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ASSIGNMENT 04 Q.1 1

Q1 find the rank of the matrix A by reducing in Row Echelon form

$$A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$$

Soln. $A = \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 4 & 3 & 2 \\ 3 & 2 & 1 & 3 \\ 6 & 8 & 7 & 5 \end{bmatrix}$

Applying $R_2 \to R_2 - 2R_1$
$R_3 \to R_3 - 3R_1$
$R_4 \to R_4 - 6R_1$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & -4 & -11 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 0 & -3 & 0 \end{bmatrix}$$

$R_4 \to R_4 - R_2$

$$\begin{bmatrix} 1 & 2 & 3 & 0 \\ 0 & 0 & -3 & 0 \\ 0 & -4 & -8 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Rank $(A) = 3$

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4.2 (2)

Q.2 let W be the vector space of all symmetric 2×2 matrices and let $T: W \to P_2$ be the linear transformation defined by $T\begin{bmatrix} a & b \\ c & d \end{bmatrix} =$

$(a-b) + (b-c)x + (c-a)x^2$. find the rank and nullity of T.

Find Rank and nullity of T.

Since the maximum degree of polynomial $T=2$, so $\dim(P_2) = 3$.

So, a is subset of kernel T is $T(A)=0$

$(a-b) + (b-c)x + (c-a)x^2 = 0$

$\boxed{a=b=c = t \text{ (let)}}$

new matrix $\begin{bmatrix} t & t \\ t & d \end{bmatrix}$

Dimension of kernel is 1 because ~~they~~ there are only one independent parameter as 't'

According to Rank Nullity Theorem $\rightarrow$

rank (T) nullity $\Rightarrow$ dimension (W)

Rank $+ 1 = 4$

Rank $= 3$

$\therefore$ Rank $= 3$ & nullity $= 1$

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Q3 let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. find the eigen values and eigen vectors of A^{-1} and $A+4I$.

Soln: $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}{2\times 2 - [(-1)(-1)]}$$

$$= \frac{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}{4-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

$$A+4I = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2+4 & -1 \\ -1 & 2+4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

Now

(i) A^{-1} [Eigen value and Eigen vectors]

$$\Rightarrow |A^{-1} - \lambda I| = 0$$

$$\Rightarrow \left|\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\Rightarrow \left|\begin{bmatrix} 2/3-\lambda & 1/3 \\ 1/3 & 2/3-\lambda \end{bmatrix}\right| = 0$$

Q3 Let $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$. Find the eigen values and eigen vectors of A^{-1} and $A+4I$.

Soln » $A = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} = \frac{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}{2\times2 - [(-1)(-1)]}$$

$$= \frac{\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}}{4-1} = \begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix}$$

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$$= \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$$

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(i) A^{-1} [Eigen value and Eigen vectors]

$$\Rightarrow |A^{-1} - \lambda I| = 0$$

$$\Rightarrow \left|\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\Rightarrow \left|\begin{bmatrix} 2/3 - \lambda & 1/3 \\ 1/3 & 2/3 - \lambda \end{bmatrix}\right| = 0$$

$$\cancel{\left(\tfrac{2}{3}-\lambda\right)^2 - \tfrac{1}{}}$$

$$\Rightarrow \left(\tfrac{2}{3}-\lambda\right)^2 - \left(\tfrac{1}{3}\right)^2 = 0$$

$$\Rightarrow \cancel{\left(\tfrac{2}{3}\right)^2 + \lambda^2 - \tfrac{2}{3}\times\lambda\times 2 - \tfrac{1}{9}}$$

$$\Rightarrow \left(\frac{2}{3}-\lambda+\frac{1}{3}\right)\left(\frac{2}{3}-\lambda-\frac{1}{3}\right)=0$$

$$\Rightarrow (1-\lambda)\left(\frac{1}{3}-\lambda\right)=0$$

$$\boxed{\lambda=1} \qquad \boxed{\lambda=\frac{1}{3}}$$

Now

$\boxed{\text{for } \lambda=1}$ Eigen vector

$$\Rightarrow [A^{-1}-\lambda I][X]=[0]$$

$$\Rightarrow \left[\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]\begin{bmatrix} x \end{bmatrix} = [0]$$

$$\Rightarrow \left[\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right]\begin{bmatrix} x \end{bmatrix} = [0]$$

$$\Rightarrow \begin{bmatrix} 2/3-1 & 1/3 \\ 1/3 & 2/3-1 \end{bmatrix}\begin{bmatrix} x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -1/3 & 1/3 \\ 1/3 & -1/3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$R_2 \to R_2 + R_1$

$$\begin{bmatrix} -1/3 & 1/3 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

