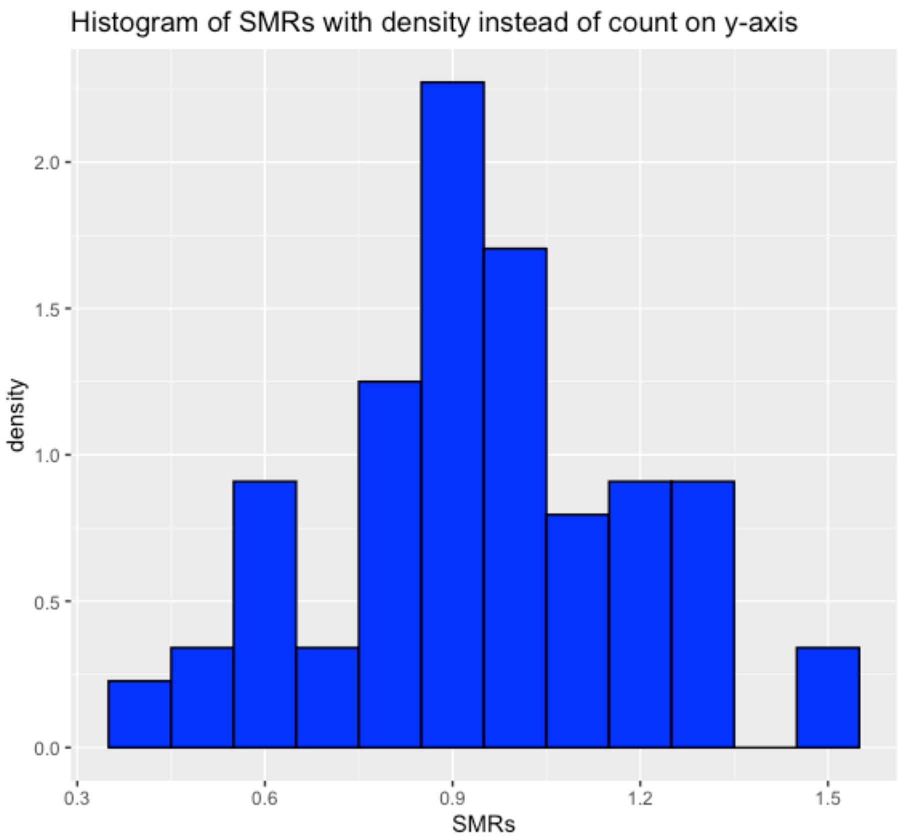
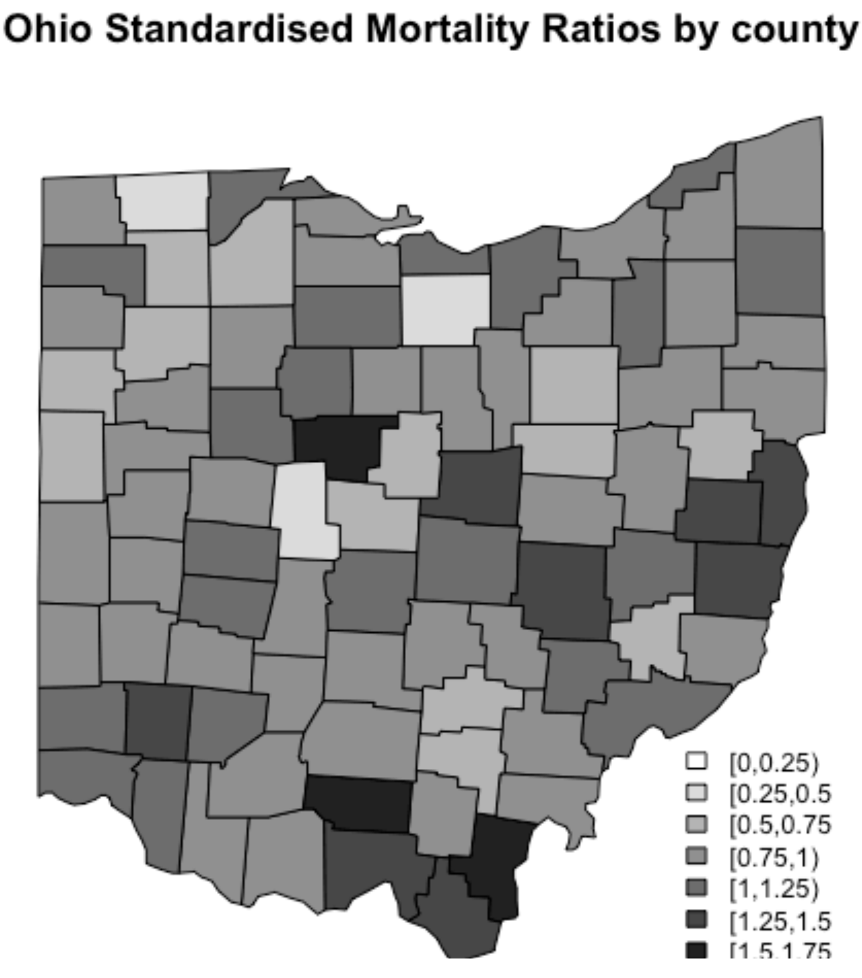
Daanish Ahsan
Advanced Topic in Statistics MTHM017
Candidate Number: 124070

Advanced Topics in Statistics Assignment 1

Q1)



Q2)



