Cicada population dynamics

David Hwang 09/20/2024

- Abstract

How does the population of cicada grow? And through its growth pattern, how long does it take to reach the maximal population capacity inside a region? To test this, our goal was to find a specific carrying capacity for a given region and use this parameter to find the time length that the cicada population takes to reach its maximal population capacity in the region. The result demonstrates that due to its clear life cycle, its population growth follows a discrete-time growth pattern, and by considering the statistical survival rate we could get a discrete-time growth model. Through this model, we were able to find the consuming time for cicadas to repopulate a given region.

- Introduction

"Magicicada septendecim is a species of periodic cicada with a 17-year life cycle: mature nymphs (fifth instar) emerge en masse from the ground when soil temperatures reach about 15◦ C and begin a final moult into the adult form; adults live for 2–4 weeks during which time mating occurs, with each female laying 200-400 eggs in small slits on the branches of trees; after 6–8 weeks, the eggs hatch and the young nymphs (first instar) burrow into the soil; for 17 years the nymphs live underground, progressing through a total of five growth stages (instars) before emerging to mate."(Green 193p) A good approach to understanding the growth patterns of cicada populations is to create mathematical models and examine graphs. Based on this information, we can say that a cicada has a 17-year life cycle. Since its growth cycle shows a clear periodical pattern, we can make a discrete-time growth model such as of Hassell for its population.

$$N_{n+1} = \frac{R_o N_n}{(1 + aN_n)^b}$$

We assume that healthy hardwood forests can support up to about 250 million cicadas per square kilometer. Also assume that a cicada population is equally split between males and females, 80% of adults are able to reproduce, each female lays 300 eggs, and 20% of eggs survive to adulthood.

- Results and Discussion

We first Imagined a newly revitalized hardwood forest area of 350 km^2 where the cicadas were extinct. When we assume that healthy hardwood forests can support up to about 250 million cicadas per square kilometer, we want to find the carrying capacity, K, of this region and per capita growth rate, R0, considering the survival rate.

```
% the life cycle of cicada is 17y, so we assume that the populations in one cycle are not added to the next cycle (All of them only can survive for one cycle)
K1 = 250000000; % carrying capacity (km^2)
K = K1*350; % carrying capacity of 350km^2 region
R0 = (1/2)*0.8*300*0.2; % growth rate per capita(in terms of the total population)
```

