Set 5: Restoration of images

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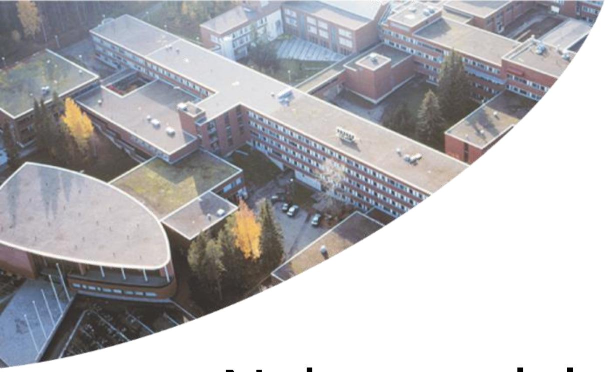


Contents

- Noise models
  - Estimation of noise parameters

- Restoration of noisy images
  - Restoration with spatial filters
  - Restoration in frequency domain
  - Degradation function *H*

Image restoration in general

- Image enhancement mostly for humans
- In image restoration the goal is recover the image from a degraded information
  - Degradation function *H* and additive noise *η*

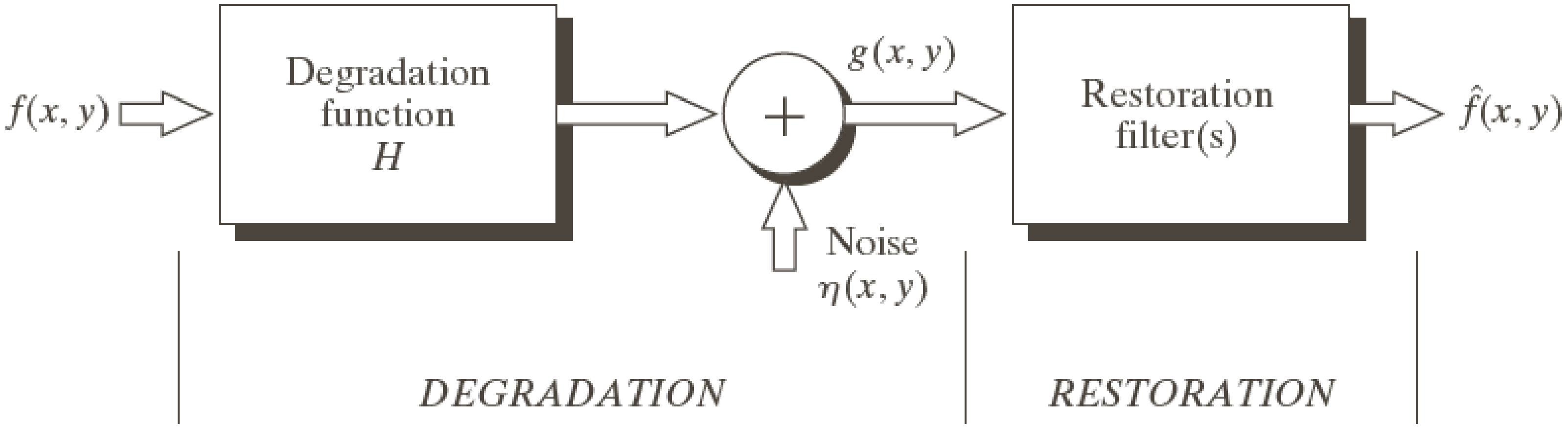


Image restoration in general

- If *H* is a linear process then in spatial domain
$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$
- Similarly in frequency domain
$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$
- Noise η from
  - Acquisition (environment, sensing elements)
  - Transmission (interference), compression
- Two options for *H:*
  - *H* is an indentity operator: noise degrades the image
  - Estimating *H* since seldom it is known completely

Noise models

- *White noise:* frequencies of noise covers the full spectrum
- *Spatially periodic noise:* from interference with electrical devices
- Noise models
  - Additive noise
    - is uncorrelated with the image itself
  - Multiplicative noise
    - Depends on the image itself

$$g(x,y) = f(x,y)n_m(x,y)\ (+\eta(x,y))$$

  - Noise is independent of spatial coordinates

Noise models

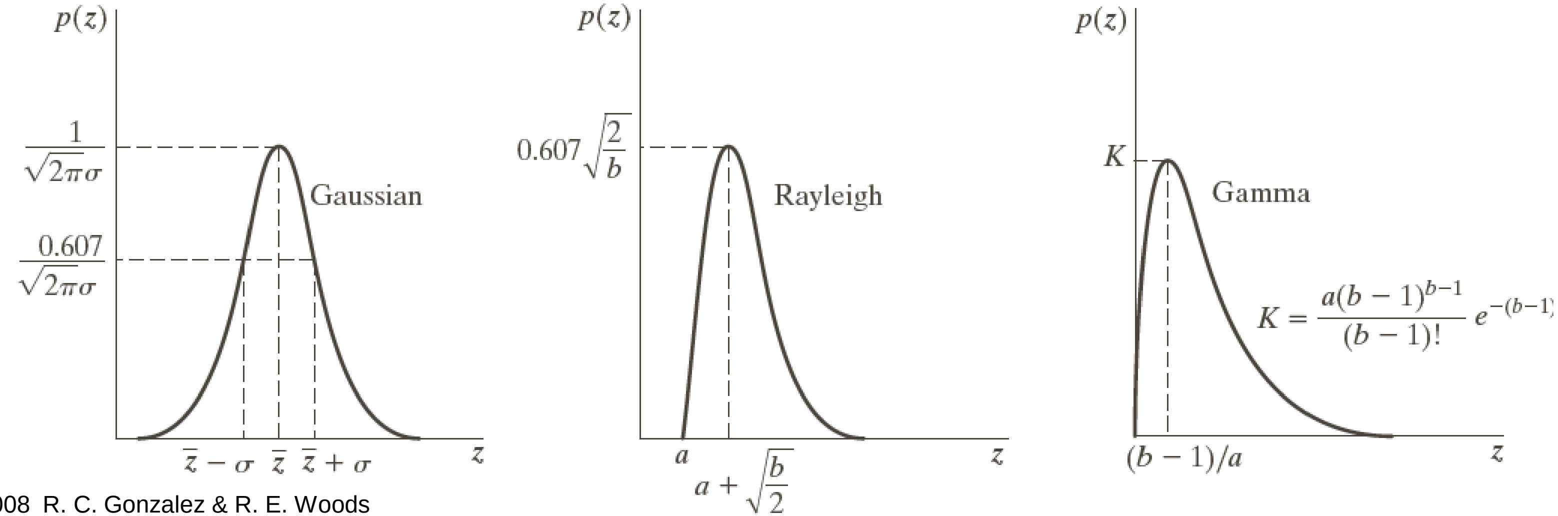
- Modeling of noise is based on a random variable with
  - a statistical behavior of the intensity values and
  - described with a PDF
  - the assumption of spatial processing
- Gaussian noise: typically AWGN
- Rayleigh noise: skewed noise model, range imaging
- Erlang (gamma) noise: electronic circuit, sensor noise
- Exponential noise: laser imaging
- Impulse noise: faults in imaging

Noise models, PDF:s

Gaussian: $p(z) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(z-\bar{z})^2/2\sigma^2}$

Rayleigh: $p(z) = \frac{2}{b}(z-a)e^{-\frac{(z-a)^2}{b2\sigma^2}}; p(z) = 0, z < a$

