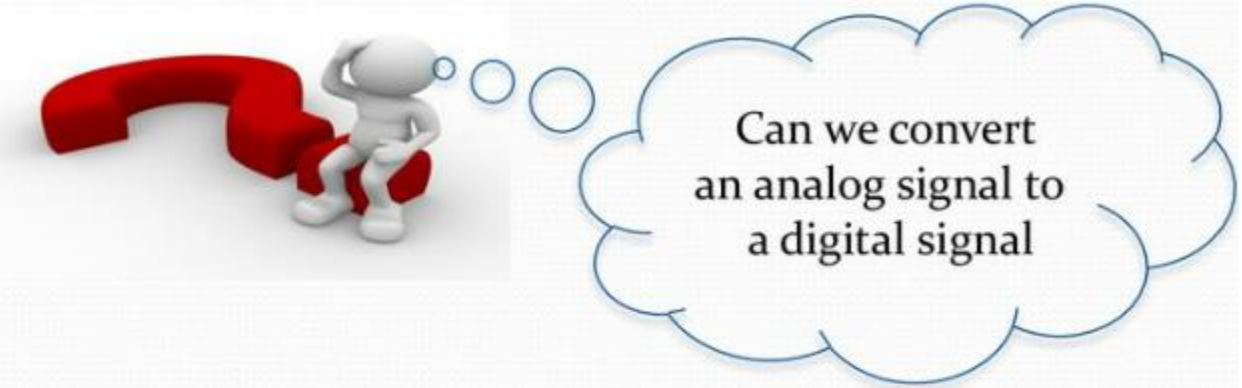
Sampling and Quantization

Introduction

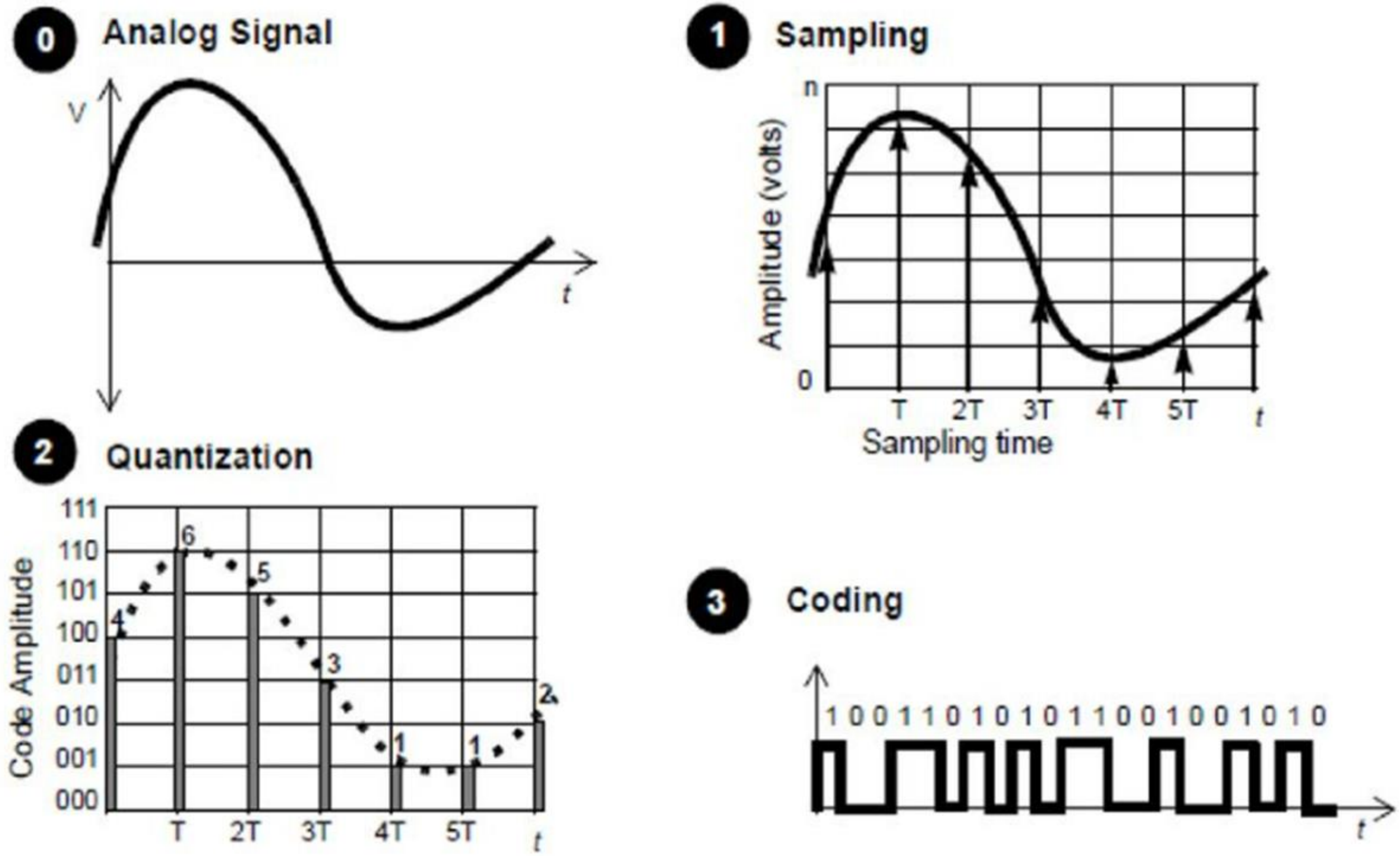
- A digital signal is superior to an analog signal because it is more robust to noise and can easily be recovered, corrected and amplified.
- For this reason, the tendency today is to change an analog signal to digital data

Introduction

- An analog signal:
  amplitude can take any value over a continuous range.
- Digital signals:
  amplitude can take only discrete and finite values.



Changing analog signal to digital signal: Sampling → Quantizing → Coding



The three steps of digitalization of a signal: sampling of the signal, quantization of the amplitude, and binary encoding.

Introduction

- One can convert an analog signal to a digital signal by sampling and quantizing (collectively called analog-to-digital conversion, or ADC).
- The processed signals are then converted back into analog signals using a reconstruction or interpolation operation (called digital-to-analog conversion, or DAC).

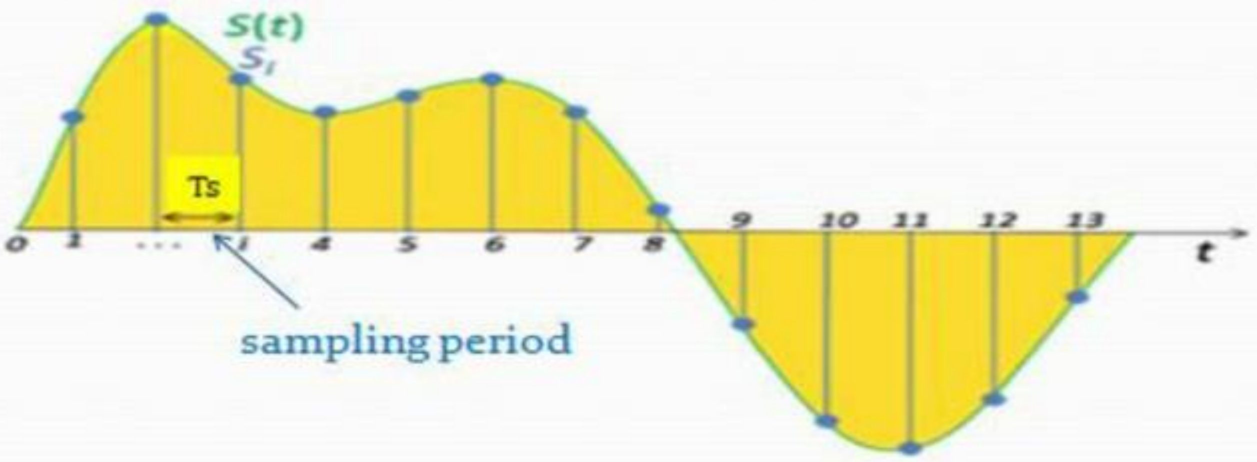
Sampling

Sampling Process

- The sampling process is a basic operation in the digital communication.
- In this process, the continuous-time analog signal is sampled by measuring its amplitude at a discrete instants.
- So, the continuous-time analog signal is converted into a corresponding sequence of samples that are usually spaced uniformly in time.
- It is necessary to choose the sampling rate properly, so the sequence of samples uniquely defines the original analog signal.

Sampling

- To sample a continuous-time signal $x(\mathrm{t})$ is to represent $x(\mathrm{t})$ at a discrete number of points, $\mathrm{t} = \mathrm{n}\mathrm{T}_\mathrm{s}$, where T_s is the sampling period.



