Wireless Communication

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1 Transmit and Receive Signal Models

In wireless communication, the transmitted and received signals are real. While the model of communication channels using a complex frequency response for simplicity, in fact the channel just introduces an amplitude and phase change at each frequency of the transmitted signal so that the received signal is also real. Real modulated and demodulated signals are often represented as the real part of a complex signal to facilitate analysis. This model gives rise to the complex baseband representation of bandpass signals. The transmitted signal is modeled as

$$\begin{aligned} s(t) &= \Re\left\{u(t)e^{j2\pi f_c t}\right\} \\ &= \Re\{u(t)\}\cos(2\pi f_c t) - \Im\{u(t)\}\sin(2\pi f_c t) \\ &= x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t), \end{aligned} \tag{1}$$

where $u(t) = x(t) + jy(t)$ is a complex baseband signal with in-phase component $x(t) = \Re\{u(t)\}$, quadrature component $y(t) = \Im\{u(t)\}$, bandwidth B_u, and power P_u. The signal $u(t)$ is called the complex envelope or complex lowpass equivalent signal of $s(t)$. $u(t)$ is the complex envelope of $s(t)$ since the magnitude of $u(t)$ is the magnitude of $s(t)$ and the phase of $u(t)$ is the phase of $s(t)$. This phase includes any carrier phase offset.

The received signal will have a similar form:

$$r(t) = \Re\left\{v(t)e^{j2\pi f_c t}\right\}, \tag{2}$$

where the complex baseband signal $v(t)$ will depend on the channel through which $s(t)$ propagates. In particular, if $s(t)$ is transmitted through a time-invariant channel then $v(t) = u(t) * c(t)$, where $c(t)$ is the equivalent lowpass channel impulse response for the channel.

2 Path Loss and Shadowing

2.1 Free-Space Path Loss

If there are no obstructions between the transmitter and receiver and the signal propagates along a straight line between the two, then this channel is called a line-of-sight

(LOS) channel. The received signal is modeled as

$$r(t) = \Re\left\{ \frac{u(t)\sqrt{G_t G_r}\lambda e^{j2\pi d/\lambda}}{4\pi d} e^{j(2\pi f_c t)} \right\}. \tag{3}$$

2.2 Simplified Path Loss Model

The following simplified model for path loss as a function of distance is commonly used for system design:

$$P_r = P_t K \left[\frac{d_0}{d}\right]^{\gamma}. \tag{4}$$

2.3 Shadow Fading

A signal transmitted through a wireless channel will typically experience random variation due to blockage from objects in the signal path, giving rise to random variations of the received power at a given distance. The most common model for this additional attenuation is log-normal shadowing. In the log-normal shadowing model the ratio of transmit-to-receive power $\psi = P_t/P_r$ is assumed random with a log-normal distribution given by

$$p(\psi) = \frac{\xi}{\sqrt{2\pi}\sigma_{\psi_{dB}}\psi} \exp\left[-\frac{\left(10\log_{10}\psi - \mu_{\psi_{dB}}\right)^2}{2\sigma^2_{\psi_{dB}}}\right], \psi > 0. \tag{5}$$

Performance in log-normal shadowing is parameterized by the log mean $\mu_{\psi_{dB}}$, which is refered to as the average dB path loss. With a change of variables we see that the distribution of the dB value of ψ is Gaussian with mean $\mu_{\psi_{dB}}$ and standard deviation $\sigma_{\psi_{dB}}$:

$$p\left(\psi_{dB}\right) = \frac{1}{\sqrt{2\pi}\sigma_{\psi_{dB}}} \exp\left[-\frac{\left(\psi_{dB} - \mu_{\psi_{dB}}\right)^2}{2\sigma^2_{\psi_{dB}}}\right]. \tag{6}$$

