17.5/20

Orbital Simulation and Analysis

This notebook simulates and analyzes the orbits of celestial bodies under classical and relativistic mechanics. It includes:

1. **Classical vs. Relativistic Orbit Comparison**: Simulates and compares the trajectories of objects under classical and relativistic gravitational models.
2. **Deviation Analysis**: Calculates and visualizes the differences between classical and relativistic orbits over time.
3. **Multi-Object Simulation**: Simulates orbits of multiple celestial bodies with varying eccentricities and visualizes their trajectories on different scales.

The simulations use numerical integration methods and visualize the results to provide insights into orbital dynamics.

Module design (1 point):

-0.25 (a) Read the instructions below and clearly outline the directory structure of your module in an **analysis.ipynb** notebook. Follow the class notes on how to structure python packages.

Not provided. Answer not provided as requested.
I found the directory structure in README.md

Code development (8 points):

Create a single python script/module **orbits.py**, adequately organised in classes and functions, that:

✓ (b) initialises the two-body problem on a 2D Cartesian grid with an option to save the initial map (if the user wishes to do so). Use the Argparse Library to facilitate user customisation. The grid should be in astronomical units, AU, and a circle denoting the Schwarzschild radius of the black hole should be added.

-0.25 Canvas does not close.

✓ (cx2) includes three ODE integration methods: two own-developed methods to carry out the **Trapezoidal Euler** and **Runge-Kutta 3** integrations, and one that uses higher order **SciPy integrators** for initial value problems. Good class structure.

✓ (dx2) includes a function for the **relativistic** and **classical** slopes given by the above equations of motion. The user should be able to select which slope to use (relativistic or classical).

Fix pylint complaints. See below.

-0.5

✓ (e) includes a **run class** to integrate the above system of ODEs for N orbital periods and saves the history of the planet's orbital motion around the black hole into an output file inside an **outputfolder**. **Note:** Both ODEs need to be integrated simultaneously, so you don't need separate functions for the integration of each.

(f) includes an **animation class** that reads the planet's orbital history and returns a GIF animation containing the planet position and velocity at different times. The user should be able to turn on a flag at runtime to indicate if the GIF animation is desired. Use the Argparse Library to add this functionality.

(g) accepts as inputs from the user: e, M, a, N, and the numerical method to update the ODE system. Use the Argparse Library to add this functionality. **Note:** Please provide an example of how I should execute your code in the README file.

pylint output:

```
pylint orbits.py
************* Module orbits.orbits
orbits.py:111:0: C0301: Line too long (102/100) (line-too-long)
orbits.py:137:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:141:80: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:176:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:180:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:184:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:208:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:212:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:215:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:218:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:222:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:234:0: C0301: Line too long (104/100) (line-too-long)
orbits.py:246:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:249:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:252:0: C0301: Line too long (106/100) (line-too-long)
orbits.py:255:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:258:75: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:283:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:288:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:292:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:296:71: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:320:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:327:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:331:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:335:88: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:336:0: C0301: Line too long (101/100) (line-too-long)
orbits.py:362:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:367:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:371:79: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:372:69: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:383:0: C0301: Line too long (115/100) (line-too-long)
orbits.py:388:0: C0301: Line too long (102/100) (line-too-long)
orbits.py:389:0: C0301: Line too long (110/100) (line-too-long)
orbits.py:393:0: C0301: Line too long (103/100) (line-too-long)
```

```
orbits.py:404:59: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:410:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:416:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:420:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:440:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:445:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:447:95: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:472:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:473:98: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:480:0: C0301: Line too long (114/100) (line-too-long)
orbits.py:481:0: C0301: Line too long (111/100) (line-too-long)
orbits.py:497:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:514:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:528:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:531:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:534:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:538:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:542:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:546:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:550:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:552:60: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:554:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:557:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:560:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:567:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:568:77: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:571:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:581:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:590:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:592:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:609:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:616:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:619:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:624:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:626:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:642:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:648:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:651:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:657:22: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:660:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:665:0: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:693:0: C0301: Line too long (101/100) (line-too-long)
orbits.py:694:0: C0301: Line too long (102/100) (line-too-long)
orbits.py:695:66: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:697:66: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:699:74: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:701:65: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:703:60: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:705:63: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:707:66: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:710:60: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:723:72: C0303: Trailing whitespace (trailing-whitespace)
```

```
orbits.py:735:69: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:741:46: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:742:79: C0303: Trailing whitespace (trailing-whitespace)
orbits.py:749:0: C0304: Final newline missing (missing-final-
newline)
orbits.py:52:0: R0402: Use 'from matplotlib import animation'
instead (consider-using-from-import)
orbits.py:97:31: C0103: Argument name "M" doesn't conform to
snake_case naming style (invalid-name)
orbits.py:97:61: C0103: Argument name "N" doesn't conform to
snake_case naming style (invalid-name)
orbits.py:97:4: R0913: Too many arguments (6/5) (too-many-
arguments)
orbits.py:97:4: R0917: Too many positional arguments (6/5) (too-
many-positional-arguments)
orbits.py:139:8: W0612: Unused variable 'fig' (unused-variable)
orbits.py:156:55: C0103: Argument name "M" doesn't conform to
snake_case naming style (invalid-name)
orbits.py:156:30: W0613: Unused argument 't' (unused-argument)
orbits.py:185:58: C0103: Argument name "M" doesn't conform to
snake_case naming style (invalid-name)
orbits.py:185:33: W0613: Unused argument 't' (unused-argument)
orbits.py:251:18: W1309: Using an f-string that does not have any
interpolated variables (f-string-without-interpolation)
orbits.py:253:18: W1309: Using an f-string that does not have any
interpolated variables (f-string-without-interpolation)
orbits.py:335:4: R0913: Too many arguments (6/5) (too-many-
arguments)
orbits.py:335:4: R0917: Too many positional arguments (6/5) (too-
many-positional-arguments)
orbits.py:364:18: W0613: Unused argument 't' (unused-argument)
orbits.py:371:4: R0913: Too many arguments (8/5) (too-many-
arguments)
orbits.py:371:4: R0917: Too many positional arguments (8/5) (too-
many-positional-arguments)
orbits.py:406:8: C0103: Variable name "M" doesn't conform to
snake_case naming style (invalid-name)
orbits.py:454:0: R0902: Too many instance attributes (13/7) (too-
many-instance-attributes)
orbits.py:473:4: R0913: Too many arguments (8/5) (too-many-
arguments)
orbits.py:473:4: R0917: Too many positional arguments (8/5) (too-
many-positional-arguments)
orbits.py:515:4: R0914: Too many local variables (17/15) (too-many-
locals)
orbits.py:530:8: W0201: Attribute 'fig' defined outside __init__
(attribute-defined-outside-init)
orbits.py:530:18: W0201: Attribute 'ax' defined outside __init__
(attribute-defined-outside-init)
orbits.py:536:8: W0201: Attribute 'planets' defined outside
__init__ (attribute-defined-outside-init)
orbits.py:537:8: W0201: Attribute 'trails' defined outside __init__
```

```
(attribute-defined-outside-init)
orbits.py:454:0: R0903: Too few public methods (1/2) (too-few-public-methods)
orbits.py:741:8: W0621: Redefining name 'animation' from outer scope (line 52) (redefined-outer-name)
orbits.py:47:0: C0411: standard import "argparse" should be placed before third party import "numpy" (wrong-import-order)
orbits.py:50:0: C0411: standard import "typing.Callable" should be placed before third party imports "numpy", "scipy.integrate.solve_ivp", "matplotlib.pyplot" (wrong-import-order)
orbits.py:52:0: C0412: Imports from package matplotlib are not grouped (ungrouped-imports)
orbits.py:50:0: W0611: Unused List imported from typing (unused-import)
orbits.py:50:0: W0611: Unused Union imported from typing (unused-import)
orbits.py:54:0: W0611: Unused Image imported from PIL (unused-import)

------------------------------------------------------------------
Your code has been rated at 4.79/10 (previous run: 4.79/10, +0.00)
```

Unit tests (2 points):

(h) Create a **test_orbits.py** file containing `pytest` unit tests. Provide 3 examples of pytest unit tests that could verify: a) correct input values from the user, b) handling of invalid input methods, and c) whether different inputs actually lead to different outputs.

-0.25

Fix pylint complaints. See below.

a) Value Error raised with negative mass, but a nan is computed?
b) Not correctly implemented. setup/teardown are missing. -0.5
c) Ok.

pylint output:

```
pylint test_orbits.py
************* Module tests.test_orbits
test_orbits.py:21:0: C0303: Trailing whitespace (trailing-whitespace)
test_orbits.py:27:0: C0303: Trailing whitespace (trailing-whitespace)
test_orbits.py:32:0: C0303: Trailing whitespace (trailing-whitespace)
test_orbits.py:45:40: C0303: Trailing whitespace (trailing-whitespace)
test_orbits.py:47:19: C0303: Trailing whitespace (trailing-whitespace)
test_orbits.py:57:0: C0303: Trailing whitespace (trailing-whitespace)
test_orbits.py:60:0: C0303: Trailing whitespace (trailing-whitespace)
test_orbits.py:70:31: C0303: Trailing whitespace (trailing-whitespace)
test_orbits.py:72:15: C0303: Trailing whitespace (trailing-whitespace)
```

```
test_orbits.py:75:0: C0303: Trailing whitespace (trailing-
whitespace)
test_orbits.py:77:31: C0303: Trailing whitespace (trailing-
whitespace)
test_orbits.py:79:15: C0303: Trailing whitespace (trailing-
whitespace)
test_orbits.py:85:0: C0303: Trailing whitespace (trailing-
whitespace)
test_orbits.py:89:0: C0304: Final newline missing (missing-final-
newline)
test_orbits.py:1:0: C0114: Missing module docstring (missing-
module-docstring)
test_orbits.py:11:0: C0116: Missing function or method docstring
(missing-function-docstring)
test_orbits.py:37:36: W0621: Redefining name 'earth_orbit' from
outer scope (line 11) (redefined-outer-name)

-----------------------------------
Your code has been rated at 5.75/10
```

docstring missing in:
def earth_orbit():

pytest output:

Canvas are not closed.

-0.25

```
pytest test_orbits.py
===================================================================
test session starts
====================================================================
platform darwin -- Python 3.9.18, pytest-8.3.4, pluggy-1.5.0
rootdir:
/Users/wbandabarragan/Library/CloudStorage/Dropbox/Yachay_Tech/Semestre
plugins: anyio-4.7.0
collected 3 items

test_orbits.py ...
[100%]

====================================================================
3 passed in 8.26s
=====================================================================
```

i) Use your module/script to run and show two simulations: one relativistic and one classical for this set of initial conditions. It may be helpful to compare the orbital history in a single plot.

Parameter	Description		Units
e	Eccentricity of the orbit	0	

Parameter	Description	Units
M	Mass of the central black hole	$5 \times 10^6\ \mathrm{M}_\odot$
a	Semi-major axis of the orbit	$1\ \mathrm{AU}$
N	Number of orbital periods to simulate	2
Method	Numerical method for ODE integration	RK3

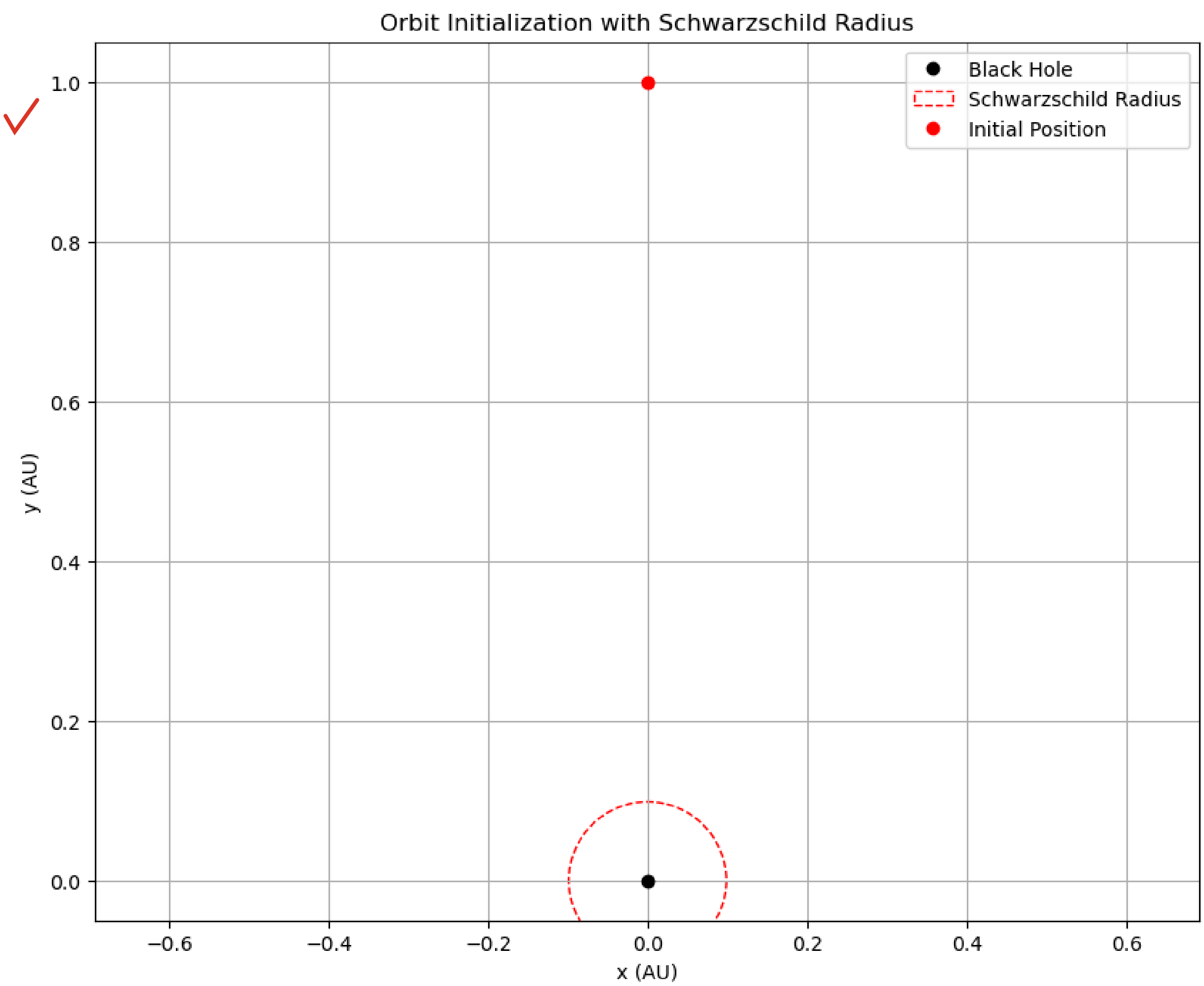
In [1]:

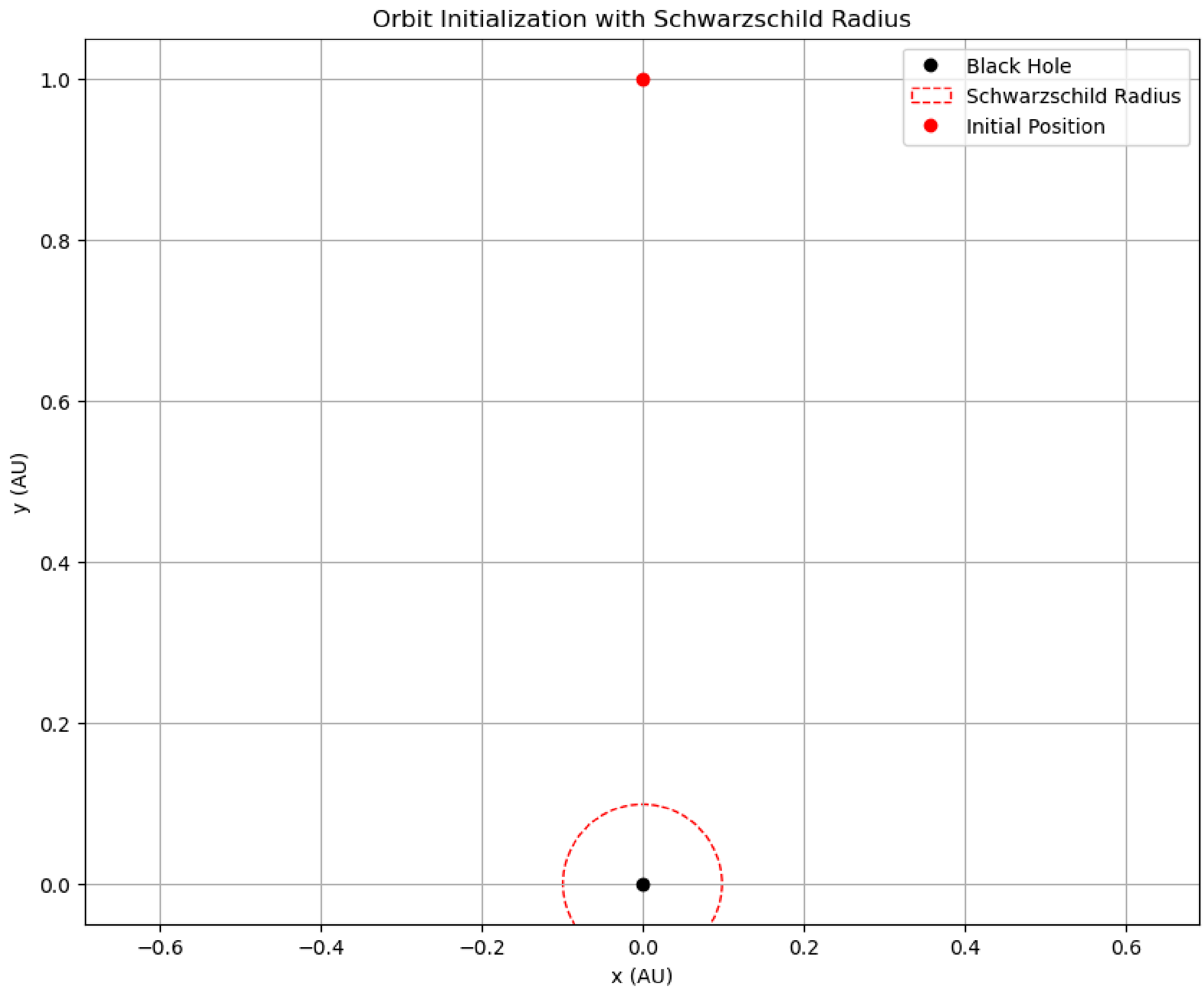
```python
from orbits import Orbits
from orbits import OrbitAnimation
import matplotlib.pyplot as plt
import numpy as np
from PIL import Image
import pandas as pd
import time
from matplotlib.ticker import ScalarFormatter
import os
```

