

Lab Notebook

Photonic Lantern Information Determination

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1 PL Information Determination

1.1 The data

There are two groups of datasets.

1.1.1 Atmospheric aberration related

There are 4 datasets composed by PSFs and their corresponding PL intensities.

PSFs The PSFs' electric fields are stored in a 3d matrix of depth 2: depth 1 and 2 represent the real and imaginary value of the electric field in a point.

- **Original sized PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 128x128x2 and 128x128 matrices respectively.

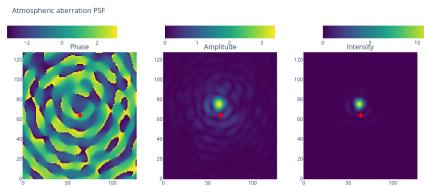


Figure 1: Example original sized PSF

- **Cropped sized PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped PSFs correspond to the central pixels from the Original sized PSFs.
- **Original sized predicted PSFs:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 128x128x2 and 128x128 matrices respectively. These predicted PSFs are the outputs of a model trained with the Original PSFs dataset and their corresponding PL intensities.

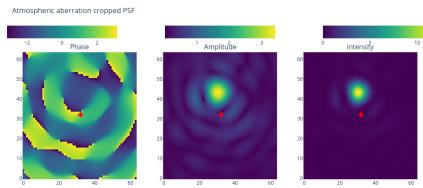


Figure 2: Example Cropped sized PSF

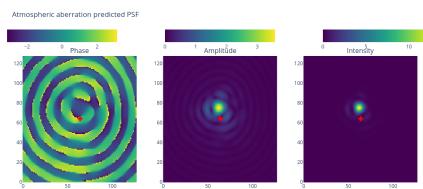


Figure 3: Example original sized predicted PSF

- **Cropped sized predicted PSF:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized PSFs dataset and their corresponding PL intensities (which are the same output intensities from the Original sized PSFs dataset).

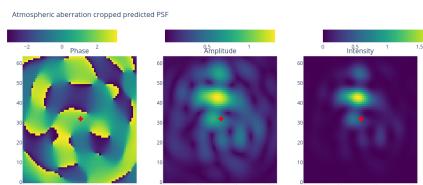


Figure 4: Example cropped sized predicted PSF

PL intensities The same dataset of PL output intensities are used for every PSF dataset. The intensities are computed multiplying the LP coefficients by the transfer matrix of the **19 mode PL**. This dataset has 70000 datapoints, each datapoint being

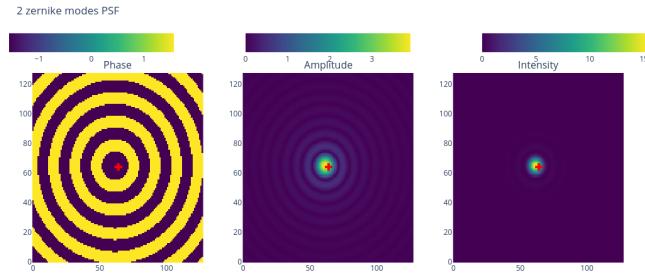
a vector of 19 elements.

1.1.2 Zernike modes related

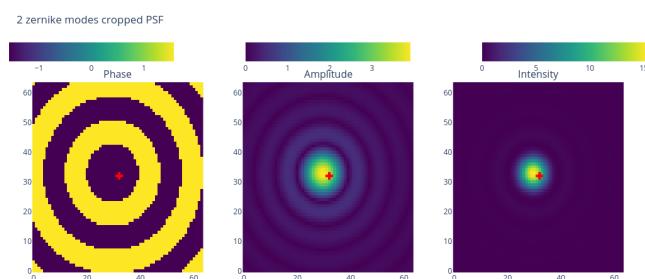
There are 5 subgroups of datasets: PSFs generated with 2, 5, 9, 14 and 20 zernike modes. Each subgroup is divided in original sized, cropped sized, predicted and cropped predicted as in the case of the atmospheric aberration PSFs.

2 Zernike modes PSFs

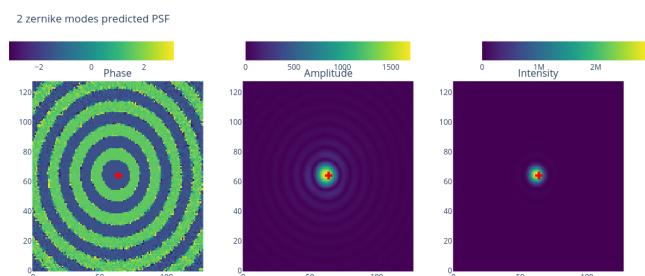
- **Original sized 2 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 128x128x2 and 128x128 matrices respectively. The aberration by a 2 modes zernike basis.
- **Cropped sized 2 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped PSFs correspond to the central pixels from the Original sized 2 modes PSFs.
- **Original sized predicted 2 modes PSFs:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 128x128x2 and 128x128 matrices respectively. These predicted PSFs are the outputs of a model trained with the Original sized 2 modes PSFs dataset and their corresponding PL intensities.
- **Cropped sized predicted 2 modes PSF:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 2 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 2 modes PSFs dataset).



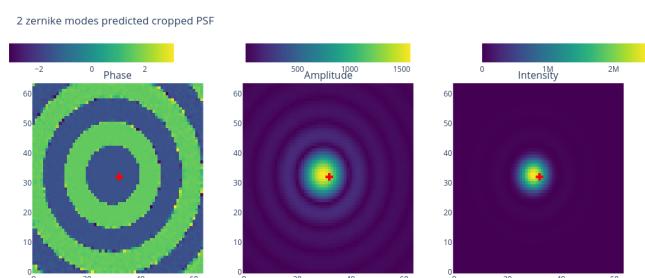
(a) Original sized 2 modes PSF example



(b) Cropped sized 2 modes PSF example



(c) Original sized predicted 2 modes PSF example

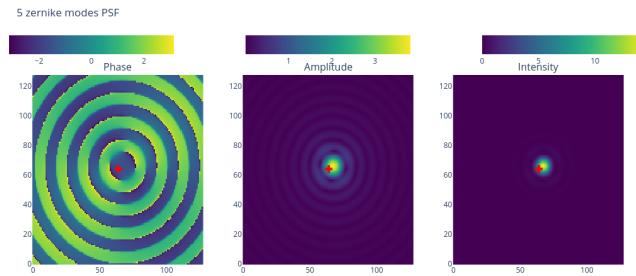


(d) cropped sized predicted 2 modes PSF example

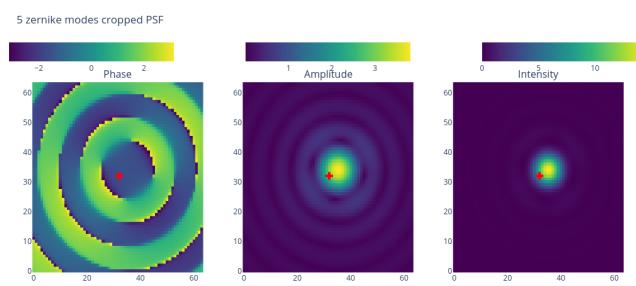
Figure 5: 2 Zernike modes PSF datasets examples

5 Zernike modes PSFs

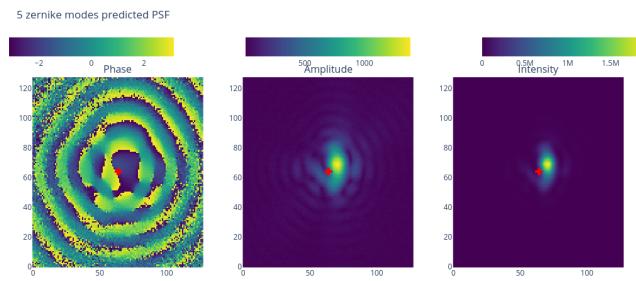
- **Original sized 5 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 128x128x2 and 128x128 matrices respectively. The aberration by a 5 modes zernike basis.
- **Cropped sized 5 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped PSFs correspond to the central pixels from the Original sized 5 modes PSFs.
- **Original sized predicted 5 modes PSFs:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 128x128x2 and 128x128 matrices respectively. These predicted PSFs are the outputs of a model trained with the Original sized 5 modes PSFs dataset and their corresponding PL intensities.
- **Cropped sized predicted 5 modes PSF:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 5 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 5 modes PSFs dataset).



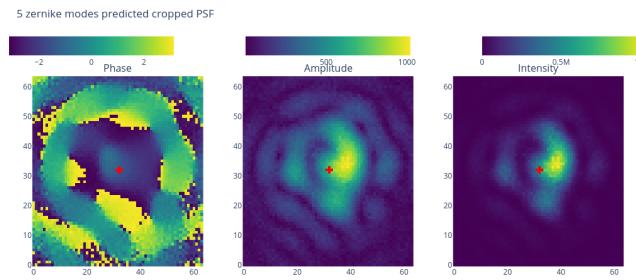
(a) Original sized 5 modes PSF example



(b) Cropped sized 5 modes PSF example



(c) Original sized predicted 5 modes PSF example

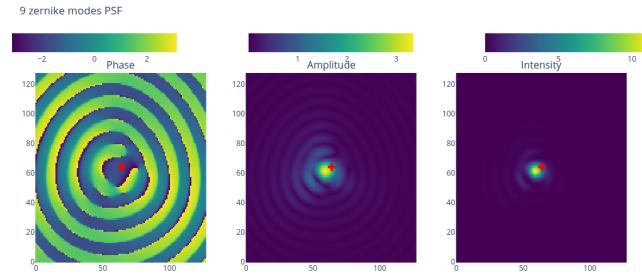


(d) cropped sized predicted 5 modes PSF example

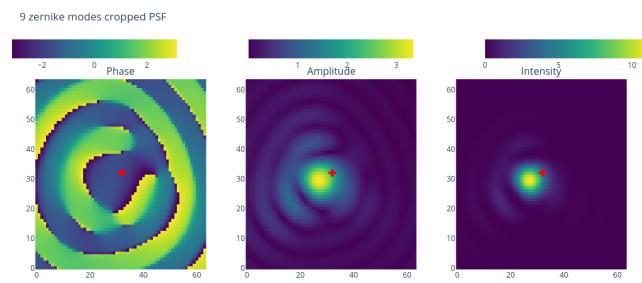
Figure 6: 5 Zernike modes PSF datasets examples

9 Zernike modes PSFs

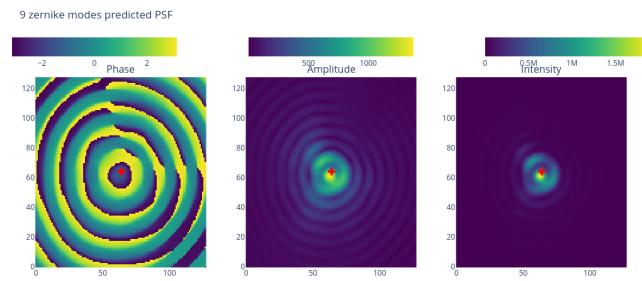
- **Original sized 9 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 128x128x2 and 128x128 matrices respectively. The aberration by a 9 modes zernike basis.
- **Cropped sized 9 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped PSFs correspond to the central pixels from the Original sized 9 modes PSFs.
- **Original sized predicted 9 modes PSFs:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 128x128x2 and 128x128 matrices respectively. These predicted PSFs are the outputs of a model trained with the Original sized 9 modes PSFs dataset and their corresponding PL intensities.
- **Cropped sized predicted 9 modes PSF:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 9 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 9 modes PSFs dataset).



