



Currently,  $c_1 = 3$ , and each isoprofit line has the form  $3x_1 + 2x_2 = \text{constant}$ , or

$$x_2 = \frac{3x_1}{2} + \frac{\text{constant}}{2}$$

and each isoprofit line has a slope of  $-\frac{3}{2}$ . From Figure 1, we see that if a change in  $c_1$  causes the isoprofit lines to be flatter than the carpentry constraint, the optimal solution will change

the current optimal solution (point B) to a new optimal solution (point A). If the profit for each soldier is  $c_1$ , the slope of each isoprofit line will be  $-\frac{c_1}{2}$ . Because the slope of the carpentry constraint is -1, the isoprofit lines will be flatter than the carpentry constraint if  $-\frac{c_1}{2} < -1$ , or  $c_1 < 2$ , and the current basis will no longer be optimal. The new optimal solution will be (0, 80), point A in Figure 1. If the isoprofit lines are steeper than the finishing constraint, then the optimal solution will change from point B to point C. The slope of the finishing constraint is -2. If  $-\frac{c_1}{2} > -2$ , or  $c_1 > 4$ , then the current basis is no longer optimal and point C, (40, 20), will be optimal. In summary, we have shown that (if all other parameters remain unchanged) the current basis remains optimal for  $2 \leq c_1 \leq 4$ , and Giapetto should still manufacture 20 soliders and 60 trains. Of course, even if  $2 \leq c_1 \leq 4$ , Giapetto's profit will change. For instance, if  $c_1 = 4$ , then Giapetto's profit will now be  $4(20) + 2(60) = 200$  instead of 180.

### Graphical Analysis of the Effect of a Change in a Right-Hand Side on the LP's Optimal Solution

A graphical analysis can also be used to determine whether a change in the right-hand side of a constraint will make the current basis no longer optimal. Let  $b_1$  be the number of available finishing hours. Currently,  $b_1 = 100$ . For what values of  $b_1$  does the current basis remain optimal? From Figure 2, we see that a change in  $b_1$  shifts the finishing constraint parallel to its current position. The current optimal solution (point B in Figure 2)