Solutions to:
Quiz 1

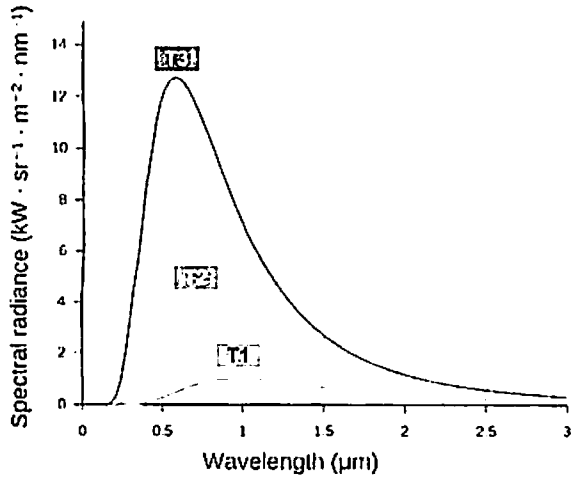
NAME: _____ SCORE:
Subject: Quantum Mechanics I
Date: Monday 6 June 2022
Duration: 30 minutes
Credits: 20 points
Number of questions: 10
Type of evaluation: Laboratory (LAB)

Part A. Choose the correct answer to each question or statement given below, and briefly justify your choice in the white space assigned to each of them.

1. **(2 points) Units of the Planck constant, h**
   If [M], [L], and [T] represent the dimensions of mass, length, and time, respectively, what are the dimensions of the Planck constant, h?

   A. $[M\,L^2\,T^{-2}]$
   (B) $[M\,L^2\,T^{-1}]$ because $E = h\nu \Rightarrow [h] = \frac{[E]}{[\nu]} = \frac{ML^2T^{-2}}{T^{-1}} = ML^2T^{-1}$
   C. $[M\,L^2\,T^{-3}]$
   D. $[M\,L^2\,T]$

2. **(2 points) Black body radiation**
   Given the figure below, what is the relation between the temperatures T1, T2, and T3?



   A. $T1 > T2 > T3$
   B. $T3 > T1 > T2$
   (C) $T3 > T2 > T1$ because $\lambda_{max} = \frac{b}{T}$ (Wien's law), so smaller λ indicate higher T.
   D. $T1 = T2 = T3$

3. **(2 points) Compton effect**
   What happens to the wavelength of a photon after it collides with an electron?

   (A) Increases because the γ loses energy $\begin{cases} E_{before} = h\frac{c}{\lambda_{before}} \\ E_{after} = h\frac{c}{\lambda_{after}} \end{cases}$
   B. Decreases
   C. Remains the same
   D. Becomes infinite

   $E_{after} < E_{before} \Rightarrow \lambda_{after} > \lambda_{before}$

4. **(2 points) Photoelectric effect**
   If the wavelength of electromagnetic radiation is doubled, the energy of photons:

   A. remains the same.
   B. doubles too.
   (C) is halved. because $E \propto \frac{1}{\lambda}$ $\begin{cases} E_1 = h\frac{c}{\lambda_1} \\ E_2 = h\frac{c}{\lambda_2} = 2E_1 \end{cases}$ $\frac{E_2}{E_1} = 2 = \frac{\lambda_1}{\lambda_2}$
   D. is infinite.

   $\Rightarrow \lambda_2 = 0.5\lambda_1$

5. (2 points) Wavelike nature of particles
The length scale at which the wavelike nature of particles emerges and becomes apparent is called:

A. the Planck's length.
B. the Bohr radius.
C. the Compton wavelength.
(D) the de Broglie wavelength. because it is the characteristic $\lambda = \frac{h}{p}$ of matter waves

6. (2 points) Quantum superposition
The state of a quantum system can be described as the superposition of two states, $|\Psi_1\rangle$ and $|\Psi_2\rangle$, as follows: $|\Psi\rangle = a|\Psi_1\rangle + b|\Psi_2\rangle$, where $|a| > |b|$. Upon measurement of the same property, $|\Psi_1\rangle$ returns A, and $|\Psi_2\rangle$ returns B, which value does $|\Psi\rangle$ return after measurement?

A. A
(B) A or B It can return either value with probabilities:
C. $aA + bB$ $\text{prob}(A) = \frac{|a|^2}{|a|^2+|b|^2}$ ^ $\text{prob}(B) = \frac{|b|^2}{|a|^2+|b|^2}$
D. None

7. (2 points) Expectation values
In Quantum Mechanics, the expectation value of the momentum of a particle represents:

A. the most probable value of its momentum.
B. the average value of the momentum measured in repeated experiments on the same particle.
C. the average of the square of the momentum measured on identical particles in the same state.
(D) the average value of the momentum measured on identical particles in the same state.
because it is not the mode, repeated experiments should agree, and C. gives wrong units $\langle p^2 \rangle$.

Part B. Provide concise answers to the following items:

8. (2 points) The Schrödinger equation
Write down the Schrödinger equation, and indicate what each term in it represents.

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

kinetic energy operator applied on Ψ

potential energy operator applied on Ψ

Complex-valued term indicating how the wavefunction Ψ changes with time

9. (2 points) de Broglie's proposal
Briefly explain what de Broglie's proposal consists of.
de Broglie's proposal states that all matter (not just photons) exhibit wave-like properties and that the wavelength of such waves are inversely proportional to their momentum.

10. (2 points) Measuring the Planck constant
Briefly explain what kind of experiment you would set up to measure the Planck constant h.
For example, we can set up a photoelectric effect experiment in which we measure the voltage needed to stop the e- current for different ν, and then carry out a linear regression. The slope of the fitted line is the Planck constant h.

Recall that: $K_{max} = h\nu - \phi$ (ν: frequency; ϕ: work function; K_{max}: Kinetic energy of e-)