Characterizes the roots of a Quadratic Equation using the Discriminant.

M9AL-Ic-1

Essential Concept

In the Quadratic formula $x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$, the radicand $b^2 - 4ac$ is the Discriminant. Thus, D = $b^2 - 4ac$

Perfect Squares are product of integers multiplied by itself.
Ex. 1, 4, 9, 16, 25, 36, 49, 64, 81,100, …

Non-Perfect Squares are numbers whose square roots contain decimal.
Ex. 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, …

Value of Discriminant	Nature/Characteristics of the Roots
D < 0	Imaginary/complex
D = 0	Real, equal
D > 0 D is a perfect square	Rational, unequal
D > 0 D is a non-perfect square	Irrational, unequal

Imaginary/Complex are numbers/roots when squared it gives the negative result.
Ex. i, 3i, 5 – 2i

Real are numbers/roots that are rational or irrational
Ex. 2, -5, $\frac{1}{2}$, $\sqrt{10}$

Rational are numbers/roots that can be written as a fraction
Ex. 3, $\frac{1}{2}$, $-\frac{3}{5}$

Irrational are numbers/roots that cannot be written as a fraction
Ex. $\sqrt{2}$, $\sqrt{10}$, 1.23456…

Steps:

1. Transform the Quadratic Equation into General Form $ax^2 + bx + c = 0$
2. Identify the values *a, b,* and *c* in the Quadratic Equation
3. Evaluate the Discriminant, D = $b^2 - 4ac$
4. Determine the nature/characteristics of the roots

Example:

Characterize the roots of the following Quadratic Equations using the Discriminant.

1. $x^2 + 6x = -9$

Solution:

Step 1: Transform

$x^2 + 6x = -9$

$x^2 + 6x + 9 = -9 + 9$

$x^2 + 6x + 9 = 0$ General Form

Step 2: Identify *a, b, c*

$x^2 + 6x + 9 = 0$

a = 1 b = 6 c = 9

Step 3: Evaluate D = $b^2 - 4ac$

a = 1 b = 6 c = 9

D = $b^2 - 4ac$

D = $6^2 - 4(1)(9)$

D = 36 – 36

D = 0

Step 4: Determine the nature of the roots

Since D = 0, therefore the roots are **Real and equal**

2. $-2x^2 + 3x = 2$

Solution:

Step 1: Transform

$-2x^2 + 3x = 2$

$-2x^2 + 3x - 2 = 2 - 2$

$-2x^2 + 3x - 2 = 0$ General Form

Step 2: Identify *a, b, c*

$-2x^2 + 3x - 2 = 0$

a = -2 b = 3 c = -2

Step 3: Evaluate D = $b^2 - 4ac$

a = -2 b = 3 c = -2

D = $b^2 - 4ac$

D = $3^2 - 4(-2)(-2)$

D = 9 – 16

D = -7

Step 4: Determine the nature of the roots

Since D = -7, and

-7 < 0, so

D < 0, therefore the roots are **Imaginary/complex.**

3. $x^2 - 5 = 4x$

Solution:

Step 1: Transform

$x^2 - 5 = 4x$

$x^2 - 5 - 4x = 4x - 4x$

$x^2 - 4x - 5 = 0$ General Form

Step 2: Identify *a, b, c*

$x^2 - 4x - 5 = 0$

a = 1 b = -4 c = -5

Step 3: Evaluate D = $b^2 - 4ac$

a = 1 b = -4 c = -5

D = $b^2 - 4ac$

D = $(-4)^2 - 4(1)(-5)$

D = 16 + 20

D = 36

Step 4: Determine the nature of the roots

Since D = 36, and

36 > 0, also 36 is a perfect square

D > 0, D is a perfect square,

therefore, the roots are **Rational and unequal**

Exercises

I. Let' do this together!

A. *Transform to a better form!*

Identify the correct transformation of Quadratic Equation into the General form.

1. $2x^2 - x = 1$
   a. $2x^2 - x + 1 = 1 + 1 \rightarrow 2x^2 - x + 1 = 0$
   b. $2x^2 - x - 1 = 1 - 1 \rightarrow 2x^2 - x - 1 = 0$

2. $x^2 + 5 = -7x$
   a. $x^2 + 5 + 7 = -7x + 7 \rightarrow x^2 + 7 + 5 = 0$
   b. $x^2 + 5 + 7x = -7x + 7x \rightarrow x^2 + 7x + 5 = 0$

