Quiz 5 - Quantum Mechanics I

NAME: ____ **SCORE:**
Date: Tuesday 7 March 2023 **Duration:** 50 minutes
Credits: 20 points (10 questions) **Type of evaluation:** LAB

Part A. Choose the correct answer to each question or statement given below, and briefly justify your choice in the white space assigned to each of them. Unjustified answers will not count to the final grade.

1. **(2 points) Schrödinger equation**
   What is the correct Dirac formulation for the time-dependent Schrödinger Equation?

   A. $\frac{\partial|\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle$
   B. $i\hbar\frac{\partial|\Psi\rangle}{\partial t} = -\frac{\hbar^2}{2m}\hat{H}|\Psi\rangle$
   C. $i\hbar\frac{\partial|\Psi\rangle}{\partial t} = \hat{H}|\Psi\rangle$
   D. $i\hbar|\frac{\partial\Psi}{\partial t}\rangle = \hat{H}|\Psi\rangle$

2. **(2 points) Eigenfunctions and eigenvalues**
   Which of the following pairs represent eigenfunctions and corresponding eigenvalues of the differential operator $\frac{d}{dx}$?

   A. e^{x^2} and $2x$
   B. e^{x^2} and 2
   C. e^{2x} and $2x$
   D. e^{2x} and 2

3. **(2 points) The hydrogen atom**
   The energy required to knock out the electron in the third orbit of a hydrogen atom is equal to:

   A. $+13.6\,\mathrm{eV}$
   B. $+\frac{13.6}{9}\,\mathrm{eV}$
   C. $-13.6\,\mathrm{eV}$
   D. $+\frac{13.6}{3}\,\mathrm{eV}$

4. **(2 points) Spectrum of hydrogen**
   Which of the following statements is true?

   A. The Lyman series is a continuous spectrum.
   B. The Paschen series is a discrete spectrum in the infrared.
   C. The Balmer series is a discrete spectrum in the ultraviolet.
   D. The Lyman series is a discrete spectrum in the optical.

5. **(2 points) Spectrum of hydrogen**
   An electron jumps from the 4th orbit to the 2nd orbit of the hydrogen atom. The frequency in Hz of the emitted radiation will be: (Recall that: $\mathcal{R} = 10^7\,\mathrm{m}^{-1}$, $c = 3\times10^8\,\mathrm{m\,s}^{-1}$)

   A. $\frac{3}{16}\times10^5$
   B. $\frac{3}{16}\times10^{15}$
   C. $\frac{9}{16}\times10^{15}$
   D. $\frac{3}{4}\times10^{15}$

6. **(2 points) Orbital angular momentum**
   Consider a single electron atom with orbital angular momentum $L = \sqrt{2}\,\hbar$. Which of the following gives the possible values of a measurement of L_z, the z-component of L?

   A. $0,\ \hbar$
   B. $-\hbar,\ 0,\ \hbar$
   C. $0,\ \hbar,\ 2\hbar$
   D. $-2\hbar,\ -\hbar,\ 0,\ \hbar,\ 2\hbar$

7. **(2 points) Spinor**
   A spin-$\frac{1}{2}$ particle is in a state described by the spinor $\chi = A \begin{pmatrix} 1+i \\ 2 \end{pmatrix}$, where A is a normalisation constant. The probability of finding the particle with spin projection $S_z = -\frac{1}{2}\hbar$ is:

   A. $\frac{1}{6}$
   B. $\frac{1}{3}$
   C. $\frac{1}{2}$
   D. $\frac{2}{3}$

Part B. Provide <u>concise answers</u> to the following items:

8. **(1.5 points) Electron in a magnetic field**
   Explain how we can experimentally demonstrate that the spin angular momentum is quantised.

9. **(2.5 points) Commutators and Spins**
   Write down the correct answers for the following operations (only (d) requires proof):
   (a) If $\hat{x}$ and $\hat{p}$ are the position and momentum operators, $[\hat{x}, \hat{p}] =$
   (b) If $\hat{S}_x$ and $\hat{S}_y$ are the spin x- and y-component operators, $[\hat{S}_x, \hat{S}_y] =$
   (c) If $\hat{\mathbf{L}}$ and $\hat{L}_z$ are the total and z-component angular momenta, $[\hat{\mathbf{L}}^{\mathbf{2}}, \hat{L}_z] =$
   (d) The net spin if we combine three quarks (spin-$\frac{1}{2}$ particles) is:

10. **(2 points) Solutions for the hydrogen atom**
    Briefly explain: (a) the terms of the potential of the hydrogen atom,

    (b) how we solved the Schrödinger equation, and

    (c) what solutions we found.