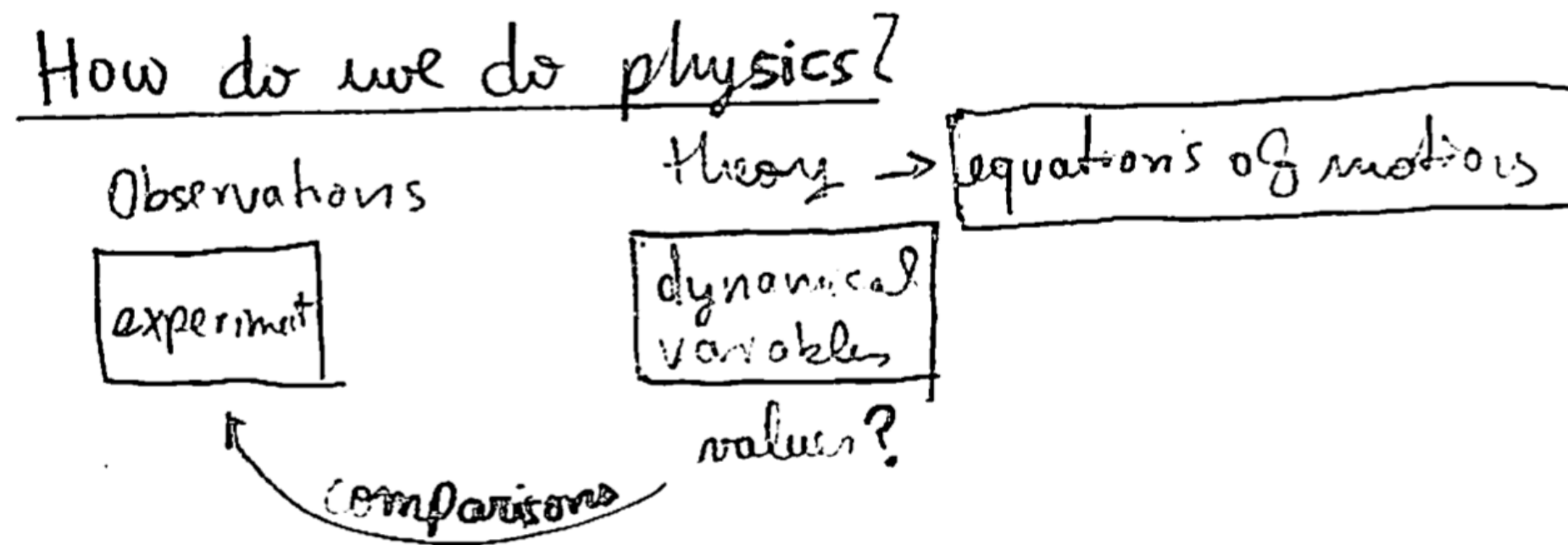


# How do we do physics? Scientific method

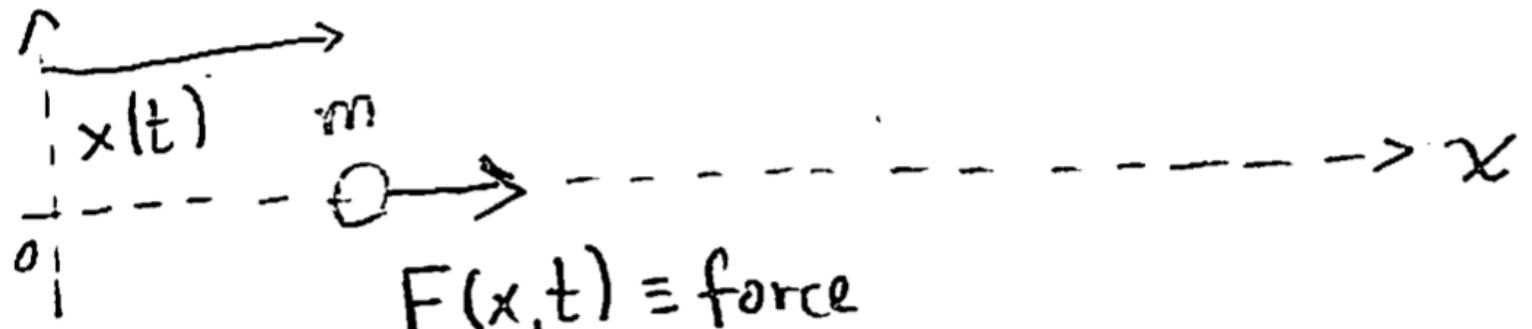
- Observations/experiments  $\leftrightarrow$  dynamical variables (theory)
- Equations of motion solve for dynamical variables.



