Subtask 3

$$p(y|x) = \frac{p(x|y)\,p(y)}{p(x)}$$, Bayes Rule.

$$p(y=1|x) = \frac{p(x|y=1)\,p(y=1)}{p(x)}. \quad \text{—(I)}$$

$$p(y) = \phi^{y}(1-\phi)^{1-y} \quad \text{—(i)}$$

$$p(x|y=0) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(x-\mu_0)^T \Sigma^{-1}(x-\mu_0)\right) \quad \text{(ii)}$$

$$p(x|y=1) = \frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}} \exp\left(\frac{-1}{2}(x-\mu_1)^T \Sigma^{-1}(x-\mu_1)\right). \quad \text{(iii)}$$

~~$p(x) = p(x|y=0) + p(x|y=1)$.~~

$$p(x) = p(x|y=1)\,p(y=1) + p(x|y=0).p(y=0). \quad \text{—(iv)}$$

Substituting the value of p(x) from eq^n (iv) in eq^n (I)

$$p(y=1|x) = \frac{p(x|y=1)\,p(y=1)}{p(x|y=1)\,p(y=1) + p(x|y=0).p(y=0)}$$

Dividing by $p(x|y=1).p(y=1)$

$$= \frac{1}{1 + \dfrac{p(x|y=0).p(y=0)}{p(x|y=1)\,p(y=1)}}$$

~~Substituting eq^n (i),(ii),(iii),~~

Substituting eq^n (i), (ii), (iii)

$$P(y=1|x) = \cfrac{1}{1 + \cfrac{(1-\phi)\cancel{\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}}\cdot \exp\left(-\frac{1}{2}(x-\mu_0)^T\Sigma^{-1}(x-\mu_0)\right)}{(\phi)\cancel{\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}}\exp\left(-\frac{1}{2}(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right)}}$$

$$= \frac{1}{1+\frac{(1-\phi)}{(\phi)}\exp\left(-\frac{1}{2}\left((x-\mu_0)^T\cdot\Sigma^{-1}(x-\mu_0) - (x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right)\right)}$$

$$= \frac{1}{\cancel{1+\left(\frac{1}{\phi}-1\right)\exp\left(-\frac{1}{2}\left((x-\mu_0)^T\cdot\Sigma^{-1}(x-\mu_0)-(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right)\right)}}$$

$$= \frac{1}{1+\exp\left(-\frac{1}{2}\left[(x-\mu_0)^T\Sigma^{-1}(x-\mu_0)-(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right]\right)\cdot\exp\left(\ln\left(\frac{1-\phi}{\phi}\right)\right)}$$

$$= \frac{1}{1+\exp\left(\left(-\frac{1}{2}\left[(x-\mu_0)^T\Sigma^{-1}(x-\mu_0)-(x-\mu_1)^T\Sigma^{-1}(x-\mu_1)\right]+\left(\ln\left(\frac{\phi}{1-\phi}\right)\right)\right)\right)}$$

$$= \frac{1}{1+\exp\left(-\left(\theta^T x+\theta_0\right)\right)}.$$

where $\theta^T x = \frac{1}{2}\left[(x-\mu_0)^T \Sigma^{-1} (x-\mu_0) - (x-\mu_1)^T \Sigma^{-1} (x-\mu_1)\right]$

$\theta_0 = \ln\left(\frac{\phi}{1-\phi}\right)$.

$\therefore$ LHS = RHS, proved.