2.4 Combined Path Loss and Shadowing

In this combined model, average dB path loss $\left(\mu_{\psi_{dB}}\right)$ is characterized by the path loss model and shadow fading, with a mean of 0 dB, creates variations about this path loss. For this combined model the ratio of received to transmitted power in dB is given by:

$$\frac{P_r}{P_t}(dB) = 10\log_{10} K - 10\gamma \log_{10} \frac{d}{d_0} - \psi_{dB}. \tag{7}$$

3 Statistical Multipath Channel Models

3.1 Introduction

An important characteristic of a multipath channel is the time delay. And another characteristic of the multipath channel is its time-varying nature. This time variation arises because either the transmitter or the receiver is moving, and therefore the location of reflectors in the transmission path, which give rise to multipath, will change over time. Due to the nature of time variation, we must characterize multipath channels statistically.

3.2 Time-Varying Channel Impulse Response

Let the transmitted signal be as in section 1 :

$$s(t) = \Re\left\{u(t)e^{j2\pi f_c t}\right\} = \Re\{u(t)\}\cos\left(2\pi f_c t\right) - \Im\{u(t)\}\sin\left(2\pi f_c t\right), \tag{8}$$

where $u(t)$ is the complex envelope of $s(t)$ with bandwidth B_u and f_c is its carrier frequency. The corresponding received signal is the sum of the LoS path and all resolvable multipath components:

$$r(t) = \mathrm{Re}\left\{\left[\sum_{n=0}^{N} \alpha_n u\left(t-\tau_n\right) e^{-j\phi_n}\right] e^{j2\pi f_c t}\right\}. \tag{9}$$

The received signal $r(t)$ is obtained by convolving the baseband input signal $u(t)$ with the equivalent lowpass time-varying channel impulse response $c(\tau, t)$ of the channel and then upconverting to the carrier frequency:

$$r(t) = \Re\left\{\left(\int_{-\infty}^{\infty} c(\tau,t)u(t-\tau)d\tau\right) e^{j2\pi f_c t}\right\}. \tag{10}$$

where the $c(\tau, t)$ can be given as

$$c(\tau,t) = \sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\delta\left(\tau - \tau_n(t)\right). \tag{11}$$

3.3 Narrowband Fading Models

Suppose the delay spread T_m of a channel is small relative to the inverse signal bandwidth B of the transmitted signal, i.e. $T_m << B^{-1}$. The spread characterizations $T_m << B^{-1}$ implies that the delay associated with the i th multipath component $\tau_i \leq T_m \forall i$, so $u(t-\tau_i) \approx u(t) \forall i$ and we can rewrite (9) as

$$r(t) = \Re\left\{u(t)e^{j2\pi f_c t}\left(\sum_n \alpha_n(t)e^{-j\phi_n(t)}\right)\right\}. \tag{12}$$

With this assumption the received signal becomes

$$r(t) = \Re\left\{\left[\sum_{n=0}^{N(t)} \alpha_n(t)e^{-j\phi_n(t)}\right]e^{j2\pi f_c t}\right\} = r_I(t)\cos 2\pi f_c t + r_Q(t)\sin 2\pi f_c t, \tag{13}$$

where the in-phase and quadrature components are given by

$$r_I(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\cos\phi_n(t), \tag{14}$$

and

$$r_Q(t) = \sum_{n=1}^{N(t)} \alpha_n(t)\sin\phi_n(t). \tag{15}$$

With the in-phase and quadrature received signal components $r_I(t)$ and $r_Q(t)$, the autocorrelation, cross correlation, power spectral density and power distribution can be derived to study the important characteristics of the narrowband channel. In the following, we would introduce the deviation of envelope and power distribution of narrowband channel as an example.