In [2]:

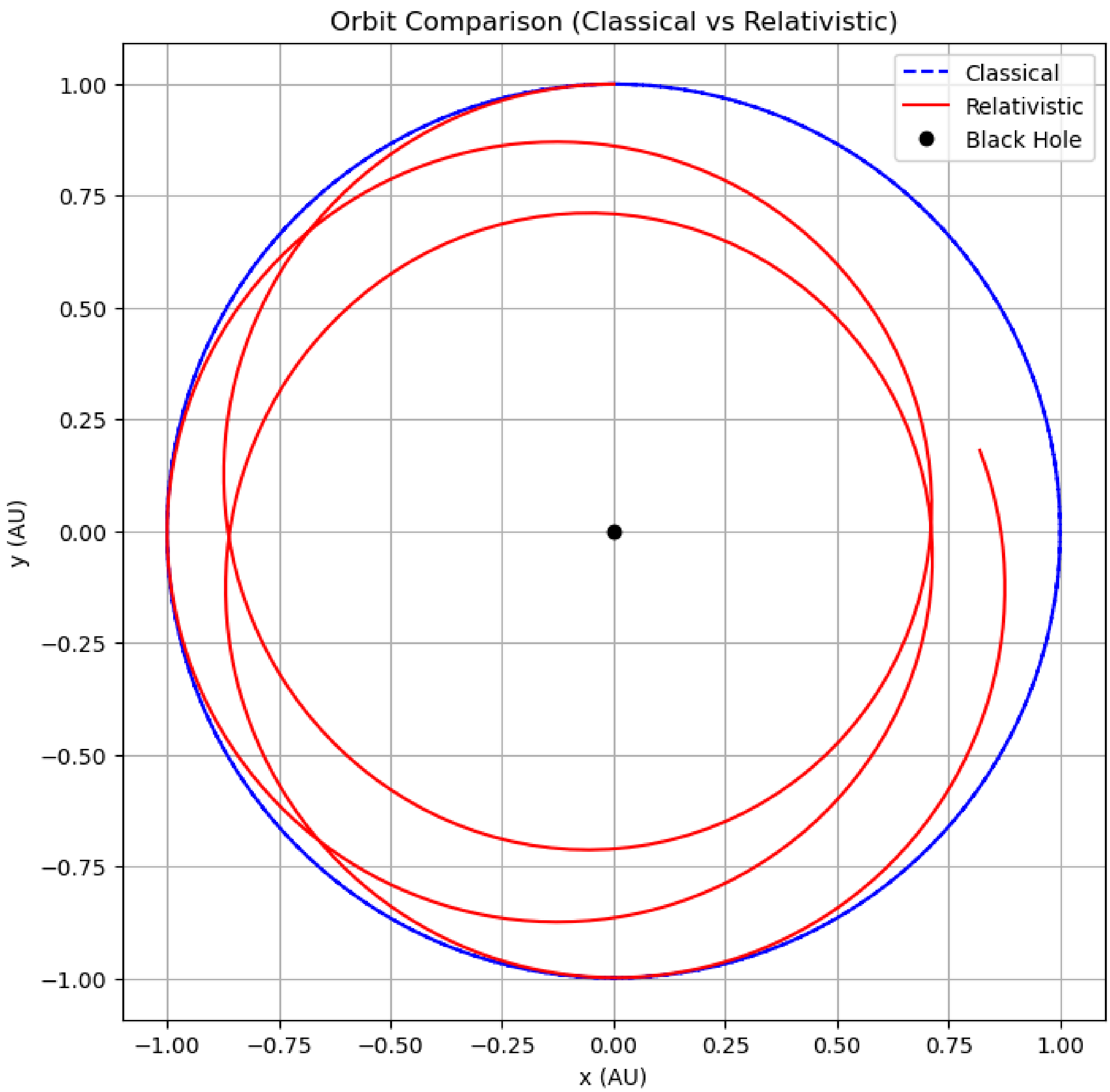
```python
# Constants
AU = 1.496e11  # meters
M_sun = 1.989e30  # kilograms
M = 5e6 * M_sun  # Mass of the central black hole
a = 1 * AU  # Semi-major axis
e = 0  # Eccentricity
N = 2  # Number of orbital periods

# Initialize orbits
orbit_classical = Orbits("classical")
orbit_classical.initialize_orbit(M, a, e, N)

orbit_relativistic = Orbits("relativistic")
orbit_relativistic.initialize_orbit(M, a, e, N)

# Calculate orbital period (Kepler's law)
T = 2 * np.pi * np.sqrt(a**3 / (6.67430e-11 * M))
t_span = (0, N * T)
t_eval = np.linspace(0, N * T, 400)

# Classical simulation
y_classical = orbit_classical.run_simulation(
    slope_function=orbit_classical.classical_slope,
    integration_method="rk3",
    t_span=t_span,
    t_eval=t_eval,
```

```
    output_folder="output"
)

# Relativistic simulation
y_relativistic = orbit_relativistic.run_simulation(
    slope_function=orbit_relativistic.relativistic_slope,
    integration_method="rk3",
    t_span=t_span,
    t_eval=t_eval,
    output_folder="output"
)

# Plot both orbits
plt.figure(figsize=(8, 8))
plt.plot(y_classical[:, 0], y_classical[:, 1], label="Classical", linestyle=
plt.plot(y_relativistic[:, 0], y_relativistic[:, 1], label="Relativistic", c
plt.plot(0, 0, 'ko', label="Black Hole")
plt.xlabel("x (AU)")
plt.ylabel("y (AU)")
plt.title("Orbit Comparison (Classical vs Relativistic)")
plt.legend(loc="upper right")
plt.grid(True)
plt.axis("equal")
plt.show()
```






```
Simulation results saved to output/classical_orbit_history.csv
Simulation results saved to output/relativistic_orbit_history.csv
```



In [3]:
```
csv_files = ["./output/classical_orbit_history.csv", "./output/relativistic_
output_filename = "orbits_i_animation.gif"
output_folder = "./output"

animation = OrbitAnimation(csv_files, output_filename=output_filename, outpu
animation.animate()

gif = Image.open('./output/orbits_i_animation.gif')
# gif.show()
```

```
Animation saved to: ./output/orbits_i_animation.gif
```

(j) Use the orbital history of both simulations to design a method that quantifies their differences and evaluates the importance of using the relativistic approach for massive objects. Do we need to worry about the relativistic corrections if we replace the black hole with our Sun?

In [4]:
```
# Set constants
AU = 1.496e11  # Astronomical Unit in meters
```

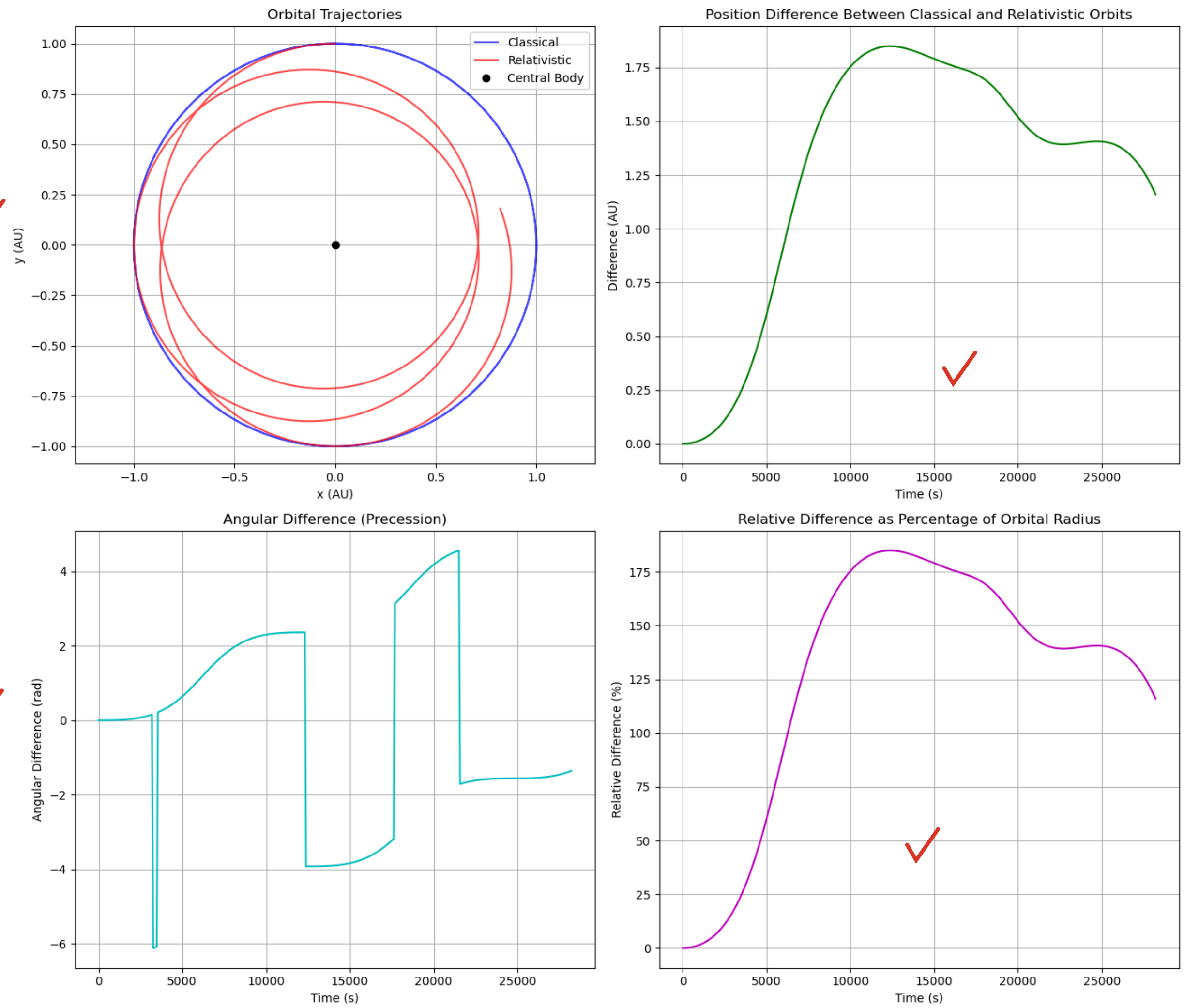
```python
M_sun = 1.989e30  # Mass of the Sun in kg
G = 6.67430e-11  # Gravitational constant
C = 3e8  # Speed of light in m/s

# Load the data from CSV files
classical_data = pd.read_csv('./output/classical_orbit_history.csv')
relativistic_data = pd.read_csv('./output/relativistic_orbit_history.csv')

print(f"Classical simulation: {len(classical_data)} timesteps")
print(f"Relativistic simulation: {len(relativistic_data)} timesteps")

# Calculate the Euclidean distance between the two trajectories at each time
position_diff = np.sqrt(
    (classical_data['x'] - relativistic_data['x'])**2 +
    (classical_data['y'] - relativistic_data['y'])**2
)

# Convert the difference to Astronomical Units
position_diff_AU = position_diff

# Calculate angular difference (precession)
def calculate_angle(x, y):
    return np.arctan2(y, x)

classical_angles = calculate_angle(classical_data['x'], classical_data['y'])
relativistic_angles = calculate_angle(relativistic_data['x'], relativistic_c
angular_diff = relativistic_angles - classical_angles

# Calculate relative difference as a percentage of orbital radius
# Using average radius as reference
avg_radius_classical = np.mean(np.sqrt(classical_data['x']**2 + classical_da
relative_diff_percent = position_diff / avg_radius_classical * 100

# Calculate maximum and average differences
max_diff_AU = np.max(position_diff_AU)
avg_diff_AU = np.mean(position_diff_AU)
max_diff_percent = np.max(relative_diff_percent)
avg_diff_percent = np.mean(relative_diff_percent)

print(f"Maximum position difference: {max_diff_AU:.6e} AU ({max_diff_percent
print(f"Average position difference: {avg_diff_AU:.6e} AU ({avg_diff_percent
```

```
Classical simulation: 400 timesteps
Relativistic simulation: 400 timesteps
Maximum position difference: 1.848917e+00 AU (184.897703%)
Average position difference: 1.283104e+00 AU (128.314559%)
```

In [5]:

```python
# Set up a figure with 2x2 subplots
fig, axs = plt.subplots(2, 2, figsize=(14, 12))

# Plot 1: Orbital trajectories
axs[0, 0].plot(classical_data['x'], classical_data['y'], 'b-', label='Classi
axs[0, 0].plot(relativistic_data['x'], relativistic_data['y'], 'r-', label='
axs[0, 0].plot(0, 0, 'ko', label='Central Body')
axs[0, 0].set_xlabel('x (AU)')
axs[0, 0].set_ylabel('y (AU)')
```

```
axs[0, 0].set_title('Orbital Trajectories')
axs[0, 0].grid(True)
axs[0, 0].axis('equal')
axs[0, 0].legend()

# Plot 2: Position difference over time
axs[0, 1].plot(classical_data['time'], position_diff_AU, 'g-')
axs[0, 1].set_xlabel('Time (s)')
axs[0, 1].set_ylabel('Difference (AU)')
axs[0, 1].set_title('Position Difference Between Classical and Relativistic
axs[0, 1].grid(True)

# Plot 3: Angular difference (precession)
axs[1, 0].plot(classical_data['time'], angular_diff, 'c-')
axs[1, 0].set_xlabel('Time (s)')
axs[1, 0].set_ylabel('Angular Difference (rad)')
axs[1, 0].set_title('Angular Difference (Precession)')
axs[1, 0].grid(True)

# Plot 4: Relative difference as percentage
axs[1, 1].plot(classical_data['time'], relative_diff_percent, 'm-')
axs[1, 1].set_xlabel('Time (s)')
axs[1, 1].set_ylabel('Relative Difference (%)')
axs[1, 1].set_title('Relative Difference as Percentage of Orbital Radius')
axs[1, 1].grid(True)

plt.tight_layout()
plt.show()
```