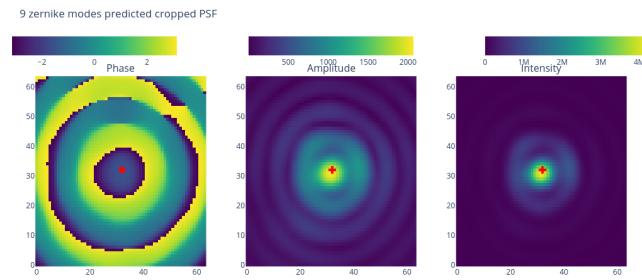
(a) Original sized 9 modes PSF example



(b) Cropped sized 9 modes PSF example



(c) Original sized predicted 9 modes PSF example

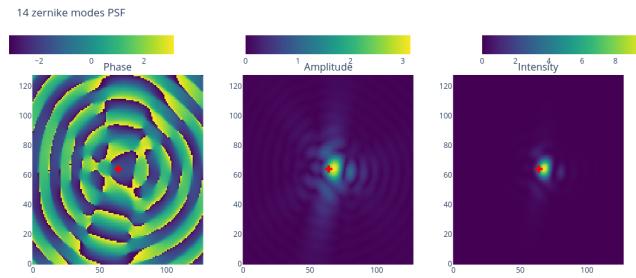


(d) cropped sized predicted 9 modes PSF example

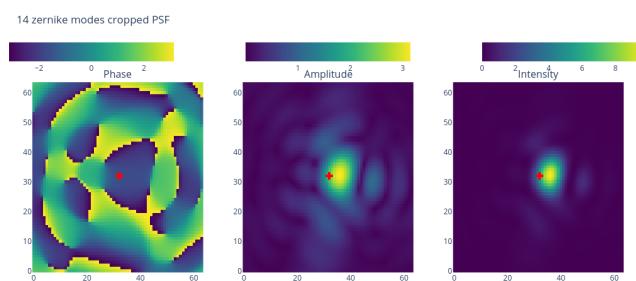
Figure 7: 9 Zernike modes PSF datasets examples

14 Zernike modes PSFs

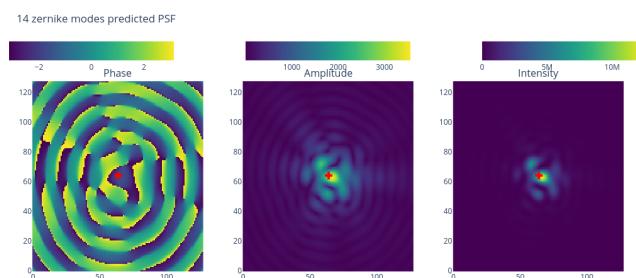
- **Original sized 14 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 128x128x2 and 128x128 matrices respectively. The aberration by a 14 modes zernike basis.
- **Cropped sized 14 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped PSFs correspond to the central pixels from the Original sized 14 modes PSFs.
- **Original sized predicted 14 modes PSFs:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 128x128x2 and 128x128 matrices respectively. These predicted PSFs are the outputs of a model trained with the Original sized 14 modes PSFs dataset and their corresponding PL intensities.
- **Cropped sized predicted 14 modes PSF:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 14 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 14 modes PSFs dataset).



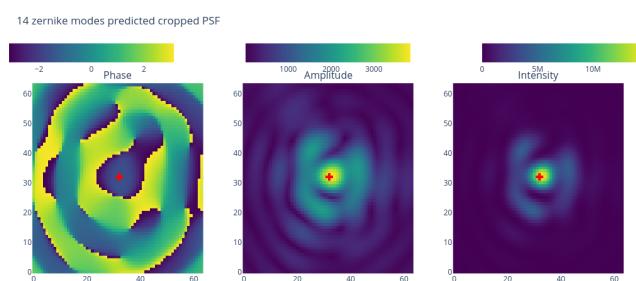
(a) Original sized 14 modes PSF example



(b) Cropped sized 14 modes PSF example



(c) Original sized 14 modes PSF example

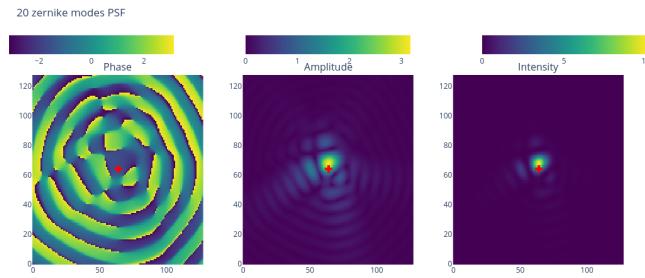


(d) cropped sized predicted 14 modes PSF example

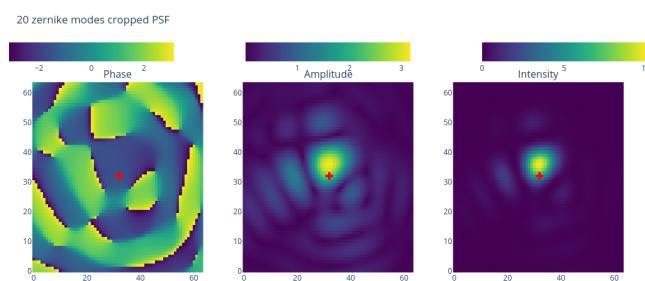
Figure 8: 14 Zernike modes PSF datasets examples

20 Zernike modes PSFs

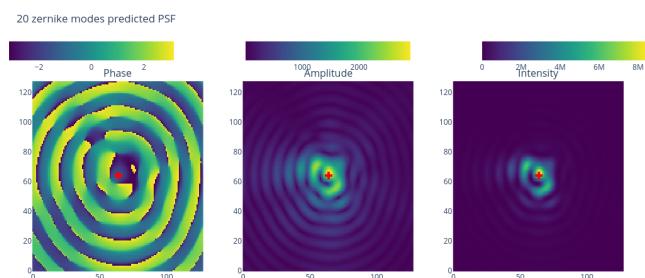
- **Original sized 20 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 128x128x2 and 128x128 matrices respectively. The aberration by a 20 modes zernike basis.
- **Cropped sized 20 modes PSFs:** Two datasets of 70000 electric fields and corresponding intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped PSFs correspond to the central pixels from the Original sized 20 modes PSFs.
- **Original sized predicted 20 modes PSFs:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 128x128x2 and 128x128 matrices respectively. These predicted PSFs are the outputs of a model trained with the Original sized 20 modes PSFs dataset and their corresponding PL intensities.
- **Cropped sized predicted 20 modes PSF:** Two datasets of 70000 predicted electric fields and predicted intensities stored in 64x64x2 and 64x64 matrices respectively. These cropped predicted PSFs are the outputs of a model trained with the Cropped sized 20 modes PSFs dataset and their corresponding PL intensities (which are the same ouput intensities from the Original sized 20 modes PSFs dataset).



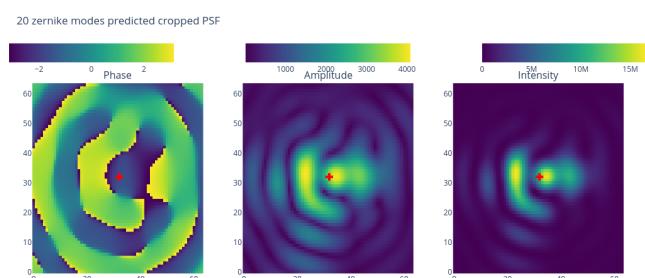
(a) Original sized 20 modes PSF example



(b) Cropped sized 20 modes PSF example



(c) Original sized 20 modes PSF example



(d) cropped sized predicted 20 modes PSF example

Figure 9: 20 Zernike modes PSF datasets examples

LP mode coefficients There are two PL intensities dataset per Zernike aberration PSF subgroup: LP modes coefficients for 2, 5, 9, 14, 20 modes PSFs. Each of the dataset has 70000 datapoints each datapoint being the complex coefficients stored in a 19x2 matrix that separates the real and imaginary part of the coefficients.

The two datasets correspond to the LP coefficients that are computed in the multimode end of Photonic Lanterns. The PLs are:

- 19 mode supporting multimode end with 19 waveguides in the single mode end.
- 42 mode supporting multimode end with 42 waveguides in the single mode end.

PL intensities There is one PL intensities dataset per Zernike aberration PSF subgroup: PL intensities for 2, 5, 9, 14, 20 modes PSFs. Each of the dataset has 70000 datapoints each datapoint being the 19 intensities corresponding to the PSF

The two datasets correspond to the single mode end intensities of Photonic Lanterns. The PLs are:

- 19 mode supporting multimode end with 19 waveguides in the single mode end.
 - 42 mode supporting multimode end with 42 waveguides in the single mode end.
-

1.2 The models

For all the datasets a model with the following configuration has been trained. The inputs of the model are the PL intensities and the outputs are the flattened matrices that represent the PSFs' complex fields.

HYPERPARAMETERS:***ARCHITECTURE HYPERPARAMETERS :**

- Fully Connected
- Input shape: 19
- Output shape:
 - 32768 for original sized PSF electric field
 - 16384 for original sized PSF intensity
 - 8192 for cropped sized PSF electric field
 - 4096 for cropped sized PSF intensity
- Hidden layers: [1024, 1024, 1024, 1024, 1024, 1024]
- Regularizer: None
- Hidden Layers Activation: relu
- Output Layer Activation: linear
- Batch Normalization: False
- Dropout: False , 0.2

***COMPILE HYPERPARAMETERS :**

- Optimizer: ADAM lr=0.001 , beta_1=0.9 , beta_2=0.999
- Loss Function: MSE
- Metric: MSE

***TRAINING HYPERPARAMETERS :**

- Epochs: 100
- Batch size: 32
- Callbacks:

```
-ReduceLROnPlateau: MSE 20 x0.1
-Early Stop: MSE 50
```

The exception is the model trained for the Atmospheric Aberration Cropped PSF which has Batch Normalization activated.

1.2.1 Atmospheric aberration related models

Original sized PSF :

```
-Train MSE: 0.004607476759701967
-Validation MSE: 0.056021399796009064
```

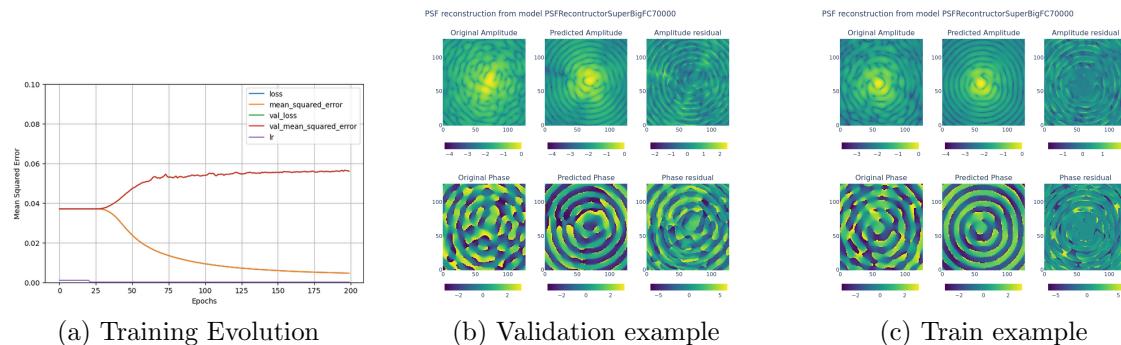


Figure 10: Results of training the model PSFReconstructorSuperBigFC70000-1

Cropped sized PSF :

```
-Train MSE: 0.008466990664601326
-Validation MSE: 0.20970138907432556
```

