

The Radial Equation

$$\frac{1}{R} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) - \frac{2mr^2}{\hbar^2} [V(r) - E] = \ell(\ell + 1);$$

The angular part of the wave function, $Y(\theta, \phi)$, is the same for *all* spherically symmetric potentials.

The actual *shape* of the potential, $V(r)$, affects only the *radial* part of the wave function, $R(r)$.

Variable change:

$$R = u/r,$$

$$u(r) \equiv r R(r) \quad \rightarrow \quad \begin{aligned} dR/dr &= [r (du/dr) - u] / r^2, \\ (d/dr) [r^2 (dR/dr)] &= r d^2u/dr^2, \end{aligned}$$

We get the **radial equation**:

$$-\frac{\hbar^2}{2m} \frac{d^2u}{dr^2} + \left[V + \frac{\hbar^2}{2m} \frac{\ell(\ell + 1)}{r^2} \right] u = Eu.$$

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It is *identical in form* to the one-dimensional Schrödinger, where:

$$V_{\text{eff}} = V + \frac{\hbar^2}{2m} \frac{\ell(\ell+1)}{r^2},$$

is the **effective potential**, which contains a **centrifugal term**: $(\hbar^2/2m) [\ell(\ell+1)/r^2]$

It tends to throw the particle outward (away from the origin), just like the centrifugal (pseudo-)force in classical mechanics.

The normalisation conditions is: $\int_0^\infty |u|^2 dr = 1.$

which is potential $V(r)$ specific.