3.3.1 Envelope and Power Distributions

If we assume a variance of σ^2 for both in-phase and quadrature components then the signal envelope

$$z(t) = |r(t)| = \sqrt{r_I^2(t) + r_Q^2(t)} \tag{16}$$

is Rayleigh-distributed with distribution

$$p_Z(z) = \frac{2z}{P_r}\exp\left[-z^2/P_r\right] = \frac{z}{\sigma^2}\exp\left[-z^2/\left(2\sigma^2\right)\right], \quad x \geq 0. \tag{17}$$

The complex lowpass equivalent signal for $r(t)$ is given by $r_{LP}(t) = r_I(t) + jr_Q(t)$ which has phase $\theta = \arctan\left(r_Q(t)/r_I(t)\right)$. For $r_I(t)$ and $r_Q(t)$ uncorrelated Gaussian random variables, θ is uniformly distributed and independent of $|r_{LP}|$. So $r(t)$ has a Rayleigh-distributed amplitude and uniform phase, and the two are mutually independent.

The above equation are based on the assumption which apply to propagation models without a dominant LOS component. If the channel has a fixed LOS component then $r_I(t)$ and $r_Q(t)$ are not zero-mean. In this case the received signal equals the superposition of a complex Gaussian component and a LOS component, the signal envelope in this case can be shown to have a Rician distribution, given by

$$p_Z(z) = \frac{z}{\sigma^2}\exp\left[\frac{-\left(z^2+s^2\right)}{2\sigma^2}\right] I_0\left(\frac{zs}{\sigma^2}\right), \quad z \geq 0. \tag{18}$$

3.4 Wideband Fading Models

In the wideband channel, a short transmitted pulse of duration T will result in a received signal that is of duration $T + T_m$. If the multipath delay spread $T_m >> T$, then each of the different multipath components can be resolved. However, these multipath components interfere with subsequently transmitted pulses. This effect is called intersymbol interference (ISI).

Although the approximation in (12) no longer satisfied, if the number of multipath components is large and the phase of each component is uniformly distributed then the received signal will still be a zero-mean complex Gaussian process with a Rayleigh-distributed envelope. However, wideband fading differs from narrowband fading in terms of the resolution of the different multipath components. Specifically, for narrowband signals, the multipath components' delay is less than the inverse of the signal bandwidth, so the multipath components characterized in Equation (11) combine at the receiver to yield the original transmitted signal with amplitude and phase characterized by random processes and omit the effect of the delay. However, with wideband signals, the received signal experiences distortion due to the delay spread of the different multipath components, so the received signal can no longer be characterized by just the amplitude and phase random processes. The effect of multipath on wideband signals must therefore take into account both the multipath delay spread and the time-variations associated with the channel.

The starting point for characterizing wideband channels is the equivalent lowpass time-varying channel impulse response $c(\tau, t)$. The statistical characterization of $c(\tau, t)$

is thus determined by its autocorrelation function, defined as

$$A_c\left(\tau_1, \tau_2; t, \Delta t\right) = E\left[c^*\left(\tau_1; t\right) c\left(\tau_2; t+\Delta t\right)\right]. \tag{19}$$

Due to the property of WSS (wide sense stationary) and US (uncorrelated scattering), the equation (19) can be simplified as

$$A_c\left(\tau_1; \Delta t\right) \delta\left[\tau_1 - \tau_2\right] \triangleq A_c(\tau; \Delta t). \tag{20}$$

The scattering function for random channels is defined as the Fourier transform of $A_c(\tau; \Delta t)$ with respect to the Δt parameter:

$$S_c(\tau, \rho) = \int_{-\infty}^{\infty} A_c(\tau, \Delta t) e^{-j2\pi\rho\Delta t} d\Delta t. \tag{21}$$

The scattering function characterizes the average output power associated with the channel as a function of the multipath delay τ and Doppler ρ.

The most important characteristics of the wideband channel, including the power delay profile, coherence bandwidth, Doppler power spectrum, and coherence time, are derived from the channel autocorrelation $A_c(\tau, \Delta t)$ or scattering function $S(\tau, \rho)$. In the following, we would introduce the deviation of coherence bandwidth as an example.

