

Homework 5 - Quantum Mechanics I

NAME/S: _____ SCORE: _____

Deadline: Friday 10th March 2023 by 11:00am (submission only on paper)

Credits: 20 points \rightarrow 20 credits **Number of problems:** 4

Type of evaluation: Formative Evaluation

- This homework consists of problems related to the hydrogen atom and angular momenta.

- You may submit this assignment either individually or in pairs. If you work in pairs, only 1 copy is needed. Submitted assignments should have maximum two authors.

- Please solve the following problems and highlight the answers.

1. (7 points) Angular momentum

(a) Let L_x , L_y , and L_z be the components of the orbital angular momentum operator \vec{L} . Calculate the value of the commutator: $[L_x L_y, L_z]$.

(b) Suppose we have a spin- $\frac{1}{2}$ particle, whose quantum state can be represented by the eigenstates $|\uparrow\rangle$ and $|\downarrow\rangle$ of the S_z operator. Using the Pauli matrix σ_x , calculate a normalised eigenstate of the S_x operator with an eigenvalue $-\frac{\hbar}{2}$.

(c) Calculate the normalised spin eigenfunctions of a system with two spin- $\frac{1}{2}$ particles. These are called singlet and triplet states, why?

(d) Calculate the orbital angular momentum eigenfunction $Y_\ell^m(\theta, \phi)$ in a quantum state for which the operator \mathbf{L}^2 has an eigenvalue $6\hbar^2$ and the operator \mathbf{L}_z has an eigenvalue $-\hbar$.

2. (4 points) Spin

Consider an electron in the spin state:

$$\chi = C \begin{bmatrix} 1 - 3i \\ 2 \end{bmatrix}$$

(a) Determine the normalisation constant C .

(b) Find the expectation values of S_x , S_y , and S_z .

(c) Find the uncertainties σ_{S_x} , σ_{S_y} , and σ_{S_z} .

(d) Are the results consistent with all three uncertainty principles?

3. (4 points) Spherical harmonics

- (a) A quantum system is known to be in the (unnormalised) state described by the wave function $\psi(\theta, \phi) = 5Y_4^3 + Y_6^3 - 2Y_6^0$, where the $Y_\ell^m(\theta, \phi)$ are the spherical harmonics. What is the probability of finding the system in a state with quantum number $m = 3$?
- (b) Construct all the possible spherical harmonics, $Y_\ell^m(\theta, \phi)$, for $\ell = 2$.
- (c) Choose two of the spherical harmonics constructed in part (b), and check that they are normalised and orthogonal.
- (d) Using your favourite programming language, make 3D plots of all of them.

4. (5 points) Hydrogen atom

- (a) Construct all the possible spatial wave functions, $\psi_{nlm}(r, \theta, \phi) = R_{nl}(r)Y_\ell^m(\theta, \phi)$, of the hydrogen atom for $(n, \ell, m) = (3, 2, m)$.
- (b) Using your favourite programming language, make density plots of all of these states.
- (c) Calculate the energy level of these states in units of eV.
- (d) In terms of the Bohr radius, find $\langle r \rangle$, $\langle x \rangle$, $\langle r^2 \rangle$ and $\langle x^2 \rangle$ for an electron in the ground state of hydrogen.
- (e) Find $\langle x^2 \rangle$ in the state $(n, \ell, m) = (3, 2, m)$ with the highest possible value of m that is allowed. How different is the result with respect to that calculated in part (d) for the ground state of hydrogen?