Erlang: $p(z) = \frac{a^b z^{b-1}}{(b-1)!} e^{-az}; p(z) = 0, z < 0$

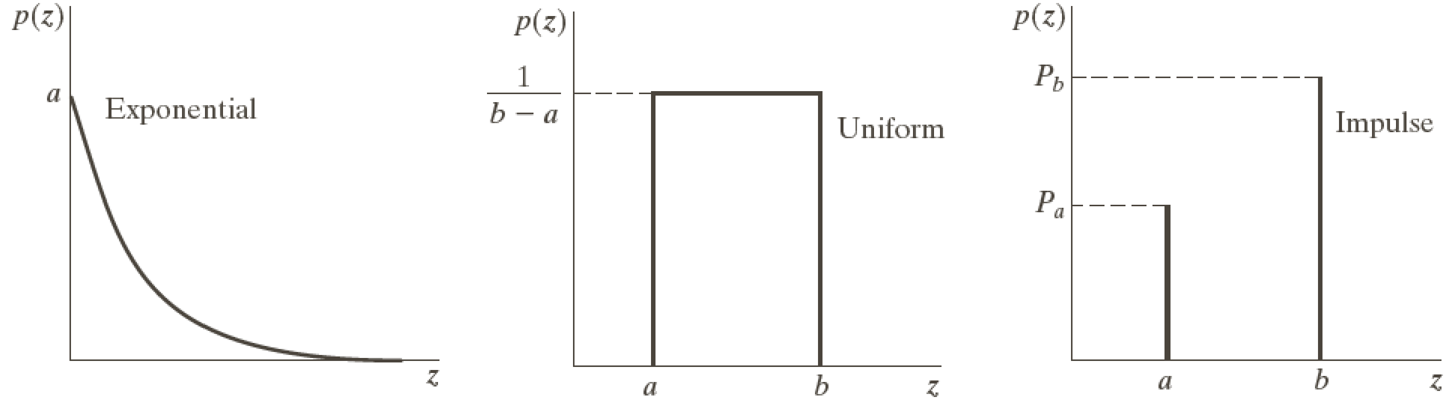


Noise models, PDF:s

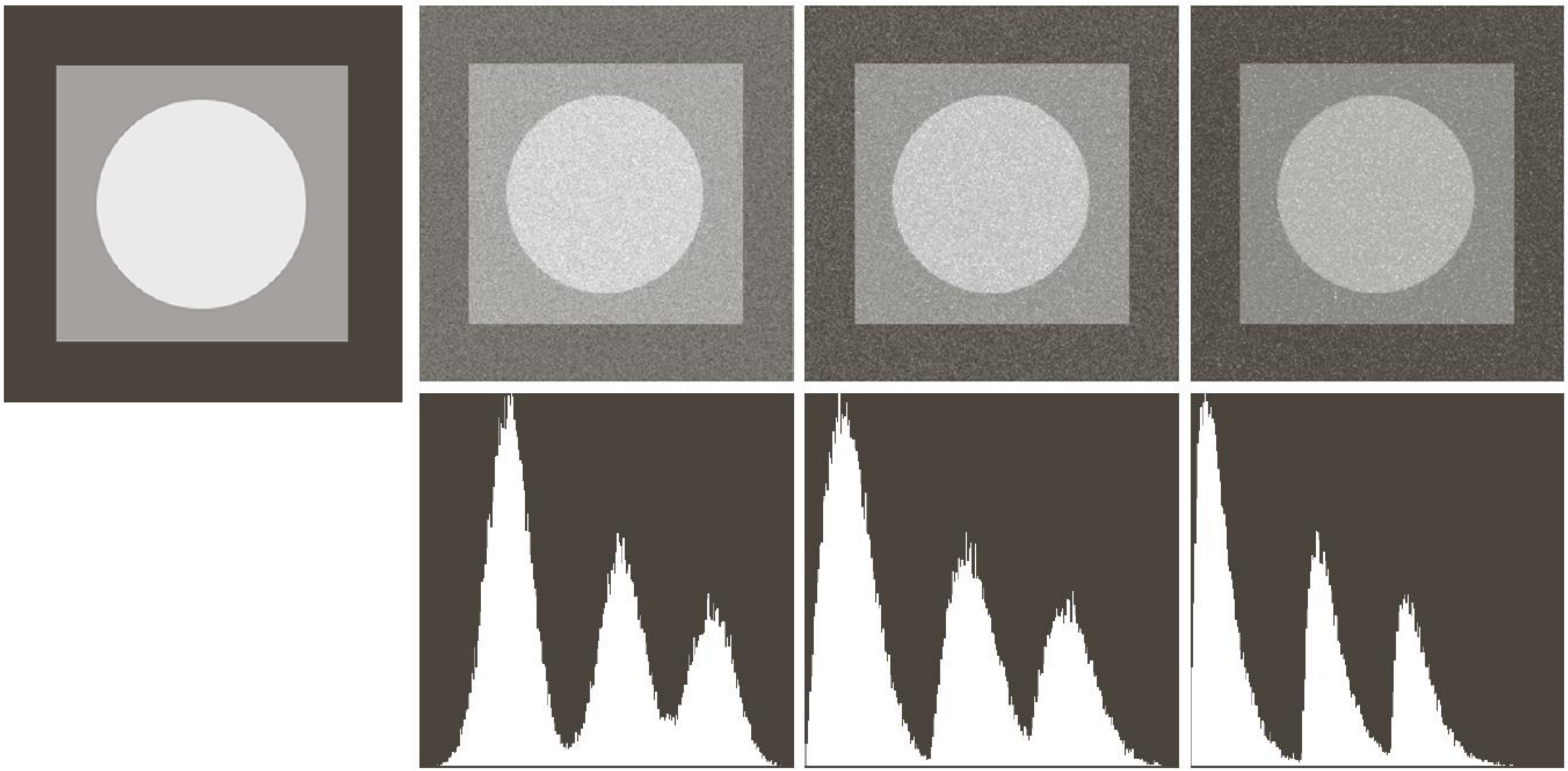
Exponential: $p(z) = ae^{-az}; p(z) = 0, z < 0$

Uniform: $p(z) = \frac{1}{b-a}; p(z) = 0$, otherwise

Impulse: $p(z) = P_{a,} p(z) = P_b$

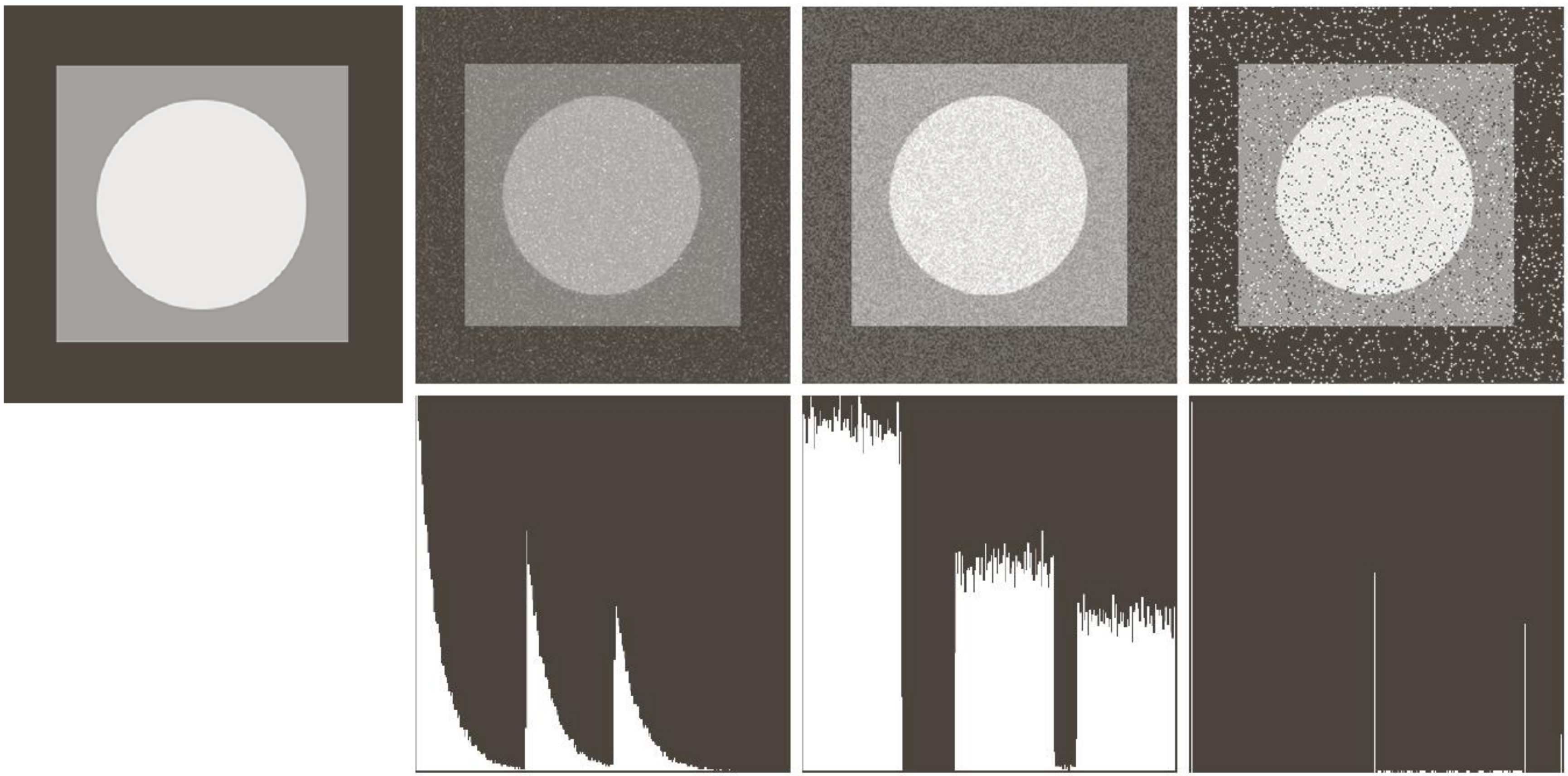


Noise models



Gaussian Rayleigh Gamma

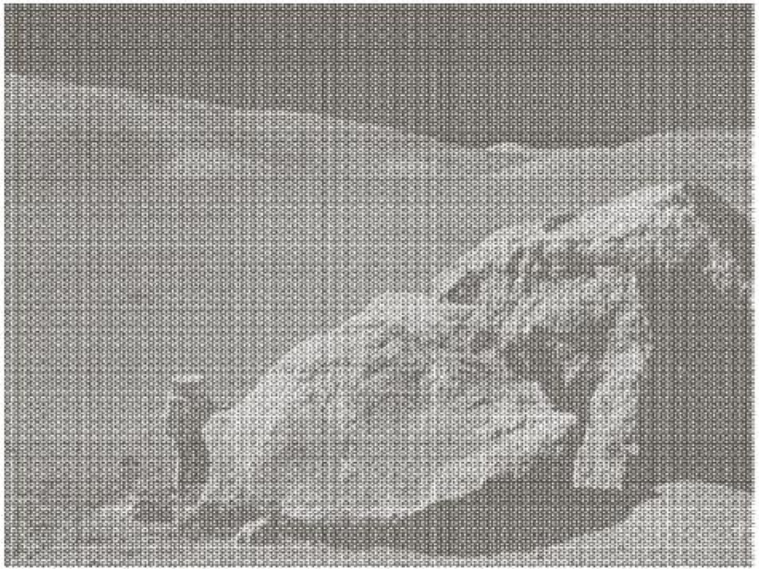
Noise models

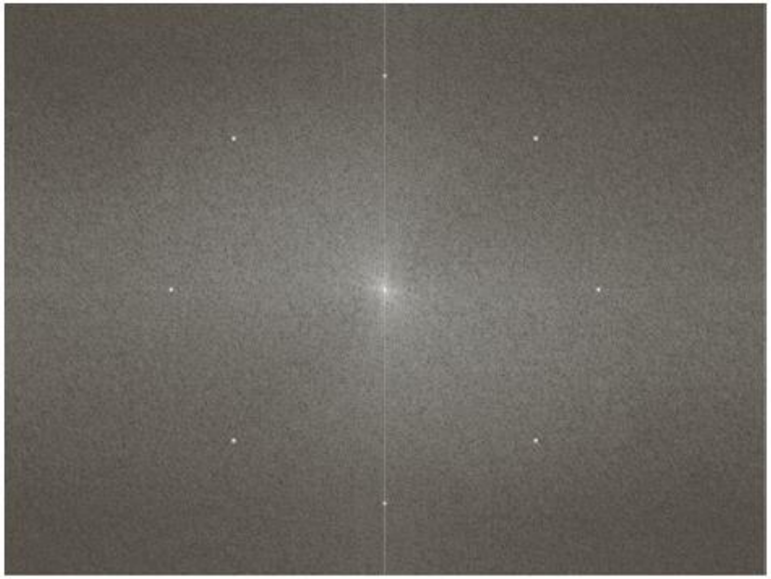


Exponential Uniform Salt & Pepper

Periodic noise

- Periodic noise originates from electrical and electromechanical interference in image acquisition
- Processing in frequency domain





