Lab Notebook

Pupil Reconstruction

Daniel Carmona

Contents

1 PL Information Determination

1.1 The data

There are two groups of datasets.

1.1.1 Atmospheric aberration related

There are 4 datasets composed by PSFs and their corresponding PL intensities.

PSFs The PSFs' electric fields are stored in a 3d matrix of depth 2: depth 1 and 2 represent the real and imaginary value of the electric field in a point.

- **Original sized PSFs**: A dataset of 70000 electric fields stored in 128x128x2 matrixes.

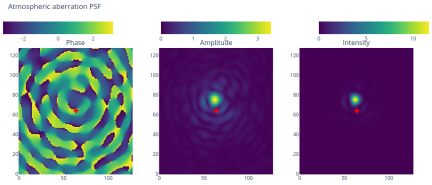


Figure 1: Example original sized PSF

- **Cropped sized PSFs**: A dataset of 70000 electric fields stored in 64x64x2 matrixes. These cropped PSFs correspond to the central pixels from the Original sized PSFs.
- **Original sized predicted PSFs**: A dataset of 70000 predicted electric fields stored in 128x128x2 matrices. These predicted PSFs are the outputs of a model trained with the Original PSFs dataset and their corresponding PL intensities.

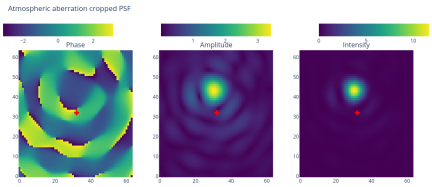


Figure 2: Example Cropped sized PSF

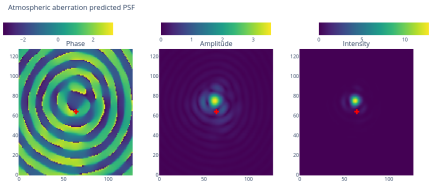


Figure 3: Example original sized predicted PSF

- **Cropped sized predicted PSF**: A dataset of 70000 predicted electric fields stored in 64x64x2 matrices. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized PSFs dataset).

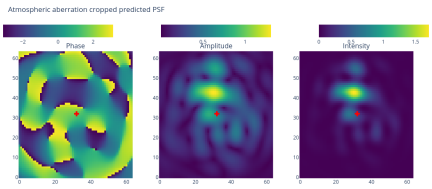


Figure 4: Example cropped sized predicted PSF

PL intensities The same dataset of PL output intensities are used for every PSF dataset. The intensities are computed multiplying the LP coefficients by the transfer matrix of the **19 mode PL**. This dataset has 70000 datapoints, each datapoint being

a vector of 19 elements.

1.1.2 Zernike modes related

There are 5 subgroups of datasets: PSFs generated with 2, 5, 9, 14 and 20 zernike modes. Each subgroup is divided in original sized, cropped sized, predicted and cropped predicted as in the case of the atmospheric aberration PSFs.

PSFs

- **Original sized 2 modes PSFs**: A dataset of 70000 electric fields stored in 128x128x2 matrixes. The aberration by a 2 modes zernike basis.

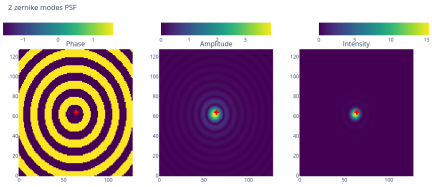


Figure 5: Example original sized 2 modes PSF

- **Cropped sized 2 modes PSFs**: A dataset of 70000 electric fields stored in 64x64x2 matrixes. These cropped PSFs correspond to the central pixels from the Original sized 2 modes PSFs.

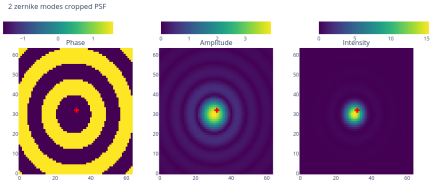


Figure 6: Example Cropped sized 2 modes PSF

- **Original sized predicted 2 modes PSFs**: A dataset of 70000 predicted electric fields stored in 128x128x2 matrices. These predicted PSFs are the outputs of a model trained with the Original sized 2 modes PSFs dataset and their corresponding PL intensities.

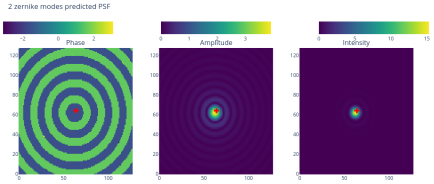


Figure 7: Example original sized predicted 2 modes PSF

- **Cropped sized predicted 2 modes PSF**: A dataset of 70000 predicted electric fields stored in 64x64x2 matrices. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 2 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 2 modes PSFs dataset).

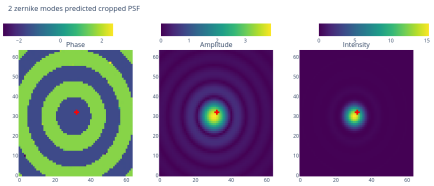


Figure 8: Example cropped sized predicted 2 modes PSF

- **Original sized 5 modes PSFs**: A dataset of 70000 electric fields stored in 128x128x2 matrixes. The aberration by a 5 modes zernike basis.
- **Cropped sized 5 modes PSFs**: A dataset of 70000 electric fields stored in 64x64x2 matrixes. These cropped PSFs correspond to the central pixels from the Original sized 5 modes PSFs.

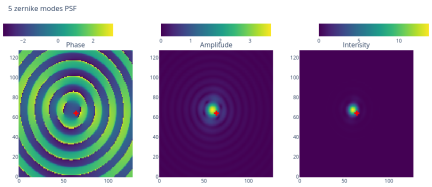


Figure 9: Example original sized 5 modes PSF

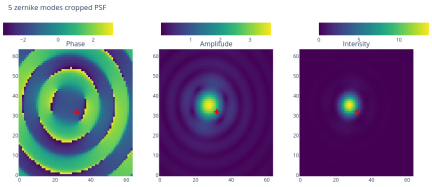


Figure 10: Example Cropped sized 5 modes PSF

- **Original sized predicted 5 modes PSFs**: A dataset of 70000 predicted electric fields stored in 128x128x2 matrices. These predicted PSFs are the outputs of a model trained with the Original sized 5 modes PSFs dataset and their corresponding PL intensities.

