CS 419M Introduction to Machine Learning	Spring 2021-22
Lecture 7: Classification	
Lecturer: Abir De	*Scribe: Group 2*

7.1 Topics covered in lecture

1. What is classification task
2. What are the classsification models
3. Support Vector Machine (not Covered in lecture)

7.2 Classification task

- Given training set $\mathcal{D} = \{(x_i, y_i) \mid y_i \in \mathcal{G}\}$, $\mathcal{G} = \{y_1, y_2\}$, $x_i \in \mathbb{R}^d$, find $m(x) \mapsto y$.
- Test set $= \{x_i \in \mathbb{R}^d\}$, y_i are not known in test set.

7.3 Probabilistic Approch

$$
\begin{aligned}
P_m(y \mid x) &= \frac{1}{1 + e^{-w^T x \cdot y}} \\
\implies \max_w &\prod_{i \in \mathcal{D}} P_m(y_i \mid x_i) \\
\implies \max_w &\sum_{i \in \mathcal{D}} \log P_m(y_i \mid x_i) \\
\implies \min_w &\sum_{i \in mathcalD} \log\left(1 + e^{w^T x_i \cdot y_i}\right)
\end{aligned}
$$

7.4 Simpler way to classify

Let $\mathcal{G} = \{+1, -1\}$, for some other labels we can convert that to +1,-1. We want some linear boundary to classify given points into $\mathcal{G}$.

$$
\begin{aligned}
&w^T x + b \geq \Delta, \quad y = 1 \\
&w^T x + b \leq -\Delta, \quad y = -1 \\
&\min_w f(w)
\end{aligned}
$$

where $f(w)$ is any regularizer. This problem can be solved when there is no overlapping between the classification variables but there isn't a solution for something like This

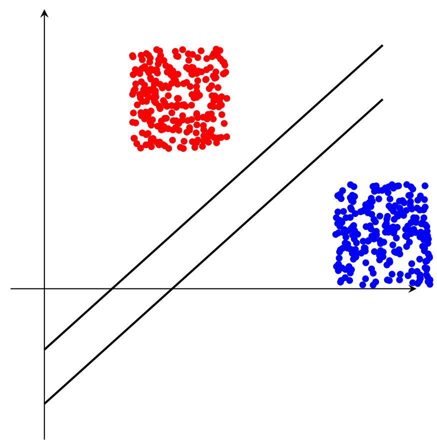

Figure 7.1: no overlapping

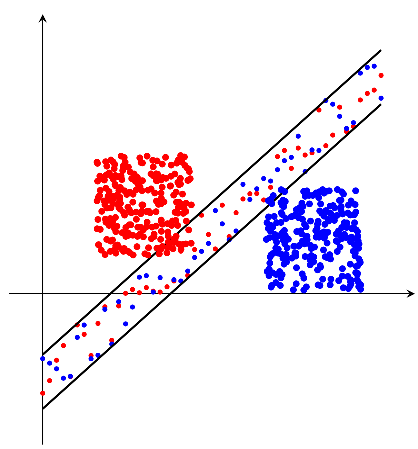

Figure 7.2: Overlapping

We can ignore the overlapping points,
Let $I^+ = \{i \mid y_i = 1\}$, $I^- = \{i \mid y_i = -1\}$, $S^+ \in I^+$,$S^- \in I^-$ such that $|S^+ \cup S^-| = n$,

$$\min_{\zeta_i} f(w) - \left(\sum_{i \in S^+} \mathbb{I}\left(w^T x_i + b \geq \Delta\right) + \sum_{iinS^-} \mathbb{I}\left(w^T x_i + b \leq \Delta\right) \right)$$

7.5 Adding Slack variable to solve the overlapping case

Modifying optimisation problem to include overlapping points,

$$
\begin{aligned}
w^T x_i + b &\geq \Delta - \zeta_i, \quad y_i = 1 \\
w^T x_i + b &\leq -\Delta + \zeta_i, \quad y_i = -1 \\
y_i \cdot \left(w^T x_i + b\right) &\geq \Delta - \zeta_i \\
\zeta_i &\geq 0
\end{aligned}
$$

with above conditions, we have to solve following

$$\min_{w,b,\zeta_i} C \sum_{i \in \mathcal{D}} \zeta_i + \lambda \mid\mid w \mid\mid^2$$

7.6 Group Details and Individual Contribution

Name	Roll number	contribution
Dadhichi Telwadkar	20D070083	7.1,7.2,7.3,7.4,7.5