$-1/3\, x_1 + 1/3\, x_2 = 0$

$1/3\, x_2 = 1/3\, x_1$

$x_1 = x_2$

$\boxed{\text{Let } x_1 = K}$

$\boxed{x_2 = K}$

$$\therefore \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = K\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$\boxed{\text{For } \lambda = \frac{1}{3}}$ Eigen vector

$\Rightarrow [A^{-1} - \lambda I][X] = [0]$

$$\Rightarrow \left[\begin{bmatrix} 2/3 & 1/3 \\ 1/3 & 2/3 \end{bmatrix} - \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right][X] = [0]$$

$$\Rightarrow \begin{bmatrix} 2/3 - 1/3 & 1/3 \\ 1/3 & 2/3 - \frac{1}{3} \end{bmatrix}[X] = [0]$$

$$\Rightarrow \begin{bmatrix} 1/3 & 1/3 \\ 1/3 & 1/3 \end{bmatrix}[X] = [0]$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} 1/3 & 1/3 \\ 0 & 0 \end{bmatrix}[X] = [0]$$

$1/3\, x_1 + 1/3\, x_2 = 0$

$1/3\, x_1 = -\frac{1}{3} x_2$

let $\boxed{x_2 = K}$

$\boxed{x_1 = -K}$

Eigen vector $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = K\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

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Now (ii) $A + 4I = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

Eigen ~~vect~~ values → $|A - \lambda I| = 0$

$$\left| \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{vmatrix} = 0$$

$$(6-\lambda)^2 - (-1)(-1) = 0$$

$$36 + \lambda^2 - 12\lambda - 1 = 0$$

$$\lambda^2 - 12\lambda + 35 = 0$$

$$\lambda^2 - 7\lambda - 5\lambda + 35 = 0$$

$$\lambda(\lambda - 7) - 5(\lambda - 7) = 0$$

$$\lambda = 5, 7.$$

$\boxed{\text{Eigen vector for } \lambda = 5}$

$$[A - \lambda I][X] = 0$$

$$\left[\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right] [X] = 0$$

$$\begin{bmatrix} 6-5 & 0 \\ 0 & 6-5 \end{bmatrix} [X] = 0$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [X] = 0 \qquad \begin{matrix} x_1 + 0x_2 = 0 & \text{(I)} \\ 0 + x_2 = 0 & \text{(II)} \end{matrix}$$

$\boxed{\text{From eq (I), } x_1 = 0}$

Put in eq (II), we get $x_2 = 0$

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Now (ii) $A+4I = \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix}$

Eigen ~~vect~~ values → $|A-\lambda I| = 0$

$$\left| \begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} 6-\lambda & -1 \\ -1 & 6-\lambda \end{vmatrix} = 0$$

$(6-\lambda)^2 - (-1)(-1) = 0$

$36 + \lambda^2 - 12\lambda - 1 = 0$

$\lambda^2 - 12\lambda + 35 = 0$

$\lambda^2 - 7\lambda - 5\lambda + 35 = 0$

$\lambda(\lambda - 7) - 5(\lambda - 7) = 0$

$\lambda = 5, 7.$

$\boxed{\text{Eigen vector for } \lambda = 5}$

$[A - \lambda I][X] = 0$

$$\left[\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} - 5 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right] [X] = 0$$

$$\begin{bmatrix} 6-5 & 0 \\ 0 & 6-5 \end{bmatrix} [X] = 0$$

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} [X] = 0$$

$x_1 + 0x_2 = 0$ — (I)

$0 + x_2 = 0$ — (II)

$\boxed{\text{From eq (I), } x_1 = 0}$

Put in eq (II), we get $x_2 = 0$

Eigen vector = $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\boxed{\text{Eigen vector for } \lambda = 7}$

