Homework 1 - Quantum Mechanics I

NAME: ____ **SCORE:** ____________

Deadline: Wednesday 24th May 2023 by 10:00am (submission only on paper)
Credits: 20 points **Number of problems:** 4
Type of evaluation: Formative Evaluation

- This homework includes problems on unit 1 of our quantum mechanics course.
- This assignment should be submitted individually by the deadline.
- Unless stated otherwise, write your answers in SI units and highlight them.

1. **(6 points) The photoelectric effect**

As reviewed in class, the photoelectric effect experiment consists of setting up an electric circuit embedding 2 metal plates. We illuminate the first plate with a beam of photons of certain wavelength, λ, and as the photons interact with the metal plate, electrons are ejected from it. The electrons then reach the second plate and an electric current emerges. To prevent these electrons from reaching the second plate, we can increase the voltage until the electric current becomes zero in the circuit. The voltage value at which this happens is called "stopping potential", V_0, which is defined as the potential needed to stop the photoelectrons with the largest kinetic energy (so $K_{max} = e\,V_0$) at a specific wavelength λ.

(a) Carry out photoelectric effect experiments for 2 different metals and collect 12 data points for each, using: `https://applets.kcvs.ca/photoelectricEffect/PhotoElectric.html` To collect the data, choose a metal, fix a wavelength, and vary the voltage until the current becomes zero. When this happens, push "record data points". Then, vary the wavelength and repeat the process. When you have 12 data points for the first metal, choose another metal and repeat the experiment. At the end, you should have a data table with 24 data points, 12 for each metal (send your data file via email by the deadline).

Using your favourite programming language (Python, Mathematica) or Spreadsheet/Excel:

(b) Open and read the data file containing the experimental results, and make two high-quality labeled scattered plots, one for each metal, with the maximum kinetic energy (K_{max}) in the Y-axis and frequency on the X-axis.

(c) For each metal, define a good model to describe the data. Carry out a regression and find the function that best fits the data. Report the fitting functions for each metal and make two labeled plots, one for each metal, containing the experimental data and their fits.

(d) Make a new figure combining the data and fitting functions for both metals. Which metal has a higher cutoff frequency? What does the slope of the curves represent?

Using the fitting functions, carry out the following calculations:

(e) Calculate the work function, ϕ, and the cutoff wavelength, λ_{cutoff}, for each metal, and the relative errors with respect to the known values (research what these values are).

2. **(4 points) Compton wavelength and scattering**

(a) Calculate the Compton wavelengths of an electron and a muon. How much energy would photons with those wavelengths have?

(b) Suppose we have an experiment in which monochromatic light is scattered by an electron at an angle of 75°. What is the fractional increase in the wavelength, $\frac{\Delta\lambda}{\lambda}$ if the incident light has: 1) a $\lambda = 580\,\mathrm{nm}$ (i.e. photons are in the visible region), and 2) a $\lambda = 0.02\,\mathrm{nm}$ (i.e. photons are in the X-ray region)? Why were X-rays used by Compton in his experiments?

3. **(5 points) Basic quantum mechanics**

(a) A typical microwave oven operates at roughly 2.5 GHz at a maximum power of 300 W. How many photons per second can it emit? What about a low-power laser operating at 10 mW at 633 nm, or a mobile phone operating at 0.25 W at 850 MHz?

(b) Calculate the de Broglie wavelengths of: 1) a particle of diameter 10 cm with a mass of $m = 2\,\mathrm{kg}$ that is moving at a speed of $25\,\mathrm{m\,s^{-1}}$, and 2) a neutron whose kinetic energy is 0.167 eV. Based on these results, are the wave properties of matter relevant for these particles? Why?

(c) Calculate the limit of accuracy with which we can locate along the X-axis: 1) a particle of mass $m = 3 \times 10^{-2}\,\mathrm{kg}$, and 2) an electron, for which the X-component of their velocity is measured to an accuracy of $\pm 10^{-8}\,\mathrm{m\,s^{-1}}$. What do these results tell us about classical and quantum mechanical particles?

4. **(5 points) Wave function, normalisation, and expectation values**

Let us assume we have a quantum particle of mass m, whose wave function is described by:

$$\Psi(x,t) = \beta\, e^{-\frac{\gamma m}{2\hbar}x^2}\, e^{-i\frac{\gamma}{2}t}$$

where β and γ are positive real constants.

(a) Find β.

(b) For what potential energy function, $V(x)$, is this a solution to the Schrödinger equation?

(c) Calculate the expectation values of x and x^2.

(d) Calculate the expectation values of p and p^2.

(e) Find σ_x and σ_p. Is their product consistent with the uncertainty principle?