ELECTROMAGNETIC INTERACTIONS

The Aharonov–Bohm Effect

In classical electrodynamics the potentials **A** and ϕ are not uniquely determined; the *physical* quantities are the *fields*, **E** and **B.** Specifically, the potentials

$$\varphi' \equiv \varphi - \frac{\partial \Lambda}{\partial t}, \quad \mathbf{A}' \equiv \mathbf{A} + \nabla \Lambda$$

yield the same fields as ϕ and **A.** Here, Λ is an arbitrary real function of position and time) This equation is called a **gauge transformation**, and the theory **gauge invariant.**

In quantum mechanics the potentials play a more direct role as they appear in:

$$\boxed{i\hbar \frac{\partial \Psi}{\partial t} = \left[\frac{1}{2m} (-i\hbar \nabla - q\mathbf{A})^2 + q\varphi \right] \Psi.}$$

They only by a *phase factor*, so they represent the same physical state, and in this sense the theory *is* gauge invariant.

$$\Psi' \equiv e^{iq\Lambda/\hbar} \Psi$$

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The Aharonov–Bohm Effect

It was believed that there could be no electromagnetic influences in regions where **E** and **B** are zero.

In 1959 **Aharonov and Bohm** showed that the vector potential *can* affect the quantum behaviour of a charged particle, ***even when the particle is confined to a region where the field itself is zero.***

Suppose a particle is moving through a region where **B** is zero (so $\nabla \times \mathbf{A} = \mathbf{0}$), but **A** itself is *not.* Below, $\mathcal{O}$ is some (arbitrarily chosen) reference point.

$$\left[\frac{1}{2m}(-i\hbar\nabla - q\mathbf{A})^2\right]\Psi = i\hbar\frac{\partial\Psi}{\partial t}, \qquad \Psi = e^{ig}\Psi', \qquad \text{where} \qquad g(\mathbf{r}) \equiv \frac{q}{\hbar}\int_{\mathcal{O}}^{\mathbf{r}} \mathbf{A}(\mathbf{r}') \cdot d\mathbf{r}',$$

$$\nabla \times \mathbf{A} = \mathbf{0}$$

The gradient of Ψ is:

$$\nabla\Psi = e^{ig}(i\nabla g)\Psi' + e^{ig}(\nabla\Psi'); \qquad \nabla g = (q/\hbar)\mathbf{A},$$