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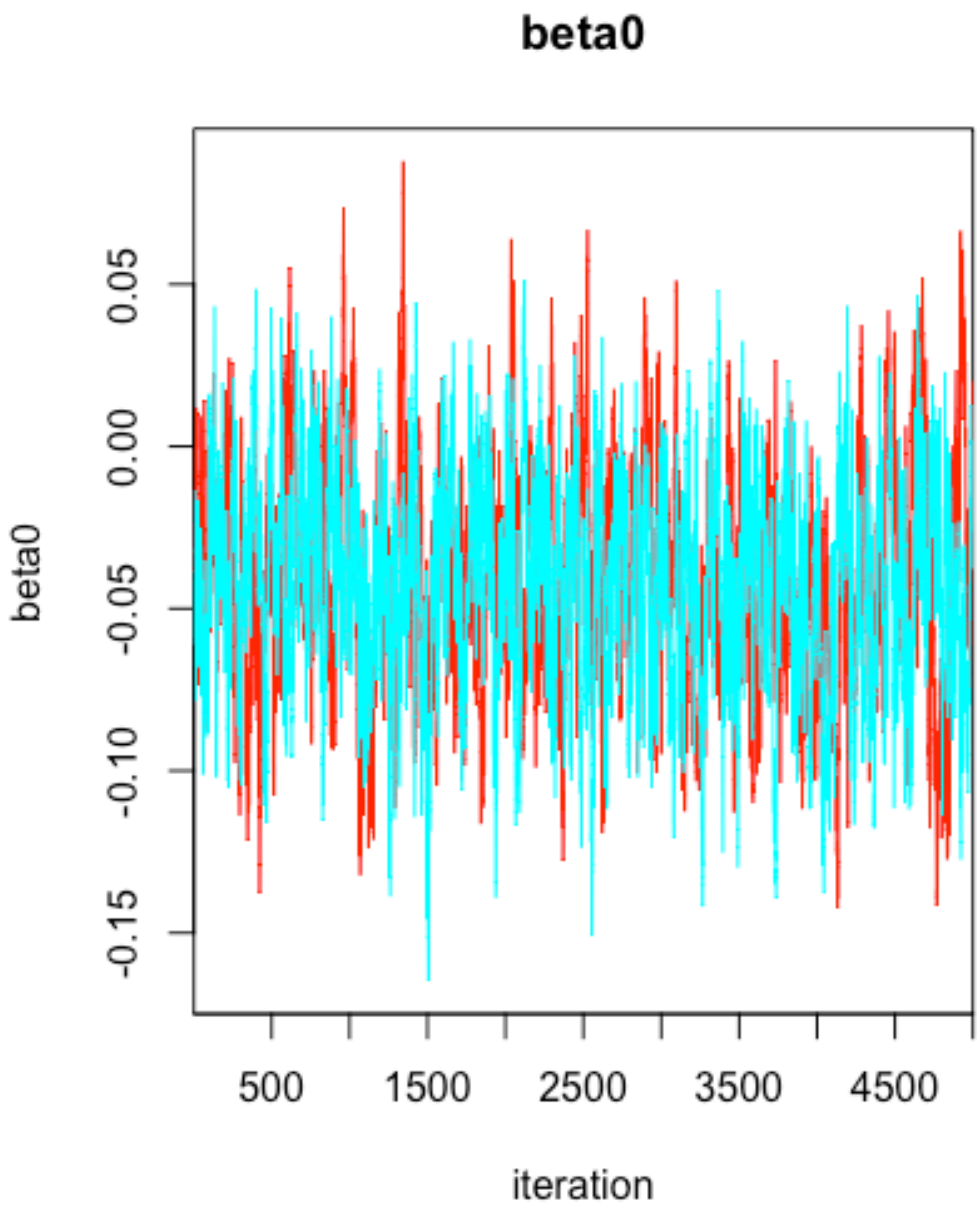
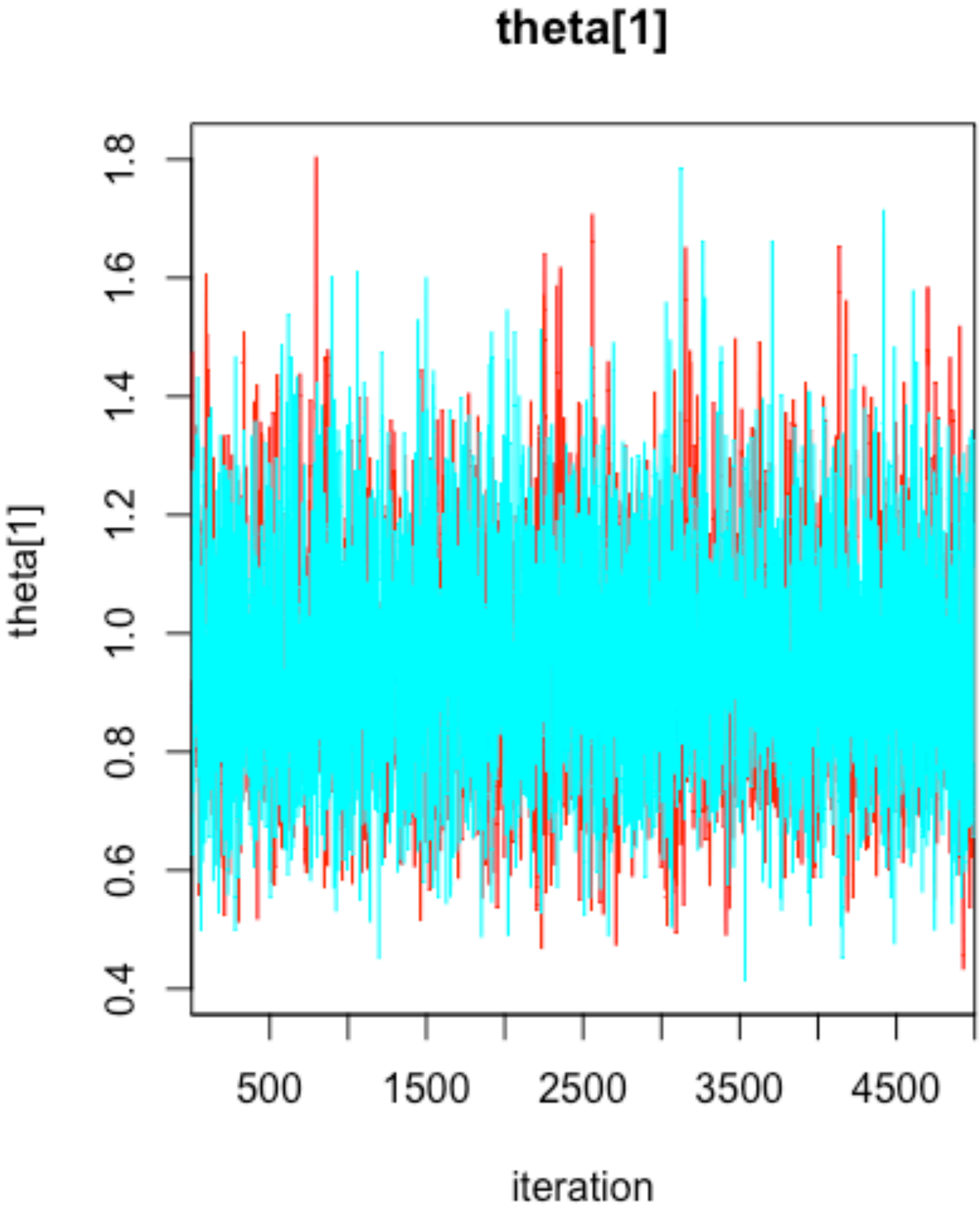
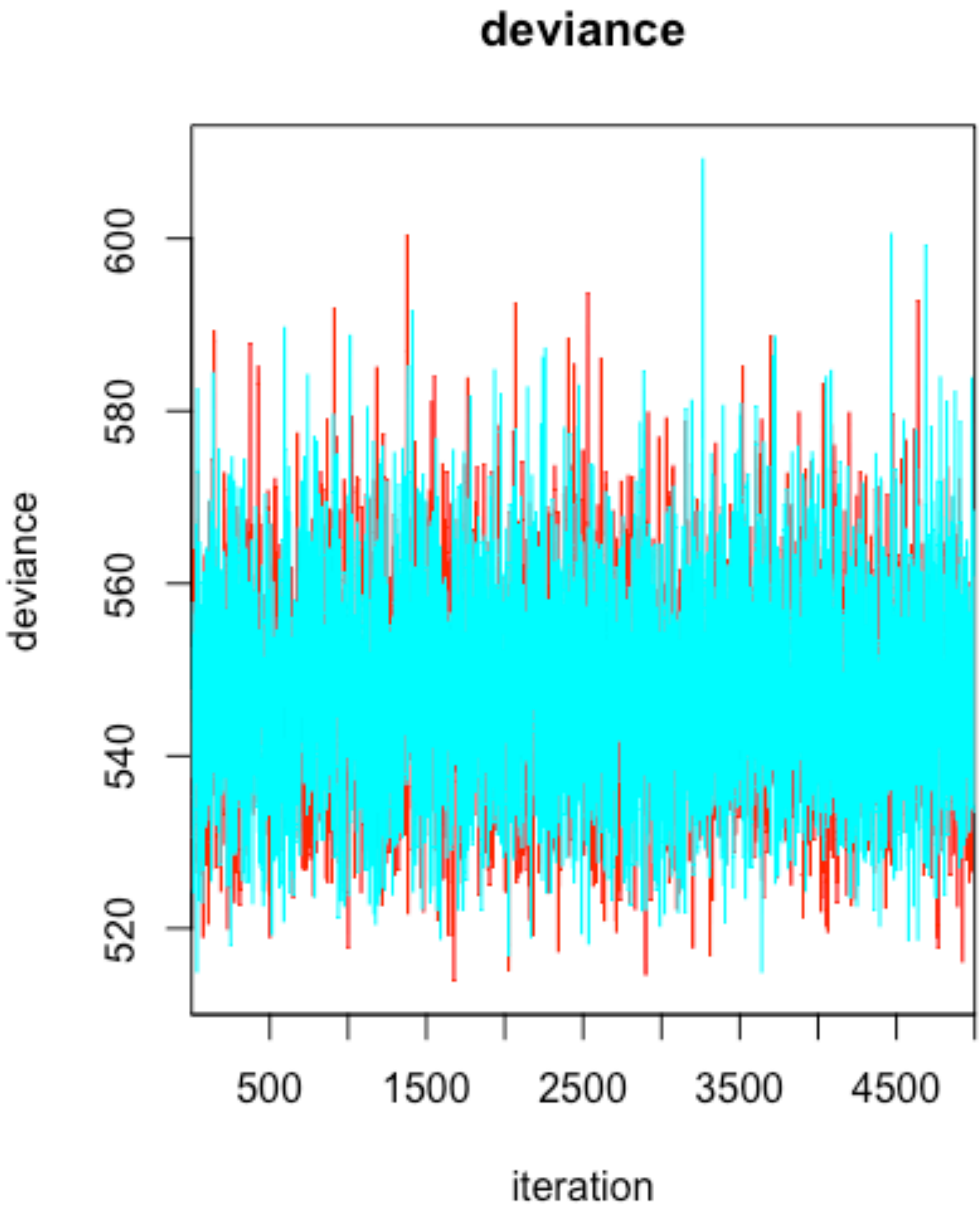
Q3)

Beta0 is the overall increase or decrease in risk across all the areas so tells us if there is a higher or lower number of observed cases than expected. The thetas tell us us how each area differs from the overall increase or decrease of beta0. Without the exponentiation of Beta0, its value is -0.0419, which suggests a 4.19% decrease in risk if 0 is taken as no change in risk. After exponentiation, Beta0 is 0.959, which again suggests a 4.1% decrease in risk, but now 1 is taken as no change in risk. Multiplying this exp(Beta0) by the thetas makes more sense, as is makes the SMRs easier to read as ratios. Otherwise, multiplying a theta value of 0.959 by -0.0419, results in -0.0401, which can still be interpreted as an RR, but is much more comparable and easier to understand if 0.959 is multiplied by the exp(Beta0) of 0.959, which results in 0.919. This way, RR values can be interpreted as being the same as the average of the county if they=1, but above or below 1 is a higher or lower relative risk respectively.

