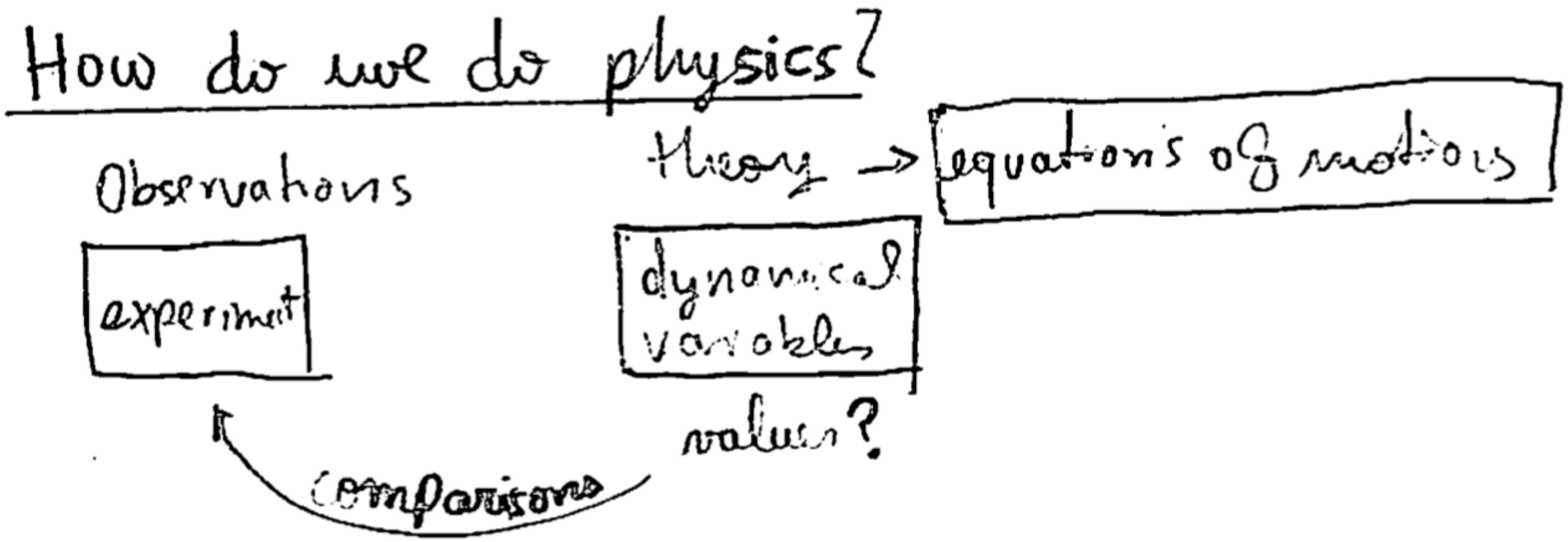
How do we do physics? Scientific method

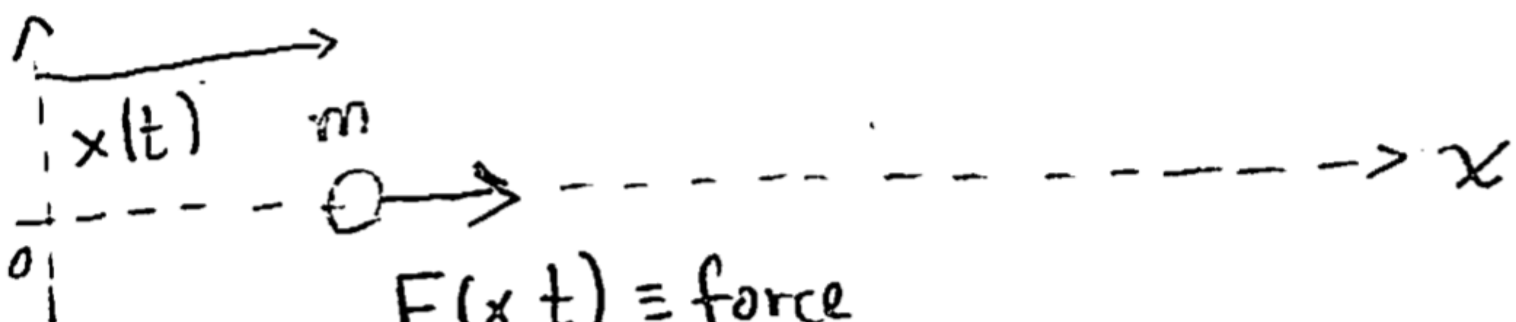
- Observations/experiments <-> dynamical variables (theory)
- Equations of motion solve for dynamical variables.