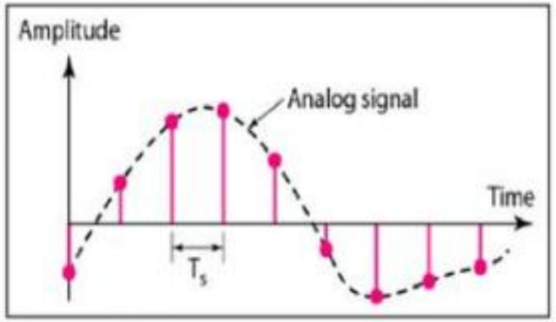
Sampling Theorem

- The sampling theorem states that a band-limited signal $x(t)$ with a bandwidth W (W is the highest frequency) can be reconstructed from its sample values if the sampling rate (frequency) $f_s = 1/T_s$ is greater than or equal to twice the bandwidth W of $x(t)$

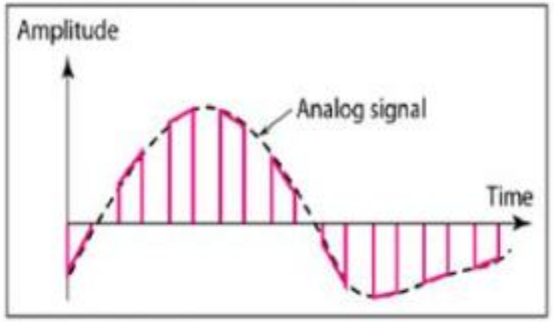
- The minimum sampling rate of f_s for an analog band-limited signal is called the Nyquist rate.

Sampling

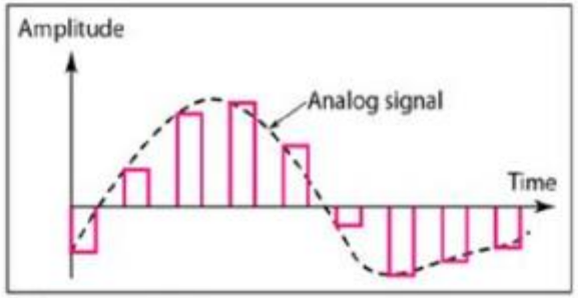
- There are 3 sampling methods:
  - Ideal - an impulse at each sampling instant
  - Natural - a pulse of short width with varying amplitude
  - Flattop - sample and hold, like natural but with single amplitude value



a. Ideal sampling



b. Natural sampling

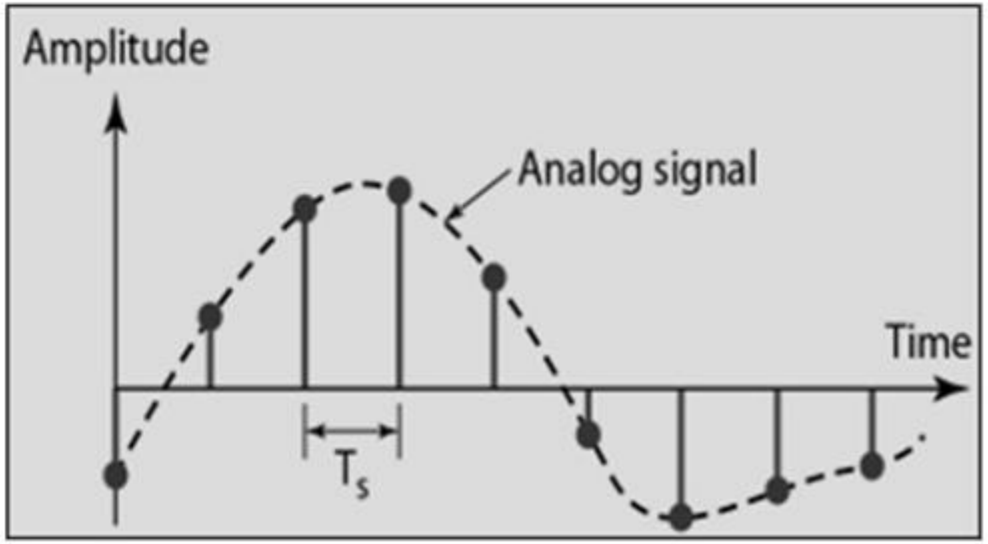


c. Flat-top sampling

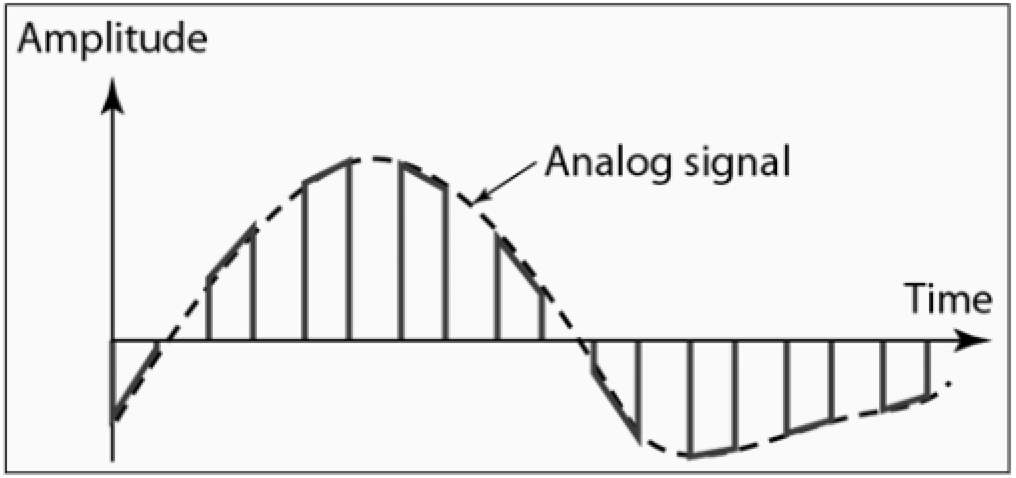
Sampling

- As long as the sampling of the analog signal is taken with a sufficiently high frequency (higher than the minimum Nyquist rate of twice the signal largest frequency), it can be shown that there is *no loss* in information as a result of taking discrete samples.

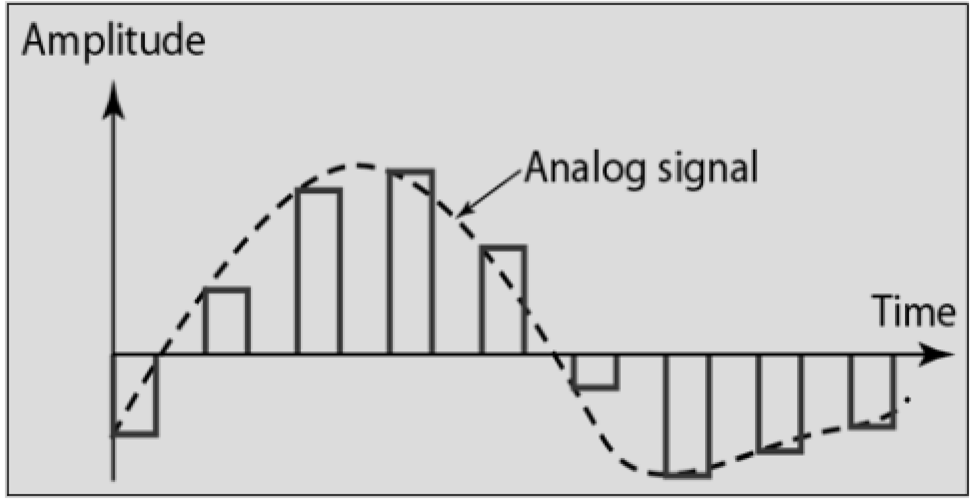
Ideal sampling - an impulse at each sampling instant



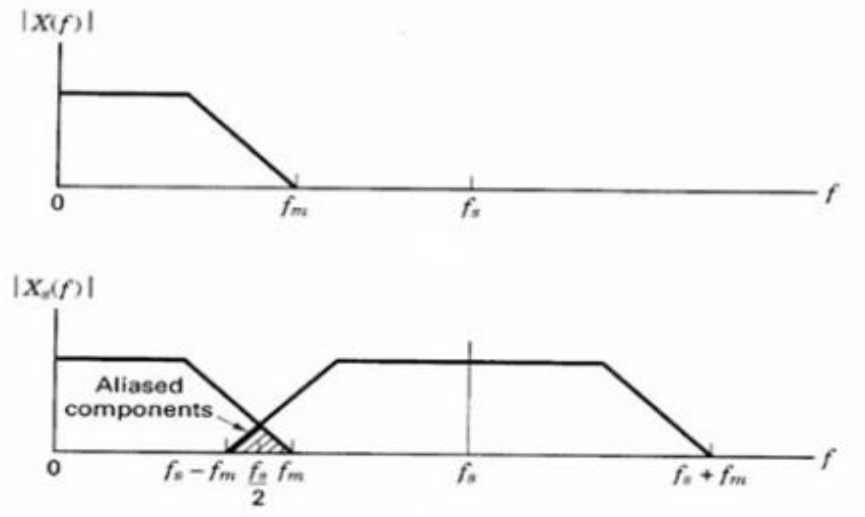
Natural sampling - a pulse of short width with varying amplitude with natural tops



Flat-top sampling - a pulse of short width with varying amplitude with flat tops



When $f_s < 2f_m$, spectral components of adjacent samples will overlap, known as aliasing



Quantization

Quantization

- In order to process the sampled signal digitally, the sample values have to be quantized to a finite number of levels, and each value can then be represented by a string of bits.
- To quantize a sample value is to round it to the nearest point among a finite set of permissible values.
- Therefore, a distortion will inevitably occur. This is called quantization noise (or error).