In [6]:
```python
# Function to calculate theoretical precession per orbit
def calculate_perihelion_precession(M, a, e):
    """
    Calculate the perihelion precession per orbit in radians

    Args:
        M: Mass of central body in kg
        a: Semi-major axis in meters
        e: Eccentricity

    Returns:
        Precession per orbit in radians
    """
    # Formula for perihelion precession in General Relativity
    precession_per_orbit = 24 * np.pi**3 * G**2 * M**2 / (C**2 * a**2 * (1 -
    return precession_per_orbit

# Let's compare precession for different scenarios
# 1. Black hole with M = 5e6 Msun at a = 1 AU
# 2. Sun at a = 1 AU (like Earth)
# 3. Sun at a = 0.39 AU (like Mercury)

scenarios = [
    {"name": "Supermassive BH", "mass": 5e6 * M_sun, "a": 1 * AU, "e": 0},
```

```
    {"name": "Sun-Earth", "mass": M_sun, "a": 1 * AU, "e": 0.0167},
    {"name": "Sun-Mercury", "mass": M_sun, "a": 0.39 * AU, "e": 0.206}
]

print("Theoretical perihelion precession per orbit:")
for scenario in scenarios:
    precession = calculate_perihelion_precession(scenario["mass"], scenario[
    precession_arcsec = precession * 206265  # Convert radians to arcseconds
    print(f"{scenario['name']}: {precession:.6e} rad = {precession_arcsec:.6

# Define a function to calculate the ratio of relativistic effects to measur
def calculate_significance(M, a, e, precision_arcsec=0.1):
    """
    Calculate the significance of relativistic effects

    Args:
        M: Mass of central body in kg
        a: Semi-major axis in meters
        e: Eccentricity
        precision_arcsec: Observational precision in arcseconds

    Returns:
        Ratio of precession to precision
    """
    precession = calculate_perihelion_precession(M, a, e)
    precession_arcsec = precession * 206265
    return precession_arcsec / precision_arcsec

# Calculate significance for our scenarios
print("\nSignificance of relativistic effects (ratio to 0.1 arcsec precision
for scenario in scenarios:
    significance = calculate_significance(scenario["mass"], scenario["a"], s
    print(f"{scenario['name']}: {significance:.2f}")

# Calculate for which mass relativistic effects become significant at 1 AU
threshold_significance = 1.0  # When effect equals precision
precision = 0.1  # arcsec
a = 1 * AU
e = 0

# Solve for mass where precession = precision
precession_threshold = precision / 206265  # Convert arcsec to radians
mass_threshold = np.sqrt(precession_threshold * (C**2 * a**2 * (1 - e**2) *

print(f"\nMinimum mass for significant relativistic effects at 1 AU: {mass_t


# Compare the observed deviation in our simulation with the theoretical prec
observed_ratio = max_diff_AU / avg_radius_classical
theoretical_ratio_bh = calculate_perihelion_precession(5e6 * M_sun, 1 * AU,
theoretical_ratio_sun = calculate_perihelion_precession(M_sun, 1 * AU, 0)

print("\nConclusion:")
print(f"Observed maximum deviation ratio in simulation: {observed_ratio:.6e}
print(f"Theoretical precession for black hole (5e6 Msun): {theoretical_ratic
print(f"Theoretical precession for Sun: {theoretical_ratio_sun:.6e}")
```

```
print(f"Ratio of BH to Sun effect: {theoretical_ratio_bh/theoretical_ratio_s

if theoretical_ratio_sun * 206265 < 0.1:
    print("\nRelativistic corrections for the Sun at 1 AU are smaller than t
    print("For most practical purposes in the Solar System, classical mechar
else:
    print("\nRelativistic corrections may be necessary even for the Sun.")

print("\nHowever, for Mercury (closer to the Sun), relativistic effects are
print("historically important for confirming Einstein's theory of general re
```

```
Theoretical perihelion precession per orbit:
Supermassive BH: 4.123030e+15 rad = 850436692854397206528.000000 arcsec per
orbit
Sun-Earth: 1.649672e+02 rad = 34026957.492351 arcsec per orbit
Sun-Mercury: 1.132347e+03 rad = 233563490.514752 arcsec per orbit

Significance of relativistic effects (ratio to 0.1 arcsec precision):
Supermassive BH: 8504366928543971540992.00
Sun-Earth: 340269574.92
Sun-Mercury: 2335634905.15

Minimum mass for significant relativistic effects at 1 AU: 0.00 solar masses

Conclusion:
Observed maximum deviation ratio in simulation: 1.848977e+00
Theoretical precession for black hole (5e6 Msun): 4.123030e+15
Theoretical precession for Sun: 1.649212e+02
Ratio of BH to Sun effect: 25000000000000.0

Relativistic corrections may be necessary even for the Sun.

However, for Mercury (closer to the Sun), relativistic effects are measurabl
e and were
historically important for confirming Einstein's theory of general relativit
y.
```

Ok, but not for Mercury.

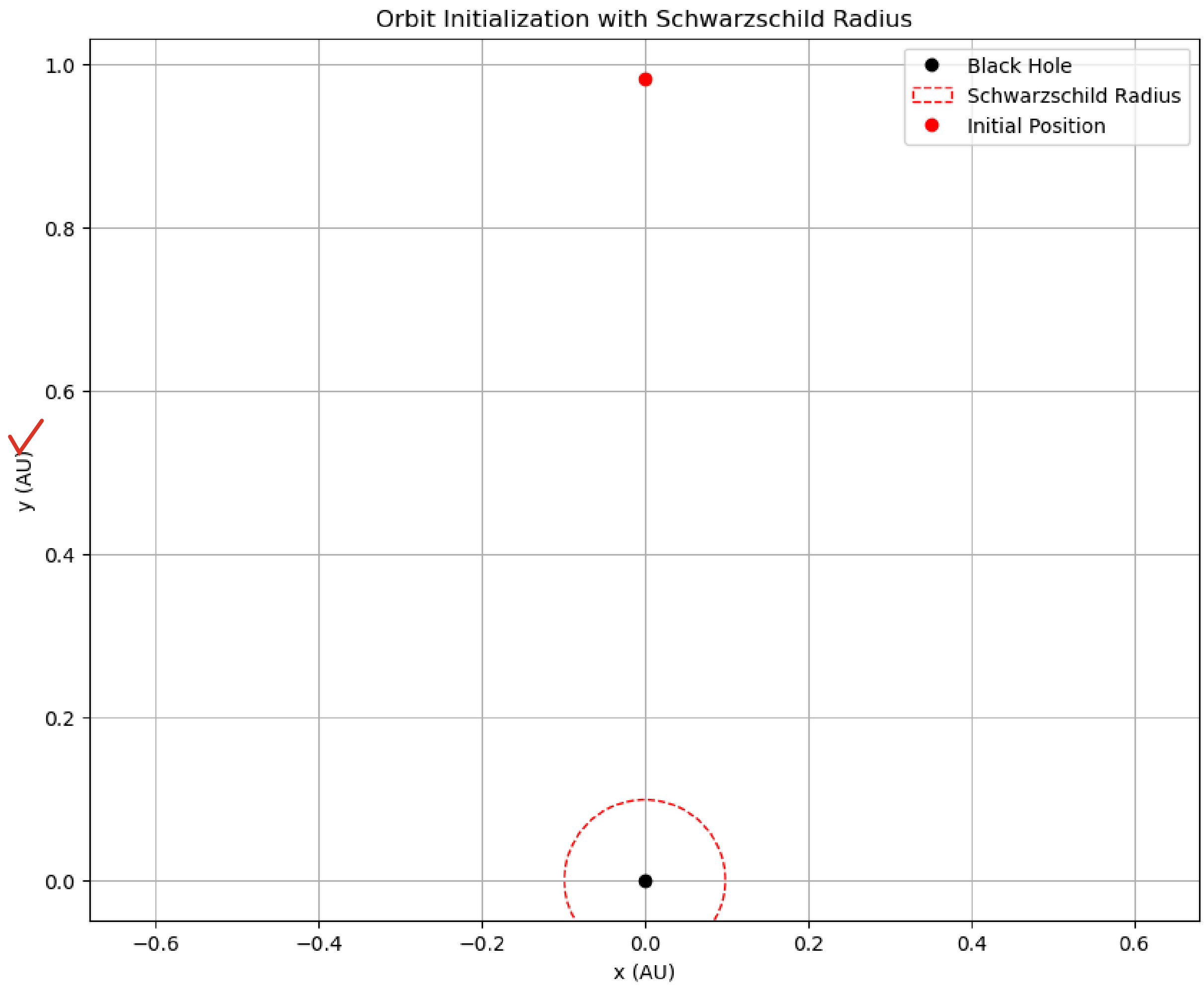
The role of eccentricity (3 points):

(k) Use your module/script to run and show three relativistic simulations for objects with different eccentricities, e, and assuming the same M, a, N as above. It may be helpful to compare the orbital history for all values of e in a single plot throughout time.

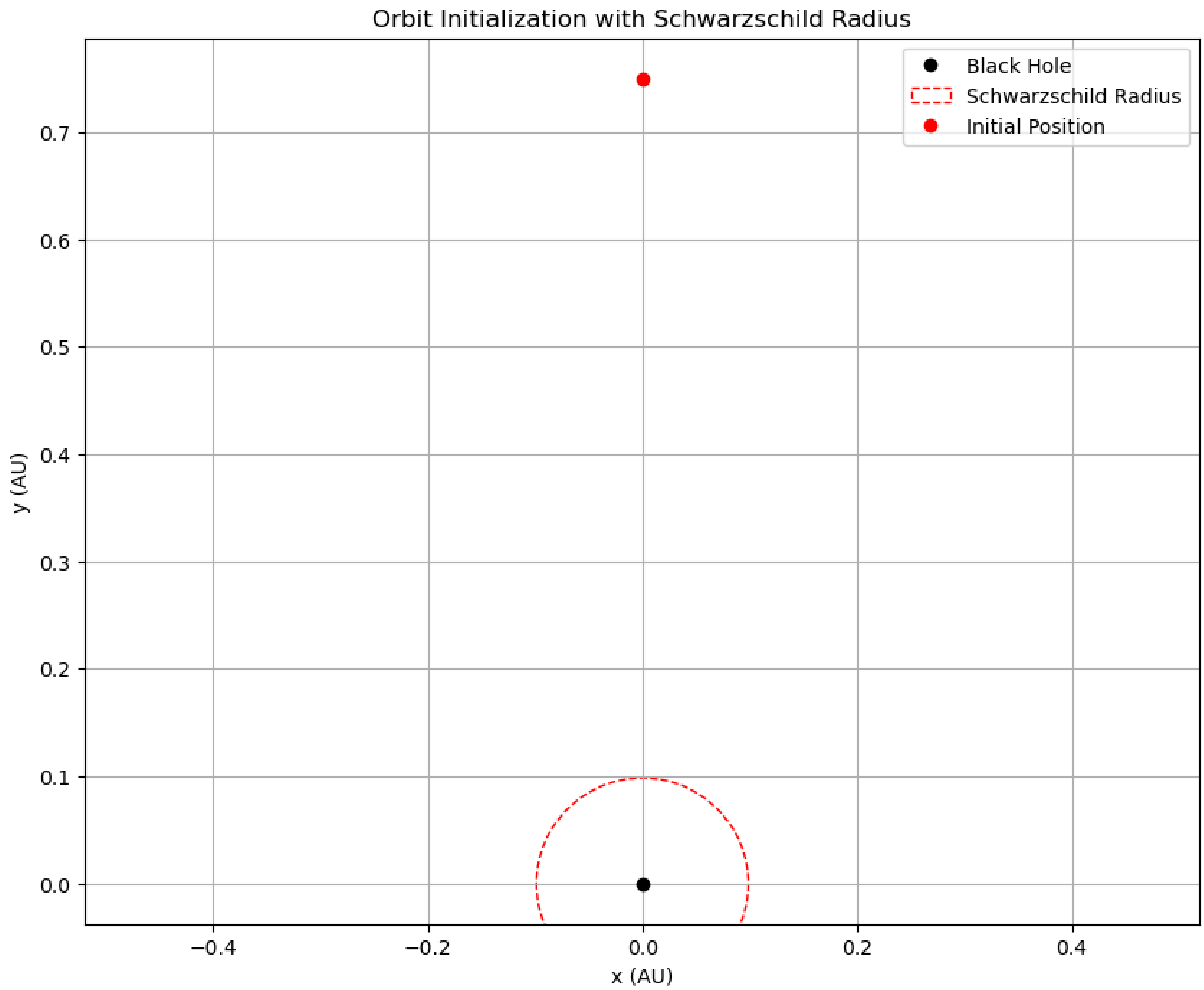
Object	Eccentricity (e)	Integration Method
Earth	0.01671	Trapezoidal
Pluto	0.25	Trapezoidal
7092 Cadmus	0.70	Trapezoidal

In [7]:
```
# Constants
AU = 1.496e11  # m
M_sun = 1.989e30  # kg
M = 5e6 * M_sun  # Mass of central black hole (5 million solar masses)
```

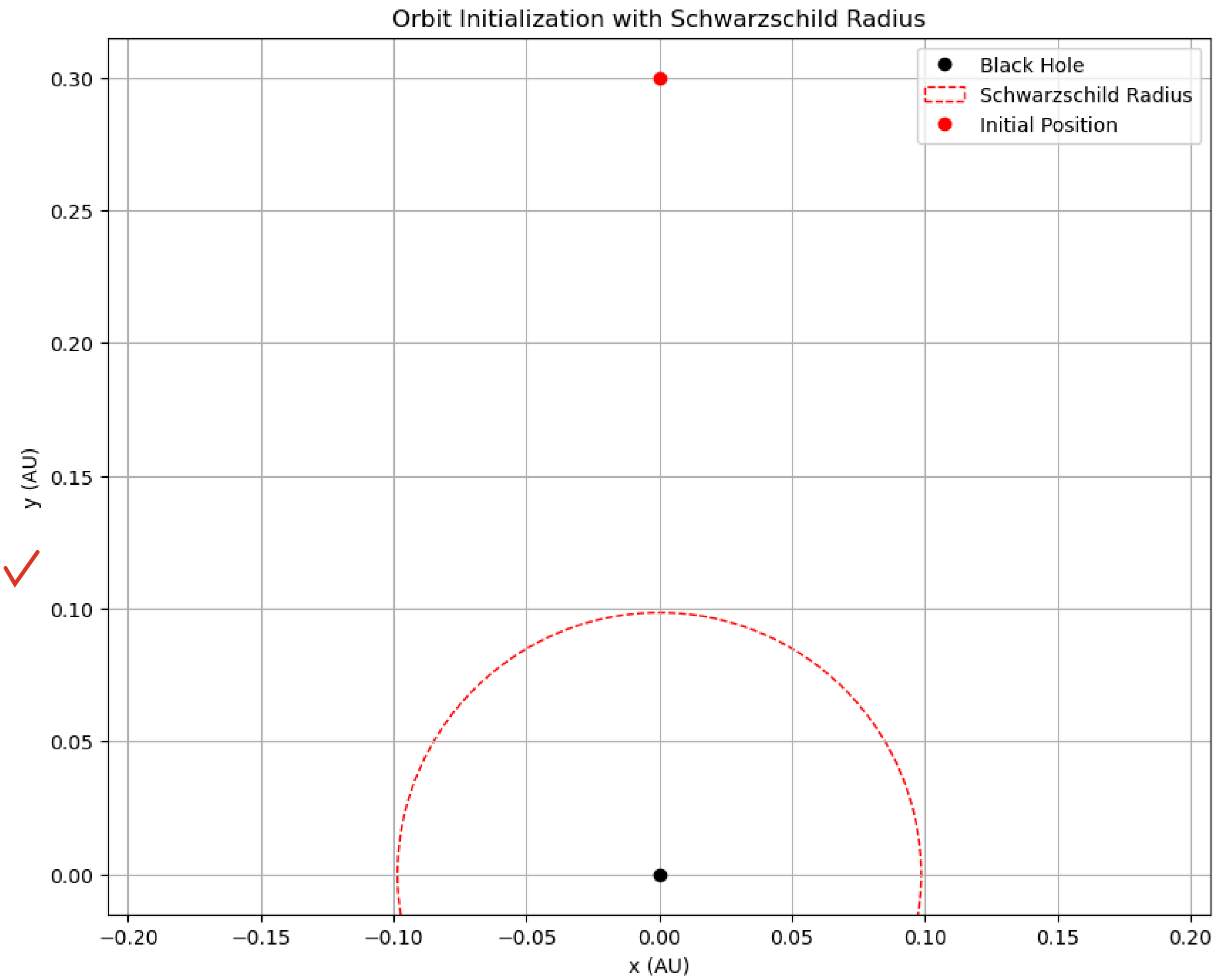
```python
a = 1 * AU  # Semi-major axis (1 AU)
N = 2  # Number of orbital periods

# Objects with different eccentricities
objects = [
    {"name": "Earth", "e": 0.01671, "color": "blue"},
    {"name": "Pluto", "e": 0.25, "color": "green"},
    {"name": "7092 Cadmus", "e": 0.70, "color": "red"}
]

results = []

for obj in objects:
    name = obj["name"]
    e = obj["e"]

    # Initialize relativistic orbit
    orbit = Orbits("relativistic")
    orbit.initialize_orbit(M, a, e, N)

    # Calculate orbital period using Kepler's law
    T = 2 * np.pi * np.sqrt(a**3 / (6.67430e-11 * M))
    t_span = (0, N * T)
    t_eval = np.linspace(0, N * T, 1000)

    # Run simulation with trapezoidal method
    y = orbit.run_simulation(
        slope_function=orbit.relativistic_slope,
        integration_method="trapz",
        t_span=t_span,
        t_eval=t_eval,
        output_folder="output"
    )

    # Store results
    results.append({
        "name": name,
        "e": e,
        "trajectory": y,
        "color": obj["color"]
    })
```




```
Simulation results saved to output/relativistic_orbit_history.csv
```