3. $-8x + 8 = -2x^2$
   a. $-8x + 8 + 2x^2 = -2x^2 + 2x^2 \rightarrow 2x^2 - 8x + 8 = 0$
   b. $-8x + 8 + 2x = -2x^2 + 2x \rightarrow 2x - 8x + 8 = 0$

4. $-x^2 - 4x = 5$
   a. $-x^2 - 4x - 5 = 5 - 5 \rightarrow -x^2 - 4x - 5 = 0$
   b. $-x^2 - 4x + 5 = 5 + 5 \rightarrow -x^2 - 4x + 5 = 0$

5. $5x^2 + 3x = 6x - 1$
   a. $5x^2 + 3x + 6x - 1 = 6x - 1 + 6x - 1 \rightarrow 5x^2 + 9x - 1 = 0$
   b. $5x^2 + 3x - 6x + 1 = 6x - 1 - 6x + 1 \rightarrow 5x^2 - 3x + 1 = 0$

B. *Know your value!*

Identify the values of *a, b,* and *c* from the Quadratic Equations

1. $2x^2 - x - 1 = 0$
   a. $a = 2, b = -1, c = -1$ b. $a = 2, b = 1, c = 1$

2. $x^2 + 7x + 5 = 0$
   a. $a = 0, b = 7, c = 5$ b. $a = 1, b = 7, c = 5$

3. $2x^2 - 8x + 8 = 0$
   a. $a = 2, b = -8, c = 8$ b. $a = 2, b = 8, c = 8$

4. $-x^2 - 4x - 5 = 0$
   a. $a = -0, b = -4, c = -5$ b. $a = -1, b = -4, c = -5$

5. $5x^2 - 3x + 1 = 0$
   a. $a = -0, b = -4, c = -10$ b. $a = -1, b = -4, c = -10$

C. *Write what's right!*

Evaluate the discriminant ($D = b^2 - 4ac$) of the Quadratic Equations.

1. $2x^2 - x - 1 = 0$; $a = 2$, $b = -1$, $c = -1$

a.
$$D = b^2 - 4ac$$
$$D = (-1)^2 - 4(2)(-1)$$
$$D = 1 - (-8)$$
$$D = 1 + 8$$
$$D = 9$$

b.
$$D = b^2 - 4ac$$
$$D = 2^2 - 4(-1)(-1)$$
$$D = 4 - 4$$
$$D = 0$$

2. $x^2 + 7x + 5 = 0$; $a = 1$, $b = 7$, $c = 5$

a.
$$D = b^2 - 4ac$$
$$D = 1^2 - 4(7)(5)$$
$$D = 1 - 140$$
$$D = -139$$

b.
$$D = b^2 - 4ac$$
$$D = 7^2 - 4(1)(5)$$
$$D = 49 - 20$$
$$D = 29$$

3. $2x^2 - 8x + 8 = 0$; $a = 2$, $b = -8$, $c = 8$

a.
$$D = b^2 - 4ac$$
$$D = 8^2 - 4(2)(8)$$
$$D = 16 - 64$$
$$D = -48$$

b.
$$D = b^2 - 4ac$$
$$D = (-8)^2 - 4(2)(8)$$
$$D = 64 - 64$$
$$D = 0$$

4. $-x^2 - 4x - 5 = 0$; $a = -1$, $b = -4$, $c = -5$

a.
$$D = b^2 - 4ac$$
$$D = (-1)^2 - 4(-4)(-5)$$
$$D = 1 - 80$$
$$D = -79$$

b.
$$D = b^2 - 4ac$$
$$D = (-4)^2 - 4(-1)(-5)$$
$$D = 16 - 20$$
$$D = -4$$

5. $5x^2 - 3x + 1 = 0$; $a = 5$, $b = -3$, $c = 1$

a.
$$D = b^2 - 4ac$$
$$D = (-3)^2 - 4(5)(0)$$
$$D = 9 - 0$$
$$D = 9$$

b.
$$D = b^2 - 4ac$$
$$D = (-3)^2 - 4(5)(1)$$
$$D = 9 - 20$$
$$D = -11$$

D. *Ensure your nature!*

Characterize the Roots of the Quadratic Equations using the Discriminant.

