}}$	$\underline{H}_{21}$	$\underline{H}_{22}$	$\frac{-\underline{C}_{21}}{\det\underline{C}}$	$\frac{\underline{C}_{11}}{\det\underline{C}}$	$\frac{-1}{\underline{A}_{22}}$	$\frac{\underline{A}_{21}}{\underline{A}_{22}}$
(C)	$\frac{\det\underline{Y}}{\underline{Y}_{22}}$	$\frac{\underline{Y}_{12}}{\underline{Y}_{22}}$	$\frac{1}{\underline{Z}_{11}}$	$\frac{-\underline{Z}_{12}}{\underline{Z}_{11}}$	$\frac{\underline{H}_{22}}{\det\underline{H}}$	$\frac{-\underline{H}_{12}}{\det\underline{H}}$	$\underline{C}_{11}$	$\underline{C}_{12}$	$\frac{\underline{A}_{21}}{\underline{A}_{11}}$	$\frac{-\det\underline{A}}{\underline{A}_{11}}$
	$\frac{-\underline{Y}_{21}}{\underline{Y}_{22}}$	$\frac{1}{\underline{Y}_{22}}$	$\frac{\underline{Z}_{21}}{\underline{Z}_{11}}$	$\frac{\det\underline{Z}}{\underline{Z}_{11}}$	$\frac{-\underline{H}_{21}}{\det\underline{H}}$	$\frac{\underline{H}_{11}}{\det\underline{H}}$	$\underline{C}_{21}$	$\underline{C}_{22}$	$\frac{1}{\underline{A}_{11}}$	$\frac{\underline{A}_{12}}{\underline{A}_{11}}$
(A)	$\frac{-\underline{Y}_{22}}{\underline{Y}_{21}}$	$\frac{-1}{\underline{Y}_{21}}$	$\frac{\underline{Z}_{11}}{\underline{Z}_{21}}$	$\frac{\det\underline{Z}}{\underline{Z}_{21}}$	$\frac{-\det\underline{H}}{\underline{H}_{21}}$	$\frac{-\underline{H}_{11}}{\underline{H}_{21}}$	$\frac{1}{\underline{C}_{21}}$	$\frac{\underline{C}_{22}}{\underline{C}_{21}}$	$\underline{A}_{11}$	$\underline{A}_{12}$
	$\frac{-\det\underline{Y}}{\underline{Y}_{21}}$	$\frac{-\underline{Y}_{11}}{\underline{Y}_{21}}$	$\frac{1}{\underline{Z}_{21}}$	$\frac{\underline{Z}_{22}}{\underline{Z}_{21}}$	$\frac{-\underline{H}_{22}}{\underline{H}_{21}}$	$\frac{-1}{\underline{H}_{21}}$	$\frac{\underline{C}_{11}}{\underline{C}_{21}}$	$\frac{\det\underline{C}}{\underline{C}_{21}}$	$\underline{A}_{21}$	$\underline{A}_{22}$

Formeln für Vierpoldeterminanten:

$$\det\underline{Y} = \underline{Y}_{11}\underline{Y}_{22} - \underline{Y}_{12}\underline{Y}_{21} = \frac{1}{\det\underline{Z}} = \frac{\underline{H}_{22}}{\underline{H}_{11}} = \frac{\underline{C}_{11}}{\underline{C}_{22}} = \frac{\underline{A}_{21}}{\underline{A}_{12}}$$

$$\det\underline{Z} = \frac{1}{\det\underline{Y}} = \underline{Z}_{11}\underline{Z}_{22} - \underline{Z}_{12}\underline{Z}_{21} = \frac{\underline{H}_{11}}{\underline{H}_{22}} = \frac{\underline{C}_{22}}{\underline{C}_{11}} = \frac{\underline{A}_{12}}{\underline{A}_{21}}$$