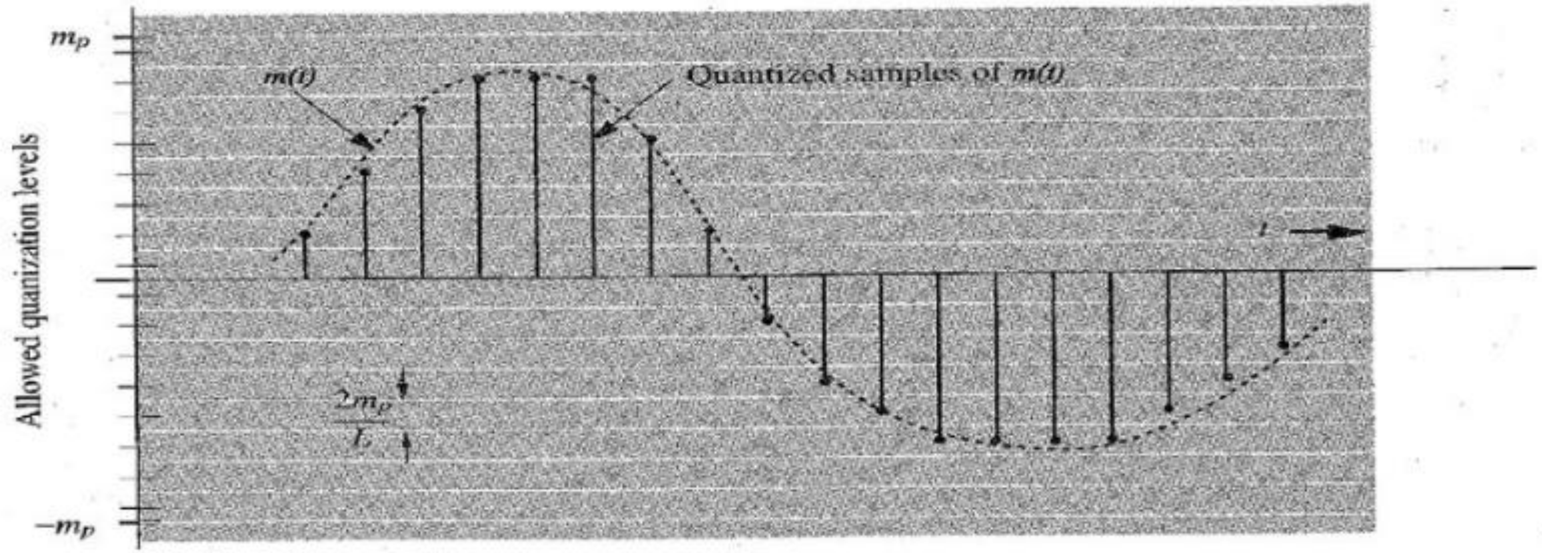
Quantization

- The sampling results is a series of pulses of varying amplitude values ranging between two limits: a min and a max.
- The amplitude values are infinite between the two limits.
- We need to map the *infinite* amplitude values onto a finite set of known values.
- This is achieved by dividing the distance between min and max into L zones, each of height Δ.

$$\Delta = (\max - \min)/L$$

Quantization Levels

- The midpoint of each zone is assigned a value from 0 to L-1 (resulting in L values)
- Each sample falling in a zone is then approximated to the value of the midpoint.



Quantization Error

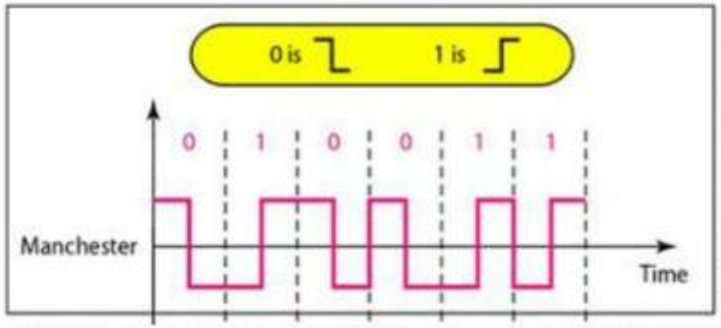
- When a signal is quantized, we introduce an error - the coded signal is an approximation of the actual amplitude value.
- The difference between actual and midpoint value is referred to as the quantization error.
- The more zones, the smaller Δ which results in smaller errors.

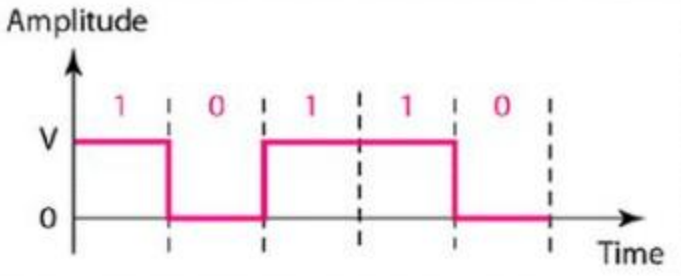
Encoding

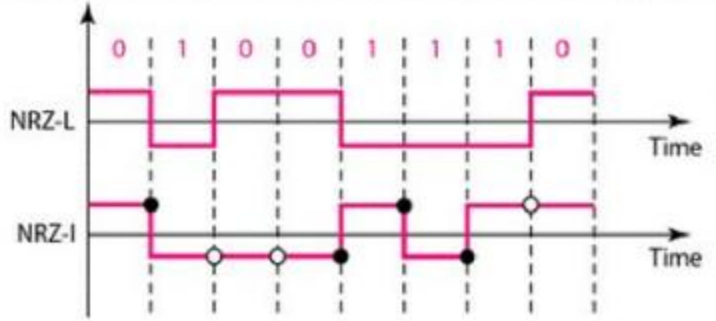
- In combining the process of sampling and quantization, the specification of the continuous-time analog signal becomes limited to a discrete set of values.
- Representing each of this discrete set of values as a code called **encoding** process.
- Code consists of a number of code elements called symbols.
- In binary coding, the symbol take one of two distinct values. in ternary coding the symbol may be one of three distinct values and so on for the other codes.

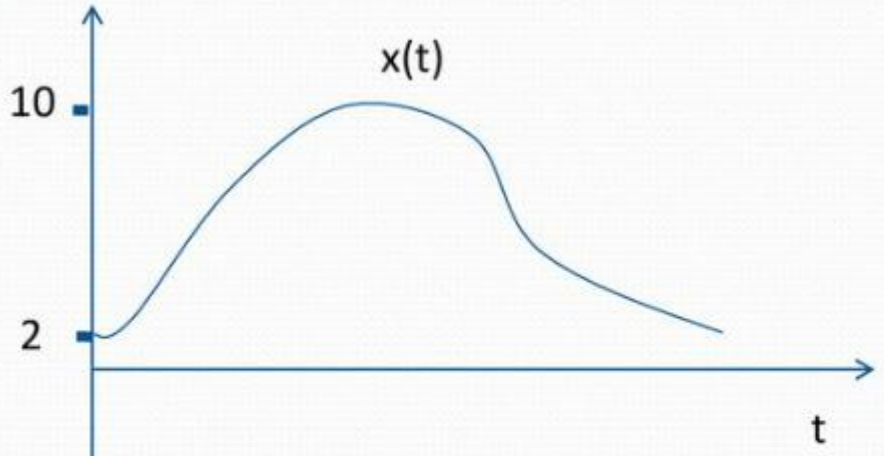
Line Coding

- Any of several line codes can be used for the electrical representation of a binary data stream.
- Examples of line coding : RZ, NRZ, and Manchester

