# Homework 1 - Quantum Mechanics I

**NAME:** ____________________________________________ **SCORE:** ____________

**Deadline:** Tuesday 5 March 2024 by 16:00 (submission on paper)
**Credits:** 20 points **Number of problems:** 4 **Type of evaluation:** Formative Evaluation

**- This homework includes problems on unit 1 of the QM course programme.**
**- This assignment should be submitted individually by the deadline.**
**- If you work with classmates, please state their names and write your own reports.**
**- Please submit a professional homework report and clearly highlight all answers.**
**- All calculation steps should be justified to get full credits.**
**- Any code used should be attached to get full credits.**

1. **(5 points) The photoelectric effect**
As reviewed in class, the photoelectric effect experiment consists of setting up an electric circuit embedding 2 metal plates. We illuminate the first plate with a beam of photons of certain wavelength, $\lambda$, and as the photons interact with the metal plate, electrons are ejected from it. The electrons then reach the second plate and an electric current emerges. To prevent these electrons from reaching the second plate, we can increase the voltage until the electric current becomes zero in the circuit. The voltage value at which this happens is called "stopping potential", $V_0$, which is defined as the potential needed to stop the photoelectrons with the largest kinetic energy (so $K_{max} = e\,V_0$) at a specific wavelength $\lambda$.

(a) Carry out photoelectric effect experiments for 2 different metals and collect 10 data points for each, using: `https://applets.kcvs.ca/photoelectricEffect/PhotoElectric.html` To collect the data, choose a metal, fix a wavelength, and vary the voltage until the current becomes zero. When this happens, push "record data points". Then, vary the wavelength and repeat the process. When you have 10 data points for the first metal, choose another metal and repeat the experiment. At the end, you should have a data table with 20 data points, 10 for each metal (**send your data file via email by the deadline**).

Using your favourite programming language (Python, Mathematica) or Spreadsheet/Excel:

(b) Open and read the data file containing the experimental results, and make two high-quality labeled scattered plots, one for each metal, with the maximum kinetic energy ($K_{\mathrm{max}}$) in the Y-axis and frequency on the X-axis.

(c) For each metal, define a good model to describe the data. Carry out a regression and find the function that best fits the data. Report the fitting functions for each metal and make two labeled plots, one for each metal, containing the experimental data and their fits.

(d) Make a new figure combining the data and fitting functions for both metals. Which metal has a higher cutoff frequency? What does the slope of the curves represent?

Using the fitting functions, carry out the following calculations:

(e) Calculate the work function, $\phi$, and the cutoff wavelength, $\lambda_{\mathrm{cutoff}}$, for each metal, and the relative errors with respect to the known values (research what these values are).

2. **(5 points) Review of classical waves**
The frictionless motion of a block with a mass of 2.5 kg, attached to a spring of constant, $k$, is described by the following one-dimensional position: $x = x(t) = 4\ \cos\left(\frac{\pi}{3}\,\mathrm{t} + \frac{\pi}{4}\right)$ m. This is generally called a "horizontal spring-mass system".

(a) Make a sketch of this system and find the remaining equations of motion, i.e., velocity $v_x$, and acceleration $a_x$.

(b) What are the amplitude, the angular frequency, the period and the frequency of the oscillation?

(c) Use your favourite programming tool to make plots of $x$ vs. $t$, $v_x$ vs. $t$, and $a_x$ vs. $t$. What are the initial position, velocity, and acceleration?

(d) What are the maximum values of velocity and acceleration of the system?

(e) Calculate the spring constant, $k$.

3. **(5 points) Compton wavelength, scattering, and diffraction**
(a) Calculate the Compton wavelengths of an electron, a muon, and a tau particle. How much energy (in both J and eV) would photons with those wavelengths have?

(b) Suppose we have an experiment in which monochromatic light is scattered by an electron at an angle of 57°. What is the fractional increase in the wavelength, $\frac{\Delta\lambda}{\lambda}$ if the incident light has: 1) a $\lambda = 500\,\mathrm{nm}$ (i.e. photons are in the visible region), and 2) a $\lambda = 0.02\,\mathrm{nm}$ (i.e. photons are in the X-ray region)? Why were X-rays used by Compton in his experiments?

(c) Light with a wavelength of 650 nm passes through two narrow slits that are 0.05 nm apart. If the screen is 7.5 m away from the slits, what is the distance between the third-order bright fringe and the central fringe?

4. **(5 points) de Broglie wavelength, black body spectrum, and complex numbers**
(a) Calculate the de Broglie wavelengths of: 1) a particle of diameter 7 cm with a mass of $m = 2.5\,\mathrm{kg}$ that is moving at a speed of $10\,\mathrm{m\,s^{-1}}$, and 2) a neutron whose kinetic energy is 0.167 eV. Based on these results, are the wave properties of matter relevant for these particles? Why?

(b) Stars are almost perfect black bodies. Imagine we study two stars of equivalent size, one is a red giant (i.e., its spectrum peaks at $\lambda_{\mathrm{red}} = 710\,\mathrm{nm}$) and the other one is a blue giant ($\lambda_{\mathrm{blue}} = 470\,\mathrm{nm}$). Calculate the surface temperature of each star. Which star is colder? Then, use your favourite programming tool to make a plot comparing their black-body spectra according to the Rayleigh-Jeans law and the Planck law. Which law is correct to describe the spectra of stars?

(c) Complex numbers are viewed as vectors in a 2D complex plane. The multiplication of a complex number by a phase (a complex number of unit magnitude) is equivalent to a rotation in the complex plane. 1) Show that the multiplication of a complex number, $z = a + i\,b$, by $i$ is equivalent to rotation by 90°. 2) Write $(i\,z)$ in terms of $a$ and $b$ and identify its real part. 3) Show that the multiplication by $e^{i\,\phi}$ is equivalent to rotating by $\phi$.