Homework 5 - Quantum Mechanics I

NAME: ____ **SCORE:** ____________

Deadline: Friday 14th June 2024 by 19h00
Credits: 20 points **Number of problems:** 4
Type of evaluation: Formative Evaluation

- This homework is individual and includes problems on units 4 and 5.
- Please send a single PDF file via email to: wbanda@yachaytech.edu.ec
- If you work with classmates, please state their names and write your own reports.
- Please submit a professional homework report and clearly highlight all answers.
- All calculation steps should be justified to get full credits.
- Any code used should be attached to get full credits.

1. **(5 points) Electron in a Magnetic Field**
   Consider an electron (at rest) embedded in an oscillating magnetic field:

$$\vec{B} = B_0 \cos(\omega t)\, \vec{k},$$

   where B_0 and ω are constants.

   (a) Construct the Hamiltonian matrix for this system. Is this Hamiltonian time-dependent or time-independent?

   (b) The electron starts out (at $t = 0$) in the spin-up state with respect to the x axis (i.e., $\chi(0) = \chi_+^{(x)}$). Determine $\chi(t)$ at any subsequent time by solving the Schrödinger equation.

   (c) Find the probability of getting $-\frac{\hbar}{2}$, if you measure S_x.

   (d) What is the minimum field strength (B_0) required to force a complete flip in S_x?

2. **(5 points) Atoms**
   The wave function of a hydrogen-like atom can be written as follows:

$$\psi(r,\theta) = \frac{1}{81}\sqrt{\frac{2}{\pi}}\, Z^{\frac{3}{2}}\,(6 - Z\,r)\,Z\,r\,e^{-\frac{Z\,r}{3}}\,\cos(\theta)$$

   where r is written in units of the Bohr radius, a_0.

   (a) Find the corresponding values of the quantum numbers n, ℓ, and m.

   (b) Construct from $\psi(r,\theta)$ another wave function with the same values of n and ℓ, but a different magnetic quantum number, $m + 1$.

   (c) Calculate the most probable value of r for an electron in the state corresponding to ψ and with $Z = 1$.

3. **(5 points) Two-particle systems**
Consider a system of two non-interacting quantum particles (both of mass m) inside a 1D infinite square well potential of width, a.

(a) Based on the analysis carried out in unit 2 of the course for a particle trapped in this potential, write down the one-particle wave function and the respective energy for each particle in the system.

(b) Write down the composite wave function of the two-particle system assuming the particles are: distinguishable (system D), identical bosons (system B), and identical fermions (system F).

(c) Based on the previous results, find the ground state of each two-particle system (D, B, and F), jointly with the respective energy. Briefly explain your findings.

(d) Find the first three excited states for each two-particle system (D, B, and F), jointly with their respective energies.

4. **(5 points) Free electron gases**
The density of copper is $8.96\,\mathrm{g\,cm^{-3}}$, and its atomic weight is $63.5\,\mathrm{g\,mole^{-1}}$.

(a) Figure out the electronic configuration of copper. How many free electrons, d, can each cooper atom provide?

(b) Calculate the Fermi energy (in electron volts) for copper. What is the corresponding electron velocity? Are the electrons in copper non-relativistic?

(c) At what (Fermi) temperature, T_F, would the characteristic thermal energy equal the Fermi energy for copper? What states would most of the electrons occupy at room temperature? Why?

(d) Calculate the degeneracy pressure of copper. How does this pressure compare to the sea-level atmospheric pressure? Is it higher, comparable, or lower? What does this mean?

(e) At low temperatures ($T \ll T_F$), helium-3 can be treated as a Fermi gas. Given a density of $82\,\mathrm{kg\,m^{-3}}$, calculate the Fermi temperature, T_F, for helium-3. How does this temperature compare to that of copper, calculated in (c)? **Note:** Helium-3 is fermion with spin 1/2, unlike the more common isotope helium-4 which is a boson.