Midterm Exam - Quantum Mechanics I

NAME: ____ **SCORE:** ______________

Date: Tuesday 24 January 2023
Duration: 120 minutes   Credits: 20 points
Number of problems: 14 in total (12 in part I, and 2 in part II)

- This exam consists of two parts. Part I is closed-book and contains concept questions and short-answer problems. Part II is open-book and contains long-answer problems.

PART I:
I.1. Choose the correct answer to each question or statement given below, and briefly justify your choice in the white space assigned to each of them.

1. **(1 point) Black body radiation and Wien's law**
   Stars are almost perfect black bodies. Imagine we study two stars of equivalent size, one is a red dwarf ($\lambda_{\text{red}} = 700\,\text{nm}$) and the other one is a yellow dwarf ($\lambda_{\text{yellow}} = 570\,\text{nm}$). Which star has a higher temperature?

   A. Both have the same temperature.
   B. The yellow dwarf star.
   C. The red dwarf star.

2. **(1 point) Compton scattering**
   How much energy is transferred to an electron (e-, initially at rest) by a photon in a Compton scattering experiment? The wavelength of the photon before the collision is $\lambda = 0.01\,\text{nm}$, and the scattering angle is $\theta = 60^\circ$ ($h = 6.626 \times 10^{-34}\,\text{J s}$, $c = 3 \times 10^8\,\text{m s}^{-1}$).

   A. $0\,\text{J}$, no energy is transferred to the e-.
   B. $2.15 \times 10^{-15}\,\text{J}$
   C. $1.99 \times 10^{-14}\,\text{J}$
   D. $1.77 \times 10^{-14}\,\text{J}$

3. **(1 point) Photoelectric effect**
   What is the maximum kinetic energy of an electron (e-) removed from a cooper (Cu) metal surface illuminated by light of $\lambda = 600\text{nm}$? The work function, Φ, of Cu is $4.7\,\text{eV}$ ($1\text{eV} = 1.6 \times 10^{-19}\,\text{J}$).

   A. The e- cannot be ejected by light of this λ
   B. $-4.22 \times 10^{-19}\,\text{J}$
   C. $7.53 \times 10^{-19}\,\text{J}$
   D. $3.32 \times 10^{-19}\,\text{J}$

4. **(1 point) de Broglie wavelength**
   What is the de Broglie wavelength of a baseball of mass 0.125 kg and size 0.08 m moving at $28\,\mathrm{m\,s^{-1}}$?

   A. 1.89×10^{-34} m
   B. 2.32×10^{-33} m
   C. 3.00×10^{-35} m
   D. 1.89×10^{34} m

5. **(1 point) Relevance of quantum mechanics**
   Based on the result above, is quantum mechanics relevant when studying the physics of a baseball? Why?

   A. Yes.
   B. No.

6. **(1 point) Expectation values**
   In quantum mechanics, the expectation value of the position of a particle represents:

   A. the most probable value of its position.
   B. the average value of the position measured in repeated experiments on the same particle.
   C. the average value of the position measured on identical particles in the same state.
   D. the only possible value of its position.

7. **(1 point) Infinite square well potential**
   What kind of stationary state solutions does an infinite square well potential allow?

   A. Only bound states.
   B. Only scattering states.
   C. Both bound and scattering states.
   D. No solutions.

8. **(1 point) Normalisation**
   Consider the wave function, $\Psi(x,t) = A\,e^{-\lambda|x|}\,e^{-i\omega t}$, where A, λ, and ω are positive real constants. What is the normalisation constant, A, equal to?

   A. λ
   B. $\sqrt{\lambda}$
   C. 2λ
   D. $\sqrt{2\lambda}$

9. **(1 point) Quantum harmonic oscillator**
The first excited state energy value, E_1, of a quantum harmonic oscillator is:

A. The harmonic oscillator does not have excited states.
B. $\frac{3}{2}\hbar\,\omega$
C. $\hbar\,\omega$
D. $\frac{1}{2}\hbar\,\omega$

I.2. Provide answers/solutions to the following items.

10. **(1 point) Principle of physical equivalence and expectation values**
(i) Explain what the principle of physical equivalence in quantum mechanics refers to.

(ii) Write down the mathematical definition of the expectation value of the kinetic energy operator.

11. **(1 point) Delta function well and energy quantisation in a finite square well**
(i) Sketch a delta function well and indicate what kind/s of solutions this potential allows.

(ii) Explain where energy quantisation comes from for the bound states of a particle in a finite square well potential.

12. **(3 points) The time-independent Schrödinger equation**
Consider a current of particles with energies $E > V_0$ moving from $x = -\infty$ to the right, under the influence of a Heaviside potential $V(x)$ given by:

$$V(x) = \begin{cases} V_0, & x \geq 0 \\ 0, & x < 0, \end{cases}$$

where A, n, and x_0 are constants.

(a) Sketch the potential and write down the time-independent Schrödinger equation.
(b) Find the stationary state solutions for each region of interest.
(c) Express the transmitted and reflected amplitudes in terms of the incident amplitude.

PART II:
Solve the following problems and highlight the answers.

13. **(3 points) Wave packet and probability density**
A free particle of mass m has the following wave function at time $t = 0$:

$$\Psi(x,0) = \frac{\sqrt{\alpha}}{(2\pi)^{3/4}} \int_{-\infty}^{\infty} e^{-\frac{\alpha^2}{4}(k-k_0)^2} e^{ikx} dk$$

(a) Calculate the time-dependent wave packet $\Psi(x,t)$.
(b) Calculate the probability density $|\Psi(x,t)|^2$.
(c) Use your favourite programming tool to plot the probability density for $t = 0$ and $t > 0$, and briefly explain your findings.

Hint: Recall that: $\int_{-\infty}^{\infty} e^{-\lambda^2(y+\mu)^2} dy = \frac{\sqrt{\pi}}{\lambda}$, where λ, $\mu \in \mathbb{C}$.

14. **(3 points) Heaviside potential: transmission and reflection coefficients**

For a current of particles travelling from $x = -\infty$ to the right with energies $E > V_0$, under the influence of the same Heaviside potential, $V(x)$, described in problem 12:

(a) Find the probability current, $j(x)$, in each region of interest.
(b) Use the results from part (a) to find the reflection and transmission coefficients, and check that $T + R = 1$. Note that the transmission coefficient is not simply $\frac{|F|^2}{|A|^2}$ (with A the incident amplitude and F the transmitted amplitude), because the transmitted wave travels at a different speed.
(c) Calculate the reflection coefficient for the case $E < V_0$, and comment on the result.