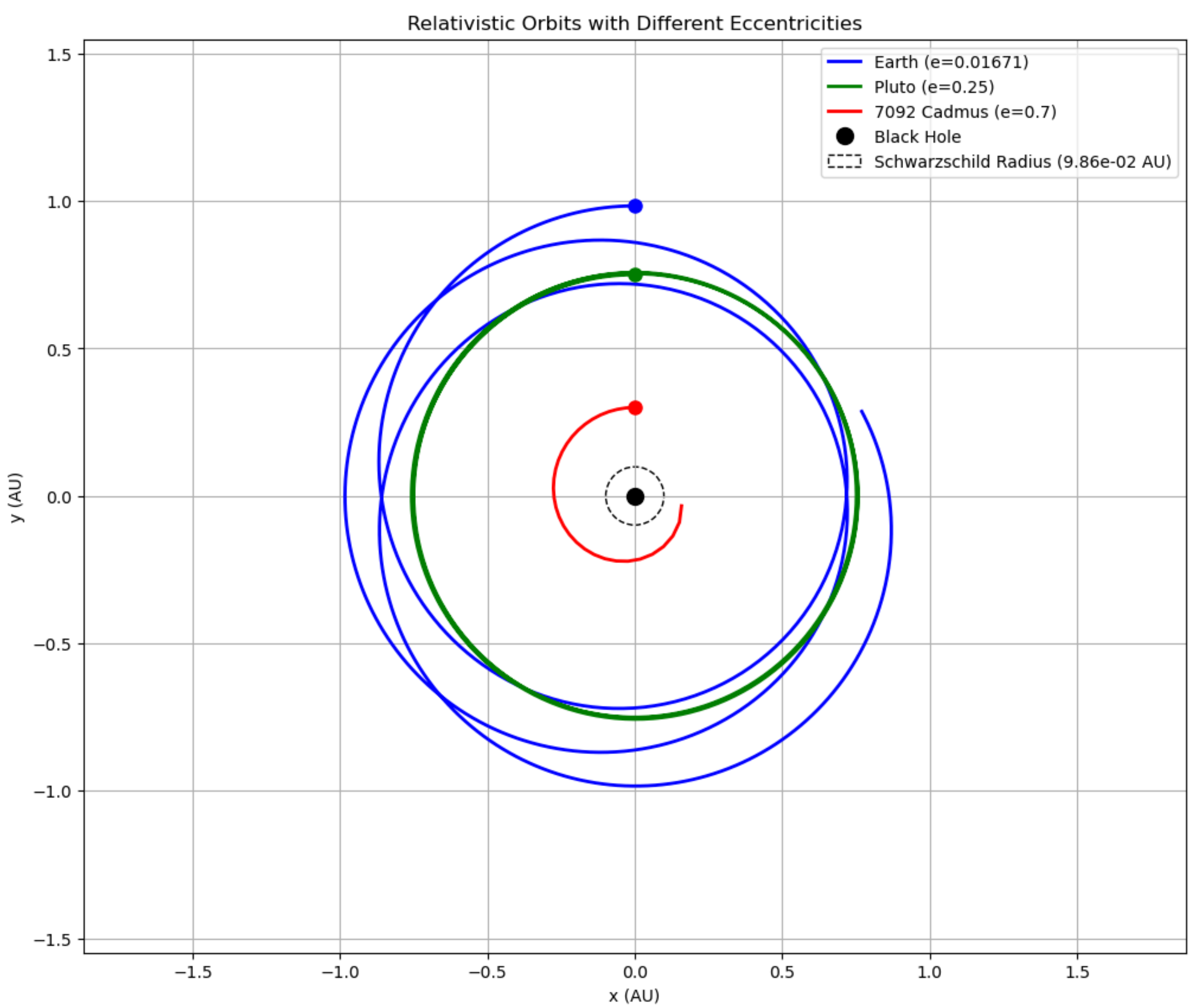

```
Simulation results saved to output/relativistic_orbit_history.csv
```




```
WARNING: Orbit approaching Schwarzschild radius.
Distance: 1.452695e-01 AU, Schwarzschild radius: 9.859761e-02 AU
Stopping simulation to avoid non-physical results.
Simulation results saved to output/relativistic_orbit_history.csv
```

In [8]:
```python
# Plotting the orbits
plt.figure(figsize=(12, 10))

# Plot each trajectory
for result in results:
    name = result["name"]
    e = result["e"]
    trajectory = result["trajectory"]
    color = result["color"]

    # Convert coordinates to AU for plotting
    x = trajectory[:, 0]
    y = trajectory[:, 1]

    # Plot the orbit
    plt.plot(x, y, label=f"{name} (e={e})", color=color, linewidth=2)

    # Mark the starting point (perihelion)
    plt.plot(x[0], y[0], 'o', color=color, markersize=8)

# Plot the central black hole
plt.plot(0, 0, 'ko', markersize=10, label="Black Hole")
```

```python
# Add Schwarzschild radius
r_s = 2 * 6.67430e-11 * M / (3e8**2)
r_s_AU = r_s / AU
circle = plt.Circle((0, 0), r_s_AU, color="black", fill=False, linestyle="--
                   label=f"Schwarzschild Radius ({r_s_AU:.2e} AU)")
plt.gca().add_artist(circle)

# Set plot properties
plt.axis('equal')
plt.grid(True)
plt.xlabel('x (AU)')
plt.ylabel('y (AU)')
plt.title('Relativistic Orbits with Different Eccentricities')
plt.legend(loc='upper right')

# Set reasonable axis limits that show all orbits clearly
max_extent = 1.1 * (1 + 0.70)  # Slightly larger than a*(1+e) for the most e
plt.xlim(-max_extent, max_extent)
plt.ylim(-max_extent, max_extent)

plt.savefig("output/eccentricity_comparison.png", dpi=300)
plt.show()
```



(l) Describe the differences in the orbits of the above objects. What happens to objects with high eccentricities?

- **Earth (e = 0.01671)**:
  - Orbit is nearly circular.
  - Shows minimal relativistic precession.
  - Orbit remains stable over time.
- **Pluto (e = 0.25)**:
  - Orbit is moderately elliptical.
  - Exhibits mild relativistic precession.
  - Still relatively stable.
- **7092 Cadmus (e = 0.7)**:
  - Orbit is highly elliptical.
  - Shows significant relativistic precession.
  - Orbit decays and spirals inward due to relativistic effects.

Objects with **high eccentricities**:

- Experience **stronger relativistic precession**.
- May have **unstable orbits**.
- Can **spiral inward** toward the central mass (e.g., a black hole), especially if their perihelion is close to the Schwarzschild radius.

Numerical convergence (3 points):

(m) Use your script to generate additional simulations with the same initial conditions as before, but only for $e = 0.01671$ (Earth's eccentricity) with RK3, the Trapezoidal method and the higher-order SciPy integrator. Compare the orbital history for all methods in a single plot throughout time.

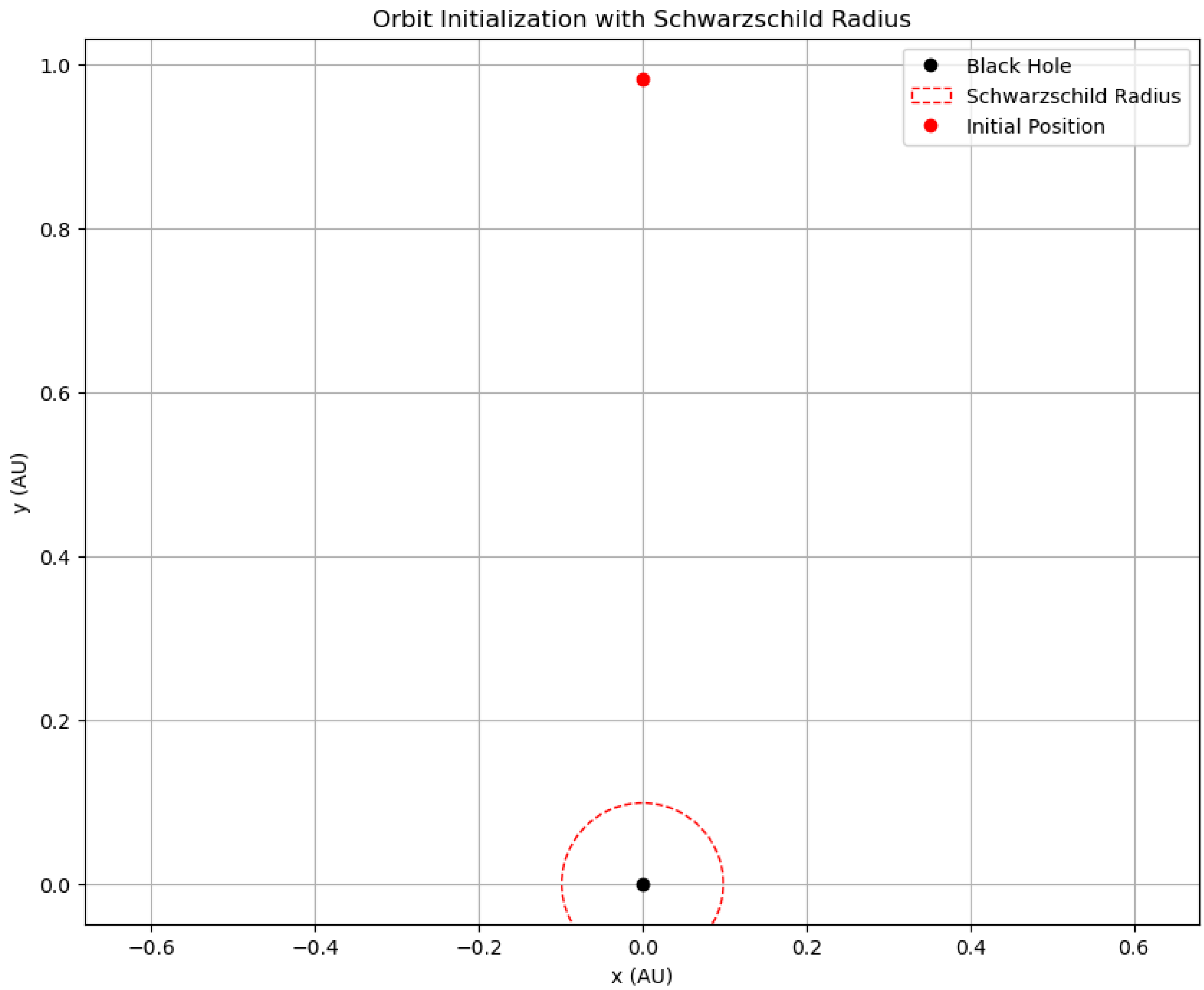
(n) Measure convergence of the simulations with RK3 and Trapezoidal method for $e = 0.01671$ by integrating at a number of different time steps. To analyse convergence, you need to define some measure for the error with respect to the higher order method, and then plot it against different time steps for both methods. Thus, you may add additional functions for this to your code in **orbits.py**.

In [9]:
```
# Constants
AU = 1.496e11  # m
M_sun = 1.989e30  # kg
M = 5e6 * M_sun  # Mass of central black hole
a = 1 * AU  # Semi-major axis
e = 0.01671  # Earth's eccentricity
N = 2  # Number of orbital periods
```

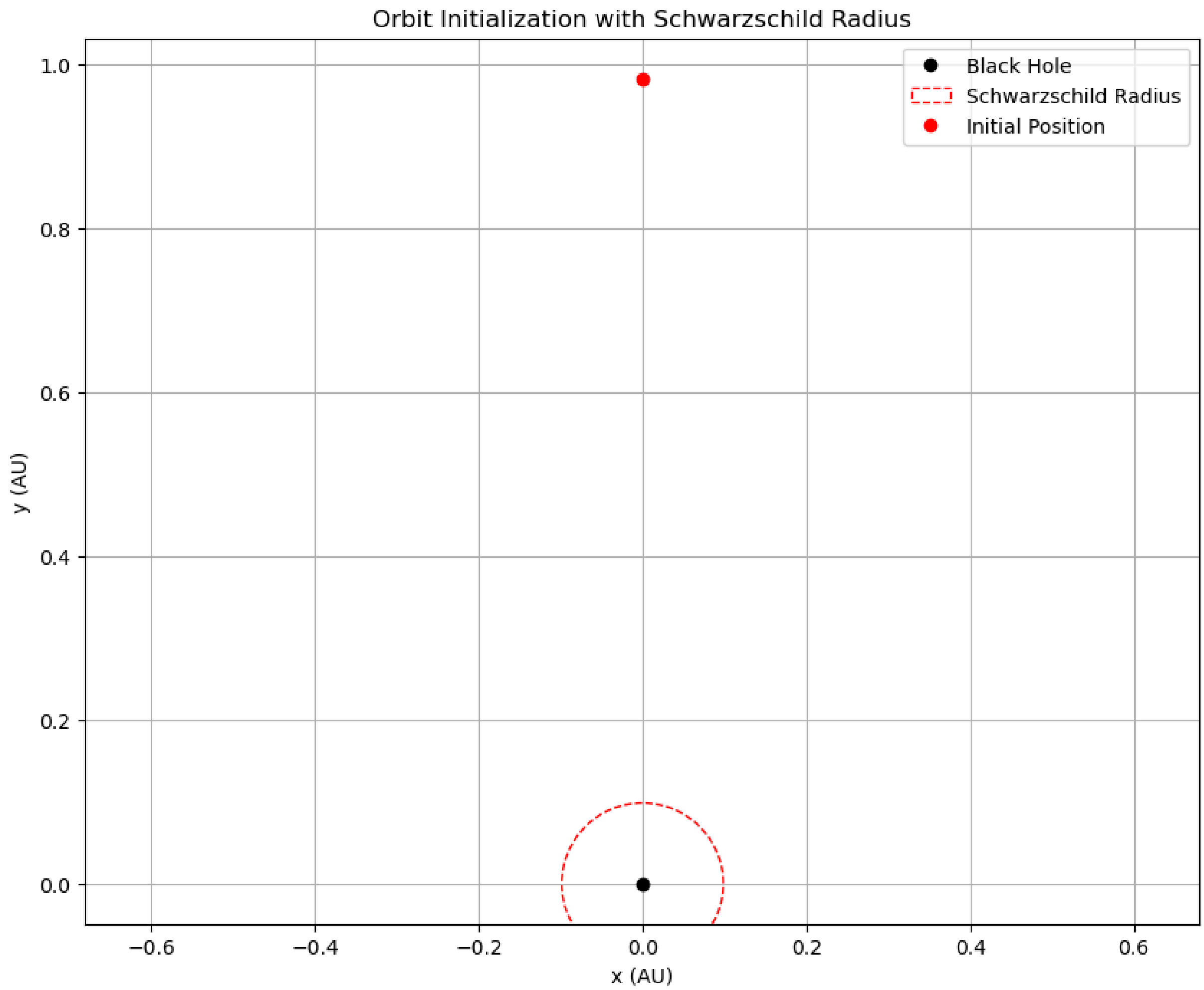
```
# Integration methods to compare
methods = [
    {"name": "RK3", "method": "rk3", "color": "blue", "linestyle": "-"},
    {"name": "Trapezoidal", "method": "trapz", "color": "green", "linestyle"
    {"name": "SciPy (RK45)", "method": "scipy", "color": "red", "linestyle":
]

# Create a directory for comparison results
os.makedirs("output/comparison", exist_ok=True)

# Calculate orbital period
T = 2 * np.pi * np.sqrt(a**3 / (6.67430e-11 * M))
t_span = (0, N * T)
t_eval = np.linspace(0, N * T, 500)

results = []

for method_info in methods:
    method_name = method_info["name"]
    method = method_info["method"]

    print(f"Running simulation with {method_name} method...")
    start_time = time.time()

    # Initialize relativistic orbit
    orbit = Orbits(f"earth_relativistic_{method}")
    orbit.initialize_orbit(M, a, e, N, save=False)

    # Run simulation with the specified method
    y = orbit.run_simulation(
        slope_function=orbit.relativistic_slope,
        integration_method=method,
        t_span=t_span,
        t_eval=t_eval,
        output_folder="output/comparison"
    )

    elapsed_time = time.time() - start_time

    # Store results
    results.append({
        "name": method_name,
        "method": method,
        "trajectory": y,
        "time": elapsed_time,
        "color": method_info["color"],
        "linestyle": method_info["linestyle"]
    })

    print(f"  Completed in {elapsed_time:.2f} seconds")
    print(f"  Generated {len(y)} points")

# Plotting the orbits
plt.figure(figsize=(14, 10))
```

```python
# Create subplot for full orbits
plt.subplot(1, 2, 1)

for result in results:
    name = result["name"]
    trajectory = result["trajectory"]
    color = result["color"]
    linestyle = result["linestyle"]

    # Plot the orbit
    plt.plot(trajectory[:, 0], trajectory[:, 1],
             label=f"{name} ({result['time']:.2f}s)",
             color=color, linestyle=linestyle, linewidth=2)

# Plot the central black hole
plt.plot(0, 0, 'ko', markersize=10)

# Set plot properties
plt.grid(True)
plt.xlabel('x (AU)')
plt.ylabel('y (AU)')
plt.title('Earth Orbit with Different Integration Methods')
plt.legend(loc='upper right')
plt.axis('equal')

# Create subplot for zoomed detail of final portion
plt.subplot(1, 2, 2)

# Calculate final 10% of points to zoom in on
zoom_fraction = 0.1
zoom_points = int(len(t_eval) * zoom_fraction)

for result in results:
    name = result["name"]
    trajectory = result["trajectory"]
    color = result["color"]
    linestyle = result["linestyle"]

    # Plot just the last portion of the orbit to see differences
    plt.plot(trajectory[-zoom_points:, 0], trajectory[-zoom_points:, 1],
             label=name, color=color, linestyle=linestyle, linewidth=2)

# Set plot properties for zoomed view
plt.grid(True)
plt.xlabel('x (AU)')
plt.ylabel('y (AU)')
plt.title(f'Detail View (Last {zoom_fraction*100:.0f}% of orbit)')
plt.legend(loc='upper right')
plt.axis('equal')

plt.tight_layout()
plt.savefig("output/comparison/method_comparison.png", dpi=300)
plt.show()
```