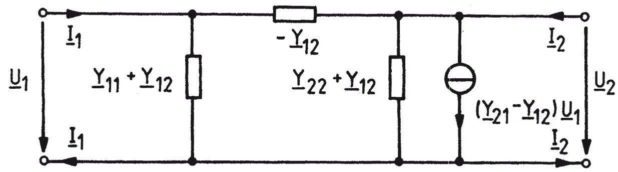
$$\det\underline{H} = \frac{\underline{Y}_{22}}{\underline{Y}_{11}} = \frac{\underline{Z}_{11}}{\underline{Z}_{22}} = \underline{H}_{11}\underline{H}_{22} - \underline{H}_{12}\underline{H}_{21} = \frac{1}{\det\underline{C}} = \frac{\underline{A}_{11}}{\underline{A}_{22}}$$

$$\det\underline{C} = \frac{\underline{Y}_{11}}{\underline{Y}_{22}} = \frac{\underline{Z}_{22}}{\underline{Z}_{11}} = \frac{1}{\det\underline{H}} = \underline{C}_{11}\underline{C}_{22} - \underline{C}_{12}C_{21} = \frac{\underline{A}_{22}}{\underline{A}_{11}}$$

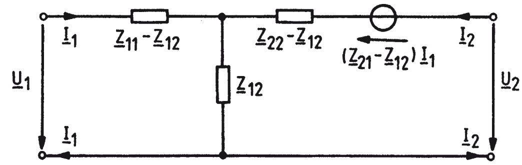
$$\det\underline{A} = \frac{\underline{Y}_{12}}{\underline{Y}_{21}} = \frac{\underline{Z}_{12}}{\underline{Z}_{21}} = -\frac{\underline{H}_{12}}{H_{21}} = -\frac{\underline{C}_{12}}{\underline{C}_{21}} = \underline{A}_{11}\underline{A}_{22} - \underline{A}_{12}\underline{A}_{21}$$

Ersatzschaltungen von Vierpolen

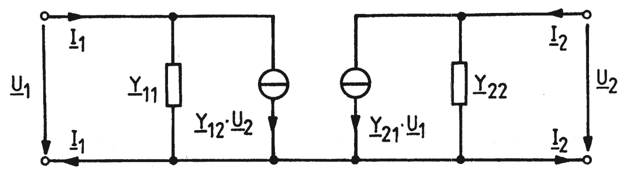
π-Ersatzschaltung:

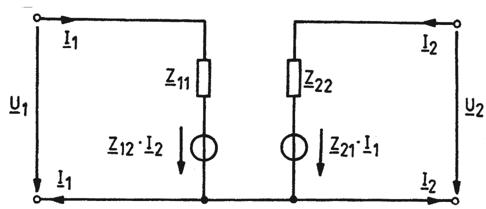


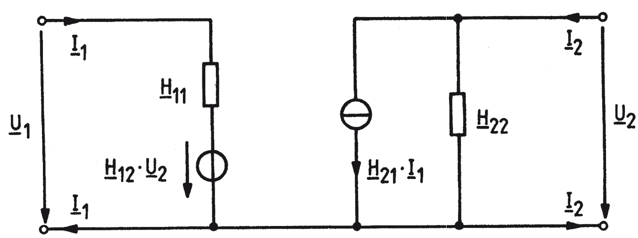
T-Ersatzschaltung:

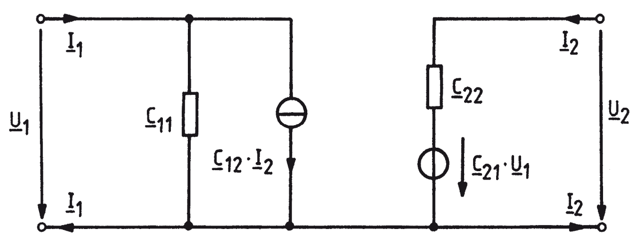


U-Ersatzschaltungen:



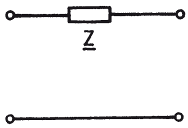
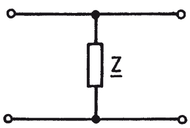
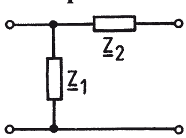


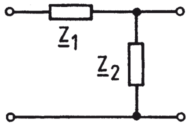
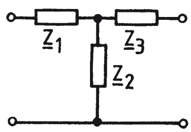
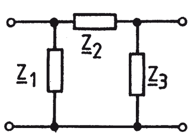