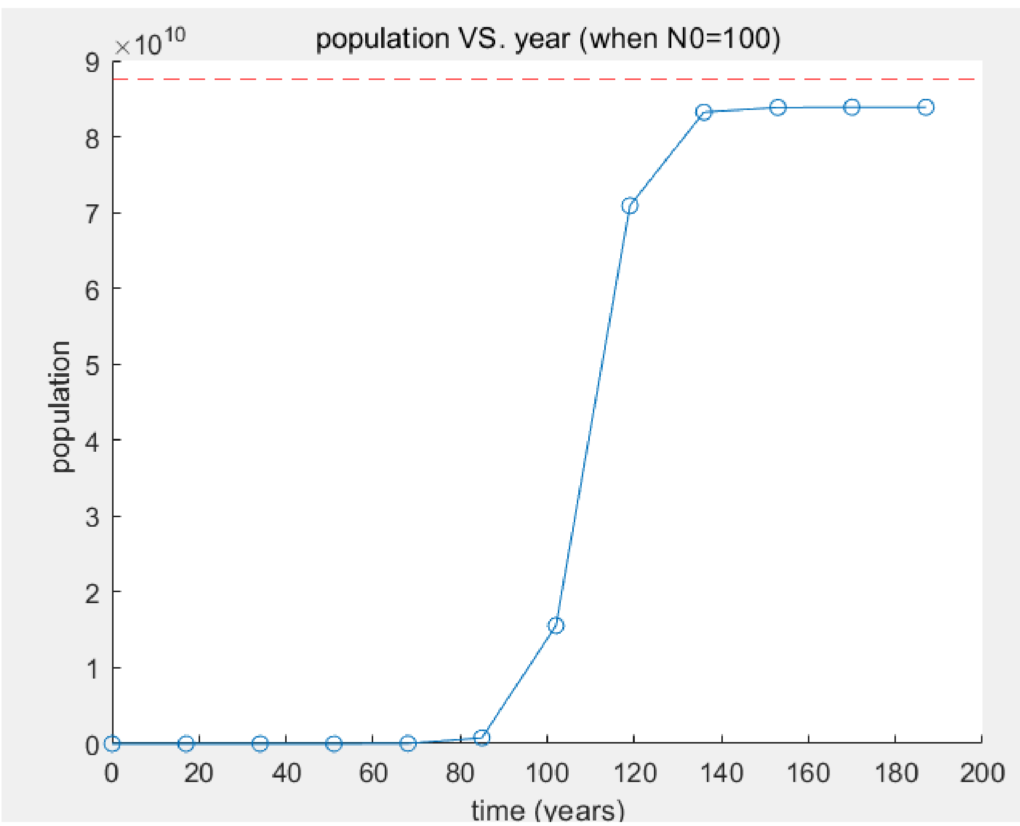
Since, in the absence of competition, a cicada population is equally split between males and females, we divide the previous cycle populations by 2. And 80% of adults are able to reproduce meaning that from the number of females in the previous cycle only 80% can reproduce. Multiplying 0.8. And lastly, since each female lays 300 eggs, and 20% of eggs survive to adulthood, we multiply 300*0.2 by the number of reproducible females. Through these codes, we got the carrying capacity of this region = K = 8.7500e+10 and the per capita growth rate = R0 = 24.

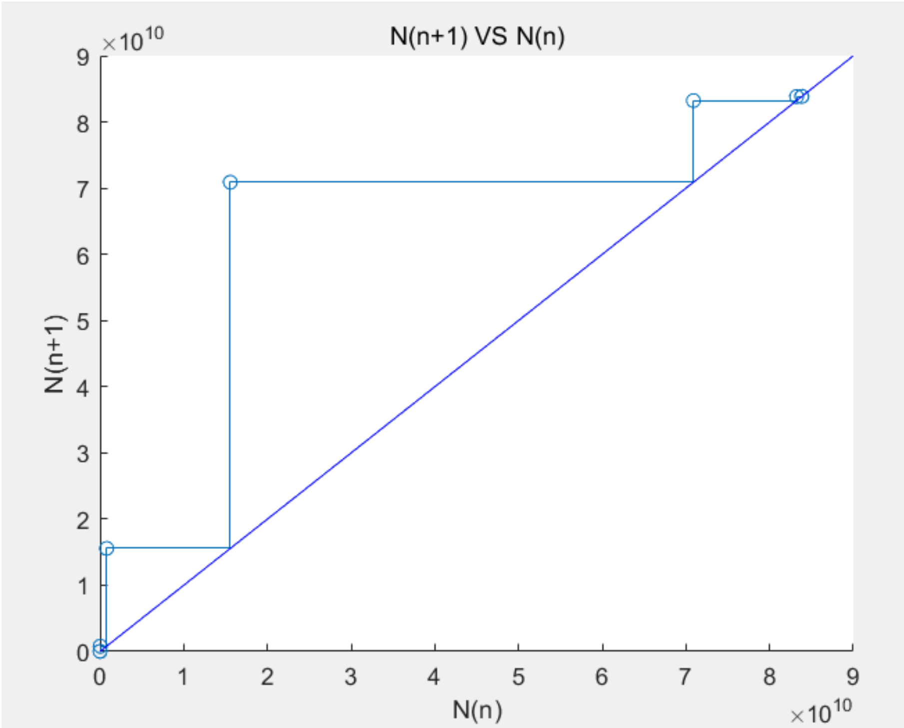
Now we are going to use the Hessell model. We assume that each discrete number 'n' of N_n means each 17 years. Thus, all populations of one cycle die after one cycle and are never calculated in the next cycle. So, we simply do not need to consider the death rate. However, we need to consider how many populations were in the previous cycle, since from that, we can determine how many eggs will hatch and survive to adulthood.

