De Broglie's proposal: matter waves

· de Broglie relations   (1924)

→ De Broglie's proposal.

— γ are particles
— γ is also a wave

— definite amount of E, packets, cannot be broken. & momentum p

therefore
$\Rightarrow$ this could be a more general property.

→ interferes / described by waves, ν

Is this universal?

de Broglie:

→ All "matter particles" behave as waves, not just the γ's.

→ There is a wave associated to a matter particle.

De Broglie's proposal: matter waves

QM:
{ probability amplitude to be somewhere
{ probability waves

Matter waves are introduced:

↳ Matter waves → probability amplitude. $\mathbb{C}$#'s

↳ Associate to a particle a wave that depends on the momentum.

For a particle of momentum p, we associate a plane

wave $\boxed{\lambda = \dfrac{h}{p}}$ which is the de Broglie λ,

QM arises as a theory

- **1925** - Schrödinger/Heisenberg wrote the governing equations of QM.

- QM is almost a 100 years old!

What is QM?

QM is a framework to do physics.

Quantum physics

- QM replaces classical mechanics CM. CM is a good approximation but it is not accurate when describing some experiments.

- **Quantum physics:** principles of QM applied to physical phenomena.

- **Branches of QM:**

  - **QED:** QM + EM

  - **QCD:** QM + Strong interaction

  - **Quantum optics:** QM + photons

  - **Quantum gravity:** QM + gravitation -> String theory (QM of gravity)

Mathematical tools for QM

- **Is QM a linear theory?**

- Why do we need complex numbers?

Linear theory:

Solution 1
Solution 2 $\Big\}$ $\Rightarrow$ New Solution 3

We can create linear combinations of known solutions to get new solutions.

Linear Operators

- $L.u = 0$

- L = linear operator, u = unknown

- Several operators applied to the same unknown: L1.u=0, L2.u=0

- Same operator applied to different unknowns: L(u1,u2,u3) =0

Properties of linear operators:

- Scale a solution: L(au) = a Lu

- Combine solutions: L(u1+u2) = L(u1) + L(u2)

Is QM a linear theory?

Linear equation :

$$L \; u = 0$$

↙   ↘ unknown (variables)

Linear operator
(eq)
↓

Properties : $\left.\begin{array}{l} L(\alpha u) = \alpha L u \\ L(u_1 + u_2) = L u_1 + L u_2 \end{array}\right\}$

Linear combinations:

$$L(\alpha u_1 + \beta u_2) = L(\alpha u_1) + L(\beta u_2) = \alpha L u_1 + \beta L u_2$$

$$\text{If } u_1, u_2 \equiv sln \Rightarrow \alpha u_1 \pm \beta u_2 \Rightarrow sln$$

Linear vs. Non-linear Theories

linear & non-linear theories.

$\textcircled{L}$ $\begin{cases} \text{EM} \\ \text{QM} \end{cases}$

much simpler

$\textcircled{NL}$ $\begin{cases} \text{G.R.} \\ \text{C.M. i.g. 3-body problem} \end{cases}$

very non-linear

QM is linear!