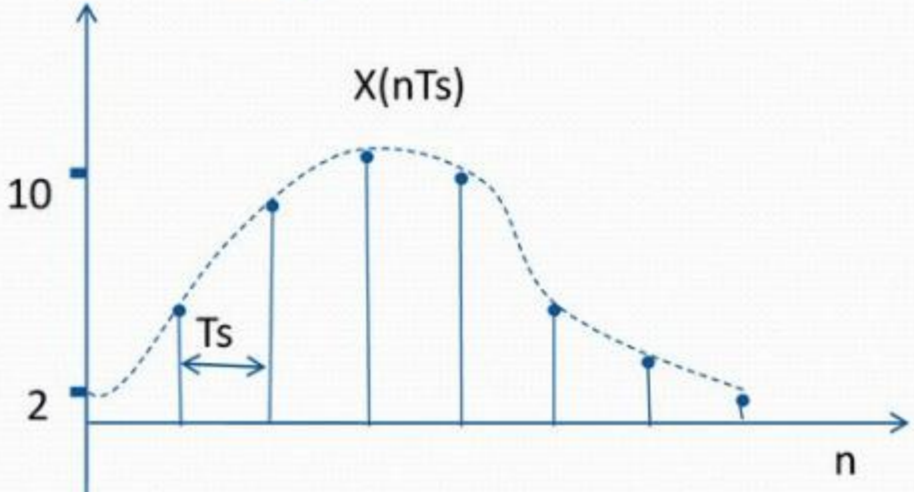




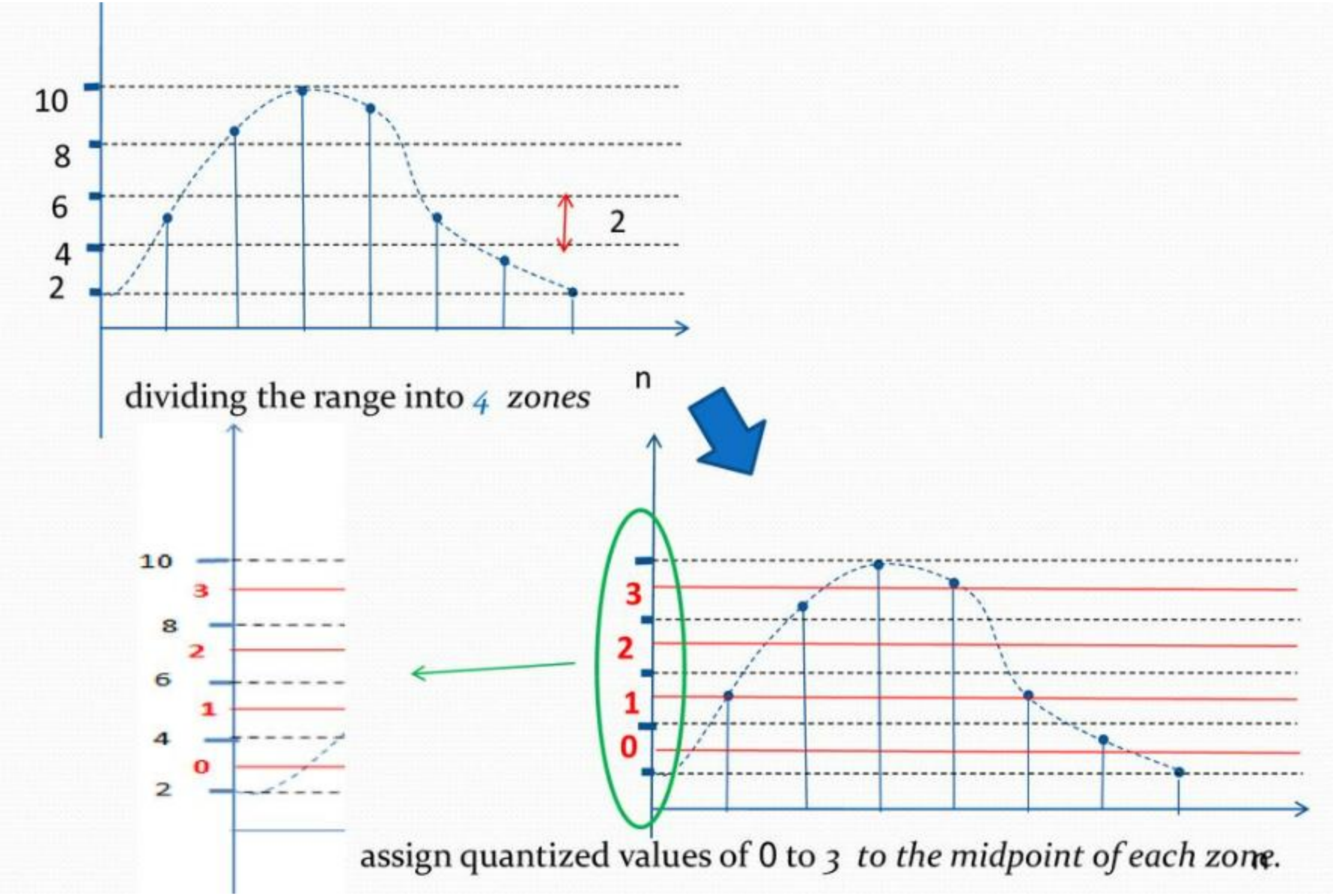


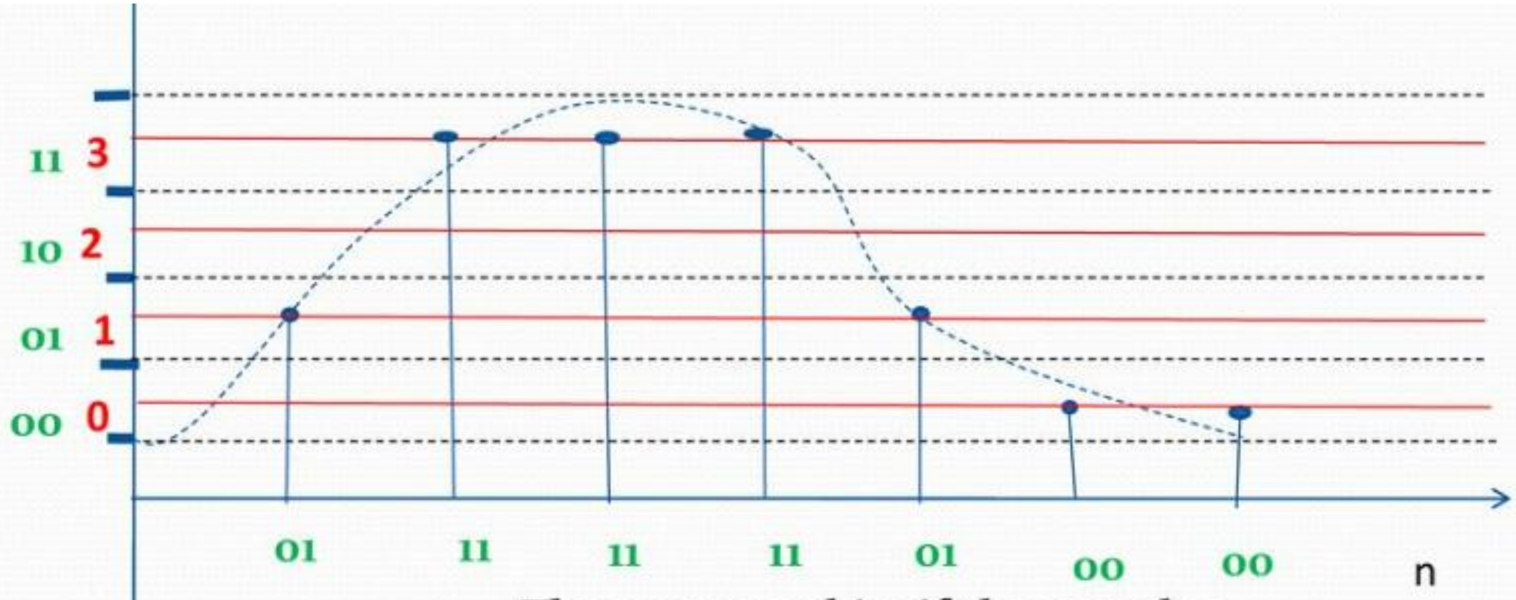


Consider the analog Signal x(t).



The signal is first sampled

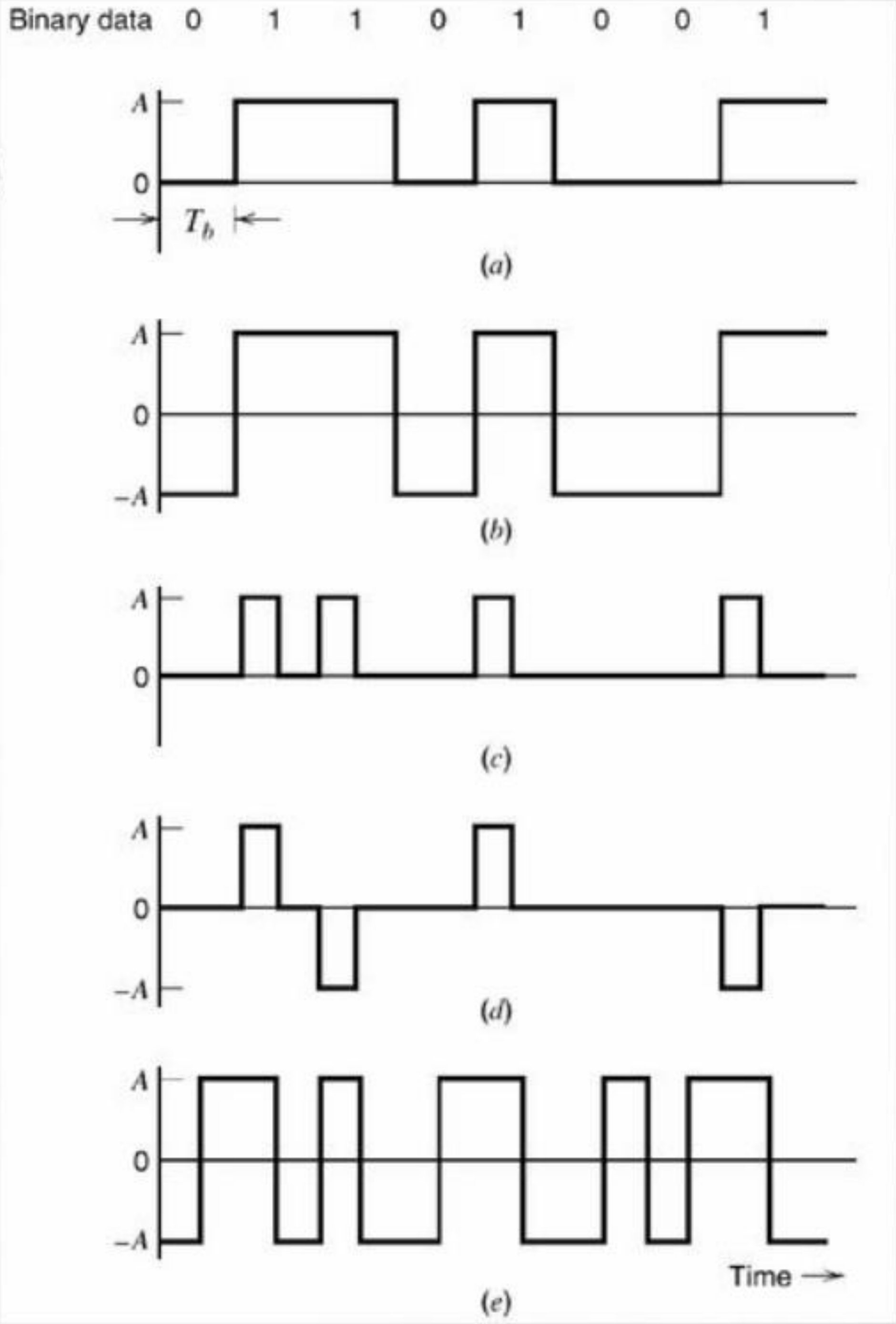

10
8
6
4
2
2
dividing the range into 4 zones
n
10
3
8
2
6
1
4
0
2
3
2
1
0
assign quantized values of 0 to 3 to the midpoint of each zone.
n