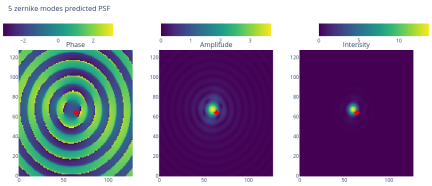


Figure 11: Example original sized predicted 5 modes PSF

- **Cropped sized predicted 5 modes PSF**: A dataset of 70000 predicted electric fields stored in 64x64x2 matrices. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 5 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 5 modes PSFs dataset).

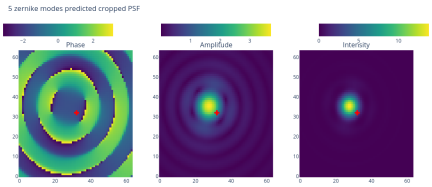


Figure 12: Example cropped sized predicted 5 modes PSF

- **Original sized 9 modes PSFs**: A dataset of 70000 electric fields stored in 128x128x2 matrixes. The aberration by a 9 modes zernike basis.

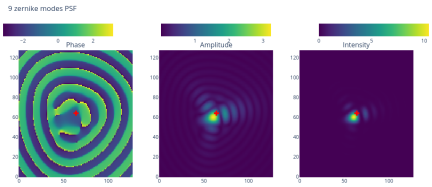


Figure 13: Example original sized 9 modes PSF

- **Cropped sized 9 modes PSFs**: A dataset of 70000 electric fields stored in 64x64x2 matrixes. These cropped PSFs correspond to the central pixels from the Original sized 9 modes PSFs.

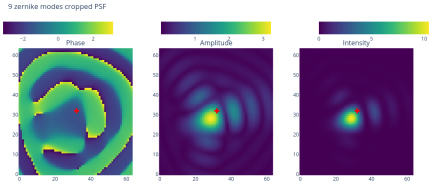


Figure 14: Example Cropped sized 9 modes PSF

- **Original sized predicted 9 modes PSFs**: A dataset of 70000 predicted electric fields stored in 128x128x2 matrices. These predicted PSFs are the

outputs of a model trained with the Original sized 9 modes PSFs dataset and their corresponding PL intensities.

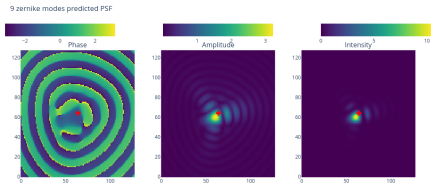


Figure 15: Example original sized predicted 9 modes PSF

- **Cropped sized predicted 9 modes PSF**: A dataset of 70000 predicted electric fields stored in 64x64x2 matrices. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 9 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 9 modes PSFs dataset).

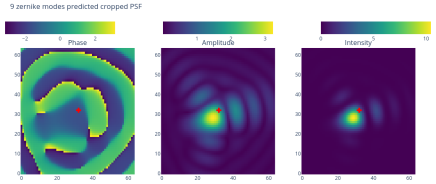


Figure 16: Example cropped sized predicted 9 modes PSF

- **Original sized 14 modes PSFs**: A dataset of 70000 electric fields stored in 128x128x2 matrixes. The aberration by a 14 modes zernike basis.
- **Cropped sized 14 modes PSFs**: A dataset of 70000 electric fields stored in 64x64x2 matrixes. These cropped PSFs correspond to the central pixels from the Original sized 14 modes PSFs.
- **Original sized predicted 14 modes PSFs**: A dataset of 70000 predicted electric fields stored in 128x128x2 matrices. These predicted PSFs are the

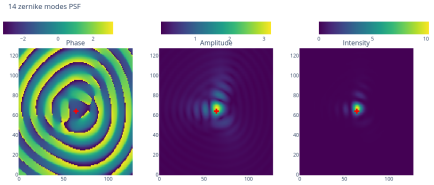


Figure 17: Example original sized 14 modes PSF

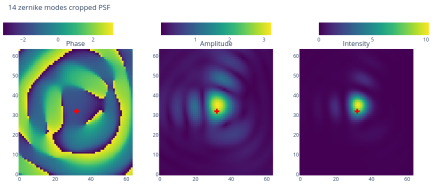


Figure 18: Example Cropped sized 14 modes PSF

outputs of a model trained with the Original sized 14 modes PSFs dataset and their corresponding PL intensities.

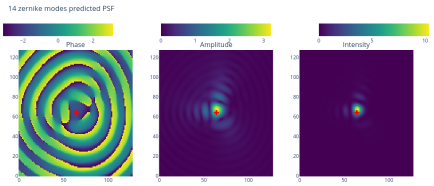


Figure 19: Example original sized predicted 14 modes PSF

- **Cropped sized predicted 14 modes PSF**: A dataset of 70000 predicted electric fields stored in 64x64x2 matrices. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 14 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 14 modes PSFs dataset).

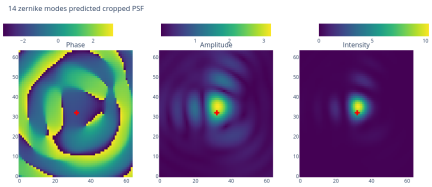


Figure 20: Example cropped sized predicted 14 modes PSF

- **Original sized 20 modes PSFs**: A dataset of 70000 electric fields stored in 128x128x2 matrixes. The aberration by a 20 modes zernike basis.

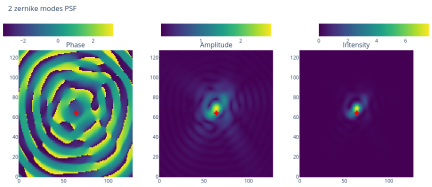


Figure 21: Example original sized 20 modes PSF

- **Cropped sized 20 modes PSFs**: A dataset of 70000 electric fields stored in 64x64x2 matrixes. These cropped PSFs correspond to the central pixels from the Original sized 20 modes PSFs.