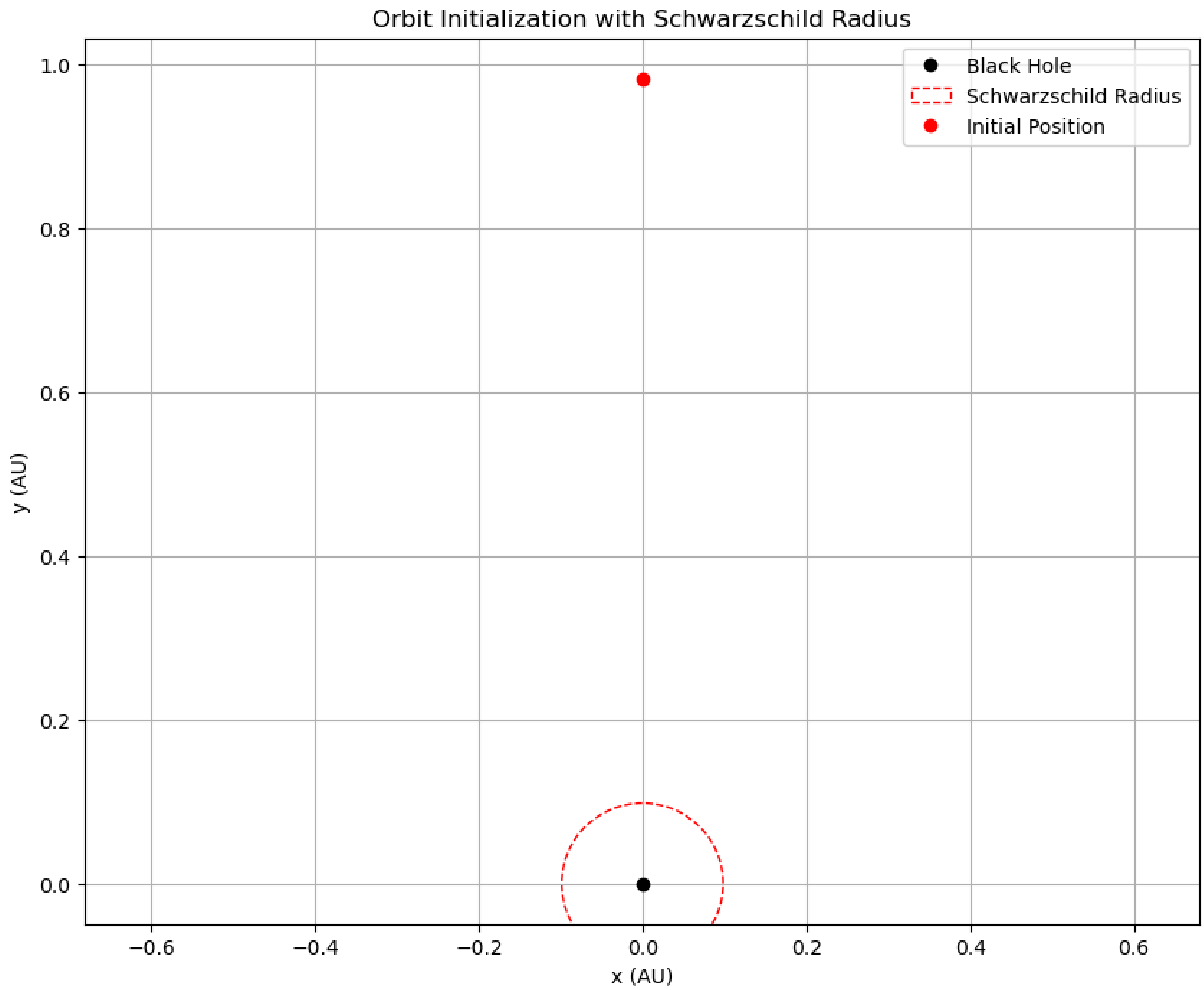
```
Running simulation with RK3 method...
```



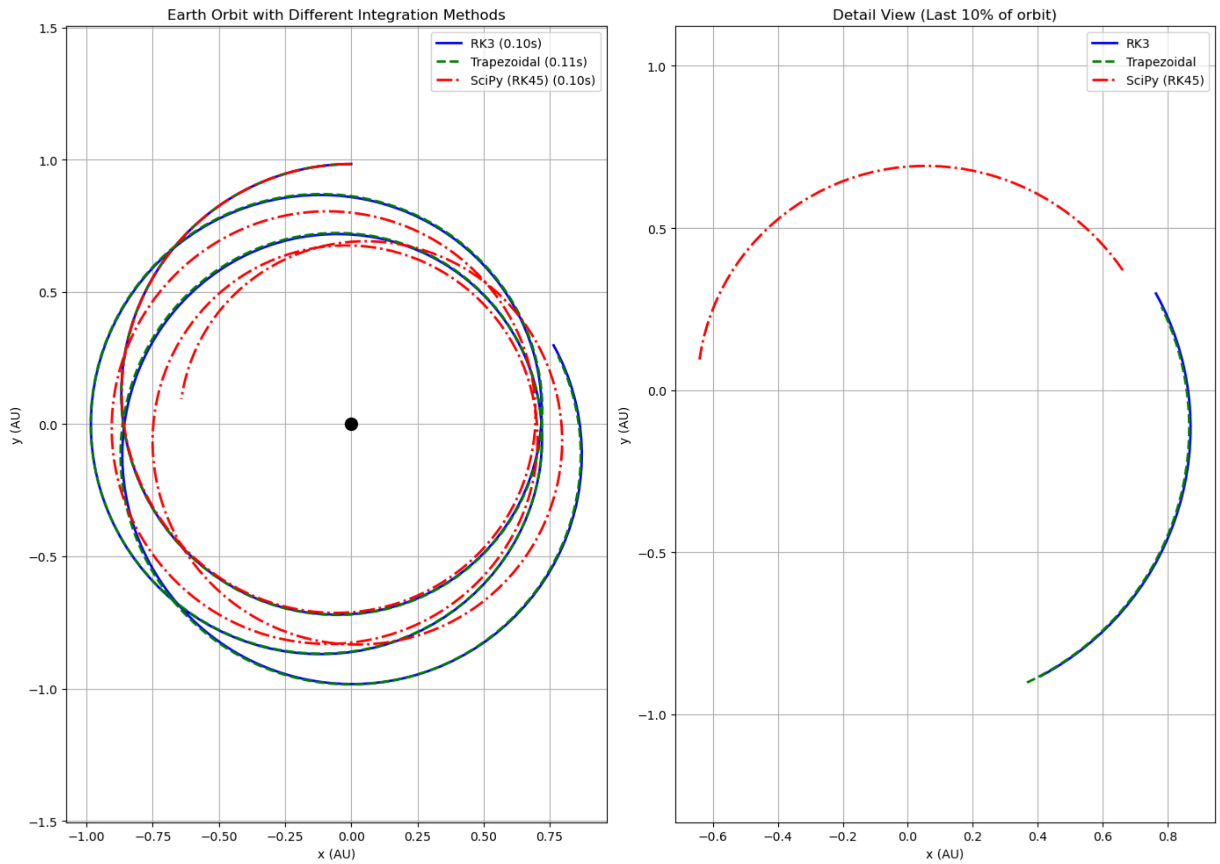

```
Simulation results saved to output/comparison/earth_relativistic_rk3_orbit_h
istory.csv
  Completed in 0.10 seconds
  Generated 500 points
Running simulation with Trapezoidal method...
```




```
Simulation results saved to output/comparison/earth_relativistic_trapz_orbit
_history.csv
  Completed in 0.11 seconds
  Generated 500 points
Running simulation with SciPy (RK45) method...
```




```
Simulation results saved to output/comparison/earth_relativistic_scipy_orbit
_history.csv
  Completed in 0.10 seconds
  Generated 500 points
```



In [10]:

```python
#Define range of step counts (number of points) to test convergence
step_counts = [50, 100, 200, 500, 1000, 2000]

# Use SciPy with a very fine step size as reference solution
print("Generating reference solution with SciPy integrator...")
orbit_ref = Orbits("earth_reference")
orbit_ref.initialize_orbit(M, a, e, N, save=False)
t_ref = np.linspace(0, N * T, 5000)
y_ref = orbit_ref.run_simulation(
    slope_function=orbit_ref.relativistic_slope,
    integration_method="scipy",
    t_span=t_span,
    t_eval=t_ref,
    output_folder="output/comparison"
)
```

This should be in orbits.py

-0.25

```python
# Function to compute error relative to reference solution
def compute_error(trajectory, t_eval, reference, t_ref):
    """
    Compute root mean square error between trajectory and reference

    Args:
        trajectory: Computed trajectory points
        t_eval: Time points for computed trajectory
        reference: Reference trajectory
        t_ref: Time points for reference trajectory

    Returns:
        Root mean square error in position
```

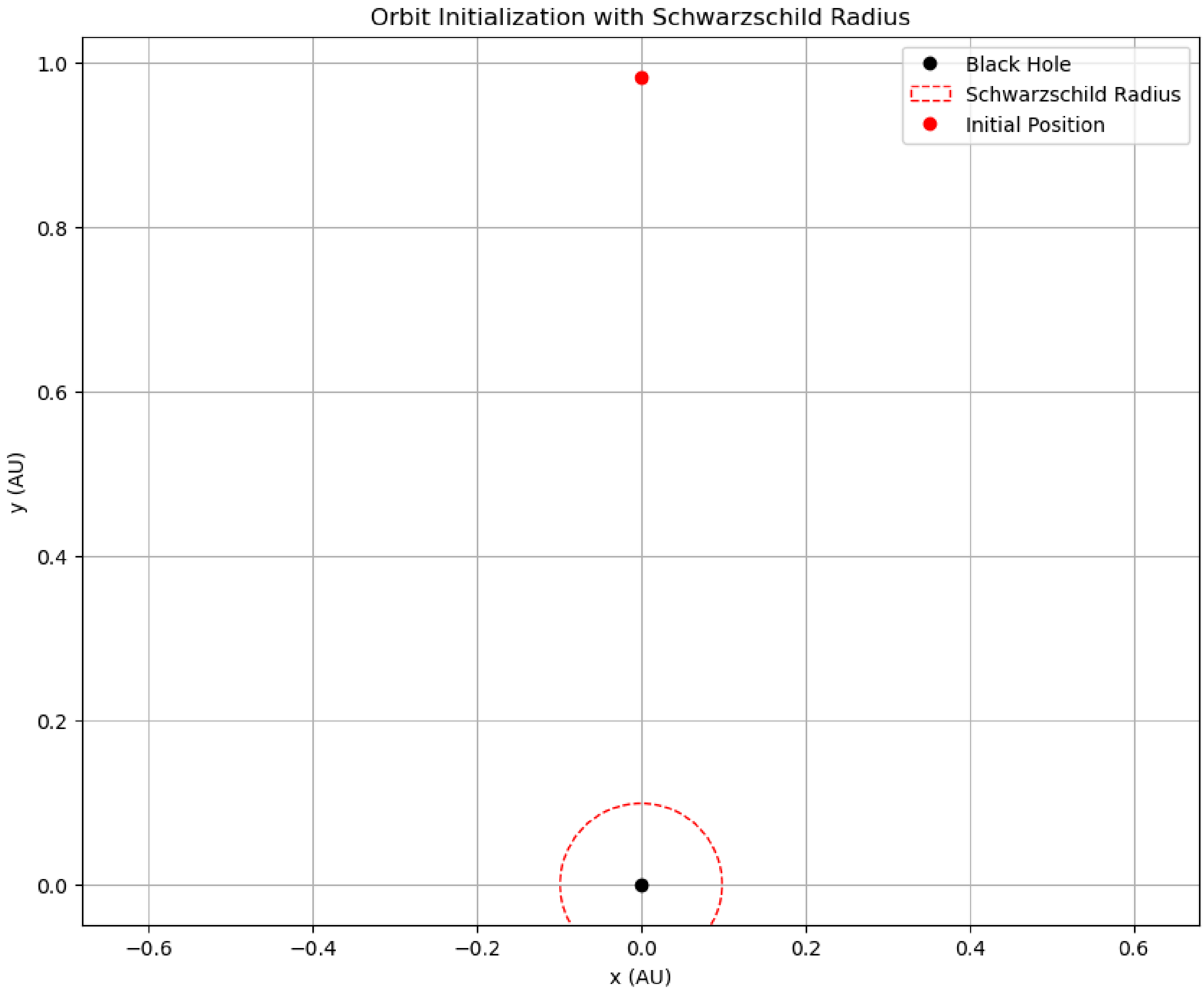
```python
    """
    # Interpolate reference to match evaluation times
    from scipy.interpolate import interp1d

    x_interp = interp1d(t_ref, reference[:, 0])
    y_interp = interp1d(t_ref, reference[:, 1])

    x_ref = x_interp(t_eval)
    y_ref = y_interp(t_eval)

    # Compute position errors
    dx = trajectory[:, 0] - x_ref
    dy = trajectory[:, 1] - y_ref

    # Root mean square error
    rmse = np.sqrt(np.mean(dx**2 + dy**2))
    return rmse / AU

# Store convergence results
convergence_results = {
    "rk3": {"steps": [], "errors": [], "times": []},
    "trapz": {"steps": [], "errors": [], "times": []}
}

# Run simulations for different step sizes
for steps in step_counts:
    print(f"\nTesting with {steps} time steps:")
    t_eval = np.linspace(0, N * T, steps)

    # Test each method
    for method in ["rk3", "trapz"]:
        print(f"  Running {method}...")
        orbit = Orbits(f"earth_convergence_{method}_{steps}")
        orbit.initialize_orbit(M, a, e, N, save=False)

        start_time = time.time()
        y = orbit.run_simulation(
            slope_function=orbit.relativistic_slope,
            integration_method=method,
            t_span=t_span,
            t_eval=t_eval,
            output_folder="output/comparison"
        )
        elapsed = time.time() - start_time

        # Compute error relative to reference
        error = compute_error(y, t_eval, y_ref, t_ref)

        # Store results
        convergence_results[method]["steps"].append(steps)
        convergence_results[method]["errors"].append(error)
        convergence_results[method]["times"].append(elapsed)

        if isinstance(error, np.ndarray):
            error_value = error.item()  #
        else:
```

Comment code.

