High Precision Servo System

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CONTENTS

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1. Introduction

- **1.1 Project**

- Worked as a full-stack engineer designing and debugging software of the system which was required to have a high positional accuracy (±1") and velocity index (accuracy: ±0.002%, fluctuation: ±0.002%).
- Modeled the object as a second-order transfer function through the Amplitude and Frequency Characteristics which can be acquired by adding white noise.
- Designed control algorithm (PID + DOB + ZPETC) to overcome the noise caused by friction and ensured the dynamic characteristic meet the requirement.

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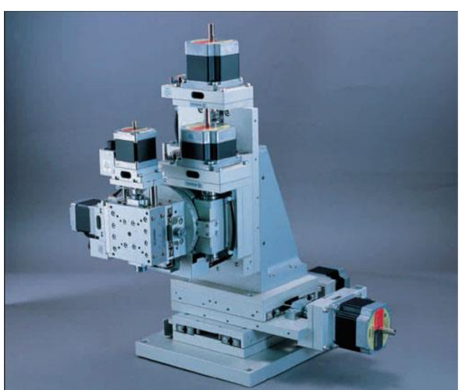
1. Introduction

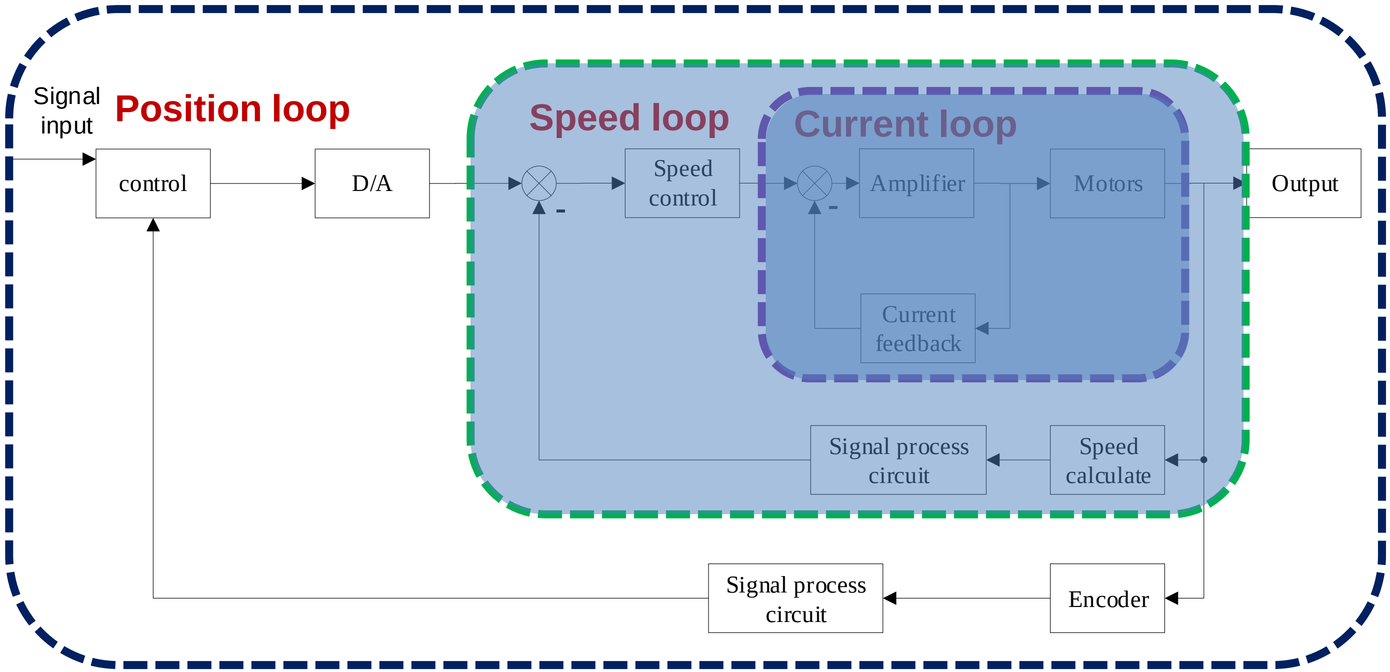
- **1.2 Servo System**
- Servo system is used to control the state(such as angle or displacement) of the object, it is a complex automated systems that can continuous and precisely reproduce the input.