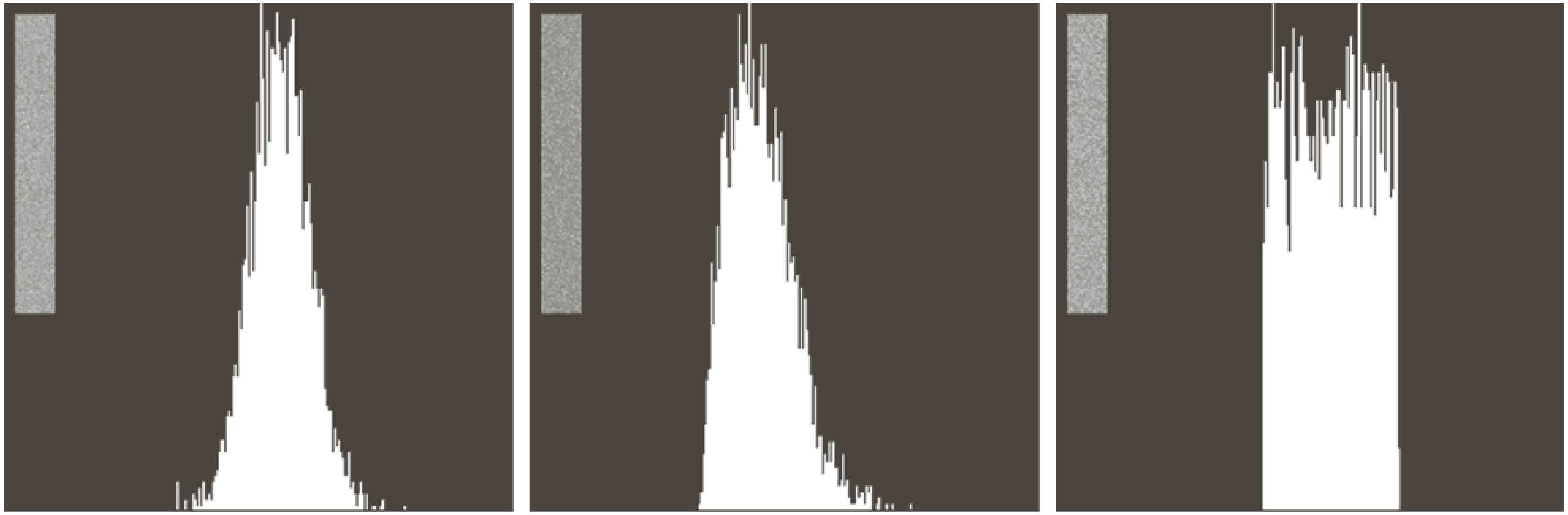
Estimation of Noise Parameters

- Finding the features of the imaging system
  - Specifications of the imaging system
  - Imaging a flat environment
- Mean and variance describe the data (partially)

$$\bar{z} = \sum_{i=0}^{L-1} z_i p_S(z_i)\,;\ \sigma^2 = \sum_{i=0}^{L-1} (z_i - \bar{z})^2 p_S(z_i)$$

  - Find flat regions from the image and then parameter values
  - Use the parameter values to find the histogram (Gaussian) or solve the corresponding paramters (*a* and *b*) for other distributions.

Estimation of Noise Parameters



- Impulse noise: find the propabilities of black and white pixels, and then the heights P_a and P_b of the peaks.
  - Again, from a relative smooth area

Restoration with spatial filters

- Basic assumption

$$g(x, y) = f(x, y) + \eta(x, y)$$

- In frequency domain it is possible to estimate periodic noise *N(u,v)* from the spectrum of *G(u,v)*
  - Signal *F(u,v)* in frequency domain can be found
- In spatial domain the additive noise in basically unknown and subtraction is not available
- Multiplicative noise ?

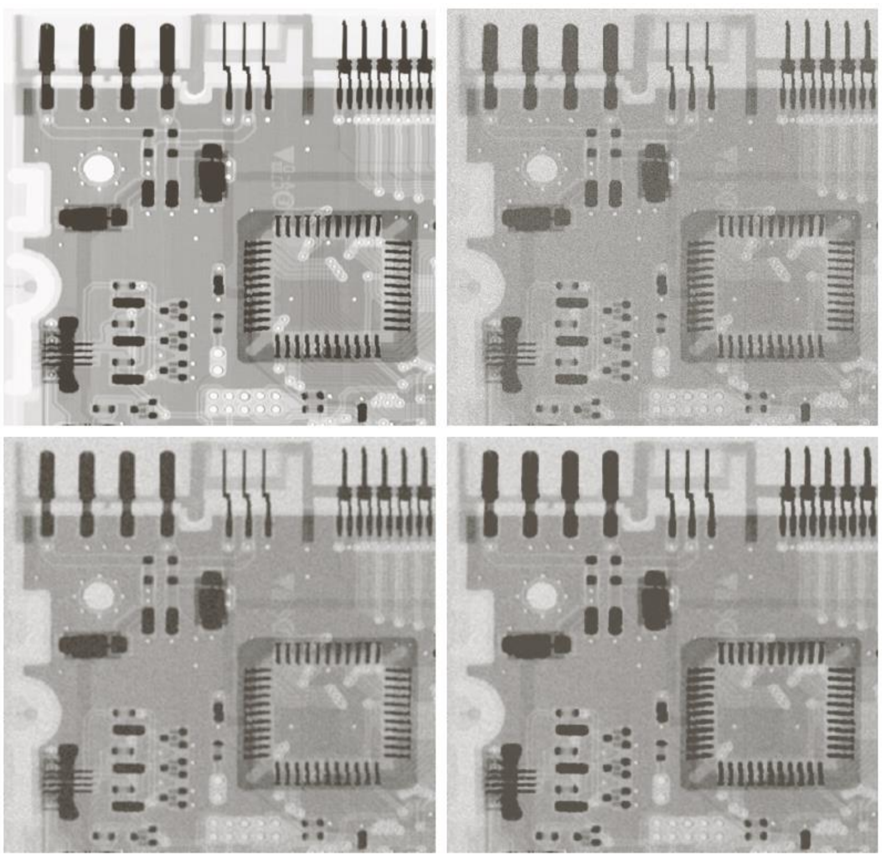
Restoration with spatial filters

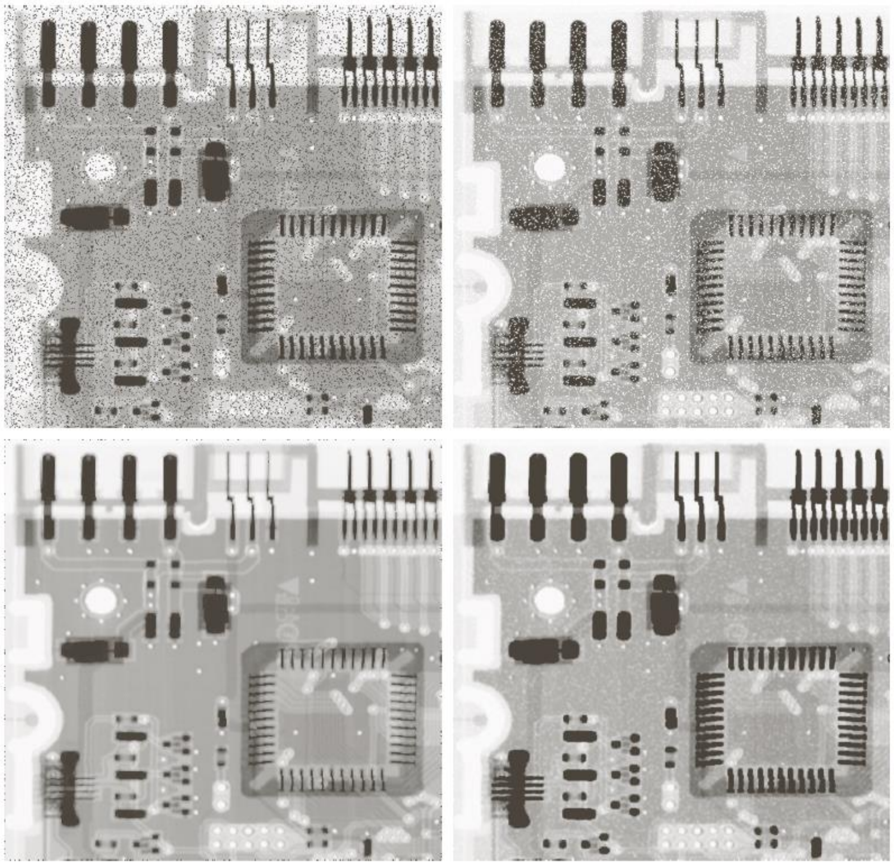
- Mean filters
  - Operating on a rectangular odd-size window
  - Arithmetic mean filter: $\hat{f}(x,y) = \frac{1}{mn}\sum_{(s,t)\in S_{xy}} g(s,t)$
  - Geometric mean filter: $\hat{f}(x,y) = \left[\prod_{(s,t)\in S_{xy}} g(s,t)\right]^{\frac{1}{mn}}$
  - Harmonic mean filter: $\hat{f}(x,y) = \frac{mn}{\sum_{(s,t)\in S_{xy}}\frac{1}{g(s,t)}}$
  - Contraharmonic mean filter: $\hat{f}(x,y) = \frac{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q+1}}{\sum_{(s,t)\in S_{xy}} g(s,t)^{Q}}$