Q4)




theta[1]
theta[1]
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
500
1500
2500
3500
4500
iteration


deviance
deviance
600
580
560
540
520
500
1500
2500
3500
4500
iteration

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Summary of parameters

n.sims = 10000 iterations saved

	mu.vect	sd.vect	2.5%	25%	50%	75%	97.5%	Rhat	n.eff
beta0	-0.024	0.024	-0.071	-0.039	-0.023	-0.008	0.020	1.007	280
theta[1]	0.984	0.120	0.761	0.903	0.981	1.060	1.230	1.002	10000
theta[2]	0.955	0.090	0.785	0.893	0.955	1.014	1.140	1.002	1900
theta[3]	0.998	0.111	0.786	0.924	0.995	1.068	1.223	1.001	10000
theta[4]	0.992	0.093	0.817	0.928	0.989	1.051	1.187	1.001	10000
theta[5]	0.948	0.110	0.738	0.874	0.947	1.018	1.172	1.003	1700
theta[6]	0.972	0.110	0.758	0.899	0.972	1.044	1.191	1.001	10000
theta[7]	1.144	0.111	0.949	1.065	1.136	1.215	1.378	1.006	270
theta[8]	0.989	0.117	0.769	0.911	0.987	1.062	1.234	1.002	2300
theta[9]	1.123	0.085	0.971	1.064	1.118	1.176	1.303	1.008	220
theta[10]	0.921	0.119	0.690	0.841	0.921	0.999	1.158	1.003	820
theta[11]	1.008	0.118	0.785	0.927	1.004	1.082	1.257	1.001	10000
theta[12]	1.063	0.089	0.896	1.002	1.059	1.122	1.247	1.001	4800
theta[13]	1.128	0.103	0.945	1.054	1.122	1.193	1.343	1.004	540
theta[14]	1.064	0.122	0.847	0.981	1.056	1.138	1.331	1.002	1400
theta[15]	0.952	0.089	0.783	0.889	0.951	1.009	1.132	1.001	10000
theta[16]	0.964	0.111	0.750	0.889	0.963	1.036	1.191	1.002	1200
theta[17]	0.979	0.109	0.770	0.904	0.977	1.049	1.200	1.001	3100
theta[18]	0.956	0.037	0.889	0.931	0.956	0.980	1.029	1.004	560
theta[19]	0.998	0.106	0.804	0.927	0.994	1.065	1.220	1.001	10000
theta[20]	1.045	0.122	0.825	0.962	1.037	1.123	1.298	1.002	2500
theta[21]	0.924	0.107	0.718	0.852	0.924	0.993	1.141	1.003	860
theta[22]	1.025	0.103	0.835	0.955	1.020	1.091	1.246	1.001	10000
theta[23]	0.939	0.095	0.756	0.876	0.938	1.000	1.133	1.003	860
theta[24]	0.997	0.119	0.771	0.917	0.992	1.070	1.249	1.001	10000
theta[25]	1.188	0.059	1.073	1.149	1.187	1.228	1.308	1.007	230
theta[26]	0.857	0.114	0.631	0.778	0.858	0.936	1.074	1.003	660
theta[27]	1.130	0.133	0.905	1.037	1.119	1.210	1.425	1.005	360
theta[28]	0.919	0.101	0.722	0.850	0.920	0.986	1.121	1.003	840
theta[29]	0.934	0.092	0.756	0.871	0.934	0.994	1.121	1.001	6500
theta[30]	1.009	0.113	0.798	0.932	1.004	1.079	1.243	1.001	5600
theta[31]	1.061	0.049	0.969	1.028	1.060	1.094	1.161	1.002	1500
theta[32]	0.951	0.102	0.751	0.885	0.951	1.017	1.160	1.001	5300
theta[33]	1.042	0.120	0.824	0.961	1.035	1.117	1.298	1.001	3700
theta[34]	1.054	0.130	0.818	0.966	1.045	1.134	1.341	1.002	2000
theta[35]	0.894	0.120	0.661	0.813	0.896	0.975	1.124	1.004	570
theta[36]	0.996	0.113	0.784	0.921	0.991	1.067	1.232	1.001	7800
theta[37]	0.949	0.118	0.720	0.870	0.946	1.025	1.190	1.001	10000
theta[38]	0.920	0.120	0.677	0.841	0.920	0.998	1.162	1.002	3000
theta[39]	0.826	0.109	0.610	0.752	0.826	0.900	1.035	1.004	520
theta[40]	0.985	0.116	0.765	0.906	0.981	1.058	1.223	1.001	6800
theta[41]	1.149	0.108	0.958	1.073	1.143	1.217	1.378	1.003	700
theta[42]	1.095	0.119	0.883	1.013	1.086	1.168	1.350	1.001	10000
theta[43]	1.171	0.093	1.004	1.107	1.165	1.232	1.363	1.008	210
theta[44]	1.124	0.116	0.916	1.042	1.115	1.199	1.373	1.002	1600

theta[45]	**1.139**	**0.104**	**0.961**	**1.064**	**1.132**	**1.207**	**1.365**	**1.003**	**580**
theta[46]	**0.982**	**0.110**	**0.774**	**0.909**	**0.979**	**1.052**	**1.210**	**1.002**	**1300**
theta[47]	**1.058**	**0.079**	**0.914**	**1.005**	**1.054**	**1.109**	**1.223**	**1.001**	**3400**
theta[48]	**1.084**	**0.063**	**0.966**	**1.040**	**1.082**	**1.125**	**1.212**	**1.003**	**880**
theta[49]	**0.978**	**0.118**	**0.758**	**0.898**	**0.975**	**1.052**	**1.221**	**1.002**	**10000**
theta[50]	**0.964**	**0.065**	**0.842**	**0.919**	**0.962**	**1.007**	**1.093**	**1.001**	**4700**
theta[51]	**1.213**	**0.132**	**0.982**	**1.120**	**1.203**	**1.298**	**1.497**	**1.006**	**290**
theta[52]	**0.910**	**0.095**	**0.727**	**0.846**	**0.909**	**0.972**	**1.100**	**1.003**	**730**
theta[53]	**0.997**	**0.120**	**0.774**	**0.916**	**0.993**	**1.071**	**1.246**	**1.001**	**10000**
theta[54]	**0.887**	**0.113**	**0.662**	**0.813**	**0.888**	**0.963**	**1.111**	**1.005**	**400**
theta[55]	**0.918**	**0.095**	**0.735**	**0.855**	**0.917**	**0.980**	**1.107**	**1.001**	**4400**
theta[56]	**0.971**	**0.124**	**0.733**	**0.890**	**0.970**	**1.048**	**1.226**	**1.002**	**8200**
theta[57]	**1.006**	**0.056**	**0.899**	**0.968**	**1.004**	**1.043**	**1.119**	**1.002**	**2100**
theta[58]	**1.033**	**0.131**	**0.791**	**0.946**	**1.024**	**1.113**	**1.315**	**1.001**	**10000**
theta[59]	**0.946**	**0.119**	**0.714**	**0.868**	**0.945**	**1.020**	**1.189**	**1.001**	**10000**
theta[60]	**1.130**	**0.111**	**0.937**	**1.049**	**1.124**	**1.202**	**1.363**	**1.002**	**2400**
theta[61]	**0.953**	**0.127**	**0.712**	**0.868**	**0.951**	**1.032**	**1.210**	**1.002**	**2000**
theta[62]	**0.966**	**0.110**	**0.759**	**0.892**	**0.962**	**1.036**	**1.190**	**1.002**	**1300**
theta[63]	**0.965**	**0.122**	**0.734**	**0.886**	**0.962**	**1.041**	**1.219**	**1.001**	**10000**
theta[64]	**0.971**	**0.116**	**0.743**	**0.894**	**0.971**	**1.043**	**1.215**	**1.001**	**5100**
theta[65]	**0.992**	**0.112**	**0.782**	**0.916**	**0.987**	**1.063**	**1.227**	**1.001**	**10000**
theta[66]	**1.108**	**0.135**	**0.879**	**1.015**	**1.098**	**1.188**	**1.404**	**1.002**	**990**
theta[67]	**0.937**	**0.093**	**0.759**	**0.874**	**0.936**	**0.999**	**1.126**	**1.001**	**9900**
theta[68]	**0.939**	**0.112**	**0.726**	**0.866**	**0.938**	**1.008**	**1.173**	**1.001**	**10000**
theta[69]	**0.901**	**0.114**	**0.677**	**0.825**	**0.903**	**0.978**	**1.124**	**1.001**	**5300**
theta[70]	**0.938**	**0.088**	**0.773**	**0.878**	**0.939**	**0.995**	**1.119**	**1.002**	**2000**
theta[71]	**0.997**	**0.102**	**0.806**	**0.929**	**0.993**	**1.061**	**1.210**	**1.001**	**10000**
theta[72]	**0.978**	**0.107**	**0.777**	**0.908**	**0.975**	**1.045**	**1.195**	**1.001**	**10000**
theta[73]	**1.169**	**0.116**	**0.963**	**1.087**	**1.160**	**1.241**	**1.413**	**1.003**	**740**
theta[74]	**1.077**	**0.113**	**0.874**	**1.000**	**1.070**	**1.148**	**1.321**	**1.002**	**1500**
theta[75]	**0.966**	**0.111**	**0.753**	**0.893**	**0.965**	**1.036**	**1.195**	**1.001**	**10000**
theta[76]	**1.002**	**0.064**	**0.883**	**0.958**	**1.000**	**1.043**	**1.135**	**1.001**	**10000**
theta[77]	**1.055**	**0.060**	**0.944**	**1.013**	**1.053**	**1.095**	**1.179**	**1.003**	**730**
theta[78]	**1.020**	**0.076**	**0.879**	**0.968**	**1.017**	**1.069**	**1.179**	**1.001**	**8700**
theta[79]	**0.978**	**0.097**	**0.794**	**0.913**	**0.975**	**1.040**	**1.178**	**1.001**	**5200**
theta[80]	**0.854**	**0.118**	**0.619**	**0.778**	**0.857**	**0.934**	**1.082**	**1.005**	**330**
theta[81]	**0.881**	**0.115**	**0.651**	**0.803**	**0.882**	**0.959**	**1.102**	**1.004**	**550**
theta[82]	**0.953**	**0.129**	**0.702**	**0.869**	**0.952**	**1.035**	**1.214**	**1.001**	**6000**
theta[83]	**1.134**	**0.113**	**0.936**	**1.055**	**1.127**	**1.206**	**1.375**	**1.008**	**240**
theta[84]	**1.046**	**0.107**	**0.849**	**0.974**	**1.039**	**1.114**	**1.267**	**1.001**	**8900**
theta[85]	**0.839**	**0.097**	**0.652**	**0.773**	**0.840**	**0.905**	**1.029**	**1.009**	**180**
theta[86]	**0.948**	**0.113**	**0.729**	**0.872**	**0.946**	**1.020**	**1.178**	**1.001**	**10000**
theta[87]	**0.835**	**0.098**	**0.643**	**0.769**	**0.837**	**0.903**	**1.027**	**1.003**	**780**
theta[88]	**1.021**	**0.126**	**0.786**	**0.936**	**1.015**	**1.098**	**1.291**	**1.001**	**10000**
deviance	**570.151**	**15.646**	**542.680**	**559.646**	**569.049**	**579.575**	**605.817**	**1.035**	**86**

For each parameter, n.eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor (at convergence, Rhat=1).