1. Introduction

- **1.3 Flight Simulation Turntable**
- Simulation turntable is a typical high-performance servo system. three axis turntable simulated various fight movement and postures of aircraft and missile in the air based on different motions of the three axis. It is the key and high-performance equipment to carry on a semi-physical simulation on the ground during the development process of aircraft and missiles.

• 1.4 System Overall



Typical control methods

- **Forward control**
- **Optimal control**
- **Sliding mode control**
- **Predictive control**

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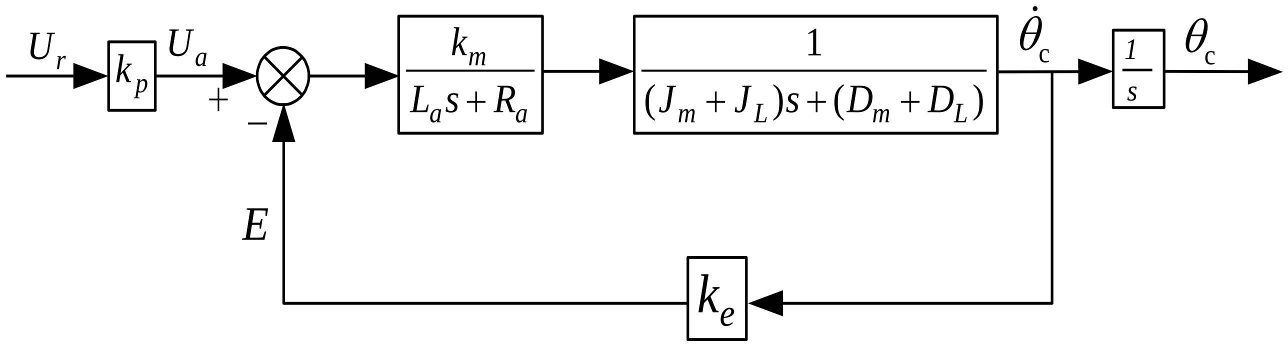
2. Formulation

- **2.1 Problem formulation**

- The ideal model for flight simulation turntable can be treated as a 2nd-order system



$$\frac{\theta_c}{U_r} = \frac{k_m k_p}{(L_a s + R_a)[(J_m + J_L)s + D_m + D_L] + k_m k_e} \times \frac{1}{s}$$

J —— 等效转动惯量

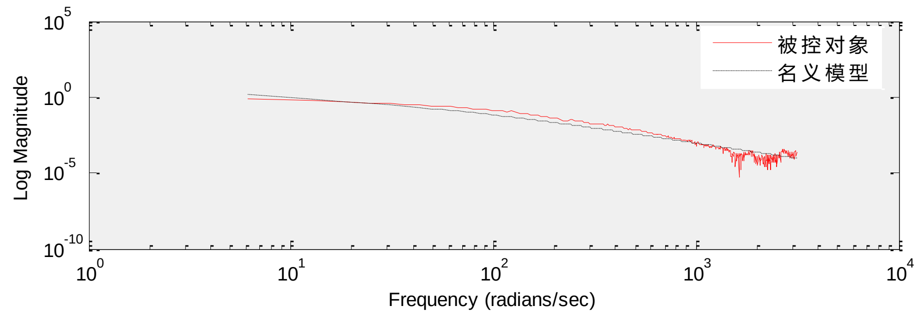
B —— 等效粘性摩擦系数

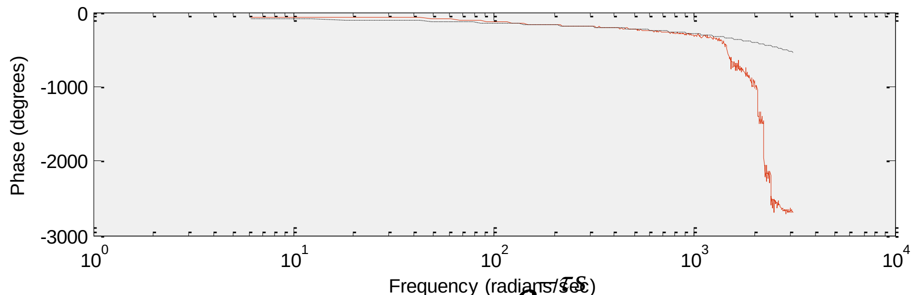
$$G_P(s) = \frac{1}{Js^2 + Bs}$$

2. Formulation

- **2.2 Open loop model**

- To model the system, white noise is added to the system, from which we can get frequency-domain analysis (Bode plot)





$$G_P(s) = \frac{e^{-\tau s}}{Js^2 + Bs}$$

$J = 0.0025v/(^\circ / s^2)$ $\quad B = 0.051v/(^\circ / s)$ $\quad \tau = 0.018s$

2. Formulation

- **2.2 Close loop model**

- If we want to get the close loop of the system, least square fitting method.

