Quantum Mechanics I

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UC1
The Schrödinger equation

UC1 contents:

- Review of quantum experiments and mathematical tools.
- The wave function and the Schrödinger equation.
- Statistical interpretation of the wave function and probability.
- Normalisation, momentum, and the uncertainty principle.

What is Quantum Mechanics (QM)?

- QM deals with small scales.
- QM can be abstract, counterintuitive and hard to grasp.
- QM is mathematically challenging.
- QM is not deterministic as it is associated with probabilities.
- Despite this, it is a linear theory, so there is harmony in the equations and there is no chaos as in Classical Mechanics (CM).

What is Quantum Mechanics (QM)?

- **Richard Feynman:** "I think I can safely say that nobody understands QM."

- "There is no general consensus as to what its fundamental principles are, how it should be taught, or what it really means."

- "QM was not created by one individual", like other theories (e.g., GR, EM).

- **The purpose of this class is to teach you how to DO and USE quantum mechanics.**

Reference: Griffiths 1995

What is Quantum Mechanics (QM)?

- **D. Griffiths:** “I do not believe one can intelligently discuss what quantum mechanics means until one has a firm sense of what quantum mechanics does."

- “Not only is quantum theory conceptually rich, it is also technically difficult.”

  e.g. Linear algebra, complex numbers, partial derivatives, Fourier analysis, classical mechanics, electrodynamics.

- “Using the right tool makes the job *easier,* not more difficult”

  e.g. Legendre, Hermite, and Laguerre polynomials, spherical harmonics, Bessel, Neumann, and Hankel functions, Airy functions, Hilbert spaces, Hermitian operators, Clebsch- Gordan coefficients, and Lagrange multipliers.

Reference: Griffiths 1995

What is Quantum Mechanics (QM)?

- “Don't let the mathematics (which, for us, is only a tool) interfere with the physics.”

- “QM represents an abrupt and revolutionary departure from classical ideas, calling forth a wholly new and radically counterintuitive way of thinking about the world.”

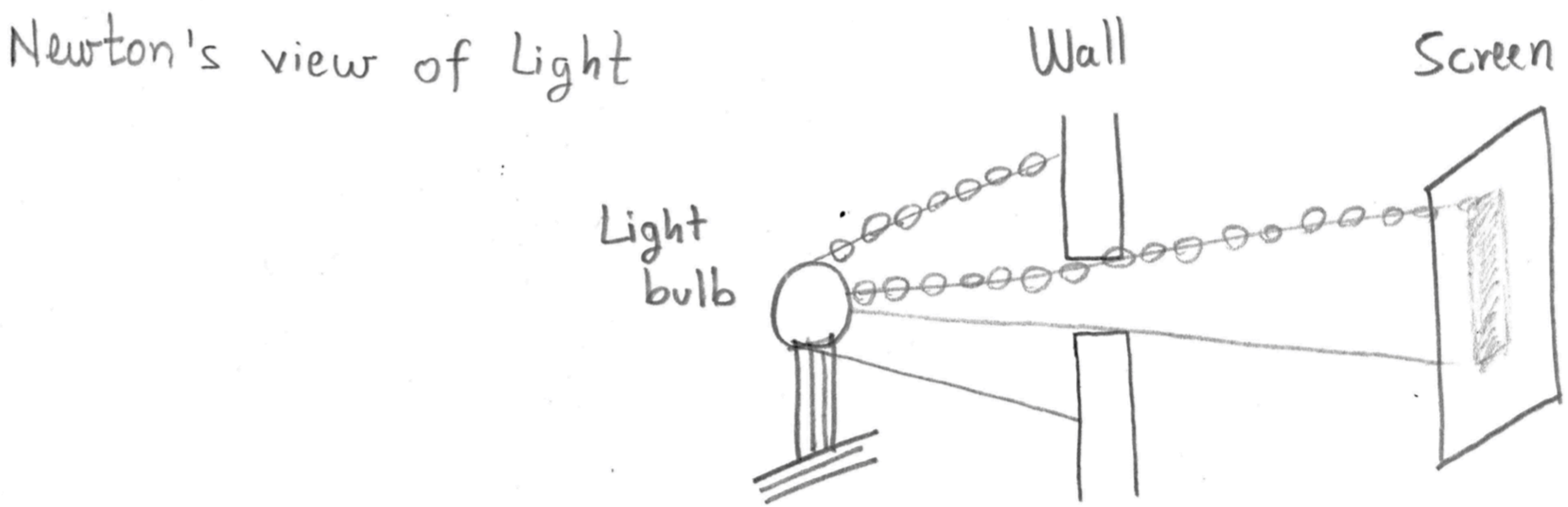
- **QM is a (mathematical) framework to do physics (at small scales).**

Reference: Griffiths 1995

Brief history of QM

Experiments and basic ideas that led to the formulation of QM:

