IDENTICAL PARTICLES

Generalised Symmetrisation Principle

For the sake of simplicity, we have assumed that:

- the particles are noninteracting,
- the spin and position are decoupled (with the combined state a product of position and spin factors), and
- the potential is time-independent.

But the fundamental symmetrisation/antisymmetrisation requirement for identical bosons/fermions is much more general.

Let us define the **exchange operator**, $\hat{P}$, which interchanges the two particles:

$$\hat{P}\,|(1,2)\rangle = |(2,1)\rangle.$$

$\hat{P}$ switches the particles ($1 \leftrightarrow 2$), exchanging their positions, their spins, and any other properties they might possess.

Here, $\hat{P}^2 = 1$, and the eigenvalues of $\hat{P}$ are ± 1. If the two particles are identical, the Hamiltonian must treat them the same:

$$m_1 = m_2 \text{ and } V(\mathbf{r}_1, \mathbf{r}_2, t) = V(\mathbf{r}_2, \mathbf{r}_1, t).$$

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It follows that $\hat{P}$ and $\hat{H}$ are compatible observables, $\left[\hat{P}, \hat{H}\right] = 0,$

and hence: $$\frac{d\left\langle \hat{P} \right\rangle}{dt} = 0.$$

If the system starts out in an eigenstate of $\hat{P}$, **symmetric** $\langle \hat{P} \rangle = 1$ or **antisymmetric** $\langle \hat{P} \rangle = -1$, it will stay that way forever.

The **symmetrisation axiom** says that for identical particles the state is not merely *allowed*, but *required* to satisfy:

$$|(1, 2)\rangle = \pm|(2, 1)\rangle,$$

with the plus sign for bosons, and the minus sign for fermions.

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If you have ***n*** identical particles, of course, the state must be symmetric or antisymmetric under the interchange of *any two*:

$$\boxed{|(1, 2, \ldots, i, \ldots, j, \ldots, n)\rangle = \pm|(1, 2, \ldots, j, \ldots, i, \ldots, n)\rangle,}$$

This is the ***generalised symmetrisation principle***, of which the following equation is a special case.

$$\psi_{\pm}(\mathbf{r}_1, \mathbf{r}_2) = A\left[\psi_a(\mathbf{r}_1)\psi_b(\mathbf{r}_2) \pm \psi_b(\mathbf{r}_1)\psi_a(\mathbf{r}_2)\right]$$

It is perfectly possible to imagine a system of two ***distinguishable particles*** (say, an electron and a positron) for which the Hamiltonian is symmetric, and yet there is no requirement that the state be symmetric (or antisymmetric).

Identical particles *have* to occupy symmetric or antisymmetric states, and this is a *new fundamental law*—on a par, logically, with Schrödinger's equation and the statistical interpretation.

Quantum mechanics allows for the *possibility* of identical particles.