$[A - \lambda I][x] = [0]$

$$\left[\begin{bmatrix} 6 & -1 \\ -1 & 6 \end{bmatrix} - 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right][x] = [0]$$

$$\begin{bmatrix} 6-7 & -1 \\ -1 & 6-7 \end{bmatrix}[x] = [0]$$

$$\begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}[x] = [0]$$

$R_2 \to R_2 - R_1$

$$\begin{bmatrix} -1 & -1 \\ 0 & 0 \end{bmatrix}[x] = [0]$$

$-1x_1 - 1x_2 = 0$

$-1x_1 = x_2$

let $x_1 = k$

$x_2 = -k$

Eigen vector = $k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Q4 Solve by Gauss - Seidel Method (Take three iterations)

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

with initial values $x(0) = 0$, $y(0) = 0$, $z(0) = 0$

Soln:- So we have three equations

$$3x - 0.1y - 0.2z = 7.85$$
$$0.1x + 7y - 0.3z = -19.3$$
$$0.3x - 0.2y + 10z = 71.4$$

It's diagonally dominant system of equations.

1st iteration

$$3x - 0.1y - 0.2z = 7.85$$
$$3x = 0.1y + 0.2z + 7.85$$
$$x = \frac{0.1y + 0.2z + 7.85}{3}$$

$\boxed{\text{Let } y, z = 0}$

$$x = \frac{7.85}{3} = 2.616$$

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$0.1x + 7y - 0.3z = -19.3$

let $z = 0$ & value of $x = 2.616$

$\Rightarrow (0.1)(2.616) + 7y - 0.3(0) = -19.3$

$\Rightarrow (0.2616) + 7y = -19.3$

$\Rightarrow 7y = -19.3 - 0.2616$

$\Rightarrow y = \frac{-19.3 - 0.2616}{7} = \frac{-19.5616}{7}$

$= -2.7945$

Now $0.3x - 0.2y + 10z = 71.4$

$0.3(2.616) - 0.2(-2.7945) + 10z = 71.4$

$z = 71.4$

$0.7848 + 0.5589 + 10z = 71.4$

$10z = 71.4 - 1.3437$

$10z = 70.0563$

$z = 7.00563$

2nd Iteration

$3x - 0.1y - 0.2z = 7.85$

$3x - 0.1(-2.7945) - 0.2(7.00563) = 7.85$

$3x = 7.85 + (0.2)(7.00563) - 0.27945$

$3x = 7.85 + 1.401126 - 0.27945$

$3x = 8.971676$

$x = \frac{8.971676}{3} = 2.99056$

Now $0.1x + 7y - 0.3z = -19.3$

$(0.1)(2.99056) + 7y - 0.3(71.4) = -19.3$

$0.299056 + 7y - 21.42 = -19.3$

$y = 0.260134$

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$0.3x - 0.2y + 10z = 71.4$

$(0.3)(2.99056) - 0.2(0.260134) + 10z = 71.4$

$10z = 70.5548588$

$z = 7.05548588$

3rd interation

$3x - 0.1y - 0.2z = 7.85$

$3x - 0.1(0.260134) - 0.2(7.05549) = 7.85$

$3x - 0.0260134 - 1.411098 = 7.85$

$3x = 0.0260134 + 1.411098 + 7.85$

$x = 3.0957038$

Now, $0.1x + 7y - 0.3z = -19.3$

$(0.1)(3.0957038) + 7y - 0.3(7.05549) = -19.3$

$y = -3.015296$

Now

$0.3x - 0.2y + 10z = 71.4$

$0.3(3.0957038) - 0.2(-3.015296) + 10z = 71.4$

$0.92871114 + 0.6030592 + 10z = 71.4$

$0.929 + 0.603 + 10z = 71.4$

$69.868 = 10z$

$z = 6.9868$

Q.5 Define consistent and inconsistent system of equations. Hence solve the following system of equations if consistent

$$x+3y+2z=0$$
$$2x-y+3z=0$$
$$3x-5y+4z=0$$
$$x+17y+4z=0$$

We have the following system of equations,

$$x+3y+2z=0$$
$$2x-y+3z=0$$
$$3x-5y+4z=0$$
$$x+17y+4z=0$$

$\Rightarrow [A][X]=[0]$

$$\Rightarrow \begin{bmatrix} 1 & 3 & 2 \\ 2 & -1 & 3 \\ 3 & -5 & 4 \\ 1 & 17 & 4 \end{bmatrix} [X] = 0$$