DIC info (using the rule, pD = var(deviance)/2)
pD = 120.9 and DIC = 691.1

Q5) The traceplots above show convergence of the chains for all the parameters; only one traceplot diagram has been included for theta as there are 88, but they all also show convergence. The Gelman.diag function reveals that virtually all values for both point estimates and confidence intervals are 1 or extremely close to one, which also indicates convergence; the values are shown below.

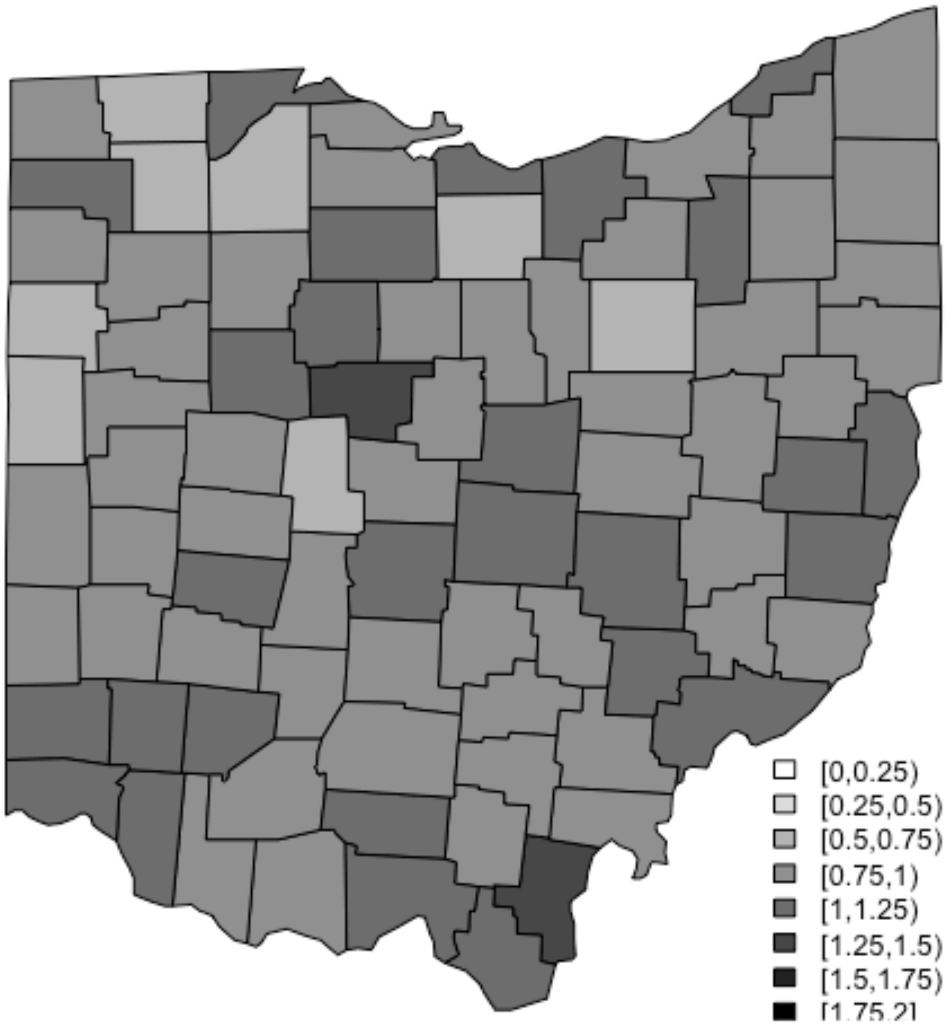
Gelman Diagnostic

Potential scale reduction factors:

	Point est.	Upper C.I.
beta0	1.01	1.04
deviance	1.00	1.00
theta[1]	1.00	1.00
theta[2]	1.00	1.00
theta[3]	1.00	1.00
theta[4]	1.00	1.00
theta[5]	1.00	1.00
theta[6]	1.00	1.01
theta[7]	1.00	1.01
theta[8]	1.00	1.01
theta[9]	1.00	1.01
theta[10]	1.00	1.00
theta[11]	1.00	1.00
theta[12]	1.00	1.00
theta[13]	1.00	1.00
theta[14]	1.00	1.00
theta[15]	1.00	1.00
theta[16]	1.00	1.00
theta[17]	1.00	1.00
theta[18]	1.00	1.02
theta[19]	1.00	1.00
theta[20]	1.00	1.00
theta[21]	1.00	1.00
theta[22]	1.00	1.01
theta[23]	1.00	1.02
theta[24]	1.00	1.01
theta[25]	1.00	1.02
theta[26]	1.00	1.01
theta[27]	1.00	1.00
theta[28]	1.00	1.01
theta[29]	1.01	1.03
theta[30]	1.00	1.00
theta[31]	1.00	1.01
theta[32]	1.00	1.01
theta[33]	1.01	1.03
theta[34]	1.00	1.00
theta[35]	1.00	1.02
theta[36]	1.00	1.00
theta[37]	1.00	1.00
theta[38]	1.00	1.00
theta[39]	1.00	1.00
theta[40]	1.00	1.00
theta[41]	1.00	1.00
theta[42]	1.00	1.02
theta[43]	1.00	1.00
theta[44]	1.00	1.00
theta[45]	1.00	1.02
theta[46]	1.00	1.00
theta[47]	1.00	1.00
theta[48]	1.01	1.03
theta[49]	1.00	1.00
theta[50]	1.00	1.00
theta[51]	1.00	1.00
theta[52]	1.00	1.00
theta[53]	1.00	1.00
theta[54]	1.00	1.00
theta[55]	1.00	1.01
theta[56]	1.00	1.00
theta[57]	1.00	1.01
theta[58]	1.00	1.00
theta[59]	1.00	1.00
theta[60]	1.00	1.00
theta[61]	1.00	1.01
theta[62]	1.00	1.00
theta[63]	1.00	1.00
theta[64]	1.00	1.00
theta[65]	1.00	1.00
theta[66]	1.00	1.00
theta[67]	1.00	1.00
theta[68]	1.00	1.00
theta[69]	1.00	1.00
theta[70]	1.00	1.02
theta[71]	1.00	1.00
theta[72]	1.00	1.00
theta[73]	1.00	1.00
theta[74]	1.00	1.00
theta[75]	1.00	1.01
theta[75]	1.00	1.01
theta[76]	1.00	1.00
theta[77]	1.00	1.01
theta[78]	1.00	1.00
theta[79]	1.00	1.00
theta[80]	1.00	1.00
theta[81]	1.00	1.01
theta[82]	1.00	1.00
theta[83]	1.00	1.00
theta[84]	1.00	1.00
theta[85]	1.00	1.00
theta[86]	1.00	1.00
theta[87]	1.00	1.00
theta[88]	1.00	1.01