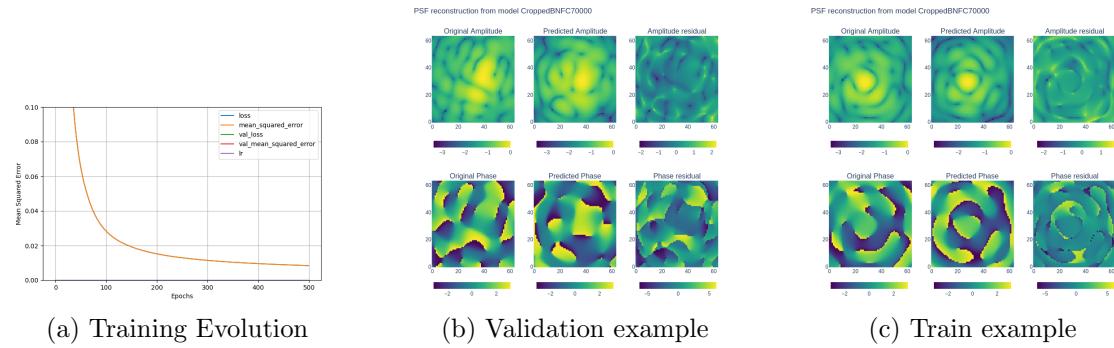


Figure 11: Results of training the model PSFReconstructorSuperBigFC70000-1

1.2.2 Zernike modes related models

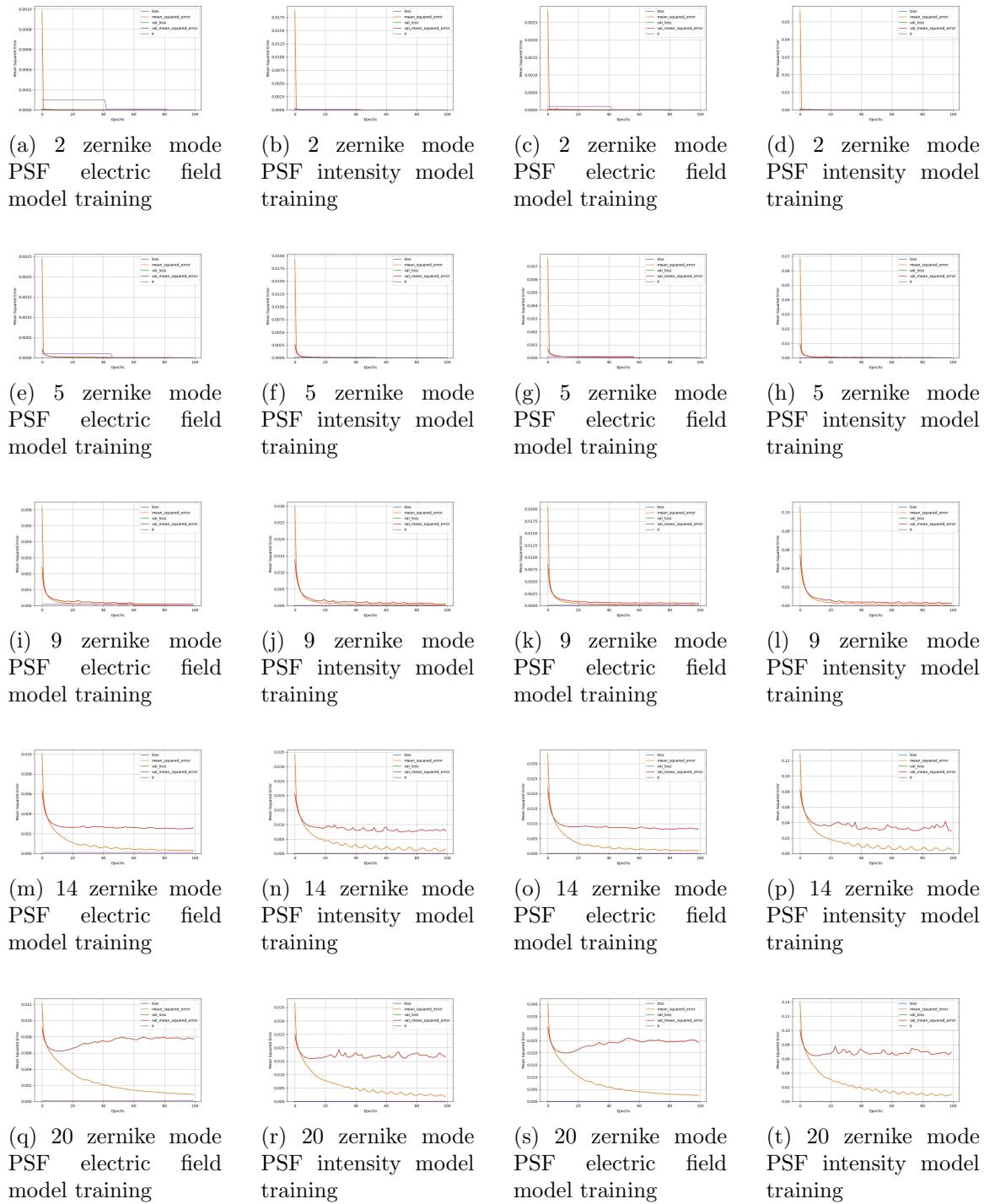


Figure 12: Training evolution comparison for the different Zernike datasets

2 modes PSF models MSE :

	Electric field	Cropped electric field	Intensity	Cropped intensity
Train MSE	6.823e-8	1.770e-7	6.212e-7	2.646e-6
Val MSE	6.053e-8	1.450e-7	5.212e-7	1.880e-6

Table 1: 2 Zernike modes related models MSE

5 modes PSF models MSE :

	Electric field	Cropped electric field	Intensity	Cropped intensity
Train MSE	1.753e-6	4.443e-6	6.019e-6	3.044e-5
Val MSE	2.529e-6	7.328e-6	1.142e-6	4.700e-5

Table 2: 5 Zernike modes related models MSE

9 modes PSF models MSE :

	Electric field	Cropped electric field	Intensity	Cropped intensity
Train MSE	1.825e-5	1.599e-4	1.025e-4	8.590e-4
Val MSE	1.025e-4	4.883e-4	4.667e-4	2.770e-3

Table 3: 9 Zernike modes related models MSE

14 modes PSF models MSE :

	Electric field	Cropped electric field	Intensity	Cropped intensity
Train MSE	3.085e-4	9.827e-4	1.597e-3	4.715e-3
Val MSE	2.602e-3	8.197e-3	7.773e-3	0.0294

Table 4: 14 Zernike modes related models MSE

20 modes PSF models MSE :

	Electric field	Cropped electric field	Intensity	Cropped intensity
Train MSE	8.804e-4	2.546e-3	1.872e-3	9.445e-3
Val MSE	7.74e-3	0.024	0.0167	0.069

Table 5: 20 Zernike modes related models MSE

A summary of the MSE evolution over the Zernike PSFs datasets is shown below. The fact that the validation MSE for 2 modes is the worse may be because the neural network is not able to understand traslations.

MSE Evolution over PSF datasets

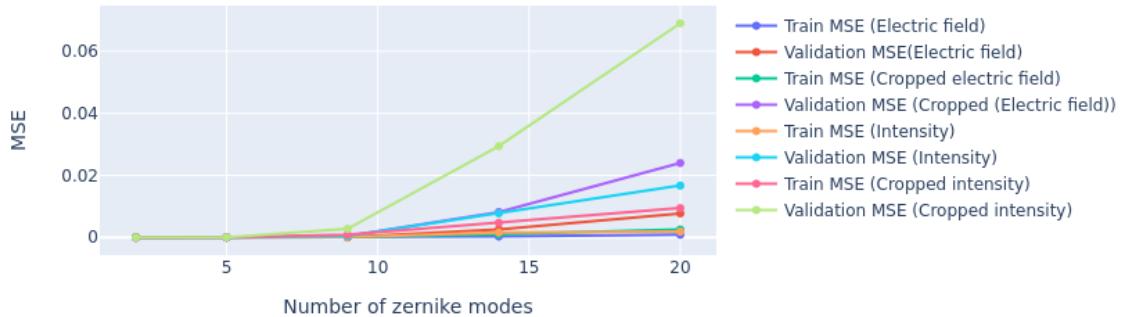


Figure 13: MSE evolution over the Zernike PSFs datasets

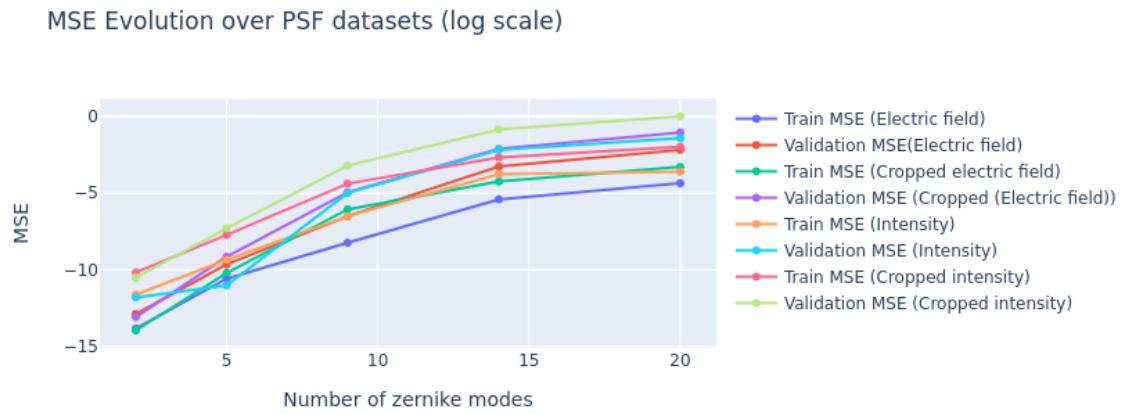
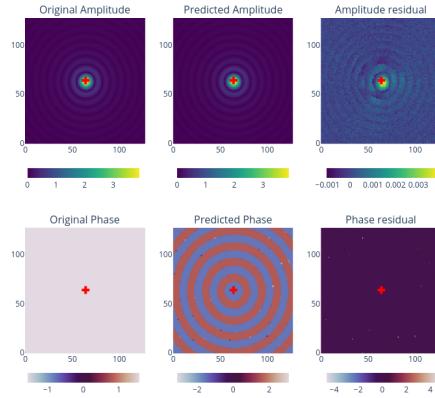


Figure 14: MSE evolution over the Zernike PSFs datasets in logarithmic scale

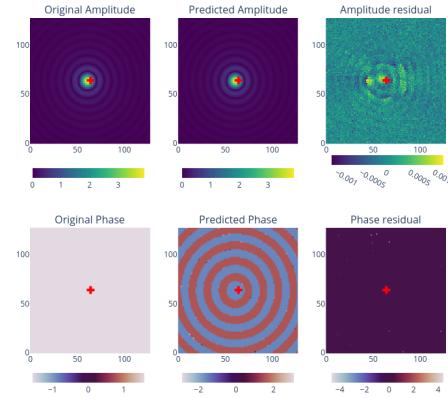
Model output examples :

PSF reconstruction from model SuperBigZernike2MFC70000



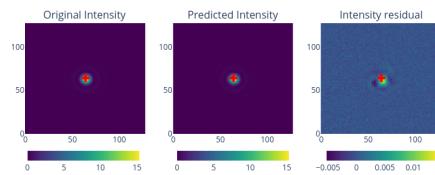
(a) Train example from electric field model

PSF reconstruction from model SuperBigZernike2MFC70000



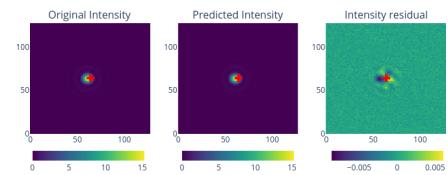
(b) Validation example from electric field model

PSF reconstruction from model SuperBigZernike2MFCIntensity70000



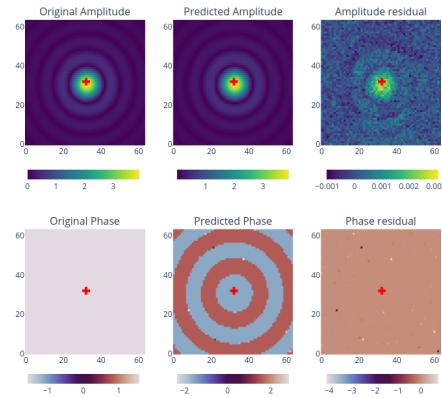
(c) Train example from intensity model

PSF reconstruction from model SuperBigZernike2MFCIntensity70000



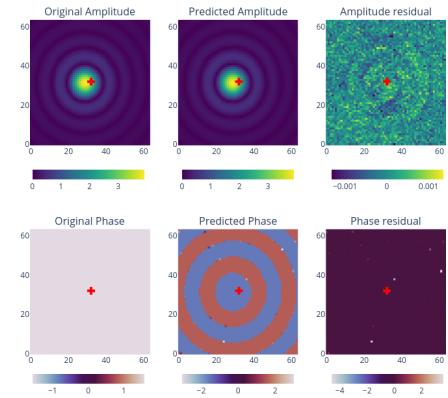
(d) Validation example from intensity model

PSF reconstruction from model SuperBigCroppedZernike2MFC70000



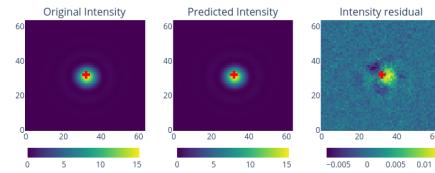
(e) Train example from cropped electric field model

PSF reconstruction from model SuperBigCroppedZernike2MFC70000



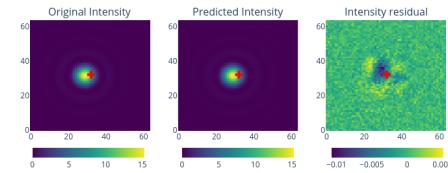
(f) Validation example cropped from electric field model

PSF reconstruction from model SuperBigCroppedZernike2MFCIntensity70000



(g) Train example from cropped intensity model

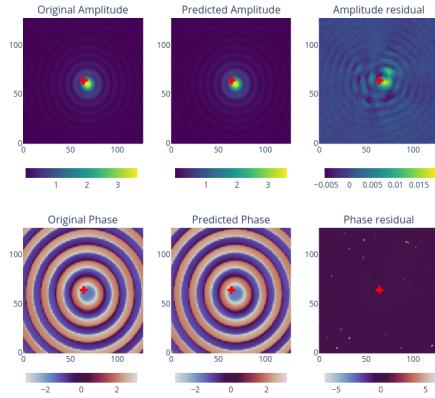
PSF reconstruction from model SuperBigCroppedZernike2MFCIntensity70000



