LECTURE 5

Axis Transformation

1

WHAT HAVE WE SEEN SO FAR?

Basic representations (point, vector)

Basic operations on points and vectors (dot product, cross products, etc.)

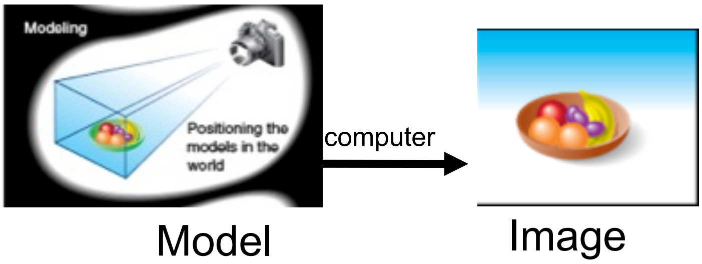
Transformation – manipulative operators on the basic representation (translate, rotate, deformations) – 4x4 matrices to “encode” all these.

2

WHY DO WE NEED THIS?

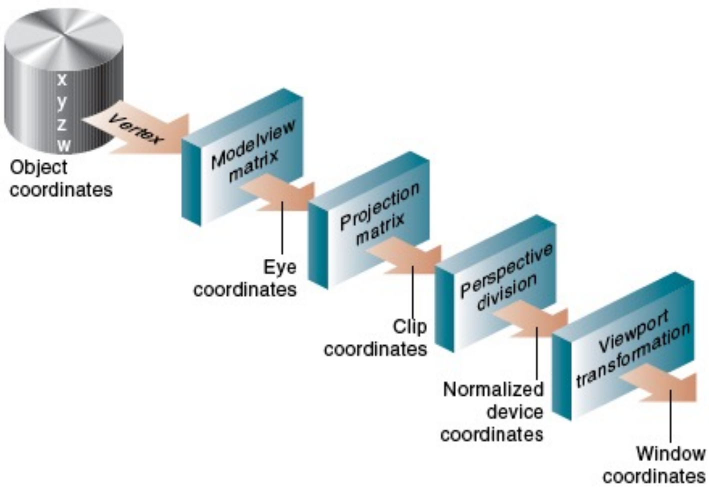
In order to generate a picture from a model, we need to be able to not only specify a model but also manipulate the model in order to create more interesting images.

From a model, how do we generate an image



3

OVERVIEW



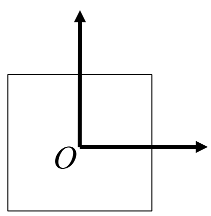
4

COORDINATE SYSTEMS

- Object coordinates
- World coordinates
- Camera coordinates
- Normalized device coordinates
- Window coordinates

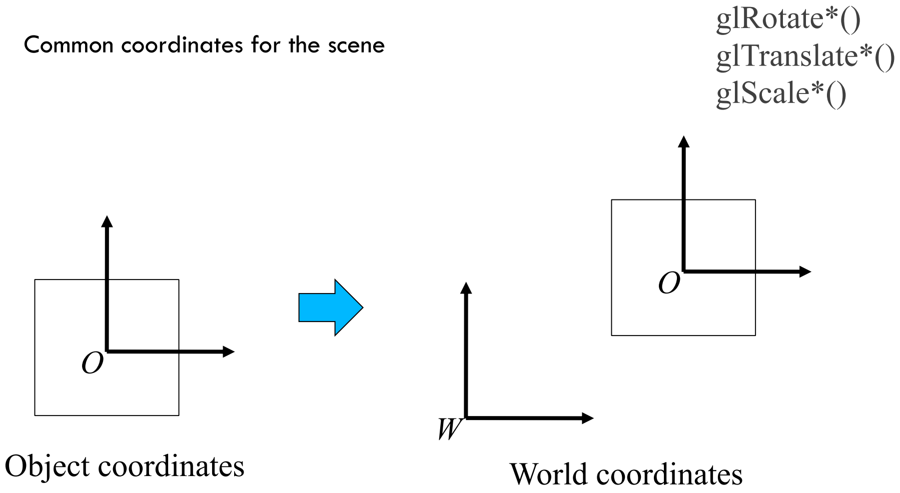
OBJECT COORDINATES

Convenient place to model the object



WORLD COORDINATES

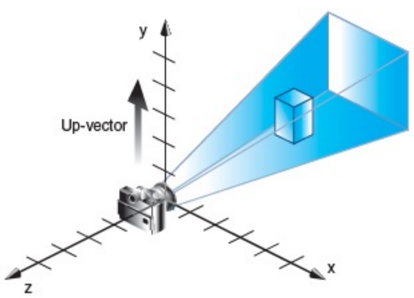
Common coordinates for the scene



7

CAMERA COORDINATES

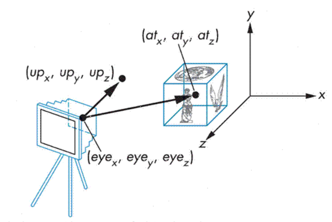
Coordinate system with the camera in a convenient pose



8

CAMERA COORDINATE

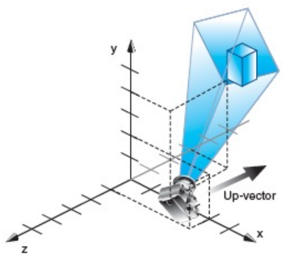
- **Viewing with gluLookAt()**
  - **looking at a scene from an arbitrary point of view.**
  - Takes 3 sets of arguments
    - specify the location of the viewpoint
    - define a reference point toward which the camera is aimed
    - indicate which direction is up.

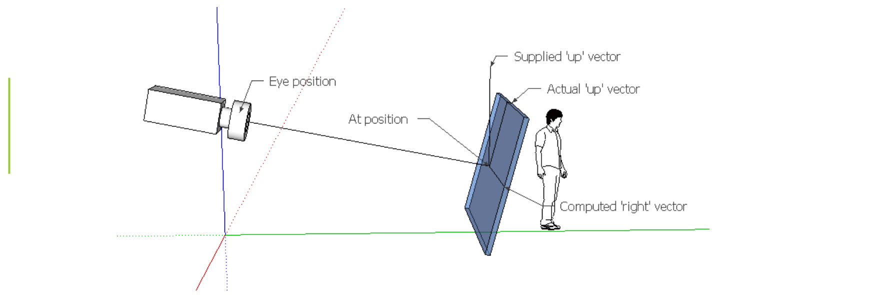



```
gluLookAt(GLdouble eyeX, GLdouble eyeY, GLdouble eyeZ,
          GLdouble atX, GLdouble atY, GLdouble atZ,
          GLdouble upX, GLdouble upY, GLdouble upZ);
```


CAMERA COORDINATES

- The effect of a gluLookAt()
  - gluLookAt(4.0, 2.0, 1.0, 2.0, 4.0, -3.0, 2.0, 2.0, -1.0)
  - The camera position (eyex, eyey, eyez) is at (4, 2, 1).
  - It looking at the model, so the reference point is at (2, 4, -3)
  - An orientation vector of (2, 2, -1) is chosen to rotate the viewpoint to this 45-degree angle.





The blue window can be thought of as the 'near-plane' that your imagery is drawn on (your monitor).

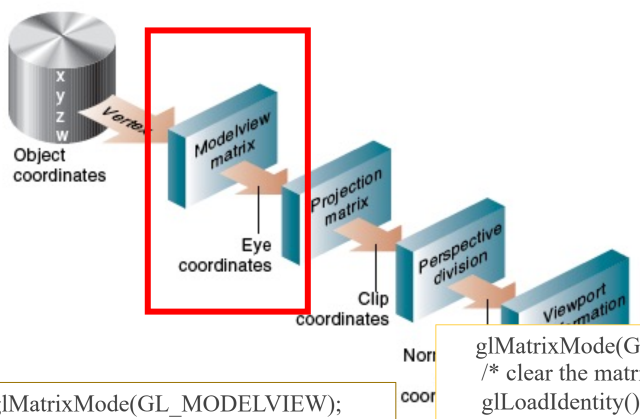
-If all you supply is the eye-point and the at-point, that window is free to spin around.

- You need to give an extra 'up' direction to pin it down.

