The hydrogen atom

Spin:

In ***classical* mechanics**, a rigid object admits two kinds of angular momentum:

orbital (**L** = **r**×**p**), associated with motion *of* the center of mass, and

spin (**S** = *I* ω), associated with motion *about* the center of mass.

In ***quantum mechanics***, the distinction is absolutely fundamental.

Orbital angular momentum, associated (in the case of hydrogen) with the motion of the electron around the nucleus (and described by the spherical harmonics).

Spin, which has nothing to do with motion in space (and not described by any function of the position variables *r*,θ,ϕ) but which is somewhat analogous to classical spin.

The electron (as far as we know) is a structureless point, and its spin angular momentum cannot be decomposed into orbital angular momenta of constituent parts.

Suffice it to say that elementary particles carry **intrinsic** angular momentum (**S**) in addition to their “extrinsic” angular momentum (**L**).

The hydrogen atom

Spin:

Fundamental commutation relations:

$$[S_x, S_y] = i\hbar S_z, \quad [S_y, S_z] = i\hbar S_x, \quad [S_z, S_x] = i\hbar S_y.$$

It follows (as before) that the eigenvectors of S^2 and S_z satisfy:

$$S^2|s\, m\rangle = \hbar^2 s\,(s+1)\,|s\, m\rangle; \quad S_z|s\, m\rangle = \hbar m|s\, m\rangle;$$

$$S_\pm\,|s\, m\rangle = \hbar\sqrt{s\,(s+1) - m\,(m \pm 1)}\;|s\;\,(m \pm 1)\rangle\,,$$

where $S_\pm \equiv S_x \pm i\, S_y$. The eigenvectors are not spherical harmonics (they're not functions of θ and ϕ at all), and there is no reason to exclude the half-integer values of *s* and *m*:

$$s = 0,\ \frac{1}{2},\ 1,\ \frac{3}{2},\ldots; \quad m = -s,\ -s+1,\ldots,s-1,\ s.$$

The hydrogen atom

Spin:

Every elementary particle has a *specific and immutable* value of s, which we call **the spin** of that particular species:

- π mesons have spin 0
- electrons have spin 1/2
- photons have spin 1
- Δ baryons have spin 3/2
- gravitons have spin 2; and so on.

By contrast, the *orbital* angular momentum quantum number l (e.g. for an electron in a hydrogen atom) can take on any (integer) value, and will change from one to another when the system is perturbed.

s is *fixed* for any given particle, so the theory of spin is comparatively simple.