(h) Validation example from cropped model

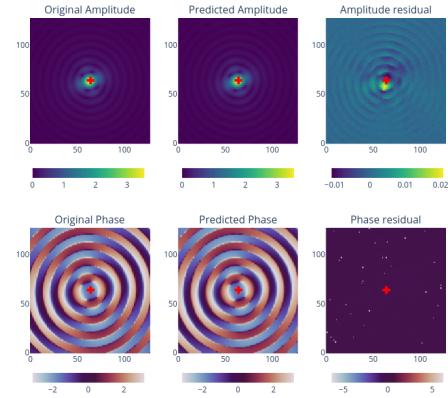
Figure 15: Model outputs for 2 mode PSF datasets

PSF reconstruction from model SuperBigZernike5MFC70000



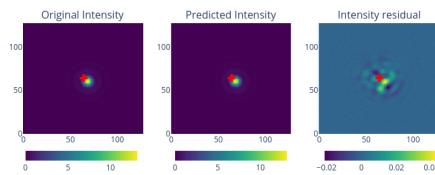
(a) Train example from electric field model

PSF reconstruction from model SuperBigZernike5MFC70000



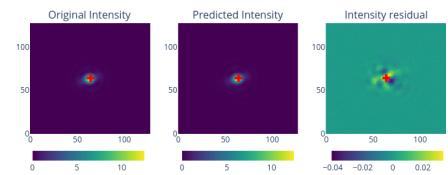
(b) Validation example from electric field model

PSF reconstruction from model SuperBigZernike5MFCIntensity70000



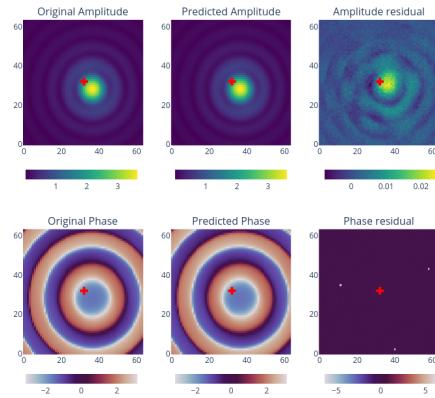
(c) Train example from intensity model

PSF reconstruction from model SuperBigZernike5MFCIntensity70000



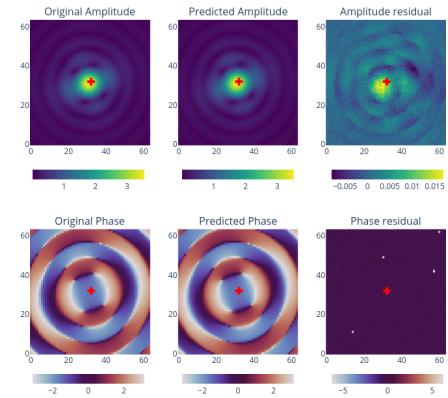
(d) Validation example from intensity model

PSF reconstruction from model SuperBigCroppedZernike5MFC70000



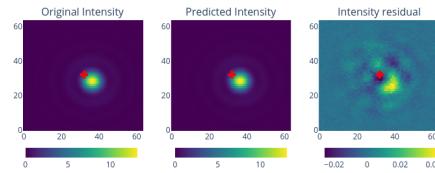
(e) Train example from cropped electric field model

PSF reconstruction from model SuperBigCroppedZernike5MFC70000



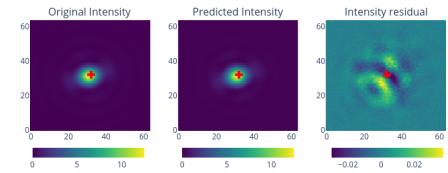
(f) Validation example cropped from electric field model

PSF reconstruction from model SuperBigCroppedZernike5MFCIntensity70000



(g) Train example from cropped intensity model

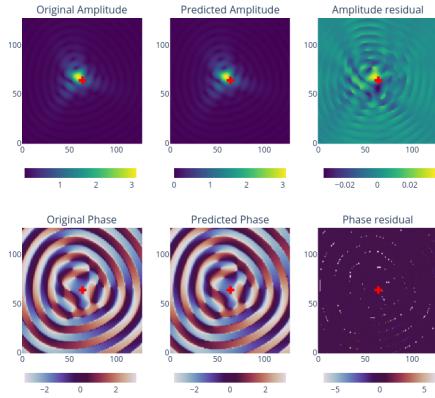
PSF reconstruction from model SuperBigCroppedZernike5MFCIntensity70000



(h) Validation example from cropped model

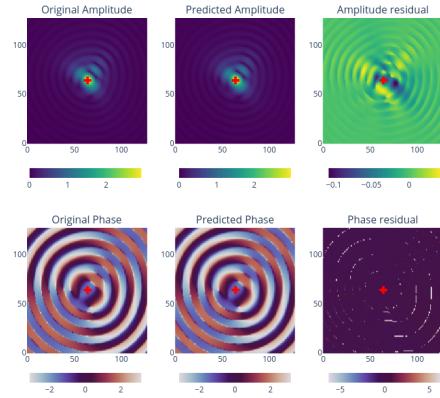
Figure 16: Model outputs for 5 mode PSF datasets

PSF reconstruction from model SuperBigZernike9MFC70000



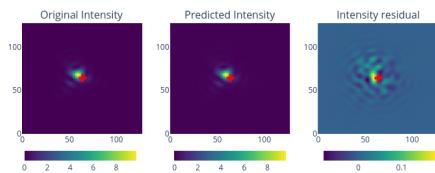
(a) Train example from electric field model

PSF reconstruction from model SuperBigZernike9MFC70000



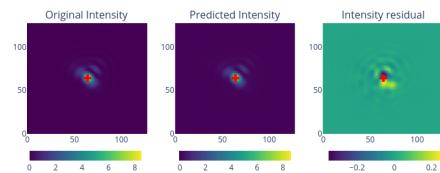
(b) Validation example from electric field model

PSF reconstruction from model SuperBigZernike9MFCIntensity70000



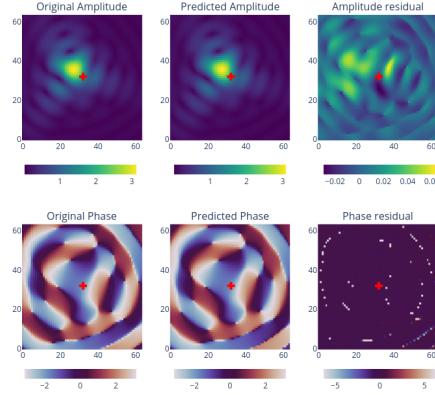
(c) Train example from intensity model

PSF reconstruction from model SuperBigZernike9MFCIntensity70000



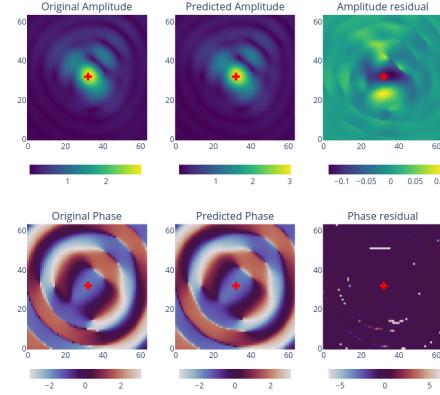
(d) Validation example from intensity model

PSF reconstruction from model SuperBigCroppedZernike9MFC70000



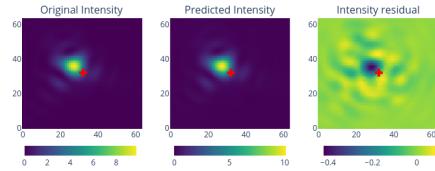
(e) Train example from cropped electric field model

PSF reconstruction from model SuperBigCroppedZernike9MFC70000



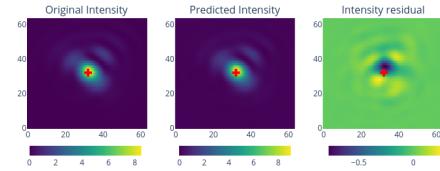
(f) Validation example cropped from electric field model

PSF reconstruction from model SuperBigCroppedZernike9MFCIntensity70000



(g) Train example from cropped intensity model

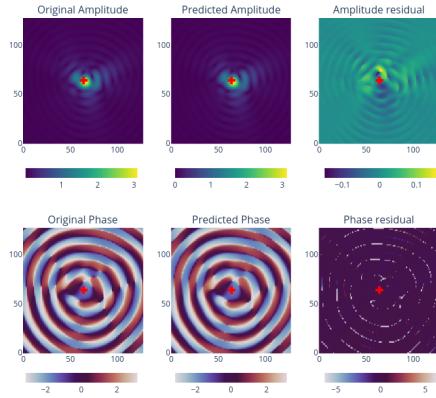
PSF reconstruction from model SuperBigCroppedZernike9MFCIntensity70000



(h) Validation example from cropped model

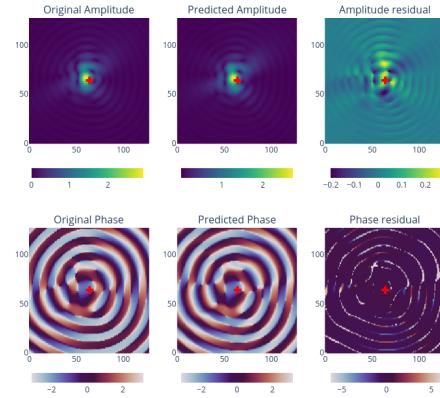
Figure 17: Model outputs for 9 mode PSF datasets

PSF reconstruction from model SuperBigZernike14MFC70000



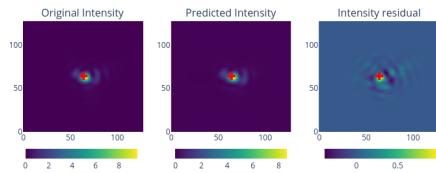
(a) Train example from electric field model

PSF reconstruction from model SuperBigZernike14MFC70000



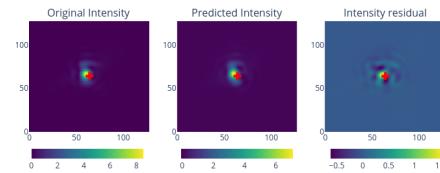
(b) Validation example from electric field model

PSF reconstruction from model SuperBigZernike14MFCIntensity70000



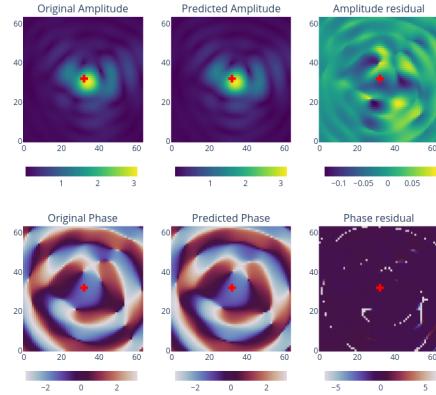
(c) Train example from intensity model

PSF reconstruction from model SuperBigZernike14MFCIntensity70000



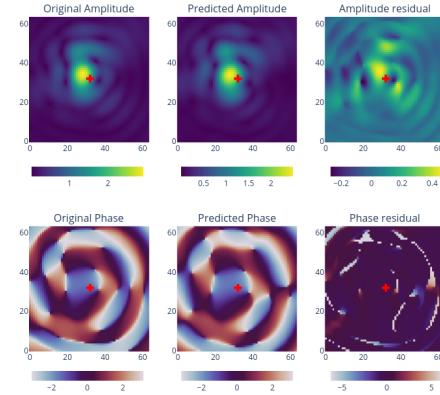
(d) Validation example from intensity model

PSF reconstruction from model SuperBigCroppedZernike14MFC70000



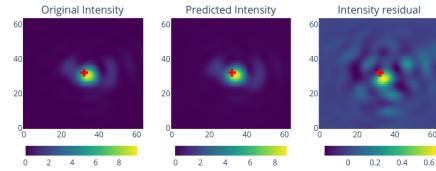
(e) Train example from cropped electric field model

PSF reconstruction from model SuperBigCroppedZernike14MFC70000



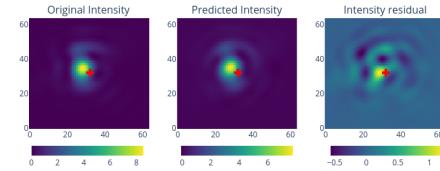
(f) Validation example cropped from electric field model