-OpenGL will project the ‘up’ vector down, so that it forms a 90 degree angle with the 'z' vector defined by *eye* and *at*

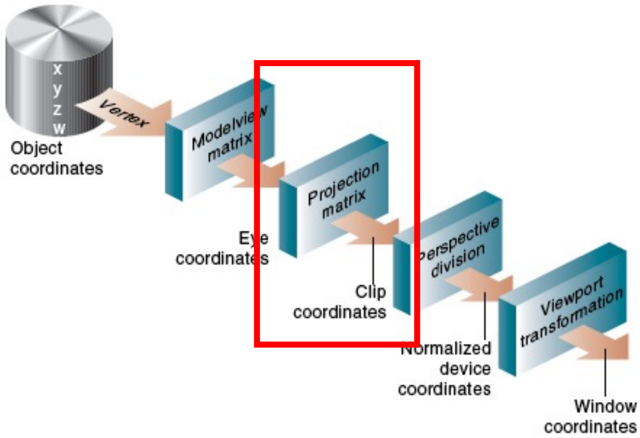
-Once 'in' (*z*) and 'up' (*y*) directions are defined, it's easy to calculate the 'right' or (*x*) direction from those two

In this figure, the 'supplied' up vector is (0,1,0) if the blue axis is in the y direction. If you were to give (1,1,1), it would most likely rotate the image by 45 degrees because that's saying that the top of the blue window should be pointed toward that direction.



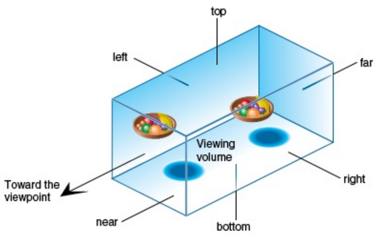

```
glMatrixMode(GL_MODELVIEW);
glLoadIdentity();
/* apply transformation */
glMultMatrixf(N);
glMultMatrixf(M);
glMultMatrixf(L);
glBegin(GL_POINTS);
glVertex3f(v);
glEnd();
```

```
glMatrixMode(GL_MODELVIEW);
/* clear the matrix */
glLoadIdentity();
/* viewing transformation */
gluLookAt(0.0, 0.0, 5.0,
          0.0, 0.0, 0.0,
          0.0, 1.0, 0.0);
/* modeling transformation */
glScalef(1.0, 2.0, 1.0);
glRotatef(45.0, 0.0,0.0,1.0);
glutWireCube(1.0);
```



ORTHOGRAPHIC PROJECTION

glOrtho(GLdouble left, GLdouble right,
GLdouble bottom, GLdouble top,
GLdouble near, GLdouble far);

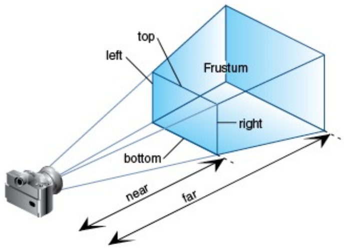


- Both near and far may be positive, negative, or even set to zero.
  - These distances are negative if the plane is to be behind the viewer
- near and far should not be the same value.

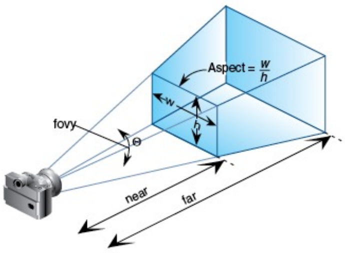
PERSPECTIVE PROJECTION

- Creates a matrix for a perspective-view frustum and multiplies the current matrix by it
- near and far:
  - the distances from the viewpoint to the near and far clipping planes.
  - They should always be positive.

void glFrustum(GLdouble left, GLdouble right,
GLdouble bottom, GLdouble top,
GLdouble near, GLdouble far);



PERSPECTIVE PROJECTION



gluPerspective(GLdouble fovy,
GLdouble aspect,
GLdouble near,
GLdouble far);

gluPerspective(60.0, 1.0, 1.5, 20.0)

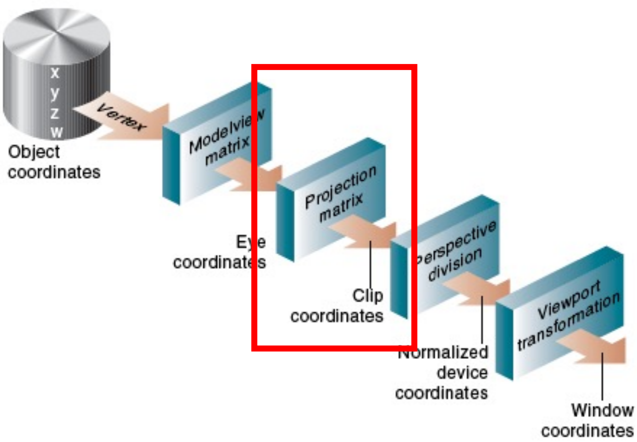
- the angle of the field of view (θ) in y-direction (y-z plane)
  - Range: (0.0~180.0)
- the aspect ratio of the width to the height (w/h)
- the distance between the viewpoint and the near and far clipping planes

→ creates a viewing volume of the same shape as glFrustum() does

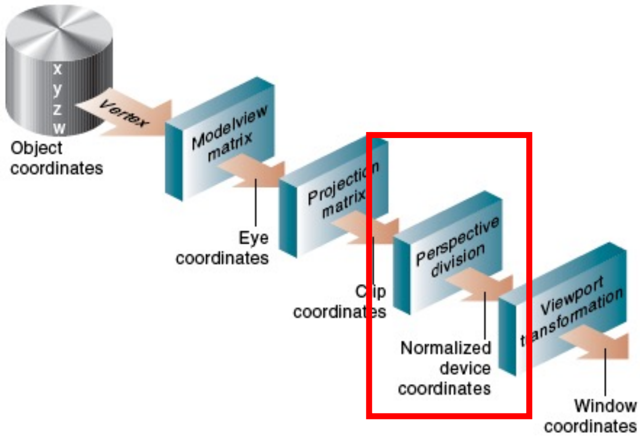
PERSPECTIVE PROJECTION

Projection Matrix

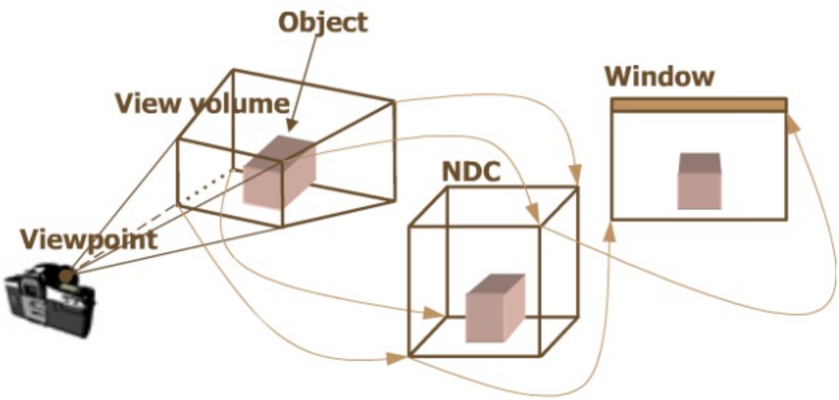
$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2\,far\cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$




```
glMatrixMode(GL_PROJECTION);
glLoadIdentity();
glFrustum(-1.0, 1.0,   // left, right
          -1.0, 1.0,   // bottom, top
           1.5, 20.0); // near, far
```

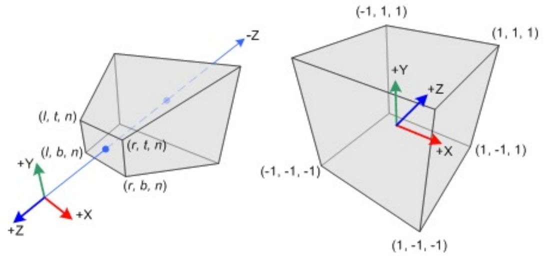

x
y
z
w
Vertex
Object
coordinates
Modelview
matrix
Eye
coordinates
Projection
matrix
Clip
coordinates
Perspective
division
Normalized
device
coordinates
Viewport
transformation
Window
coordinates

OVERVIEW


Object
View volume
Window
NDC
Viewpoint

PERSPECTIVE PROJECTION

- In perspective projection, a 3D point in a truncated pyramid frustum (eye coordinates) is mapped to a cube (NDC)
  - the range of x-coordinate from [left, right] to [-1, 1]
  - the y-coordinate from [bottom, top] to [-1, 1]
  - the z-coordinate from [-near, -far] to [-1, 1]

