Solutions to:

Quantum mechanics Quiz 2

Subject: Quantum Mechanics I
Deadline: Wednesday 22 June 2022 (by 4pm)
Duration: 24 hours
Credits: 20 points
Number of questions: 5
Type of evaluation: Laboratory (LAB)

Write the correct answer to each question and/or briefly explain your answer.

* Required

1. Email *

2. Name: *

3. 1. (4 points) Is the wave function Ψ(x,t) Galilean invariant? Explain. *

No, Ψ(x,t) is not Galilean invariant. Two measurements of the wavelength of QM waves carried out by two distinct observers (in constant relative motion with respect to one another) do not agree. This implies that the phase of Ψ(x,t) is not Galilean invariant.

4. 2. (4 points) Cite two differences between ordinary (classical) waves and quantum mechanical waves. *

1. As opposed to QM waves, two observers would agree on the value of the phase of classical waves (which are Galilean invariant).

2. Quantum wave functions are Complex and as a result cannot be directly measured, while classical wave functions are real and can be measured directly.

3. (bonus) Classical and quantum mechanical waves satisfy different wave equations. The former satisfies a partial differential equation that is second order in time, while the latter satisfies a partial differential equation (the Schrödinger equation) that is first order in time.

5. 3. (4 points) Why do we need to introduce wave packets to describe free particles in quantum mechanics? *

We need to introduce wave packets to describe free particles because the stationary states of free particles (separable solutions for x) are not physically-realisable states on their own as they have two problems:

1. They are not normalisable.
2. They move at 1/2 of the speed of the particle they are supposed to describe.

Introducing wave packets solves both problems at the same time as they are normalisable and move at the speed of the particle they describe.

6. 4. (4 points) Mathematically speaking, what are wave packets composed of? (Don't include equations, explain in words.) *

Wave packets consist of:

1. The ‘ripples’, which are superimposed sinusoidal functions moving at the phase velocity (i.e. at 1/2 of the particle velocity).

2. The ‘envelope’, which is a function that modulates the amplitudes of the ripples (i.e. it encapsulates the sinusoidal functions) and moves at the group velocity (i.e. at the particle velocity).

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7. 5. (4 points) Mention 3 properties that separable solutions of the 1D Schrödinger equation have. *

1. They represent stationary states whose probability densities do not depend on time.

2. The expectation values of any quantity associated with them is constant in time.

3. They have definite total energy because the variance of the Hamiltonian H is zero.

4. (Bonus) A linear combination of them defines a general solution to the time-dependent Schrödinger equation.