- Input $y_d = A_m sin(\omega t)$

- output $y(t) = A_f sin(\omega t + \varphi) = A_f cos(\varphi) sin(\omega t) + A_f sin(\varphi) cos(\omega t) = \begin{bmatrix} sin(\omega t) & cos(\omega t) \end{bmatrix} \begin{bmatrix} A_f cos(\varphi) \\ A_f sin(\varphi) \end{bmatrix}$

- Sample the output $Y^T = \begin{bmatrix} y(0) & y(h) & \cdots & y(nh) \end{bmatrix}$

$$\Psi^T = \begin{bmatrix} sin(w0) & sin(wh) & \cdots & sin(wnh) \\ cos(w0) & cos(wh) & \cdots & cos(wnh) \end{bmatrix}$$

$$c_1 = A_f cos(\varphi) \qquad c_2 = A_f sin(\varphi)$$

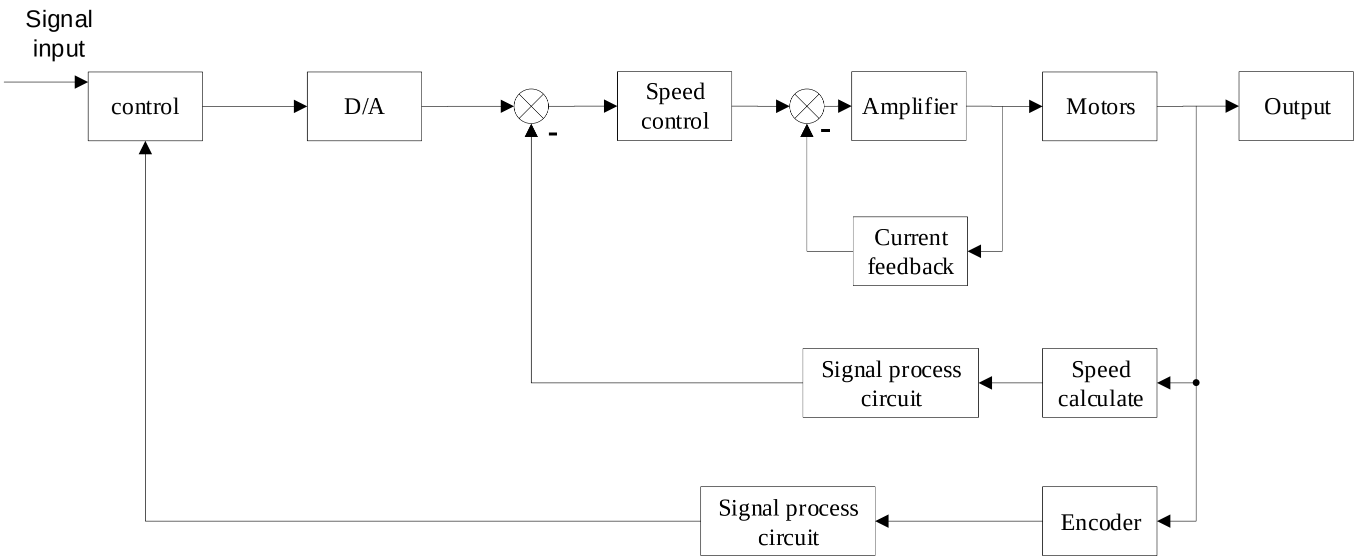
- so $\Psi \cdot \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = Y$ $\qquad \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \left(\Psi^T \Psi\right)^{-1} \Psi^T Y$

- The amplify and angle can be obtained as

$$20Lg\left(\frac{A_f}{A_m}\right) = 20Lg\left(\frac{\sqrt{c_1^2 + c_2^2}}{A_m}\right) \qquad \varphi = tg^{-1}\left(\frac{c_2}{c_1}\right)$$