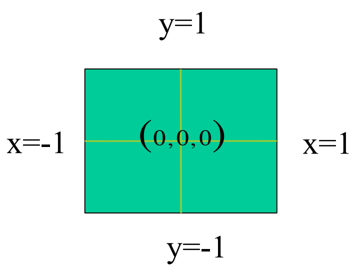


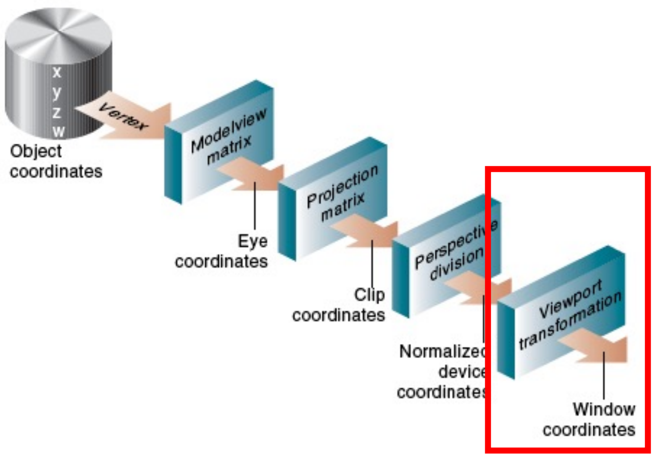
NORMALIZED DEVICE COORDINATES

Device independent coordinates

Visible coordinate usually range from:

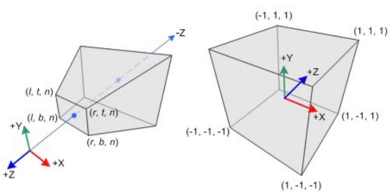
$$-1 \le x \le 1$$
$$-1 \le y \le 1$$
$$-1 \le z \le 1$$

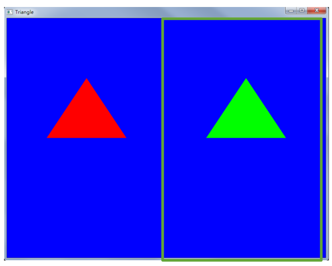




//gluPerspective(fovy, 1.0, near, far);
glViewport(0, 0, 400, 200);

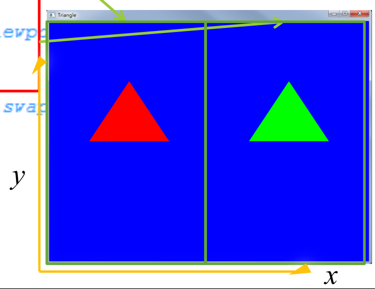
WINDOW COORDINATE





SAMPLE VIEW—MULTIVIEWPORT

```
void RenderScene(void)
{// Clear the window with current clearing color
    glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT | GL_STENCIL_BUFFER_BIT);
    glLoadIdentity();
    glViewport(0,0,width/2,height);//viewport1
    glColor3fv(vColor[0]);
    DrawTriangle();
    glLoadIdentity();
    glViewport(width/2,0,width/2,height);//viewp
    glColor3fv(vColor[1]);
    DrawTriangle();
    glutSwapBuffers(); // Perform the buffer swap
}
```



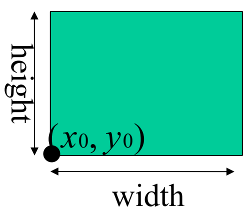
WINDOW COORDINATES

Adjusting the NDC to fit the window:

(x_0, y_0) is the lower left of the window

$$x_w = (x_{nd} + 1)\left(\frac{width}{2}\right) + x_0$$

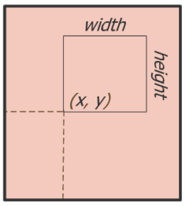
$$y_w = (y_{nd} + 1)\left(\frac{height}{2}\right) + y_0$$

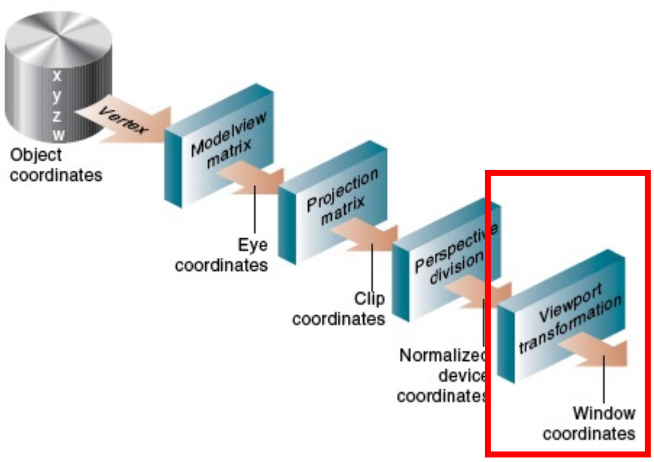


X_w: window coordinate,
X_{nd}: normalized device coordinate, $-1 \le x_{nd} \le 1$

VIEWPORT TRANSFORMATIONS

- 將投影轉換後得到的二維影像圖對應到螢幕上呈現的某個視窗中的位置(視窗坐標軸)。
- 此轉換為轉換到視窗上的最後一次轉換。
- glViewport(x, y, w, h);





//gluPerspective(fovy, 1.0, near, far);
glViewport(0, 0, 400, 200);

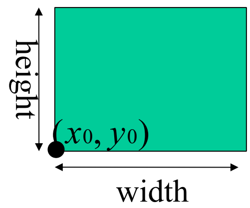
WINDOW COORDINATES

Adjusting the NDC to fit the window:

(x_0, y_0) is the lower left of the window

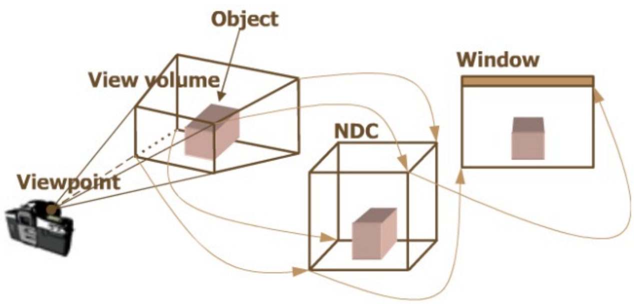
$$x_w = (x_{nd} + 1)\left(\frac{width}{2}\right) + x_0$$

$$y_w = (y_{nd} + 1)\left(\frac{height}{2}\right) + y_0$$



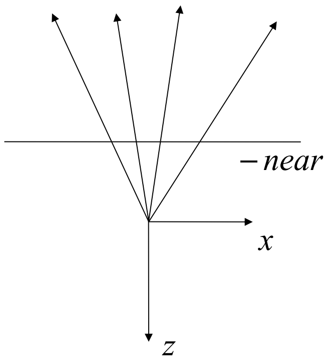
X_w: window coordinate,
X_{nd}: normalized device coordinate, $-1 \le x_{nd} \le 1$

Back to Projection…



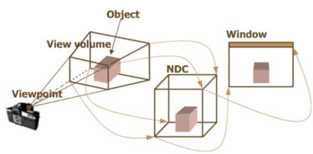
PERSPECTIVE PROJECTION

Taking the camera coordinates to NDC



PERSPECTIVE PROJECTION

Taking the camera coordinates to **NDC**

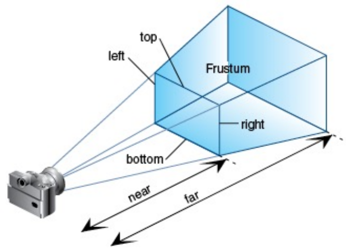


$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2\,far\cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

PERSPECTIVE PROJECTION

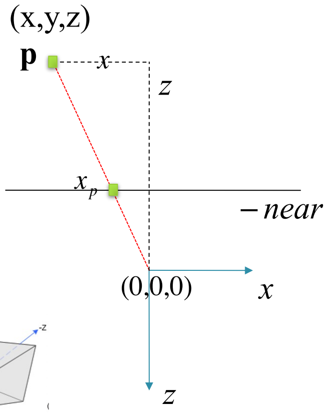
- Creates a matrix for a perspective-view frustum and multiplies the current matrix by it
- near and far:
  - the distances from the viewpoint to the near and far clipping planes.
  - They should always be positive.

void glFrustum(GLdouble left, GLdouble right,
GLdouble bottom, GLdouble top,
GLdouble near, GLdouble far);



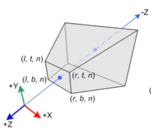
void glFrustum(GLdouble left, GLdouble right,
GLdouble bottom, GLdouble top,
GLdouble near, GLdouble far);

