Homework 1 - Quantum Mechanics I

NAME/S: _____ **SCORE:** _____

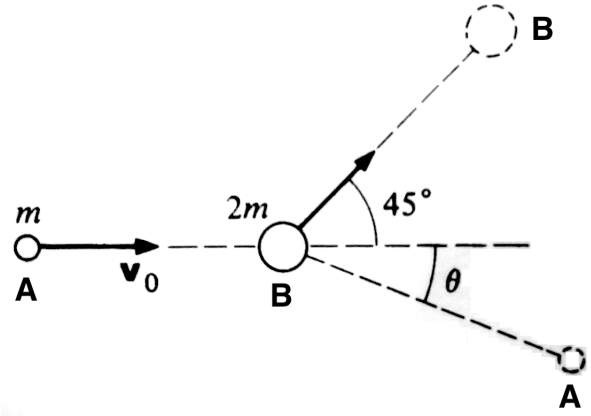
Deadline: Tuesday 13th December 2022 by 10:00am (submission only on paper)
Credits: 24 points → 20 credits **Number of problems:** 5
Type of evaluation: Formative Evaluation

- This homework consists of problems related to the concepts reviewed in class about quantum mechanics, plus some review problems on classical mechanics.

- You may submit this assignment either individually or in pairs. If you work in pairs, only 1 copy is needed. Submitted assignments should have maximum two authors.

- Unless stated otherwise, write your answers in SI units, and consider all bolded quantities as vector quantities. Please highlight the answers.

1. **(4 points) Review of classical problems: classical collision**
   Consider the problem of a particle A of mass m, moving at an initial velocity $\mathbf{v_0}$, that collides elastically with another particle B of mass $2m$, initially at rest. After the collision, the particles follow the trajectories shown in the figure below. Write down the relevant dynamical equations and find the value of the angle θ.



2. **(5 points) Review of classical problems: group velocity of classical waves**
   Waveguides are structures that guide waves (e.g. sound or electromagnetic waves) with minimal energy loss as they restrict their transmission to a single direction. The wavelength in a waveguide is considered as a wavelength in the direction of wave propagation and is defined as follows:

$$\lambda_{\text{wg}} = \frac{\lambda_0}{\sqrt{1 - \left(\frac{\lambda_0}{\lambda_c}\right)^2}},$$

   where λ_0 is a wavelength in free space at a given frequency, ν_0, and λ_c is the cutoff wavelength at frequency ν_c. The latter is related to the dimensions of the waveguide.

   (a) Write an expression for λ_{wg} in terms of the frequencies ν_0 and ν_c.
   (b) Write an expression for the group velocity, v_g, of a waveguide in terms of the light speed, c, and the phase velocity, v_p.

(c) Find the group velocity of water waves in deep water, which have a frequency given by $\nu = \sqrt{\frac{g}{2\,\pi\,\lambda}}$, where g is the acceleration by gravity.
(d) Find the group velocity of water waves in shallow water, which have a frequency given by $\nu = \sqrt{\frac{2\,\pi\,T}{\rho\,\lambda^3}}$, where T is the surface tension and ρ is the density.
(e) Compare the results found in (c) and (d), and briefly explain why they are different.

3. **(5 points) Compton scattering**
Suppose we have an experiment in which monochromatic light is scattered by an electron.
(a) Find the shift in the wavelength of the light when the scattering angle is 30°.
(b) Make a sketch of the experiment.
(c) If the incident light has a $\lambda = 600\,\text{nm}$ (i.e. photons are in the visible region), what is the fractional increase in the wavelength, $\frac{\Delta\lambda}{\lambda}$?
(d) If the incident light has a $\lambda = 0.05\,\text{nm}$ (i.e. photons are in the X-ray region), what is the fractional increase in the wavelength, $\frac{\Delta\lambda}{\lambda}$?
(e) Why were X-rays used by Compton in his experiments?

4. **(4 points) Compton and de Broglie wavelengths**
(a) Calculate the Compton wavelength of an electron.
(b) What can happen if you shine that electron with a photon with that wavelength? Why?
(c) Calculate the de Broglie wavelength of a (macro)particle of diameter $10\,\mu\text{m}$ with a mass of $m = 10^{-12}\,\text{kg}$ that is moving at a speed of $50\,\text{mm}\,\text{s}^{-1}$.
(d) Based on the result above, are the wave properties of matter relevant in the macroscopic world? Why?

5. **(6 points) Wave function and normalisation**
Let us assume we have a quantum particle of mass m, whose wave function is described by:

$$\Psi(x,t) = \alpha\, e^{-\beta\left(\frac{m\,x^2}{\hbar} + i\,t\right)}$$

where α and β are positive real constants.
(a) Find α.
(b) For what potential energy function, $V(x)$, is this a solution to the Schrödinger equation?
(c) Calculate the expectation values of x and x^2.
(d) Calculate the expectation values of p and p^2.
(e) Find σ_x and σ_p.
(f) Is their product consistent with the uncertainty principle?