Transient analysis

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$$I = I_c \sin\theta + R^{-1}\dot{\theta} + C\ddot{\theta} + \eta$$

$$M(I - I_s) = 0 \quad \leftarrow \text{KCL}$$

$$V = \dot{\theta} + L\dot{I} + V_s$$

$$AV + 2\pi\dot{\Phi}_e = 0 \quad \leftarrow \text{KVL}$$

$$\theta_s \equiv \int_0^t V_s(t')\,\partial t'$$

IKVL(z)

$$A\left[\theta + LI + \theta_s\right] + 2\pi\Phi_e = 2\pi z$$

Time discretization:

$$t_n = n\Delta t$$

$$\dot{\theta}_n = \sum_{i=0}^{m} a_i\theta_{n-i} + O(\Delta t^{m+1})$$

$$\ddot{\theta}_n = \sum_{i=0}^{m} b_i\theta_{n-i} + O(\Delta t^{m+1})$$

$$\sin\theta_n = \sin\left(\sum_{i=1}^{m} c_i\theta_{n-i}\right) + O(\Delta t^{m+1})$$

$$\theta_n^s \approx \Delta t\left(-\frac{V_n^s + V_0^s}{2} + \sum_{i=0}^{n} V_i^s\right) \quad \text{Trapezoid rule}$$

$$I_n \equiv \sum_{i=0}^{m} C_i\theta_{n-i} + I_c\sin\left(\sum_{i=1}^{m} c_i\theta_{n-i}\right) + \eta_n$$

$$I_n = A^T J_n + I_n^s$$

$$y_n \equiv \sum_{i=0}^{m} C_i\theta_{n-i} + I_c\sin\left(\sum_{i=1}^{m} c_i\theta_{n-i}\right) + \eta_n = I_n - C_0\theta_n$$

$$AC_0^{-1} I_n = A\theta_n + AC_0^{-1} y_n = AC_0^{-1}A^T J_n + I_n^s$$

$$= -AL(A^T J_n + I_n^s) - A\theta_s - 2\pi(\Phi_n^{ext} - z) + AC_0^{-1} y_n$$

$$A(C_0^{-1} + L)A^T J_n = A\left[C_0^{-1} y_n - LI_n^s - \theta_s\right] - 2\pi(\Phi_n^{ext} - z)$$

$$\begin{cases} J_n = \left[A(C_0^{-1} + L)A^T\right]^{-1} A\left[C_0^{-1} y_n - LI_n^s - \theta_s\right] - 2\pi(\Phi_n^{ext} - z) \\ I_n = A^T J_n + I_n^s \\ \theta_n = C_0^{-1}(I_n - y_n) \end{cases}$$