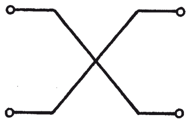
10.3 Vierpolparameter passiver Vierpole

(Band 3, S.186-188)

Längswiderstand 	**($\underline{Y}$)** $\begin{pmatrix} \frac{1}{\underline{Z}} & -\frac{1}{\underline{Z}} \\ -\frac{1}{\underline{Z}} & \frac{1}{\underline{Z}} \end{pmatrix}$	**($\underline{Z}$)** ($\underline{Z}$) existiert nicht (Matrixelemente sind unendlich)
($\underline{A}$)	**($\underline{H}$)**	**($\underline{C}$)**
$\begin{pmatrix} 1 & \underline{Z} \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} \underline{Z} & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ 1 & \underline{Z} \end{pmatrix}$
Querwiderstand 	**($\underline{Y}$)** ($\underline{Y}$) existiert nicht (Matrixelemente sind unendlich)	**($\underline{Z}$)** $\begin{pmatrix} \underline{Z} & \underline{Z} \\ \underline{Z} & \underline{Z} \end{pmatrix}$
($\underline{A}$)	**($\underline{H}$)**	**($\underline{C}$)**
$\begin{pmatrix} 1 & 0 \\ \frac{1}{\underline{Z}} & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ -1 & \frac{1}{\underline{Z}} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\underline{Z}} & -1 \\ 1 & 0 \end{pmatrix}$
Γ-Vierpol I 	**($\underline{Y}$)** $\begin{pmatrix} \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2} & -\frac{1}{\underline{Z}_2} \\ -\frac{1}{\underline{Z}_2} & \frac{1}{\underline{Z}_2} \end{pmatrix}$	**($\underline{Z}$)** $\begin{pmatrix} \underline{Z}_1 & \underline{Z}_1 \\ \underline{Z}_1 & \underline{Z}_1 + \underline{Z}_2 \end{pmatrix}$
($\underline{A}$)	**($\underline{H}$)**	**($\underline{C}$)**
$\begin{pmatrix} 1 & \underline{Z}_2 \\ \frac{1}{\underline{Z}_1} & 1 + \frac{\underline{Z}_2}{\underline{Z}_1} \end{pmatrix}$	$\begin{pmatrix} \frac{\underline{Z}_1 \cdot \underline{Z}_2}{\underline{Z}_1 + \underline{Z}_2} & \frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} \\ -\frac{\underline{Z}_1}{\underline{Z}_1 + \underline{Z}_2} & \frac{1}{\underline{Z}_1 + \underline{Z}_2} \end{pmatrix}$	$\begin{pmatrix} \frac{1}{\underline{Z}_1} & -1 \\ 1 & \underline{Z}_2 \end{pmatrix}$