PERSPECTIVE PROJECTION



$$\frac{x_p}{-near} = \frac{x}{z}$$

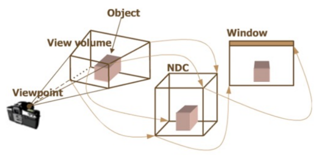
$$x_p = -near\frac{x}{z}$$



near is positive
→ -near is the coordinate

x_p：P點投影至-near平面之座標 (未轉換至NDC座標)

PERSPECTIVE PROJECTION



Method1:

$$x_p = -near\frac{x}{z}$$ ← x投影至-near平面之座標 (未轉換至NDC座標)

轉換至NDC 座標:

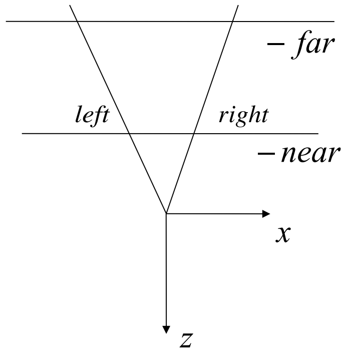
Map (left,right) to (-1,1) when z = -near

$$-near\frac{x}{z} - left$$

$$\frac{2}{right-left}\left(-near\frac{x}{z} - left\right)$$

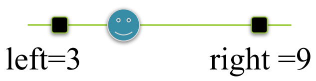
$$\frac{2}{right-left}\left(-near\frac{x}{z} - left\right) - 1$$

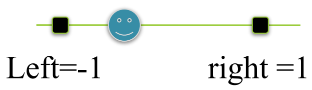
$$\Rightarrow \frac{-2near}{right-left}\frac{x}{z} - \frac{right+left}{right-left}$$ ← x投影至-near平面,並轉換至NDC座標



- A simple mapping example:

$$\frac{2}{right-left}\left(-near\frac{x}{z} - left\right) - 1$$





x’=5

x'_{ndc}=?

$$(5-3)\frac{1-(-1)}{9-3} + (-1)$$

$$= \frac{2}{9-3}(5-3) - 1$$

Method2:

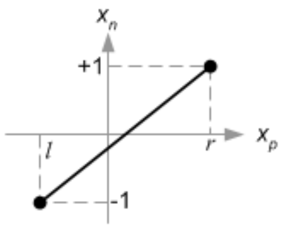
PERSPECTIVE PROJECTION

Taking the camera coordinates to NDC

Map (left,right) to (-1,1)

$$x_p = -near\frac{x}{z}$$

$$x_n = \frac{2}{r-l}x_p - \frac{r+l}{r-l}$$



Mapping from x_p to x_n

$$\begin{aligned} x_n &= \frac{2}{r-l}(-near\frac{x}{z}) - \frac{r+l}{r-l} \\ &= \frac{-2near}{r-l}(\frac{x}{z}) - \frac{r+l}{r-l} \\ &= (\frac{2near}{r-l}x + \frac{r+l}{r-l}z)/(-z) \end{aligned}$$

x_p : x_e is projected to x_p

x_n : linearly mapped x_p to NDC

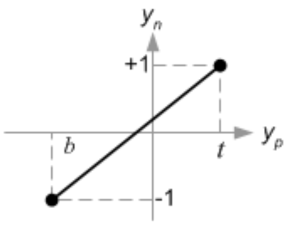
PERSPECTIVE PROJECTION

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ & & & \\ & & & \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$x_n = (\frac{2near}{r-l}x + \frac{r+l}{r-l}z)/(-z) = \frac{x_c}{w_c} \qquad y_n = (\frac{2near}{t-b}y + \frac{t+b}{t-b}z)/(-z) = \frac{y_c}{w_c}$$

The eye coordinates are transformed by multiplying PROJECTION matrix.
The clip coordinates are still a homogeneous coordinates.
It finally becomes the normalized device coordinates (NDC) by divided by the w-component of the clip coordinates

PERSPECTIVE PROJECTION



Mapping from y_p to y_n

Taking the camera coordinates to NDC

Map (bottom,top) to (-1,1)

$$y_n = ?$$

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ & & & \\ & & & \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

x_p : x_e is projected to x_p

x_n : linearly mapped x_p to NDC

PERSPECTIVE PROJECTION

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ & & & \\ & & & \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$x_n = \left(\frac{2near}{r-l}x + \frac{r+l}{r-l}z\right)/(-z) = \frac{x_c}{w_c}$$

The eye coordinates are transformed by multiplying PROJECTION matrix.
The clip coordinates are still a homogeneous coordinates.
It finally becomes the normalized device coordinates (NDC) by divided by the w-component of the clip coordinates

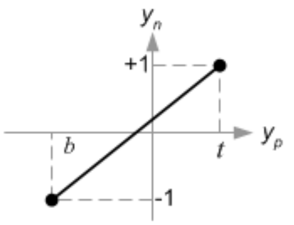
PERSPECTIVE PROJECTION

Taking the camera coordinates to NDC

Map (bottom,top) to (-1,1)

$$y_p = -near\frac{y}{z}$$

$$y_n = \frac{2}{t-b}y_p - \frac{t+b}{t-b}$$



Mapping from y_p to y_n

$$\begin{aligned} y_n &= \frac{2}{t-b}(-near\frac{y}{z}) - \frac{t+b}{t-b} \\ &= \frac{-2near}{t-b}(\frac{y}{z}) - \frac{t+b}{t-b} \\ &= (\frac{2near}{t-b}y + \frac{t+b}{t-b}z)/(-z) \end{aligned}$$

x_p : x_e is projected to x_p

x_n : linearly mapped x_p to NDC

PERSPECTIVE PROJECTION

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ ? & ? & ? & ? \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$x_n = (\frac{2near}{r-l}x + \frac{r+l}{r-l}z)/(-z) = \frac{x_c}{w_c} \qquad y_n = (\frac{2near}{t-b}y + \frac{t+b}{t-b}z)/(-z) = \frac{y_c}{w_c}$$

The eye coordinates are transformed by multiplying PROJECTION matrix.
The clip coordinates are still a homogeneous coordinates.
It finally becomes the normalized device coordinates (NDC) by divided by the w-component of the clip coordinates

HOMOGENEOUS COORDINATES

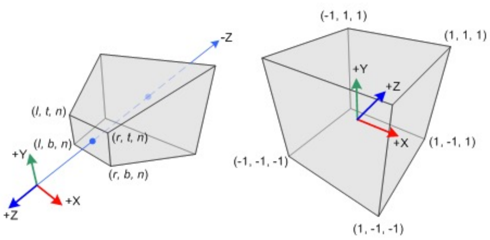
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ can be represented as } \begin{bmatrix} X \\ Y \\ Z \\ w \end{bmatrix} \qquad \text{e.g., } \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

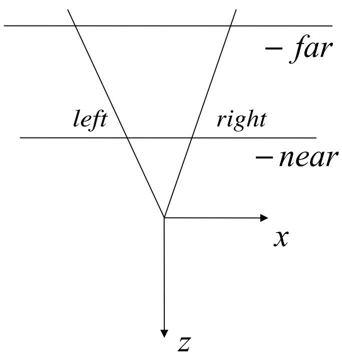
where $x = \dfrac{X}{w}, \quad y = \dfrac{Y}{w}, \quad z = \dfrac{Z}{w}$

PSEUDO DEPTH

Finding z':

Map (-near,-far) to (-1,1)





Finding z_n is a little different!
Because z_e in eye space is always projected to -near on the near plane.
But we need unique z value for the clipping and depth test.

PERSPECTIVE PROJECTION

- we don't want z **depend on x or y value**

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & ? & ? \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

- we borrow w-component to find the relationship between z_n , z_e
  → specify the 3rd row of Perspective Projection matrix like the following:

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & A & B \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

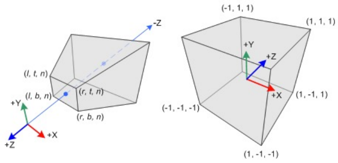
Reference: http://www.songho.ca/opengl/gl_projectionmatrix.html

PERSPECTIVE PROJECTION

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & A & B \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$z_n = \frac{z_c}{w_c} = (Az + Bw)/(-z)$$ …in eye space, w equals to 1.

