Static configurations

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$$\boxed{\begin{aligned} & I = I_c \sin\theta \\ & M(I - I_s) = 0 \qquad \leftarrow \text{KCL} \\ & A[\theta + LI] + 2\pi\Phi_e = 2\pi z \qquad \leftarrow \text{IKVL}(z) \end{aligned}}$$

Josephson Vortex defintion:

vortex configuration

$$A[pv(\theta) + LI] + 2\pi\Phi_e = 2\pi n$$

$$pv(x) \in [-\pi, \pi)$$

$$A[pv(\theta) - \theta] = 2\pi(n - z)$$

$$A \, \text{round}(\theta/2\pi) = n - z$$

Theorem:

if $z = n \rightarrow \theta = pv(\theta)$ (one-line proof)

Theorem:

For a loop with M junctions; $n \in \left[f - M\left(\frac{1}{2} + \frac{LI_c}{2\pi}\right), f + M\left(\frac{1}{2} + \frac{LI_c}{2\pi}\right)\right)$

London approximation

Assume $\sin\theta \approx \theta$, $z = n$

$$M(I_c\theta - I_s) = 0 \Rightarrow I_c\theta - I_s = A^T J \rightarrow \theta = I_c^{-1}(I_s + A^T J)$$

$$A[I_c^{-1}(I_s + A^T J) + L(I_s + A^T J)] = 2\pi(n - \Phi_e)$$

$$A[I_c^{-1} + L]A^T J = 2\pi(n - \Phi_e) - A[I_c^{-1} + L]I_s$$

$$J = (A[I_c^{-1} + L]A^T)^{-1}\left[2\pi(n - \Phi_e) - A[I_c^{-1} + L]I_s\right]$$

London approximation

$$\boxed{\theta = I_c^{-1}A^T(A[I_c^{-1} + L]A^T)^{-1}\left[2\pi(n - \Phi_e) - A[I_c^{-1} + L]I_s\right] + I_c^{-1}I_s}$$

Newton iteration:

$$\begin{cases} I = I_c \sin\theta \\ M(I - I_s) = 0 \\ A[\theta + LI] + 2\pi\Phi_e = 2\pi z \end{cases}$$

$$F(\theta) = \begin{bmatrix} M(I_c \sin\theta - I_s) \\ A(\theta + LI_c \sin\theta) + 2\pi(\Phi_e - z) \end{bmatrix} \approx 0$$

$$J = \nabla_\theta F = \begin{bmatrix} MI_c \cos\theta \\ A(I + LI_c \cos\theta) \end{bmatrix}$$

Newton $\rightarrow$ $\theta_{n+1} = \theta_n - J(\theta_n)^{-1}F(\theta_n)$

Solution $\left\{ y_n = \cos\theta_n^{-1}(\sin\theta_n - I_c^{-1}I_s) \right.$

properties:

- Second order accurate
- Operate in cycle space

Solution in cycle space

$$\begin{cases} y_n = \cos\theta_n^{-1}\left(\sin\theta_n - I_c^{-1} I_s\right) \\ J_n = \left(A\left[\cos\theta_n^{-1} I_c^{-1} + L\right]A^T\right)^{-1}\left(A\left[\theta_n - y_n - L I_s\right] + 2\pi\left(\Phi_e - z\right)\right) \\ \theta_{n+1} = \theta_n - \cos\theta_n^{-1} I_c^{-1} A^T J_n - y_n \end{cases}$$

- Second order accurate
- Operate in cycle space (The matrix that needs to be inverted is N_f by N_f square matrix, which is smaller than J so faster)

Derivation:

$\boxed{\text{step 1}}$ solve $Jx = F$

$$\begin{cases} M I_c \cos\theta\, x = F_M & \text{I} \\ A(I + L I_c \cos\theta) x = F_A & \text{II} \end{cases}$$

$I_c(\cos\theta\, x - \sin\theta) + I_s = A^T J$ (follows from I)

$x = \cos\theta^{-1}\left[I_c^{-1}\left(A^T J - I_s\right) + \sin\theta\right]$ III (note: $\theta_{n+1} = \theta_n - x(\theta_n)$)

$\boxed{\text{Step 2}}$ solve for J: (plug x into II)

$$A(\cos\theta^{-1} + L I_c)\left[I_c^{-1}\left(A^T J - I_s\right) + \sin\theta\right] = A(\theta + L I_c \sin\theta) + 2\pi(\Phi_e - z)$$

$$J = \left(A\left[\cos\theta^{-1} I_c^{-1} + L\right]A^T\right)^{-1}\left(A\left[\theta - \cos\theta^{-1}\left(\sin\theta - I_c^{-1} I_s\right) - L I_s\right] + 2\pi\left(\Phi_e - z\right)\right)$$

$\boxed{\text{Step 3}}$ obtain θ_{n+1} (plug J into III)

$$\theta_{n+1} = \theta_n - \cos\theta_n^{-1}\left[I_c^{-1} A^T \left(A\left[\cos\theta_n^{-1} I_c^{-1} + L\right]A^T\right)^{-1}\left(A\left[\theta_n - \cos\theta_n^{-1}\left(\sin\theta_n - I_c^{-1} I_s\right) - L I_s\right] + 2\pi\left(\Phi_e - z\right)\right) + \sin\theta_n - I_c^{-1} I_s\right]$$