Γ-Vierpol II 	(**Y**) $\begin{matrix} \frac{1}{\underline{Z}_1} & -\frac{1}{\underline{Z}_1} \\ -\frac{1}{\underline{Z}_1} & \frac{1}{\underline{Z}_1}+\frac{1}{\underline{Z}_2} \end{matrix}$	(**Z**) $\begin{matrix} \underline{Z}_1+\underline{Z}_2 & \underline{Z}_2 \\ \underline{Z}_2 & \underline{Z}_2 \end{matrix}$
(**A**) $\begin{matrix} 1+\frac{\underline{Z}_1}{\underline{Z}_2} & \underline{Z}_1 \\ \frac{1}{\underline{Z}_2} & 1 \end{matrix}$	(**H**) $\begin{matrix} \underline{Z}_1 & 1 \\ -1 & \frac{1}{\underline{Z}_2} \end{matrix}$	(**C**) $\begin{matrix} \frac{1}{\underline{Z}_1+\underline{Z}_2} & -\frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \\ \frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} & \frac{\underline{Z}_1\cdot\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \end{matrix}$
T-Schaltung  mit $\underline{K}=\underline{Z}_1\underline{Z}_2+\underline{Z}_1\underline{Z}_3+\underline{Z}_2\underline{Z}_3$	(**Y**) $\begin{matrix} \frac{\underline{Z}_2+\underline{Z}_3}{\underline{K}} & -\frac{\underline{Z}_2}{\underline{K}} \\ -\frac{\underline{Z}_2}{\underline{K}} & \frac{\underline{Z}_1+\underline{Z}_2}{\underline{K}} \end{matrix}$	(**Z**) $\begin{matrix} \underline{Z}_1+\underline{Z}_2 & \underline{Z}_2 \\ \underline{Z}_2 & \underline{Z}_2+\underline{Z}_3 \end{matrix}$
(**A**) $\begin{matrix} 1+\frac{\underline{Z}_1}{\underline{Z}_2} & \underline{Z}_1+\underline{Z}_3+\frac{\underline{Z}_1\underline{Z}_3}{\underline{Z}_2} \\ \frac{1}{\underline{Z}_2} & 1+\frac{\underline{Z}_3}{\underline{Z}_2} \end{matrix}$	(**H**) $\begin{matrix} \frac{\underline{K}}{\underline{Z}_2+\underline{Z}_3} & \frac{\underline{Z}_2}{\underline{Z}_2+\underline{Z}_3} \\ -\frac{\underline{Z}_2}{\underline{Z}_2+\underline{Z}_3} & \frac{1}{\underline{Z}_2+\underline{Z}_3} \end{matrix}$	(**C**) $\begin{matrix} \frac{1}{\underline{Z}_1+\underline{Z}_2} & -\frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \\ \frac{\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} & \underline{Z}_3+\frac{\underline{Z}_1\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \end{matrix}$
π-Schaltung 	(**Y**) $\begin{matrix} \frac{1}{\underline{Z}_1}+\frac{1}{\underline{Z}_2} & -\frac{1}{\underline{Z}_2} \\ -\frac{1}{\underline{Z}_2} & \frac{1}{\underline{Z}_2}+\frac{1}{\underline{Z}_3} \end{matrix}$	(**Z**) $\begin{matrix} \frac{\underline{Z}_1(\underline{Z}_2+\underline{Z}_3)}{\underline{Z}_1+\underline{Z}_2+\underline{Z}_3} & \frac{\underline{Z}_1\underline{Z}_3}{\underline{Z}_1+\underline{Z}_2+\underline{Z}_3} \\ \frac{\underline{Z}_1\underline{Z}_3}{\underline{Z}_1+\underline{Z}_2+\underline{Z}_3} & \frac{\underline{Z}_3(\underline{Z}_1+\underline{Z}_2)}{\underline{Z}_1+\underline{Z}_2+\underline{Z}_3} \end{matrix}$
(**A**) $\begin{matrix} 1+\frac{\underline{Z}_2}{\underline{Z}_3} & \underline{Z}_2 \\ \frac{1}{\underline{Z}_1}+\frac{1}{\underline{Z}_3}+\frac{\underline{Z}_2}{\underline{Z}_1\underline{Z}_3} & 1+\frac{\underline{Z}_2}{\underline{Z}_1} \end{matrix}$	(**H**) $\begin{matrix} \frac{\underline{Z}_1\cdot\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} & \frac{\underline{Z}_1}{\underline{Z}_1+\underline{Z}_2} \\ -\frac{\underline{Z}_1}{\underline{Z}_1+\underline{Z}_2} & \frac{\underline{Z}_1+\underline{Z}_2+\underline{Z}_3}{\underline{Z}_3(\underline{Z}_1+\underline{Z}_2)} \end{matrix}$	(**C**) $\begin{matrix} \frac{\underline{Z}_1+\underline{Z}_2+\underline{Z}_3}{\underline{Z}_1(\underline{Z}_2+\underline{Z}_3)} & -\frac{\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3} \\ \frac{\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3} & \frac{\underline{Z}_2\underline{Z}_3}{\underline{Z}_2+\underline{Z}_3} \end{matrix}$

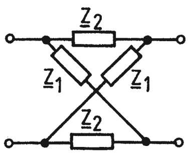
Umpoler



	(**Y**)	(**Z**)
	existiert nicht	existiert nicht

(**A**)	(**H**)	(**C**)
$\begin{matrix} -1 & 0 \\ 0 & -1 \end{matrix}$	$\begin{matrix} 0 & -1 \\ 1 & 0 \end{matrix}$	$\begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}$