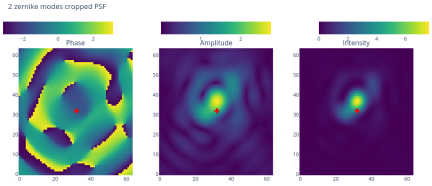


Figure 22: Example Cropped sized 20 modes PSF

- **Original sized predicted 20 modes PSFs**: A dataset of 70000 predicted electric fields stored in 128x128x2 matrices. These predicted PSFs are the

outputs of a model trained with the Original sized 20 modes PSFs dataset and their corresponding PL intensities.

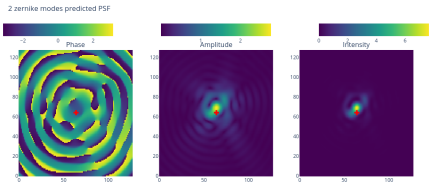


Figure 23: Example original sized predicted 20 modes PSF

- **Cropped sized predicted 20 modes PSF**: A dataset of 70000 predicted electric fields stored in 64x64x2 matrices. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 20 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 20 modes PSFs dataset).

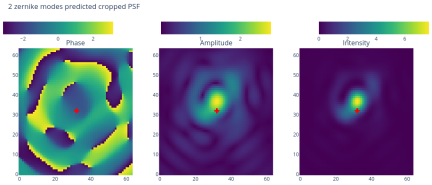


Figure 24: Example cropped sized predicted 20 modes PSF

LP mode coefficients There is one PL intensities dataset per Zernike aberration PSF subgroup: LP modes coefficients for 2, 5, 9, 14, 20 modes PSFs. Each of the dataset has 70000 datapoints each datapoint being the complex coefficients stored in a 19x2 matrix that separates the real and imaginary part of the coefficients.

PL intensities There is one PL intensities dataset per Zernike aberration PSF subgroup: PL intensities for 2, 5, 9, 14, 20 modes PSFs. Each of the dataset has

70000 datapoints each datapoint being the 19 intensities corresponding to the PSF

1.2 The models

For all the datasets a model with the following configuration has been trained. The inputs of the model are the PL intensities and the outputs are the flattened matrices that represent the PSFs' complex fields.

HYPERPARAMETERS:

```
*ARCHITECTURE HYPERPARAMETERS:
  -Fully Connected
  -Input shape: 19
  -Output shape: 32768
  -Hidden layers: [1024, 1024, 1024, 1024, 1024, 1024]
  -Regularizer: None
  -Hidden Layers Activation: relu
  -Output Layer Activation: linear
  -Batch Normalization: False
  -Dropout: False, 0.2

*COMPILATION HYPERPARAMETERS:
  -Optimizer: ADAM lr=0.001, beta_1=0.9, beta_2=0.999
  -Loss Function: MSE
  -Metric: MSE
```

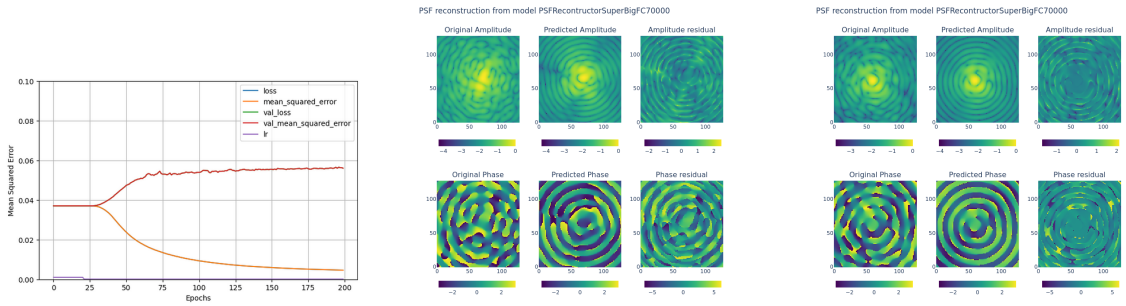
```
*TRAINING HYPERPARAMETERS:
  -Epochs: 200
  -Batch size: 32
  -Callbacks:
    -ReduceLROnPlateau: MSE 20 x0.1
    -Early Stop: MSE 50
```

The exception is the model trained for the Atmospheric Aberration Cropped PSF which has Batch Normalization activated.

1.2.1 Atmospheric aberration related models

Original sized PSF :

```
          -Train MSE: 0.004607476759701967
          -Validation MSE: 0.056021399796009064
```

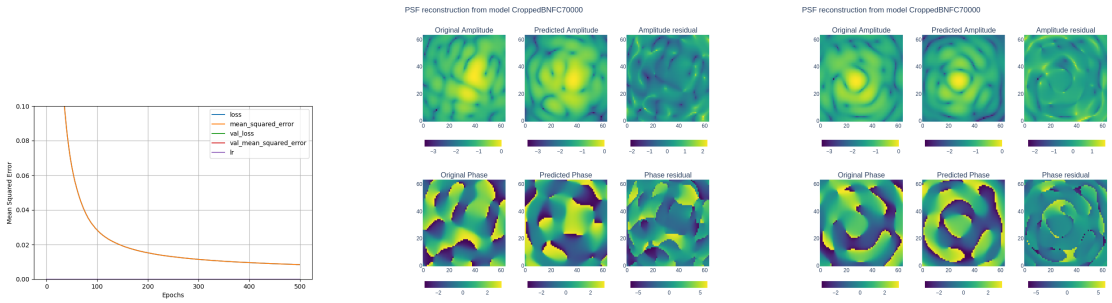


(a) Training Evolution (b) Validation example (c) Train example

Figure 25: Results of training the model PSFRecontructorSuperBigFC70000-1

Cropped sized PSF :

```
-Train MSE: 0.008466990664601326
-Validation MSE: 0.20970138907432556
```



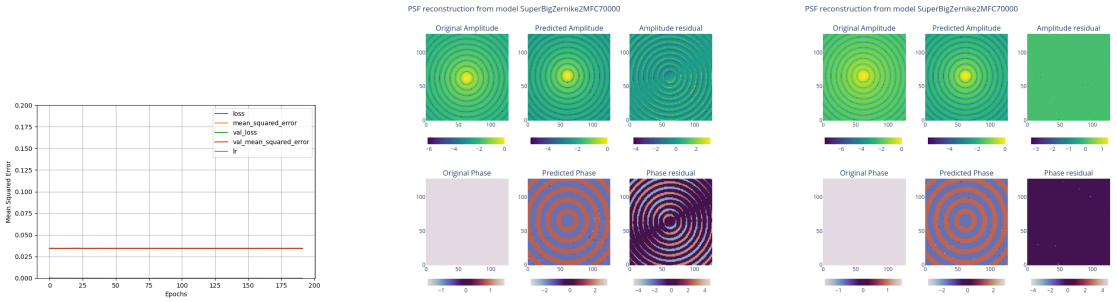
(a) Training Evolution (b) Validation example (c) Train example