3. Controller

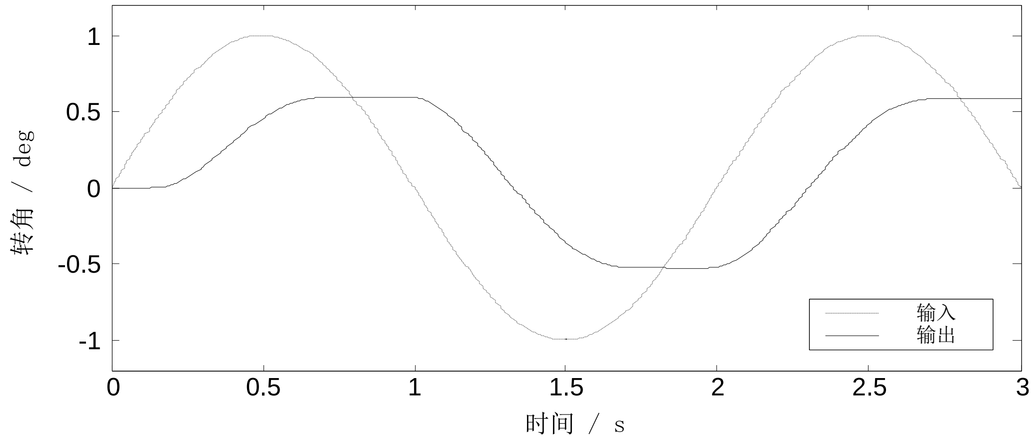
- **3.1 PD+DOB+ZPETC**



3. Controller

- **3.2 PD**

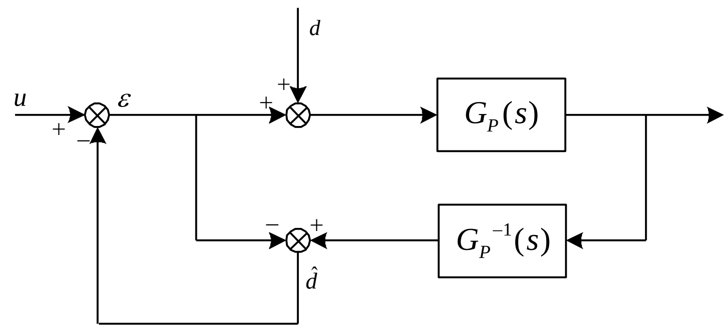
- proportional–derivative (PD) controller
- Do not rely on the model
- But have difficulties in dealing with disturbance.



3. Controller

- **3.3 DOB**

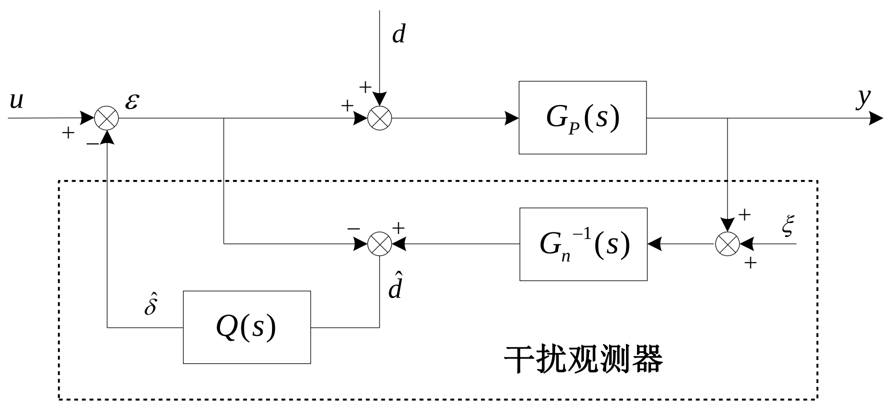
- Disturbance observer: to effectively overcome disturbance



$$\hat{d} = (\varepsilon + d)\cdot G_P(s)\cdot G_P^{-1}(s) - \varepsilon$$
$$= d$$

$G_p^{-1}(s)$ is hard to realize

Try to design Q(s)