1. $5x^2 - 3x + 1 = 0$; $D = 9$

a. Since D = 9,
D < 0,
therefore, the roots are **Imaginary/ complex**

b. Since D = 9,
D = 0,
therefore, the roots are **Real and equal**

c. Since D = 9,
D > 0, and D is a perfect square,
therefore, the roots are **Rational and unequal.**

d. Since D = 9,
D > 0, and D is a non-perfect square,
therefore, the roots are **Irrational and unequal.**

2. $x^2 + 7x + 5 = 0$; *D = 29*

a. Since D = 29,
D < 0,
therefore, the roots are **Imaginary/ complex**

b. Since D = 29,
D = 0,
therefore, the roots are **Real and equal**

c. Since D = 29,
D > 0, and D is a perfect square,
therefore, the roots are **Rational and unequal.**

d. Since D = 29,
D > 0, and D is a non-perfect square,
therefore, the roots are **Irrational and unequal.**

3. $2x^2 – 8x + 8 = 0$; *D = 0*

a. Since D = 0,
D < 0,
therefore, the roots are **Imaginary/ complex**

c. Since D = 0,
D = 0,
therefore, the roots are **Real and equal**

d. Since D = 0,
D > 0, and D is a perfect square,
therefore, the roots are **Rational and unequal.**

e. Since D = 0,
D > 0, and D is a non-perfect square,
therefore, the roots are **Irrational and unequal.**

4. $-x^2 – 4x – 5 = 0$; *D = -4*

a. Since D = -4,
D < 0,
therefore, the roots are **Imaginary/ complex**

b. Since D = -4,
D = 0,
therefore, the roots are **Real and equal**

c. Since D = -4,
D > 0, and D is a perfect square,
therefore, the roots are **Rational and unequal.**

d. Since D = -4,
D > 0, and D is a non-perfect square,
therefore, the roots are **Irrational and unequal.**

5. $5x^2 – 3x + 1 = 0$; *D = -11*

a. Since D = -11,
D < 0,
therefore, the roots are **Imaginary/ complex**

b. Since D = -11,
D = 0,
therefore, the roots are **Real and equal**

c. Since D = -11,
D > 0, and D is a perfect square, therefore, the roots are **Rational and unequal.**

d. Since D = -11,
D > 0, and D is a non-perfect square, therefore, the roots are **Irrational and unequal.**

II. You are smart independent!

Characterize the roots of the Quadratic Equations using the discriminant.
Type in the value of the discriminant and select the letter that best describes the nature of the roots.

1. $4x^2 + x = -1$; *D =* ___

   a. Roots are imaginary/complex
   b. Roots are Real and Equal
   c. Roots are Rational and Unequal
   d. Roots are Irrational and Unequal

2. $x^2 + 8 = 2x$; *D =* ___

   a. Roots are imaginary/complex
   b. Roots are Real and Equal
   c. Roots are Rational and Unequal
   d. Roots are Irrational and Unequal

3. $6x - 3 = 3x^2$; *D =* ___

   a. Roots are imaginary/complex
   b. Roots are Real and Equal
   c. Roots are Rational and Unequal
   d. Roots are Irrational and Unequal

4. $10x^2 - 6x = 2x^2 + 2$; *D =* ___

   a. Roots are imaginary/complex
   b. Roots are Real and Equal
   c. Roots are Rational and Unequal
   d. Roots are Irrational and Unequal

5. $3(2x^2 - x + 1) = 0$; *D =* ___

   a. Roots are imaginary/complex
   b. Roots are Real and Equal
   c. Roots are Rational and Unequal
   d. Roots are Irrational and Unequal

Describes the Relationship between the coefficients and the sum and product of the roots of a Quadratic Equation.
M9AL-Ic-2

Essential Concept

The Quadratic Equation $ax^2 + bx + c = 0$ with coefficients a, b, and c, has two roots x_1 and x_2 that can be obtained using Quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Thus, $x_1 = \frac{-b + \sqrt{b^2-4ac}}{2a}$ and $x_2 = \frac{-b - \sqrt{b^2-4ac}}{2a}$.

Since we are to describe the relationship between the coefficients and the sum and the product of the roots of a Quadratic Equation, then we will get the sum ($x_1 + x_2$) and the product ($x_1 \bullet x_2$) of the roots in terms of the coefficients *a, b,* and *c.*

That means,

$$x_1 + x_2 = \frac{-b + \sqrt{b^2-4ac}}{2a} + \frac{-b - \sqrt{b^2-4ac}}{2a}$$

$$x_1 + x_2 = \frac{-b + \sqrt{b^2-4ac}}{2a} + \frac{-b - \sqrt{b^2-4ac}}{2a}$$

$$x_1 + x_2 = \frac{-2b}{2a}$$

$$\boldsymbol{x_1 + x_2 = \frac{-b}{a}}$$

and,

$$x_1 \bullet x_2 = \frac{-b + \sqrt{b^2-4ac}}{2a} \bullet \frac{-b - \sqrt{b^2-4ac}}{2a}$$