Symmetrische X-Schaltung



(**Y**)

$$\begin{matrix} \frac{1}{2}\left(\frac{1}{\underline{Z}_1}+\frac{1}{\underline{Z}_2}\right) & \frac{1}{2}\left(\frac{1}{\underline{Z}_1}-\frac{1}{\underline{Z}_2}\right) \\ \frac{1}{2}\left(\frac{1}{\underline{Z}_1}-\frac{1}{\underline{Z}_2}\right) & \frac{1}{2}\left(\frac{1}{\underline{Z}_1}+\frac{1}{\underline{Z}_2}\right) \end{matrix}$$

(**Z**)

$$\begin{matrix} \frac{1}{2}(\underline{Z}_1+\underline{Z}_2) & \frac{1}{2}(\underline{Z}_1-\underline{Z}_2) \\ \frac{1}{2}(\underline{Z}_1-\underline{Z}_2) & \frac{1}{2}(\underline{Z}_1+\underline{Z}_2) \end{matrix}$$

(**A**)

$$\begin{matrix} \frac{\underline{Z}_1+\underline{Z}_2}{\underline{Z}_1-\underline{Z}_2} & \frac{2\cdot\underline{Z}_1\cdot\underline{Z}_2}{\underline{Z}_1-\underline{Z}_2} \\ \frac{2}{\underline{Z}_1-\underline{Z}_2} & \frac{\underline{Z}_1+\underline{Z}_2}{\underline{Z}_1-\underline{Z}_2} \end{matrix}$$

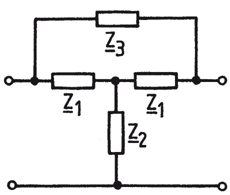
(**H**)

$$\begin{matrix} \frac{2\cdot\underline{Z}_1\cdot\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} & \frac{\underline{Z}_1-\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \\ -\frac{\underline{Z}_1-\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} & \frac{2}{\underline{Z}_1+\underline{Z}_2} \end{matrix}$$

(**C**)

$$\begin{matrix} \frac{2}{\underline{Z}_1+\underline{Z}_2} & -\frac{\underline{Z}_1-\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \\ \frac{\underline{Z}_1-\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} & \frac{2\cdot\underline{Z}_1\cdot\underline{Z}_2}{\underline{Z}_1+\underline{Z}_2} \end{matrix}$$

Symmetrischer Brücken-T-Vierpol



(**Y**)

$$\begin{matrix} \frac{\underline{Z}_1+\underline{Z}_2}{\underline{Z}_1^{\,2}+2\cdot\underline{Z}_1\cdot\underline{Z}_2}+\frac{1}{\underline{Z}_3} & -\left(\frac{\underline{Z}_2}{\underline{Z}_1^{\,2}+2\cdot\underline{Z}_1\cdot\underline{Z}_2}+\frac{1}{\underline{Z}_3}\right) \\ -\left(\frac{\underline{Z}_2}{\underline{Z}_1^{\,2}+2\cdot\underline{Z}_1\cdot\underline{Z}_2}+\frac{1}{\underline{Z}_3}\right) & \frac{\underline{Z}_1+\underline{Z}_2}{\underline{Z}_1^{\,2}+2\cdot\underline{Z}_1\cdot\underline{Z}_2}+\frac{1}{\underline{Z}_3} \end{matrix}$$

(**Z**)

$$\begin{matrix} \frac{\underline{Z}_1^{\,2}+\underline{Z}_1\cdot\underline{Z}_3}{2\cdot\underline{Z}_1+\underline{Z}_3}+\underline{Z}_2 & \frac{\underline{Z}_1^{\,2}}{2\cdot\underline{Z}_1+\underline{Z}_3}+\underline{Z}_2 \\ \frac{\underline{Z}_1^{\,2}}{2\cdot\underline{Z}_1+\underline{Z}_3}+\underline{Z}_2 & \frac{\underline{Z}_1^{\,2}+\underline{Z}_1\cdot\underline{Z}_3}{2\cdot\underline{Z}_1+\underline{Z}_3}+\underline{Z}_2 \end{matrix}$$

10.4 Betriebskenngrößen von Vierpolen

(Band 3, S.189-202)