Restoration with spatial filters

Original image; Gaussian noise
Arith. mean f.; Geom. m. f., *3x3*

Pepper noise (0.1); salt noise (0.1);
contrah. f., *3x3*, *Q=1.5*; *Q=-1.5*





Restoration with spatial filters

- Arithmetic mean filter
  - smooths local variations, noise removed, blurring as a result. For uniform and Gaussian noise.
- Geometric mean filter
  - similar to previous, smoothing, more details kept
- Harmonic mean filter
  - removes salt noise, not pepper, also Gaussian noise
- Contraharmonic mean filter
  - salt and pepper noise (pepper noise with $Q>0$, thinning and blurring dark areas; salt noise with $Q<0$, similar effect in light areas). Special filters with $Q=0$ and $Q=-1$. For impulse noise.

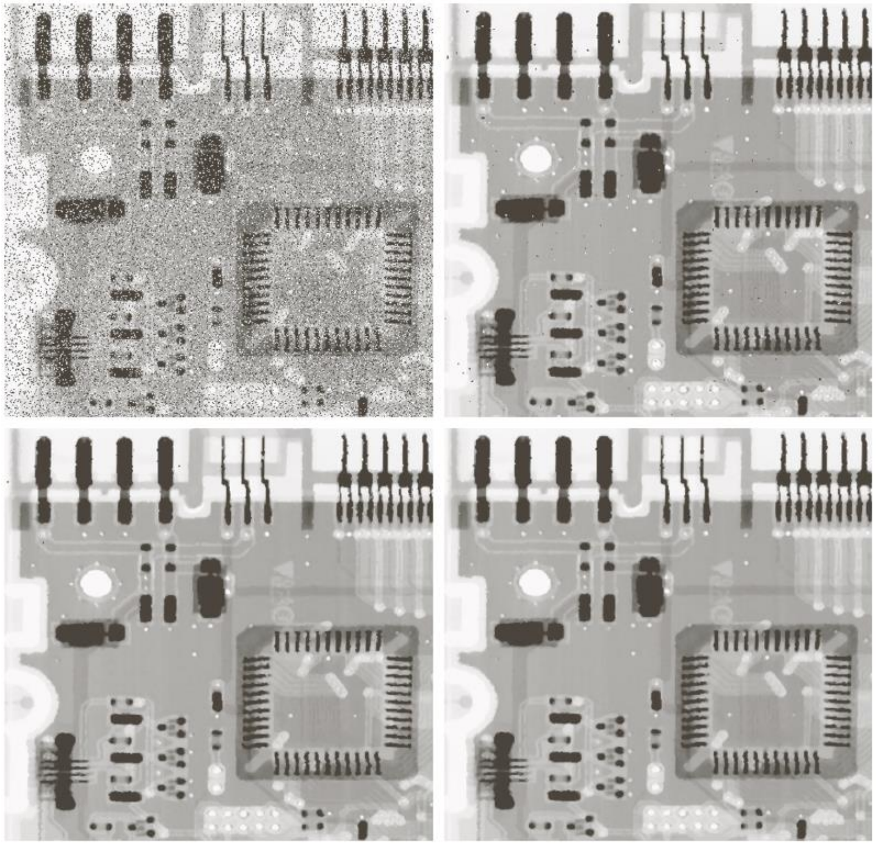
Restoration with spatial filters

- Order-statistics filters
  - Response is based on the ordering (ranking) of the values of the pixels in the neighbourhood
  - Median filter: $\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\text{median}}\{g(s,t)\}$
  - Max and min filters: $\hat{f}(x,y) = \underset{(s,t)\in S_{xy}}{\max}\{g(s,t)\}, \underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\}$
  - Midpoint filter: $\hat{f}(x,y) = \frac{1}{2}\left[\underset{(s,t)\in S_{xy}}{\max}\{g(s,t)\} + \underset{(s,t)\in S_{xy}}{\min}\{g(s,t)\}\right]$
  - Alpha-trimmed mean filter: $\hat{f}(x,y) = \frac{1}{mn-d}\sum_{(s,t)\in S_{xy}} g_r(s,t)$

Restoration with spatial filters

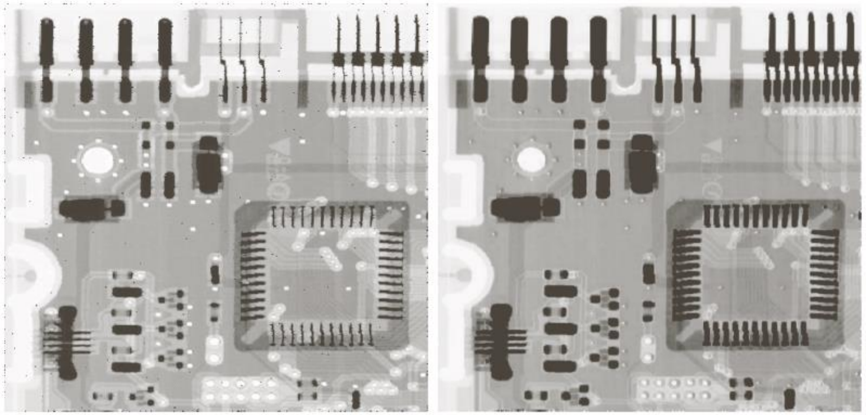
Salt-and-pepper noise $P_a = P_b = 0.1$;median filter *3x3;*

Processing with median filter, round 2; round 3; (multiple rounds removes more impulses but results in blurring)

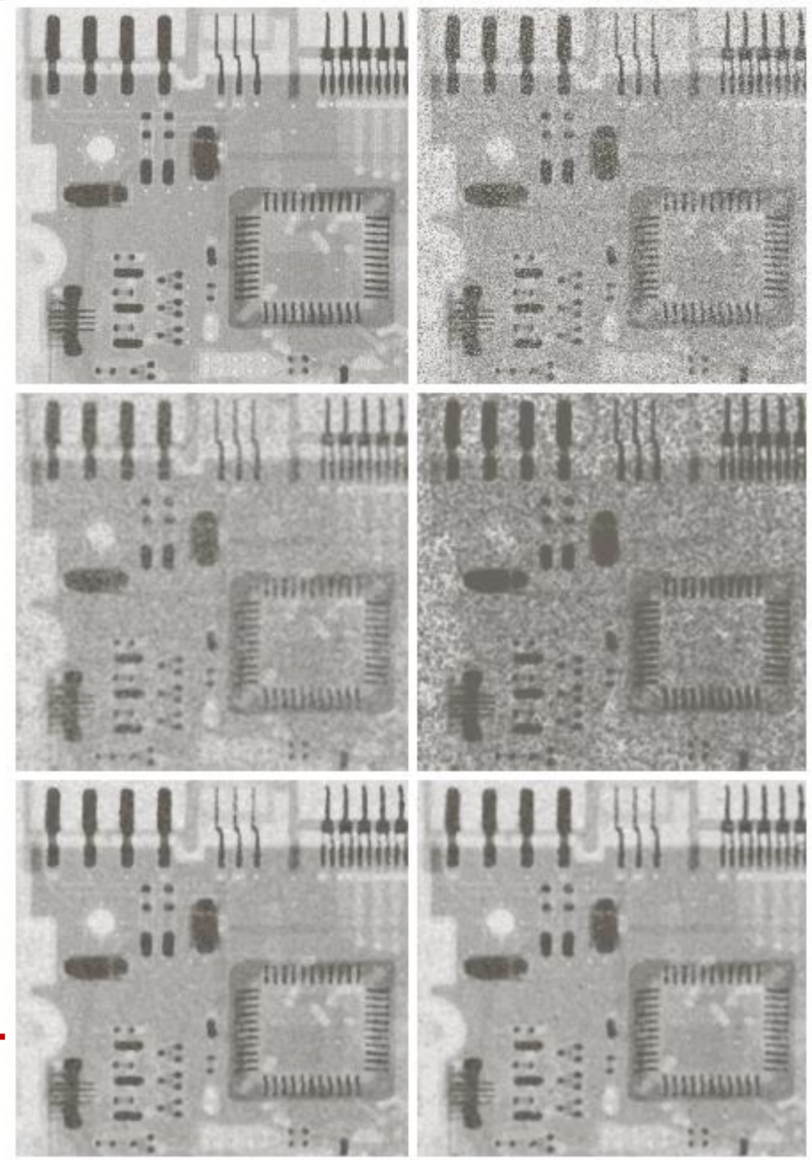


Pepper noise, max filter, *3x3;*

Salt noise, min filter, *3x3*



Restoration with spatial filters



- Additive uniform noise, *m=0, s=800*; Additive uniform noise with additive salt-and-pepper noise, $P_a = P_b = 0.1$

- Arithmetic mean filter; geometric mean filter, *5x5*

- Median filter; alpha-trimmed mean filter, *d=5, 5x5*

Restoration with spatial filters

- Median filter
  - Less blurring, effective for bipolar and unipolar impulses
- Max and min filters
  - 100th percentile, finding brightest points, pepper noise
  - 0th percentile, finding darkest points, salt noise
  - Removes also other extreme intensities
- Alpha-trimmed mean filter
  - Removing *d/2* highest and *d/2* smallest intensity values from ranking
  - *d=0:* mean filter; *d=md-1*: median filter
  - Multiple types of noise, e.g. combination of salt-and-pepper and Gaussian noise

Restoration with spatial filters

- Adaptive filters
  - The filter adapts to the image characteristics inside the $m \times n$ window
  - Filtering capabilities are better on the cost that the filters become more complex
- Adaptive, local noise reduction filters response depends on
  - $g(x, y)$, the corrupted image
  - σ_η^2 as the variance of the noise corrupting the image
  - m_L, the local mean; σ_L^2, the local variance

Restoration with spatial filters

- Design of the adaptive mean filter
  - If $\sigma_\eta^2 = 0$ the response becomes $g(x,y)$ (= $f(x,y)$)
  - If σ_L^2 is large compared to σ_η^2 the response is close to $g(x,y)$ (large local variations, like edges)
  - $\sigma_L^2 = \sigma_\eta^2$, the output comes from the mean filtering (properties in the local area is similar to the whole image)

$$\hat{f}(x,y) = g(x,y) - \frac{\sigma_\eta^2}{\sigma_L^2}[g(x,y) - m_L], \quad \sigma_\eta^2 \leq \sigma_L^2$$

  - How to find σ_η^2?

