Homework 2 - Quantum Mechanics I

NAME/S: ____ **SCORE:** _________

Deadline: Friday 23rd December 2022 by 08:00am (submission only on paper)
Credits: 20 points → 20 credits **Number of problems:** 4
Type of evaluation: Formative Evaluation

- This homework consists of problems related to the concepts reviewed in class about quantum mechanics, plus some review problems on classical mechanics.

- You may submit this assignment either individually or in pairs. If you work in pairs, only 1 copy is needed. Submitted assignments should have maximum two authors.

- Unless stated otherwise, write your answers in SI units, and consider all bolded quantities as vector quantities. Please highlight the answers.

1. **(5 points) Quantum experiments: photoelectric effect**
   In a photoelectric effect experiment, a piece of potassium (K) metal is illuminated by light beams with two different wavelengths $\lambda_1 = 300\,\mathrm{nm}$ and $\lambda_2 = 700\,\mathrm{nm}$. If a piece of potassium (K) metal has a cutoff wavelength of $\lambda_{\mathrm{cutoff}} = 558\,\mathrm{nm}$, calculate:

   (a) The work function, ϕ, for the potassium metal.
   (b) The maximum kinetic energy and speed of the electrons ejected from the metal piece for each beam.
   (c) The de Broglie wavelength of the ejected electrons.
   (d) Use programming tools to make a plot of K_{max} versus λ, including the data points for both light beams.
   (e) Use programming tools to make a plot of K_{max} versus ν, including the data points for both light beams.

   Recall that the Planck constant is $h = 6.626\times10^{-34}\,\mathrm{J\,s}$, the speed of light is $c = 3\times10^{8}\,\mathrm{m\,s^{-1}}$, and the electron mass is $m_e = 9.11\times10^{-31}\,\mathrm{kg}$.

2. **(5 points) Probability densities**
   Consider the log-normal distribution:

   $\rho(x) = \frac{A}{x}\, e^{-\frac{(\ln(x)-\mu)^2}{2\,\sigma^2}}$,

   where A, μ, and σ are positive real constants.
   (a) Determine A.
   (b) Find $\langle x\rangle$.
   (c) Find $\langle x^2\rangle$, and σ_x.
   (d) What do A, μ, and σ represent?
   (e) Plug some fiducial numbers for these constants, and sketch the graph of $\rho(x)$ using your favourite programming language.

3. **(4 points) Probability current**
   As we reviewed in class, the wave function of a particle is a complex-valued probability amplitude that depends on position, x, and time, t. As time progresses, the wave function changes and the probability of finding a particle in certain position also changes with it. Since the sum of all probabilities should always be 1, this means that the probability 'flows' from one region to another one, akin to a fluid or a current. This 'flow' can be described mathematically by the so-called probability current j, which for the wave function Ψ of a non-relativistic particle of mass m in 1D is defined as:

$$j(x,t) = \frac{\hbar}{2\,m\,i}\left(\Psi^*\,\frac{\partial\Psi}{\partial x} - \Psi\,\frac{\partial\Psi^*}{\partial x}\right)$$

   Find the probability current, j, of a superposition of 2 currents of particles of mass m, momentum p, and energy $\frac{p^2}{2\,m}$, moving in opposite directions. The amplitudes of the particle currents are α and β, respectively. Hint: Write the wave function for the superposition first.

4. **(6 points) Wave functions, normalisation, and expectation values**
   The wave function of a particle at time $t = 0$ is given by the following piecewise function:

$$\Psi(x, t=0) = \begin{cases} C\,\frac{x}{\alpha}, & 0 \le x \le \alpha \\ C\,\frac{\beta - x}{\beta - \alpha}, & \alpha \le x \le \beta \\ 0, & \text{everywhere else,} \end{cases}$$

   where C, α, and β are positive constants.

   (a) Find an expression for C.
   (b) Plug some fiducial numbers for the constants, and use programming tools to sketch $\Psi(x, t=0)$ as a function of x.
   (c) Where is the particle most likely to be found at $t = 0$?
   (d) What is the probability of finding the particle to the left of α? Check your result in the limiting cases $\beta = \alpha$ and $\beta = 2\,\alpha$.
   (e) What is the probability of finding the particle between α and β?
   (f) What is the expectation value of x?