$$x_1 \bullet x_2 = \frac{(-b)^2 - \left(\sqrt{b^2-4ac}\right)^2}{2a \bullet 2a}$$

$$x_1 \bullet x_2 = \frac{b^2 - (b^2-4ac)}{4a^2}$$

$$x_1 \bullet x_2 = \frac{4ac}{4a^2}$$

$$\boldsymbol{x_1 \bullet x_2 = \frac{c}{a}}$$

Therefore, in a quadratic equation $ax^2 + bx + c = 0$ where coefficients are a, b, and c, the sum of the roots is described as the negative ratio of coefficients *b* and *a* while the product of the roots is the ratio of coefficients *c* and *a.* In symbols,

$$\boldsymbol{x_1 + x_2 = \frac{-b}{a}} \quad \textbf{and} \quad \boldsymbol{x_1 \bullet x_2 = \frac{c}{a}}$$

Now we can use this information to formulate the quadratic equation given the sum and product of its roots and vice versa.

Finding the sum and product of the roots of the quadratic equation

Example:

Find the sum and product of the roots of a quadratic equation using its coefficients.

1. $x^2 + 6x + 8 = 0$

Step 1 : Identify the coefficients

$a = 1, \quad b = 6, \quad c = 8$

Step 2: Use the information we have derived to get the sum and product of the roots

$$x_1 + x_2 = \frac{-b}{a} \qquad x_1 \bullet x_2 = \frac{c}{a}$$
$$x_1 + x_2 = \frac{-6}{1} \qquad x_1 \bullet x_2 = \frac{8}{1}$$
$$\boldsymbol{x_1 + x_2 = -6} \qquad \boldsymbol{x_1 \bullet x_2 = 8}$$

Step 3: Verify by finding and using the roots of the equation

$x^2 + 6x + 8 = 0, \quad a = 1, \; b = 6, \; c = 8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-6 \pm \sqrt{6^2 - 4(1)(8)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{36 - 32}}{2}$$

$$x_1 = \frac{-6 + \sqrt{4}}{2} \qquad x_2 = \frac{-6 - \sqrt{4}}{2}$$

$$x_1 = \frac{-6 + 2}{2} \qquad x_2 = \frac{-6 - 2}{2}$$

$$x_1 = -2 \qquad x_2 = -4$$

Now let's solve for the sum and product of the roots

$$x_1 + x_2 = -2 + (-4) \qquad x_1 \bullet x_2 = -2 \bullet -4$$
$$\boldsymbol{x_1 + x_2 = -6} \qquad \boldsymbol{x_1 \bullet x_2 = 8}$$

Therefore, it is verified that the sum and product of the roots of $x^2 + 6x + 8 = 0$ are -6 and 8 respectively. Further, it is shown that the formula

$$x_1 + x_2 = \frac{-b}{a} \text{ and } x_1 \bullet x_2 = \frac{c}{a}$$

for the sum and product of the roots in terms of the coefficients are true.

Quadratic Equation	Sum of the Roots	Product of the Roots	Roots
2. $x^2 - 5x + 6 = 0$ a = 1 b = -5 c = 6	$x_1 + x_2 = \frac{-b}{a}$ $x_1 + x_2 = \frac{-(-5)}{1}$ $\boldsymbol{x_1 + x_2 = 5}$	$x_1 \bullet x_2 = \frac{c}{a}$ $x_1 \bullet x_2 = \frac{6}{1}$ $\boldsymbol{x_1 \bullet x_2 = 6}$	$x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$ $x = \frac{-(-5)\pm\sqrt{(-5)^2-4(1)(6)}}{2(1)}$ $x = \frac{5 \pm\sqrt{25-24}}{2}$ $x = \frac{5 \pm\sqrt{1}}{2}$ $x_1 = \frac{5+1}{2}$ $x_2 = \frac{5-1}{2}$ $\boldsymbol{x_1 = 3}$ $\boldsymbol{x_2 = 2}$ Verify, $x_1 + x_2 = 3 + 2 = 5$ $x_1 \bullet x_2 = 3 \bullet 2 = \boldsymbol{6}$
3. $2x^2 - 8x - 10 = 0$ a = 2 b = -8 c = -10	$x_1 + x_2 = \frac{-b}{a}$ $x_1 + x_2 = \frac{-(-8)}{2}$ $\boldsymbol{x_1 + x_2 = 4}$	$x_1 \bullet x_2 = \frac{c}{a}$ $x_1 \bullet x_2 = \frac{-10}{2}$ $\boldsymbol{x_1 \bullet x_2 = -5}$	$x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$ $x = \frac{-(-8)\pm\sqrt{(-8)^2-4(2)(-10)}}{2(2)}$ $x = \frac{8 \pm\sqrt{64-(-80)}}{4}$ $x = \frac{8 \pm\sqrt{144}}{4}$ $x_1 = \frac{8+12}{4}$ $x_2 = \frac{8-12}{4}$ $\boldsymbol{x_1 = 5}$ $\boldsymbol{x_2 = -1}$ Verify, $x_1 + x_2 = 5 + (-1) = \boldsymbol{4}$ $x_1 \bullet x_2 = 5 \bullet (-1) = \boldsymbol{-5}$
4. $4x^2 + 8x + 3 = 0$ a = 4 b = 8 c = 3	$x_1 + x_2 = \frac{-b}{a}$ $x_1 + x_2 = \frac{-8}{4}$ $\boldsymbol{x_1 + x_2 = -2}$	$x_1 \bullet x_2 = \frac{c}{a}$ $\boldsymbol{x_1 \bullet x_2 = \frac{3}{4}}$	$x = \frac{-b \pm\sqrt{b^2-4ac}}{2a}$ $x = \frac{-8\pm\sqrt{8^2-4(4)(3)}}{2(4)}$ $x = \frac{-8 \pm\sqrt{64-48}}{8}$ $x = \frac{-8 \pm\sqrt{16}}{8}$