We plotted the Hessell equation growth model using the growth rate derived above and a starting population of 100 adults. Also, assume that b = 1 and a = R0/K. We ran this simulation 11 times which is 187 years in total and plotted the population numbers as a function of time and plotted the populations as a graph of Nn+1 vs Nn, and add to this the line of y = x and the Hassell function with given parameters.

```
%%%%%%%%%%%%%%%%%%%%
%% Discrete growth model(Hassell equation)
%%%%%%%%%%%%%%%%%%%%
% main parameters
N0 = 100; % initial population of cicada
N = N0; % N is a number; N0 is a initial value
R0 = (1/2)*0.8*300*0.2; % growth rate per capita
K = 250000000*350; % carrying capacity of 350km^2 region
a = R0/K; % a parameter of Hassell equation
b = 1; % a parameter of Hassell equation
% simulation
timev = 0:17:187; % time vector for 187years which is 11 cycles
sim = zeros(length(timev),1); % vector to store the values of N during
simulation
k=0; % counter
for t=timev
   k = k+1;
   % main equation
   if k~=1 N=(R0*N)/((1+a*N)^b); end
   % store value of N for plotting
   sim(k)=N;
end
% plot
```

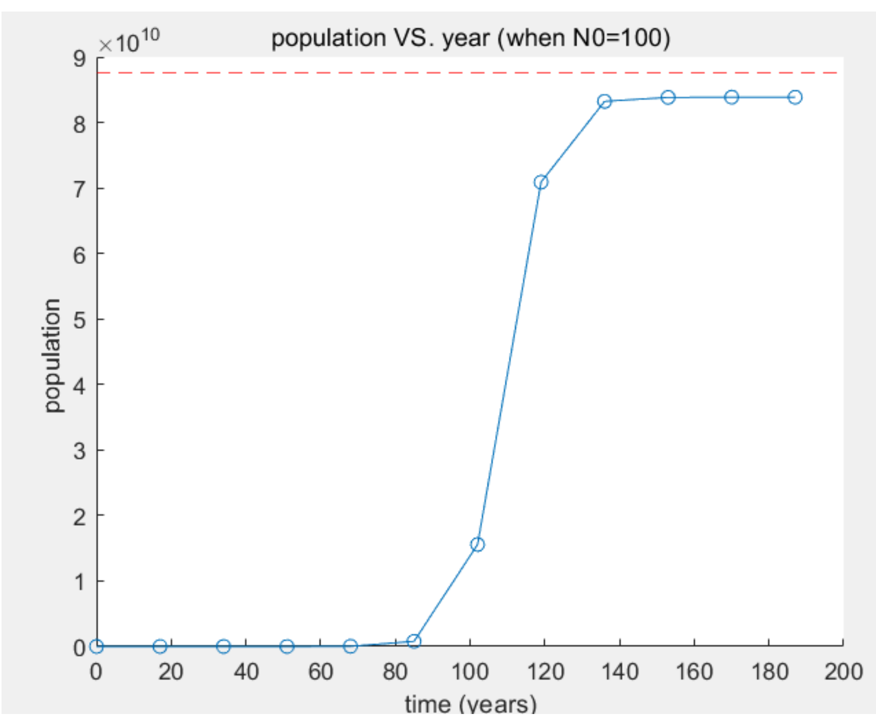
```
figure(1); clf
hold on
stairs(sim(1:k-1),sim(2:k),'-o')
plot(xlim,ylim,'-b')
hold off
xlabel('N(n)');ylabel('N(n+1)')
title('N(n+1) VS N(n)')

figure(2); clf
hold on
plot(timev,sim,'-o')
yline(K,'--r');
xlabel('time (years)');ylabel('population ')
title('population VS. year (when N0=100)')
```