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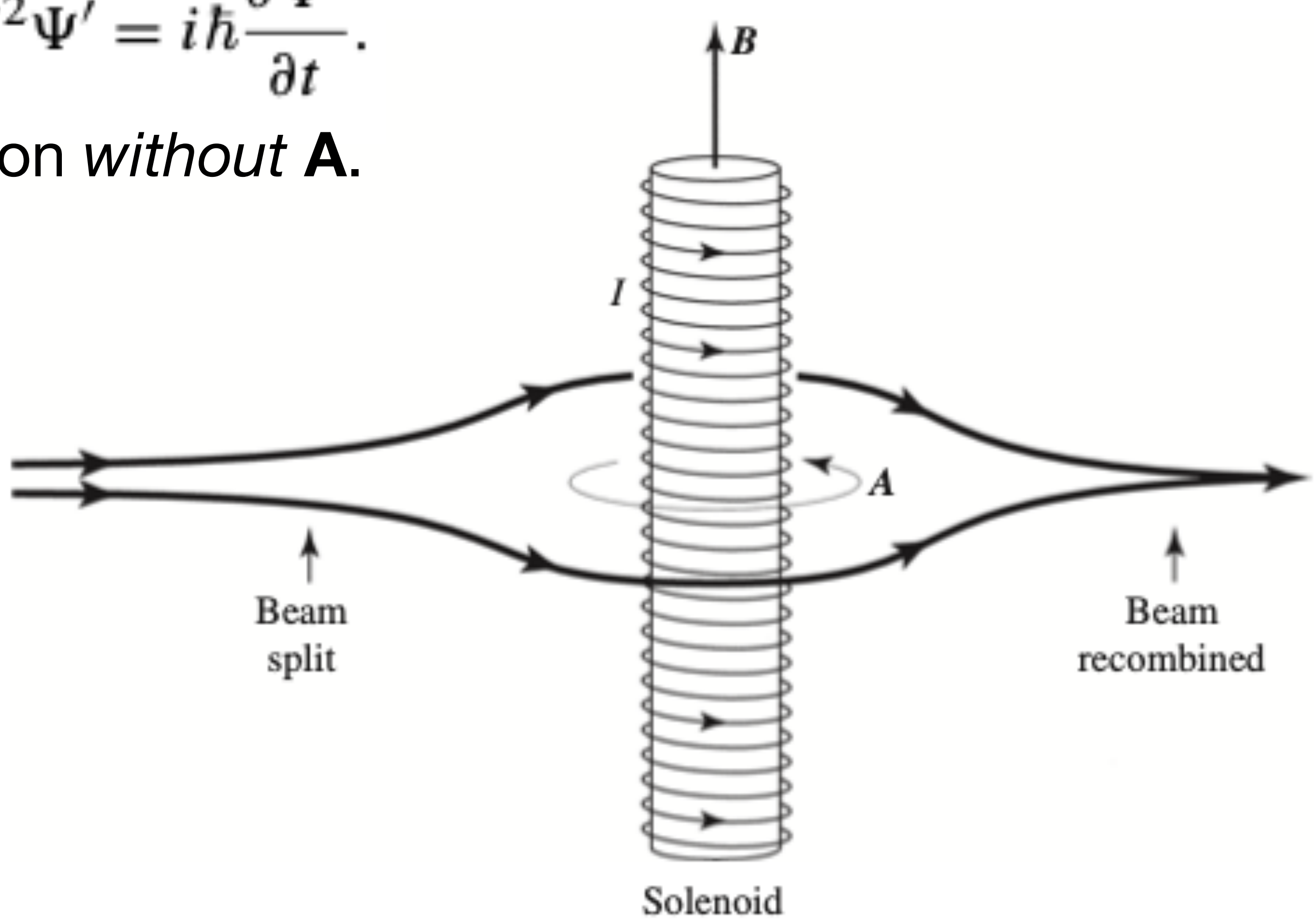
$$(-i\hbar\nabla - q\mathbf{A})\,\Psi = -i\hbar e^{ig}\nabla\Psi',$$

$$(-i\hbar\nabla - q\mathbf{A})^2\,\Psi = -\hbar^2 e^{ig}\nabla^2\Psi'$$

$$\Rightarrow \quad -\frac{\hbar^2}{2m}\nabla^2\Psi' = i\hbar\frac{\partial\Psi'}{\partial t}.$$

Evidently Ψ' satisfies the Schrödinger equation *without* **A**.



The Aharonov–Bohm Effect

Aharonov and Bohm proposed an experiment in which a beam of electrons is split in two, and they pass either side of a long solenoid before recombining.

The beams are kept well away from the solenoid itself, so they encounter only regions where **B** = **0**. But **A** is *not* zero, and the two beams arrive with *different phases*:

