Homework 1 - Quantum Mechanics I

NAME: ____ **SCORE:** _____________

Deadline: Friday 13 September 2024 by 17h00 **Format:** PDF and CSV files via email
Credits: 20 points **Number of problems:** 3 **Type of evaluation:** Formative Evaluation

- This homework includes problems on unit 1 of the QM course programme.
- This assignment should be submitted individually by the deadline.
- If you work with classmates, please state their names and write your own reports.
- Please submit a professional homework report and clearly highlight all answers.
- All calculation steps should be justified to get full credits.
- Any code used should be included within the PDF report to get full credits.

1. **(7 points) The photoelectric effect**
As reviewed in class, the photoelectric effect experiment consists of setting up an electric circuit embedding 2 metal plates. We illuminate the first plate with a beam of photons of certain wavelength, λ, and as the photons interact with the metal plate, electrons are ejected from it. The electrons then reach the second plate and an electric current emerges. To prevent these electrons from reaching the second plate, we can increase the voltage until the electric current becomes zero in the circuit. The voltage value at which this happens is called "stopping potential", V_0, which is defined as the potential needed to stop the photoelectrons with the largest kinetic energy (so $K_{\mathrm{max}} = e\,V_0$) at a specific wavelength λ.

(a) Carry out photoelectric effect experiments for 2 different metals and collect 10 data points for each, using: https://applets.kcvs.ca/photoelectricEffect/PhotoElectric.html
To collect the data, choose a metal, fix a wavelength, and vary the voltage until the current becomes zero. When this happens, push "record data points". Then, vary the wavelength and repeat the process. When you have 10 data points for the first metal, choose another metal and repeat the experiment. At the end, you should have a data table with 20 data points, 10 for each metal (**send your CSV data file via email by the deadline**).

Using your favourite programming language (Python, Mathematica) or Spreadsheet/Excel:

(b) Open and read the data file containing the experimental results, and make two high-quality labeled scattered plots, one for each metal, with the maximum kinetic energy (K_{max}) in the Y-axis and frequency on the X-axis.

(c) For each metal, define a good model to describe the data. Carry out a regression and find the function that best fits the data. Report the fitting functions for each metal and make two labeled plots, one for each metal, containing the experimental data and their fits.

(d) Make a new figure combining the data and fitting functions for both metals. Which metal has a higher cutoff frequency? What does the slope of the curves represent?

Using the fitting functions, carry out the following calculations:

(e) Calculate the work function, ϕ, and the cutoff wavelength, $\lambda_{\mathrm{cutoff}}$, for each metal, and the relative errors with respect to the known values (research what these values are).

2. **(7 points) Black body radiation**
This problem consists of determining the temperature of our Sun (which is an almost perfect black body) based on a regression performed on its spectrum. The Solar spectrum that you will analyse was obtained from satellite instruments, so it does not contain atmospheric absorption. Please download the data file from: `https://github.com/wbandabarragan/physics-teaching-data/blob/main/1D-data/sun_spectra.csv`

This data file contains 6 columns, but we will use only the first two:

1. Wavelength (λ in nm)
2. Extraterrestrial ($J_\lambda \equiv$ spectral radiosity in $\mathrm{W\,m^{-2}\,nm^{-1}}$)

As you know from quantum theory, the spectral radiance of a black body is described by Planck's law, so this would be a natural fitting model to be used for the regression. Planck's law reads:

$$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{\exp\left(\frac{hc}{\lambda k_B T}\right) - 1}$$

where B_λ is in units of $\mathrm{W\,m^{-2}\,nm^{-1}\,sr^{-1}}$ in the SI system. To obtain the spectral radiosity, you need to integrate the spectral radiance over the solid angle (Ω, in units of steradian, sr) subtended by the emitting object (which in this case is the Sun), so:

$$J_\lambda = \int_\Omega B_\lambda \, d\Omega$$

We are far away from the Sun, so you should consider the Sun size on the sky to obtain the correct solid angle. For a celestial body, the solid angle (in units of sr) can be computed from of its radius, $R = 696340\,\mathrm{km}$ and the average Earth-Sun distance (i.e. the distance from the observer to the object), $d = 151.35 \times 10^6\,\mathrm{km}$, using:

$$\Omega = 2\pi\left(1 - \frac{\sqrt{d^2 - R^2}}{d}\right)$$

(a) Calculate the solid angle Ω subtended by the Sun in units of sr from the Sun radius, R, and the Earth-Sun distance, d.