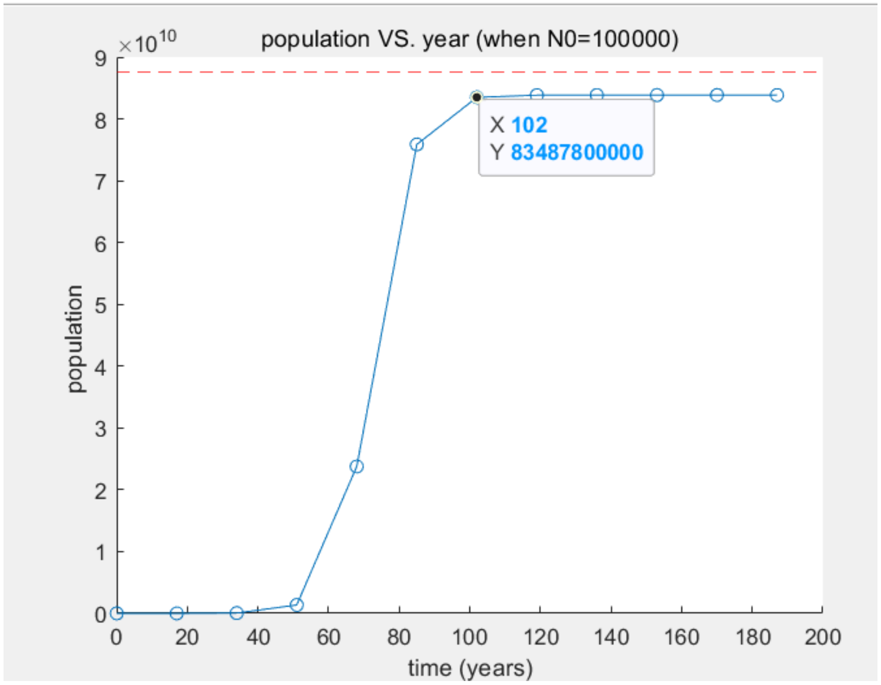
We can also check how many years are needed for the population to roughly reach the carrying capacity of the forest. We repeated the above simulation with different starting populations to determine the minimum starting populations that will allow the population to roughly reach the carrying capacity within (i) 102 and (ii) 51 years. The result graphs are below.

<Initial population is 100>



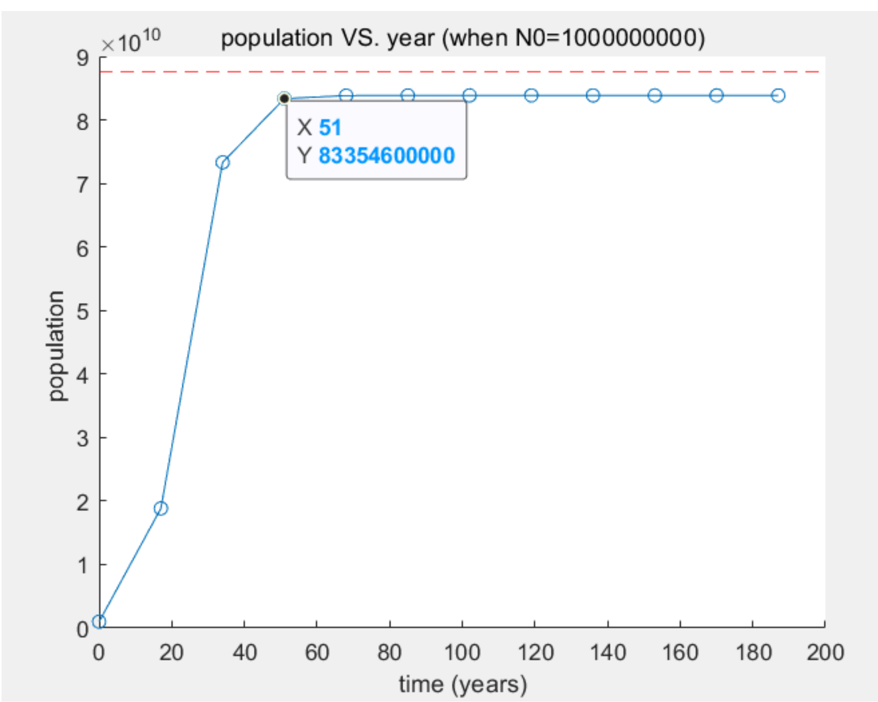
When we set the initial population to 100, we can roughly approach the carrying capacity of the forest after 150 years.