# How do we do physics? Scientific method

- Motion of classical particles

In classical mechanics: 1D motion (non relativistic  $v \ll c$ )



The diagram shows a horizontal dashed line representing the x-axis. A particle, represented by a small circle with a dot, is positioned on this axis. Above the particle, the label  $x(t)$  is written, with a vertical dashed line extending from it to the x-axis. To the right of the particle, the letter  $m$  is written. Below the particle, the text  $F(x,t) \equiv \text{force}$  is written. A solid arrow points to the right from the particle, indicating its direction of motion. The x-axis is labeled with  $x$  at its right end. The origin of the axis is marked with a vertical dashed line and labeled  $0$  at the bottom.

$$\left. \begin{aligned} x(t) = ? \rightarrow v_x &= \frac{dx}{dt} \rightarrow a_x = \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \\ \vec{p}_x &= m \vec{v}_x \\ T &= \frac{1}{2} m v_x^2 \end{aligned} \right\} \text{dynamical} \\ \text{variables}$$

How do we determine  $x(t)$ ?

# How do we do physics? Scientific method

- Motion of classical particles

Theory: Newton's 2<sup>nd</sup> law  $F = ma = -\frac{\partial V}{\partial x}$  (conservative system)  
 $V \equiv$  potential energy function

$$\Rightarrow m \frac{d^2 x}{dt^2} = -\frac{\partial V}{\partial x}$$

If we know the initial conditions, eg.  $x(t=0)$ ,  $v(t=0)$   
 $\Rightarrow x(t)$  ✓

e.g. elastic potential energy:

$$V = \frac{1}{2} k x^2 \Rightarrow \frac{\partial V}{\partial x} = kx$$

# How do we do physics? Scientific method

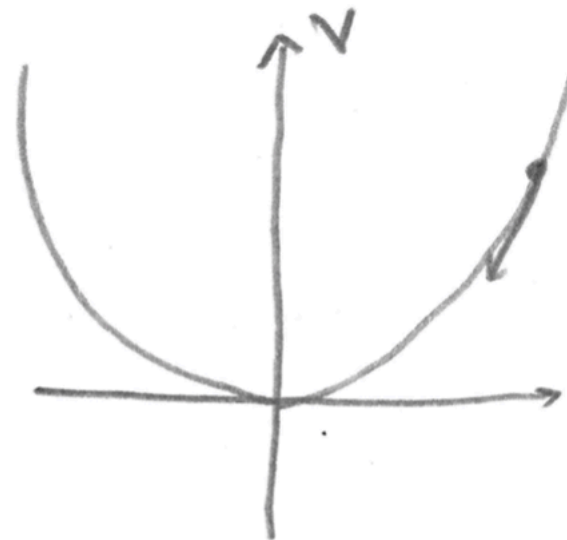
- Motion of classical particles

Motion in 1D: dynamical variable  $x(t)$

$$m\ddot{x} = -\frac{\partial V}{\partial x}$$

↳ derivatives  
are linear

non-linear



e.g. elastic potential energy:

$$V = \frac{1}{2} k x^2 \Rightarrow \frac{\partial V}{\partial x} = kx$$

**Solution is:**

$$\Rightarrow m\ddot{x} = -kx$$

$$\Rightarrow \boxed{m\ddot{x} + kx = 0} \quad \text{ODE}$$

$$x(t) = A \sin(\omega t) + B \cos(\omega t)$$

$$\omega = \sqrt{\frac{k}{m}}$$

(Angular frequency)

# How do we do quantum mechanics?

- Motion in quantum mechanics:

QM is linear: dynamical variable  $\Psi$  (wavefunction)  $\rightarrow$  sai  
 $\hookrightarrow$  describes dynamics of the Q system

- The equation of motion is Schrödinger's equation:

$$\boxed{i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi} \quad \text{Schrödinger's equation.}$$

$\hookrightarrow$  Hamiltonian, linear operator.