PSF reconstruction from model SuperBigCroppedZernike14MFCIntensity70000



(g) Train example from cropped intensity model

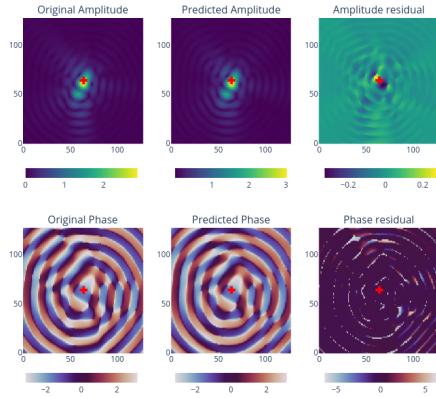
PSF reconstruction from model SuperBigCroppedZernike14MFCIntensity70000



(h) Validation example from cropped model

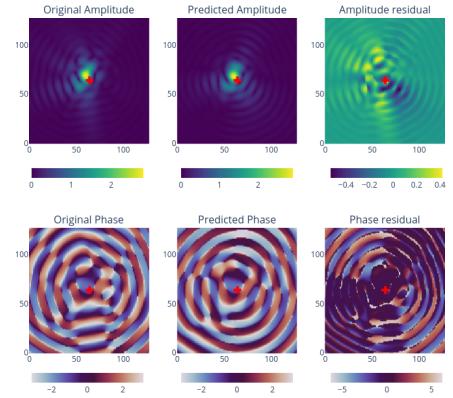
Figure 18: Model outputs for 14 mode PSF datasets

PSF reconstruction from model SuperBigZernike20MFC70000



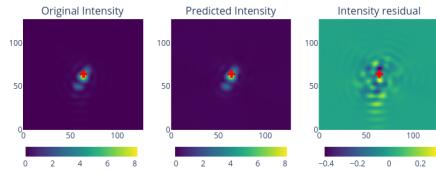
(a) Train example from electric field model

PSF reconstruction from model SuperBigZernike20MFC70000



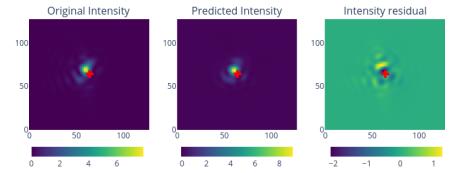
(b) Validation example from electric field model

PSF reconstruction from model SuperBigZernike20MFCIntensity70000



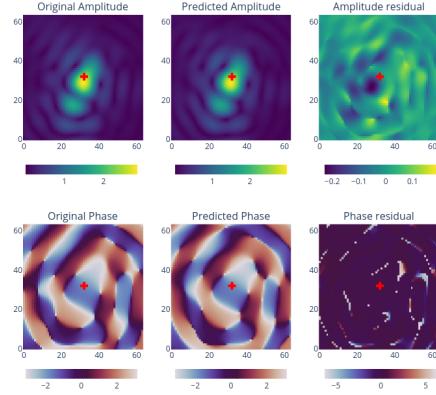
(c) Train example from intensity model

PSF reconstruction from model SuperBigZernike20MFCIntensity70000



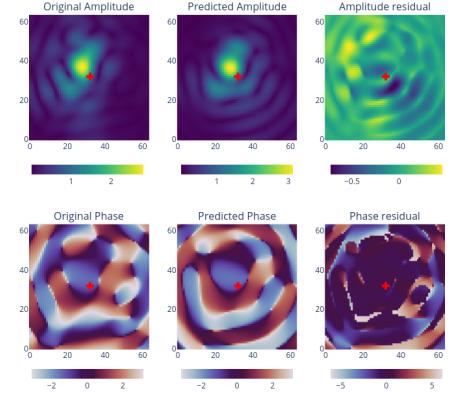
(d) Validation example from intensity model

PSF reconstruction from model SuperBigCroppedZernike20MFC70000



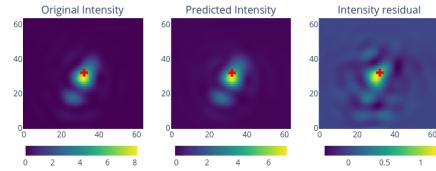
(e) Train example from cropped electric field model

PSF reconstruction from model SuperBigCroppedZernike20MFC70000



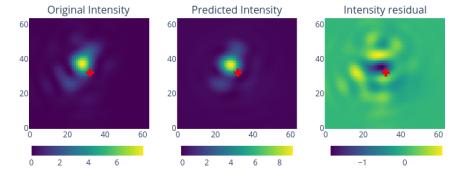
(f) Validation example cropped from electric field model

PSF reconstruction from model SuperBigCroppedZernike20MFCIntensity70000



(g) Train example from cropped intensity model

PSF reconstruction from model SuperBigCroppedZernike20MFCIntensity70000



(h) Validation example from cropped model

Figure 19: Model outputs for 20 mode PSF datasets

1.3 Euclidean distances analysis for atmospheric aberration PSFs

1.3.1 Preprocessing

- The PSF electric fields are converted to a matrix of intensities of 128x128 size and the flattened to calculate the euclidean distances between them.
- 70000 datapoint pairs are defined for which the euclidean distances will be calculated.

1.3.2 Results

After performing an ANOVA test on the euclidean distances from the selected pairs of the 4 datasets obtaining a p-value of 0 and F-statistic of 4789.1531.

1.3.3 Analysis

The correlation is 0.3 which indicates a slightly positive linear relationship between the PL flux and PSF in all cases except for the cropped predictions which has a 0.2 correlation rate. This makes sense as the model that predicted those PSFs is more overfitted than the model that predicts the original sized PSFs. The clouds are dispersed almost equally from the center of mass which may indicate that a 19 mode PL may not be enough to encode all PSF information.

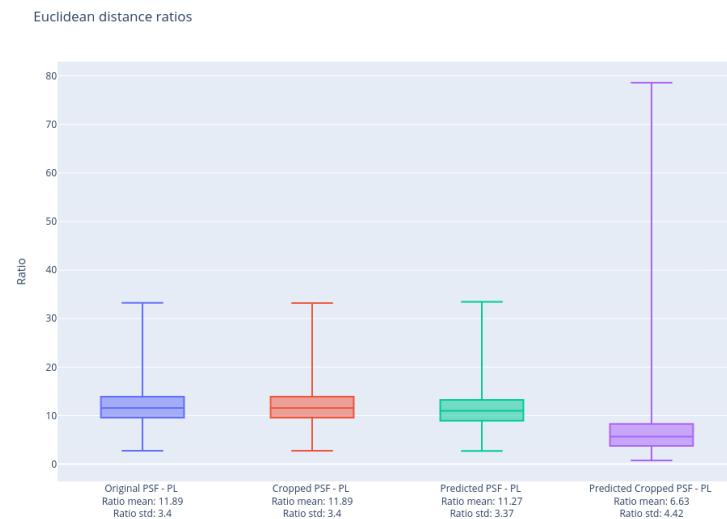
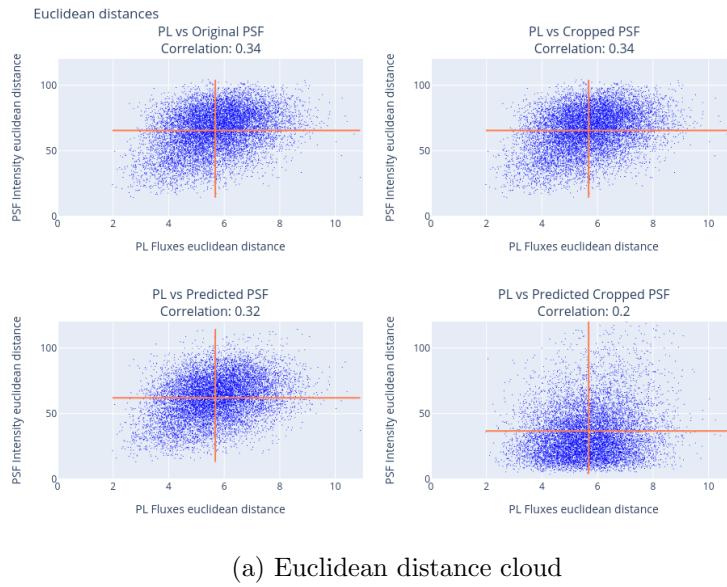


Figure 20: Euclidean distances ratios between PL and PSF pairs

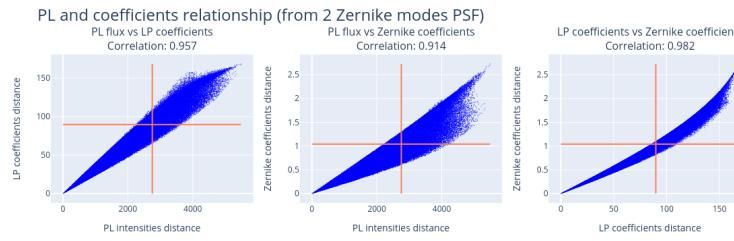
1.4 Euclidean distances analysis for Zernike modes PSFs

1.4.1 Preprocessing

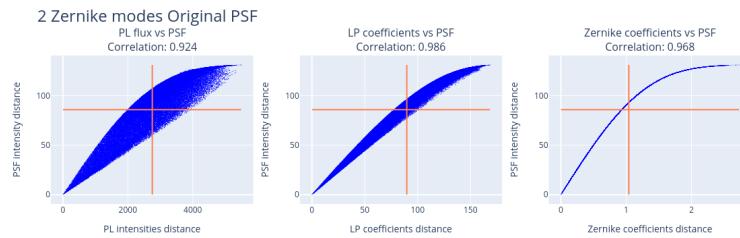
- The PSF electric fields matrices are flattened to compute the euclidean distances between 1d vectors.

- 70000 datapoint pairs for each zernike datasets are randomly defined. The euclidean distances will be calculated for these selected pairs.
- In this case, LP coefficients are also analysed.

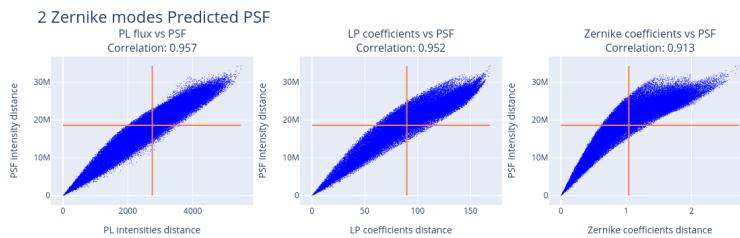
1.4.2 Euclidean distances comparison per number of zernike modes



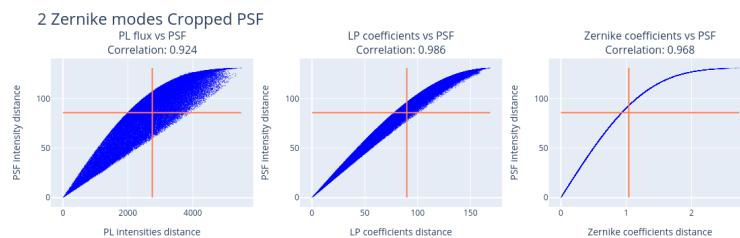
(a) Euclidean distance comparison between coefficients and PL flux



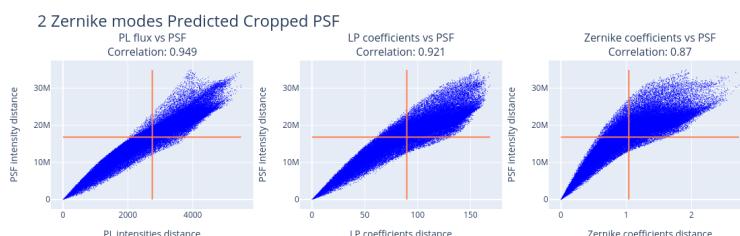
(b) Euclidean distances for PSF intensity