Figure 26: Results of training the model PSFRecontructorSuperBigFC70000-1

1.2.2 Zernike modes related models

Original sized 2 modes PSFs :

```
-Train MSE: 0.00000000895368188480461
-Validation MSE: 0.034455109387636185
```

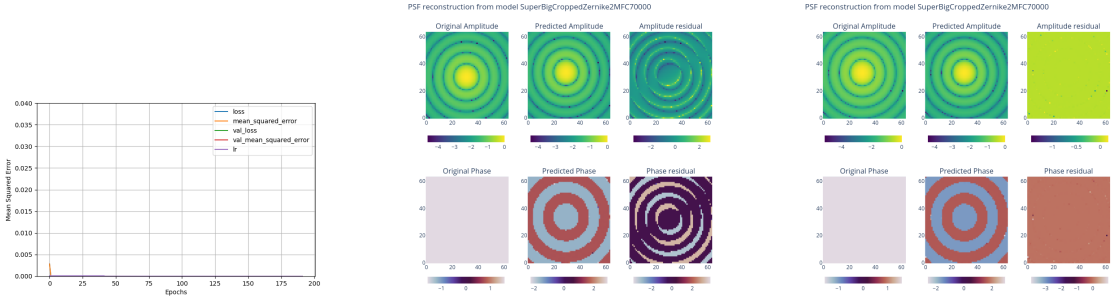


(a) Training Evolution (b) Validation example (c) Train example

Figure 27: Model training for original sized 2 zernike modes PSFs

Cropped sized 2 modes PSFs :

```
-Train MSE: 0.00000024662290343258064
-Validation MSE: 0.13220909237861633
```

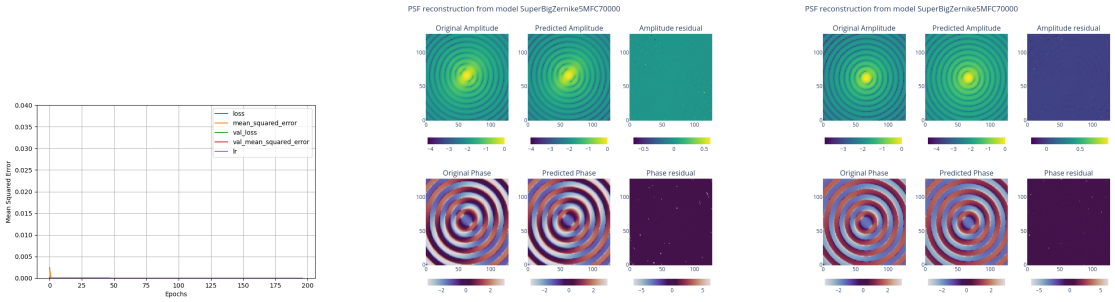


(a) Training Evolution (b) Validation example (c) Train example

Figure 28: Model training for cropped sized 2 zernike modes PSFs

Original sized 5 modes PSFs :

```
-Train MSE: 0.000001832015527725161 5
-Validation MSE: 0.0000026587338197714416
```

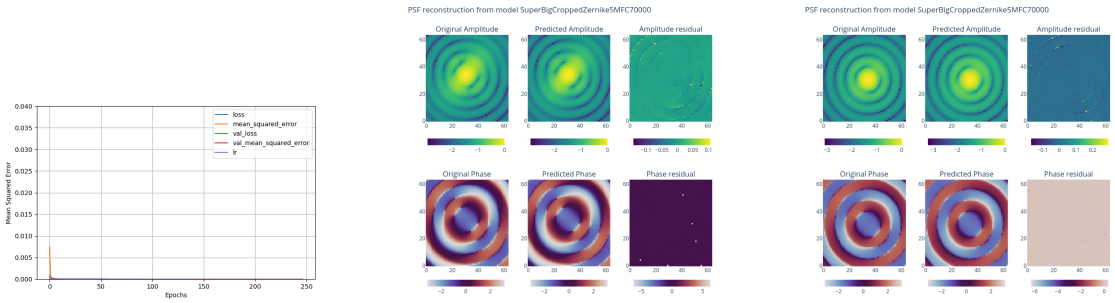


(a) Training Evolution (b) Validation example (c) Train example

Figure 29: Model training for original sized 5 zernike modes PSFs

Cropped sized 5 modes PSFs :

```
-Train MSE: 0.000004930785507895052 4
-Validation MSE: 0.00000881233972904738
```

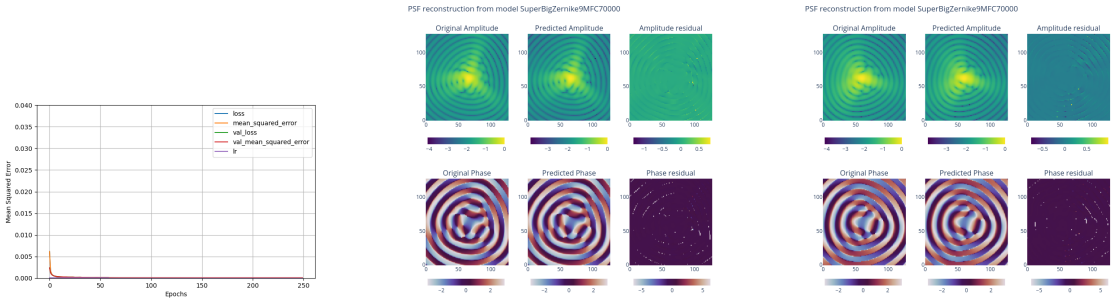


(a) Training Evolution (b) Validation example (c) Train example

Figure 30: Model training for cropped sized 5 zernike modes PSFs

Original sized 9 modes PSFs :

```
-Train MSE: 0.00001687798430793918 7
-Validation MSE: 0.00010731885413406417
```

