De Broglie's proposal: matter waves

· de Broglie relations (1924)

↳ De Broglie's proposal.

- γ are particles — definite amount of E, packets, cannot be broken. & momentum p
- γ is also a wave — interferes / described by waves, ν

therefore ⟹ this could be a more general property.

Is this universal?

de Broglie:

↳ All "matter particles" behave as waves, not just the γ's.

↳ There is a wave associated to a matter particle.

De Broglie's proposal: matter waves

QM: $\Big\{$ probability amplitude to be somewhere
probability waves

Matter waves are introduced:

↳ Matter waves → probability amplitudes. $\mathbb{C}$#'s
↳ Associate to a particle a wave that depends on the momentum.

For a particle of momentum p, we associate a plane wave $\boxed{\lambda = \frac{h}{p}}$ which is the de Broglie λ.

QM arises as a theory

- **1925** - Schrödinger/Heisenberg wrote the governing equations of QM.
- QM is almost a 100 years old!

What is QM?

QM is a framework to do physics.

Quantum physics

- QM replaces classical mechanics CM. CM is a good approximation but it is not accurate when describing some experiments.
- **Quantum physics:** principles of QM applied to physical phenomena.
- **Branches of QM:**
  - **QED:** QM + EM
  - **QCD:** QM + Strong interaction
  - **Quantum optics:** QM + photons
  - **Quantum gravity:** QM + gravitation -> String theory (QM of gravity)

Mathematical tools for QM

- **Is QM a linear theory?**
- Why do we need complex numbers?

linear theory:

$$\left.\begin{array}{l}\text{Solution 1}\\ \text{Solution 2}\end{array}\right\} \Rightarrow \begin{array}{c}\text{New}\\ \text{Solution 3}\end{array}$$

We can create linear combinations of known solutions to get new solutions.

Linear Operators

- L.u =0
- L = linear operator, u = unknown
- Several operators applied to the same unknown: L1.u=0, L2.u=0
- Same operator applied to different unknowns: L(u1,u2,u3) =0

Properties of linear operators:

- Scale a solution: L(au) = a Lu
- Combine solutions: L(u1+u2) = L(u1) + L(u2)

EM theory is linear

Example: EM

$\frac{q}{V}$ $\quad$ $\frac{q}{A\cdot t}$

$(\vec{E}, \vec{B}, \rho, \vec{J})$ is a solution

$\overset{L}{\Rightarrow} (\alpha\vec{E}, \alpha\vec{B}, \alpha\rho, \alpha\vec{J})$ is also a solution, $\alpha \in \mathbb{R}$

$$\left.\begin{array}{l}(E_1, B_1, \rho_1, J_1) \\ (E_2, B_2, \rho_2, J_2)\end{array}\right\}$$ are solutions:

$\overset{L}{\Rightarrow} (E_1+E_2, B_1+B_2, \rho_1+\rho_2+ J_1+J_2)$ is a sln.

Is QM a linear theory?

Linear equation:

$$L\, u = 0$$

Linear operator (eq)

unknown (variable)

Properties:

$$\begin{cases} L(\alpha u) = \alpha L u \\ L(u_1 + u_2) = L u_1 + L u_2 \end{cases}$$

Linear combinations:

$$L(\alpha u_1 + \beta u_2) = L(\alpha u_1) + L(\beta u_2) = \alpha L u_1 + \beta L u_2$$

If $u_1, u_2 \equiv$ sln $\Rightarrow$ $\alpha u_1 + \beta u_2 \Rightarrow$ sln

Linear vs. Non-linear Theories

linear & non-linear theories.

(L) $\left\{ \begin{array}{l} \text{EM} \\ \text{QM} \end{array} \right.$

much simpler

(NL) $\left\{ \begin{array}{l} \text{G.R.} \\ \text{C.M} \quad \text{e.g. 3-body problem} \end{array} \right.$

very non-linear

QM is linear!