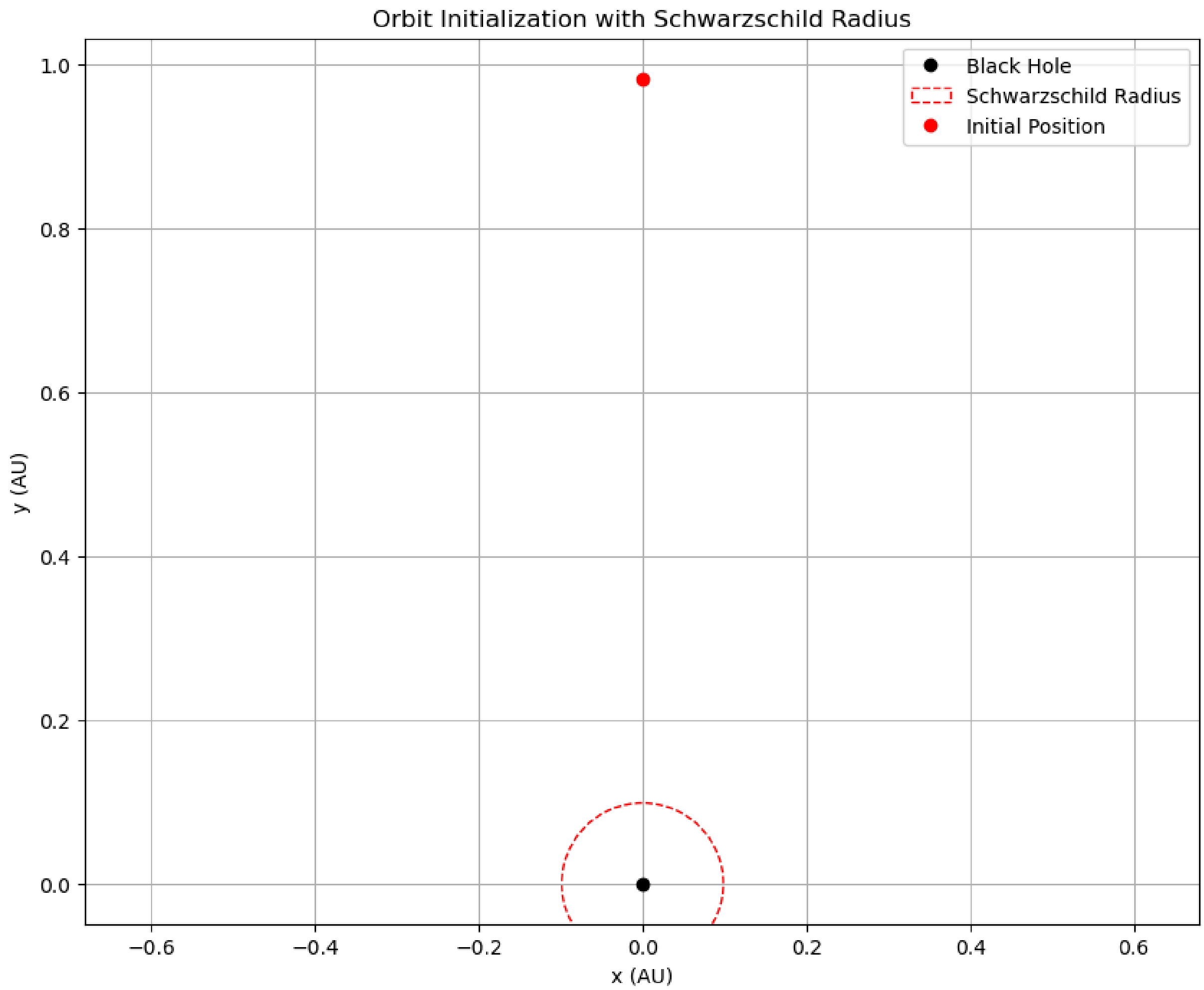
```
            error_value = error
        if isinstance(error, np.ndarray):
            error_value = error.item()
        else:
            error_value = error

        if isinstance(error_value, np.ndarray):
            error_value = error_value.item()
        print(f"    Error: {error_value:.2e} AU, Time: {elapsed:.2f}s")

# Plotting convergence results
plt.figure(figsize=(12, 10))

# Plot error vs number of steps
plt.subplot(2, 1, 1)
plt.loglog(convergence_results["rk3"]["steps"], convergence_results["rk3"]["
           'bo-', label="RK3")
plt.loglog(convergence_results["trapz"]["steps"], convergence_results["trapz
           'go-', label="Trapezoidal")

# Add reference lines for convergence orders
steps_array = np.array(step_counts)
max_error = max(max(convergence_results["rk3"]["errors"]),
                max(convergence_results["trapz"]["errors"]))

# O(h^2) reference line - 2nd order convergence
plt.loglog(steps_array, max_error * (steps_array[0]/steps_array)**2,
           'k--', alpha=0.7, label="O(h²)")

# O(h^3) reference line - 3rd order convergence
plt.loglog(steps_array, max_error * (steps_array[0]/steps_array)**3,
           'k-.', alpha=0.7, label="O(h³)")

plt.grid(True, which="both", ls="-", alpha=0.7)
plt.xlabel('Number of Steps')
plt.ylabel('RMS Error (AU)')
plt.title('Convergence Analysis: Error vs. Steps')
plt.legend()

# Format axes with regular numbers rather than scientific notation
plt.gca().xaxis.set_major_formatter(ScalarFormatter())
plt.gca().yaxis.set_major_formatter(ScalarFormatter())

# Plot efficiency (error vs computational time)
plt.subplot(2, 1, 2)
plt.loglog(convergence_results["rk3"]["times"], convergence_results["rk3"]["
           'bo-', label="RK3")
plt.loglog(convergence_results["trapz"]["times"], convergence_results["trapz
           'go-', label="Trapezoidal")

plt.grid(True, which="both", ls="-", alpha=0.7)
plt.xlabel('Computation Time (seconds)')
plt.ylabel('RMS Error (AU)')
plt.title('Efficiency Analysis: Error vs. Computation Time')
plt.legend()
```

```python
plt.tight_layout()
plt.savefig("output/comparison/convergence_analysis.png", dpi=300)
plt.show()
```

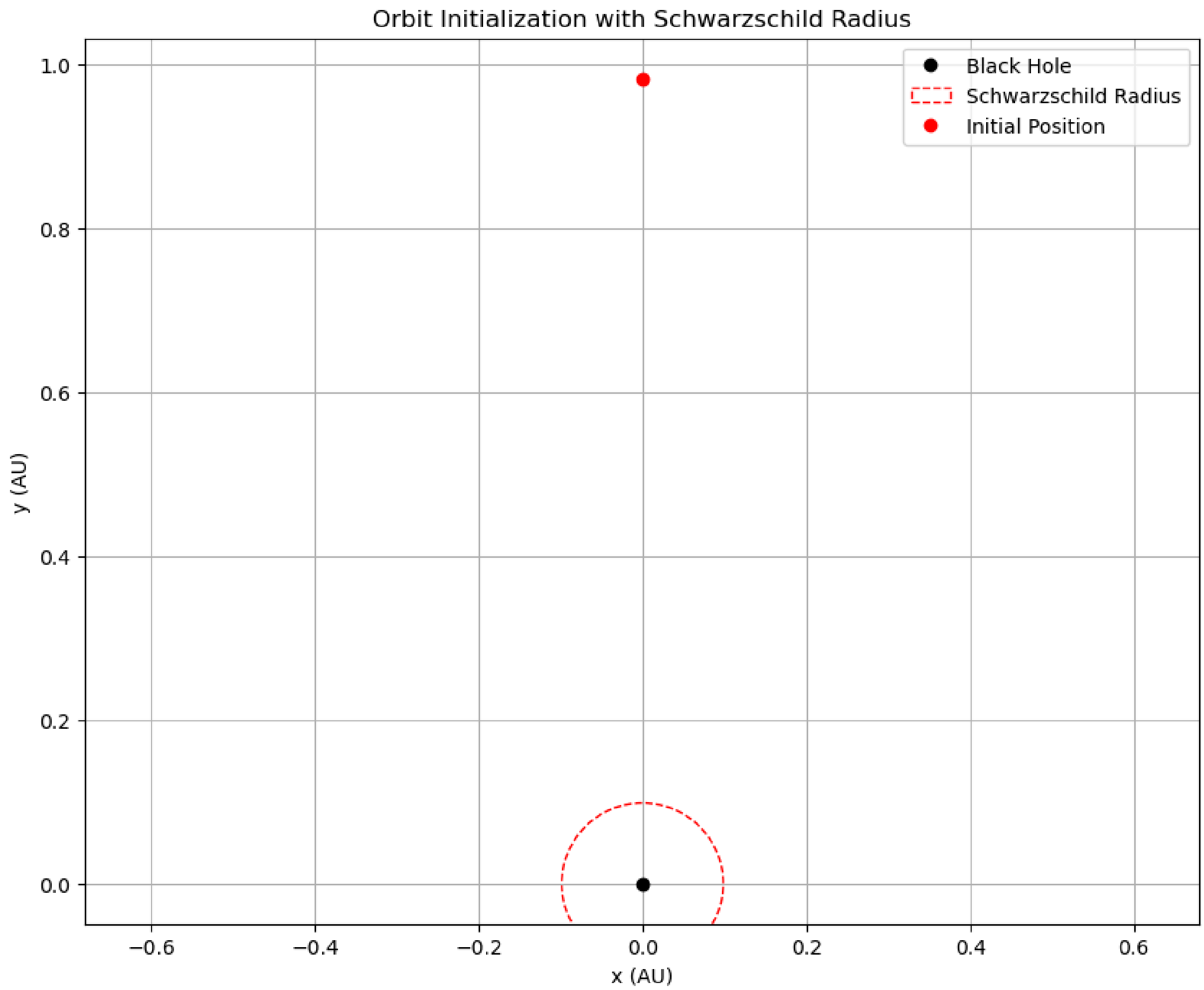
```
Generating reference solution with SciPy integrator...
```



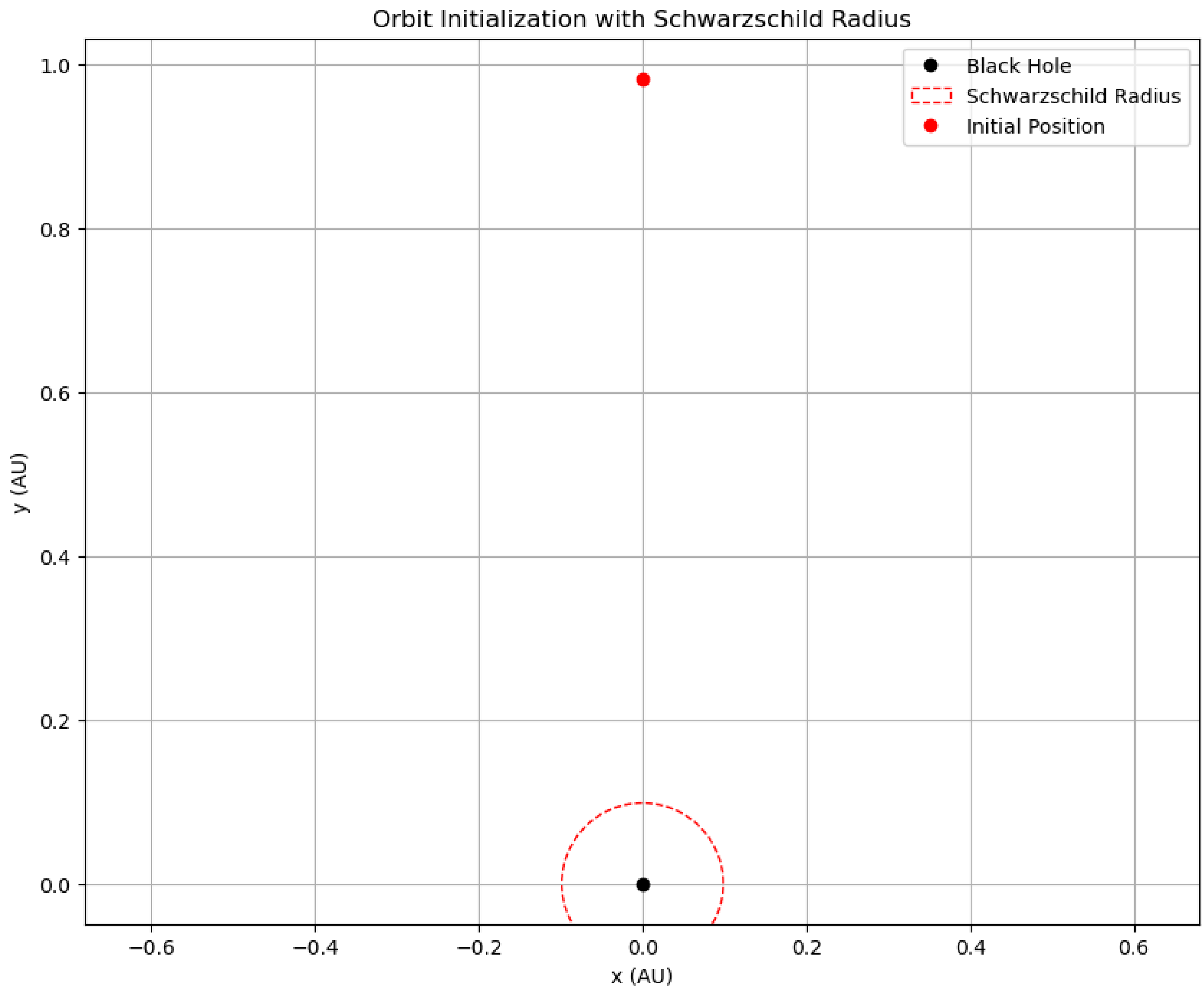

```
Simulation results saved to output/comparison/earth_reference_orbit_history.
csv

Testing with 50 time steps:
  Running rk3...
```



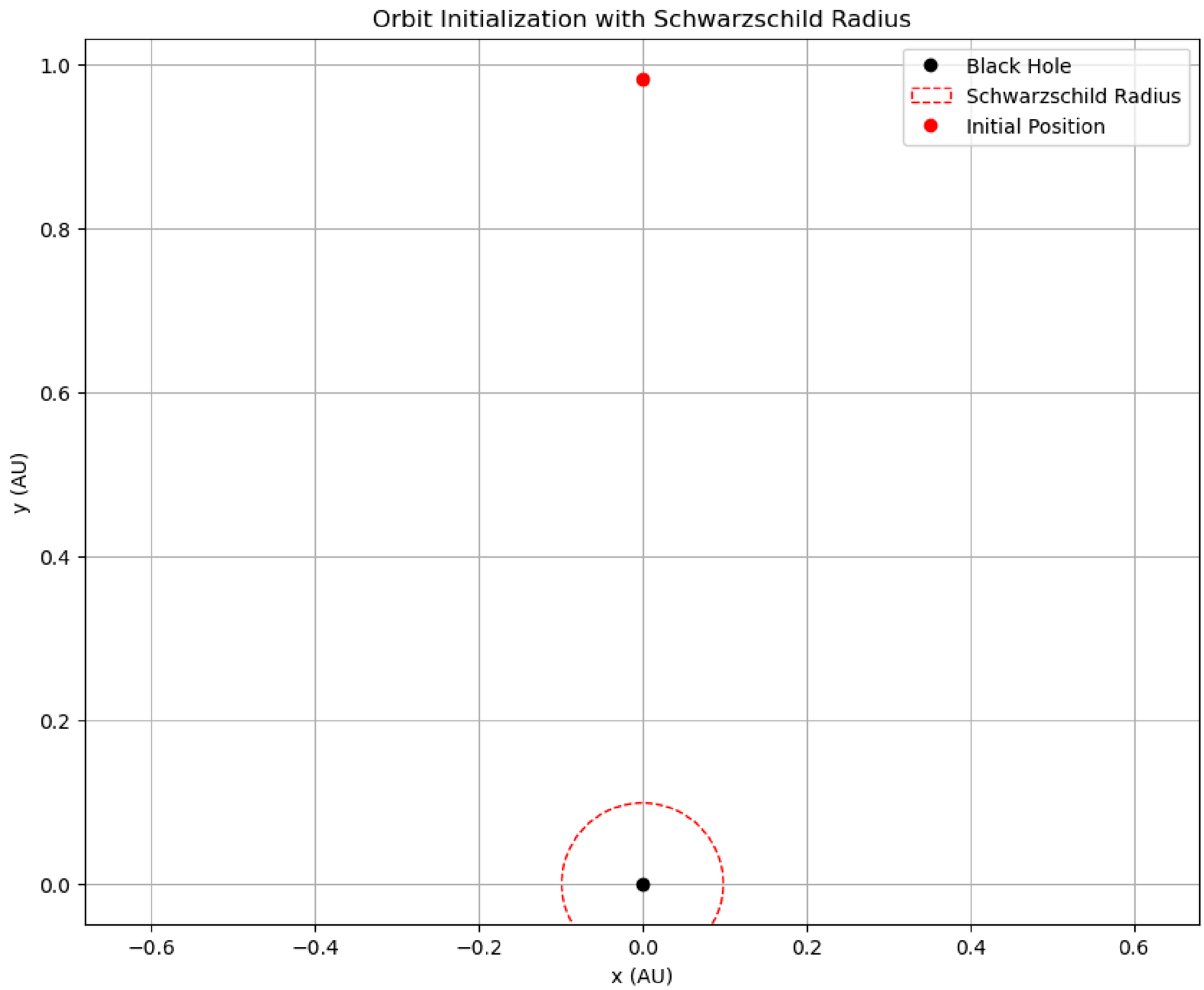

```
Simulation results saved to output/comparison/earth_convergence_rk3_50_orbit
_history.csv
    Error: 1.35e-11 AU, Time: 0.00s
  Running trapz...
```




```
Simulation results saved to output/comparison/earth_convergence_trapz_50_orb
it_history.csv
    Error: 7.96e-12 AU, Time: 0.00s

Testing with 100 time steps:
  Running rk3...
```