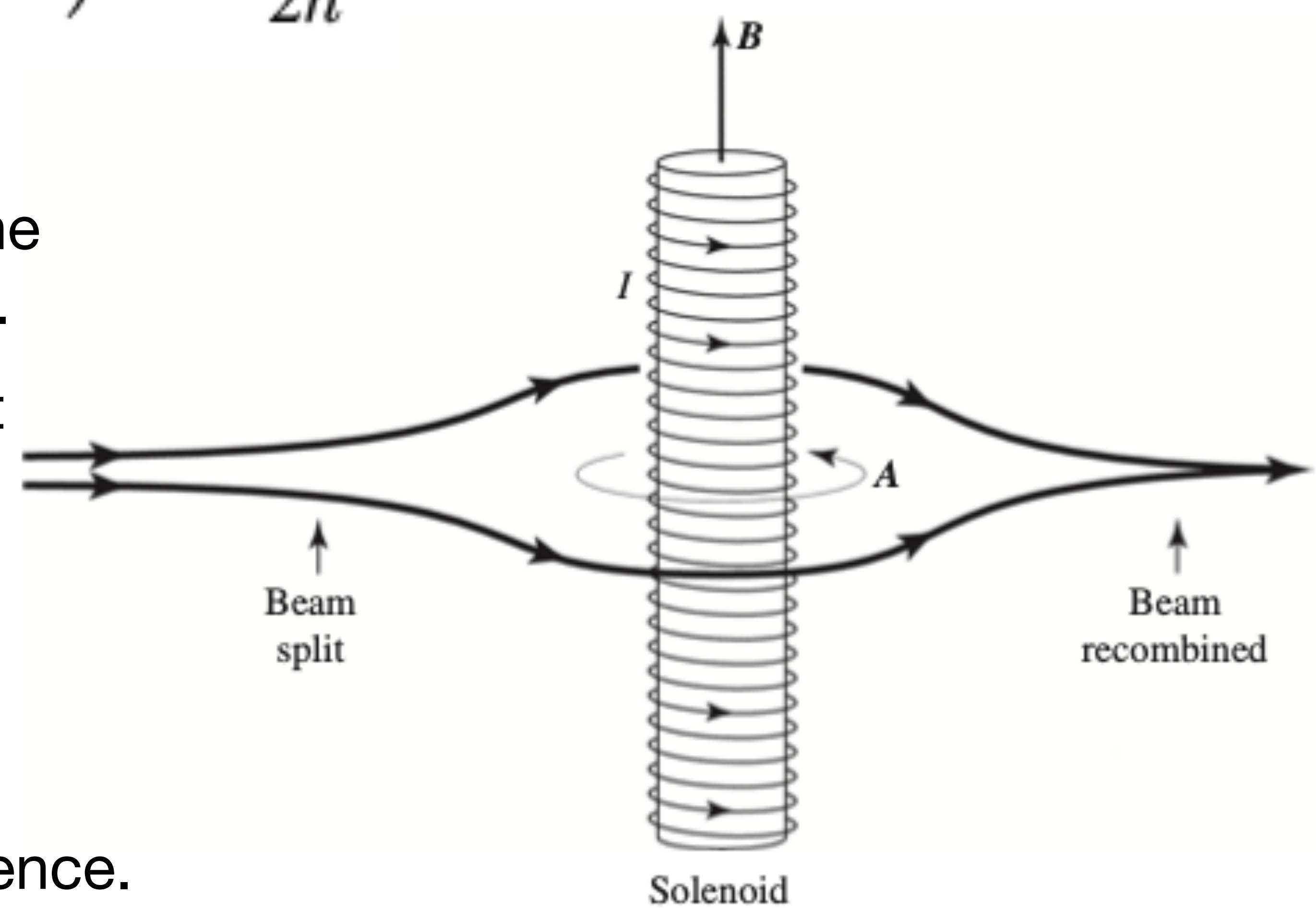
$$g = \frac{q}{\hbar}\int \mathbf{A}\cdot d\mathbf{r} = \frac{q\Phi}{2\pi\hbar}\int\left(\frac{1}{r}\hat{\phi}\right)\cdot\left(r\hat{\phi}\,d\phi\right) = \pm\frac{q\Phi}{2\hbar}.$$

The plus sign applies to the electrons traveling in the same direction as **A**, i.e., in the same direction as the current in the solenoid.

The beams arrive out of phase by an amount proportional to the magnetic flux their paths encircle:

$$\text{phase difference} = \frac{q\Phi}{\hbar}.$$

This phase shift leads to measurable interference.



There *can* be electromagnetic effects in regions where the fields are zero.