(c) Euclidean distances for predicted PSF intensity

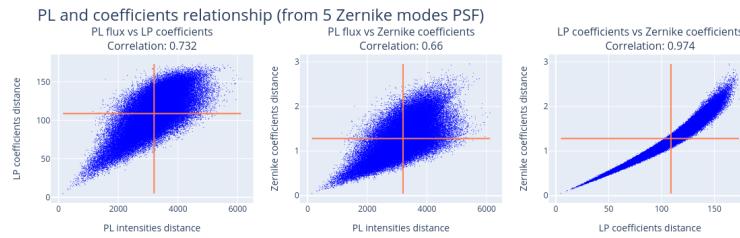


(d) Euclidean distances for cropped PSF intensity

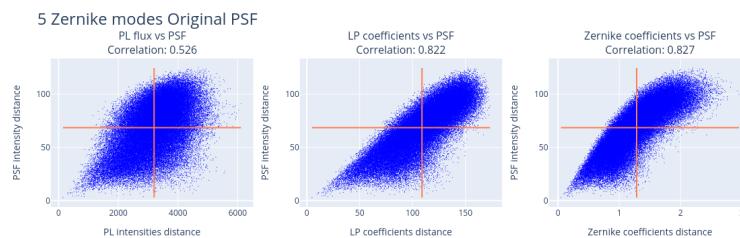


(e) Euclidean distances for predicted PSF intensity

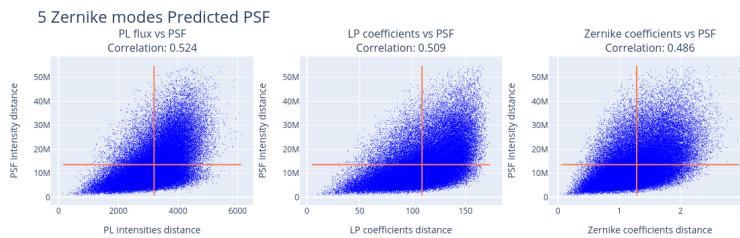
Figure 21: Euclidean distances comparison for 2 zernike modes related datasets



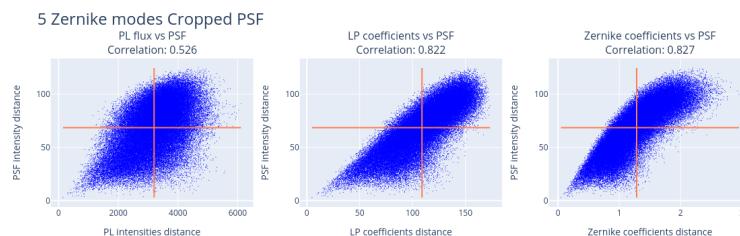
(a) Euclidean distance comparison between coefficients and PL flux



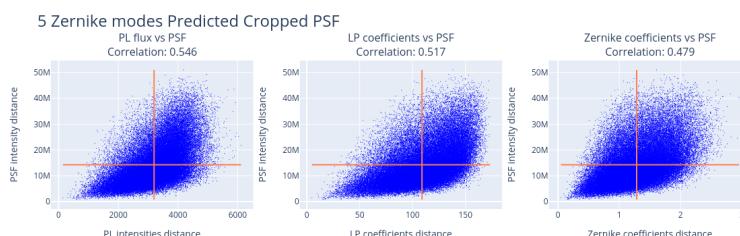
(b) Euclidean distances for PSF intensity



(c) Euclidean distances for predicted PSF intensity

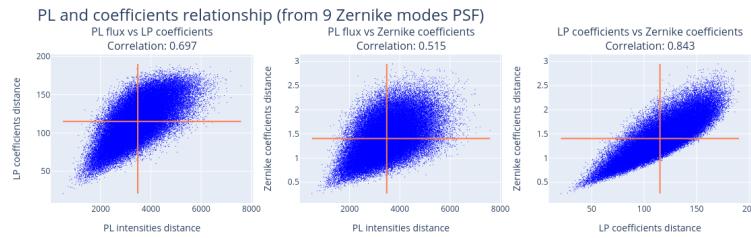


(d) Euclidean distances for cropped PSF intensity

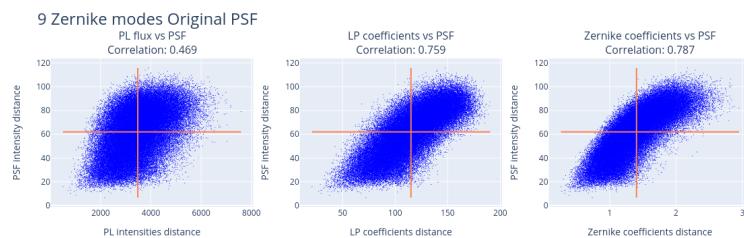


(e) Euclidean distances for predicted PSF intensity

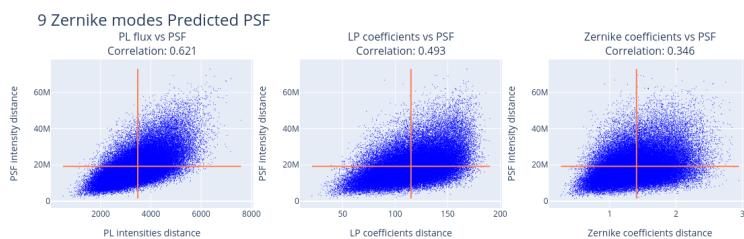
Figure 22: Euclidean distances comparison for 5 zernike modes related datasets



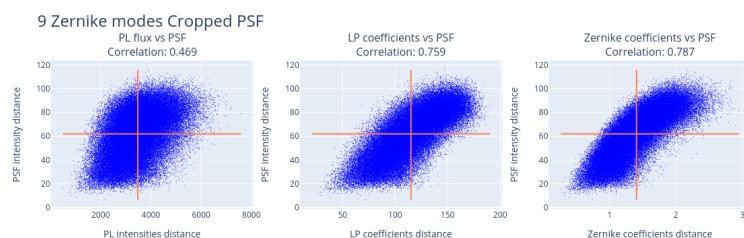
(a) Euclidean distance comparison between coefficients and PL flux



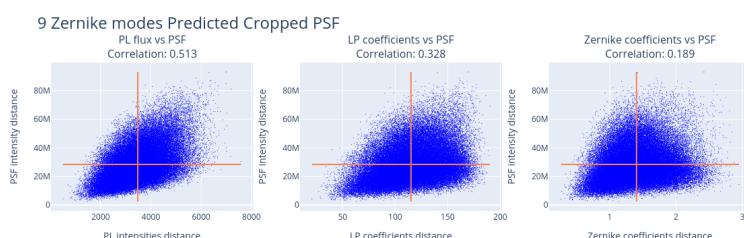
(b) Euclidean distances for PSF intensity



(c) Euclidean distances for predicted PSF intensity

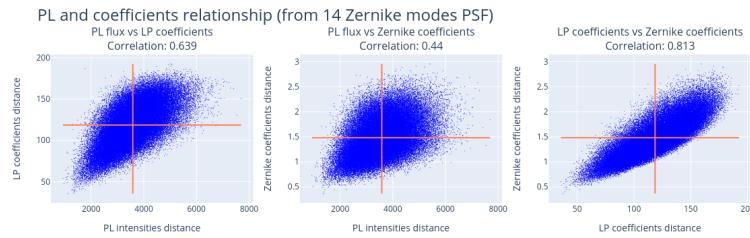


(d) Euclidean distances for cropped PSF intensity

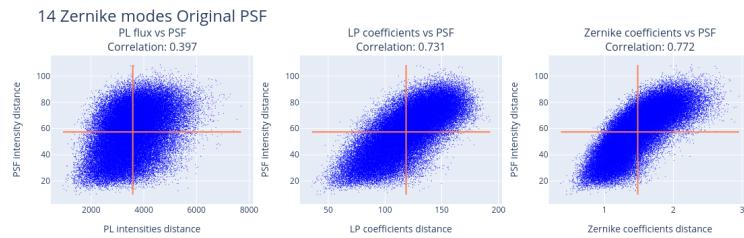


(e) Euclidean distances for predicted PSF intensity

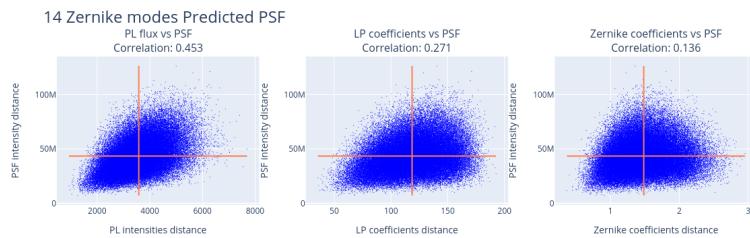
Figure 23: Euclidean distances comparison for 9 zernike modes related datasets



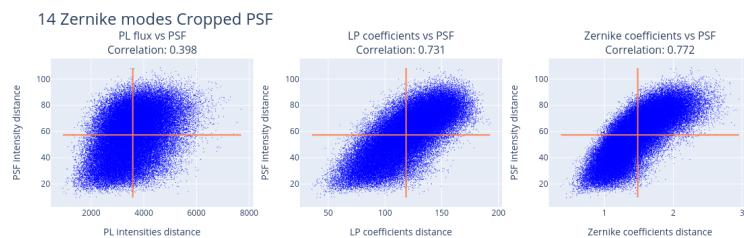
(a) Euclidean distance comparison between coefficients and PL flux



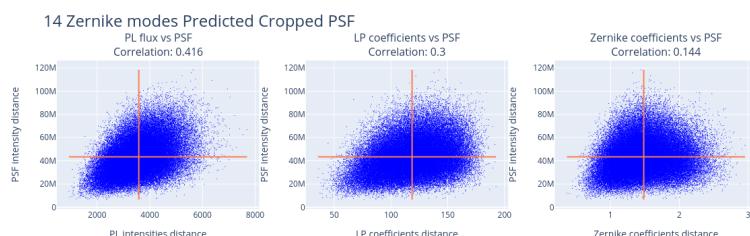
(b) Euclidean distances for PSF intensity



(c) Euclidean distances for predicted PSF intensity

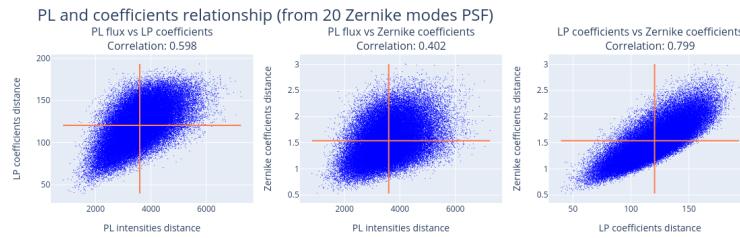


(d) Euclidean distances for cropped PSF intensity

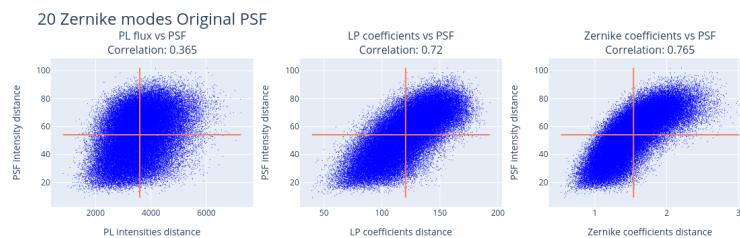


(e) Euclidean distances for predicted PSF intensity

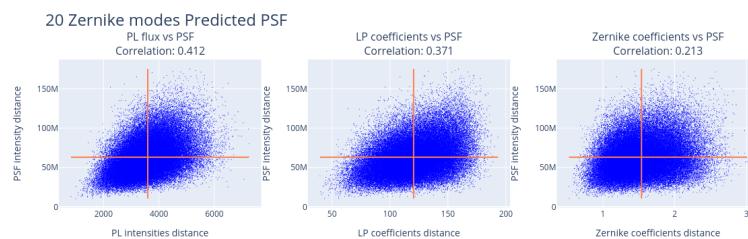
Figure 24: Euclidean distances comparison for 14 zernike modes related datasets



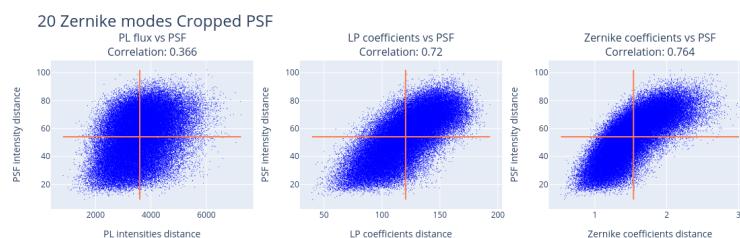
(a) Euclidean distance comparison between coefficients and PL flux



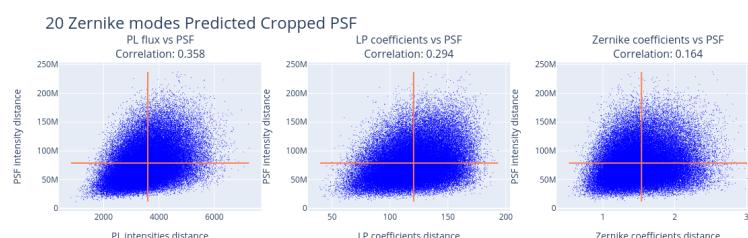
(b) Euclidean distances for PSF intensity