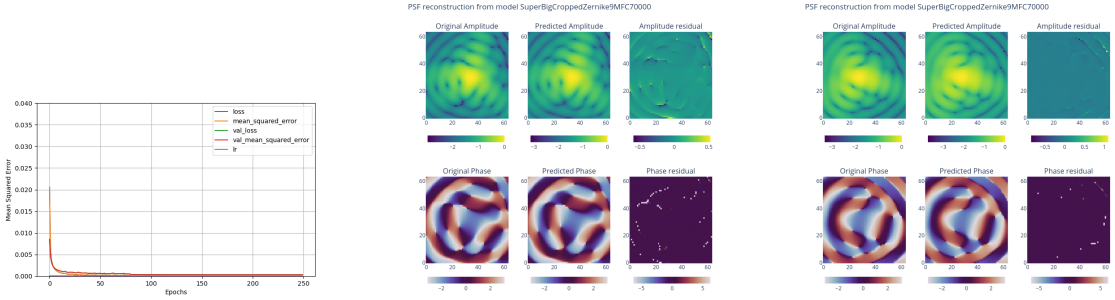


(a) Training Evolution (b) Validation example (c) Train example

Figure 31: Model training for original sized 9 zernike modes PSFs

Cropped sized 9 modes PSFs :

```
-Train MSE: 0.00003678303983178921
-Validation MSE: 0.00034940888872370124
```

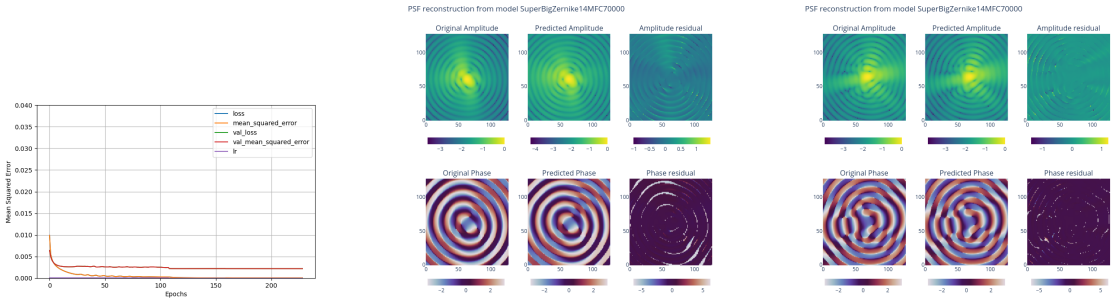


(a) Training Evolution (b) Validation example (c) Train example

Figure 32: Model training for cropped sized 9 zernike modes PSFs

Original sized 14 modes PSFs :

```
-Train MSE: 0.00008819704817142338
-Validation MSE: 0.002211778424680233
```

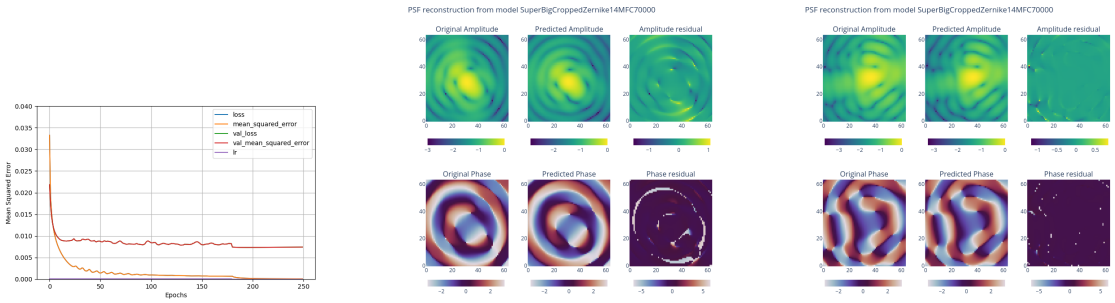


(a) Training Evolution (b) Validation example (c) Train example

Figure 33: Model training for original sized 14 zernike modes PSFs

Cropped sized 14 modes PSFs :

```
-Train MSE: 0.00009449137724004686
-Validation MSE: 0.007428955752402544
```

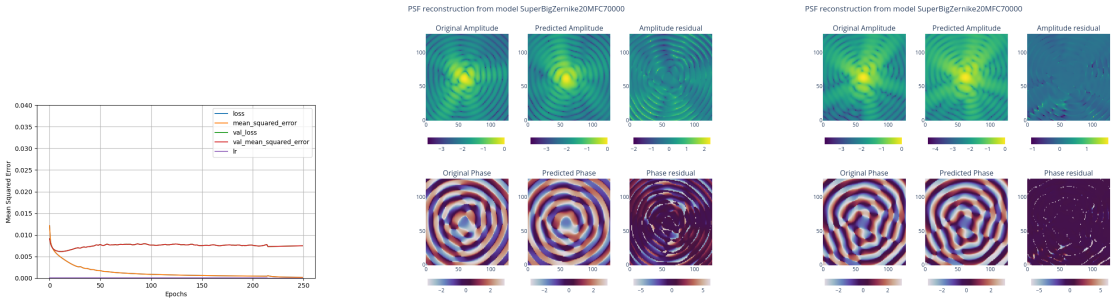


(a) Training Evolution (b) Validation example (c) Train example

Figure 34: Model training for cropped sized 14 zernike modes PSFs

Original sized 20 modes PSFs :

```
-Train MSE: 0.0002106916217599064
-Validation MSE: 0.007495625875890255
```