⇨ $$z_n = (Az + B)/(-z)$$



- To find the coefficients, *A and B*

$$z_n = (Az + B)/(-z)$$

- *substitute the* (z, z_n) *to* (-near, -1) *and* (-far, 1)

$$\begin{cases} \dfrac{-An+B}{n} = -1 \\ \dfrac{-Af+B}{f} = 1 \end{cases} \rightarrow \begin{cases} -An+B=-n & (1) \\ -Af+B=f & (2) \end{cases}$$

$$\begin{cases} -An+B=-n & (1) \\ -Af+B=f & (2) \end{cases}$$

- To solve the equations for *A and B,*
- *rewrite eq.(1) for B*

  → *B = An-n* -----(1')
- *Substitute eq.(1') to B in eq.(2), then solve for A*

→-*Af +An-n = f* -----(1')

→*A = (f+n)/(n-f) = - (f+n)/(f-n)*

- *Put A into eq.(1) to find B*

→*n(f+n)/(f-n) +B = -n*

→*B = -n-n(f+n)/(f-n)*

→*B = -2fn/(f-n)*

$$z_n = (Az + B)/(-z)$$

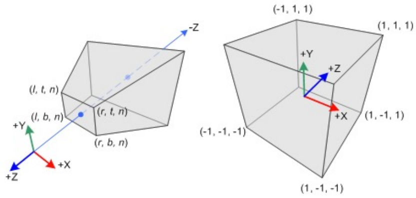
$$z_n = \left(\frac{-(f+n)}{f-n}z + \frac{-2fn}{f-n}\right)/(-z)$$

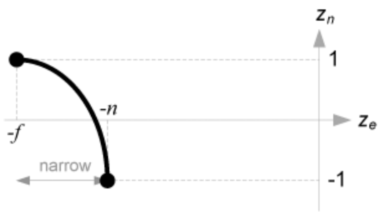
PSEUDO DEPTH

Finding z':

Map (-near,-far) to (-1,1)

$$z_n = \left(\frac{-(f+n)}{f-n} z + \frac{-2fn}{f-n}\right) / (-z)$$





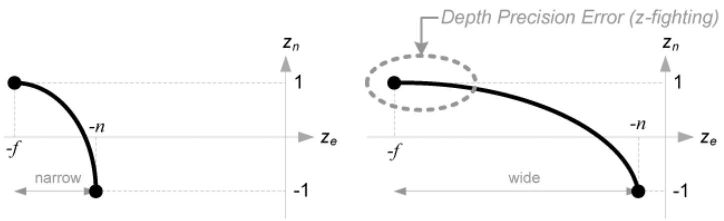
PERSPECTIVE PROJECTION

$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2far \cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}$$

$$z_n = (Az + B) / (-z)$$

$$z_n = \left(\frac{-(f+n)}{f-n} z + \frac{-2fn}{f-n}\right) / (-z)$$

- There is very high precision at the *near* plane, but very little precision at the *far* plane.
  - If the range [-n, -f] is getting larger, it causes a depth precision problem (z-fighting)
  - a small change of z_e around the *far* plane does not affect on z_n value.
  - The distance between *n* and *f* should be short as possible to minimize the depth buffer precision problem.



PERSPECTIVE PROJECTION

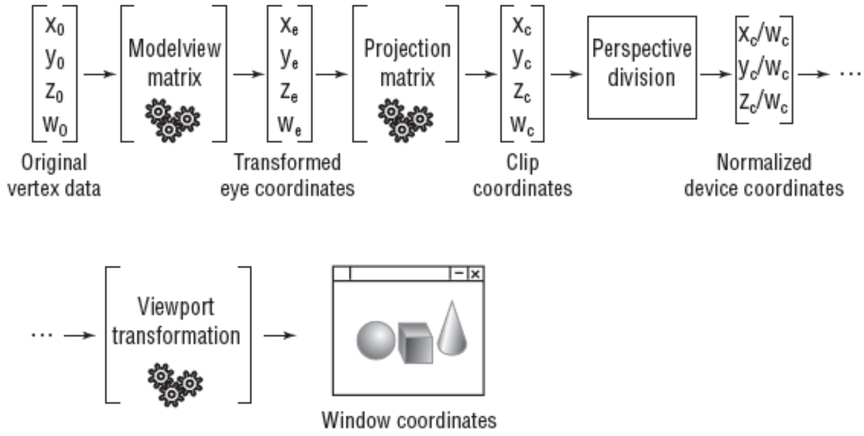
$$\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} = \begin{bmatrix} \frac{2near}{right-left} & 0 & \frac{right+left}{right-left} & 0 \\ 0 & \frac{2near}{top-bottom} & \frac{top+bottom}{top-bottom} & 0 \\ 0 & 0 & -\frac{far+near}{far-near} & -\frac{2\,far\cdot near}{far-near} \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \\ w_e \end{bmatrix}$$

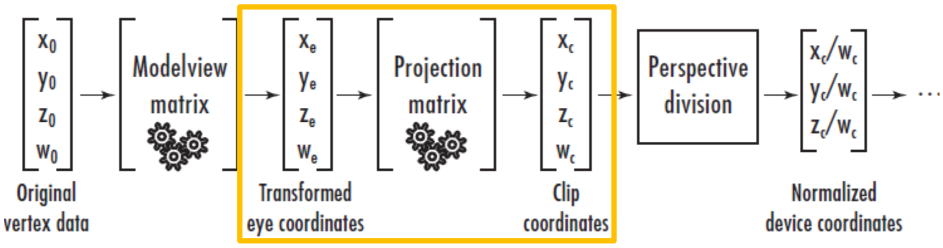
$$x_n = \frac{x_c}{w_c} = (\frac{2near}{r-l}x + \frac{r+l}{r-l}z)/(-z)$$

$$y_n = \frac{y_c}{w_c} = (\frac{2near}{t-b}y + \frac{t+b}{t-b}z)/(-z)$$

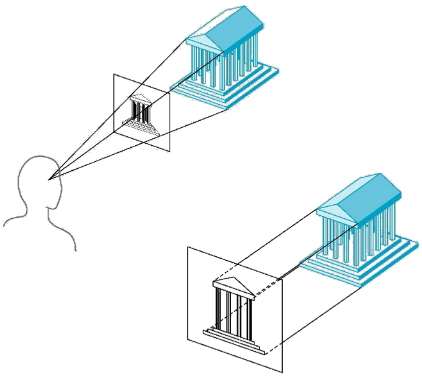
$$z_n = \frac{z_c}{w_c} = (\frac{-(f+n)}{f-n}z + \frac{-2fn}{f-n})/(-z)$$

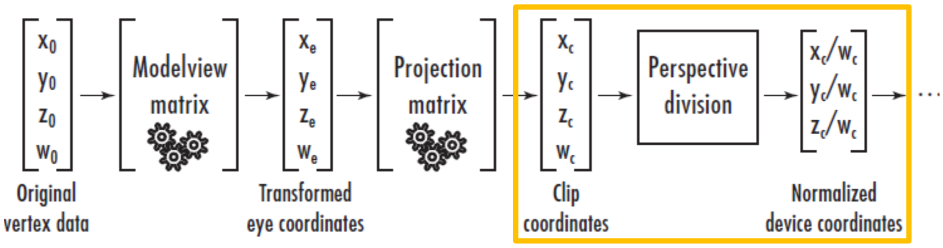
TRANSFORMATION PIPELINE





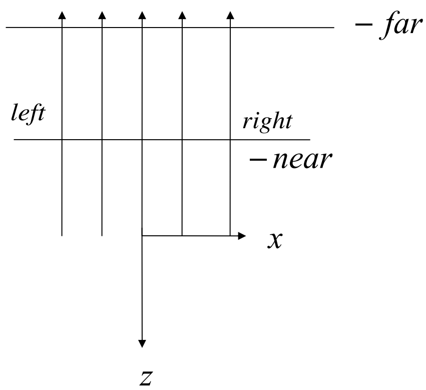
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix}$$





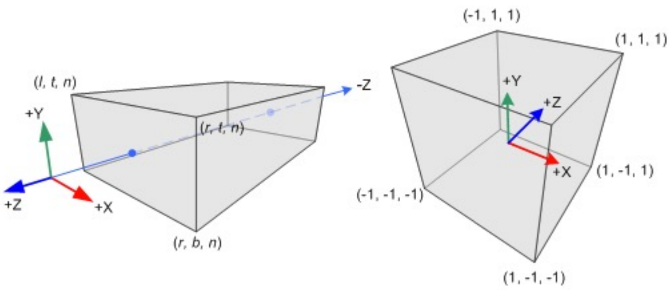
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} \Rightarrow \begin{bmatrix} x_c/w_c \\ y_c/w_c \\ z_c/w_c \\ w_c/w_c \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

ORTHOGRAPHIC PROJECTION



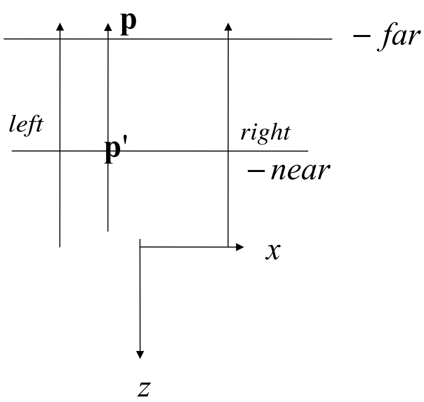
ORTHOGRAPHIC PROJECTION

- Constructing Orthographic Projection matrix for orthographic projection is much simpler!
  - We just need to scale a rectangular volume to a cube, then move it to the origin.