Kenngrößen eines Vierpols im Vorwärtsbetrieb

Betriebskenngröße		Leerlauf	Kurzschluss
Eingangsleitwert	$\underline{Y}_{in} = \frac{\underline{I}_1}{\underline{U}_1}$	$\underline{C}_{11}$	$\underline{Y}_{11}$
Eingangswiderstand	$\underline{Z}_{in} = \frac{\underline{U}_1}{\underline{I}_1}$	$\underline{Z}_{11}$	$\underline{H}_{11}$
Übertragungsleitwert vorwärts	$\underline{Y}_{üf} = \frac{\underline{I}_2}{\underline{U}_1}$	0	$\underline{Y}_{21} = -\frac{1}{\underline{A}_{12}}$
Übertragungswiderstand vorwärts	$\underline{Z}_{üf} = \frac{\underline{U}_2}{\underline{I}_1}$	$\underline{Z}_{21} = \frac{1}{\underline{A}_{21}}$	0
Spannungsübersetzung vorwärts	$\underline{V}_{uf} = \frac{\underline{U}_2}{\underline{U}_1}$	$\underline{C}_{21} = \frac{1}{\underline{A}_{11}}$	0
Stromübersetzung vorwärts	$\underline{V}_{if} = \frac{\underline{I}_2}{\underline{I}_1}$	0	$\underline{H}_{21} = -\frac{1}{\underline{A}_{22}}$

Kenngrößen eines Vierpols im Rückwärtsbetrieb

Betriebskenngröße		Leerlauf	Kurzschluss
Ausgangsleitwert	$\underline{Y}_{out} = \frac{\underline{I}_2}{\underline{U}_2}$	$\underline{H}_{22}$	$\underline{Y}_{22}$
Ausgangswiderstand	$\underline{Z}_{out} = \frac{\underline{U}_2}{\underline{I}_2}$	$\underline{Z}_{22}$	$\underline{C}_{22}$
Übertragungsleitwert rückwärts	$\underline{Y}_{ür} = \frac{\underline{I}_1}{\underline{U}_2}$	0	$\underline{Y}_{12}$
Übertragungswiderstand rückwärts	$\underline{Z}_{ür} = \frac{\underline{U}_1}{\underline{I}_2}$	$\underline{Z}_{12}$	0
Spannungsrückwirkung	$\underline{V}_{ur} = \frac{\underline{U}_1}{\underline{U}_2}$	$\underline{H}_{12}$	0
Stromrückwirkung	$\underline{V}_{ir} = \frac{\underline{I}_1}{\underline{I}_2}$	0	$\underline{C}_{12}$