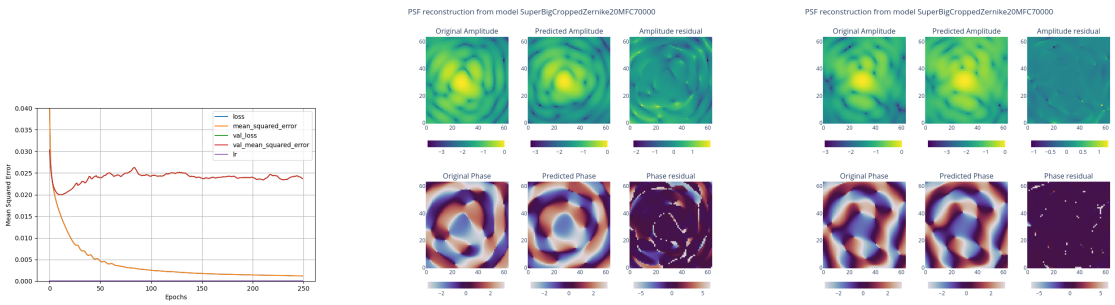


(a) Training Evolution (b) Validation example (c) Train example

Figure 35: Model training for original sized 20 zernike modes PSFs

Cropped sized 20 modes PSFs :

```
-Train MSE: 0.0012251562438905 24
-Validation MSE: 0.023689033463597298
```



(a) Training Evolution (b) Validation example (c) Train example

Figure 36: Model training for cropped sized 20 zernike modes PSFs

A summary of the MSE evolution over the Zernike PSFs datasets is shown below. The fact that the validation MSE for 2 modes is the worse may be because the neural network is not able to understand traslations.

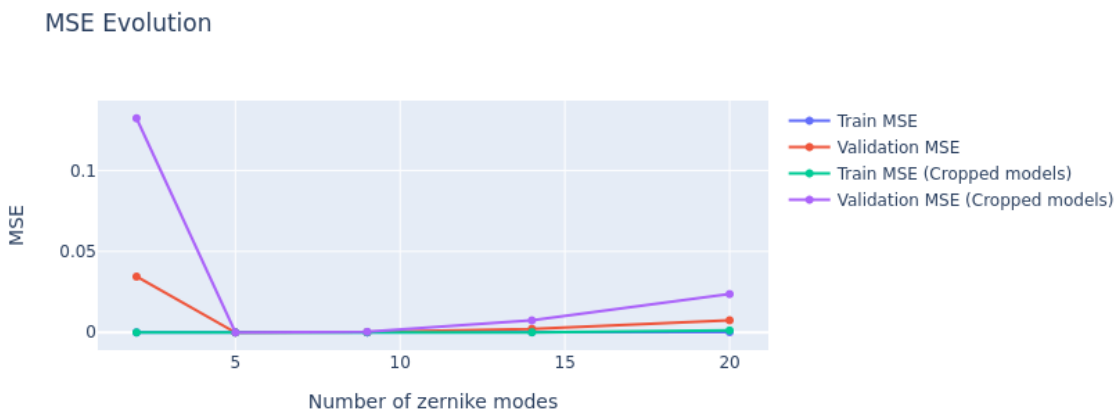


Figure 37: MSE evolution over the Zernike PSFs datasets

1.3 Euclidean distances analysis for atmospheric aberration PSFs

1.3.1 Preprocessing

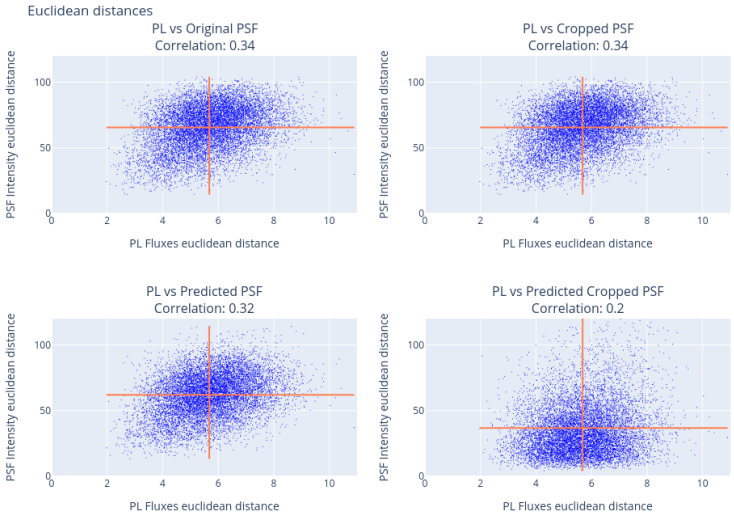
- The PSF electric fields matrices are flattened to compute the euclidean distances between 1d vectors.
- 70000 datapoint pairs are defined for which the euclidean distances will be calculated.

1.3.2 Results

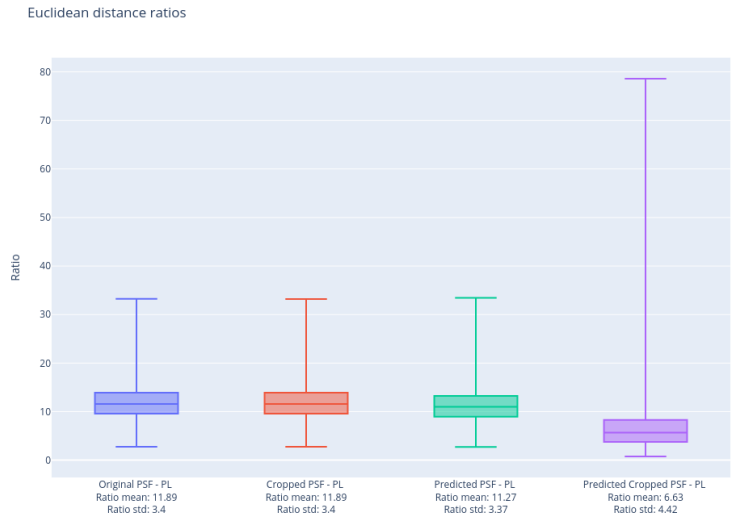
After performing an ANOVA test on the euclidean distances from the selected pairs of the 4 datasets obtaining a p-value of 0 and F-statistic of 4789.1531.

1.3.3 Analysis

The correlation is 0.3 which indicates a slightly positive linear relationship between the PL flux and PSF in all cases except for the cropped predictions which has a



(a) Euclidean distance cloud



(b) Euclidean distance ratios

Figure 38: Euclidean distances ratios between PL and PSF pairs

0.2 correlation rate. This makes sense as the model that predicted those PSFs is more overfitted than the model that predicts the original sized PSFs.The clouds are dispersed almost equally from the center of mass which may indicate that a 19 mode

PL may not be enough to encode all PSF information.