Augmented Matrix

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 2 & -1 & 3 & 0 \\ 3 & -5 & 4 & 0 \\ 1 & 17 & 4 & 0 \end{bmatrix} \xrightarrow[\substack{R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \\ R_4 \to R_4 - R_1}]{} \begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & -14 & -2 & 0 \\ 0 & 14 & 2 & 0 \end{bmatrix}$$

$\downarrow$ $R_4 \to R_4 + R_3$, $R_3 \to R_3 - 2R_2$

$$\begin{bmatrix} 1 & 3 & 2 & 0 \\ 0 & -7 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A)=2$

$\rho(A:B)=2$

No. of unknown $=3 \Rightarrow$ Infinite sol^n

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$-7x_2 - x_3 = 0$

$-7x_2 = x_3$

$x_1 + 3x_2 + 2x_3 = 0$

let $x_3 = k$

$x_2 = \frac{-k}{7}$

$\therefore x_1 + 3\left(\frac{-k}{7}\right) + 2(k) = 0$

$x_1 - \frac{3k}{7} + 2k = 0$

$x_1 = \frac{3k}{7} - 2k = \frac{-11k}{7}$

Soln $\Rightarrow$ $k\begin{bmatrix} \frac{-11}{7} \\ \frac{-1}{7} \\ 1 \end{bmatrix}$

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Q6 Determine whether the function $T: P_2 \to P_2$ is linear transformation or not,
where $T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

$T(a + bx + cx^2) = (a+1) + (b+1)x + (c+1)x^2$

i) Additive →

$T(u+v) \Rightarrow T(u) + T(v)$

$u = a_1 + b_1 x + c_1 x^2$

$v = a_2 + b_2 x + c_2 x^2$

$T(u+v) \Rightarrow T((a_1 + a_2) + (b_1 + b_2)x + (c_1 + c_2)x^2)$

$\Rightarrow (a_1 + a_2 + 1) + (b_1 + b_2 + 1)x + (c_1 + c_2 + 1)x^2$

$\Rightarrow (a_1 + 1) + (b_1 + 1)x + (c_1 + 1)x^2 +$

$(a_2 + 1) + (b_2 + 1)x + (c_2 + 1)x^2$

$\Rightarrow T(u) + T(v)$ proved

ii) Homogeneity

$\Rightarrow T(ku) \Rightarrow k\,T(u)$

$\Rightarrow T(k(a + bx + cx^2))$

$\Rightarrow T(ka + xbx + kcx^2)$

$\Rightarrow (ka + kb + kc + 1) + (ka + kb + kc + 1)x$

$+ (ka + kb + kc + 1)x^2$

$= k(a+1) + k(b+1)x + k(c+1)x^2$

$k\,T(u)$ proved

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4.14 (76)

Q7 Determine the whether the set $S = \{(1,2,3), (3,1,0), (-2,1,3)\}$ is a basis of $V_3(R)$. [In case S is not a basis determine the dimension and the basis of the sub space spanned by S.

we have the set of vector spaces in R^3

$$S = \{(1,2,3), (3,1,0), (-2,1,3)\}$$

Checking whether the set S is a basis of R^3.

$$\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 0 \\ -2 & 1 & 3 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$
$R_3 \to R_3 + 2R_1$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 5 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 5 & 9 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \\ 0 & 0 & 0 \end{bmatrix}$$

Here, $\rho(S) = 2$

$\therefore$ Linearly dependency Exist.

& the vectors does not span R^3

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In this case,

S is not a basis.

Basis of the sub-space spanned by S

$$= \begin{bmatrix} 1 & 2 & 3 \\ 0 & -5 & -9 \end{bmatrix}$$

Dimension of the sub-space $= 2$

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Q.8 Using Jacobi's method (perform 3 iterations) solve

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

with initial values $x_0 = 1$, $y_0 = 1$, $z_0 = 1$.