Restoration with spatial filters

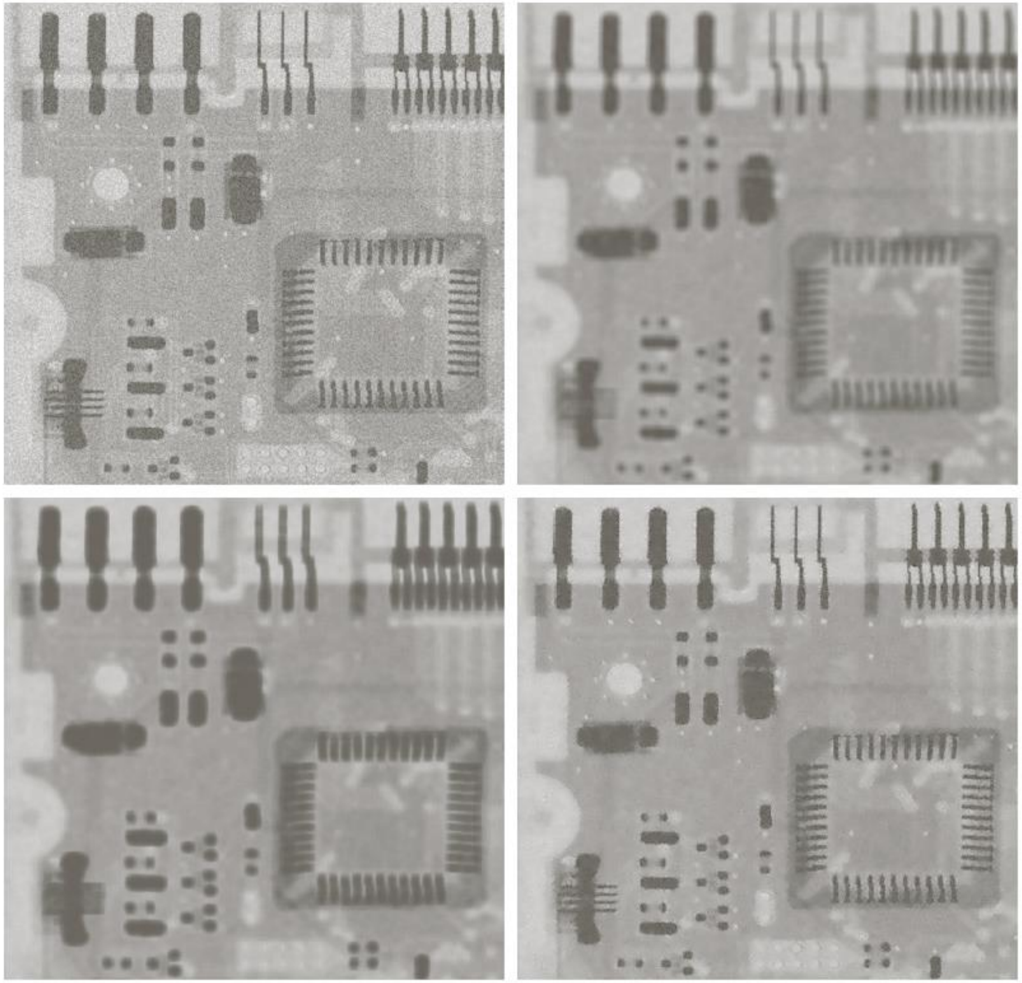
Corrupted image, additive Gaussian noise,
$m = 0,\ \sigma_\eta^2 = 1000$

Arithmetic mean filtering

Geometric mean filtering

Adaptive noise reduction

Window size: 7x7



Restoration with spatial filters

- How to find σ_η^2?
  - Estimated value too low: output close to $g(x, y)$
  - Estimated value too high: ratio values clipped to 1; (nonlinear filtering)
    - Subtraction of the local mean too often
- Negative outputs accepted
  - image becomes rescaled and
  - the dynamic range becomes limited
- Impulse noise?

Restoration with spatial filters

- Design of the adaptive median filter
  - Removing salt-and-pepper noise, preserve details
  - Smoothing other noise types
  - Reduce excessive thinning or thickening of objects
  - Basic assumption: $P_a, P_b < 0.2$
  - Parameters
    - z_{min},z_{max},z_{med}: min, max, median values in S_{xy}
    - z_{xy}: intensity at (x, y)
    - S_{max}: max size of S_{xy}
  - The window size is increased during filtering process

Restoration with spatial filters

- Operation in two stages
- Stage A: check if z_{med} is an impulse or not

  A1 = z_{med}-z_{min}
  A2 = z_{med}-z_{max}
  If *A1* > *0* AND *A2* < *0* go to Stage B; % not an impulse
  Else increase the window size. % an impulse found
  If window size ≤ S_{max} repeat Stage A;
  Else output z_{med}.

- Stage B: check if z_{xy} is an impulse or not

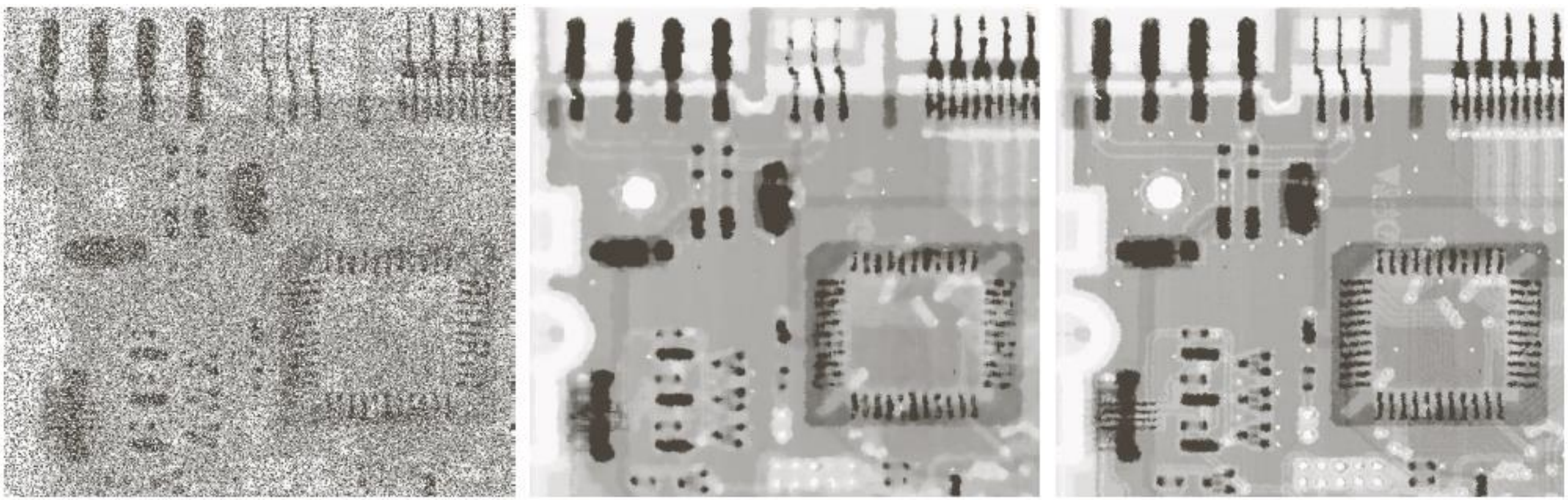
  B1 = z_{xy}-z_{min}
  B1 = z_{xy}-z_{max}
  If *B1* > *0* AND *B2* < *0* output z_{xy}; % not an impulse
  Else output z_{med}. % an impulse found