(c) Euclidean distances for predicted PSF intensity



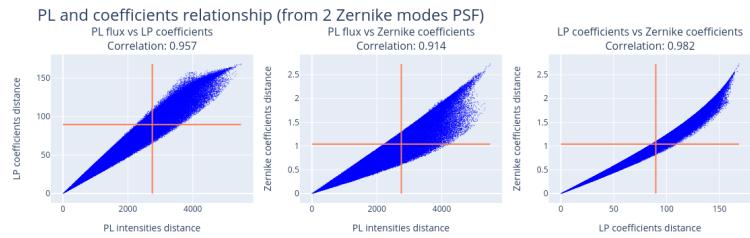
(d) Euclidean distances for cropped PSF intensity



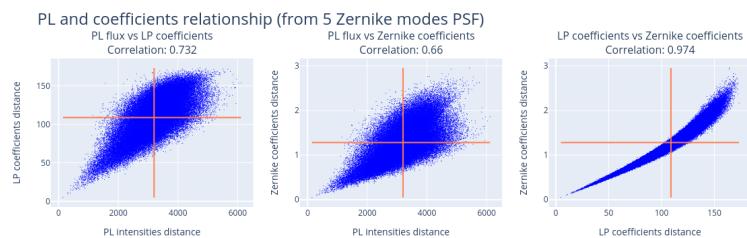
(e) Euclidean distances for predicted PSF intensity

Figure 25: Euclidean distances comparison for 20 zernike modes related datasets

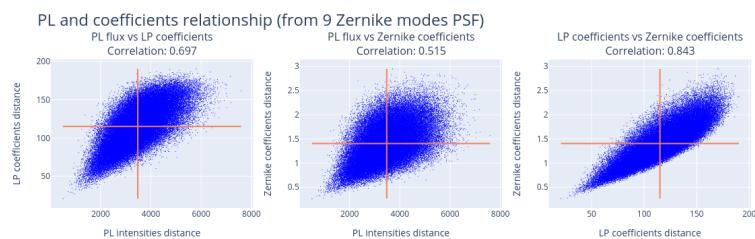
1.4.3 Euclidean distances comparison evolution over number of zernike modes



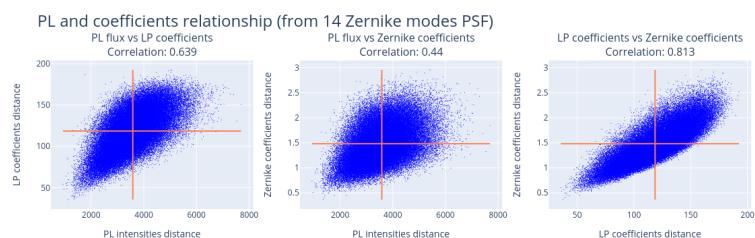
(a) Coefficients vs PL flux for 2 zernike modes



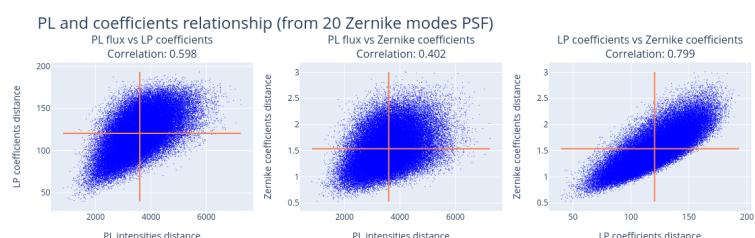
(b) Coefficients vs PL flux for 5 zernike modes



(c) Coefficients vs PL flux for 9 zernike modes

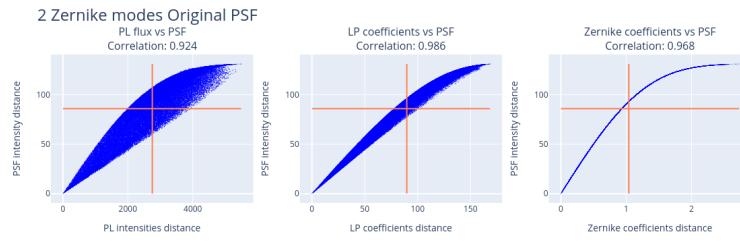


(d) Coefficients vs PL flux for 14 zernike modes

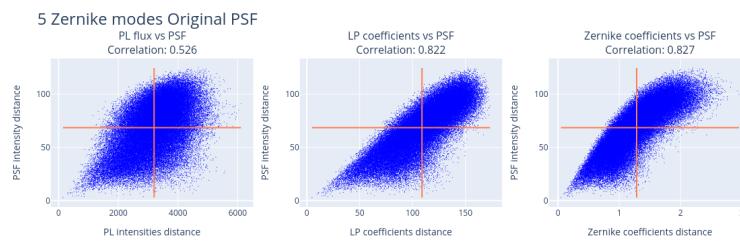


(e) Coefficients vs PL flux for 20 zernike modes

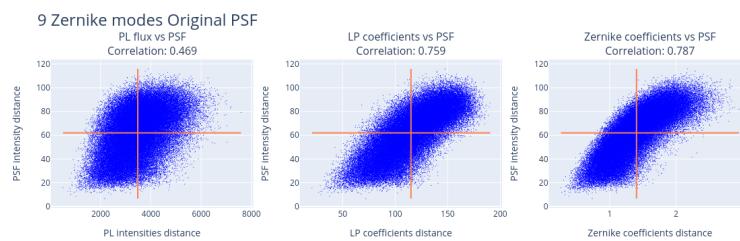
Figure 26: Euclidean distance comparison between coefficients and PL flux for different number of modes



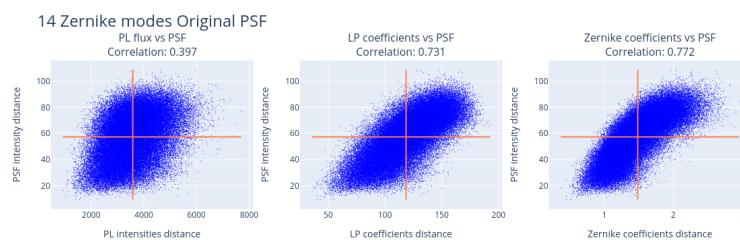
(a) Euclidean distances for 2 zernike modes PSF intensity



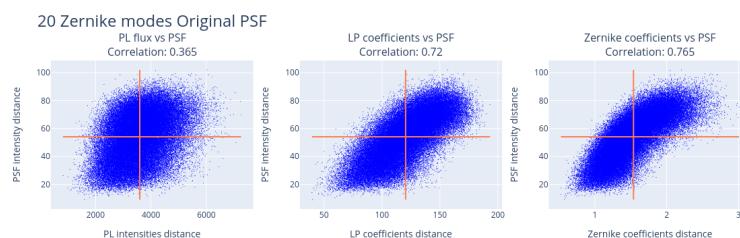
(b) Euclidean distances for 5 zernike modes PSF intensity



(c) Euclidean distances for 9 zernike modes PSF intensity

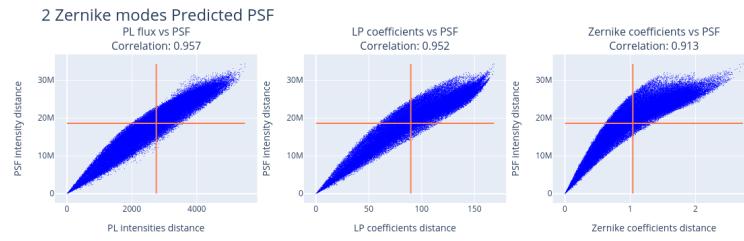


(d) Euclidean distances for 14 zernike modes PSF intensity

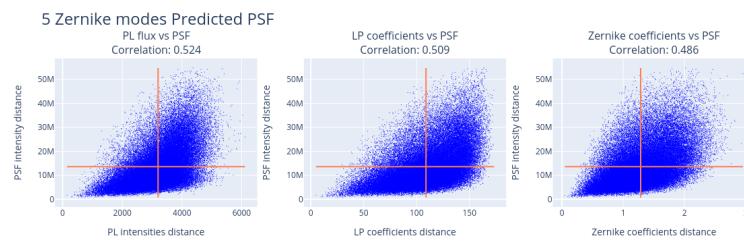


(e) Euclidean distances for 20 zernike modes PSF intensity

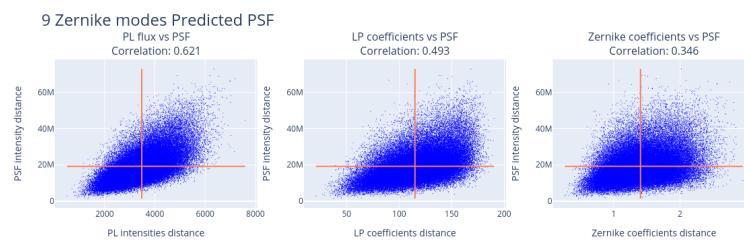
Figure 27: Euclidean distance comparison for PSF intensity for different number of modes



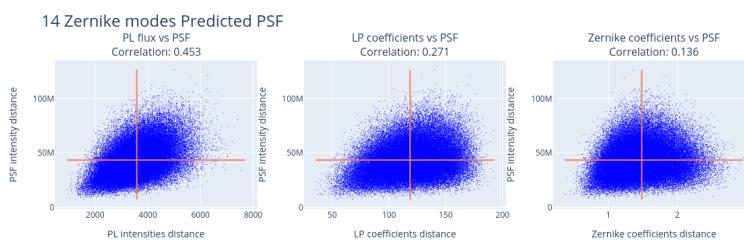
(a) Euclidean distances for 2 zernike modes predicted PSF intensity



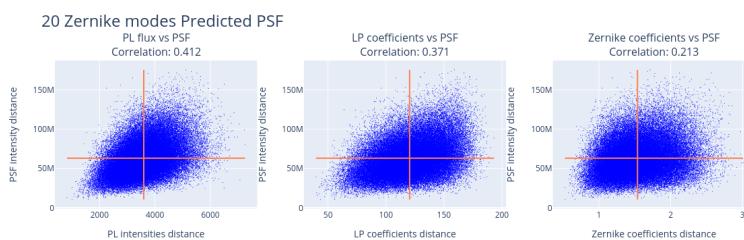
(b) Euclidean distances for 5 zernike modes predicted PSF intensity



(c) Euclidean distances for 9 zernike modes predicted PSF intensity

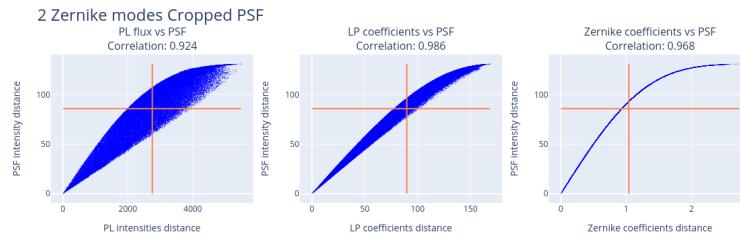


(d) Euclidean distances for 14 zernike modes predicted PSF intensity

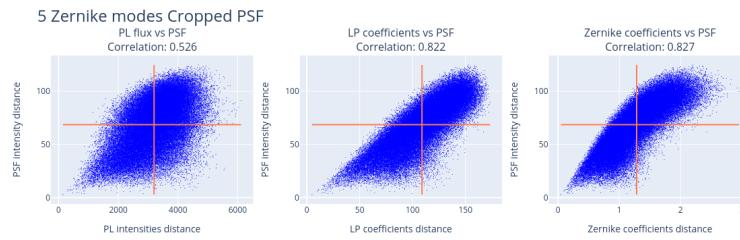


(e) Euclidean distances for 20 zernike modes predicted PSF intensity

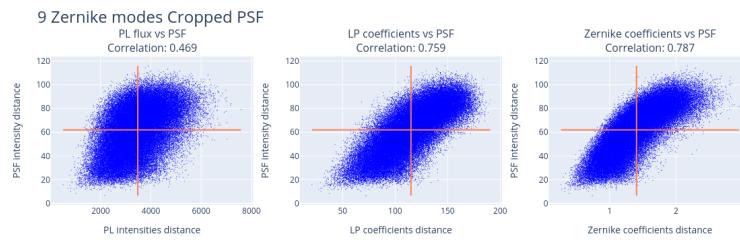
Figure 28: Euclidean distance comparison for predicted PSF intensity for different number of modes



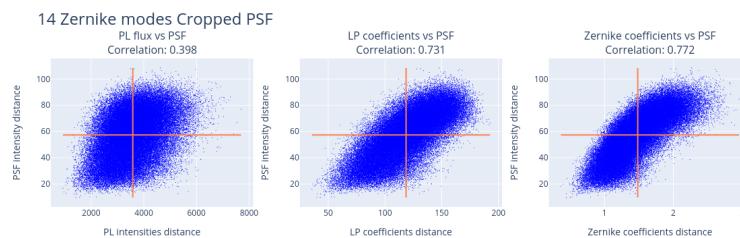
(a) Euclidean distances for 2 zernike modes cropped PSF intensity