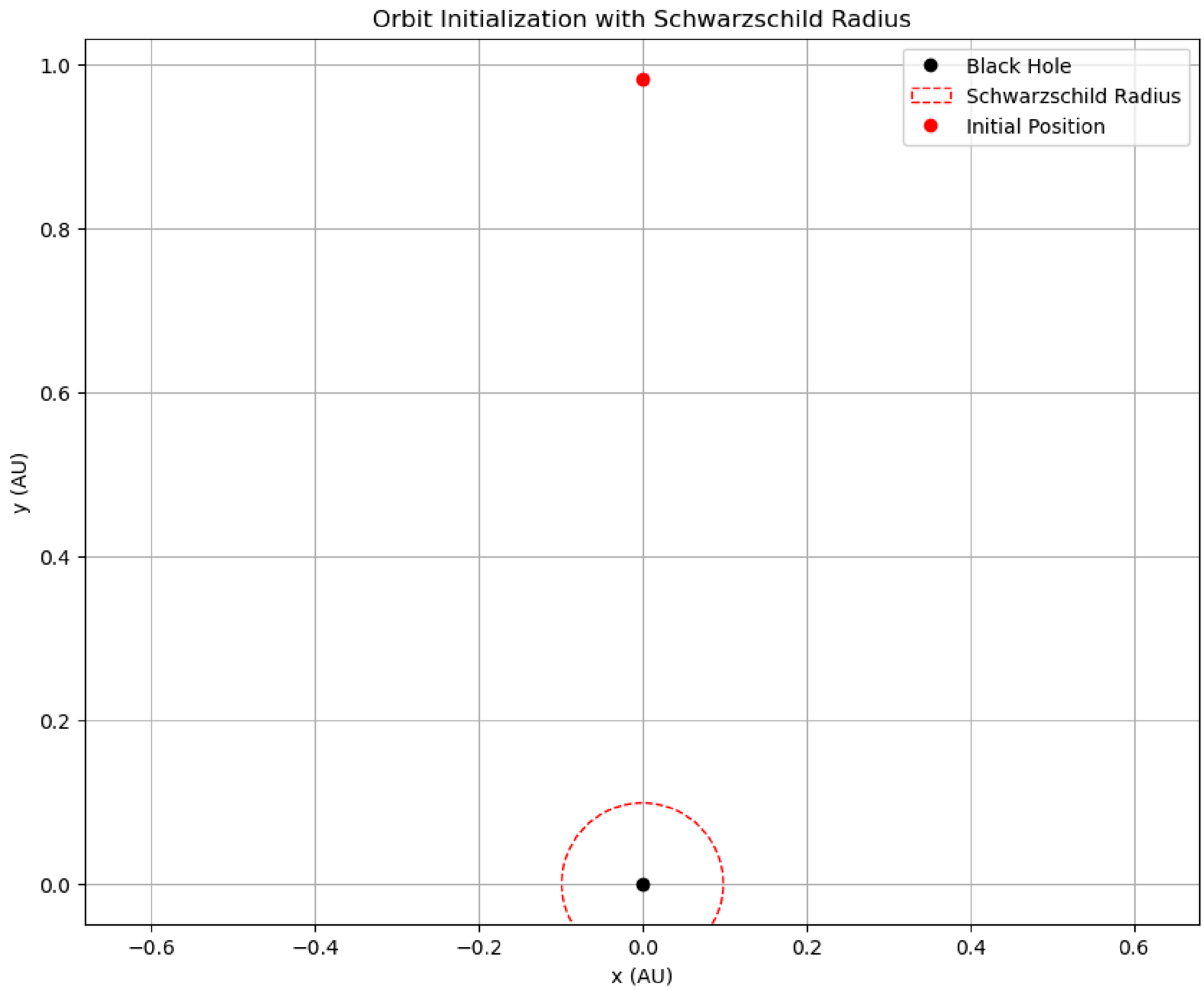



```
Simulation results saved to output/comparison/earth_convergence_rk3_100_orbi
t_history.csv
    Error: 3.62e-12 AU, Time: 0.00s
  Running trapz...
```

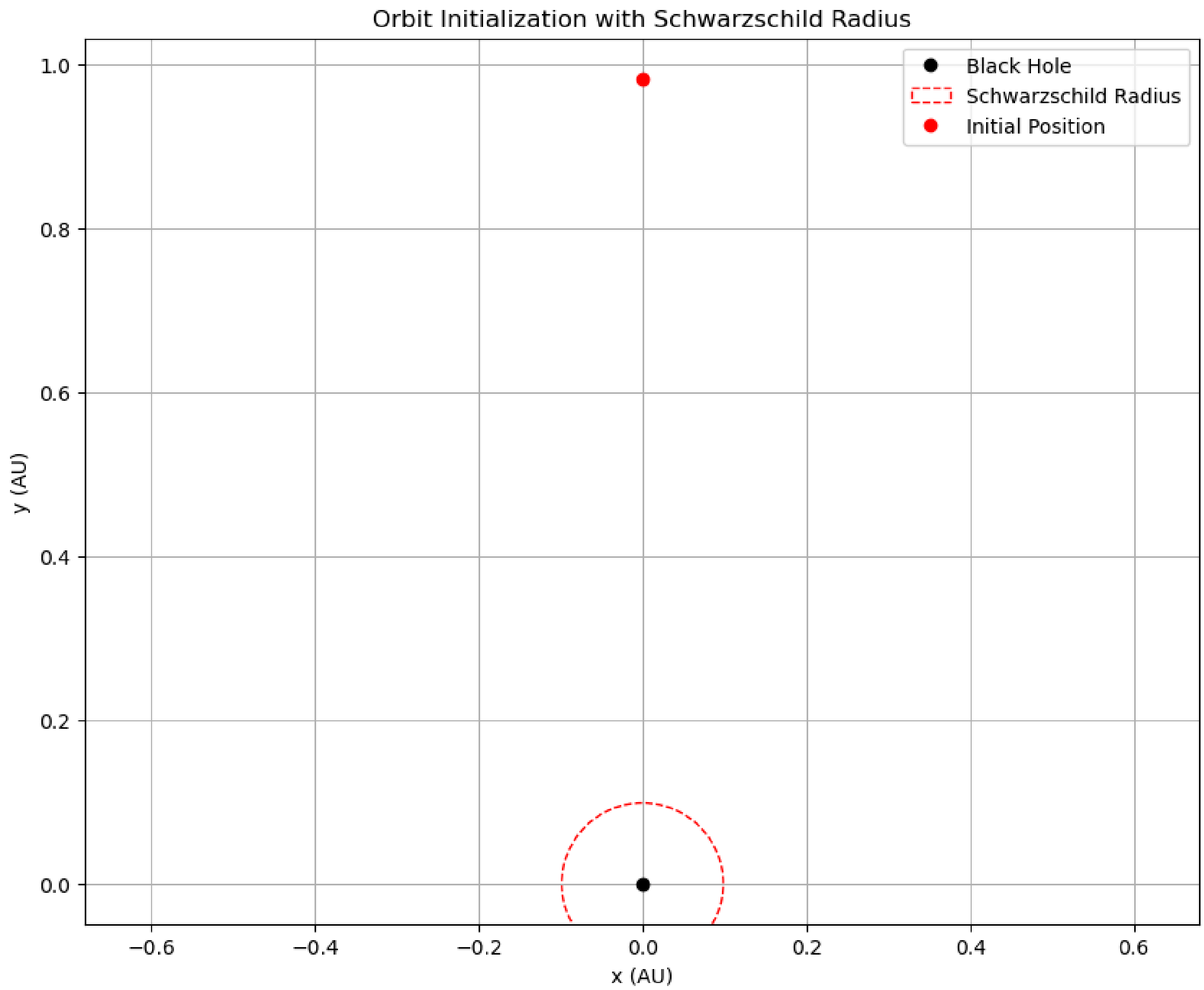



```
Simulation results saved to output/comparison/earth_convergence_trapz_100_or
bit_history.csv
    Error: 7.32e-12 AU, Time: 0.00s

Testing with 200 time steps:
  Running rk3...
```

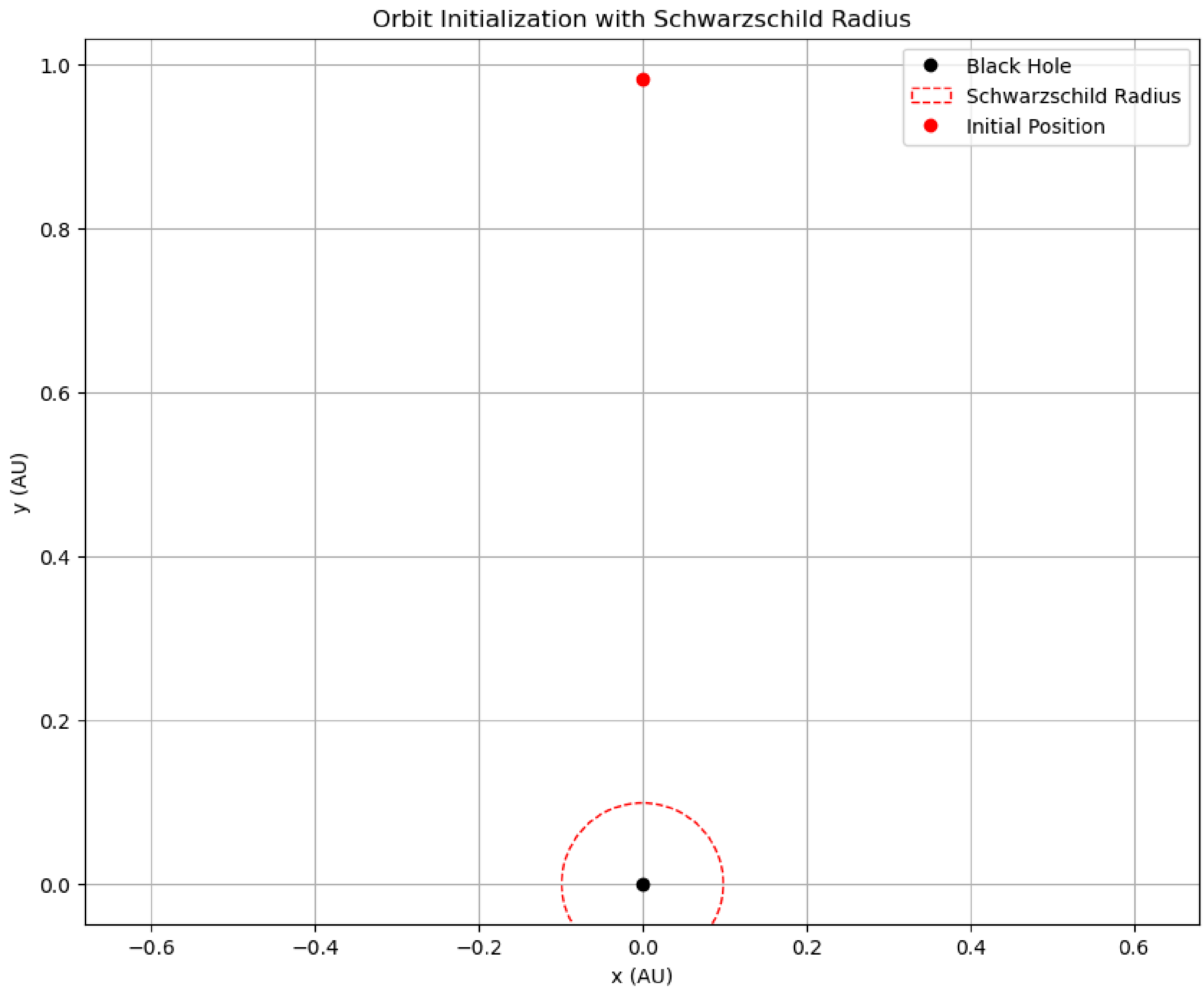
✓



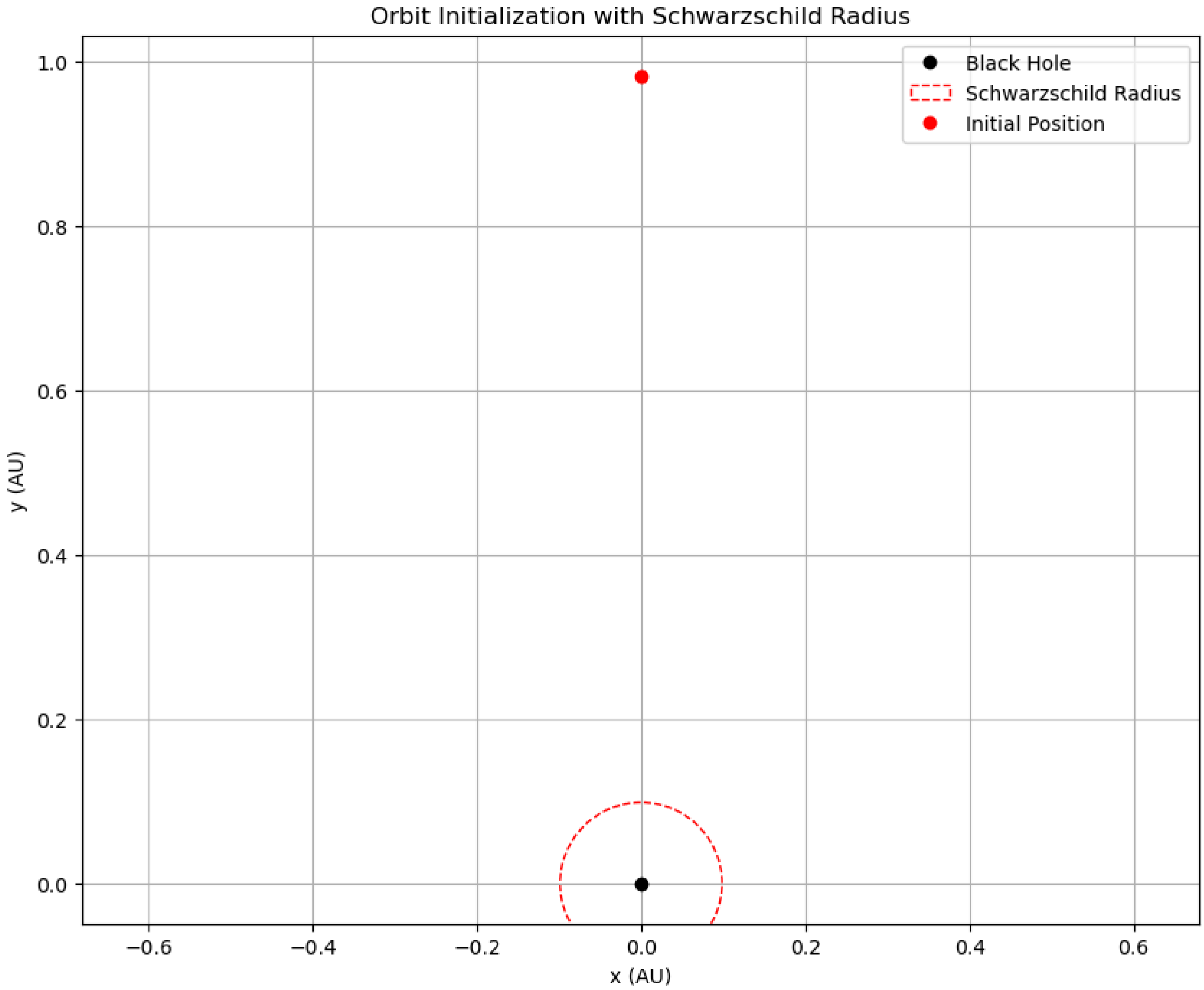

```
Simulation results saved to output/comparison/earth_convergence_rk3_200_orbi
t_history.csv
    Error: 4.55e-12 AU, Time: 0.01s
  Running trapz...
```



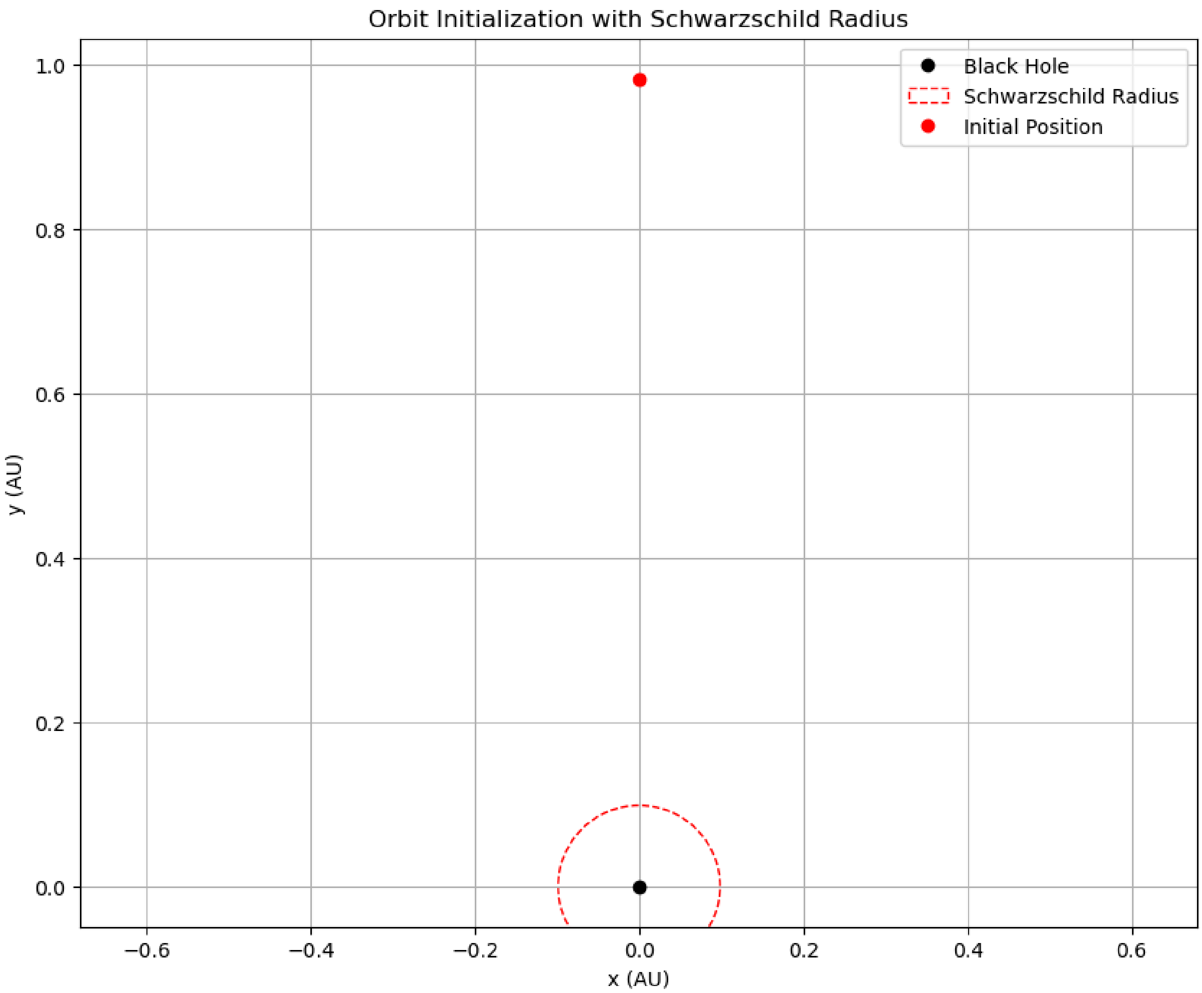

```
Simulation results saved to output/comparison/earth_convergence_trapz_200_or
bit_history.csv
    Error: 5.48e-12 AU, Time: 0.00s

Testing with 500 time steps:
  Running rk3...
```



