Knowledge Representation and Reasoning - H02C3A

Exercise Session 3

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This exercise session is about the formal semantics of FO(ID).

1 FO(ID): semantics

Assignment Check if the following definition Δ can be stratified. (hint: $P \leftarrow$ is the same as $P \leftarrow true$.) If so, construct its model by iterated rule application along its dependency graph. Also, compute the justification tree for T relative to this definition. Do all defined atoms have a two-valued supported value?

Note: a *well-founded interpretation* of a definition is one such that the value of each defined atom is equal to its supported value. The well-founded interpretation is called a *model* of the definition if it is 2-valued. Does this definition have a model?

$$\Delta = \left\{ \begin{array}{l} P \leftarrow \\ Q \leftarrow \neg P \\ Q \leftarrow R \\ R \leftarrow Q \\ S \leftarrow \neg Q \\ T \leftarrow S \wedge \neg U \\ U \leftarrow \neg P \vee Q \end{array} \right\}$$

1.1 Auxiliary exercise (homework):

Assignment Build a justification tree for $Even(4)$ given the definition below and given domain of natural numbers $D = \{0, 1, 2, ..., \infty\}$ and having that $0^I = 0$ (in interpretation I constant 0 is mapped to zero) and $S^I = \{(0,1), (1,2), (2,3), ...,\}$ (successor function).

$$\left\{ \begin{array}{c} Even(x) \leftarrow x = 0 \\ Even(S(x)) \leftarrow Odd(x) \\ Odd(S(x)) \leftarrow Even(x) \end{array} \right\}$$

2 FO(ID): semantics, (in)consistent definitions

Assignment Given the following inductive definitions.

$$\Delta_1 = \left\{ \begin{array}{l} P \leftarrow Q \\ Q \leftarrow P \end{array} \right\}$$

$$\Delta_2 = \left\{ \begin{array}{l} P \leftarrow Q \\ Q \leftarrow \neg P \end{array} \right\}$$

$$\Delta_3 = \begin{Bmatrix} P \leftarrow \neg Q \\ Q \leftarrow \neg P \end{Bmatrix}$$

What are the well-founded interpretations of these definitions? Which definitions have a model? Which definitions can be stratified? (Also construct justification trees).

3 FO(ID): semantics, default reasoning

Assignment Given the following definition, determine whether Coco and/or Skippy fly. Compute the model of the definition to find out (as always, use a graph representation to find out dependencies and stratify the definition).

$$\left\{ \begin{array}{rl} Fly(x) \leftarrow & Bird(x) \wedge \neg AbFly(x) \\ AbFly(x) \leftarrow & Ostrich(x) \wedge \neg AbOstrich(x) \\ AbOstrich(x) \leftarrow & SuperOstrich(x) \\ AbFly(x) \leftarrow & YoungSuperOstrich(x) \\ Bird(x) \leftarrow & Ostrich(x) \\ Bird(x) \leftarrow & Parrot(x) \\ Ostrich(x) \leftarrow & SuperOstrich(x) \\ SuperOstrich(x) \leftarrow & YoungSuperOstrich(x) \\ Parrot(Coco) \leftarrow & \\ YoungSuperOstrich(Skippy) \leftarrow & \end{array} \right\}$$

4 Packing

Assignment Given a rectangular area of a known dimension (width x height) and a set of squares, each of which has a known dimension, the problem is to pack all the squares into the rectangular area such that no two squares overlap. There can be empty spaces in the rectangular area. (For an example solution, see figure 1.)

The rectangular area forms a coordinate plane where the left upper corner of the area is the origin, the top edge is the x-axis, and the left edge is the y-axis. Suppose that we want to put the largest square in the top-left corner.

- Choose symbols for representing the position and size of squares and for representing the measurements of the total area. Choose a symbol for representing the largest block. Think about the necessary types.
- Give definitions for the following concepts:
  - a predicate $ValidPos : Square \rightarrow Bool$: indicates that the position of a square is within the bounds of the rectangular area.
  - predicates $LeftOf : (Square * Square) \rightarrow Bool$ and $Above : (Square * Square) \rightarrow Bool$ to specify relative positions of squares

Figure 1: Graphical representation of one solution to the packing problem.

  - a predicate $NoOverlap : (Square * Square) \rightarrow Bool$ which indicates that two square do not overlap

- Make a specification of correct packings as a FO(.) theory in this vocabulary.
- A set expression is of the form $\{(x_1, \ldots, x_n) : \phi\}$. Write a query (in this case, a set expression) to denote the set of all blocks above block A and not left of block B.
- Think of how to use the theory to solve at least two types of problems that require different forms of inference. (Note: you will probably not be able to solve this; this will be seen in lecture 5).