Restoration with spatial filters

Image corrupted with salt-and-pepper noise, $P_a = P_b = 0.25$

Median filter, $7x7$

Adaptive filter, $S_{max} = 7$



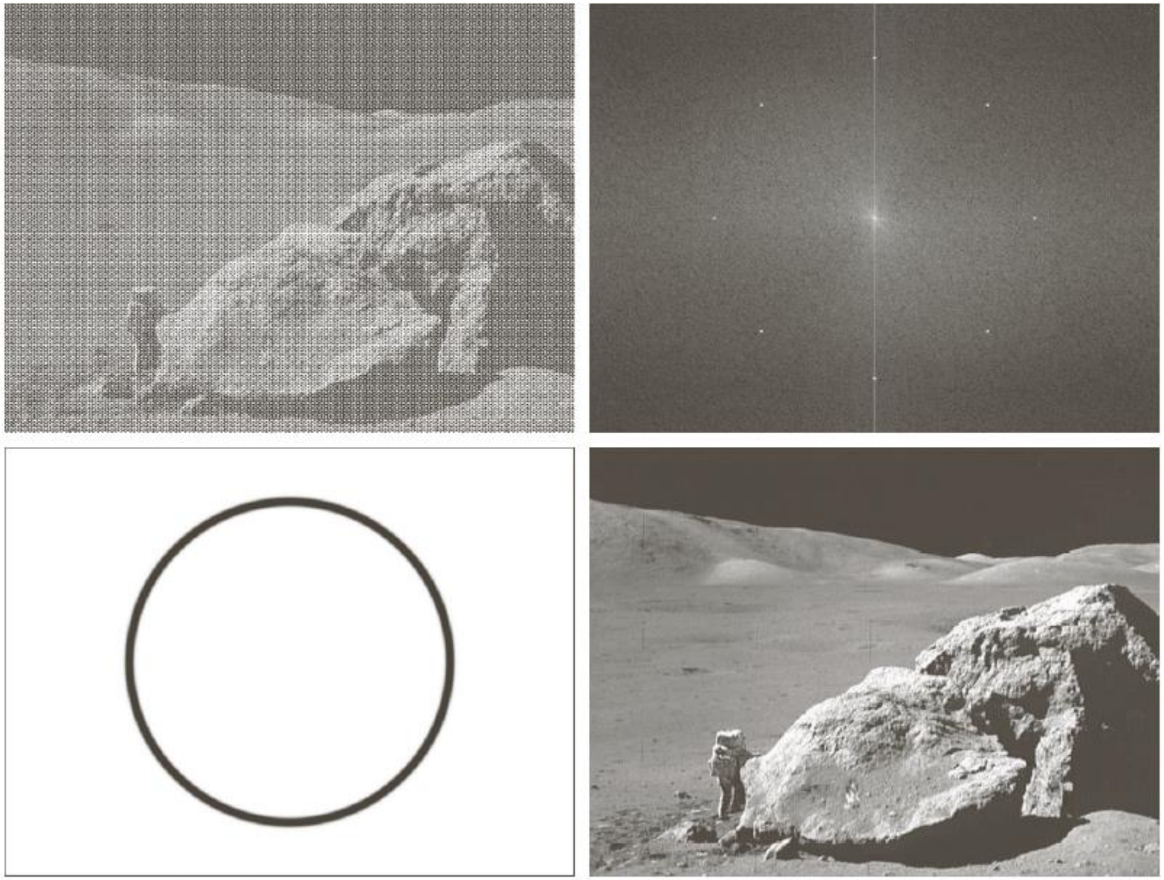
Restoration in frequency domain

- Periodic noise analysis and reduction is possible in frequency domain
- Bandreject filters for known noise

Ideal	Butterworth	Gaussian
$H(u, v) = \begin{cases} 1 & \text{if } D(u, v) \leq D_0 \\ 0 & \text{if } D(u, v) > D_0 \end{cases}$	$H(u, v) = \dfrac{1}{1 + [D(u, v)/D_0]^{2n}}$	$H(u, v) = e^{-D^2(u,v)/2D_0^2}$

Restoration in frequency domain

- Bandreject filter
- Noise appears as peaks in specific frequencies

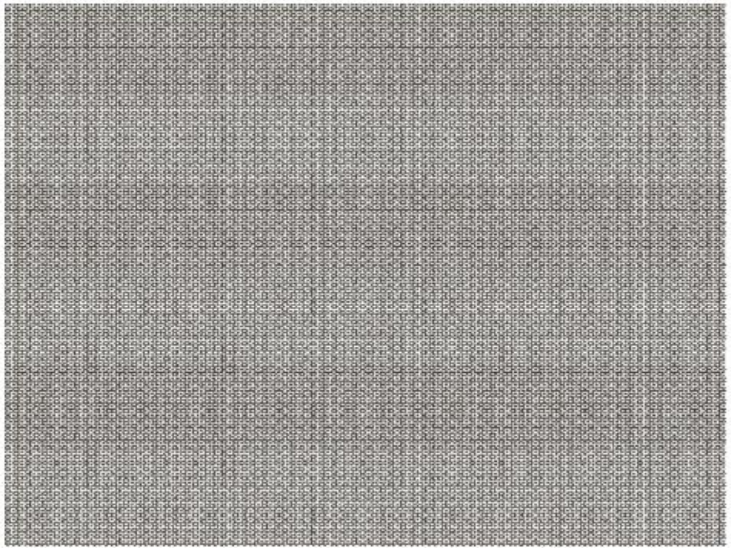


Restoration in frequency domain

- Bandpass filters

$$H_{BP}(u,v) = 1 - H_{BR}(u,v)$$

- Bandpass filtering as such removes essential information from the image
- Practical uses in detecting/isolating the noise pattern
  - Filter the image with BP filter
  - Use inverse transform to construct the noise pattern



Restoration in frequency domain

- Notch filters
  - selective filters: pass or reject frequencies defined as a neighbourhood of a center

$$H_{NR}(u,v) = \prod_{k=1}^{Q} H_k(u,v) H_{-k}(u,v)$$
$$H_{NP}(u,v) = 1 - H_{NR}(u,v)$$

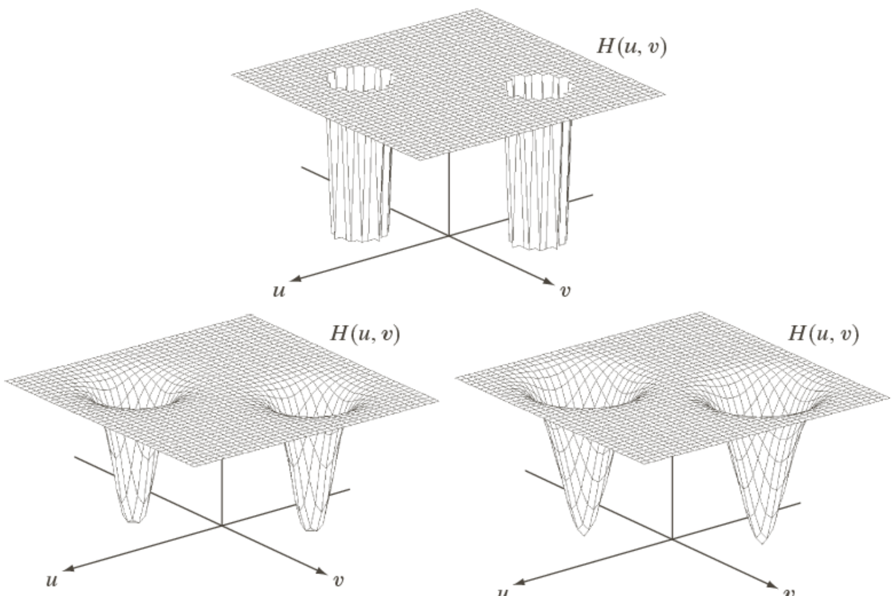
  where $H_k(u,v)$ is a highpass filter centered at (u_k, v_k)

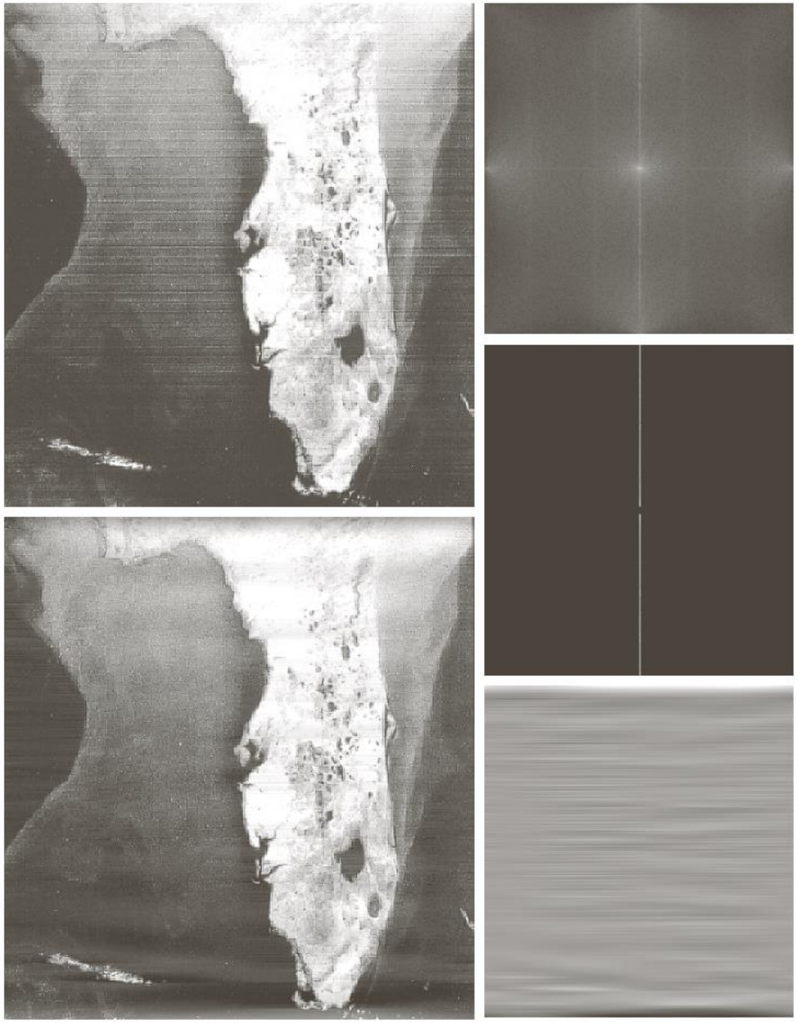
  - Notch filters are designed as symmetric pairs to avoid
    - No phase-shift and
    - Meaningful results as outcome

