Quiz 4 - Computational Physics II

NAME: _____ **SCORE:**
Date: Thursday 29 May 2025 (17h00) **Duration:** 45 minutes
Credits: 20 points (4 questions) **Type of evaluation:** LAB

Provide concise answers to the following items:

1. **(4 points) Partial differential equations (PDEs) in Fourier space**
   (a) Write down the 1D heat equation and the 1D one-way wave equation in Fourier space.
   (b) Explain the difference between diffusion and advection processes.

2. **(6 points) Numerical Stability**
   Explain 3 different methods by which we can determine the stability of a numerical scheme.

3. **(5 points) Finite-difference methods for PDEs**
   Write down the 3D Poisson equation and its central-difference approximation including errors.

4. **(5 points) Boundary Conditions for advection problems**
   Consider the advection equation, $u_t + c\,u_x = 0$ (with $c > 0$) on the domain $x \in [0, L]$ with N_x physical grid points plus 2 ghost zones (one at each end). The array `u` has size $N_x + 2$, and the physical points have indices 1 to N_x. The CFL number is defined as $\text{CFL} = c\,\Delta t/\Delta x$. The upwind scheme for interior points is: `u_new[1:Nx+1] = u[1:Nx+1] - CFL * (u[1:Nx+1] - u[0:Nx])`. Provide the Python code lines to set the ghost zone values of `u_new` for each boundary condition type below:

   1. **Periodic boundaries:** The domain wraps around, so $u(0, t) = u(L, t)$.
   2. **Dirichlet boundaries:** The boundaries have fixed values, so $u(0, t) = 0.5$ and $u(L, t) = 0.0$.
   3. **Neumann boundaries:** The boundary gradients are set, so $u_x(0, t) = 0.0$ and $u_x(L, t) = 1.0$.