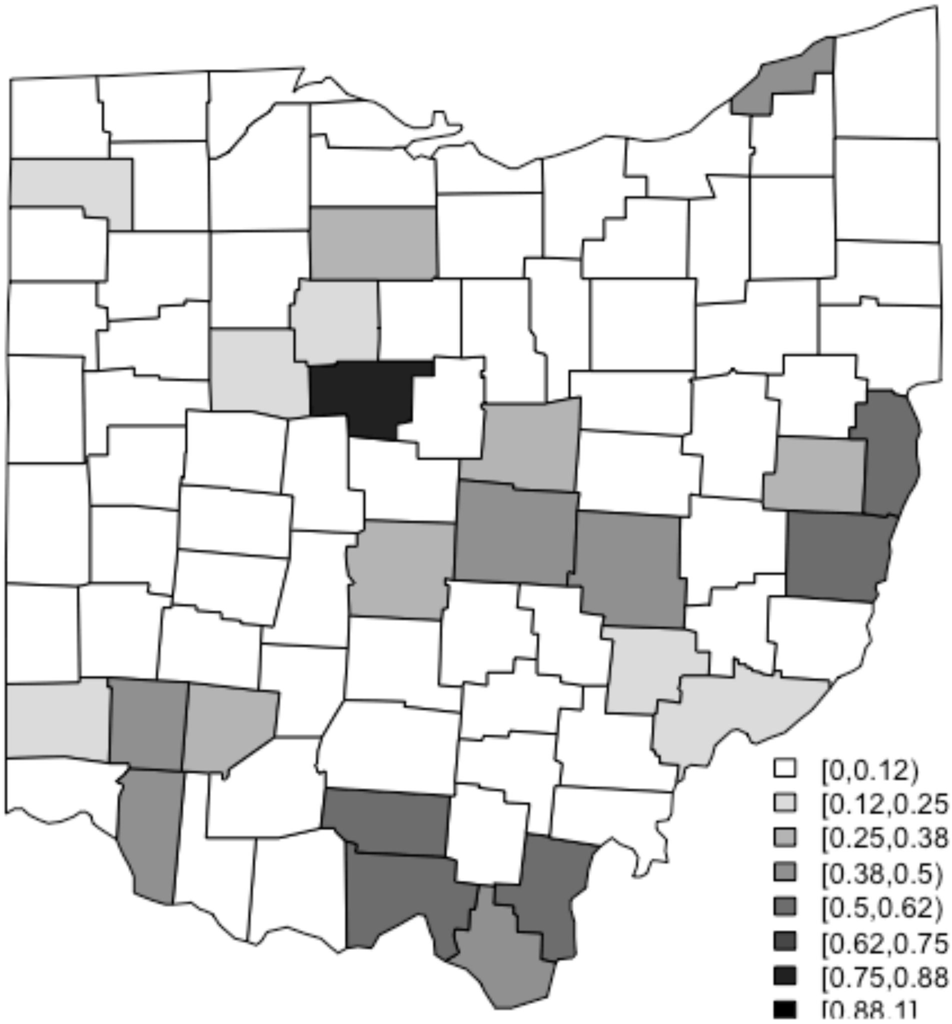
Q6)

Ohio Posterior Means for Relative Risks



Q7)

Posterior probabilities that relative risk in each area exceeds 1.2



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Q8) I have changed the prior values for alpha to (0.001, 0.001), and for beta0 to dnorm ~ (0,1). The prior probability is the probability of an event before new data is collected; these are revised using Bayes Theorem to become the posterior probability once new data is available. I adjusted these priors as based on the previous model beta0 seems more likely to be normally distributed around 0, rather than the vague uniform distribution of (-100, 100). I chose these prior values for alpha as this distribution favours smaller values over larger ones.

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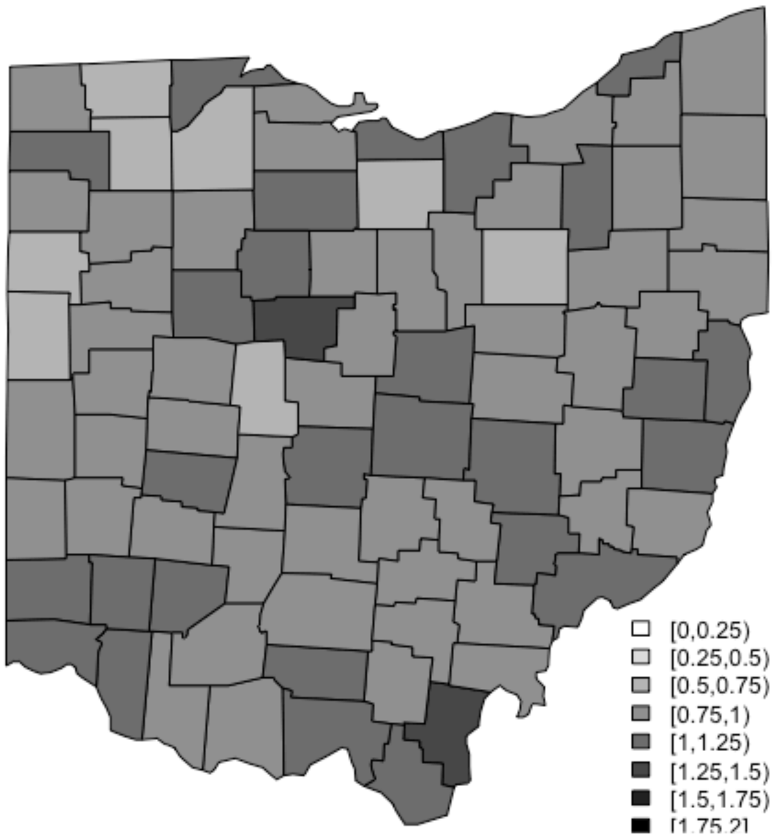
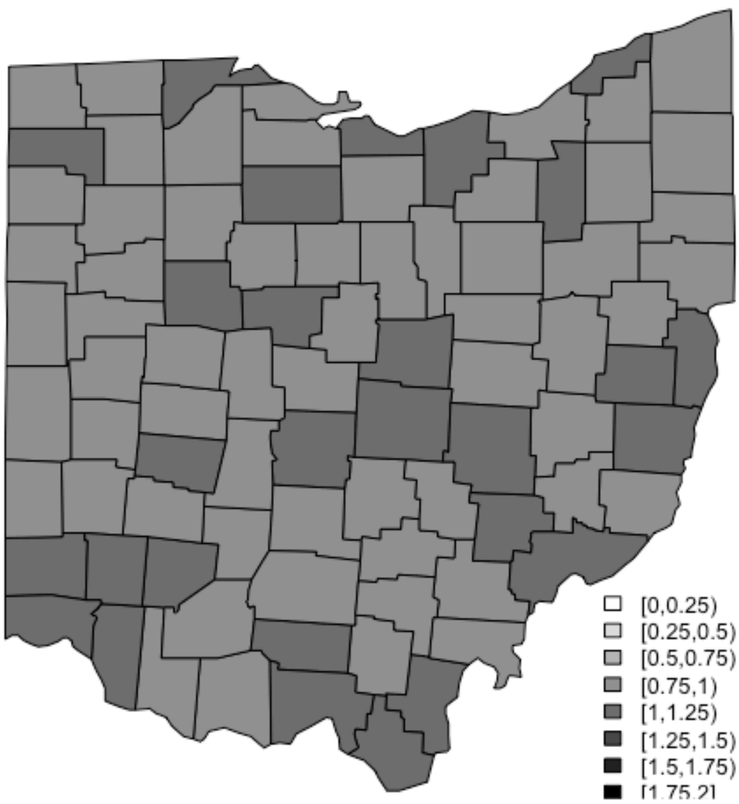
Q9)

Ohio Posterior Means for Relative Risks (from q6, unchanged priors)



Ohio Posterior Means for Relative Risks with adjusted priors for alpha and beta0



Summary of parameters; unchanged model from Q6

n.sims = 10000 iterations saved

	mu.vect	sd.vect	2.5%	25%	50%	75%	97.5%	Rhat	n.eff
beta0	-0.024	0.024	-0.071	-0.039	-0.023	-0.008	0.020	1.007	280
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theta[6]	0.972	0.110	0.758	0.899	0.972	1.044	1.191	1.001	10000
theta[7]	1.144	0.111	0.949	1.065	1.136	1.215	1.378	1.006	270
theta[8]	0.989	0.117	0.769	0.911	0.987	1.062	1.234	1.002	2300
theta[9]	1.123	0.085	0.971	1.064	1.118	1.176	1.303	1.008	220
theta[10]	0.921	0.119	0.690	0.841	0.921	0.999	1.158	1.003	820
theta[11]	1.008	0.118	0.785	0.927	1.004	1.082	1.257	1.001	10000
theta[12]	1.063	0.089	0.896	1.002	1.059	1.122	1.247	1.001	4800
theta[13]	1.128	0.103	0.945	1.054	1.122	1.193	1.343	1.004	540
theta[14]	1.064	0.122	0.847	0.981	1.056	1.138	1.331	1.002	1400
theta[15]	0.952	0.089	0.783	0.889	0.951	1.009	1.132	1.001	10000
theta[16]	0.964	0.111	0.750	0.889	0.963	1.036	1.191	1.002	1200
theta[17]	0.979	0.109	0.770	0.904	0.977	1.049	1.200	1.001	3100
theta[18]	0.956	0.037	0.889	0.931	0.956	0.980	1.029	1.004	560
theta[19]	0.998	0.106	0.804	0.927	0.994	1.065	1.220	1.001	10000
theta[20]	1.045	0.122	0.825	0.962	1.037	1.123	1.298	1.002	2500
theta[21]	0.924	0.107	0.718	0.852	0.924	0.993	1.141	1.003	860
theta[22]	1.025	0.103	0.835	0.955	1.020	1.091	1.246	1.001	10000
theta[23]	0.939	0.095	0.756	0.876	0.938	1.000	1.133	1.003	860
theta[24]	0.997	0.119	0.771	0.917	0.992	1.070	1.249	1.001	10000
theta[25]	1.188	0.059	1.073	1.149	1.187	1.228	1.308	1.007	230
theta[26]	0.857	0.114	0.631	0.778	0.858	0.936	1.074	1.003	660
theta[27]	1.130	0.133	0.905	1.037	1.119	1.210	1.425	1.005	360
theta[28]	0.919	0.101	0.722	0.850	0.920	0.986	1.121	1.003	840
theta[29]	0.934	0.092	0.756	0.871	0.934	0.994	1.121	1.001	6500
theta[30]	1.009	0.113	0.798	0.932	1.004	1.079	1.243	1.001	5600
theta[31]	1.061	0.049	0.969	1.028	1.060	1.094	1.161	1.002	1500
theta[32]	0.951	0.102	0.751	0.885	0.951	1.017	1.160	1.001	5300
theta[33]	1.042	0.120	0.824	0.961	1.035	1.117	1.298	1.001	3700
theta[34]	1.054	0.130	0.818	0.966	1.045	1.134	1.341	1.002	2000
theta[35]	0.894	0.120	0.661	0.813	0.896	0.975	1.124	1.004	570
theta[36]	0.996	0.113	0.784	0.921	0.991	1.067	1.232	1.001	7800
theta[37]	0.949	0.118	0.720	0.870	0.946	1.025	1.190	1.001	10000
theta[38]	0.920	0.120	0.677	0.841	0.920	0.998	1.162	1.002	3000
theta[39]	0.826	0.109	0.610	0.752	0.826	0.900	1.035	1.004	520
theta[40]	0.985	0.116	0.765	0.906	0.981	1.058	1.223	1.001	6800
theta[41]	1.149	0.108	0.958	1.073	1.143	1.217	1.378	1.003	700
theta[42]	1.095	0.119	0.883	1.013	1.086	1.168	1.350	1.001	10000
theta[43]	1.171	0.093	1.004	1.107	1.165	1.232	1.363	1.008	210
theta[44]	1.124	0.116	0.916	1.042	1.115	1.199	1.373	1.002	1600