The sequence bits if the samples
01111111010000

Use one of the line code scheme to get the digital signal

- Types of Coding:
  - Uni polar NRZ (fig a)
  - Polar NRZ (fig b)
  - Uni polar RZ (fig c)
  - Bipolar RZ (fig d)
  - Manchester Code (fig e)

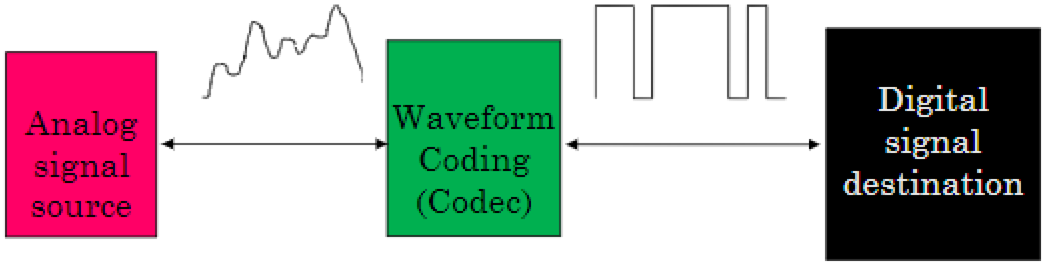


Line Codes...

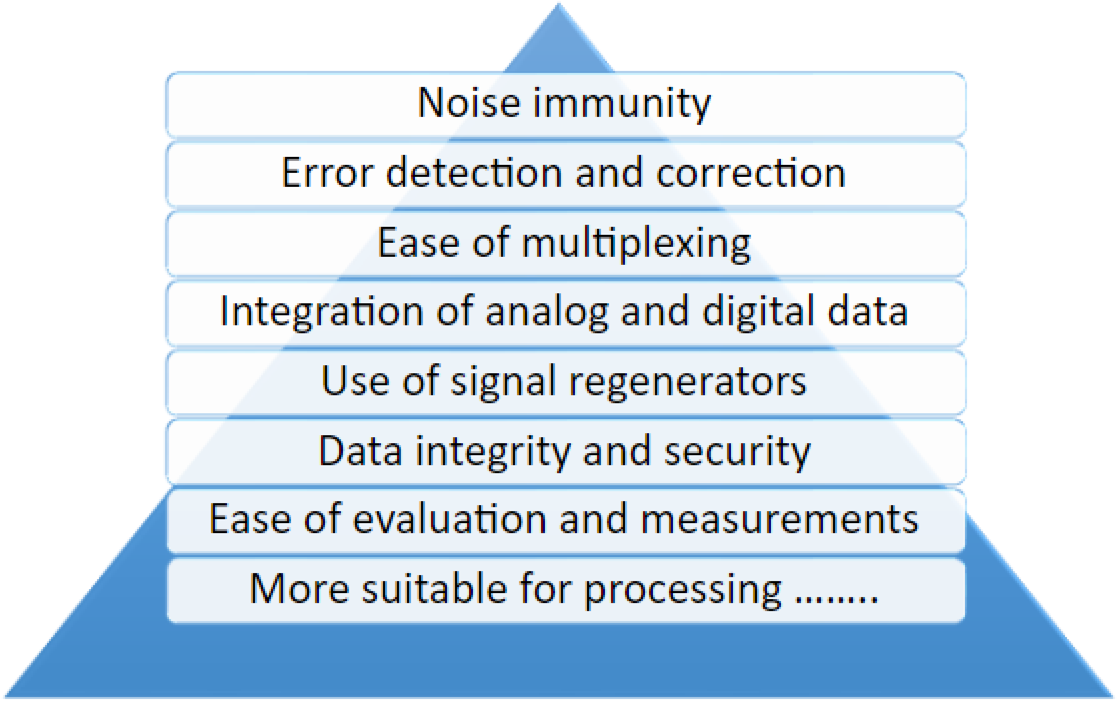
PCM, DM, and ADM

Introduction

- Digital representation of analog signals



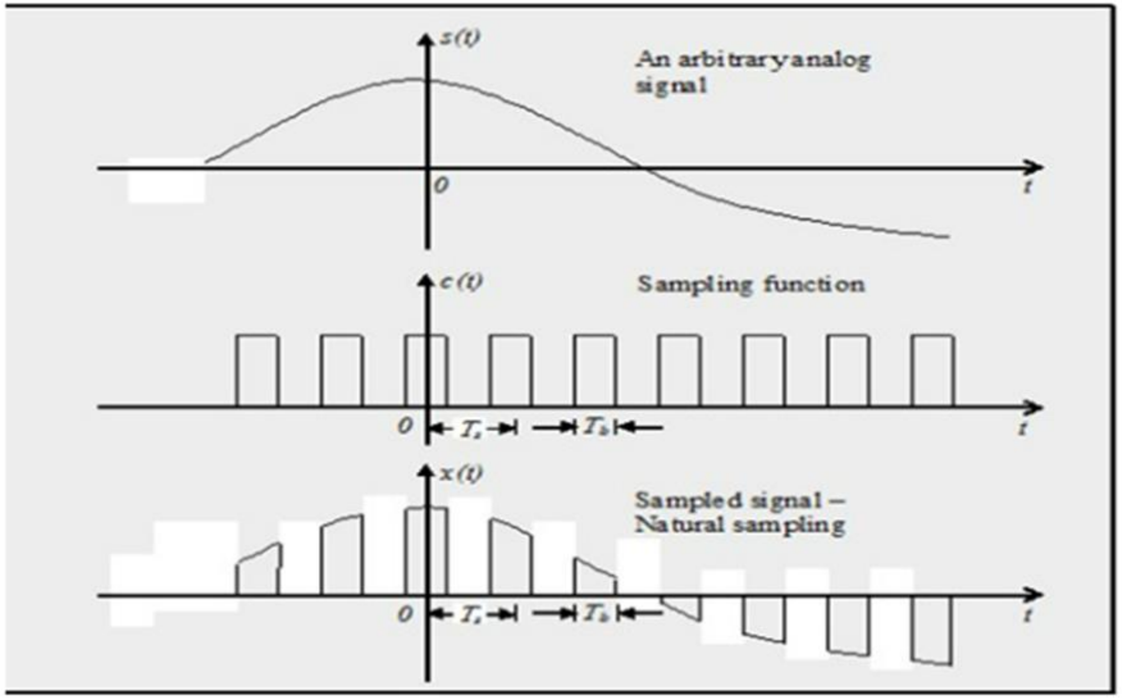
Advantages of Digital Transmissions



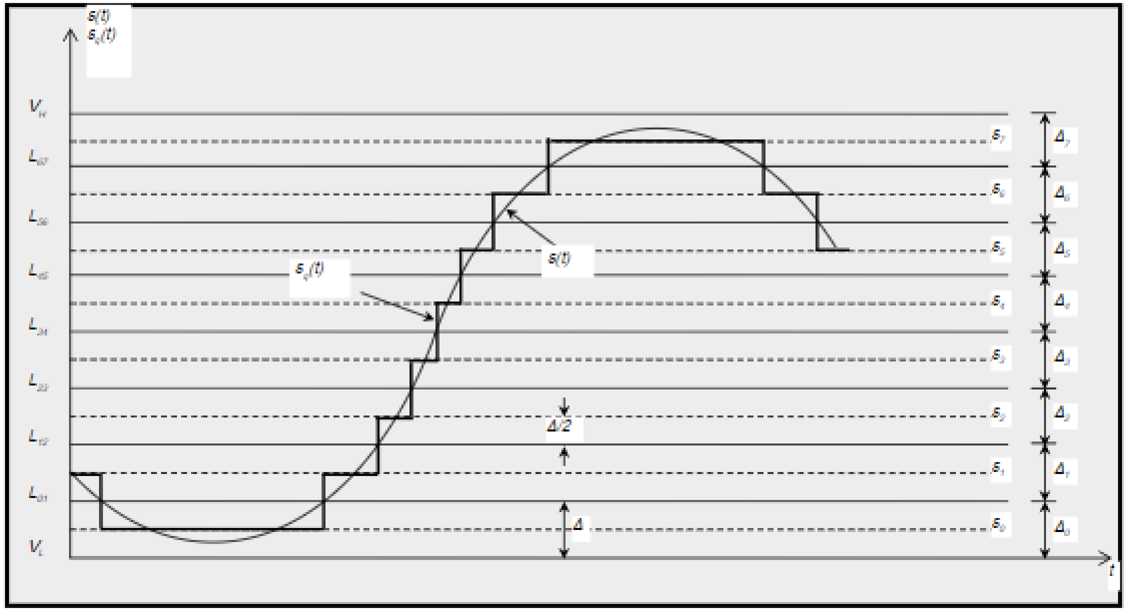
Disadvantages of Digital Transmissions

- More bandwidth requirement
- Need of precise time synchronization
- Additional hardware for encoding/decoding
- Integration of analog and digital data
- Sudden degradation in QoS
- Incompatible with existing analog facilities