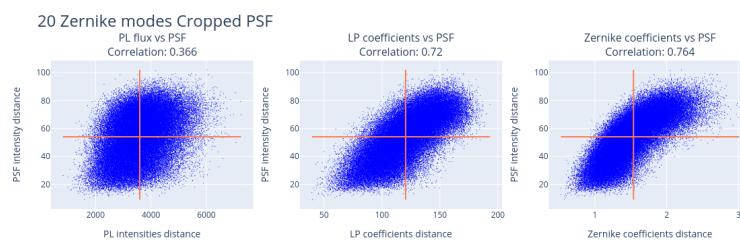
(b) Euclidean distances for 5 zernike modes cropped PSF intensity



(c) Euclidean distances for 9 zernike modes cropped PSF intensity

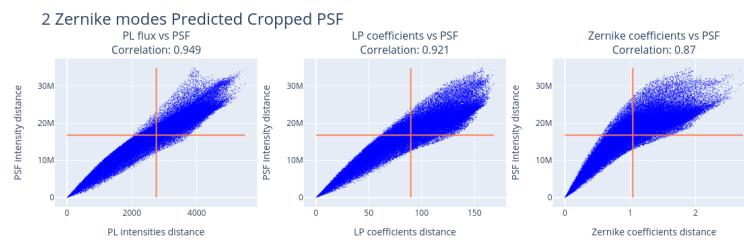


(d) Euclidean distances for 14 zernike modes cropped PSF intensity

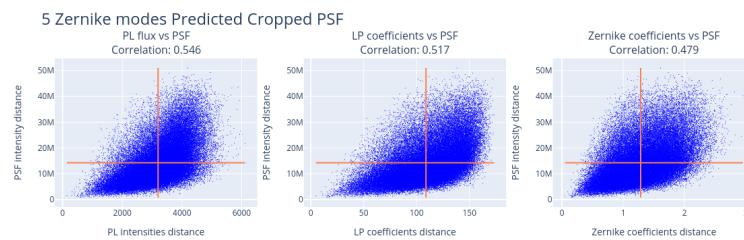


(e) Euclidean distances for 20 zernike modes cropped PSF intensity

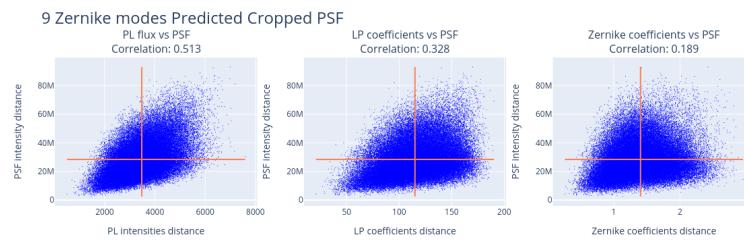
Figure 29: Euclidean distance comparison for cropped PSF intensity for different number of modes



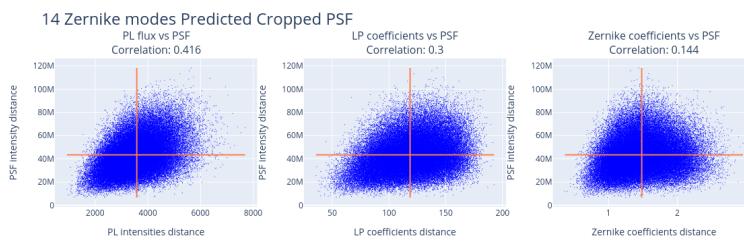
(a) Euclidean distances for 2 zernike modes predicted cropped PSF intensity



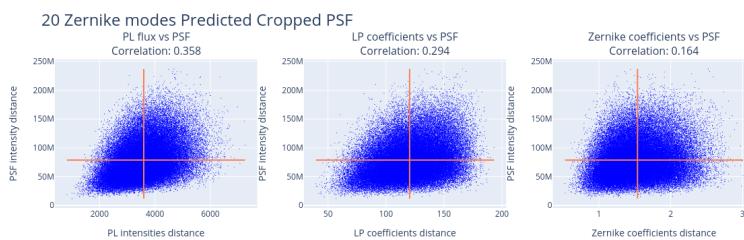
(b) Euclidean distances for 5 zernike modes predicted cropped PSF intensity



(c) Euclidean distances for 9 zernike modes predicted cropped PSF intensity



(d) Euclidean distances for 14 zernike modes predicted cropped PSF intensity



(e) Euclidean distances for 20 zernike modes predicted cropped PSF intensity

Figure 30: Euclidean distance comparison for predicted cropped PSF intensity for different number of modes

1.4.4 Analysis

- The correlations between PSF distances and PL flux distances decay as the number of modes increases while the correlation between PSF distances and LP mode coefficients from the overlap integral are constant.
- The predicted psfs from the train datasets create a similar cloud of points to the original psfs which indicates that the models are capturing the information of the PL.
- The model trained for the dataset of 2 zernike terms PSF is the one that has the most overfit as the table shows (False, False, None indicate that no dropout, no batch normalization and no regularizer has been used), the validation mse just flatlines over the training.
- For 5 terms PSFs the overfitting is reduced significantly although it increases with the number of zernike terms used. When using 20 terms the validation mse flatlines again.
- PL intensities vs PSF datasets evolution is practically the same for original, predicted, cropped and cropped predicted datasets. This indicates that the models are capturing accurately the relationship existing between the original PSF datasets and PL intensities datasets.
- LP coeffs vs PSF datasets evolution is practically the same for original, predicted, cropped and cropped predicted datasets.

1.5 Zernike modes PSFs Clustering

1.5.1 UMAPS

Before clustering, UMAPS for flattened PSF matrices, flattend LP coefficients matrices and PL intensities are processed. The same configuration is used for the different

number of modes.

Dataset type	Number of neighbors	Min distance	Number of components
Zernike modes PSF	500	0.5	1000

Table 6: UMAP parameter configurations for PSFs

1.5.2 Clustering

Using DBSCAN I create clusters for the UMAP representations.

2 Zernike modes :

	ϵ	neighbors	Clusters	CDM	CDV	Non noise points
Zernike coeffs	0.01	6	1535	41.12	92.65	63130
LP coeffs	0.99	5	1470	44.00	631.91	64686
PL fluxes	34.7	5	1431	45.46	739.78	65065

Table 7: DBSCAN clustering for 2 Zernike modes datasets

5 Zernike modes :

	ϵ	neighbors	Clusters	CDM	CDV	Non noise points
Zernike coeffs	0.128	4	1448	38.10	1204.20	55183
LP coeffs	12.8	4	1551	34.98	1109.08	54265
PL fluxes	410	4	1500	35.54	1131.51	53324

Table 8: DBSCAN Clustering for 5 Zernike modes datasets

9 Zernike modes :

	ϵ	neighbors	Clusters	CDM	CDV	Non noise points
Zernike coeffs	0.29	4	1428	30.30	953.86	43275
LP coeffs	26	4	1485	28.01	889.60	43397
PL fluxes	770	3	1616	31.85	1099.66	51474

Table 9: DBSCAN Clustering for 9 Zernike modes datasets

14 Zernike modes :

	ϵ	neighbors	Clusters	CDM	CDV	Non noise points
Zernike coeffs	0.424	3	1639	25.89	880.85	42439
LP coeffs	35.5	3	1445	32.26	1064.67	46626
PL fluxes	950	3	1669	24.31	814.44	40577

Table 10: DBSCAN Clustering for 14 Zernike modes datasets

20 Zernike modes :

	ϵ	neighbors	Clusters	CDM	CDV	Non noise points
Zernike coeffs	26.32	3	1431	26.32	841.71	37668
LP coeffs	40.05	3	1558	24.44	801.25	38079
PL fluxes	1010	3	1647	18.25	566.01	30067

Table 11: DBSCAN Clustering for 20 Zernike modes datasets

1.5.3 Normalised Mutual Information

After running an NMI on the clusters these are the results:

```
NMI from datasets created with 2 Zernike Modes
- LP coeffs and PL intensities:
  0.8164576430824315
- LP coeffs and PSF:
  0.8175003226815989
```

```
- PL intensities and PSF:  
0.7994322323774545  
  
- LP coeffs and predicted PSF:  
0.8175646434293075  
  
- PL intensities and predicted PSF:  
0.7983573747301721  
  
- Predicted PSF and original PSF:  
0.8095814407687527
```

NMI from datasets created with 5 Zernike Modes

```
- LP coeffs and PL intensities:  
0.2546247849754754  
  
- LP coeffs and PSF clusters:  
0.2761463896986409  
  
- PL intensities and PSF clusters:  
0.2169490063500632  
  
- LP coeffs and predicted PSF clusters:  
0.28175438653504503  
  
- PL intensities and predicted PSF clusters:  
0.21635800151194817  
  
- Predicted PSF and original PSF clusters:  
0.5464235488639057
```

NMI from datasets created with 9 Zernike Modes

```
- LP coeffs and PL intensities:
```

```
0.19594140953604774
- LP coeffs and PSF clusters:
0.19331293998931096
- PL intensities and PSF clusters:
0.14033346512809078
- LP coeffs and predicted PSF clusters:
0.18823201153015062
- PL intensities and predicted PSF clusters:
0.13583799714028102
- Predicted PSF and original PSF clusters:
0.4208027610913512
```

NMI from datasets created with 14 Zernike Modes

```
- LP coeffs and PL intensities:
0.2061877271415178
- LP coeffs and PSF clusters:
0.22489998453619262
- PL intensities and PSF clusters:
0.15638010188248286
- LP coeffs and predicted PSF clusters:
0.22450523757936153
- PL intensities and predicted PSF clusters:
0.15124830319698243
- Predicted PSF and original PSF clusters:
0.3978566533293418
```

```
NMI from datasets created with 20 Zernike Modes
```

- LP coeffs and PL intensities:
0.15590863566718433
- LP coeffs and PSF clusters:
0.2096232888801945
- PL intensities and PSF clusters:
0.11871048630190495
- LP coeffs and predicted PSF clusters:
0.20997226177915623
- PL intensities and predicted PSF clusters:
0.1269907541345761
- Predicted PSF and original PSF clusters:
0.2983128702057625

- There is a stronger relationship between LP coefficients and PL outputs than PSF to LP coefficients.
- The relationship between PSF and PL outputs is the weakest.

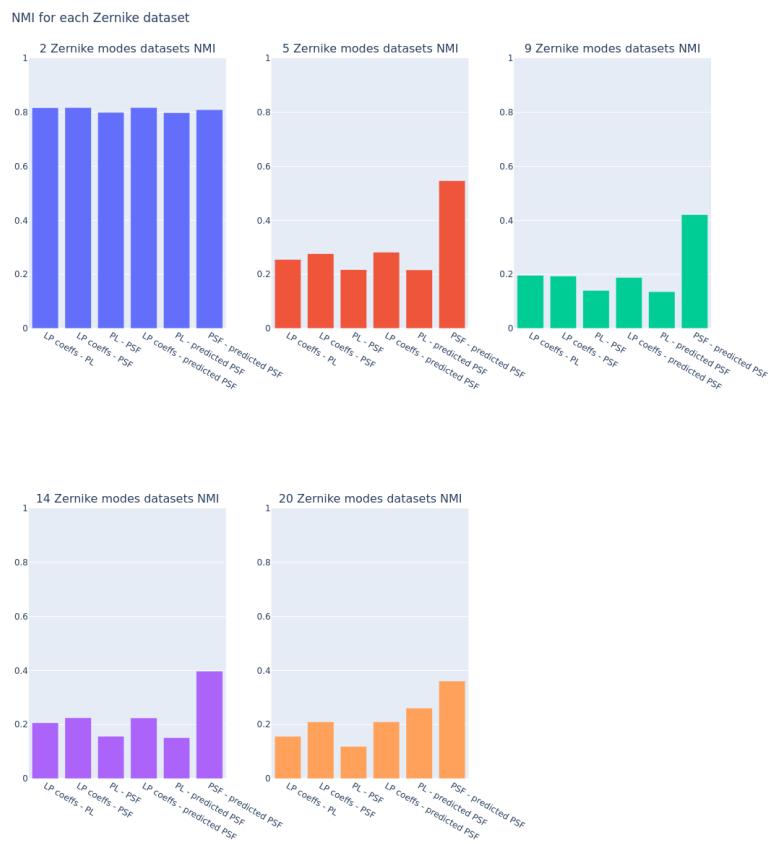


Figure 31: NMI for each of the Zernike related datasets