theta[45]	**1.139**	**0.104**	**0.961**	**1.064**	**1.132**	**1.207**	**1.365**	**1.003**	**580**
theta[46]	**0.982**	**0.110**	**0.774**	**0.909**	**0.979**	**1.052**	**1.210**	**1.002**	**1300**
theta[47]	**1.058**	**0.079**	**0.914**	**1.005**	**1.054**	**1.109**	**1.223**	**1.001**	**3400**
theta[48]	**1.084**	**0.063**	**0.966**	**1.040**	**1.082**	**1.125**	**1.212**	**1.003**	**880**
theta[49]	**0.978**	**0.118**	**0.758**	**0.898**	**0.975**	**1.052**	**1.221**	**1.002**	**10000**
theta[50]	**0.964**	**0.065**	**0.842**	**0.919**	**0.962**	**1.007**	**1.093**	**1.001**	**4700**
theta[51]	**1.213**	**0.132**	**0.982**	**1.120**	**1.203**	**1.298**	**1.497**	**1.006**	**290**
theta[52]	**0.910**	**0.095**	**0.727**	**0.846**	**0.909**	**0.972**	**1.100**	**1.003**	**730**
theta[53]	**0.997**	**0.120**	**0.774**	**0.916**	**0.993**	**1.071**	**1.246**	**1.001**	**10000**
theta[54]	**0.887**	**0.113**	**0.662**	**0.813**	**0.888**	**0.963**	**1.111**	**1.005**	**400**
theta[55]	**0.918**	**0.095**	**0.735**	**0.855**	**0.917**	**0.980**	**1.107**	**1.001**	**4400**
theta[56]	**0.971**	**0.124**	**0.733**	**0.890**	**0.970**	**1.048**	**1.226**	**1.002**	**8200**
theta[57]	**1.006**	**0.056**	**0.899**	**0.968**	**1.004**	**1.043**	**1.119**	**1.002**	**2100**
theta[58]	**1.033**	**0.131**	**0.791**	**0.946**	**1.024**	**1.113**	**1.315**	**1.001**	**10000**
theta[59]	**0.946**	**0.119**	**0.714**	**0.868**	**0.945**	**1.020**	**1.189**	**1.001**	**10000**
theta[60]	**1.130**	**0.111**	**0.937**	**1.049**	**1.124**	**1.202**	**1.363**	**1.002**	**2400**
theta[61]	**0.953**	**0.127**	**0.712**	**0.868**	**0.951**	**1.032**	**1.210**	**1.002**	**2000**
theta[62]	**0.966**	**0.110**	**0.759**	**0.892**	**0.962**	**1.036**	**1.190**	**1.002**	**1300**
theta[63]	**0.965**	**0.122**	**0.734**	**0.886**	**0.962**	**1.041**	**1.219**	**1.001**	**10000**
theta[64]	**0.971**	**0.116**	**0.743**	**0.894**	**0.971**	**1.043**	**1.215**	**1.001**	**5100**
theta[65]	**0.992**	**0.112**	**0.782**	**0.916**	**0.987**	**1.063**	**1.227**	**1.001**	**10000**
theta[66]	**1.108**	**0.135**	**0.879**	**1.015**	**1.098**	**1.188**	**1.404**	**1.002**	**990**
theta[67]	**0.937**	**0.093**	**0.759**	**0.874**	**0.936**	**0.999**	**1.126**	**1.001**	**9900**
theta[68]	**0.939**	**0.112**	**0.726**	**0.866**	**0.938**	**1.008**	**1.173**	**1.001**	**10000**
theta[69]	**0.901**	**0.114**	**0.677**	**0.825**	**0.903**	**0.978**	**1.124**	**1.001**	**5300**
theta[70]	**0.938**	**0.088**	**0.773**	**0.878**	**0.939**	**0.995**	**1.119**	**1.002**	**2000**
theta[71]	**0.997**	**0.102**	**0.806**	**0.929**	**0.993**	**1.061**	**1.210**	**1.001**	**10000**
theta[72]	**0.978**	**0.107**	**0.777**	**0.908**	**0.975**	**1.045**	**1.195**	**1.001**	**10000**
theta[73]	**1.169**	**0.116**	**0.963**	**1.087**	**1.160**	**1.241**	**1.413**	**1.003**	**740**
theta[74]	**1.077**	**0.113**	**0.874**	**1.000**	**1.070**	**1.148**	**1.321**	**1.002**	**1500**
theta[75]	**0.966**	**0.111**	**0.753**	**0.893**	**0.965**	**1.036**	**1.195**	**1.001**	**10000**
theta[76]	**1.002**	**0.064**	**0.883**	**0.958**	**1.000**	**1.043**	**1.135**	**1.001**	**10000**
theta[77]	**1.055**	**0.060**	**0.944**	**1.013**	**1.053**	**1.095**	**1.179**	**1.003**	**730**
theta[78]	**1.020**	**0.076**	**0.879**	**0.968**	**1.017**	**1.069**	**1.179**	**1.001**	**8700**
theta[79]	**0.978**	**0.097**	**0.794**	**0.913**	**0.975**	**1.040**	**1.178**	**1.001**	**5200**
theta[80]	**0.854**	**0.118**	**0.619**	**0.778**	**0.857**	**0.934**	**1.082**	**1.005**	**330**
theta[81]	**0.881**	**0.115**	**0.651**	**0.803**	**0.882**	**0.959**	**1.102**	**1.004**	**550**
theta[82]	**0.953**	**0.129**	**0.702**	**0.869**	**0.952**	**1.035**	**1.214**	**1.001**	**6000**
theta[83]	**1.134**	**0.113**	**0.936**	**1.055**	**1.127**	**1.206**	**1.375**	**1.008**	**240**
theta[84]	**1.046**	**0.107**	**0.849**	**0.974**	**1.039**	**1.114**	**1.267**	**1.001**	**8900**
theta[85]	**0.839**	**0.097**	**0.652**	**0.773**	**0.840**	**0.905**	**1.029**	**1.009**	**180**
theta[86]	**0.948**	**0.113**	**0.729**	**0.872**	**0.946**	**1.020**	**1.178**	**1.001**	**10000**
theta[87]	**0.835**	**0.098**	**0.643**	**0.769**	**0.837**	**0.903**	**1.027**	**1.003**	**780**
theta[88]	**1.021**	**0.126**	**0.786**	**0.936**	**1.015**	**1.098**	**1.291**	**1.001**	**10000**
deviance	**570.151**	**15.646**	**542.680**	**559.646**	**569.049**	**579.575**	**605.817**	**1.035**	**86**

For each parameter, n.eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor (at convergence, Rhat=1).