PCM Sampling



Quantization of Sampled Signal



Quantization Error and Classification

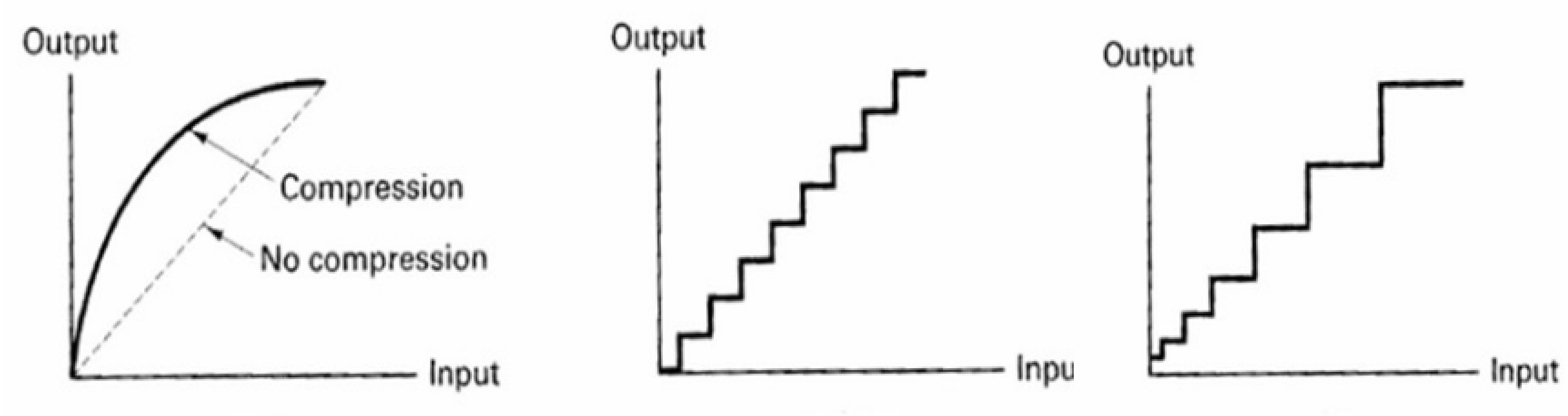
Quantization is the conversion of an analog sample of the information signal into discrete form. Thus, an infinite number of possible levels are converted to a finite number of conditions.

Quantization error is defined as the difference between rounding off sample values of an analog signal to the nearest permissible level of the quantizer during the process of quantization.

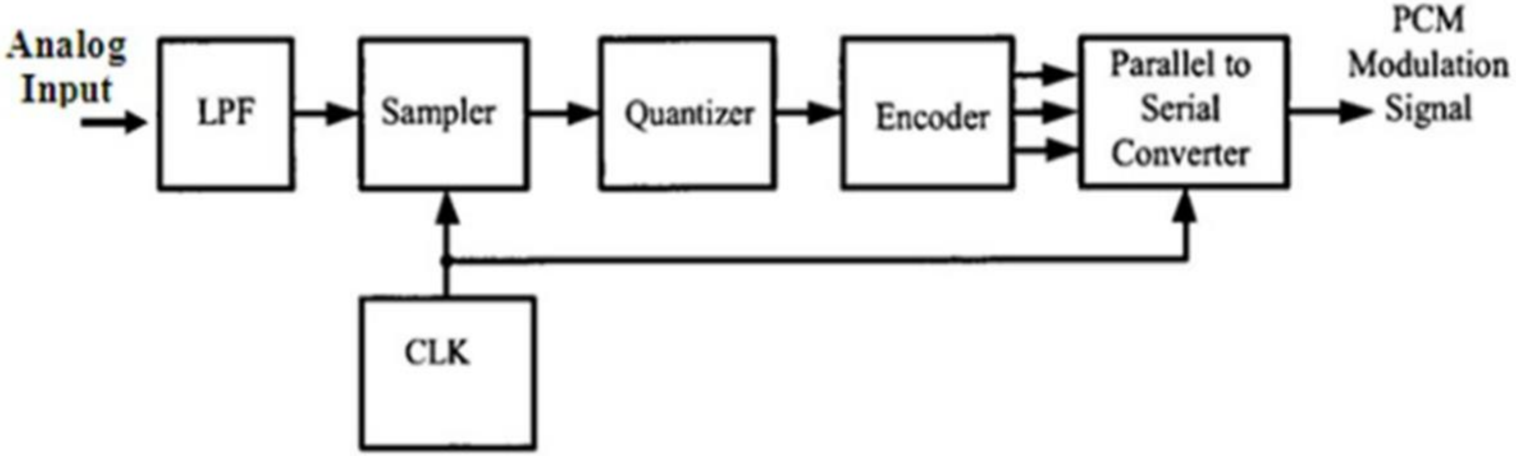
Classification of Quantization Process

- Uniform quantization
- Non-uniform quantization

Characteristics of Compressor, Uniform and Non-uniform Quantizer



Encoding of Quantized Sampled Signal



PCM – Functional Blocks

PCM Data Rate (bps) = $2nf_m$

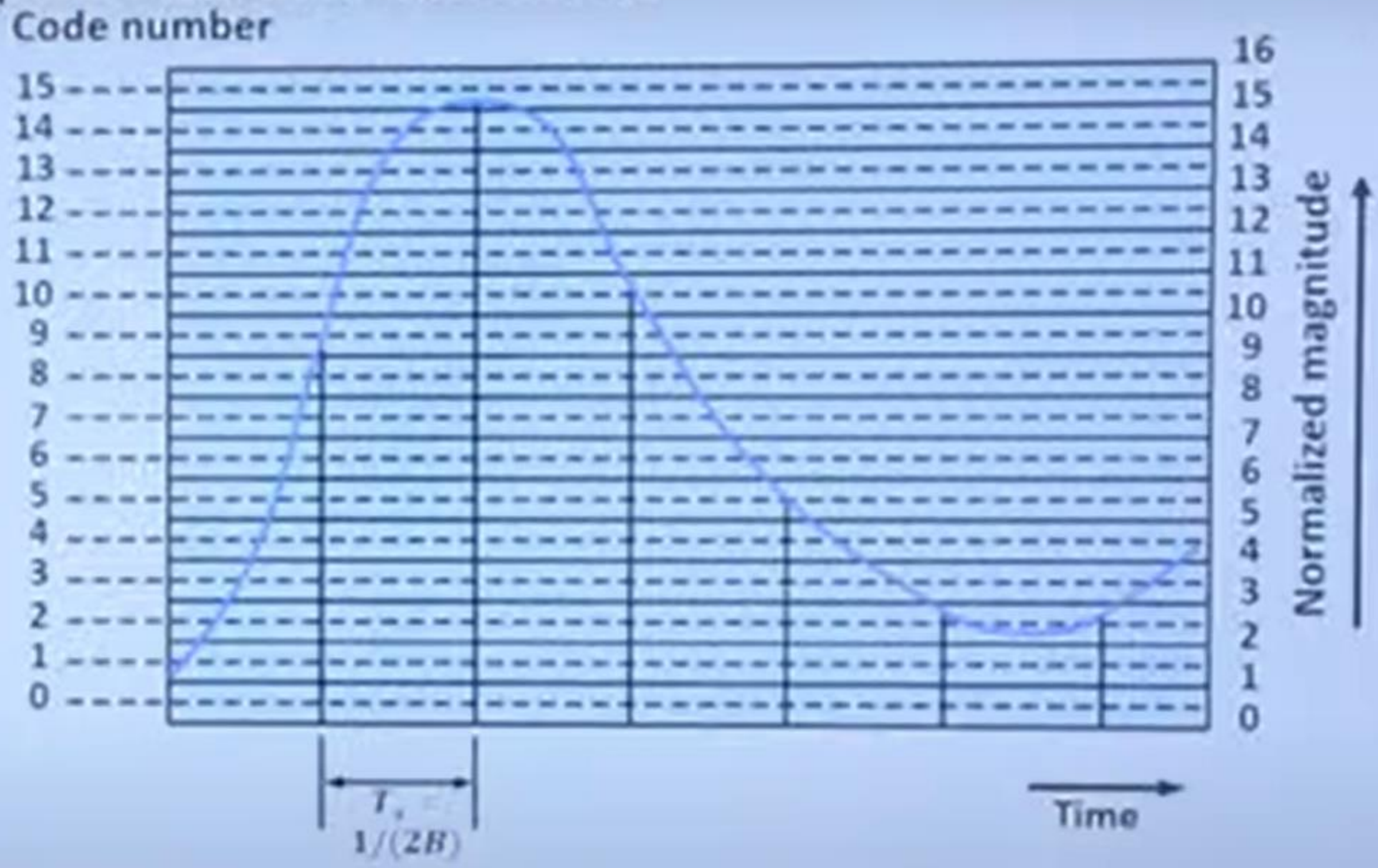
PCM Bandwidth (Hz) = (1/2) PCM Data Rate = nf_m