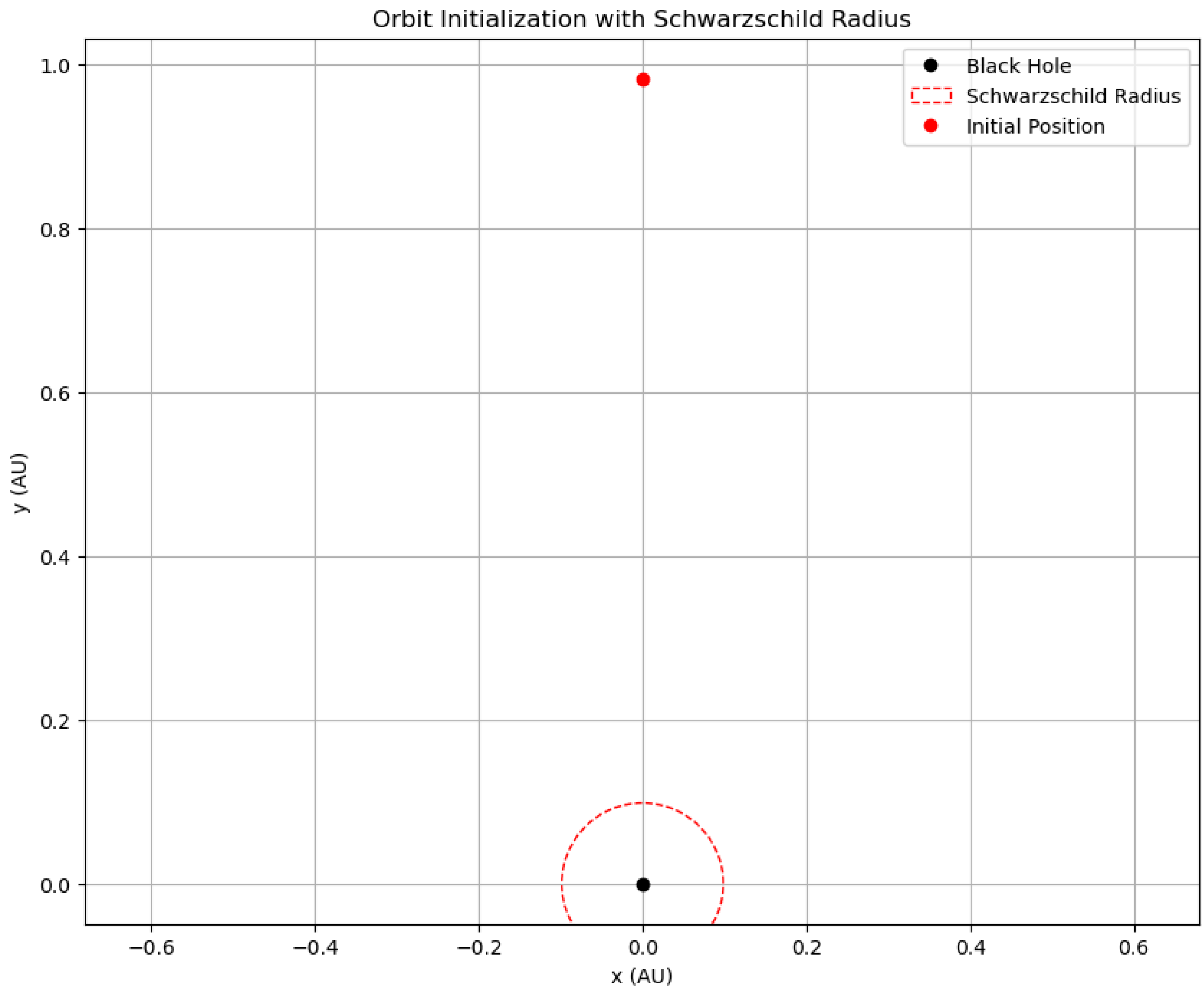

```
Simulation results saved to output/comparison/earth_convergence_rk3_500_orbi
t_history.csv
    Error: 4.65e-12 AU, Time: 0.01s
  Running trapz...
```



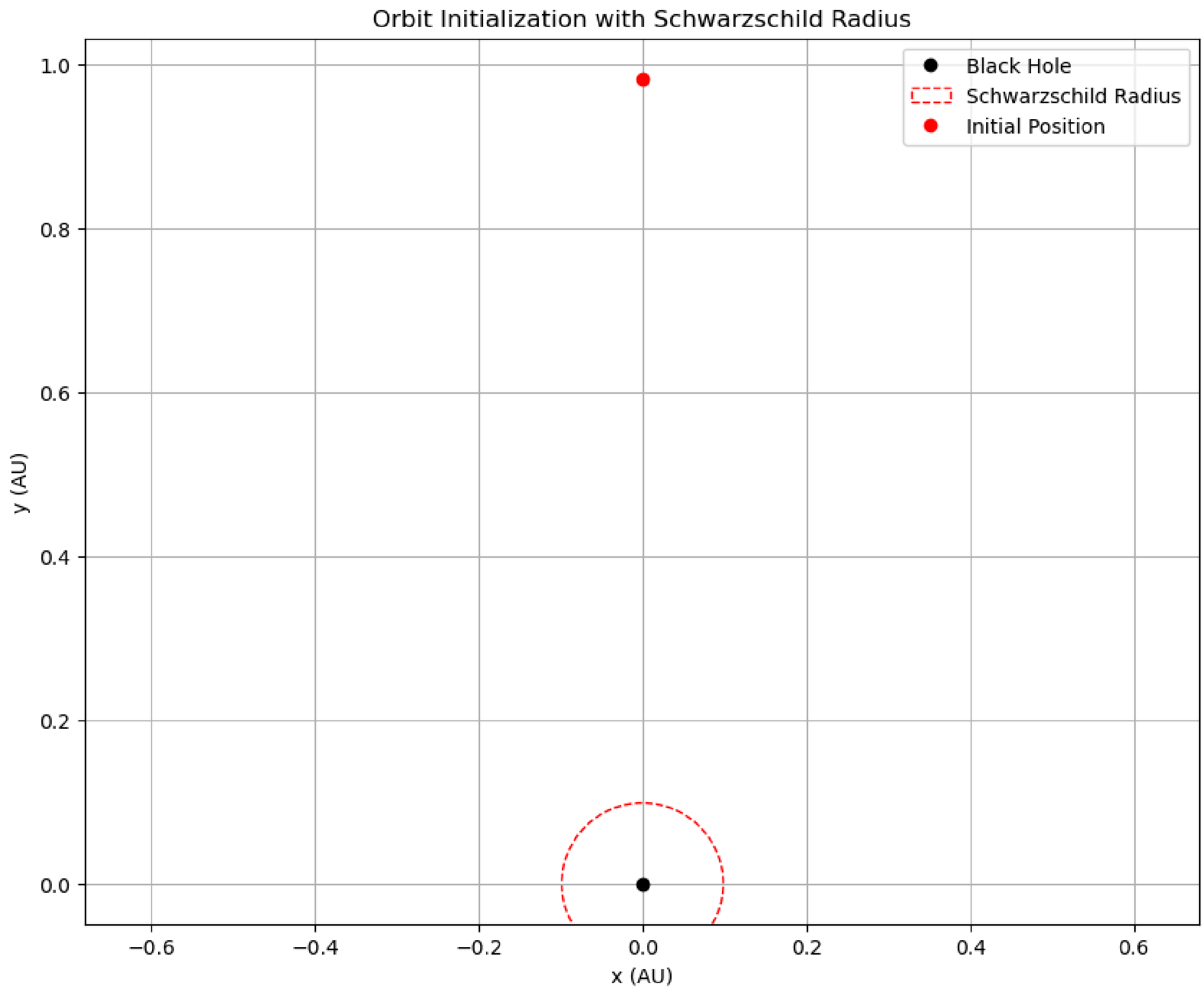

```
Simulation results saved to output/comparison/earth_convergence_trapz_500_or
bit_history.csv
    Error: 4.77e-12 AU, Time: 0.01s

Testing with 1000 time steps:
  Running rk3...
```



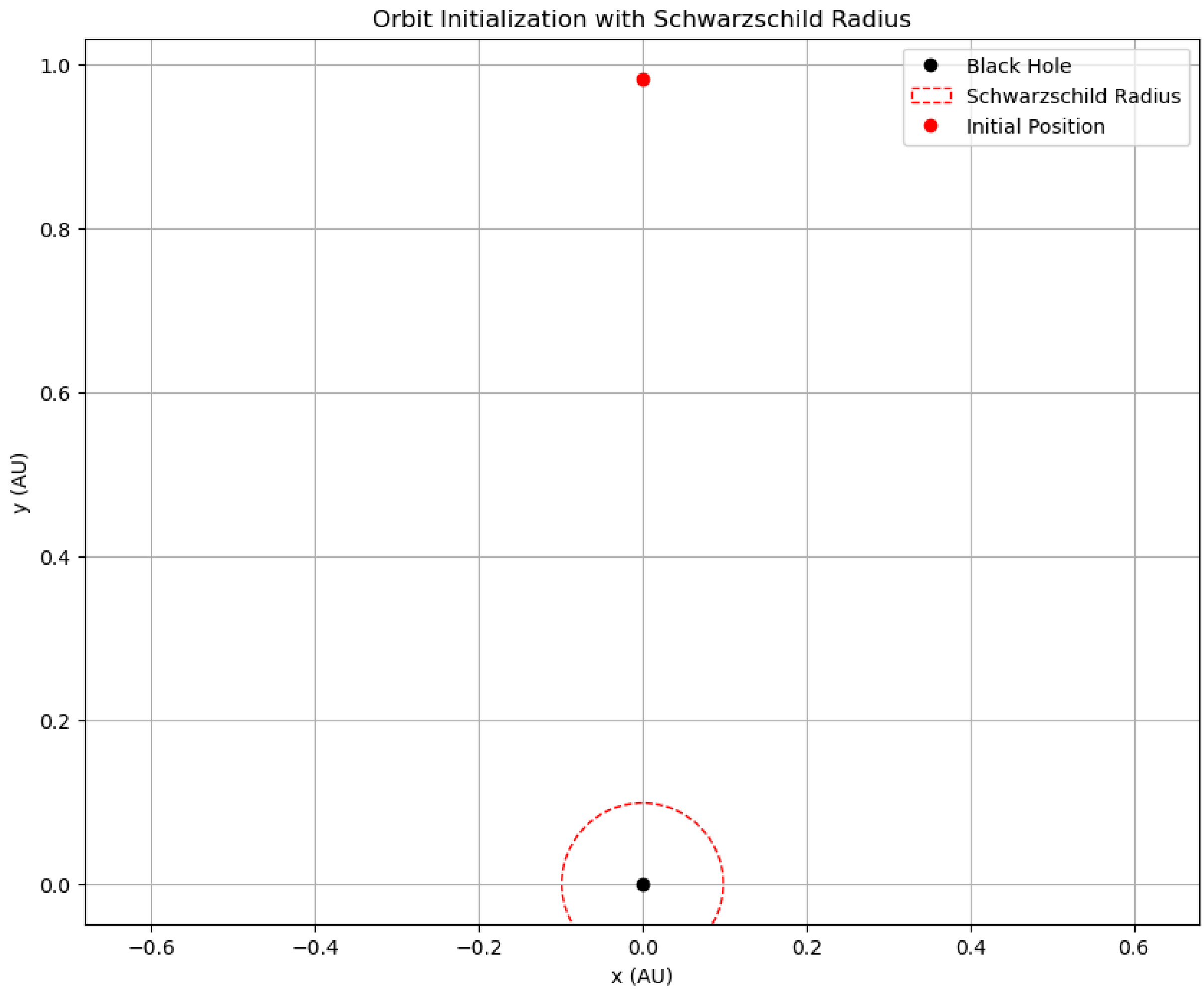

```
Simulation results saved to output/comparison/earth_convergence_rk3_1000_orb
it_history.csv
    Error: 4.65e-12 AU, Time: 0.02s
  Running trapz...
```



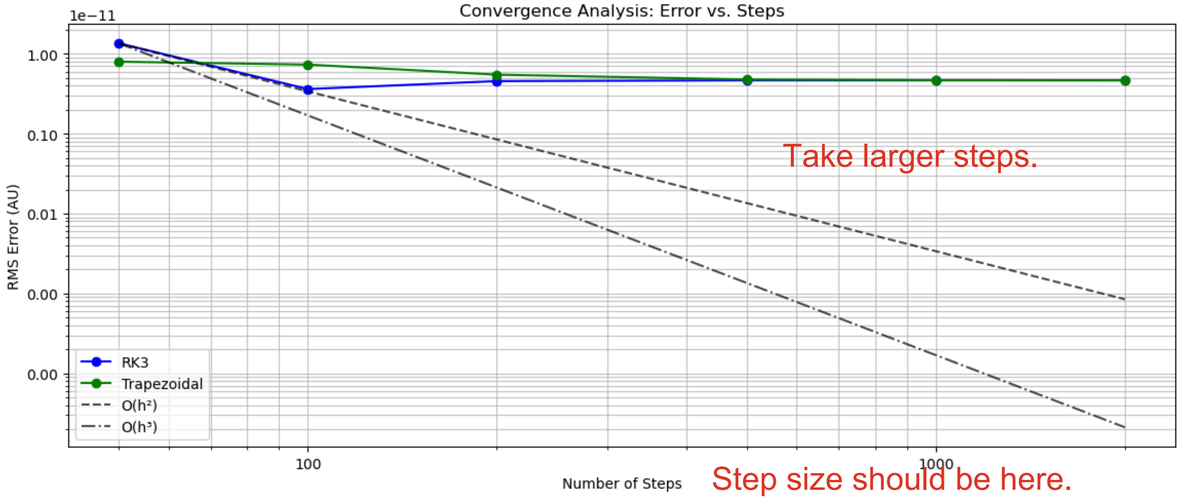

```
Simulation results saved to output/comparison/earth_convergence_trapz_1000_o
rbit_history.csv
    Error: 4.68e-12 AU, Time: 0.02s

Testing with 2000 time steps:
  Running rk3...
```




```
Simulation results saved to output/comparison/earth_convergence_rk3_2000_orb
it_history.csv
    Error: 4.65e-12 AU, Time: 0.04s
  Running trapz...
```



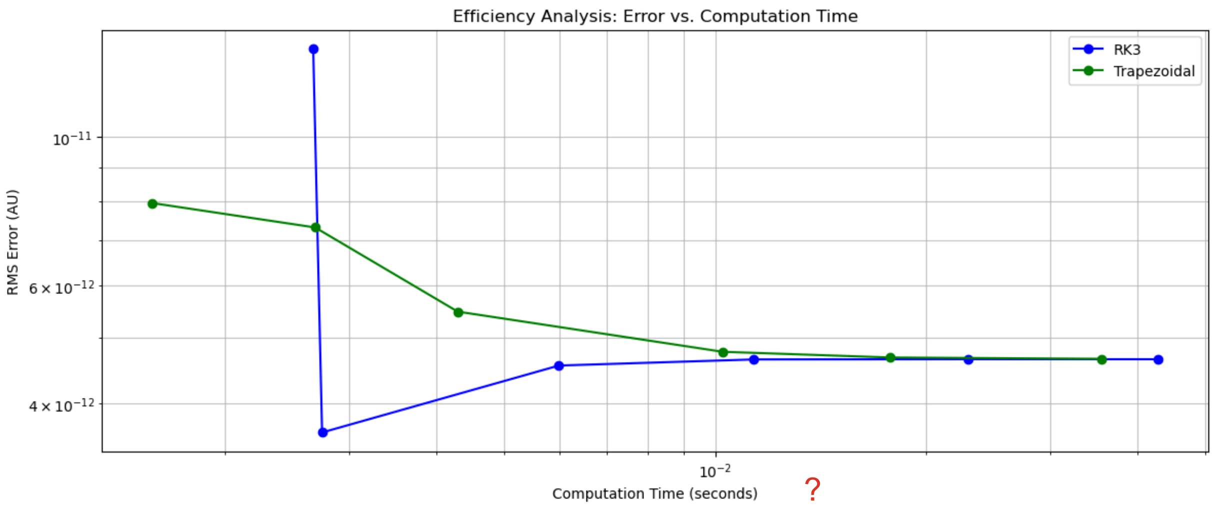

```
Simulation results saved to output/comparison/earth_convergence_trapz_2000_o
rbit_history.csv
    Error: 4.66e-12 AU, Time: 0.04s
```



Take larger steps.

-0.25

Step size should be here.



?

What does this plot represent?