DIC info (using the rule, pD = var(deviance)/2)
pD = 120.9 and DIC = 691.1

Summary of parameters; adjusted prior model

n.sims = 10000 iterations saved

	mu.vect	sd.vect	2.5%	25%	50%	75%	97.5%	Rhat	n.eff
beta0	-0.024	0.024	-0.073	-0.040	-0.023	-0.009	0.020	1.002	10000
theta[1]	0.981	0.124	0.748	0.898	0.977	1.059	1.240	1.001	10000
theta[2]	0.957	0.092	0.782	0.895	0.955	1.018	1.147	1.001	10000
theta[3]	0.996	0.113	0.786	0.919	0.992	1.070	1.227	1.001	5900
theta[4]	0.992	0.094	0.813	0.928	0.990	1.053	1.185	1.001	10000
theta[5]	0.950	0.112	0.730	0.876	0.948	1.022	1.179	1.001	2600
theta[6]	0.972	0.113	0.760	0.897	0.969	1.043	1.202	1.001	10000
theta[7]	1.152	0.111	0.955	1.074	1.145	1.221	1.393	1.002	1300
theta[8]	0.989	0.117	0.769	0.910	0.986	1.064	1.233	1.001	7200
theta[9]	1.124	0.083	0.967	1.067	1.121	1.178	1.294	1.002	1800
theta[10]	0.922	0.116	0.693	0.845	0.921	0.996	1.159	1.002	2100
theta[11]	1.010	0.118	0.794	0.929	1.004	1.084	1.265	1.001	10000
theta[12]	1.066	0.090	0.899	1.004	1.063	1.123	1.249	1.001	10000
theta[13]	1.131	0.107	0.940	1.056	1.124	1.200	1.357	1.002	2100
theta[14]	1.069	0.124	0.847	0.984	1.059	1.145	1.341	1.001	10000
theta[15]	0.952	0.092	0.779	0.889	0.949	1.012	1.138	1.002	10000
theta[16]	0.966	0.116	0.741	0.889	0.963	1.040	1.207	1.001	10000
theta[17]	0.978	0.110	0.776	0.903	0.975	1.049	1.205	1.002	1400
theta[18]	0.957	0.037	0.885	0.931	0.956	0.981	1.031	1.001	10000
theta[19]	0.999	0.109	0.791	0.925	0.994	1.070	1.226	1.001	3400
theta[20]	1.046	0.122	0.822	0.963	1.040	1.124	1.301	1.001	10000
theta[21]	0.921	0.107	0.711	0.849	0.920	0.992	1.134	1.003	1400
theta[22]	1.027	0.103	0.837	0.958	1.023	1.090	1.243	1.002	2100
theta[23]	0.938	0.096	0.754	0.873	0.937	1.001	1.131	1.001	10000
theta[24]	1.000	0.121	0.779	0.918	0.994	1.074	1.260	1.001	10000
theta[25]	1.192	0.057	1.082	1.153	1.190	1.229	1.310	1.001	10000
theta[26]	0.852	0.115	0.623	0.775	0.854	0.930	1.079	1.001	10000
theta[27]	1.134	0.136	0.906	1.038	1.121	1.216	1.438	1.001	10000
theta[28]	0.918	0.104	0.716	0.847	0.916	0.985	1.130	1.001	10000
theta[29]	0.932	0.091	0.754	0.871	0.932	0.992	1.115	1.001	10000
theta[30]	1.012	0.118	0.795	0.932	1.006	1.086	1.253	1.001	8200
theta[31]	1.062	0.049	0.969	1.028	1.060	1.094	1.162	1.001	10000
theta[32]	0.945	0.106	0.740	0.873	0.942	1.013	1.165	1.001	6600
theta[33]	1.045	0.124	0.821	0.961	1.039	1.121	1.308	1.001	5000
theta[34]	1.058	0.132	0.826	0.966	1.050	1.140	1.347	1.001	6000
theta[35]	0.891	0.116	0.664	0.814	0.892	0.968	1.116	1.001	5200
theta[36]	0.993	0.113	0.782	0.917	0.990	1.064	1.229	1.002	2000
theta[37]	0.947	0.123	0.706	0.867	0.945	1.025	1.197	1.001	10000
theta[38]	0.916	0.120	0.687	0.835	0.915	0.992	1.160	1.001	7200
theta[39]	0.823	0.110	0.609	0.749	0.822	0.896	1.037	1.001	3800
theta[40]	0.986	0.119	0.764	0.907	0.982	1.061	1.233	1.001	3700
theta[41]	1.151	0.109	0.956	1.075	1.144	1.221	1.384	1.002	1000
theta[42]	1.095	0.120	0.881	1.012	1.088	1.168	1.359	1.001	5300
theta[43]	1.175	0.091	1.009	1.111	1.171	1.236	1.367	1.002	2400
theta[44]	1.127	0.119	0.914	1.043	1.120	1.204	1.378	1.001	10000
theta[45]	1.143	0.103	0.953	1.072	1.137	1.208	1.363	1.003	920

theta[46]	**0.980**	**0.111**	**0.769**	**0.906**	**0.976**	**1.050**	**1.212**	**1.001**	**10000**
theta[47]	**1.059**	**0.078**	**0.913**	**1.006**	**1.057**	**1.110**	**1.219**	**1.001**	**10000**
theta[48]	**1.086**	**0.063**	**0.967**	**1.042**	**1.085**	**1.127**	**1.214**	**1.002**	**1100**
theta[49]	**0.981**	**0.120**	**0.754**	**0.900**	**0.978**	**1.059**	**1.231**	**1.001**	**10000**
theta[50]	**0.963**	**0.066**	**0.836**	**0.918**	**0.963**	**1.007**	**1.095**	**1.001**	**10000**
theta[51]	**1.217**	**0.132**	**0.988**	**1.123**	**1.208**	**1.298**	**1.509**	**1.002**	**2400**
theta[52]	**0.909**	**0.096**	**0.727**	**0.843**	**0.908**	**0.971**	**1.102**	**1.001**	**7300**
theta[53]	**0.996**	**0.123**	**0.764**	**0.912**	**0.993**	**1.072**	**1.248**	**1.001**	**10000**
theta[54]	**0.885**	**0.115**	**0.659**	**0.808**	**0.884**	**0.961**	**1.109**	**1.001**	**10000**
theta[55]	**0.915**	**0.096**	**0.727**	**0.850**	**0.913**	**0.979**	**1.105**	**1.001**	**10000**
theta[56]	**0.974**	**0.125**	**0.738**	**0.891**	**0.971**	**1.054**	**1.239**	**1.001**	**5100**
theta[57]	**1.007**	**0.055**	**0.903**	**0.970**	**1.006**	**1.044**	**1.115**	**1.001**	**2600**
theta[58]	**1.037**	**0.130**	**0.801**	**0.948**	**1.030**	**1.115**	**1.313**	**1.001**	**4700**
theta[59]	**0.948**	**0.120**	**0.720**	**0.868**	**0.944**	**1.022**	**1.197**	**1.001**	**10000**
theta[60]	**1.133**	**0.111**	**0.936**	**1.054**	**1.125**	**1.203**	**1.371**	**1.001**	**10000**
theta[61]	**0.949**	**0.129**	**0.692**	**0.865**	**0.949**	**1.031**	**1.213**	**1.002**	**1700**
theta[62]	**0.965**	**0.108**	**0.762**	**0.892**	**0.961**	**1.035**	**1.188**	**1.001**	**10000**
theta[63]	**0.967**	**0.125**	**0.729**	**0.881**	**0.962**	**1.047**	**1.225**	**1.001**	**10000**
theta[64]	**0.972**	**0.120**	**0.746**	**0.892**	**0.969**	**1.046**	**1.216**	**1.001**	**10000**
theta[65]	**0.990**	**0.115**	**0.781**	**0.912**	**0.985**	**1.062**	**1.225**	**1.001**	**10000**
theta[66]	**1.112**	**0.135**	**0.870**	**1.020**	**1.103**	**1.196**	**1.407**	**1.001**	**6000**
theta[67]	**0.936**	**0.091**	**0.760**	**0.875**	**0.935**	**0.996**	**1.121**	**1.001**	**5600**
theta[68]	**0.937**	**0.113**	**0.719**	**0.862**	**0.937**	**1.010**	**1.172**	**1.002**	**2400**
theta[69]	**0.901**	**0.116**	**0.677**	**0.823**	**0.901**	**0.978**	**1.135**	**1.001**	**10000**
theta[70]	**0.939**	**0.086**	**0.777**	**0.882**	**0.938**	**0.996**	**1.115**	**1.001**	**3700**
theta[71]	**0.998**	**0.103**	**0.802**	**0.929**	**0.993**	**1.063**	**1.215**	**1.001**	**3900**
theta[72]	**0.982**	**0.106**	**0.779**	**0.910**	**0.980**	**1.049**	**1.198**	**1.001**	**10000**
theta[73]	**1.170**	**0.116**	**0.964**	**1.090**	**1.162**	**1.241**	**1.422**	**1.001**	**3400**
theta[74]	**1.081**	**0.115**	**0.871**	**1.002**	**1.076**	**1.152**	**1.328**	**1.001**	**8600**
theta[75]	**0.965**	**0.115**	**0.746**	**0.891**	**0.963**	**1.037**	**1.202**	**1.001**	**10000**
theta[76]	**1.002**	**0.064**	**0.882**	**0.958**	**1.000**	**1.045**	**1.131**	**1.001**	**8300**
theta[77]	**1.056**	**0.058**	**0.949**	**1.016**	**1.055**	**1.094**	**1.174**	**1.001**	**10000**
theta[78]	**1.020**	**0.076**	**0.881**	**0.968**	**1.017**	**1.071**	**1.175**	**1.001**	**10000**
theta[79]	**0.976**	**0.096**	**0.794**	**0.911**	**0.974**	**1.036**	**1.173**	**1.001**	**4100**
theta[80]	**0.855**	**0.120**	**0.618**	**0.775**	**0.856**	**0.934**	**1.087**	**1.001**	**3500**
theta[81]	**0.881**	**0.120**	**0.643**	**0.801**	**0.883**	**0.961**	**1.113**	**1.001**	**10000**
theta[82]	**0.953**	**0.124**	**0.712**	**0.870**	**0.952**	**1.034**	**1.203**	**1.001**	**9700**
theta[83]	**1.133**	**0.114**	**0.933**	**1.054**	**1.124**	**1.204**	**1.384**	**1.001**	**10000**
theta[84]	**1.044**	**0.109**	**0.840**	**0.970**	**1.040**	**1.112**	**1.276**	**1.001**	**10000**
theta[85]	**0.833**	**0.097**	**0.642**	**0.768**	**0.834**	**0.899**	**1.021**	**1.001**	**3700**
theta[86]	**0.948**	**0.115**	**0.726**	**0.870**	**0.946**	**1.023**	**1.181**	**1.002**	**2100**
theta[87]	**0.834**	**0.096**	**0.649**	**0.767**	**0.835**	**0.899**	**1.019**	**1.002**	**2400**
theta[88]	**1.018**	**0.125**	**0.789**	**0.933**	**1.011**	**1.096**	**1.288**	**1.001**	**10000**
deviance	**569.152**	**14.397**	**542.821**	**558.791**	**568.750**	**578.904**	**597.878**	**1.004**	**580**

For each parameter, n.eff is a crude measure of effective sample size,
and Rhat is the potential scale reduction factor (at convergence, Rhat=1).

