Q:1 HW2

a) $f(n) = 4n\log n$
$g(n) = n\log n + 5$

$$\lim_{n\to\infty} \frac{4n\log n}{n\log n+5} = 4 < \infty$$

$f(n) = O(g(n)) \leftarrow 4 < \infty$
$f(n) = \Omega(g(n)) \xleftarrow{e.g} n=10, c=\frac{1}{1000} \quad |f(n)| \geq c|g(n)|$

$f(n) = \Theta(g(n)) \leftarrow f(n) = O(g(n)), f(n) = \Omega(g(n))$

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b)
$f(n) = 2^{\log n}$
$g(n) = (\log n)^2$

Note:
$f(n) = \omega(g(n))$
Implies that
$f(n) = \Omega(g(n))$

$$\lim_{n\to\infty} \frac{2^{\log n}}{(\log n)^2} = \infty$$

So
$f(n) = \omega(g(n))$
$f(n) = \Omega(g(n))$

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c)
$f(n) = 4{,}000{,}000^{2000}$
$g(n) = \log_{58}(n)$

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 < \infty$$

So

$$f(n) = O(g(n))$$
$$f(n) = o(g(n))$$

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d)

$$f(n) = 100000 = \text{constant}$$
$$g(n) = \sum_{i=0}^{\infty} \frac{30}{5^i} = 30 \sum_{i=0}^{\infty} \frac{1}{5^i} = 30 \times \frac{5}{4} = \frac{150}{4} = \text{Constant}$$

So

$$f(n) = O(g(n))$$
$$f(n) = \Omega(g(n))$$
$$f(n) = \Theta(g(n))$$

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e)

$$f(n) = \sum_{i=0}^{n} \frac{200}{n} = \text{Constant} \rightarrow \text{Harmonic series}$$

$$g(n) = \sqrt{n} = \text{Not a constant}$$

So

$f(n) = O(g(n))$ Constant will always be lower

$$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0 \rightarrow f(n) = o(g(n))$$

A	B	O	o	Θ	ω	Ω
4n log n	n log n + 5	Yes	No	Yes	No	Yes
$2^{\log n}$	$(\log n)^2$	No	No	No	Yes	Yes
$4{,}000{,}000^{2000}$	$\text{Log}_{58}(n)$	Yes	Yes	No	No	No
100,000	$\sum_{i=0}^{\infty} \frac{30}{5^i}$	Yes	No	Yes	No	Yes
$\sum_{i=0}^{n} \frac{200}{n}$	$\sqrt{n}$	Yes	Yes	No	No	No

Q:2

$$\prod_{n=1}^{n} \left(1 - \frac{1}{k^3}\right) = 0$$

$$\left(1 + \frac{1}{n}\right)^n = e$$

$$\sum_{k=1}^{\log(n)} \frac{n}{3^k} \in \Theta\left(n - n\left(\frac{1}{3^{\log n}}\right)\right) = \Theta(n) \text{ because } \frac{n}{3^{\log n}} < n$$

for $n > 1$

$(n+1)^n \rightarrow$ Dominent term $= n^n$

So

$$0 = \prod_{n=1}^{n} \left[1 - \frac{1}{k^3}\right] < 1 = \left(1 + \frac{1}{n}\right)^n < \ln(n) < \sqrt[3]{n}$$

$$< \sum_{k=1}^{\log(n)} \frac{n}{3^k} = \frac{n}{4} < n^3 < n^9 + n + 2 < (n+1)^n < n^{n^n}$$