Restoration in frequency domain

- Example in notch pass filtering
- Notch reject filters:

  ideal, Butterworth, Gaussian





Restoration in frequency domain

- Design of optimum notch filters
  - Finding and isolating the interference pattern $N(u,v)$, e.g. using a notch pass filter
$$N(u,v) = H_{NP}(u,v)G(u,v)$$
  - Find the corresponding spatial filter (IFFT)
$$\eta(x,y) = F^{-1}\{H_{NP}(u,v)G(u,v)\}$$
  - Obtain the spatial image as
$$f(x,y) = g(x,y) - \eta(x,y)$$
  - The noise pattern is an estimate, so the weight $w(x,y)$ is applied
$$\hat{f}(x,y) = g(x,y) - w(x,y)\eta(x,y)$$

Restoration in frequency domain

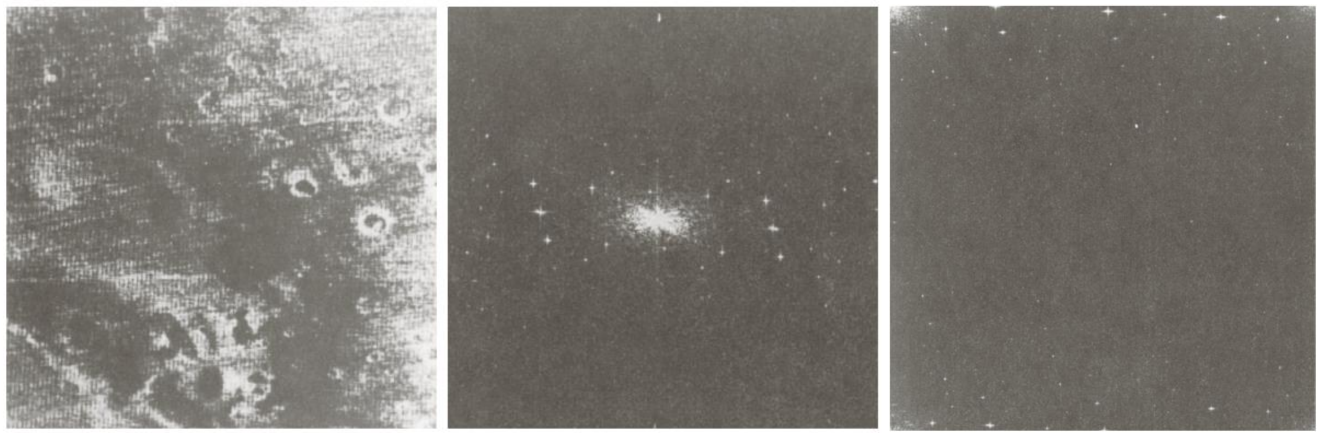
- Design of optimum notch filters
  - The weight $w(x,y)$ is found through minimizing the variance of $\hat{f}(x,y)$ in the neighbourhood of (x,y)
  - As a result of the optimization $w(x,y)$ is found

$$w(x,y) = \frac{\overline{g(x,y)\eta(x,y)} - \bar{g}(x,y)\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

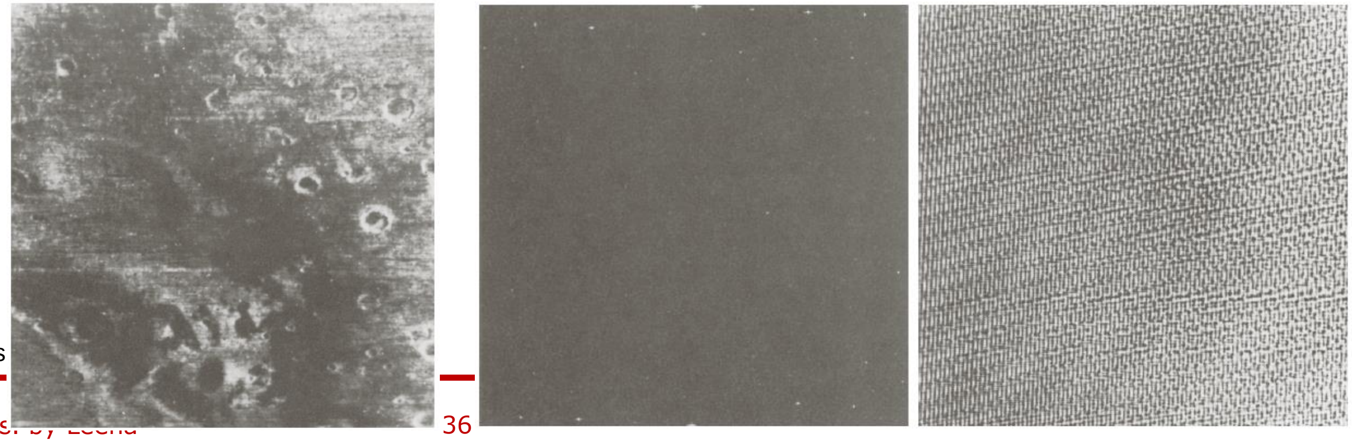
  - $w(x,y)$ is assumed to remain constant in the neighbourhood
    - Simplifications in the computations (finding $w(x,y)$ only once for each center point of the non-overlapping neighbourhood)

Restoration in frequency domain

- Orginal image; Fourier spectrum; Spectrum without shifting



- Result from filtering; Fourier spectrum of $N(u, v)$; noise pattern $\eta(x, y)$



Degradations

- The degradation affects to the input, the image
- The output becomes a degraded image

$$g(x,y) = H[f(x,y)] + \eta(x,y)$$

- So far, H has been identity operator
- Now $\eta(x,y) = 0$ and then $g(x,y) = H[f(x,y)]$
- In modeling H is assumed to be a *linear operator* (additivity, homogeneity) and also *position invariant*

$$H[f(x-a,y-b)] = g(x-a,y-b)$$

- Finally, the output $g(x,y)$ is found as a convolution integral

$$g(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\alpha,\beta)h(x-\alpha,y-\beta)d\alpha d\beta$$

Degradations

- If the impulse response is known, then the output g can be found for any input f, i.e. the convolution of impulse response and the input function
- With a noise component

$$g(x,y) = h(x,y) * f(x,y) + \eta(x,y)$$

And

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

- Next, the task is to estimate the degradation function H

Degradations

- Blind deconvolution
  - True degradation is and remains unknown

- Estimation by image observation
  - Seeking for specific features from the image: blurring, edges, high contrast areas

$$H_s(u,v) = \frac{G_s(u,v)}{\hat{F}_s(u,v)}$$

  - and assuming position invariance
  - Requires manual work (in the first step)

Degradation

- Estimation by experimentation
  - Imaging with a similar system as with which the original image was obtained
  - Using various settings of the system, one can acquire a similar image as the original one
  - Then, imaging an impulse with the those settings describes the degradation

$$H(u,v) = \frac{G(u,v)}{A}$$

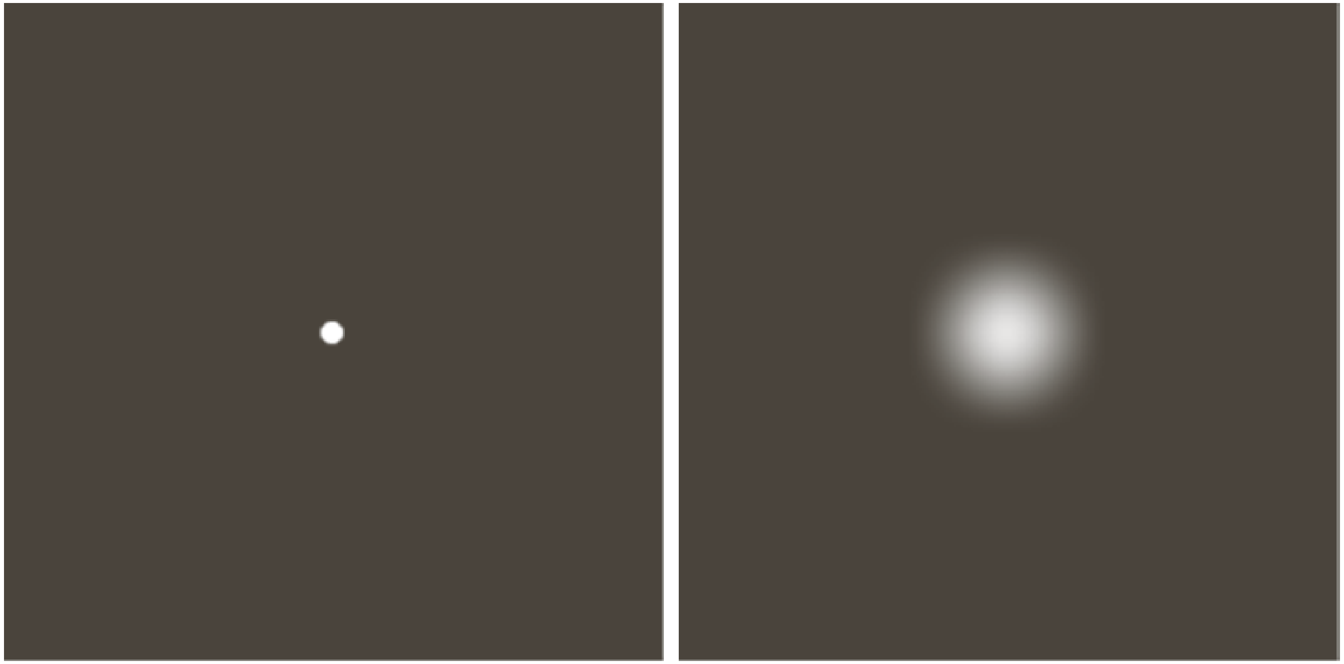
where $G(u,v)$ is the FT of the observed image and A is the strength of the impulse.