3.4.1 Coherence Bandwidth

Define the random process

$$C(f; t) = \int_{-\infty}^{\infty} c(\tau; t) e^{-j2\pi f\tau} d\tau. \tag{22}$$

Since $c(\tau; t)$ is WSS, its integral $C(f; t)$ is as well. Thus, the autocorrelation of (22) is given by

$$A_C\left(f_1, f_2; \Delta t\right) = E\left[C^*\left(f_1; t\right) C\left(f_2; t+\Delta t\right)\right]. \tag{23}$$

We can simplify $A_C\left(f_1, f_2; \Delta t\right)$ as

$$\begin{aligned} A_C\left(f_1, f_2; \Delta t\right) &= \int_{-\infty}^{\infty} A_c(\tau, \Delta t) e^{-j2\pi(f_2-f_1)\tau} d\tau \\ &= A_C(\Delta f; \Delta t). \end{aligned} \tag{24}$$

If we define $A_C(\Delta f) \triangleq A_C(\Delta f; 0)$ then from (24),

$$A_C(\Delta f) = \int_{-\infty}^{\infty} A_c(\tau) e^{-j2\pi\Delta f\tau} d\tau. \tag{25}$$

It characterize the correlation of channel as time t for two different frequencies. The frequency B_c where $A_C(\Delta f) \approx 0$ for all $\Delta f > B_c$ is called the coherence bandwidth of the channel.

If we are transmitting a narrowband signal with bandwidth $B << B_c$, then fading across the entire signal bandwidth is highly correlated, i.e. the fading is roughly equal across the entire signal bandwidth. This is usually referred to as flat fading. On the other hand, if the signal bandwidth $B >> B_c$, then the channel amplitude values at frequencies separated by more than the coherence bandwidth are roughly independent. Thus, the channel amplitude varies widely across the signal bandwidth. In this case the channel is called frequency-selective.

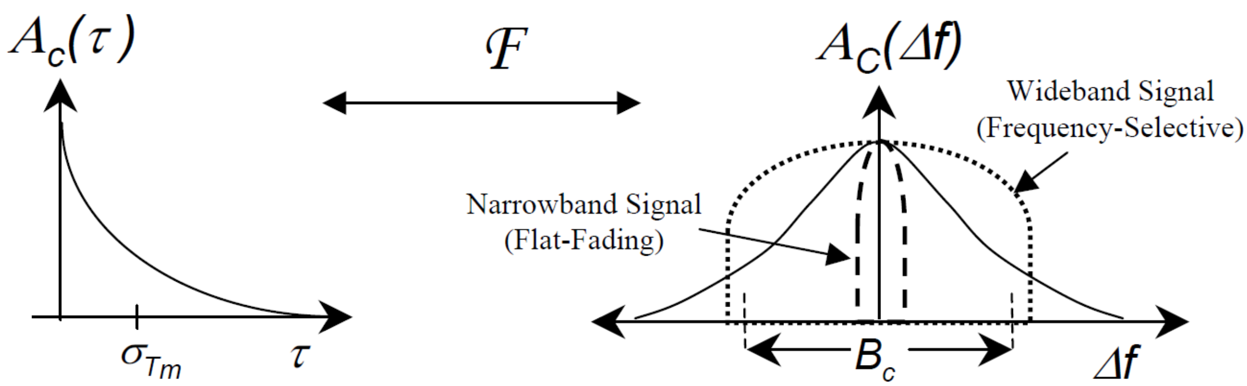

Figure 1: Power Delay Profile, RMS Delay Spread, and Coherence Bandwidth.

4 Capacity of Wireless Channels

4.1 Shannon Capacity

Consider a discrete-time AWGN channel , the capacity of this channel is given by Shannon's formula:

$$C = B \log_2(1 + \gamma). \tag{26}$$

It represents the maximum mutual information of a channel. The channel capacity is not dependent on transmission or reception techniques or limitation.

4.2 Capacity of Flat-Fading Channels

The capacity of this channel depends on what is known about the channel gain at the transmitter and receiver. In general, there are three different scenarios which refers to CDI, receiver CSI and transmitter and receiver CSI.

