HW 1

Q1

a) $\sum_{i=12}^{n} 5^i = \sum_{i=0}^{n} 5^i - \sum_{i=0}^{11} 5^i$

$= \frac{1-5^{n+1}}{1-5} - \frac{1-5^{12}}{1-5}$

$= \frac{-5^{n+1} + 5^{12}}{4}$

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b) $\sum_{i=0}^{\infty} \frac{3}{11^i} = 3\sum_{i=0}^{\infty} \frac{1}{11^i}$

$= 3\left(\frac{1}{1-\frac{1}{11}}\right)$

$= 3\left(\frac{1}{\frac{11-1}{11}}\right)$

$= 3\left(\frac{11}{10}\right)$

$= \frac{33}{10}$

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c) $\sum_{i=1}^{N} (6i^3 + 3i - 9) = \sum_{i=1}^{N} 6i^3 + \sum_{i=1}^{N} 3i - \sum_{i=1}^{N} 9$

$= 6\sum_{i=1}^{N} i^3 + 3\sum_{i=1}^{N} i - 9\sum_{i=1}^{N} 1$

$= \frac{3}{2}n^2(n+1)^2 + \frac{3}{2}n(n+1) - 9n$

$= \frac{3n^4 + 6n^3 + 6n^2 + 3n}{2} - 9n$

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d) $\sum_{i=91}^{761} \frac{1}{i} = \ln\left(\frac{760}{90}\right) + \text{const}$

Remember the formula for the sum of Harmonic Series.

e) $\sum_{i=0}^{\infty} \frac{i}{79^i}$

$\rightarrow \sum_{i=0}^{\infty} \frac{1}{79} i = \frac{1}{1-\frac{1}{79}}$

Take derivative on both sides

$\sum_{i=0}^{\infty} i \left(\frac{1}{79}\right)^{i-1} = \frac{1}{\left(1-\frac{1}{79}\right)^2}$

Multiply both sides by $\frac{1}{79}$

$\sum_{i=0}^{\infty} \frac{i}{79^i} = \frac{\frac{1}{79}}{\left(1-\frac{1}{79}\right)^2}$

Q: 2

a) $x^1 \cdot x^2 \cdot x^3 \cdots x^{43} = x^{\sum_{i=1}^{43} i}$

$= x^{1+2+3\cdots+43}$

$= x^{\frac{43(43+1)}{2}}$

$= x^{946}$

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b) $\log_4 (19 \cdot 19 \cdot 19 \cdot 19) = \log_4 19^4$

$= 4 \log_4 19$

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c) $32^{\log_{32} 841} = 841$

$\therefore$ log rule : $a^{\log_a b} = b$

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d) $\log_{49} ((7x)^y) = y \log_{49} (7x)$

$= y (\log_{49} 7 + \log_{49} x)$

$= y \log_{49} 7 + y \log_{49} x$

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e) $\sum_{i=1}^{3^N} \log_{18} i = \log_{18} 1 + \log_{18} 2 \cdots\cdots + \log_{18} 3^N$

$= \log_{18} (1 \cdot 2 \cdot 3 \cdots\cdots 3^N)$

$= \log_{18} (3^N!)$

Q: 3

10 choices for each place

Total 12 places

Answer is $\boxed{10^{12}}$

Q: 4

- 28 options for first position
- 27 options for second position
- 26 option for third position
- Only one order is valid out of six

Answer : $\frac{28 \cdot 27 \cdot 26}{3 \cdot 2}$