Algoritmo del par más cercano

(Algorithm for the closest pair)

Aduanich Rguez Rguez
Alejandro David Carrillo Padrón
Daniel Darias Sánchez

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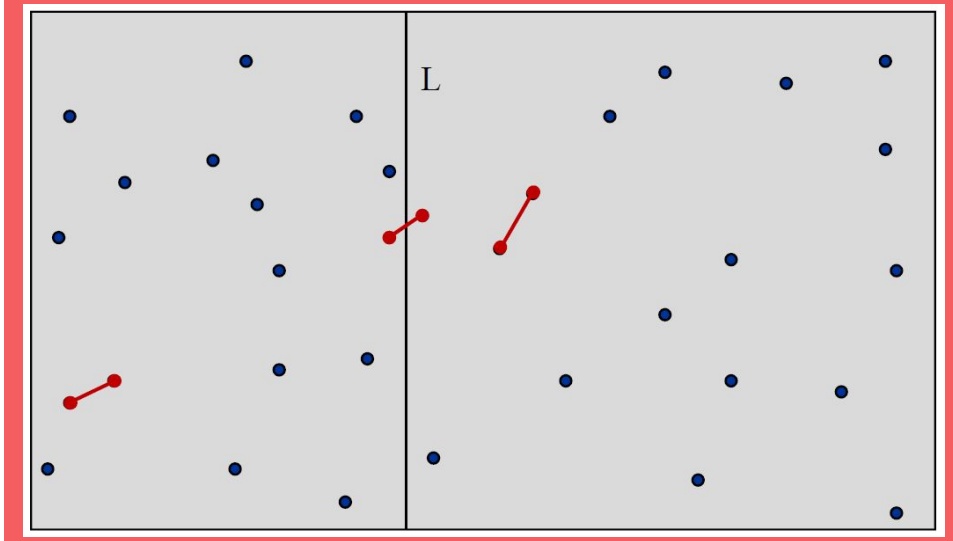
Introducción

Proporciona una manera de encontrar los dos puntos más cercanos entre sí de un conjunto de puntos dados en dos dimensiones en un caso planar.



Explicación

Método no óptimo
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Se realiza la comparación de cada punto dentro del vector de puntos con el resto hasta encontrar el caso con el par más cercano.

Método óptimo
DIVIDE Y VENCERÁS

Se divide el vector n veces hasta que el tamaño de cada sección sea 2 que es el tamaño mínimo para hacer comparaciones de dos puntos del algoritmo.

Pseudocódigo

```
bruteForceClosestPair of P(1), P(2), ... P(N)
if N < 2 then
  return ∞
else
  minDistance ← |P(1) - P(2)|
  minPoints ← { P(1), P(2) }
  foreach i ∈ [1, N-1]
    foreach j ∈ [i+1, N]
      if |P(i) - P(j)| < minDistance then
        minDistance ← |P(i) - P(j)|
        minPoints ← { P(i), P(j) }
      endif
    endfor
  endfor
  return minDistance, minPoints
 endif
```

```
closestPair of (xP, yP)
               where xP is P(1) .. P(N) sorted by x coordinate, and
                     yP is P(1) .. P(N) sorted by y coordinate (ascending order)
if N ≤ 3 then
  return closest points of xP using brute-force algorithm
else
  xL ← points of xP from 1 to ⌈N/2⌉
  xR ← points of xP from ⌈N/2⌉+1 to N
  xm ← xP(⌈N/2⌉)x
  yL ← { p ∈ yP : px ≤ xm }
  yR ← { p ∈ yP : px > xm }
  (dL, pairL) ← closestPair of (xL, yL)
  (dR, pairR) ← closestPair of (xR, yR)
  (dmin, pairMin) ← (dR, pairR)
  if dL < dR then
    (dmin, pairMin) ← (dL, pairL)
  endif
  yS ← { p ∈ yP : |xm - px| < dmin }
  nS ← number of points in yS
  (closest, closestPair) ← (dmin, pairMin)
  for i from 1 to nS - 1
    k ← i + 1
    while k ≤ nS and yS(k)y - yS(i)y < dmin
      if |yS(k) - yS(i)| < closest then
        (closest, closestPair) ← (|yS(k) - yS(i)|, {yS(k), yS(i)})
      endif
      k ← k + 1
    endwhile
  endfor
  return closest, closestPair
endif
```

