Appendix D

Computational Projects

These projects are designed around the computational implementation of one or more mathematical models and the use of that implementation to explore the properties of the model, and are intended to be done individually, rather than in groups. The projects should be treated as a computational experiments, with results written in the style of a lab report that is aimed at a third party with little background in the subject. The results thus need to be fully explained with such an audience in mind. A recommended structure would include:

Title: Project title, author and date.

Abstract: A *brief* summary of the key questions, results and conclusions. In general this should not be longer than a half page, double spaced.

Introduction: A description of the problem(s) under consideration, and any relevant background information. This section should include both a summary of the biological problem(s) being considered, as well as the mathematical modeling frameworks that will be used.

Results and Discussion: An integrated presentation of the results of each "experiment" and any relevant discussion. This should be written in narrative prose (that is, complete sentences that walk the reader through everything that was done), with any generated graphics and code inserted at the appropriate location.

Conclusion: A brief summary of the key insights obtained, and perhaps any unanswered questions that remain.

References: A list of all resources used in completion of the project. If a resource is used in generating a specific result or in support of specific point made in the introduction or discussion, this should be noted with a citation at the appropriate place in the report text. A specific format for citations/references is not required, but the format used should be consistent.

Note that the projects were created with the use of MATLAB in mind, and in my classes this is expected. However, the projects are not dependent on this choice, and could be handled equally well with other programming languages.

D.1 Yeast Growth

The yeast species *Saccharomyces cerevisiae* is a budding yeast that is used extensively around the world in baking and brewing, and is one of the best studied model organisms. As a budding yeast, cell division is assymetric, with smaller daughter cells grow out from the surface of a mature parent cell, ultimately separating into a distinct individual. Under optimal growth conditions, it can have a doubling time as fast as 90 minutes.

As a continuously growing species, the growth of yeast may be modeled either with an unrestricted (exponential) growth model:

$$\frac{dN}{dt} = R_o N(t) \rightarrow N(t) = N_0 e^{R_o t} = N_o 2^{\left(\frac{t}{\tau_2}\right)}$$

or with the logistic equation:

$$\frac{dN}{dt} = R_o N(t) \left(1 - \frac{N(t)}{K}\right) \rightarrow N(t) = \frac{K N_0 e^{R_o t}}{K - N_0 + N_0 e^{R_o t}}$$

- Imagine beginning with a single yeast cell and growing it under ideal conditions for three days. Using an unrestricted growth model, make a plot of the number of cells expected to be present every 3 hours; how many cells are expected to be present at the end of the three days?

- Yeast cells are approximately spherical, with a typical diameter of about 6 μm; estimate the volume of a single yeast cell in litres, noting that the volume of a sphere of radius r is given by $\frac{4}{3}\pi r^3$, and that 1 L = 10^{-3} m^3. How many litres would the yeast occupy after 3 days (assuming no extra space is required)?

- Now, consider an experiment biologist growing a culture of yeast cells — she begins with a small number of yeast cells (perhaps 10,000) selected from an agar plate and transfers them to a 2 L flask containing 1 L of liquid growth medium. Using the results of the above model, how long do you estimate she should wait to reach the maximal number of yeast cells possible in her system: about an hour, about one working day (8 hours), overnight (about 16 hours), a whole day or several days?

- The above growth model neglected competition for resources and the impact of limited environmental capacity. For yeast, the maximal density achievable in liquid culture is about 2×10^8 cells per mL; use this to derive a carrying capacity for a 1 L total volume. How much of this total volume would be occupied by yeast cells?

- Using this carrying capacity with a logistic growth model, make a plot of the growth per hour for the experimental system described above. Use this plot to revise your estimate of the expected waiting time to reach a maximal number of yeast cells, and discuss the differences.

D.2 Cicada population dynamics

Magicicada septendecim is a species of periodic cicada with a 17-year life cycle: mature nymphs (fifth instar) emerge *en masse* from the ground when soil temperatures reach about 15° C and begin a final moult into the adult form; adults live for 2–4 weeks during which time mating occurs, with each female laying 200-400 eggs in small slits on the branches of trees; after 6–8 weeks, the eggs hatch and the young nymphs (first instar) burrow into the soil; for 17 years the nymphs live underground, progressing through a total of five growth stages (instars) before emerging to mate.

With a clearly periodic growth cycle in which reproduction occurs in synchronized bursts, growth is best modeled with a discrete-time growth model such as that of Hassell:

$$N_{n+1} = \frac{R_o N_n}{(1 + aN_n)^b}$$

- Imagine a newly revitalized hardwood forest area of 350 km^2 from which cicadas have gone locally extinct. If healthy hardwood forests can support up to about 250 million cicadas per square kilometer, what is the carrying capacity, K, of this region? If, in absence of competition, a cicada population is equally split between males and females, 80% of adults are able to reproduce, each female lays 300 eggs, and 20% of eggs survive to adulthood, what is the effective *per capita* growth rate, R_o, in terms of the total (male and female) population?

- Implement an discrete growth model based on the Hassell equation, using the growth rate derived above and a starting population of 100 adults; assume that $b = 1$ and $a = \frac{R_0}{K}$. Run the simulation for at least 10 breeding cycles (170 years), and plot the population numbers as a function of time. Additionally, plot the populations as a graph of N_{n+1} vs N_n, and add to this the line of $y = x$ and the Hassell function with given parameters.

- How many years are needed for the population to roughly reach the carrying capacity of the forest? Repeat your simulation with different starting populations to determine the minimum starting populations that will allow the population to roughly reach the carrying capacity within (i) 102 and (ii) 51 years.

- It can be difficult to intuitively assess the effective size of large numbers of small organisms. Given that an adult cicada weighs approximately 2 grams, convert the populations found above into kg, and then into the number of people with an equivalent mass (taking 70 kg as the average mass of a person). Do the same with the carrying capacity of the forest.

- Do you think introducing cicadas at a level to repopulate the forest in 51 years would be feasible? What about the levels required to repopulate the forest in 102 years? Discuss your reasoning.