1) Within 102y



When we set the initial population to around 100,000, we can roughly approach the forest's carrying capacity within 102 years.

2) Within 51y



When we set the initial population to around 1000000000, we can roughly approach the carrying capacity of the forest within 51 years.

Since it may be difficult to intuitively assess the effective size of large numbers of small organisms. So, assume that an adult cicada weighs approximately 2 grams, we converted the populations found above into kg, and then into the number of people with an equivalent mass (taking 70 kg as the average mass of a person). We know that 2 grams = 0.002 kilograms and 70kg=1person.

```
% main parameters % we set the initial population to 100,000
N0 = 100000; % initial population of cicada
N = N0; % N is a number; N0 is a initial value
R0 = (1/2)*0.8*300*0.2; % growth rate per capita
K = 250000000*350; % carrying capacity of 350km^2 region
a = R0/K; % a parameter of Hassell equation
b = 1; % a parameter of Hassell equation
% simulation
timev = 0:17:187; % time vector for 170years which is 11 cycles
sim = zeros(length(timev),1); % vector to store the values of N during
simulation
k=0; % counter
for t=timev
   k = k+1;
   % main equation
   if k~=1 N=(R0*N)/((1+a*N)^b); end
   % store value of N for plotting
   sim(k)=N;
end
total_g = sim*2; % populaion vector in respect to grams
total_kg = total_g/1000; % populaion vector in respect to kgrams
total_p = fix(total_kg/70); % vector for the number of people with an
equivalent mass
```

If we convert the population into kg and the number of people with an equivalent mass, the increasing process of the kg and the number of people look like this.

total_kg =

```
   1.0e+08 *
    0.0000
    0.0000
    0.0012
    0.0272
    0.4754
    1.5173
    1.6698
    1.6768
    1.6771
    1.6771
    1.6771
    1.6771
```

```
total_p(people)=

          2
         68
       1644
      38856
     679200
    2167567
    2385366
    2395395
    2395815
    2395832
    2395833
    2395833
```

Lastly, with this model, when we think about introducing cicadas at a level to repopulate the forest, how long it would take?

```
timev(7) % 102 years
```

```
total_kg(7) % total kg after 102 years
total_p(7) % the number of people after 102 years
```

From above, we came to know that starting with the initial number of around 100,000 cicadas, we can roughly repopulate the forest within 102 years. And also we came to know that starting with the initial number of around 1,000,000,000(one billion) cicadas, we can roughly repopulate the forest within 51 years. Thus, In the real world, I do not think it is feasible to repopulate the forest with cicadas within 51 years and also 102 years. This is because it is very hard to obtain more than 100,000 cicadas without harming the natural environment.

- Conclusion

This project shows the population dynamics of cicadas and the feasibility of reintroducing this species into a previously extinct habitat. By considering the life cycle of the cicadas, we concluded that it follows the Hassell discrete-time growth model. Also, we determined that a newly revitalized hardwood forest has approximately 87.5 billion cicadas. Our simulations revealed that starting with an initial population of around 100,000 cicadas would make the population to reach carrying capacity within 102 years, while an initial population of 1 billion cicadas could reach within 51 years.

However, reintroducing such large numbers of cicadas can raise real-world concerns. Acquiring and releasing a billion cicadas can be very difficult without harming the environment. Thus, while our model illustrates the potential recovery for the cicada population within a period, the feasibility of reintroducing them starting from a big population within the proposed timelines remains questionable.

- References

David Green (2024). *Cicada population dynamics*

I have only used the information inside the homework paper