Código y Explicación

```cpp
std::pair<double, points_t> find_closest_brute(const std::vector<point_t>& points) {
	if (points.size() < 2) {
		return { -1, { { 0, 0 }, { 0, 0 } } };
	}
	auto minDistance = std::abs(distance_between(points.at(0), points.at(1)));
	points_t minPoints = { points.at(0), points.at(1) };
	for (auto i = std::begin(points); i != (std::end(points) - 1); ++i) {
		for (auto j = i + 1; j < std::end(points); ++j) {
			auto newDistance = std::abs(distance_between(*i, *j));
			if (newDistance < minDistance) {
				minDistance = newDistance;
				minPoints.first = *i;
				minPoints.second = *j;
			}
		}
	}
	return { minDistance, minPoints };
}
```

```cpp
std::pair<double, points_t> find_closest_optimized(const std::vector<point_t>& xP,
	const std::vector<point_t>& yP) {
	if (xP.size() <= 3) {
		return find_closest_brute(xP);
	}
	auto N = xP.size();
	auto xL = std::vector<point_t>();
	auto xR = std::vector<point_t>();
	std::copy(std::begin(xP), std::begin(xP) + (N / 2), std::back_inserter(xL));
	std::copy(std::begin(xP) + (N / 2), std::end(xP), std::back_inserter(xR));
	auto xM = xP.at(N / 2).first;
	auto yL = std::vector<point_t>();
	auto yR = std::vector<point_t>();
	std::copy_if(std::begin(yP), std::end(yP), std::back_inserter(yL), [&xM](const point_t& p) {
		return p.first <= xM;
	});
	std::copy_if(std::begin(yP), std::end(yP), std::back_inserter(yR), [&xM](const point_t& p) {
		return p.first > xM;
	});
	auto p1 = find_closest_optimized(xL, yL);
	auto p2 = find_closest_optimized(xR, yR);
	auto minPair = (p1.first <= p2.first) ? p1 : p2;
	auto yS = std::vector<point_t>();
	std::copy_if(std::begin(yP), std::end(yP), std::back_inserter(yS), [&minPair, &xM](const point_t& p) {
		return std::abs(xM - p.first) < minPair.first;
	});
	auto result = minPair;
	for (auto i = std::begin(yS); i != (std::end(yS) - 1); ++i) {
		for (auto k = i + 1; k != std::end(yS) &&
		 ((k->second - i->second) < minPair.first); ++k) {
			auto newDistance = std::abs(distance_between(*k, *i));
			if (newDistance < result.first) {
				result = { newDistance, { *k, *i } };
			}
		}
	}
	return result;
}
```

