Homework 5 - Quantum Mechanics I

NAME: ____ **SCORE:** _____________

Deadline: Thursday 3rd August 2023 by 11:00am (submission only on paper)
Credits: 20 points **Number of problems:** 5
Type of evaluation: Formative Evaluation

- This homework includes problems on units 4 and 5 of the QM course.
- This assignment should be submitted individually by the deadline.
- Please justify all calculations and highlight your answers.

1. **(4 points) Spinor**
   An electron is in the spin state: $\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$, where A is a normalisation constant.

   (a) Determine the normalisation constant A.

   (b) Find the expectation values of S_x, S_y, and S_z.

   (c) Find the uncertainties (standard deviations) for S_x, S_y, and S_z.

   (d) Is the uncertainty principle satisfied?

2. **(4 points) Particle in a 3D infinite potential well**
   Suppose we have a particle in a 3D spherical and infinite potential well:

$$V(r) = \begin{cases} 0 & 0 \le r \le a \\ \infty & r > a \end{cases}$$

   where (r, θ, ϕ) are the radial and angular (spherical) coordinates.

   (a) Write down the time-dependent Schrödinger equation for the particle in spherical coordinates. Use variable separation to split the spatial and temporal terms.

   (b) Apply variable separation to the time-independent Schrödinger equation and write the differential equations of the radial and angular parts.

   (c) Solve the angular equation.

   (d) Compute the energy levels and the stationary wave function, $\psi(r, \theta, \phi)$ for $\ell = 0$.

3. **(4 points) Radial wave function of atoms**
   The wave function of an electron in a hydrogenic atom with an atomic number $Z = 70$ and mass number $A = 173.05$ is given by:

$$\psi(r) = Be^{-r/a}$$

where $a = \frac{a_0}{Z}$ and $a_0 = 0.53$ Å is the Bohr radius. This (Ytterbium, Yb) atom contains only one electron with charge e. The charge and radius of its nucleus are eZ and $R = 1.2(A^{1/3})$ fm, respectively.

(a) Normalise the wave function.

(b) Calculate the probability that the electron is found in the nucleus.

(c) What is the probability that the electron is in the region $y < 0$?

(d) What is the probability that the electron is in the region $x, y, z > 0$?

4. **(4 points) Electron in a Magnetic Field**
Consider an electron (at rest) embedded in an oscillating magnetic field:

$$\vec{B} = B_0 \cos(\omega t)\, \vec{k},$$

where B_0 and ω are constants.

(a) Construct the Hamiltonian matrix for this system. Is this Hamiltonian time-dependent or time-independent?

(b) The electron starts out (at $t = 0$) in the spin-up state with respect to the x axis (i.e., $\chi(0) = \chi_+^{(x)}$). Determine $\chi(t)$ at any subsequent time by solving the Schrödinger equation.

(c) Find the probability of getting $-\frac{\hbar}{2}$, if you measure S_x.

(d) What is the minimum field strength (B_0) required to force a complete flip in S_x?

5. **(4 points) Two-particle systems**
Consider a system of two non-interacting quantum particles (both of mass m) inside a 1D infinite square well potential of width, a.

(a) Based on the analysis carried out in unit 2 of the course for a particle trapped in this potential, write down the one-particle wave function and the respective energy for each particle in the system.

(b) Write down the composite wave function of the two-particle system assuming the particles are: distinguishable (system D), identical bosons (system B), and identical fermions (system F).

(c) Based on the previous results, find the ground state of each two-particle system (D, B, and F), jointly with the respective energy. Briefly explain your findings.

(d) Find the first three excited states for each two-particle system (D, B, and F), jointly with their respective energies.