Orthographic Volume and Normalized Device Coordinates (NDC)

ORTHOGRAPHIC PROJECTION



$$x' = x$$

ORTHOGRAPHIC PROJECTION

Taking the camera coordinates to NDC

Map (left,right) to (-1,1)

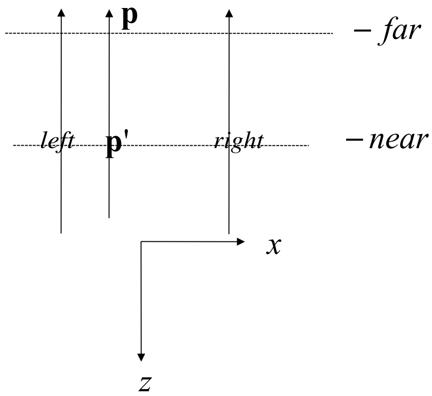
$$x' = x$$

$$x - left$$

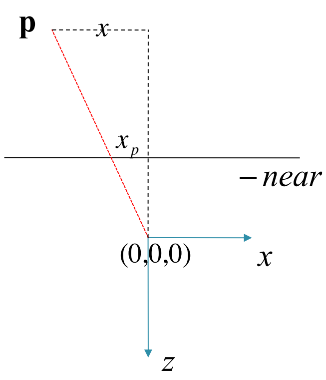
$$\frac{2}{right - left}(x - left)$$

$$\frac{2}{right - left}(x - left) - 1$$

$$\frac{2}{right - left}x - \frac{right + left}{right - left}$$



PERSPECTIVE PROJECTION



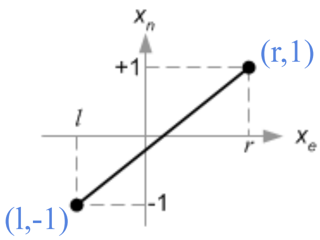
$$\frac{x_p}{-near} = \frac{x}{z}$$

$$x_p = -near\frac{x}{z}$$

ORTHOGRAPHIC PROJECTION

Taking the camera coordinates to NDC

Map (left,right) to (-1,1)



Mapping from x_e to x_n

$$x_p = x_e$$

$$x_n = \frac{1-(-1)}{r-l}x_e + \beta \quad \text{, 代入 } x_e = r,\ x_n = 1$$

$$1 = \frac{1-(-1)}{r-l}r + \beta$$

$$\beta = 1 - \frac{2}{r-l}r = \frac{r-l-2r}{r-l} = -\frac{r+l}{r-l}$$

$$x_n = \frac{2}{r-l}x_e - \frac{r+l}{r-l}$$

$$x_n = \frac{2}{r-l}x - \frac{r+l}{r-l}$$

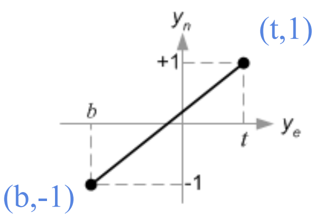
x_e : x in eye space

x_n : linearly mapped x_e to NDC

ORTHOGRAPHIC PROJECTION

Taking the camera coordinates to NDC

Map (bottom,top) to (-1,1)



Mapping from y_e to y_n

$$y_p = y_e$$

$$y_n = \frac{1-(-1)}{t-b}y_e + \beta \quad \text{, 代入 } y_e = t,\ y_n = 1$$

$$1 = \frac{1-(-1)}{t-b}t + \beta$$

$$\beta = 1 - \frac{2}{t-b}t = \frac{t-b-2t}{t-b} = -\frac{t+b}{t-b}$$

$$y_n = \frac{2}{t-b}y_e - \frac{t+b}{t-b}$$

$$y_n = \frac{2}{t-b}y - \frac{t+b}{t-b}$$

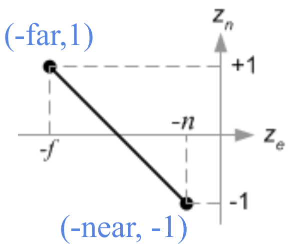
y_e : y in eye space

y_n : linearly mapped y_e to NDC

ORTHOGRAPHIC PROJECTION

Taking the camera coordinates to NDC

Map (-near,-far) to (-1,1)



Mapping from z_e to z_n

$$z_n = \frac{1-(-1)}{-f-(-n)} z_e + \beta$$

$$1 = \frac{2}{-f+n}(-f) + \beta \quad \text{，代入 } z_e = -f,\ z_n = 1$$

$$\beta = 1 - \frac{2}{f-n} f = \frac{f-n-2f}{f-n} = -\frac{f+n}{f-n}$$

$$z_n = \frac{-2}{f-n} z_e - \frac{f+n}{f-n}$$

$$z_n = \frac{-2}{f-n} z - \frac{f+n}{f-n}$$

z_e : z in eye space

z_n : linearly mapped z_e to NDC

ORTHOGRAPHIC PROJECTION

Orthographic projection matrix:

$$P = \begin{bmatrix} \frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\ 0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\ 0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PERSPECTIVE PROJECTION

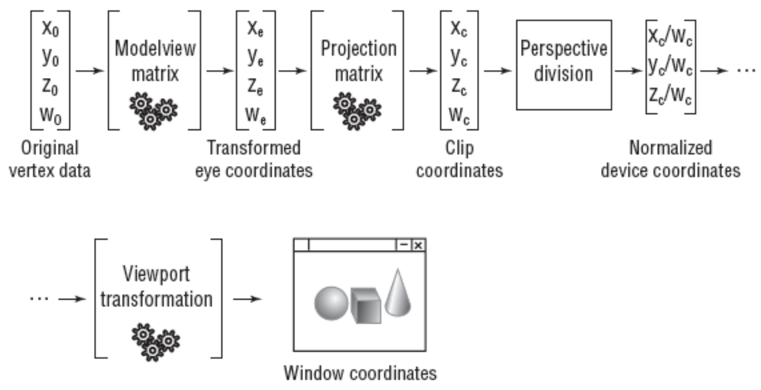
$$
\begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} =
\begin{bmatrix}
\frac{2}{right-left} & 0 & 0 & -\frac{right+left}{right-left} \\
0 & \frac{2}{top-bottom} & 0 & -\frac{top+bottom}{top-bottom} \\
0 & 0 & \frac{-2}{far-near} & -\frac{far+near}{far-near} \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix}
$$

$$x_n = \frac{x_c}{w_c} = \frac{2}{right-left}x - \frac{right+left}{right-left}$$