DIC info (using the rule, pD = var(deviance)/2)
pD = 103.5 and DIC = 672.6
DIC is an estimate of expected predictive error (lower deviance is better).

The two maps of relative risks are clearly different, with the adjusted priors map being much less informative in terms of relative risk than the non adjusted model map. This is because the priors for alpha and beta0 are distributed differently and so beta0 takes on a different value; as do the thetas, resulting in differing relative risks. In terms of the summaries of the model, the DIC (estimate of expected predictive error) is 691.1 for the unchanged model, but 672.6 for the adjusted priors model. The Rhat for both models' parameters indicate convergence, but marginally more so for the second model, where almost all parameters have a Rhat value of 1.001 or 1.002. For the unadjusted model, the Rhat varies more among parameters between 1.001 and 1.007; there is still convergence, but marginally less so than in the adjusted model. Mu of Beta0 for both models is -0.024, but it is clear that the theta values differ, if only by around 0.001. However, this change in the theta values is clearly enough of a difference to affect the RR calculations, as indicated by the differences in maps of RR on page 11.

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Advanced Topic in Statistics MTHM017
Candidate Number: 124070
Code Appendix

```
#Advanced Topic in Statistics Assignment 1

library(R2jags)
library(coda)
library(lattice)
library(MCMCpack)
library(runjags)
library(ggplot2)
library(MASS)
library(mcmc)
library(coda)
library(mcmcplots)
library(superdiag)
library(ggmcmc)
library(maps)
model.file <- system.file(package = "R2jags", "model", "schools.txt")

#reading in the data
Ohio <- read.csv('Ohio_data.csv')

#1

SMRs <- Ohio$Obs/Ohio$Exp

Ohio <- cbind(Ohio, SMRs)

ggplot(Ohio, aes(x=SMRs)) +
  geom_histogram(aes(y = ..density..),      # Histogram with density
 instead of count on y-axis
                 binwidth = 0.1,
                 colour = "black",
                 fill="blue") +
  ggtitle('Histogram of SMRs with density instead of count on y-axis')
  geom_density(alpha = 0.2, fill="#FF6656")

#2

#map function
OhioMap <-function(data, ncol=5, figmain="", digits=5, type="e",
                   lower=NULL, upper=NULL) {
  if (is.null(lower)) lower <- min(data)
  if (is.null(upper)) upper <- max(data)

  if (type=="q"){p <- seq(0,1,length=ncol+1)
  br <- round(quantile(data,probs=p),2)}
  if (type=="e"){br <- round(seq(lower,upper,length=ncol+1),2)}
  shading <- gray((ncol-1):0/(ncol-1))
  data.grp <- findInterval(data,vec=br,rightmost.closed=T,all.inside=T)
  data.shad <- shading[data.grp]
```

```
  map("county", "ohio", fill=TRUE, col=data.shad)
  leg.txt<-paste("[",br[ncol],",",br[ncol+1],"]",sep="")
  for(i in (ncol-1):1){
    leg.txt<-append(leg.txt,paste("[",br[i],",",br[i+1],")",sep=""),)
  }
  leg.txt<-rev(leg.txt)
  legend(-81.4,39.4,legend=leg.txt,fill=shading,bty="n",ncol=1,cex=.8)
  title(main=figmain,cex=1.5)
  invisible()
}

#plotting SMRs onto map
OhioMap(Ohio$SMRs, ncol=8,type="e",figmain="Standardised Mortality
 Ratios",lower=0,upper=2)

#3

#4
#Coding poisson Gamma model
set.seed(2020)
Ohio <- as.matrix(Ohio)

X <- Ohio[,1]
Obs <- Ohio[,2]
Exp <- Ohio[,3]
SMR <- Ohio[,4]

Ohio.data <- list("X","Obs","Exp","SMR")

bayes.mod <- function() {
  for(i in 1:88){
    Obs[i] ~ dpois(mu[i])
    theta[i] ~ dgamma(alpha, alpha)
    log(mu[i]) <- log(Exp[i])+ beta0+log(theta[i])
  }
  alpha ~ dgamma(1,1)
  beta0 ~ dunif(-100, 100)

}

jags.param <- c("beta0", 'theta')

inits1 <- list("beta0" = 20)
inits2 <- list("beta0"=-100)
jags.inits <- list(inits1, inits2)

bayes.mod.fit <- jags(data = Ohio.data, inits = jags.inits,
                     parameters.to.save = jags.param, n.chains = 2, n.iter
 = 10000,
                     n.burnin = 5000,n.thin=1,model.file = bayes.mod)
```

```
print(bayes.mod.fit)

plot(bayes.mod.fit)

#checking convergence and producing traceplots
traceplot(bayes.mod.fit)

bayes.mod.fit.mcmc <- as.mcmc(bayes.mod.fit)
summary(bayes.mod.fit.mcmc)

#checking convergence
gelman.diag(bayes.mod.fit.mcmc)

densityplot(bayes.mod.fit.mcmc)

denplot(bayes.mod.fit.mcmc)

mcmcplot(bayes.mod.fit.mcmc)

caterplot(bayes.mod.fit.mcmc)

bayes.mod.fit.gg <- ggs(bayes.mod.fit.mcmc)

ggs_density(bayes.mod.fit.gg)

#5

#yes chains for all parameters have converged as gelman thing is close to 1

#6 Extracting and mapping RR means

B0mu <- bayes.mod.fit$BUGSoutput$mean$beta0

B0mu <- exp(B0mu)

Thetai <- bayes.mod.fit$BUGSoutput$mean$theta

RR <- 0.959*Thetai

RR <- as.data.frame(RR)

Ohio <- cbind(Ohio, RR)

OhioMap(Ohio$RR, ncol=8,type="e",figmain="Ohio ",lower=0,upper=2)

#7
#Coding model to calculate probability that RR>1.2 in each area

RR.mod <- function() {
  for(i in 1:88){
    Obs[i] ~ dpois(mu[i])
```

```
    theta[i] ~ dgamma(alpha, alpha)
    log(mu[i]) <- log(Exp[i])+ beta0+log(theta[i])


  }
  alpha ~ dgamma(1,1)
  beta0 ~ dunif(-100, 100)
  RR <- exp(beta0)*(theta)
  p70 <- ifelse(RR>1.2,1,0)


}


jags.param <- c('p70', 'RR')

inits1 <- list("beta0" = 20)
inits2 <- list("beta0"=-100)
jags.inits <- list(inits1, inits2)

RR.mod.fit <- jags(data = Ohio.data, inits = jags.inits,
                      parameters.to.save = jags.param, n.chains = 2, n.iter
 = 10000,
                      n.burnin = 5000,n.thin=1,model.file = RR.mod)

print(RR.mod.fit)

p70 <- colMeans(RR.mod.fit$BUGSoutput$sims.list$p70)
plot(1:88,p70)

#mapping probabilities

probsRR <- p70

probsRR <- as.data.frame(probsRR)

Ohio <- cbind(Ohio, probsRR)

OhioMap(Ohio$probsRR, ncol=8,type="e",figmain="Probability of
 RR>1.2",lower=0,upper=1)

#8 RR but with diff priors from 6

bayes.mod <- function() {
  for(i in 1:88){
    Obs[i] ~ dpois(mu[i])
    theta[i] ~ dgamma(alpha, alpha)
    log(mu[i]) <- log(Exp[i])+ beta0+log(theta[i])
  }
  alpha ~ dgamma(0.001, 0.001)
  beta0 ~ dnorm(0, 1)
```

```
}

jags.param <- c("beta0", 'theta')

inits1 <- list("beta0" = 20)
inits2 <- list("beta0"=-100)
jags.inits <- list(inits1, inits2)

bayes.mod.fit <- jags(data = Ohio.data, inits = jags.inits,
                      parameters.to.save = jags.param, n.chains = 2, n.iter
 = 10000,
                      n.burnin = 5000,n.thin=1,model.file = bayes.mod)

B0mu <- bayes.mod.fit$BUGSoutput$mean$beta0

B0mu <- exp(B0mu)

Thetai  <- bayes.mod.fit$BUGSoutput$mean$theta

RR <- 0.952*Thetai

RR <- as.data.frame(RR)

Ohio <- cbind(Ohio, RR)

OhioMap(Ohio$RR, ncol=8,type="e",figmain="Ohio ",lower=0,upper=2)
```