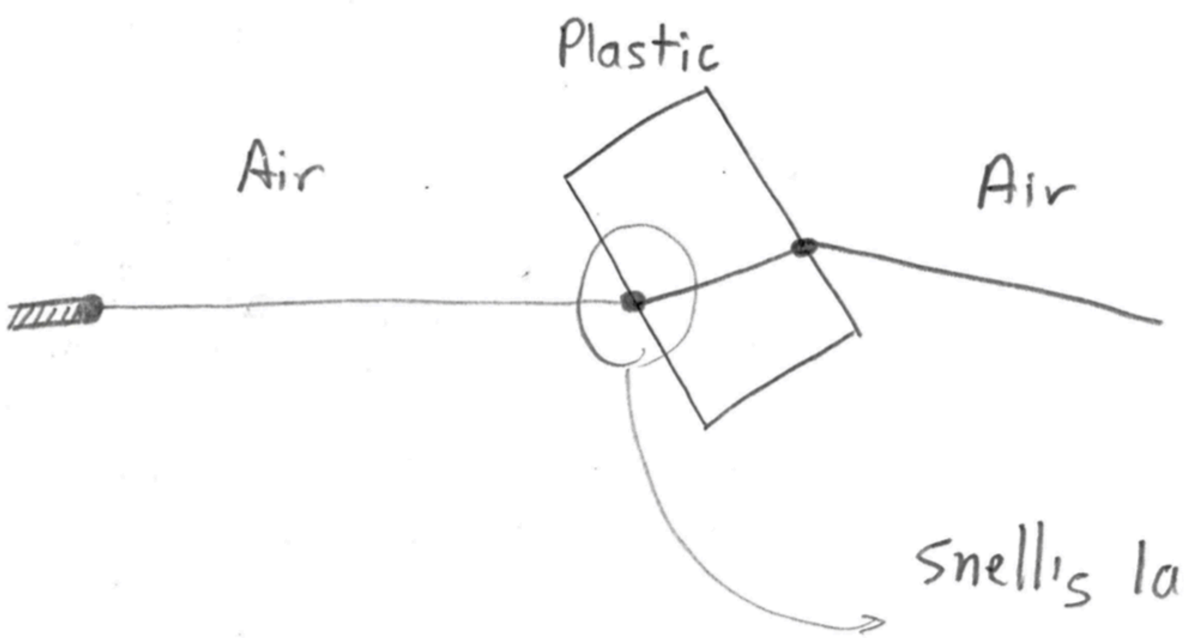
- The earliest ideas that would eventually lead to the formulation of QM emerged from trying to understand the nature of light.
- In the 1600's, I. Newton proposes light is made of a beam of particles, based on this experiment:



Brief history of QM

- Also in the **1600's,** R. Hooke proposes light is made of waves based on refraction experiments. Refraction can be explained by considering light as composed of waves.

Refraction:



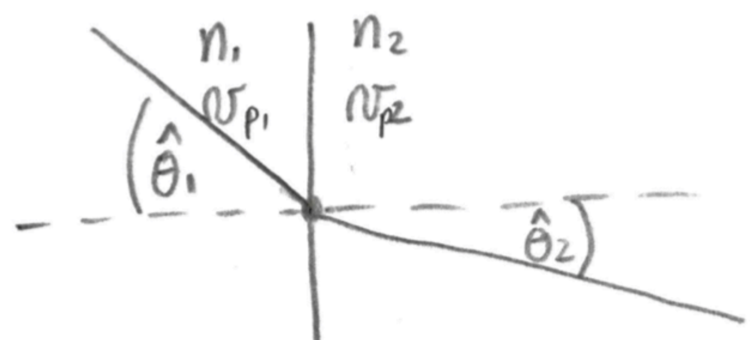
$n \equiv$ refractive indices $\equiv \frac{c}{v_p} = \frac{ck}{\omega}$

$v_p \equiv$ phase velocities $= \frac{\omega}{k} = \frac{\lambda}{T}$

$\hat{\theta}_1 \equiv$ incidence angle

$\hat{\theta}_2 \equiv$ refraction angle

Snell's law



$$\boxed{\frac{\sin \hat{\theta}_1}{\sin \hat{\theta}_2} = \frac{v_{p1}}{v_{p2}} = \frac{n_2}{n_1}}$$

$$\omega = \frac{ck}{n} \Rightarrow \frac{d\omega}{dk} = \overset{v_p}{\frac{c}{n}} - \frac{c}{n^2}\frac{dn}{dk} \equiv \text{group velocity}$$

Brief history of QM

- **1800's** - Experiments on interference and diffraction probe Hooke's ideas correct.
- **1800's** - K. Maxwell compiles the EM equations. Light is EM radiation.

Gauss' law:

$$\nabla \cdot \mathbf{E} = \frac{\rho}{\varepsilon_0}$$

Gauss' law for magnetism:

$$\nabla \cdot \mathbf{B} = 0$$

Maxwell–Faraday equation (Faraday's law of induction):

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

Maxwell–Ampère equation (circuital law):

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

Reference: https://www.cantorsparadise.com/maxwells-equations-7484212839b1