			$x_1 = \frac{-8+4}{8}$ $x_2 = \frac{-8-4}{8}$ $x_1 = \frac{-4}{8}$ $x_2 = \frac{-12}{8}$ $\boldsymbol{x_1} = \frac{-1}{2}$ $\boldsymbol{x_2} = \frac{-3}{2}$ Verify, $x_1 + x_2 = \frac{-1}{2} + \left(\frac{-3}{2}\right) = \frac{-4}{2} = \mathbf{-2}$ $x_1 \bullet x_2 = \frac{-1}{2} \bullet \left(\frac{-3}{2}\right) = \mathbf{\frac{3}{4}}$

Exercises

I. Let' do this together!

Find the sum and product of each quadratic equation below using its coefficients.

1. $x^2 + 4x + 3 = 0$

   Sum of the roots:

   a. $x_1 + x_2 = \frac{-b}{a}$
      $x_1 + x_2 = \frac{4}{1}$
      $\boldsymbol{x_1 + x_2 = 4}$

   b. $x_1 + x_2 = \frac{-b}{a}$
      $x_1 + x_2 = \frac{-4}{1}$
      $\boldsymbol{x_1 + x_2 = -4}$

   Product of the roots:

   a. $x_1 \bullet x_2 = \frac{c}{a}$
      $x_1 \bullet x_2 = \frac{3}{1}$
      $\boldsymbol{x_1 \bullet x_2 = 3}$

   b. $x_1 \bullet x_2 = \frac{c}{a}$
      $x_1 \bullet x_2 = \frac{3}{3}$
      $\boldsymbol{x_1 \bullet x_2 = 1}$

2. $x^2 - 5x - 6 = 0$

   Sum of the roots:

   a. $x_1 + x_2 = \frac{-b}{a}$
      $x_1 + x_2 = \frac{-5}{1}$
      $\boldsymbol{x_1 + x_2 = -5}$

   b. $x_1 + x_2 = \frac{-b}{a}$
      $x_1 + x_2 = \frac{-(-5)}{1}$
      $\boldsymbol{x_1 + x_2 = 5}$

   Product of the roots:

   a. $x_1 \bullet x_2 = \frac{c}{a}$
      $x_1 \bullet x_2 = \frac{-6}{1}$
      $\boldsymbol{x_1 \bullet x_2 = -6}$

   b. $x_1 \bullet x_2 = \frac{c}{a}$
      $x_1 \bullet x_2 = \frac{6}{1}$
      $\boldsymbol{x_1 \bullet x_2 = 6}$

3. $2x^2 + 3x - 2 = 0$

Sum of the roots:

a. $x_1 + x_2 = \frac{-b}{a}$

$\boldsymbol{x_1 + x_2 = \frac{-3}{2}}$

b. $x_1 + x_2 = \frac{-b}{a}$

$x_1 + x_2 = \frac{-3}{2}$

$\boldsymbol{x_1 + x_2 = -1}$

Product of the roots:

a. $x_1 \bullet x_2 = \frac{c}{a}$

$x_1 \bullet x_2 = \frac{-2}{2}$

$\boldsymbol{x_1 \bullet x_2 = -1}$

b. $x_1 \bullet x_2 = \frac{c}{a}$

$x_1 \bullet x_2 = \frac{2}{2}$

$\boldsymbol{x_1 \bullet x_2 = 1}$

4. $3x^2 + 3x - 36 = 0$

Sum of the roots:

