University of Glasgow | School of Computing Science

CSC1103 Laboratory/Tutorial 2 : Problem Solving Solution

1. Decomposition:

Input variable: $x_1, x_2 \cdots x_N$ and the number of variables, N

Process: apply mathematical function $\frac{1}{N}\sum_{n=1}^{N} x_n$

Output variable: the mean $\bar{x}$ of the N number of variables x

Pattern Recognition

The N variable $x_1, x_2 \cdots x_N$ are the same data type (mostly floating point type) and can be different from the data type of N which is integer type

Generalization/Abstraction

One formula to perform summation to sum all input variable x and then divide number of variables, N to obtain the $\bar{x}$

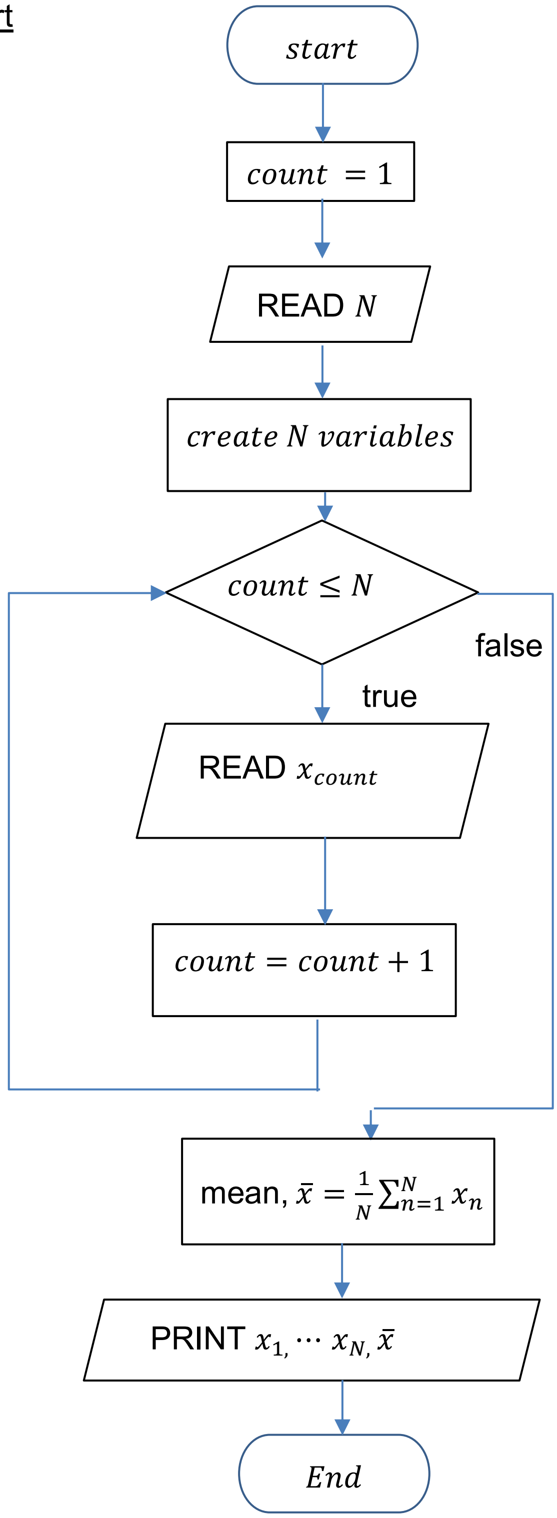
Algorithm

Pseudocode

BEGIN
- $Count \leftarrow 1$
- READ N
- N number of x variable $\leftarrow 0$
- WHILE $Count \leq N$
    - READ x_{count}
    - $count \leftarrow count + 1$
- ENDWHILE
- $mean \leftarrow x_1 + x_2 + \cdots x_N$
- $mean \leftarrow mean/N$
- PRINT $x_1, x_2 \cdots, x_N$ and $mean$

END

Flowchart


$start$
$count = 1$
READ N
$create\ N\ variables$
$count \leq N$
false
true
READ x_{count}
$count = count + 1$
mean, $\bar{x} = \frac{1}{N}\sum_{n=1}^{N} x_n$
PRINT $x_1, \cdots x_N, \bar{x}$
End

2. Decomposition:

The binomial expansion of

$$
\begin{aligned}
(x+y)^n &= \textstyle\sum_{k=0}^{n} a_k x^b y^c = a_o x^{n-0} y^0 + a_1 x^{n-1} y^1 + a_2 x^{n-2} y^2 + \cdots a_n x^{n-n} y^n \\
&= \binom{n}{0} x^{n-0} y^0 + \binom{n}{1} x^{n-1} y^1 + \binom{n}{2} x^{n-2} y^2 + \cdots + \binom{n}{n} x^{n-n} y^n \\
&= \sum_{k=0}^{n} \binom{n}{k} x^{n-k} y^k
\end{aligned}
$$

where

$$\binom{n}{k} = \frac{n!}{k!(n-k)!} = \frac{n\times(n-1)\times(n-2)\cdots(n-k+1)}{1\times 2\cdots k}$$

Input variable: The exponent, n

Process: apply mathematical function to obtain all terms $x^{n-k}y^k$ and coefficient $\binom{n}{k}$ where $b = n - k$ and $c = k$ in the definition of $a_k x^b y^c$ and a_k is coefficient $\binom{n}{k}$

Output variable: print out the binomial expansion with all coefficients $\binom{n}{k}$ and the respective term $x^{n-k}y^k$

*note: ignore any overflow problems that can occur if factorial apply directly

Pattern Recognition

a. Each coefficient is obtained in similar way of calculation. For example $\binom{n}{2}$ and $\binom{n}{3}$ are the same $= \frac{n\times(n-1)}{1\times 2} = \frac{n\times(n-1)\times(n-2)}{1\times 2\times 3}$. The rest are simply a division of numerator and denominator with different number of terms.
b. Each term is obtained in similar way of calculation. For example, $x^{n-2}y^2$ and $x^{n-4}y^4$ are obtained in a similar fashion with just increasing or decreasing exponent power

Generalization/Abstraction

Two generic formulae:

a. $\binom{n}{k}$;

b. $x^{n-k}y^k$

Algorithm

Pseudocode

```
BEGIN
    k ← 0
    READ n
    READ x
    READ y
    n + 1 number of a (stands for each term coefficient) ← 0
    n +1 number of x and y (stands for each term with different
    exponent power for x and y) ← 0
    WHILE k ≤ n
        IF (k = 0)
            a_0 ← 1
            x_o ← x^n
            y_0 ← y^0
        ELSE
            IF (k = n)
                a_n ← 1
                x_n ← x^0
                y_n ← y^n
            ELSE
                a_k ← (n×(n−1)×⋯×(n−k+1)) / (1×2⋯×k)
                x_k ← x^(n−k)
                y_k ← y^k
            ENDIF
        ENDIF
            k ← k + 1
    ENDWHILE
    PRINT a_0 ⋯ a_n    and x^n y^0, x^(n−1) y^1 ⋯ x^0 y^n
END
```