How do we do physics? Scientific method

- Motion of classical particles

In classical mechanics: 1D motion (non relativistic $v \ll c$)



$F(x,t) \equiv$ force

$$x(t) = ? \rightarrow v_x = \frac{dx}{dt} \rightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2}$$

$$\vec{p}_x = m\vec{v}_x$$

$$T = \frac{1}{2} m v_x^2$$

dynamical variables

How do we determine $x(t)$?

How do we do physics? Scientific method

- Motion of classical particles

Theory: Newton's 2nd law $F = ma = -\frac{\partial V}{\partial x}$ (conservative system)

$V \equiv$ potential energy function

$$\Rightarrow \boxed{m \frac{d^2 x}{dt^2} = -\frac{\partial V}{\partial x}}$$

If we know the initial conditions, eg. $x(t=0)$, $v(t=0)$

$\Rightarrow x(t)$ ✓

e.g. elastic potential energy:

$$V = \frac{1}{2} k x^2 \Rightarrow \frac{\partial V}{\partial x} = kx$$

How do we do physics? Scientific method

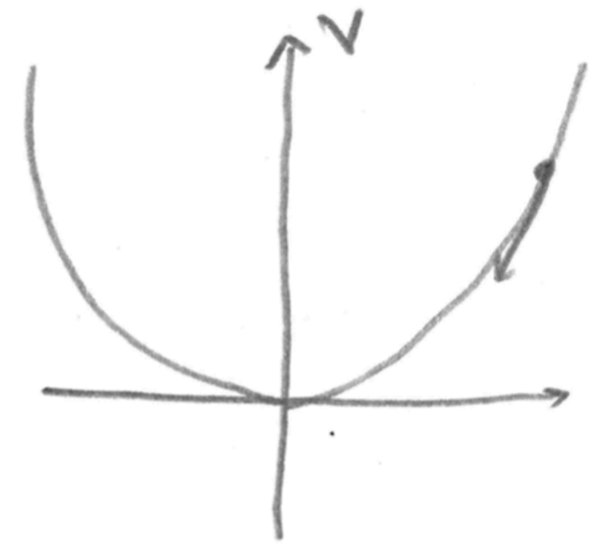
- Motion of classical particles

Motion in 1D : dynamical variable $x(t)$)

$$m\ddot{x} = -\frac{\partial V}{\partial x}$$

↳ derivatives are linear

non-linear



e.g. elastic potential energy :

$$V = \frac{1}{2}kx^2 \Rightarrow \frac{\partial V}{\partial x} = kx$$

Solution is:

$$\Rightarrow m\ddot{x} = -kx$$

$$\Rightarrow \boxed{m\ddot{x} + kx = 0} \quad \text{ODE}$$

$$x(t) = A\sin(\omega t) + B\cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

(Angular frequency)

How do we do quantum mechanics?

- Motion in quantum mechanics:

QM is linear: dynamical variable $\Psi \xrightarrow{} \text{sai}$ (wavefunction)

↳ describes dynamics of the Q system

- The equation of motion is Schrödinger's equation:

$$\boxed{i\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi}$$ Schrödinger's equation.

↳ Hamiltonian, linear operator.

$$\hat{H} = \hat{T} + \hat{V}$$

$$L\Psi = 0$$

$$L \equiv i\hbar \frac{\partial}{\partial t} - \hat{H}$$ is linear.

