Regular expressions: derivations

Which of the following statements is true? If it is true, give a derivation; if not, explain.

1. $a \in \mathsf{L}(a + b)$

2. $ab \in \mathsf{L}((a + b))$

3. $ab \in \mathsf{L}((a + b)(a + b))$

4. $aa \in \mathsf{L}(a + a)$

5. $\varepsilon \in \mathsf{L}(b*)$

6. $b \in \mathsf{L}(b*)$

7. $bb \in \mathsf{L}(b*)$

8. $\in \mathsf{L}()$

Regular expressions: properties

Two regular expressions r and r' are equivalent if forall xs, $xs \in \mathsf{L}(r)$ if and only if $xs \in \mathsf{L}(r')$.

Prove the following regular expressions are equivalent, for all regular expression a, b, c.

1. a and $a + 0$

2. $a + a$ and a

3. $a + b$ and $b + a$

4. $a + (b + c)$ and $(a + b) + c$

5. $1a$ and a

6. $(a^*)^*$ and a^*

Evaluation of lambda terms

Given the following definitions:

$$I = \lambda x.x$$
$$K = \lambda xy.x$$
$$S = \lambda xyz.(xz)(yz)$$

Give a derivation of:

1. Ia

2. $KIab$

3. $(IK)(II)$

4. $S(K(Ka))(Kb)c$

The typed lambda calculus

Let Γ be an environment including:

- one : N

- isEven : $N-> B$

- not : $B-> B$

- add : $N-> N-> N$

Give typing derivations for the following terms:

1. isEven one

2. add one one

3. $\lambda x : B.\, \text{not}(\text{not}\ x)$

4. $\lambda x : N.\, \text{one}$

5. $\lambda x : N.\lambda y : N.\text{isEven}\ x$

6. $\lambda x : (N \to N).\text{not}$