Soln: Here, it's a diagonally dominant matrix.

$$3x - 6y + 2z = 23$$
$$-4x + y - z = -15$$
$$x - 3y + 7z = 16$$

So we can use Jacobi method for the above set of system of equations

$$x = \frac{23 + 6y - 2z}{3}$$

$$y = \frac{-15 - y + z}{-4}$$

$$z = \frac{16 - x + 3y}{7}$$

$x_0 = 1$, $y_0 = 1$, $z_0 = 1$

First iteration

$$x = \frac{23 + 6(1) - 2(1)}{3} = \frac{23 - 2 + 6}{3} = 9$$

$$y = \frac{-15 - 1 + 1}{-4} = \frac{15}{4} = 3.75$$

$$z = \frac{16 - 1 + 3}{7} = \frac{18}{7} = 2.571428$$

Second iteration

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Second iteration $x = 9,\ y = 3.75,\ z = 2.571428$

$$x = \frac{23 + 6y - 2z}{3} = \frac{23 + 6(3.75) - 2(2.571428)}{3} = 13.45238$$

$$y = \frac{-15 - y + z}{-4} = \frac{-15 - 3.75 + 2.571428}{-4} = 4.044643$$

$$z = \frac{16 - x + 3y}{7} = \frac{16 - 9 + 3(3.75)}{7} = \frac{18.25}{7} =$$

$$= 2.607142$$

Third iteration $x = 13.45238,\ y = 4.044643,$

$z = 2.607142$

$$x = \frac{23 + 6y - 2z}{3} = \frac{23 + 6(4.044643) - 2(2.607142)}{3}$$

$$= 14.017858$$

$$y = \frac{-15 - y + z}{-4} = \frac{-15 - 4.044643 + 2.607142}{-4}$$

$$= 4.109375$$

$$z = \frac{16 - x + 3y}{7} = \frac{16 - 13.45238 + 3(4.044643)}{7}$$

$$= 2.097364$$

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Q5 Explain one application of matrix operations in image processing with example.

Soln:

Image processing is one such example of matrix operations say multiplication.

Every picture is made up of small-small box and the complete combination of those boxes makes up the complete image. and each boxes in known as pixels.

Every pixel has its own value and that value decides the color of the pixel. The greater valued-pixel make the picture more clear. And lesser the value of pixel makes the picture unclear.

Let assume a cat's picture of black colour in a screen.

Let pixel value = 0 [black]
pixel ~~tota~~ value = 1 [white]

When we collect all values, we set a big matrix.

Different operation in matrix (image) results in different matrix (different image)

We must have listen, we have 120 x 110 px image, that image have 120 rows & 110 columns, ~~to~~

Each image have its own matrix in the form of pixels.

The Sobel matrices, $S_1 = \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$ and

$$S_2 = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ -1 & -2 & -1 \end{bmatrix}$$

provide a method for measuring these intensity changes.

In this way, we multiply with the different matrices of different images in the form of pixels to form new image of newer pixels.

Also, if we have one image have its pixels in the form of matrices.

Then we rotate the image by matrix multiplication.

Rotation About x-axis

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

Rotation About y-axis

$$\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

Rotation About z-axis

$$\begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Q10 Give a brief description of Linear Transformation for computer vision for rotating 2-D image.

Soln Certainly! Linear Transformations play a crucial role in computer vision, including tasks like image processing and geometric manipulation. let's focus on rotating 2D images using linear transformations.

1. Image Translation
> Translation involves shifting an image by a certain distance along the x and y axes
> In Python with open CV, you can achieve translation using the warpAffine function.

Example

```
import numpy as ~~np~~ np
import cv2 as cv

img = cv.imread('girlImage.jpg', 0)

rows, cols = img.shape

M = np.float32([[1, 0, 100],
                [0, 1, 50]])

dst = cv.warpAffine(img, M,
                    (cols, rows))
```

Here, we shift the image 100 units to the right (x-axis) and 50 units downwards (y-axis)

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2. Image Reflection

> Reflection flips an image either vertically or horizontally.

> To flip horizontally.

Example M = np. float32 ([[1, 0, 0],
[0, -1, rows]
[0, 0, 1]])

reflected_img = cv.warpPerspective
(img, M, (cols, rows))

> To flip vertically

M = np. float32 ([[-1, 0, cols],
[0, 1, 0],
[0, 0, 1]])

3. Image Rotation

> Rotation is a linear transformation that rotates data points around a central point (usually the origin)

> In OpenCV, you can rotate an image using the

getRotationMatrix2D
function :

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center = (width/2, height/2)

rotate_matrix = cv. getRotationMatrix2D
(centre = centre, angle = 45,
scale = 1)

rotated_img = cv. warpAffine (img,
rotate_matrix, (cols,
rows))

Here, we rotate the image by 45
degrees clockwise