a. $x_1 + x_2 = \frac{b}{a}$

$x_1 + x_2 = \frac{3}{3}$

$\boldsymbol{x_1 + x_2 = 1}$

b. $x_1 + x_2 = \frac{-b}{a}$

$x_1 + x_2 = \frac{-3}{3}$

$\boldsymbol{x_1 + x_2 = -1}$

Product of the roots:

a. $x_1 \bullet x_2 = \frac{c}{a}$

$x_1 \bullet x_2 = \frac{36}{3}$

$\boldsymbol{x_1 \bullet x_2 = 12}$

b. $x_1 \bullet x_2 = \frac{c}{a}$

$x_1 \bullet x_2 = \frac{36}{3}$

$\boldsymbol{x_1 \bullet x_2 = 36}$

5. $2x^2 + 7x - 4 = 0$

Sum of the roots:

a. $x_1 + x_2 = \frac{b}{a}$

$\boldsymbol{x_1 + x_2 = \frac{7}{2}}$

b. $x_1 + x_2 = \frac{-b}{a}$

$\boldsymbol{x_1 + x_2 = \frac{-7}{2}}$

Product of the roots:

a. $x_1 \bullet x_2 = \frac{c}{a}$

$x_1 \bullet x_2 = \frac{-4}{2}$

$\boldsymbol{x_1 \bullet x_2 = -2}$

b. $x_1 \bullet x_2 = \frac{c}{a}$

$x_1 \bullet x_2 = \frac{4}{2}$

$\boldsymbol{x_1 \bullet x_2 = 4}$

Formulating Quadratic Equation given the sum and product of the roots

Quadratic Equation $ax^2 + bx + c = 0$ can also be written as $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$. So, to formulate the quadratic equation given the sum and product of the roots, take note that:

$x_1 + x_2 = \frac{-b}{a}$ and $x_1 \bullet x_2 = \frac{c}{a}$

Example:

Formulate the quadratic equation using the sum and product of its roots.

1. $\boldsymbol{x_1 + x_2 = -4}$ $\boldsymbol{x_1 \bullet x_2 = 3}$

We know that:

$$x_1 + x_2 = \frac{-b}{a} \qquad x_1 \bullet x_2 = \frac{c}{a}$$

So,

$$-4 = \frac{-b}{a} \qquad 3 = \frac{c}{a}$$

$$4 = \frac{b}{a}$$

Substitute the sum and product of the roots to the equation

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$\boldsymbol{x^2 + 4x + 3 = 0}$$

Therefore, the quadratic equation whose roots have sum and product of -4 and 3 respectively is $\boldsymbol{x^2 + 4x + 3 = 0.}$

2. $\boldsymbol{x_1 + x_2 = 5}$ $\boldsymbol{x_1 \bullet x_2 = -6}$

We know that:

$$x_1 + x_2 = \frac{-b}{a} \qquad x_1 \bullet x_2 = \frac{c}{a}$$

So,

$$5 = \frac{-b}{a} \qquad -6 = \frac{c}{a}$$

$$-5 = \frac{b}{a}$$

Substitute the sum and product of the roots to the equation

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$\boldsymbol{x^2 - 5x - 6 = 0}$$

Therefore, the quadratic equation whose roots have sum and product of -5 and -6 respectively is $\boldsymbol{x^2 - 5x - 6 = 0.}$

3. $\boldsymbol{x_1 + x_2 = -1}$ $\qquad$ $\boldsymbol{x_1 \bullet x_2 = -6}$

$$x_1 + x_2 = \frac{-b}{a} \qquad x_1 \bullet x_2 = \frac{c}{a}$$
$$-1 = \frac{-b}{a} \qquad -6 = \frac{c}{a}$$
$$1 = \frac{b}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$\boldsymbol{x^2 + x - 6 = 0}$$

4. $\boldsymbol{x_1 + x_2 = -\frac{3}{2}}$ $\qquad$ $\boldsymbol{x_1 \bullet x_2 = -1}$

$$x_1 + x_2 = \frac{-b}{a} \qquad x_1 \bullet x_2 = \frac{c}{a}$$
$$-\frac{\mathbf{3}}{\mathbf{2}} = \frac{-b}{a} \qquad -1 = \frac{c}{a}$$
$$\frac{\mathbf{3}}{\mathbf{2}} = \frac{b}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{\mathbf{3}}{\mathbf{2}}x - 1 = 0$$
$$2\left(x^2 + \frac{\mathbf{3}}{\mathbf{2}}x - 1 = 0\right)$$ *multiply the whole equation by the denominator 2.*
$$\boldsymbol{2x^2 + 3x - 2 = 0}$$