4.3 Channel Side Information at Receiver

Shannon capacity of a fading channel with receiver CSI for an average power constraint $\bar{P}$ can be obtained from

$$C = \int_0^\infty B \log_2(1+\gamma)p(\gamma)d\gamma. \tag{27}$$

Since only the receiver knows the instantaneous SNR $\gamma[i]$, and therefore the data rate transmitted over the channel is constant, regardless of γ. By Jensen's inequality,

$$\mathbf{E}\left[B \log_2(1+\gamma)\right] = \int B \log_2(1+\gamma)p(\gamma)d\gamma \le B \log_2(1+\mathbf{E}[\gamma]) = B \log_2(1+\bar{\gamma}) \tag{28}$$

where $\bar{\gamma}$ is the average SNR on the channel. From equation (28) we found that the fading reduces Shannon capacity when only the receiver has CSI.

4.4 Channel Side Information at Transmitter and Receiver

When both the transmitter and receiver have CSI, the transmitter can adapt its transmission strategy relative to this CSI. We allow the transmit power $P(\gamma)$ to vary with γ, subject to an average power constraint $\bar{P}$:

$$\int_0^\infty P(\gamma)p(\gamma)d\gamma \le \bar{P}. \tag{29}$$

This motivates defining the fading channel capacity with average power constraint as

$$C = \max_{P(\gamma):\int P(\gamma)p(\gamma)d\gamma=\bar{P}} \int_0^\infty B \log_2\left(1+\frac{P(\gamma)\gamma}{\bar{P}}\right)p(\gamma)d\gamma. \tag{30}$$

Solving for $P(\gamma)$ with the constraint that $P(\gamma) > 0$ yields the optimal power adaptation that maximizes (30) as

$$\frac{P(\gamma)}{\bar{P}} = \begin{cases} \frac{1}{\gamma_0} - \frac{1}{\gamma} & \gamma \ge \gamma_0 \\ 0 & \gamma < \gamma_0. \end{cases} \tag{31}$$

where the cutoff value γ_0 must satisfy

$$\int_{\gamma_0}^\infty \left(\frac{1}{\gamma_0} - \frac{1}{\gamma}\right)p(\gamma)d\gamma = 1. \tag{32}$$

5 Digital Modulation and Performance over Wireless Channel

5.1 Passband Modulation Principles

In general, modulated carrier signals encode information in the amplitude $\alpha(t)$, frequency $f(t)$, or phase $\theta(t)$ of a carrier signal. Thus, the modulated signal can be represented as

$$s(t) = \alpha(t)\cos\left[2\pi\left(f_c + f(t)\right)t + \theta(t) + \phi_0\right] = \alpha(t)\cos\left(2\pi f_c t + \phi(t) + \phi_0\right), \tag{33}$$

where $\phi(t) = 2\pi f(t)t + \theta(t)$ and ϕ_0 is the phase offset of the carrier. We can rewrite the right-hand side of in terms of its in-phase and quadrature components as:

$$\begin{aligned} s(t) &= \alpha(t)\cos\phi(t)\cos\left(2\pi f_c t\right) - \alpha(t)\sin\phi(t)\sin\left(2\pi f_c t\right) \\ &= s_I(t)\cos\left(2\pi f_c t\right) - s_Q(t)\sin\left(2\pi f_c t\right), \end{aligned} \tag{34}$$

where $s_I(t) = \alpha(t)\cos\phi(t)$ is called the in-phase component of $s(t)$ and $s_Q(t) = \alpha(t)\sin\phi(t)$ is called its quadrature component. We can also write $s(t)$ in its complex baseband representation as

$$s(t) = \Re\left\{u(t)e^{j(2\pi f_c t)}\right\}, \tag{35}$$

where $u(t) = s_I(t) + js_Q(t)$.