1.4 Euclidean distances analysis for Zernike modes PSFs

1.4.1 Preprocessing

- The PSF electric fields matrices are flattened to compute the euclidean distances between 1d vectors.
- 70000 datapoint pairs for each zernike datasets are randomly defined. The euclidean distances will be calculated for these selected pairs.
- In this case, LP coefficients are also analysed.

1.4.2 Results

1.4.3 Analysis

- The correlations between PSF distances and PL flux distances decay as the number of modes increases while the correlation between PSF distances and LP mode coefficients from the overlap integral are constant.
- The predicted psfs from the train datasets create a similar cloud of points to the original psfs which indicates that the models are capturing the information of the PL.
- The model trained for the dataset of 2 zernike terms PSF is the one that has the most overfit as the table shows (False, False, None indicate that no dropout, no batch normalization and no regularizer has been used), the validation mse just flatlines over the training.
- For 5 terms PSFs on the overfitting is reduced significantly although it increases with the number of zernike terms used. When using 20 terms the validation mse flatlines again.

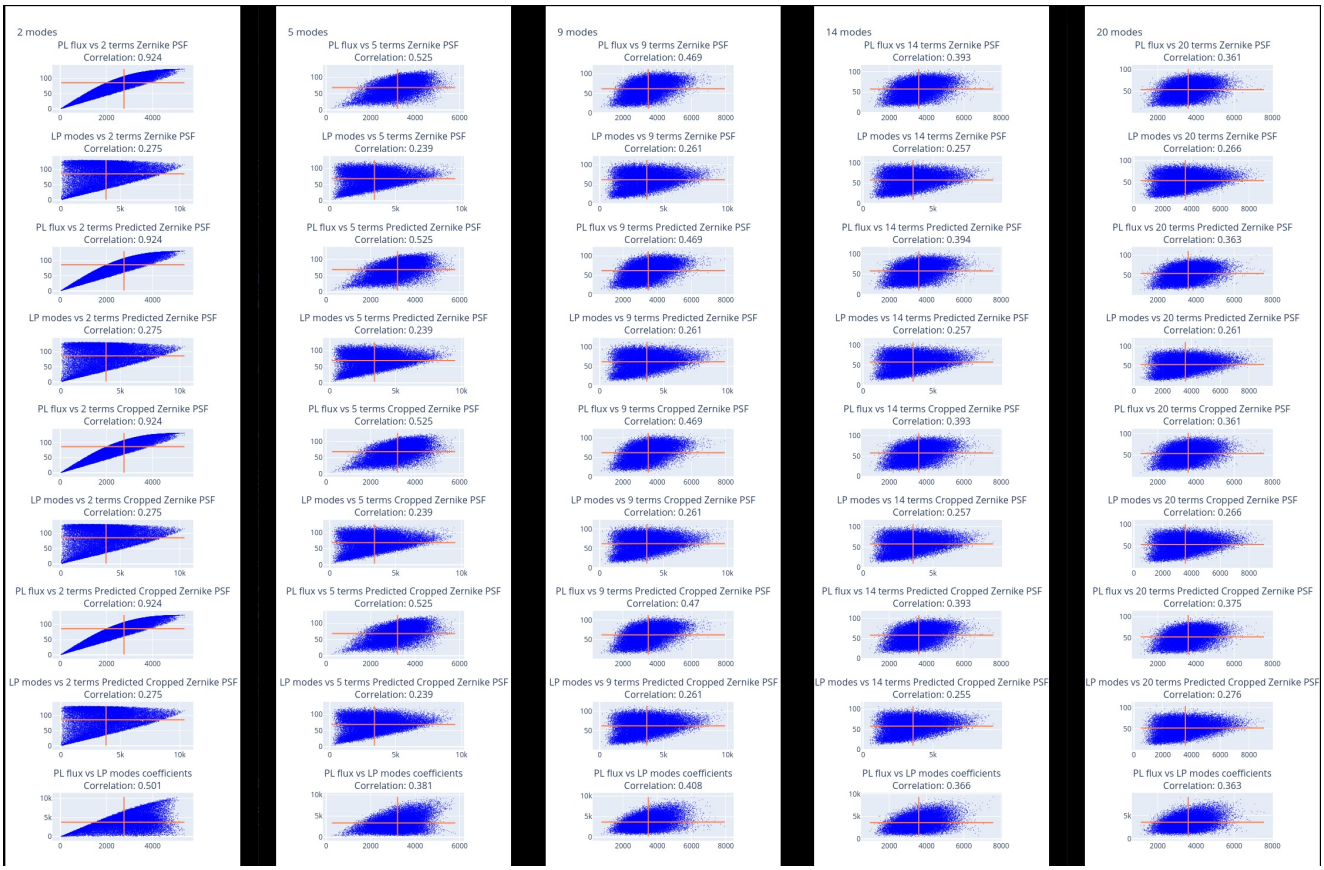


Figure 39: Euclidean distances relationship between the Zernike PSFs datasets

- PL intensities vs PSF datasets evolution is practically the same for original, predicted, cropped and cropped predicted datasets. This indicates that the models are capturing accurately the relationship existing between the original PSF datasets and PL intensities datasets.

- LP coeffs vs PSF datasets evolution is practically the same for original, predicted, cropped and cropped predicted datasets.