5. $\boldsymbol{x_1 + x_2 = -\frac{7}{2}}$ $\qquad$ $\boldsymbol{x_1 \bullet x_2 = -2}$

$$x_1 + x_2 = \frac{-b}{a} \qquad x_1 \bullet x_2 = \frac{c}{a}$$
$$-\frac{7}{2} = \frac{-b}{a} \qquad -2 = \frac{c}{a}$$
$$\frac{7}{2} = \frac{b}{a}$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{\mathbf{7}}{\mathbf{2}}x - 2 = 0$$
$$2\left(x^2 + \frac{\mathbf{7}}{\mathbf{2}}x - 2 = 0\right)$$ *multiply the whole equation by the denominator 2.*
$$\boldsymbol{2x^2 + 7x - 4 = 0}$$

Exercises

I. Let' do this together!

A. Solve for the sum and product of the roots.

1. $\boldsymbol{x_1 = 2 \quad and \quad x_2 = -4}$

$x_1 + x_2 = \mathbf{2} + (-4)$ $\qquad$ $x_1 \bullet x_2 = \mathbf{2} \bullet (-4)$

$x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

2. $\boldsymbol{x_1 = 5 \quad and \quad x_2 = 3}$

$x_1 + x_2 = \mathbf{5} + 3$ $\qquad$ $x_1 \bullet x_2 = \mathbf{5} \bullet 3$

$x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

3. $\boldsymbol{x_1 = -6 \quad and \quad x_2 = -6}$

$x_1 + x_2 = \mathbf{-6} + (-6)$ $\qquad$ $x_1 \bullet x_2 = \mathbf{-6} \bullet (-6)$

$x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

4. $\boldsymbol{x_1 = \frac{2}{3} \quad and \quad x_2 = 1}$

$x_1 + x_2 = \boldsymbol{\frac{2}{3}} + 1$ $\qquad$ $x_1 \bullet x_2 = \boldsymbol{\frac{2}{3}} \bullet 1$

$x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

5. $\boldsymbol{x_1 = -3 \quad and \quad x_2 = \frac{3}{2}}$

$x_1 + x_2 = -3 + \boldsymbol{\frac{3}{2}}$ $\qquad$ $x_1 \bullet x_2 = \mathbf{-3} \bullet \boldsymbol{\frac{3}{2}}$

$x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

B. Formulate the Quadratic Equation using the sum and product of its root.

1. $\boldsymbol{x_1 + x_2 = 2}$ **and** $\boldsymbol{x_1 \bullet x_2 = -6}$

a. $x_1 + x_2 = \frac{-b}{a}$ $\quad$ $x_1 \bullet x_2 = \frac{c}{a}$

$2 = \frac{-b}{a}$ $\quad$ $-6 = \frac{c}{a}$

$-2 = \frac{b}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\boldsymbol{x^2 - 2x - 6 = 0}$

b. $x_1 + x_2 = \frac{-b}{a}$ $\quad$ $x_1 \bullet x_2 = \frac{c}{a}$

$2 = \frac{-b}{a}$ $\quad$ $6 = \frac{c}{a}$

$2 = \frac{b}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\boldsymbol{x^2 + 2x + 6 = 0}$

2. $\boldsymbol{x_1 + x_2 = 8}$ **and** $\boldsymbol{x_1 \bullet x_2 = 15}$

a. $x_1 + x_2 = \frac{-b}{a}$ $\quad$ $x_1 \bullet x_2 = \frac{c}{a}$

$8 = \frac{-b}{a}$ $\quad$ $15 = \frac{c}{a}$

$8 = \frac{b}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\boldsymbol{x^2 + 8x + 15 = 0}$

b. $x_1 + x_2 = \frac{-b}{a}$ $\quad$ $x_1 \bullet x_2 = \frac{c}{a}$

$8 = \frac{-b}{a}$ $\quad$ $15 = \frac{c}{a}$

$-8 = \frac{b}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\boldsymbol{x^2 - 8x + 15 = 0}$