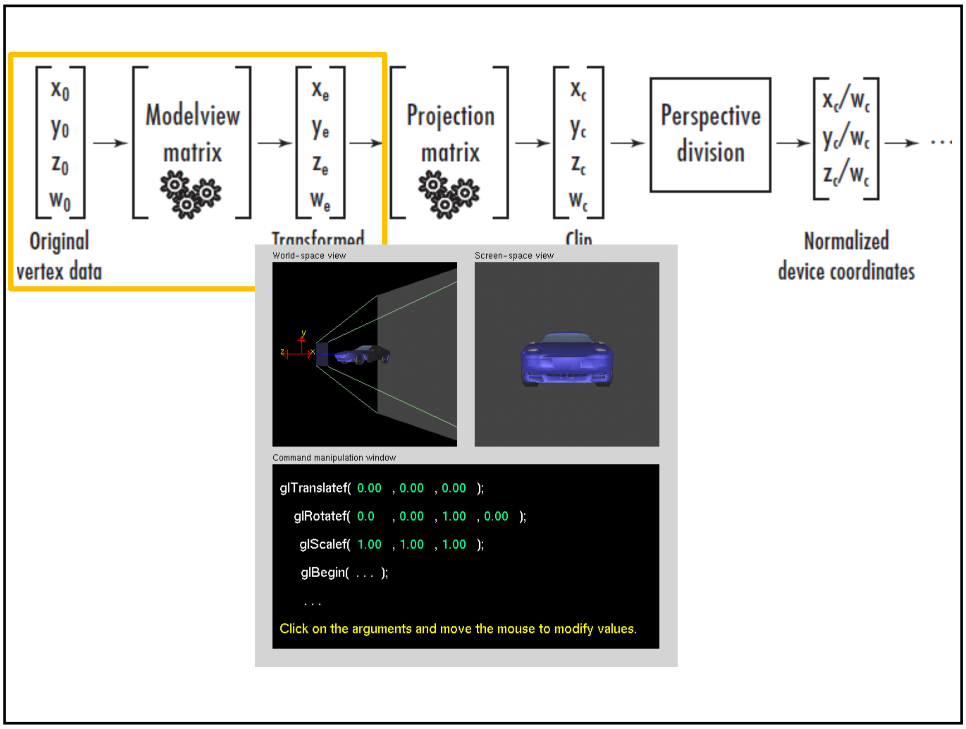
$$y_n = \frac{y_c}{w_c} = \frac{2}{top-bottom}y - \frac{top+bottom}{top-botton}$$

$$z_n = \frac{z_c}{w_c} = \frac{-2}{far-near}z - \frac{far+near}{far-near}$$

REVIEW



Transformation pipeline



Modelview matrix
Projection matrix
Perspective division
Original vertex data
Transformed
Clip
Normalized device coordinates
World-space view
Screen-space view
Command manipulation window
glTranslatef(0.00 , 0.00 , 0.00);
glRotatef(0.0 , 0.00 , 1.00 , 0.00);
glScalef(1.00 , 1.00 , 1.00);
glBegin(. . .);
. . .
Click on the arguments and move the mouse to modify values.

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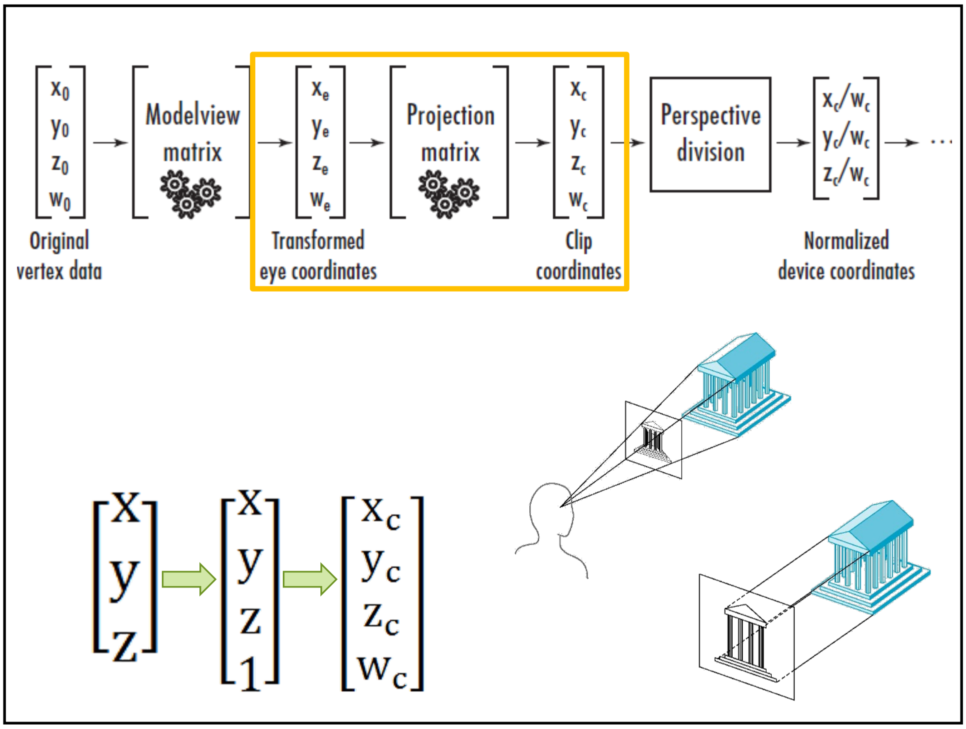

Modelview matrix
Projection matrix
Perspective division
Original vertex data
Transformed eye coordinates
Clip coordinates
Normalized device coordinates

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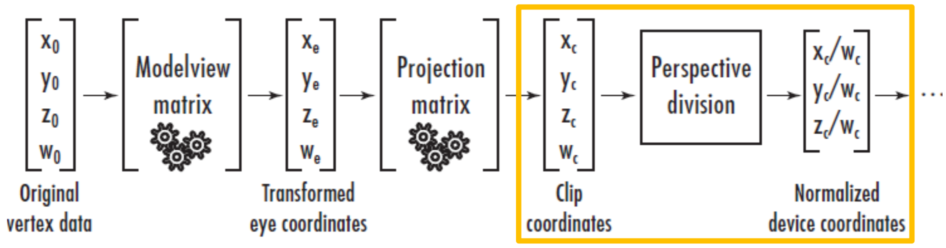
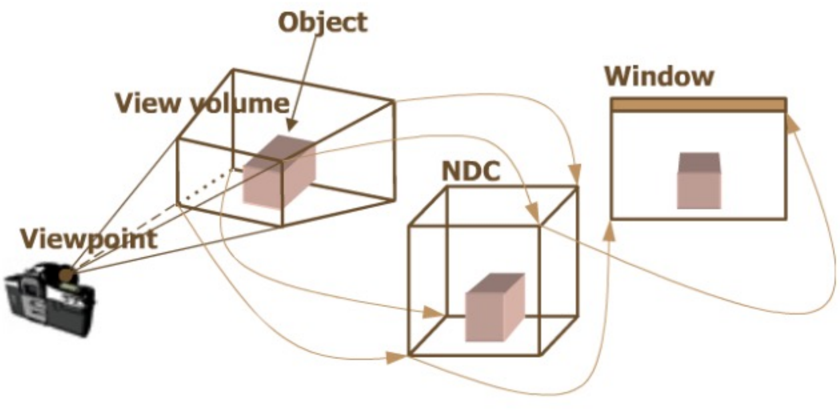


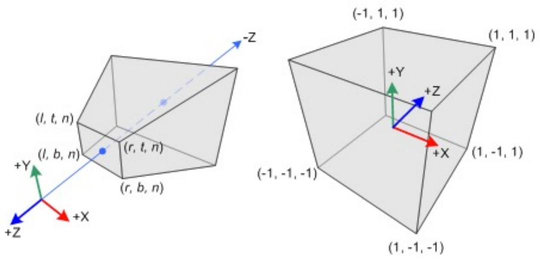
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x_c \\ y_c \\ z_c \\ w_c \end{bmatrix} \Rightarrow \begin{bmatrix} x_c/w_c \\ y_c/w_c \\ z_c/w_c \\ w_c/w_c \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix}$$

OVERVIEW



PERSPECTIVE PROJECTION

- In perspective projection, a 3D point in a truncated pyramid frustum (eye coordinates) is mapped to a cube (NDC)
  - the range of x-coordinate from [left, right] to [-1, 1]
  - the y-coordinate from [bottom, top] to [-1, 1]
  - the z-coordinate from [-near, -far] to [-1, 1]



NORMALIZED DEVICE COORDINATES

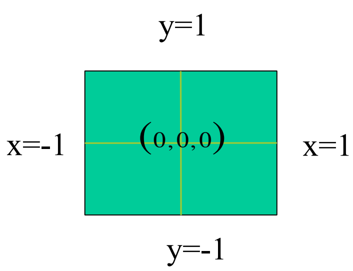
Device independent coordinates

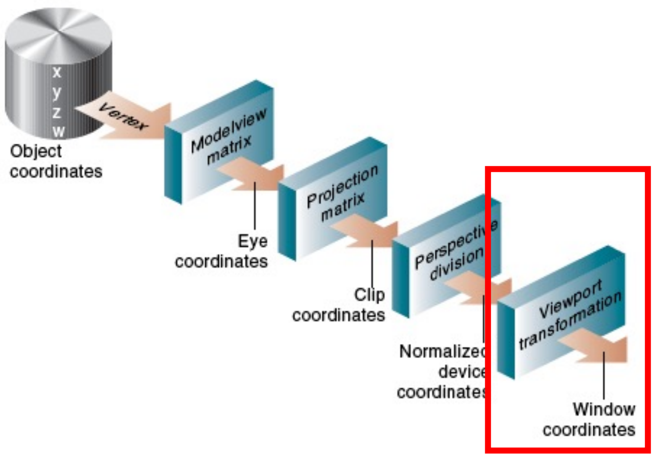
Visible coordinate usually range from:

$$-1 \le x \le 1$$

$$-1 \le y \le 1$$

$$-1 \le z \le 1$$






```
//gluPerspective(fovy, 1.0, near, far);
glViewport(0, 0, 400, 200);
```