(b) Use your favourite programming tool to open the above CSV data file and take the first two columns from it, i.e. the wavelength (λ in nm) and spectral radiosity (J_λ in $\mathrm{kW\,m^{-2}\,nm^{-1}}$). Then, make a high-quality labeled plot of spectral radiosity (in the Y-axis) versus wavelength (in the X-axis). Does the relation between the two look linear? Hints: The conversion to kW will help later on.

(c) Define a physically-motivated fitting function (i.e. J_λ in units of $\mathrm{kW\,m^{-2}\,nm^{-1}}$ as obtained from the Planck law). Which variable should be the free parameter for the regression? Hints: Be very careful in handling the units and do not forget to use the Sun's solid angle computed in (a) to obtain the spectral radiosity from the spectral radiance.

(d) Carry out the regression using your favourite programming tool. Report the best-fit function, what is the temperature of the Sun? Hint: since the fitting function is not a simple polynomial function, providing an initial guess for the regression may help some regression algorithms in Mathematica or Python.

(e) Make a high-quality labeled plot of spectral radiosity (in the Y-axis) versus wavelength (in the X-axis) showing both the experimental data and the best-fit model (obtained from Planck's law for the fitted temperature).

(f) Finally, you will compare the results to predictions based on classical theory. As you know, the black body radiation in classical theory is described by the Rayleigh-Jeans law. Calculate the spectral radiosity based on temperature, but now according to the Rayleigh-Jeans function given below. Hint: As before, be careful with the units and do not forget to use the Sun's solid angle computed in (a) to obtain the spectral radiosity, J_λ, from the spectral radiance:

$$B_\lambda = \frac{2ck_BT}{\lambda^4}$$

(g) Make a high-quality labeled plot of spectral radiance (in the Y-axis) versus wavelength (in the X-axis) showing the experimental data, the best-fit model (i.e. Planck's law for the fitted temperature), and the classical model obtained in (f). Does classical theory correctly describe the black body spectrum of the Sun? Hint: You may wish to limit the Y-axis of the plot to be able to compare the lines.

3. **(6 points) Quantum concepts and experiments**
(a) Calculate the Compton wavelengths of an electron, a muon, and a tau particle. How much energy would photons with those wavelengths have?

(b) 1) We wish a uranium-238 nucleus to have enough energy so that its de Broglie wavelength is equal to its nuclear radius, which is 6.8 fm. How much energy is required? Take the nuclear mass to be 238 u. 2) A non-relativistic particle is moving three times as fast as an electron. The ratio of their de Broglie wavelengths, particle to electron, is 1.813×10^{-4}. Identify the particle. Hint: Research tables with rest masses of particles.

(c) Suppose we have an experiment in which monochromatic light is scattered by an electron at an angle of 15°. What is the fractional increase in the wavelength, $\frac{\Delta\lambda}{\lambda}$ if the incident light has: 1) a $\lambda = 660$ nm (i.e. photons are in the visible region), and 2) a $\lambda = 0.07$ nm (i.e. photons are in the X-ray region)? Why were X-rays used by Compton in his experiments?

(d) A typical microwave oven operates at roughly 2.5 GHz at a maximum power of 350 W. How many photons per second can it emit? What about a low-power laser operating at 8 mW at 625 nm, or a mobile phone operating at 0.25 W at 850 MHz?

(e) In a double-slit experiment, electrons are fired one by one toward a pair of slits separated by a distance $d = 5.0 \times 10^{-7}$ m. The electron source emits electrons with a kinetic energy of 90 eV. The screen is placed $L = 1.1$ m behind the slits, and an interference pattern is observed. 1) What is the de Broglie wavelength of the electrons? 2) Determine the positions y_1 and y_2 of the first and second bright fringes on the screen.

(f) Let $z_1 = -3 + 7i$ and $z_2 = 1 - 5i$, where $i \equiv \sqrt{-1}$. 1) Calculate the quotient: $\frac{z_1}{z_2}$. 2) Show that the multiplication of either z_1 by $e^{i\phi}$ or z_2 by $e^{i\phi}$ is equivalent to rotating them by ϕ in a 2D complex plane.