How do we do quantum mechanics?

- Schrödinger's equation: $$i\hbar \frac{\partial \psi}{\partial t} = \hat{H} \psi$$

$$i = \sqrt{-1}$$

$$\hbar \equiv \text{Planck's constant} = \frac{h}{2\pi} = 1.054573 \times 10^{-34}\ J \cdot s$$

- QM is linear, so in some sense it is simpler than CM.

- We can scale solutions, and add/combine solutions to create superpositions, which become new solutions.

The necessity of complex numbers

- Why do we need complex numbers?

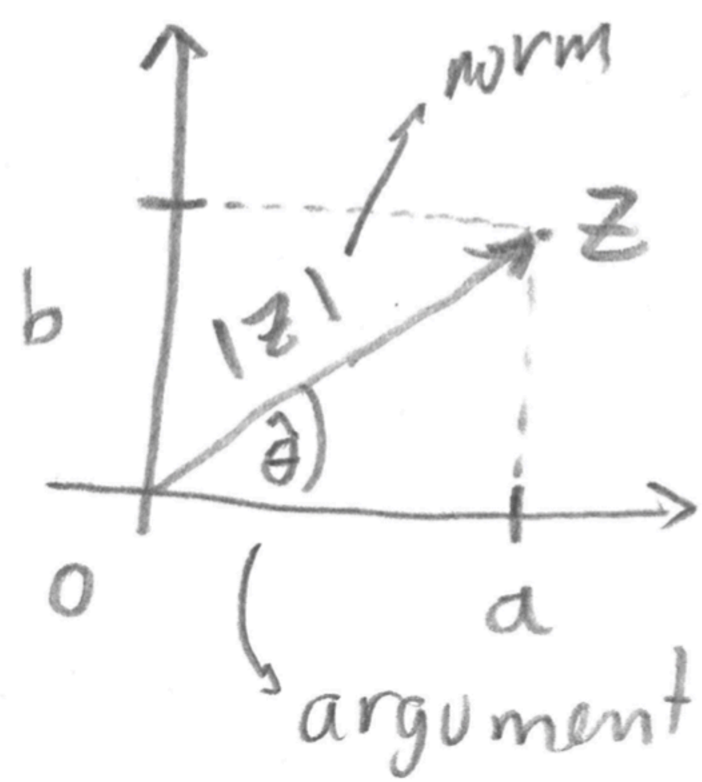
Complex numbers: $x^2 = 1 \Rightarrow x = \sqrt{-1}$

$i = \sqrt{-1}$

$\boxed{z = a + ib}$; $a, b \in \mathbb{R}$

$z \in \mathbb{C}$

$$\begin{cases} Re(z) = a \\ Im(z) = b \end{cases}$$



The necessity of complex numbers

- Why do we need complex numbers?

Complex conjugate of z:

$\boxed{z^* = a - ib}$ $(\bar{z}, \hat{z})$

Norm of a complex #: $\in \mathbb{R}$

$|z| = \sqrt{a^2+b^2}$, $|z|^2 = a^2+b^2 = z z^*$

Summation:

$(a+bi) + (c+di) = (a+c) + (b+d)i$

Multiplication:

$(a+bi)(c+di) = (ac-bd) + (bc+ad)i$

The necessity of complex numbers

- Why do we need complex numbers?

In polar coordinates:

$$a = |z| \cos \hat{\theta}$$
$$b = |z| \sin \hat{\theta}$$
$$\Rightarrow \quad z = |z|(\cos \hat{\theta} + i \sin \hat{\theta}) = |z| e^{i\theta}$$

Identity: $e^{i\hat{\theta}} = \cos \hat{\theta} + i \sin \hat{\theta}$ (Euler formula)

$$e^{-i\theta} = \cos \hat{\theta} - i \operatorname{sen} \hat{\theta}$$

$$\boxed{\text{In QM } \Psi \in \mathbb{C}}$$

We need $\mathbb{C}$ numbers.

- The wave function, Ψ, has to be a complex number to satisfy Schrödinger equation.