Degradation

- Estimation by experimentation

a b

FIGURE 5.24 Degradation estimation by impulse characterization. (a) An impulse of light (shown magnified). (b) Imaged (degraded) impulse.



Degradation

- Estimation by modeling
  - The degradation function is modeled based on the physical phenomena
  - E.g atmospheric turbulence (similar to uniform blurring)

$$H(u,v) = e^{-k(u^2+v^2)^{5/6}}$$

  - E.g. uniform linear timedependent motion $(x_0(t), y_0(t))$

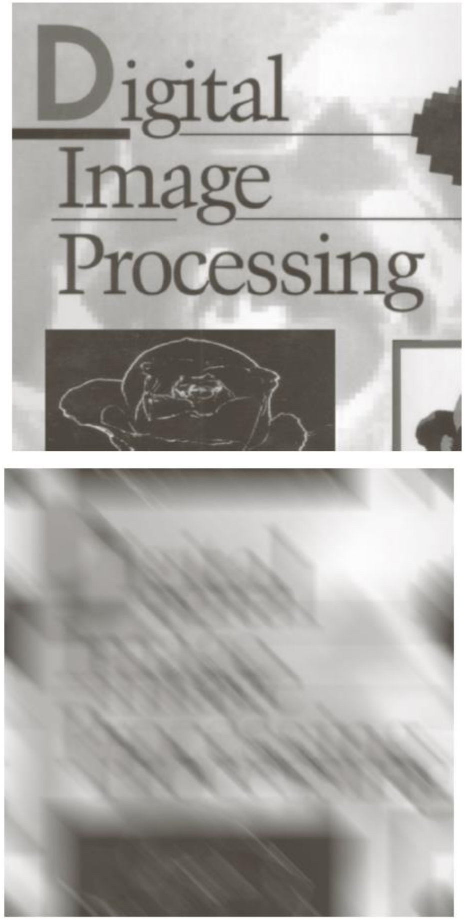
$$H(u,v) = \frac{T}{\pi(ua+vb)} \sin[\pi(ua+vb)] e^{-j\pi(ua+vb)}$$

where $(x_0(t), y_0(t)) = (at/T, bt/T)$

Degradation

a b
c d

FIGURE 5.25 Illustration of the atmospheric turbulence model. (a) Negligible turbulence. (b) Severe turbulence, $k = 0.0025$. (c) Mild turbulence, $k = 0.001$. (d) Low turbulence, $k = 0.00025$. (Original image courtesy of NASA.)



a b

FIGURE 5.26 (a) Original image. (b) Result of blurring using the function in Eq. (5.6-11) with $a = b = 0.1$ and $T = 1$.

Restoration of images with H

- When degradation function H is known then an estimate of the image is

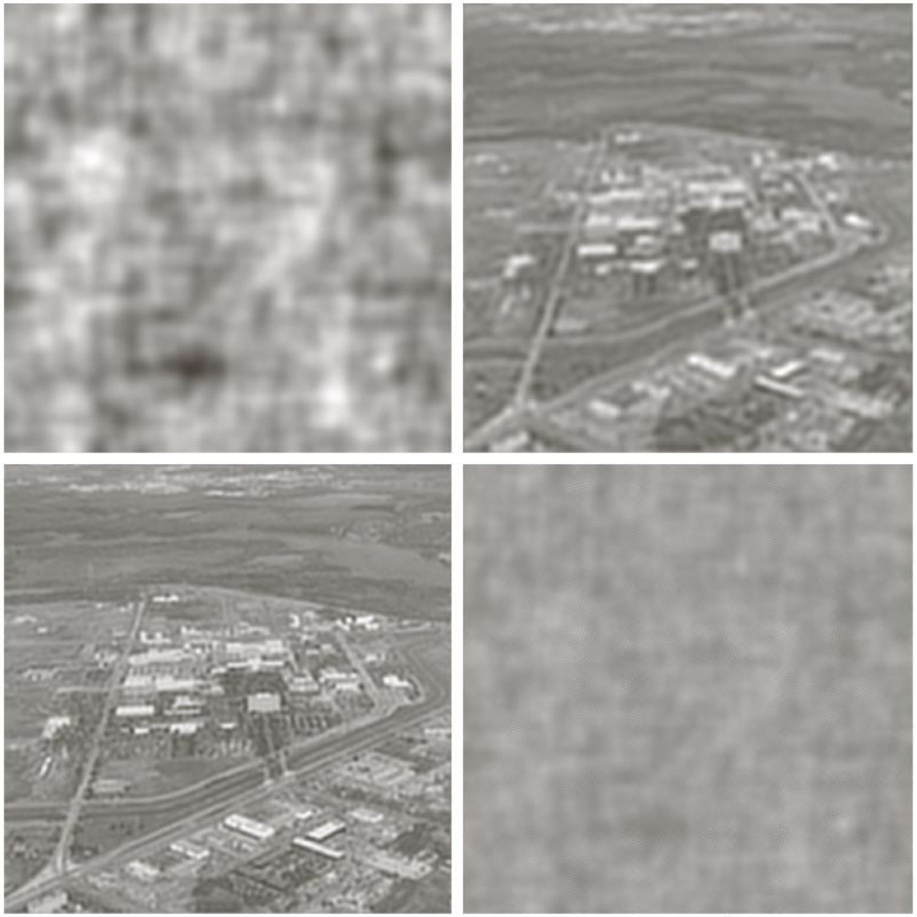
$$\hat{F}(u,v) = \frac{G(u,v)}{H(u,v)} = F(u,v) + \frac{N(u,v)}{H(u,v)}$$

  - Again, one should be able to describe the noise
  - If the degradation function has zeros or very small values, then the latter term dominates the output values
    - Solution: concentrate on values close to $H(0,0)$

Restoration of images with H

a b
c d

FIGURE 5.27 Restoring Fig. 5.25(b) with Eq. (5.7-1). (a) Result of using the full filter. (b) Result with H cut off outside a radius of 40; (c) outside a radius of 70; and (d) outside a radius of 85.



- Degraded image
- Restored images:

  best visual results

  with radius $r=70$

Restoration of images with H

- If the characteristics of noise are available then that can be incorporated in restoration process
- Minimum mean square error filtering (Wiener filtering)
  - The goal is to minimize

$$e^2 = E\{(f - \hat{f})^2\}$$

    where the image and noise are uncorrelated, at least one has zero mean

  - It can be shown that at optimum, the estimate is

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)} \frac{|H(u,v)|^2}{|H(u,v)|^2 + S_\eta(u,v)/S_f(u,v)}\right] G(u,\mathrm{v})$$

  - In general, $S_f(u,v)$ seldom is unknown

Restoration of images with H

- One way to use approximation of $S_f(u,v)$ and then

$$\hat{F}(u,v) = \left[\frac{1}{H(u,v)}\frac{|H(u,v)|^2}{|H(u,v)|^2+K}\right]G(u,\mathrm{v})$$

a b c

FIGURE 5.28 Comparison of inverse and Wiener filtering. (a) Result of full inverse filtering of Fig. 5.25(b). (b) Radially limited inverse filter result. (c) Wiener filter result.

Restoration of images with H

- Top row: high noise variance
- Center row: intermediate noise variance (magnitude less)
- Bottom row: low noise variance (5 orders of magnitude less)
- Left column: noisy image
- Center column:direct inverse filtering
- Right column: Wiener filtering

Restoration of images with H

- Constrained least squares filtering
  - Filtering results from considering smoothness such as the second derivative $P(u,v)$ of the image

$$\min C; \text{ subject to } \left\|g - H\hat{f}\right\|^2 = \|\eta\|^2$$

and

$$C = \sum_{x=0}^{M-1} \sum_{y=0}^{N-1} [\nabla^2 f(x,y)]^2$$

  - finally

$$\hat{F}(u,v) = \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \gamma |P(u,v)|^2}\right] G(u,v)$$

Restoration of images with H

- Constrained least squares filtering
  - E.g. $p(x, y)$ is Laplacian: $p(x,y) = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 0 \end{bmatrix}$
- Various constant, manually selected, values for γ (page -2)

Restoration of images with $\boldsymbol{H}$

- Constrained least squared filtering
  - The value of parameter γ can be found also iteratively
    - The goal is to minimize residual $\boldsymbol{r} = \boldsymbol{g} - \boldsymbol{H}\hat{\boldsymbol{f}}$
    - the mean and the variance of noise is needed

a b

FIGURE 5.31 (a) Iteratively determined constrained least squares restoration of Fig. 5.16(b), using correct noise parameters. (b) Result obtained with wrong noise parameters.

Restoration of images with *H*

- Geometric mean filter
  - Generalization of the Wiener filter

$$\hat{F}(\mathrm{u,v}) = \left[\frac{H^*(u,v)}{|H(u,v)|^2}\right]^{\alpha} \left[\frac{H^*(u,v)}{|H(u,v)|^2 + \beta\left[\frac{S_\eta(u,v)}{S_f(u,v)}\right]}\right]^{1-\alpha} G(u,v)$$

  - Two constants α and β, with special cases:
    - $\alpha = 1$: inverse filter
    - $\alpha = 0$: parametric Wiener filter, $\beta = 1$: Wiener filter;
    - $\alpha = 1/2$: geometric mean of the components
    - $\beta = 1, \alpha < 1/2$, closer to inverse filter
    - $\beta = 1, \alpha > 1/2$, closer to Wiener filter

Summary

- Many approaches assume linear, position invariant process, and additive noise
- What is ”optimal”? Computational sense or HVS sense?
- Modeling of restoration filters in frequency domain
- Practical restoration applies convolution with a mask in the spatial domain