$$L\Psi = 0$$

$$L \equiv i\hbar \frac{\partial}{\partial t} - \hat{H} \text{ is linear.}$$

$$\hat{H} = \hat{T} + \hat{V}$$

# How do we do quantum mechanics?

- Schrödinger's equation:

$$i\hbar \frac{\partial \Psi}{\partial t} = \hat{H} \Psi$$

$$i = \sqrt{-1}$$

$$\hbar \equiv \text{Planck's constant} = \frac{h}{2\pi} = 1.054573 \times 10^{-34} \text{ J}\cdot\text{s}$$

- QM is linear, so in some sense it is simpler than CM.
- We can scale solutions, and add/combine solutions to create superpositions, which become new solutions.

# The necessity of complex numbers

- Why do we need complex numbers?

Complex numbers:

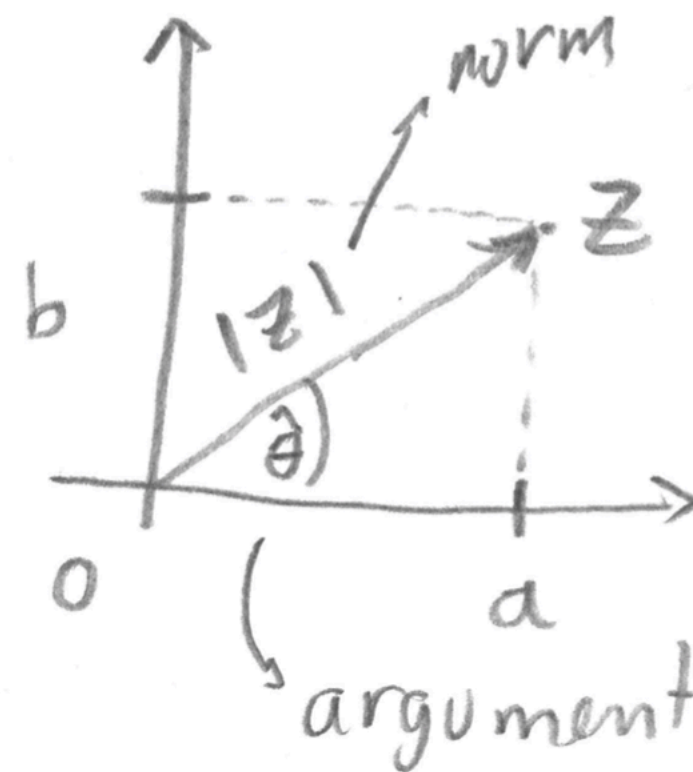
$$x^2 = -1 \Rightarrow x = \sqrt{-1}$$

$$i = \sqrt{-1}$$

$$\boxed{z = a + ib} ; \quad a, b \in \mathbb{R}$$

$z \in \mathbb{C}$

$$\begin{cases} \operatorname{Re}(z) = a \\ \operatorname{Im}(z) = b \end{cases}$$



# The necessity of complex numbers

- Why do we need complex numbers?

Complex conjugate of  $z$ :

$$\boxed{z^* = a - ib} \quad (\bar{z}, \hat{z})$$

Norm of a complex #:  $\in \mathbb{R}$

$$|z| = \sqrt{a^2 + b^2}, \quad |z|^2 = a^2 + b^2 = z z^*$$

Summation:

$$(a+bi) + (c+di) = (a+c) + (b+d)i$$

Multiplication:

$$(a+bi)(c+di) = (ac-bd) + (bc+ad)i$$



# The necessity of complex numbers

- Why do we need complex numbers?

In polar coordinates:

$$a = |z| \cos \hat{\theta}$$

$$b = |z| \sin \hat{\theta}$$

$$\Rightarrow z = |z|(\cos \hat{\theta} + i \sin \hat{\theta}) = |z|e^{i\theta}$$

Identity:  $e^{i\hat{\theta}} = \cos \hat{\theta} + i \sin \hat{\theta}$  (Euler formula)

$$e^{-i\theta} = \cos \hat{\theta} - i \sin \hat{\theta}$$

In QM:  $\Psi \in \mathbb{C}$

We need  $\mathbb{C}$  numbers.

- The wave function,  $\Psi$ , has to be a complex number to satisfy Schrödinger equation.