5.2 Average Probability of Error in Flat Fading Channel

If we can assume that γ_s is roughly constant over a symbol time, then the averaged probability of error is computed by integrating the error probability in AWGN over the fading distribution:

$$\bar{P}_s = \int_0^\infty P_s(\gamma)p_{\gamma_s}(\gamma)d\gamma, \tag{36}$$

where $P_s(\gamma)$ is the probability of symbol error in AWGN with SNR γ.

6 Multiple Antennas and Space-Time Communications

6.1 Narrowband MIMO Model

A narrowband point-to-point communication system of M_t transmit and M_r receive antennas can be represented by the following discrete time model:

$$
\begin{bmatrix} y_1 \\ \vdots \\ y_{M_r} \end{bmatrix} = \begin{bmatrix} h_{11} & \cdots & h_{1M_t} \\ \vdots & \ddots & \vdots \\ h_{M_r 1} & \cdots & h_{M_r M_t} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_{M_t} \end{bmatrix} + \begin{bmatrix} n_1 \\ \vdots \\ n_{M_r} \end{bmatrix}, \tag{37}
$$

or simply as $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$.

The optimal decoding of the received signal requires ML demodulation. If the symbols modulated onto each of the M_t transmit antennas are chosen from an alphabet of size $|\mathcal{X}|$, then because of the cross-coupling between transmitted symbols at the receiver antennas, ML demodulation requires an exhaustive search over all $|\mathcal{X}|^{M_t}$ possible input vector of M_t symbols.

6.2 Parallel Decomposition of the MIMO Channel

Consider a MIMO channel with $M_r \times M_t$ channel gain matrix $\mathbf{H}$ known to both the transmitter and the receiver. Let R_H denote the rank of $\mathbf{H}$. From matrix theory, for any matrix $\mathbf{H}$ we can obtain its SVD as

$$
\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\mathbf{H}} \tag{38}
$$

The transmit precoding and receiver shaping transform the MIMO channel into R_H parallel single-input single-output (SISO) channels with input $\mathbf{x}$ and output $\mathbf{y}$. From the SVD, we have that

$$
\begin{aligned}
\tilde{\mathbf{y}} &= \mathbf{U}^H(\mathbf{H}\mathbf{x} + \mathbf{n}) \\
&= \mathbf{U}^H\left(\mathbf{U}\mathbf{V}\mathbf{V}^H\tilde{\mathbf{x}} + \mathbf{n}\right) \\
&= \Sigma\tilde{\mathbf{x}} + \tilde{\mathbf{n}}
\end{aligned} \tag{39}
$$

Thus, the transmit precoding and receiver shaping transform the MIMO channel into R_H parallel independent channels where the i th channel has input $\tilde{x}_i$, output $\tilde{y}_i$, noise $\tilde{n}_i$, and channel gain σ_i. Since the parallel channels do not interfere with each other. the optimal ML demodulation complexity is linear in R_H.

6.3 MIMO Diversity Gain: Beamforming

The multiple antennas at the transmitter and receiver can be used to obtain diversity gain and capacity gain. To obtain a diversity gain, the same symbol, weighted by a complex scale factor, is sent over each transmit antenna, so that the input covariance matrix has unit rank. This scheme is referred to as MIMO beamforming.

The transmit symbol x is sent over the i th antenna with weight v_i. On the receive side, the signal received on the i th antenna is weighted by u_i. Both transmit and receive weight vectors are normalized so that $|u|=|v|=1$. The resulting received signal is given by

$$y = \mathbf{u}^*\mathbf{H}\mathbf{v}x + \mathbf{u}^*\mathbf{n}. \tag{40}$$

The diversity gain of MIMO beamforming depends on whether or not the channel is known at the transmitter. When the channel matrix $\mathbf{H}$ is known, the received SNR is optimized by choosing $\mathbf{u}$ and $\mathbf{v}$ as the principal left and right singular vectors of the channel matrix $\mathbf{H}$. When the channel is not known at the transmitter, the transmit antenna weights are all equal, so the received SNR equals $\gamma = |\mathbf{H}\mathbf{u}^*|$, where $\mathbf{u}$ is chosen to maximize γ.