Complejidad

Método no óptimo
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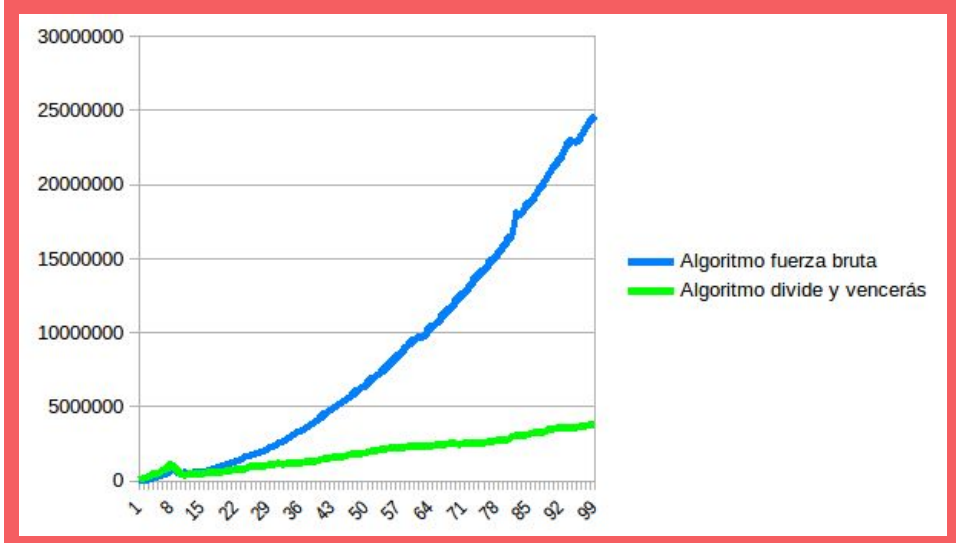
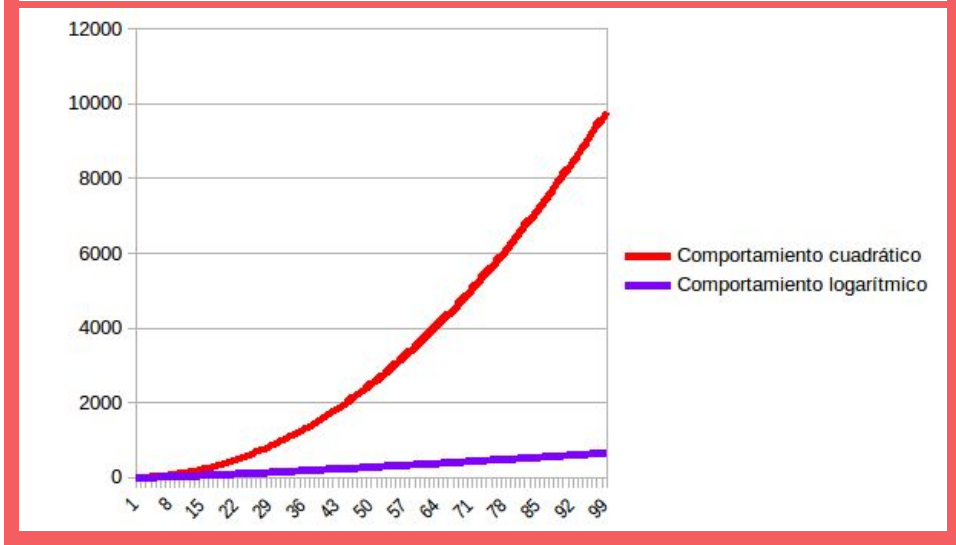
$O(n^m)$
m = 2 ó 3 según las dimensiones del problema.
n = número de puntos del problema.

```
Closest-Pair (p1, …, pn)
{
   Compute separation line L such that half the points          O(n log n)
   are on one side and half on the other side.

   δ1 = Closest-Pair(left half)                                 2T(n / 2)
   δ2 = Closest-Pair(right half)
   δ  = min(δ1, δ2)

   Delete all points further than δ from separation line L      O(n)

   Sort remaining points by y-coordinate.                       O(n log n)

   Scan points in y-order and compare distance between          O(n)
   each point and next 11 neighbors. If any of these
   distances is less than δ, update δ.

   return δ.
}
```

$$T(n) \leq 2T(n/2) + O(n) \Rightarrow T(n) = O(n \log n)$$

Tiempos de Ejecución





Tamaño del Problema	Alg. Fuerza Bruta	Alg. Div y Vencerás	Comportamiento Cuadrático	Comportamiento Logarítmico
10	421677	464150	100	33.2192
19	960821	590863	361	80.7106
29	2.21E+06	1.09E+06	841	140.8814
39	4.01E+06	1.35E+06	1521	206.1306
49	6.32E+06	1.84E+06	2401	275.1207
59	9.22E+06	2.29E+06	3481	347.0759
69	1.20E+07	2.51E+06	4761	421.4881
79	1.58E+07	2.77E+06	6241	497.998
89	2.07E+07	3.44E+06	7921	576.340

Conclusiones

- Efectividad
  - Menor número de comparaciones
  - Menor número de iteraciones
  - Particularmente eficaz en resolución del problema en un plano Euclidiano
- Ventajas de uso.
  - Evitar colisión entre puntos
  - Control de elementos en entorno espacial

¿Preguntas?

Bibliografía

https://en.wikipedia.org/wiki/Closest_pair_of_points_problem

http://www.geeksforgeeks.org/closest-pair-of-points/