Dynamic Range (dB) = $20 \log (2^n - 1)$

Coding Efficiency (%) = [(minimum bits)/(actual bits)] x 100

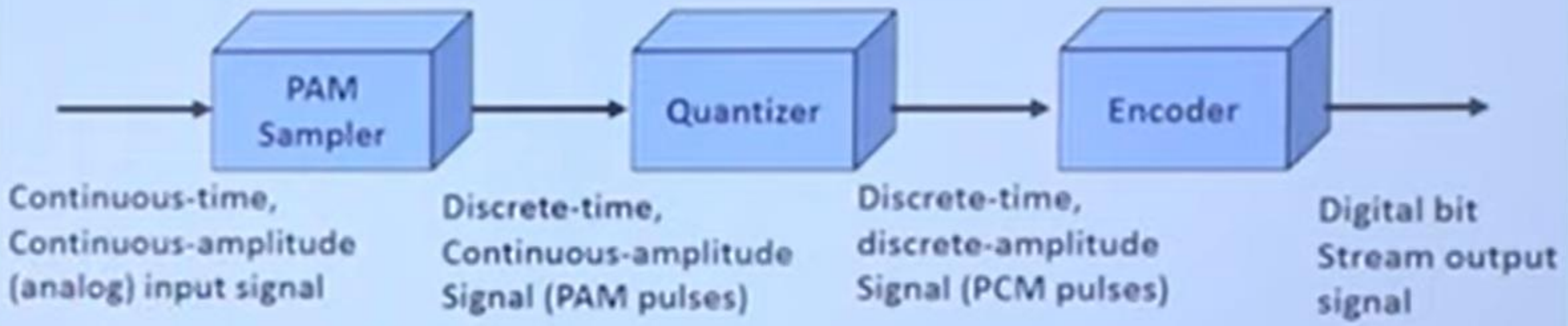
Where n is number of PCM encoding bits and f_m is the highest frequency component of information signal

Example of Pulse Code Modulation?



PAM value	1.1	9.2	15.2	10.8	5.6	2.8	2.7
Quantized code number	1	9	15	10	5	2	2
PCM code	0001	1001	1111	1010	0101	0010	0010

Block diagram of PCM –



Delta Modulation

Essence of Delta Modulation (DM)

Delta modulation (DM) uses a single-bit DPCM code to achieve digital transmission of analog signals

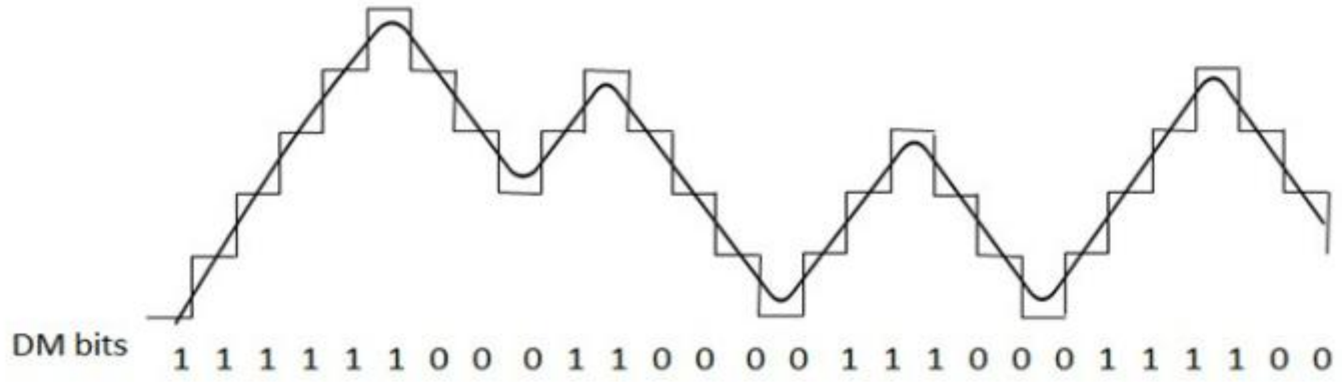
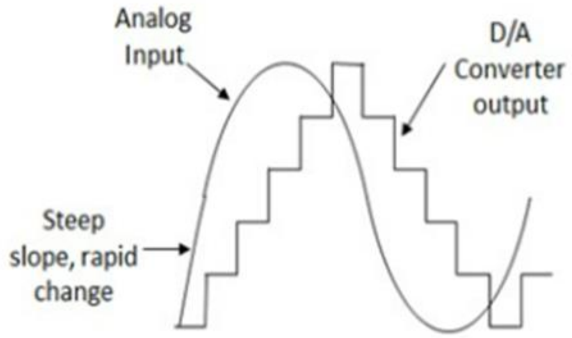
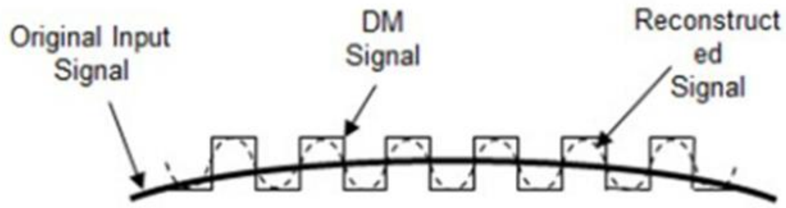


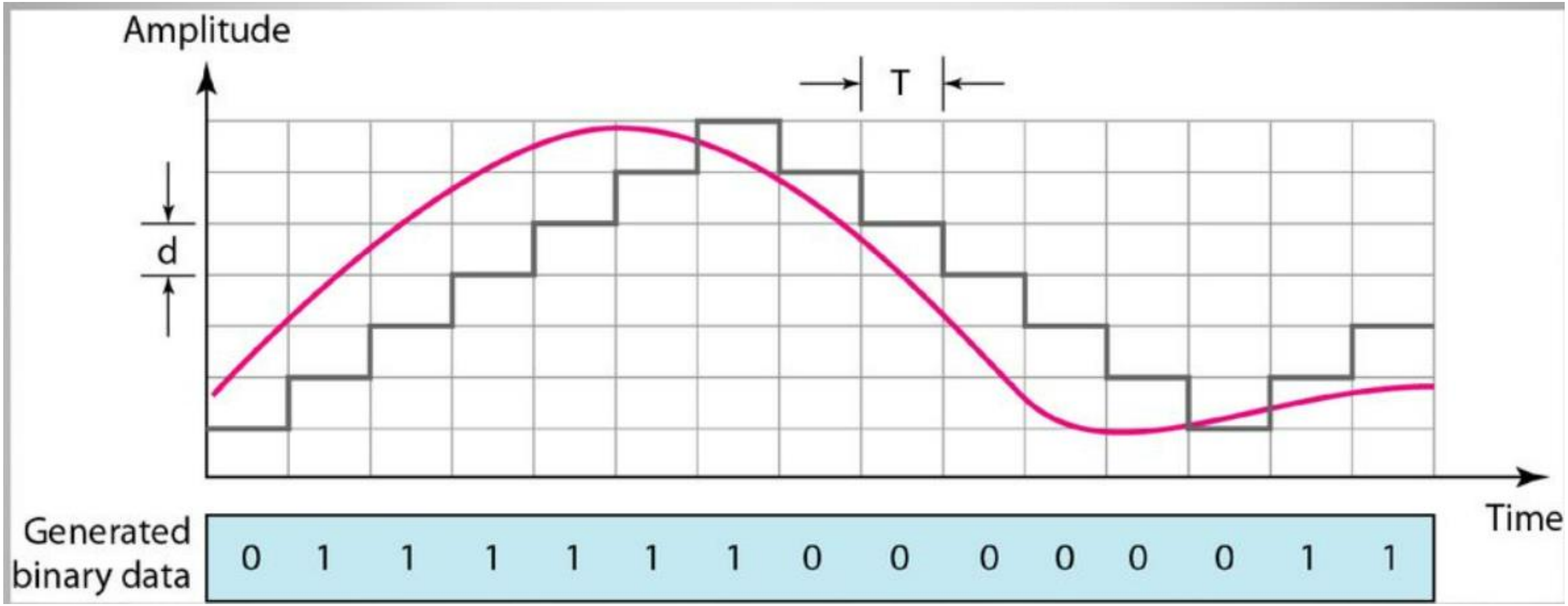
Figure 1.4.1 An Ideal Delta Modulation Waveform

Slope Overload and Granular Noise





REMEMBER: In DM, the step size is related to the sampling frequency. In order to avoid slope overload distortion, the maximum slope of the staircase approximation must be equal to or greater than the maximum slope of the signal.


