QM applications

Stern-Gerlach experiment

The Stern–Gerlach experiment: In an *inhomogeneous* magnetic field, there is not only a *torque*, but also a *force*, on a magnetic dipole:[48]

$$\mathbf{F} = \nabla(\boldsymbol{\mu} \cdot \mathbf{B}). \tag{4.168}$$

This force can be used to separate out particles with a particular spin orientation. Imagine a beam of heavy neutral atoms,[49] traveling in the y direction, which passes through a region of static but inhomogeneous magnetic field (Figure 4.15)—say

$$\mathbf{B}(x, y, z) = -\alpha x \hat{\imath} + (B_0 + \alpha z)\hat{k}, \tag{4.169}$$

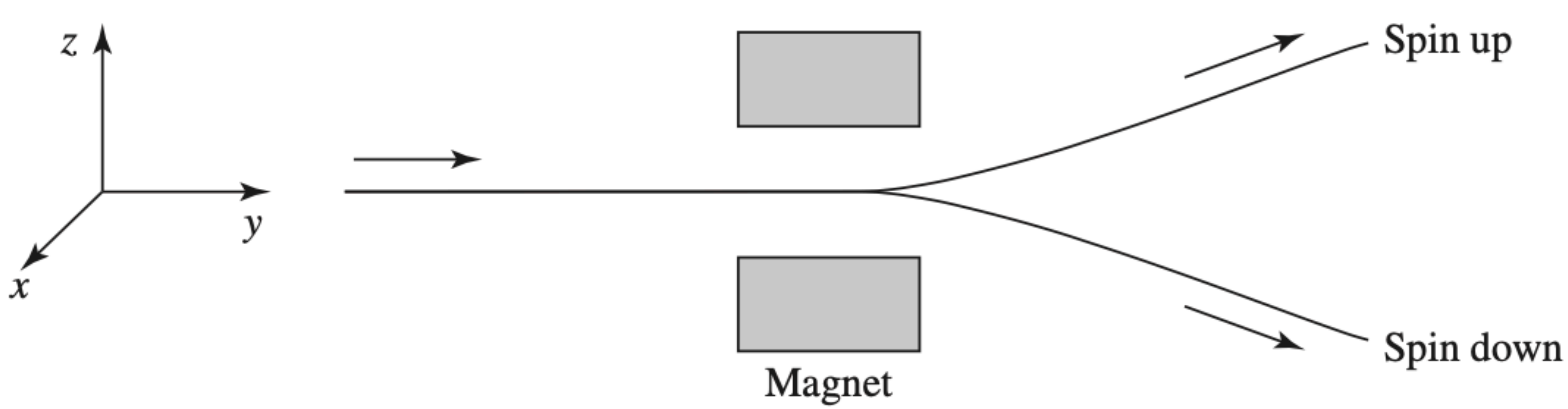

Figure 4.15: The Stern–Gerlach apparatus.

QM applications

Stern-Gerlach experiment

where B_0 is a strong uniform field and the constant α describes a small deviation from homogeneity. (Actually, what we'd *prefer* is just the z component of this field, but unfortunately that's impossible—it would violate the electromagnetic law $\nabla \cdot \mathbf{B} = 0$; like it or not, the x component comes along for the ride.) The force on these atoms is[50]

$$\mathbf{F} = \gamma\alpha\left(-S_x\hat{\imath} + S_z\hat{k}\right).$$

But because of the Larmor precession about $\mathbf{B}_0$, S_x oscillates rapidly, and *averages* to zero; the *net* force is in the z direction:

$$F_z = \gamma\alpha S_z, \tag{4.170}$$

and the beam is deflected up or down, in proportion to the z component of the spin angular momentum. *Classically* we'd expect a *smear* (because S_z would not be quantized), but in fact the beam splits into $2s+1$ separate streams, beautifully demonstrating the quantization of angular momentum. (If you use silver atoms, all the inner electrons are paired, in such a way that their angular momenta cancel. The net spin is simply that of the outermost—unpaired—electron, so in this case $s = 1/2$, and the beam splits in two.)

QM applications

Stern-Gerlach experiment

The Stern–Gerlach experiment has played an important role in the philosophy of quantum mechanics, where it serves both as the prototype for the preparation of a quantum state and as an illuminating model for a certain kind of quantum measurement. We tend casually to assume that the *initial* state of a system is *known* (the Schrödinger equation tells us how it subsequently evolves)—but it is natural to wonder how you get a system into a particular state in the first place. Well, if you want to prepare a beam of atoms in a given spin configuration, you pass an unpolarized beam through a Stern–Gerlach magnet, and select the outgoing stream you are interested in (closing off the others with suitable baffles and shutters). Conversely, if you want to *measure* the z component of an atom's spin, you send it through a Stern–Gerlach apparatus, and record which bin it lands in. I do not claim that this is always the most *practical* way to do the job, but it is *conceptually* very clean, and hence a useful context in which to explore the problems of state preparation and measurement.