Kenngrößen des beschalteten Vierpols im Vorwärtsbetrieb

	($\underline{Y}$)	($\underline{Z}$)	($\underline{H}$)	($\underline{C}$)	($\underline{A}$)
$\underline{Y}_{in}$	$\frac{\det\underline{Y} + \underline{Y}_{11}\cdot\underline{Y}_a}{\underline{Y}_{22} + \underline{Y}_a}$	$\frac{1+\underline{Z}_{22}\cdot\underline{Y}_a}{\underline{Z}_{11}+\underline{Y}_a\cdot\det\underline{Z}}$	$\frac{\underline{H}_{22}+\underline{Y}_a}{\det\underline{H}+\underline{H}_{11}\cdot\underline{Y}_a}$	$\frac{\underline{C}_{11}+\underline{Y}_a\cdot\det\underline{C}}{1+\underline{C}_{22}\cdot\underline{Y}_a}$	$\frac{\underline{A}_{21}+\underline{A}_{22}\cdot\underline{Y}_a}{\underline{A}_{11}+\underline{A}_{12}\cdot\underline{Y}_a}$
$\underline{Z}_{in}$	$\frac{\underline{Y}_{22} + \underline{Y}_a}{\det\underline{Y} + \underline{Y}_{11}\cdot\underline{Y}_a}$	$\frac{\underline{Z}_{11}+\underline{Y}_a\cdot\det\underline{Z}}{1+\underline{Z}_{22}\cdot\underline{Y}_a}$	$\frac{\det\underline{H}+\underline{H}_{11}\cdot\underline{Y}_a}{\underline{H}_{22}+\underline{Y}_a}$	$\frac{1+\underline{C}_{22}\cdot\underline{Y}_a}{\underline{C}_{11}+\underline{Y}_a\cdot\det\underline{C}}$	$\frac{\underline{A}_{11}+\underline{A}_{12}\cdot\underline{Y}_a}{\underline{A}_{21}+\underline{A}_{22}\cdot\underline{Y}_a}$
$\underline{Y}_{üf}$	$\frac{\underline{Y}_{21}\cdot\underline{Y}_a}{\underline{Y}_{22}+\underline{Y}_a}$	$\frac{-\underline{Z}_{21}\cdot\underline{Y}_a}{\underline{Z}_{11}+\underline{Y}_a\cdot\det\underline{Z}}$	$\frac{\underline{H}_{21}\cdot\underline{Y}_a}{\det\underline{H}+\underline{H}_{11}\cdot\underline{Y}_a}$	$\frac{-\underline{C}_{21}\cdot\underline{Y}_a}{1+\underline{C}_{22}\cdot\underline{Y}_a}$	$\frac{-\underline{Y}_a}{\underline{A}_{11}+\underline{A}_{12}\cdot\underline{Y}_a}$
$\underline{Z}_{üf}$	$\frac{-\underline{Y}_{21}}{\det\underline{Y}+\underline{Y}_{11}\cdot\underline{Y}_a}$	$\frac{\underline{Z}_{21}}{1+\underline{Z}_{22}\cdot\underline{Y}_a}$	$\frac{-\underline{H}_{21}}{\underline{H}_{22}+\underline{Y}_a}$	$\frac{\underline{C}_{21}}{\underline{C}_{11}+\underline{Y}_a\cdot\det\underline{C}}$	$\frac{1}{\underline{A}_{21}+\underline{A}_{22}\cdot\underline{Y}_a}$
$\underline{V}_{uf}$	$\frac{-\underline{Y}_{21}}{\underline{Y}_{22}+\underline{Y}_a}$	$\frac{\underline{Z}_{21}}{\underline{Z}_{11}+\underline{Y}_a\cdot\det\underline{Z}}$	$\frac{-\underline{H}_{21}}{\det\underline{H}+\underline{H}_{11}\cdot\underline{Y}_a}$	$\frac{\underline{C}_{21}}{1+\underline{C}_{22}\cdot\underline{Y}_a}$	$\frac{1}{\underline{A}_{11}+\underline{A}_{12}\cdot\underline{Y}_a}$
$\underline{V}_{if}$	$\frac{\underline{Y}_{21}\cdot\underline{Y}_a}{\det\underline{Y}+\underline{Y}_{11}\cdot\underline{Y}_a}$	$\frac{-\underline{Z}_{21}\cdot\underline{Y}_a}{1+\underline{Z}_{22}\cdot\underline{Y}_a}$	$\frac{\underline{H}_{21}\cdot\underline{Y}_a}{\underline{H}_{22}+\underline{Y}_a}$	$\frac{-\underline{C}_{21}\cdot\underline{Y}_a}{\underline{C}_{11}+\underline{Y}_a\cdot\det\underline{C}}$	$\frac{-\underline{Y}_a}{\underline{A}_{21}+\underline{A}_{22}\cdot\underline{Y}_a}$