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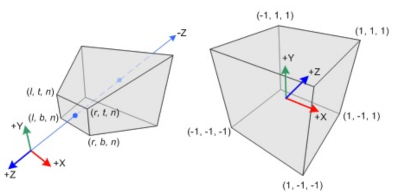
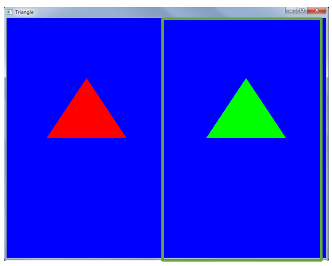
WINDOW COORDINATE





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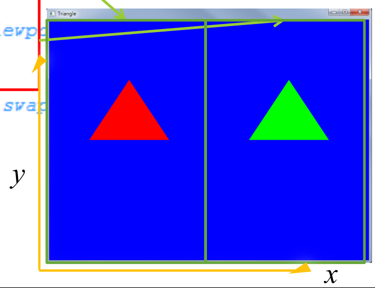
SAMPLE VIEW—MULTIVIEWPORT

```
void RenderScene(void)
{// Clear the window with current clearing color
    glClear(GL_COLOR_BUFFER_BIT | GL_DEPTH_BUFFER_BIT | GL_STENCIL_BUFFER_BIT);
    glLoadIdentity();
    glViewport(0,0,width/2,height);//viewport1
    glColor3fv(vColor[0]);
    DrawTriangle();
    glLoadIdentity();
    glViewport(width/2,0,width/2,height);//viewp
    glColor3fv(vColor[1]);
    DrawTriangle();
    glutSwapBuffers(); // Perform the buffer swap
}
```



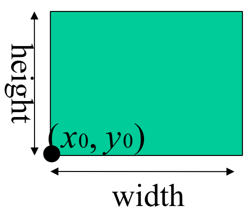
WINDOW COORDINATES

Adjusting the NDC to fit the window:

(x_0, y_0) is the lower left of the window

$$x_w = (x_{nd} + 1)\left(\frac{width}{2}\right) + x_0$$

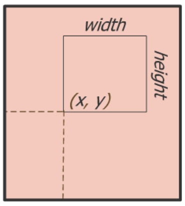
$$y_w = (y_{nd} + 1)\left(\frac{height}{2}\right) + y_0$$

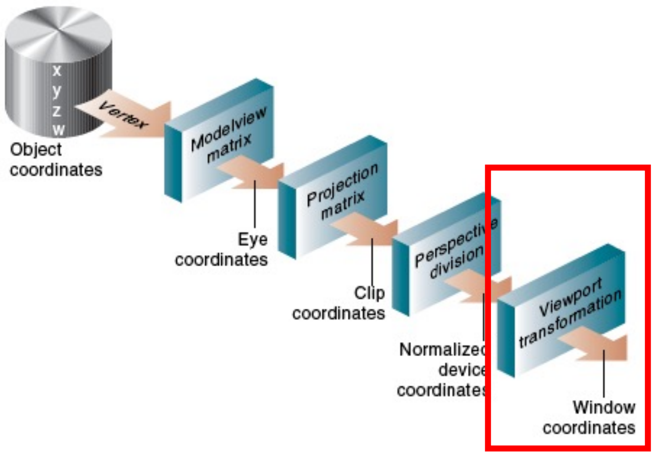


X_w: window coordinate,
X_{nd}: normalized device coordinate, $-1 \le x_{nd} \le 1$

VIEWPORT TRANSFORMATIONS

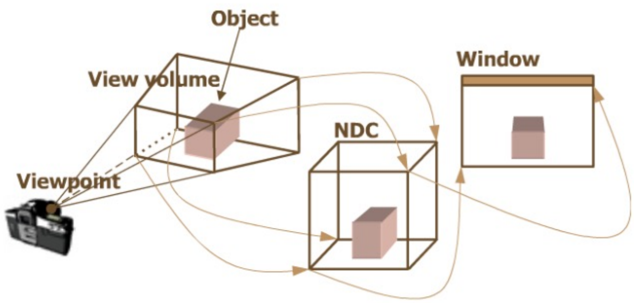
- 將投影轉換後得到的二維影像圖對應到螢幕上呈現的某個視窗中的位置(視窗坐標軸)。
- 此轉換為轉換到視窗上的最後一次轉換。
- glViewport(x, y, w, h);





//gluPerspective(fovy, 1.0, near, far);
glViewport(0, 0, 400, 200);

Back to Projection…



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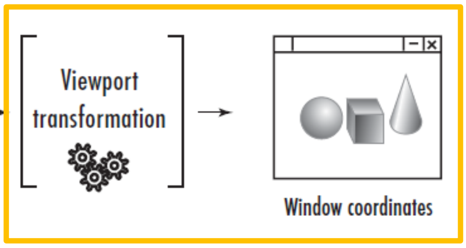

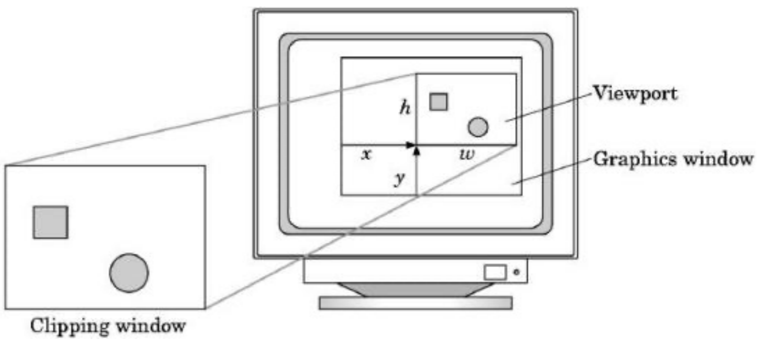

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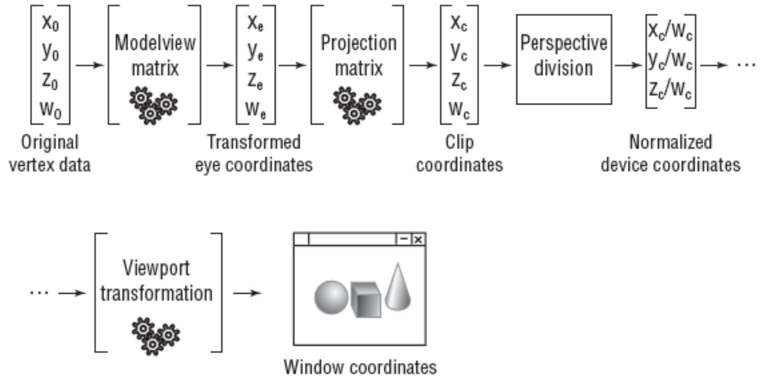
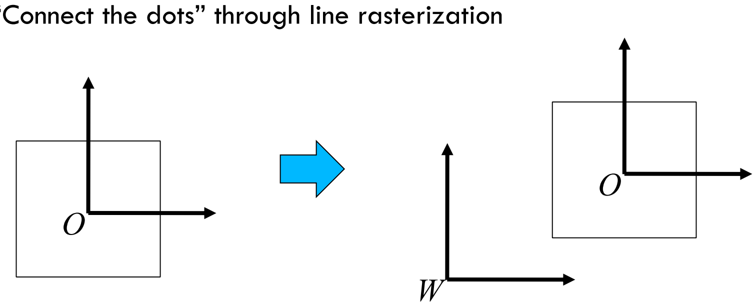
TRANSFORMATION PIPELINE



PUTTING IT ALL TOGETHER!!

- Take your representation (points) and transform it from Object Space to World Space
- Take your World Space point and transform it to Camera Space
- Perform the remapping and projection onto the image plane in Normalized Device Coordinates
- Perform this set of transformations on each point of the polygonal object
- “Connect the dots” through line rasterization



Object coordinates

World coordinates

INTUITIVELY