Amplitude
T
d
Time
Generated binary data
0 1 1 1 1 1 1 0 0 0 0 0 0 1 1

- Important to note:

(1) step size (δ) – should not be too small, nor too large

- small δ + signal changes rapidly $\Rightarrow$ underestimation
- large δ + signal changes slowly $\Rightarrow$ overestimation

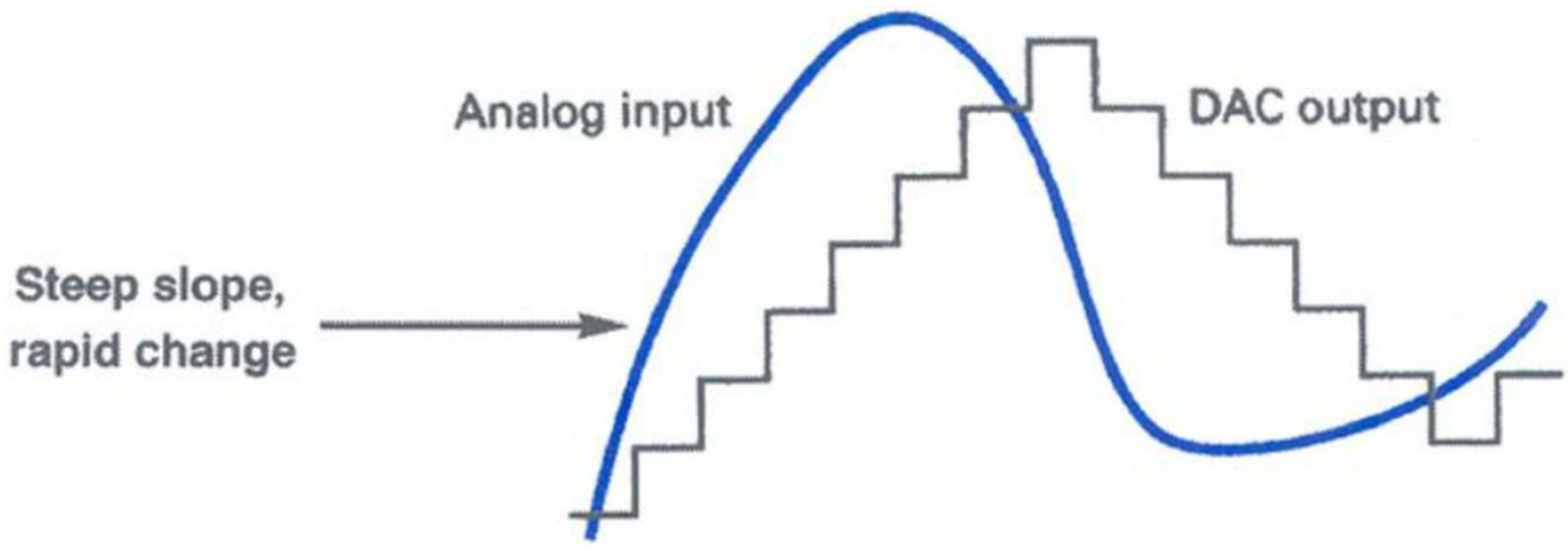
(2) sampling time (T)

- smaller **T** increase overall accuracy
- but, small **T** increases output data rate, i.e. # of bps

- **Delta-modulation rule: smaller $\delta \Rightarrow$ smaller T, larger $\delta \Rightarrow$ larger T**

- **Slope overload** - when the analog input signal changes at a faster rate than the DAC can maintain. The slope of the analog signal is greater than the delta modulator can maintain and is called *slope overload*.

- *Increasing the clock frequency* reduces the probability of slope overload occurring. Another way to prevent slope overload is to *increase the magnitude of the minimum step size*.



Slope overload distortion

- Granular noise. It can be seen that when the original analog input signal has a relatively constant amplitude, the reconstructed signal has variations that were not present in the original signal. This is called *granular noise*. Granular noise in delta modulation is analogous to quantization noise in conventional PCM.

- Granular noise can be reduced by *decreasing the step size*. Therefore, to reduce the granular noise, a *small resolution* is needed, and to reduce the possibility of *slope overload occurring, a large resolution is required*. Obviously, a compromise is necessary.

- Granular noise is more prevalent in analog signals that have gradual slopes and whose amplitudes vary only a small amount. Slope overload is more prevalent in analog signals that have steep slopes or whose amplitudes vary rapidly.

Adaptive Delta Modulation (ADM)

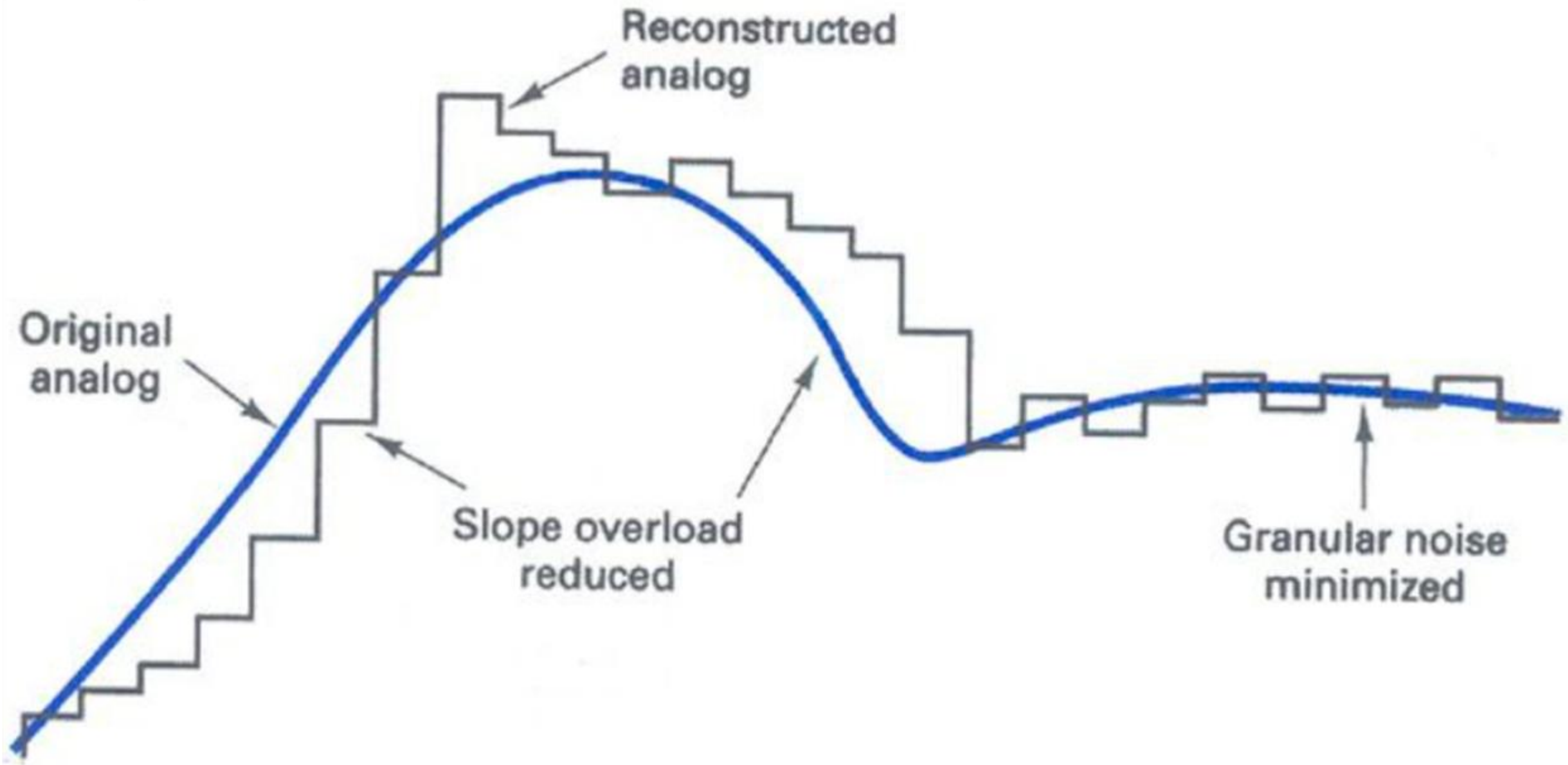
The step size is automatically varied, depending on the level of the derivative of the input analog signal

The receiver must be able to adapt step sizes in exactly the same manner as the transmitter

A common algorithm followed for an ADM is that when three consecutive 1s or 0s occur, the step size is increased or decreased by a factor of 1.5.

NOTE: Continuous Variable Slope Delta Modulation (CVSDM) is an improvement over ADM.

- *Adaptive delta modulation* is a delta modulation system where the step size of the DAC is automatically varied, depending on the amplitude characteristics of the analog input signal.



Adaptive delta modulation

S. No.	Parameter	PCM	DPCM	DM	ADM
1.	Number of bits per sample	4/8/16 bits	More than one bit but less than PCM	One bit	One bit
2.	Number of levels	Depends on number of bits	Fixed number of levels	Two levels	Two levels
3.	Step size	Fixed or variable	Fixed or variable	Fixed	Variable
4.	Transmission bandwidth	More bandwidth needed	Lesser than PCM	Lowest	Lowest
5.	Feedback	Does not exist	Exists	Exists	Exists
6.	Quantization noise/distortion	Quantization noise depends on number of bits	Quantization noise & slope overload	slope overload & granular noise	Quantization noise only
7.	Complexity of implementation	Complex	Simple	Simple	Simple