Kenngrößen des beschalteten Vierpols im Rückwärtsbetrieb

	($\underline{Y}$)	($\underline{Z}$)	($\underline{H}$)	($\underline{C}$)	($\underline{A}$)
$\underline{Y}_{out}$	$\frac{\det\underline{Y}+\underline{Y}_{22}\cdot\underline{Y}_i}{\underline{Y}_{11}+\underline{Y}_i}$	$\frac{1+\underline{Z}_{11}\cdot\underline{Y}_i}{\underline{Z}_{22}+\underline{Y}_i\cdot\det\underline{Z}}$	$\frac{\underline{H}_{22}+\underline{Y}_i\cdot\det\underline{H}}{1+\underline{H}_{11}\cdot\underline{Y}_i}$	$\frac{\underline{C}_{11}+\underline{Y}_i}{\det\underline{C}+\underline{C}_{22}\cdot\underline{Y}_i}$	$\frac{\underline{A}_{21}+\underline{A}_{11}\cdot\underline{Y}_i}{\underline{A}_{22}+\underline{A}_{12}\cdot\underline{Y}_i}$
$\underline{Z}_{out}$	$\frac{\underline{Y}_{11}+\underline{Y}_i}{\det\underline{Y}+\underline{Y}_{22}\cdot\underline{Y}_i}$	$\frac{\underline{Z}_{22}+\underline{Y}_i\cdot\det\underline{Z}}{1+\underline{Z}_{11}\cdot\underline{Y}_i}$	$\frac{1+\underline{H}_{11}\cdot\underline{Y}_i}{\underline{H}_{22}+\underline{Y}_i\cdot\det\underline{H}}$	$\frac{\det\underline{C}+\underline{C}_{22}\cdot\underline{Y}_i}{\underline{C}_{11}+\underline{Y}_i}$	$\frac{\underline{A}_{22}+\underline{A}_{12}\cdot\underline{Y}_i}{\underline{A}_{21}+\underline{A}_{11}\cdot\underline{Y}_i}$
$\underline{Y}_{ür}$	$\frac{\underline{Y}_{12}\cdot\underline{Y}_i}{\underline{Y}_{11}+\underline{Y}_i}$	$\frac{-\underline{Z}_{12}\cdot\underline{Y}_i}{\underline{Z}_{22}+\underline{Y}_i\cdot\det\underline{Z}}$	$\frac{-\underline{H}_{12}\cdot\underline{Y}_i}{1+\underline{H}_{11}\cdot\underline{Y}_i}$	$\frac{\underline{C}_{12}\cdot\underline{Y}_i}{\det\underline{C}+\underline{C}_{22}\cdot\underline{Y}_i}$	$\frac{-\underline{Y}_i\cdot\det\underline{A}}{\underline{A}_{22}+\underline{A}_{12}\cdot\underline{Y}_i}$
$\underline{Z}_{ür}$	$\frac{-\underline{Y}_{12}}{\det\underline{Y}+\underline{Y}_{22}\cdot\underline{Y}_i}$	$\frac{\underline{Z}_{12}}{1+\underline{Z}_{11}\cdot\underline{Y}_i}$	$\frac{\underline{H}_{12}}{\underline{H}_{22}+\underline{Y}_i\cdot\det\underline{H}}$	$\frac{-\underline{C}_{12}}{\underline{C}_{11}+\underline{Y}_i}$	$\frac{\det\underline{A}}{\underline{A}_{21}+\underline{A}_{11}\cdot\underline{Y}_i}$
$\underline{V}_{ur}$	$\frac{-\underline{Y}_{12}}{\underline{Y}_{11}+\underline{Y}_i}$	$\frac{\underline{Z}_{12}}{\underline{Z}_{22}+\underline{Y}_i\cdot\det\underline{Z}}$	$\frac{\underline{H}_{12}}{1+\underline{H}_{11}\cdot\underline{Y}_i}$	$\frac{-\underline{C}_{12}}{\det\underline{C}+\underline{C}_{22}\cdot\underline{Y}_i}$	$\frac{\det\underline{A}}{\underline{A}_{22}+\underline{A}_{12}\cdot\underline{Y}_i}$
$\underline{V}_{ir}$	$\frac{\underline{Y}_{12}\cdot\underline{Y}_i}{\det\underline{Y}+\underline{Y}_{22}\cdot\underline{Y}_i}$	$\frac{-\underline{Z}_{12}\cdot\underline{Y}_i}{1+\underline{Z}_{11}\cdot\underline{Y}_i}$	$\frac{-\underline{H}_{12}\cdot\underline{Y}_i}{\underline{H}_{22}+\underline{Y}_i\cdot\det\underline{H}}$	$\frac{\underline{C}_{12}\cdot\underline{Y}_i}{\underline{C}_{11}+\underline{Y}_i}$	$\frac{-\underline{Y}_i\cdot\det\underline{A}}{\underline{A}_{21}+\underline{A}_{11}\cdot\underline{Y}_i}$