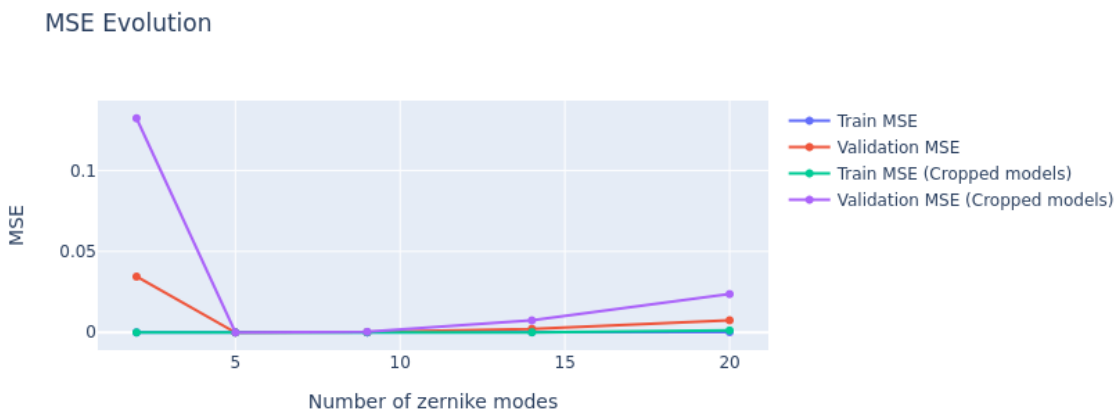


Figure 40: MSE evolution over the Zernike PSFs datasets

1.5 Zernike modes PSFs Clustering

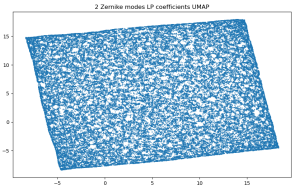
1.5.1 UMAPS

Before clustering, UMAPS for flattened PSF matrices, flattend LP coefficients matrices and PL intensities are processed. The same configuration is used for the different number of modes.

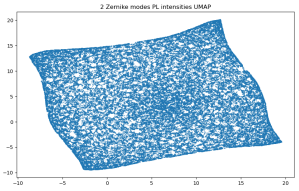
Dataset type	Number of neighbors	Min distance	Number of components
Zernike modes PSF	500	0.3	3
LP coefficients	500	0.1	2
PL intensities	500	0.1	2

Table 1: UMAP parameter configurations for each of the dataset type

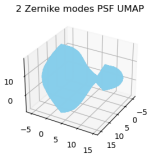
The resulting projections are the following:



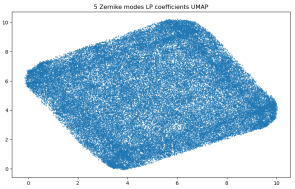
(a) 2 Zernike modes LP coefficients UMAP



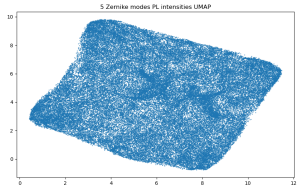
(b) 2 Zernike modes PL intensities UMAP



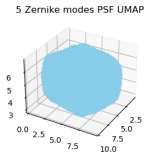
(c) 2 Zernike modes PSF UMAP



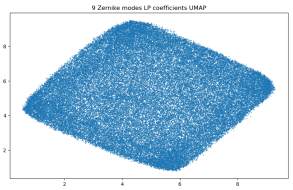
(d) 5 Zernike modes LP coefficients UMAP



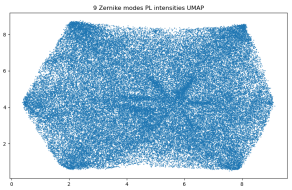
(e) 5 Zernike modes PL intensities UMAP



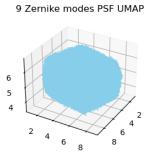
(f) 5 Zernike modes PSF UMAP



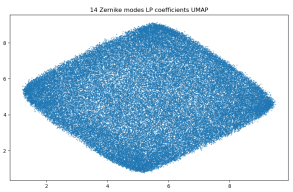
(g) 9 Zernike modes LP coefficients UMAP



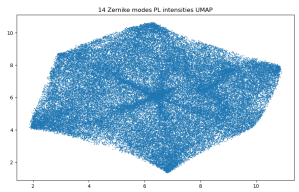
(h) 9 Zernike modes PL intensities UMAP



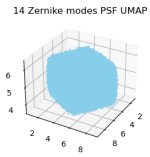
(i) 9 Zernike modes PSF UMAP



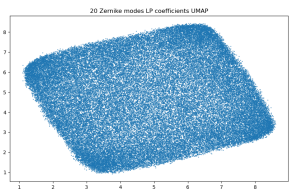
(j) 14 Zernike modes LP coefficients UMAP



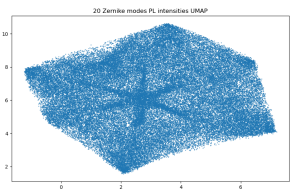
(k) 14 Zernike modes PL intensities UMAP



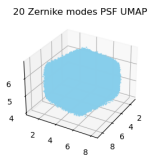
(l) 14 Zernike modes PSF UMAP



(m) 20 Zernike modes LP coefficients UMAP



(n) 20 Zernike modes PL intensities UMAP



(o) 20 Zernike modes PSF UMAP

Figure 41: Datasets UMAPs

1.5.2 Clustering

Using DBSCAN, clusters are defined in each dataset.

2 Zernike modes :

	PSF	LP coeffs	PL intensities	Pred PSF
DBSCAN ϵ		0.11	0.102 0.102	
DBSCAN neighbors		6	4	
Number of clusters		1447	1553	
Cluster density mean		42.37	41.59	
Cluster density variance		157.86	64594	
Non noise points		61313		

Table 2: Clustering for 2 Zernike modes datasets

5 Zernike modes :

	PSF	LP coeffs	PL intensities	Pred PSF
DBSCAN ϵ		0.0385	0.041	
DBSCAN neighbors		5	5	
Number of clusters		1601	1527	
Cluster density mean		39.42	41.43	
Cluster density variance		348.27	273.60	
Non noise points		63120	63266	

Table 3: Clustering for 5 Zernike modes datasets

9 Zernike modes :

14 Zernike modes :

20 Zernike modes :

	PSF	LP coeffs	PL intensities
DBSCAN ϵ		0.0335	
DBSCAN neighbors		6	
Number of clusters		1649	
Cluster density mean		37.05	
Cluster density variance		239.01	
Non noise points		61106	

Table 4: Clustering for 9 Zernike modes datasets

	PSF	LP coeffs	PL intensities
DBSCAN ϵ		0.0322	
DBSCAN neighbors		6	
Number of clusters		1472	
Cluster density mean		41.82	
Cluster density variance		330.54	
Non noise points		61570	

Table 5: Clustering for 14 Zernike modes datasets

	PSF	LP coeffs	PL intensities
DBSCAN ϵ		0.031	
DBSCAN neighbors		6	
Number of clusters		1596	
Cluster density mean		38.09	
Cluster density variance		305.13	
Non noise points		60804	

Table 6: Clustering for 20 Zernike modes datasets