

Regular expressions: derivations

Which of the following statements is true? If it is true, give a derivation; if not, explain.

1. $a \in L(a + b)$
2. $ab \in L((a + b))$
3. $ab \in L((a + b)(a + b))$
4. $aa \in L(a + a)$
5. $\varepsilon \in L(b^*)$
6. $b \in L(b^*)$
7. $bb \in L(b^*)$

Regular expressions: properties

Two regular expressions r and r' are equivalent if for all xs , $xs \in L(r)$ if and only if $xs \in L(r')$.

Prove the following regular expressions are equivalent, for all regular expression a, b, c .

Clearly state how the proof is constructed, either by using rule induction or applying rules. When using rule induction, state the cases and hypotheses available at every step:

1. a and $a + 0$
2. $a + a$ and a
3. $a + b$ and $b + a$
4. $a + (b + c)$ and $(a + b) + c$
5. $1a$ and a
6. $(a^*)^*$ and a^*

Evaluation of lambda terms

Given the following definitions:

$$\begin{aligned} I &= \lambda x.x \\ K &= \lambda xy.x \\ S &= \lambda xyz.(xz)(yz) \end{aligned}$$

Given a derivation of following terms to a normal form, using the rules presented in class:

1. Ia
2. $KIab$
3. $(IK)(II)$
4. $S(K(Ka))(Kb)c$

The typed lambda calculus

Let Γ be an environment including:

- $\text{one} : N$
- $\text{isEven} : N \rightarrow B$
- $\text{not} : B \rightarrow B$
- $\text{add} : N \rightarrow N \rightarrow N$

Give typing derivations for the following terms using the rules presented in class:

1. isEven one
2. add one one
3. $\lambda x : B. \text{not}(\text{not } x)$
4. $\lambda x : N. \text{one}$
5. $\lambda x : N. \lambda y : N. \text{isEven } x$
6. $\lambda x : (N \rightarrow N). \text{not}$