3. $\boldsymbol{x_1 + x_2 = -12}$ **and** $\boldsymbol{x_1 \bullet x_2 = 36}$

a. $x_1 + x_2 = \frac{-b}{a}$ $\qquad$ $x_1 \bullet x_2 = \frac{c}{a}$

$-12 = \frac{-b}{a}$ $\qquad$ $36 = \frac{c}{a}$

$12 = \frac{b}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\boldsymbol{x^2 + 36x + 12 = 0}$

b. $x_1 + x_2 = \frac{-b}{a}$ $\qquad$ $x_1 \bullet x_2 = \frac{c}{a}$

$-12 = \frac{-b}{a}$ $\qquad$ $36 = \frac{c}{a}$

$12 = \frac{b}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$\boldsymbol{x^2 + 12x + 36 = 0}$

4. $\boldsymbol{x_1 + x_2 = \frac{5}{3}}$ **and** $\boldsymbol{x_1 \bullet x_2 = \frac{2}{3}}$

a. $x_1 + x_2 = \frac{-b}{a}$ $\qquad$ $x_1 \bullet x_2 = \frac{c}{a}$

$\frac{5}{3} = \frac{-b}{a}$ $\qquad$ $\frac{2}{3} = \frac{c}{a}$

$-\frac{5}{3} = \frac{b}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 + \frac{5}{3}x - \frac{2}{3} = 0$

$\boldsymbol{3x^2 + 5x - 2 = 0}$

b. $x_1 + x_2 = \frac{-b}{a}$ $\qquad$ $x_1 \bullet x_2 = \frac{c}{a}$

$\frac{5}{3} = \frac{-b}{a}$ $\qquad$ $\frac{2}{3} = \frac{c}{a}$

$-\frac{5}{3} = \frac{b}{a}$

$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$

$x^2 - \frac{5}{3}x + \frac{2}{3} = 0$

$\boldsymbol{3x^2 - 5x + 2 = 0}$

5. $\boldsymbol{x_1 = -3 \quad and \quad x_2 = \frac{3}{2};}$ $\qquad$ $\boldsymbol{x_1 + x_2 = -\frac{3}{2}}$ **and** $\boldsymbol{x_1 \bullet x_2 = -\frac{9}{2}}$

a. $x_1 + x_2 = \frac{-b}{a}$ $\qquad$ $x_1 \bullet x_2 = \frac{c}{a}$

$\frac{\mathbf{3}}{\mathbf{2}} = \frac{-b}{a}$ $\qquad$ $-\frac{\mathbf{9}}{\mathbf{2}} = \frac{c}{a}$

$-\frac{\mathbf{3}}{\mathbf{2}} = \frac{b}{a}$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 + \frac{\mathbf{9}}{\mathbf{2}}x - \frac{\mathbf{3}}{\mathbf{2}} = 0$$
$$\mathbf{2x^2 + 9x - 3 = 0}$$

b. $x_1 + x_2 = \frac{-b}{a}$ $\qquad$ $x_1 \bullet x_2 = \frac{c}{a}$

$\frac{\mathbf{3}}{\mathbf{2}} = \frac{-b}{a}$ $\qquad$ $-\frac{\mathbf{9}}{\mathbf{2}} = \frac{c}{a}$

$-\frac{\mathbf{3}}{\mathbf{2}} = \frac{b}{a}$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$
$$x^2 - \frac{\mathbf{3}}{\mathbf{2}}x - \frac{\mathbf{9}}{\mathbf{2}} = 0$$
$$\mathbf{2x^2 - 3x - 9 = 0}$$

III. You are smart independent!

A. Solve for the sum and product of each quadratic equation below using its coefficients.

1. $x^2 + 7x - 3 = 0$
   $x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

2. $x^2 - 5x + 4 = 0$
   $x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

3. $2x^2 + 10x - 6 = 0$
   $x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

4. $4x^2 - 8x - 2 = 0$
   $x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

5. $3x^2 - 5x + 6 = 0$
   $x_1 + x_2 =$ ____ $\qquad$ $x_1 \bullet x_2 =$ ____

B. Formulate the Quadratic Equation given the sum and product of the roots.

1. $x_1 + x_2$ = -7 $x_1 \bullet x_2$ = -3

2. $x_1 + x_2$ = 5 $x_1 \bullet x_2$ = 4

3. $x_1 + x_2$ = -5 $x_1 \bullet x_2$ = -3

4. $x_1 + x_2$ = 2 $x_1 \bullet x_2 = -\frac{1}{2}$

5. $x_1 + x_2 = -\frac{5}{3}$ $x_1 \bullet x_2$ = 2

Analyzes the effects of changing the values of a, h and k in the equation y = a(x – h)2 + k of a quadratic function on its graph
M9AL-Ii-2

Essential Concept
The Quadratic Equation $ax^2 + bx + c = 0$ with coefficients a, b, and c, has two roots x_1 and x_2 that