$$y = G_{UY}(s)u + G_{DY}(s)d + G_{\xi Y}(s)\xi$$

$$G_{UY}(s) = \frac{G_P(s)G_n(s)}{G_n(s) + [G_P(s) - G_n(s)]Q(s)}$$

$$G_{DY}(s) = \frac{G_P(s)G_n(s)[1 - Q(s)]}{G_n(s) + [G_P(s) - G_n(s)]Q(s)}$$

$$G_{\xi Y}(s) = \frac{G_P(s)Q(s)}{G_n(s) + [G_P(s) - G_n(s)]Q(s)}$$

3. Controller

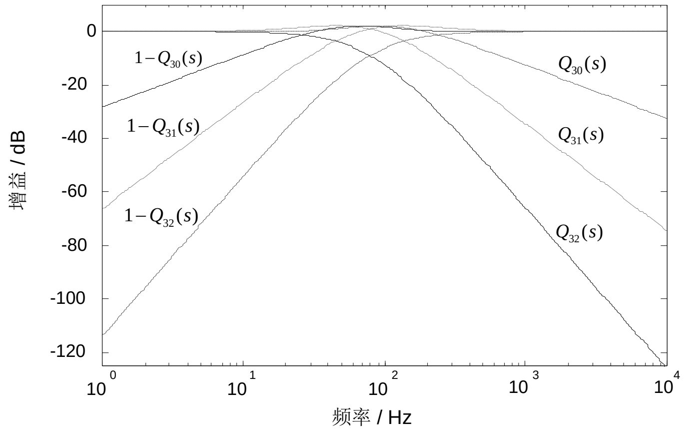
- **3.3 DOB**

- Q(s) can be regarded as a filter

$$Q_{NM}(s)=\frac{\sum_{k=0}^{M}\alpha_k(\tau s)^k}{(\tau s+1)^N}\quad (M=0,1,\cdots,N-1)\qquad \alpha_k=\frac{N!}{(N-k)!k!}$$

- Different order has different function

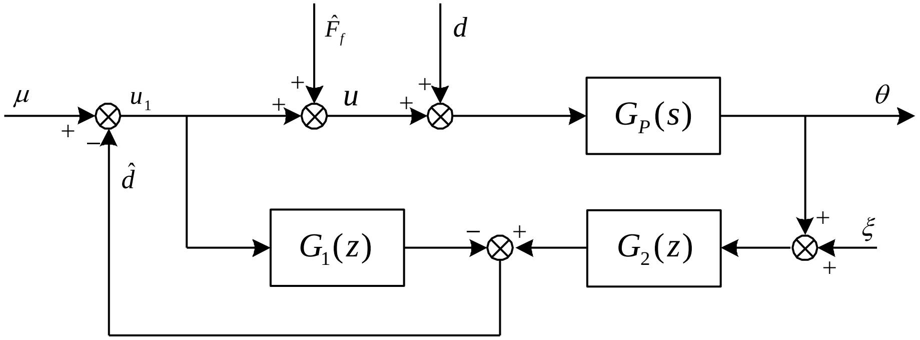
$\lVert Q_{NM}(j\omega)\rVert_\infty$	Q_{20}	Q_{21}	Q_{30}	Q_{31}	Q_{32}	Q_{40}	Q_{41}	Q_{42}	Q_{43}
Value	1.00	1.16	1.00	1.30	1.29	1.00	1.43	1.61	1.38



3. Controller

- **3.3 DOB**

- In turntable, we adopt the following system



- Objective model $$G_n(s) = \frac{1}{J_n s^2 + B_n s}$$
- DOB (Q(s)) $$Q(s) = Q_{31}(s) = \frac{3\tau s + 1}{\tau^3 s^3 + 3\tau^2 s^2 + 3\tau s + 1}$$

- So, in the system $$G_1(z) = Q(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}} \qquad G_2(z) = Q(s)G_n^{-1}(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}}$$

- Finally, the error can be described as $$\hat{d}(k) = G_2(z)\theta(k) - G_1(z)u_1(k)$$

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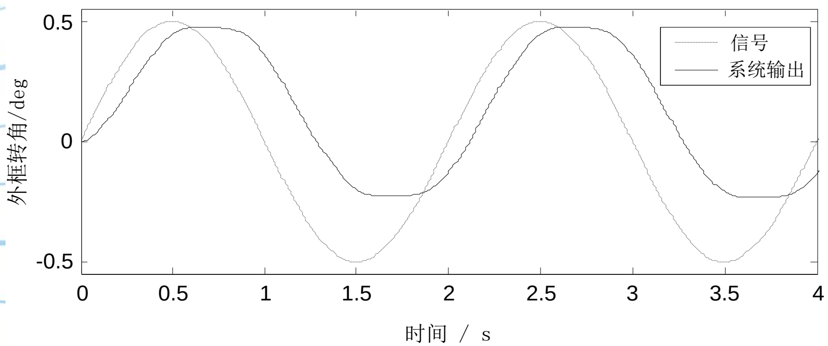
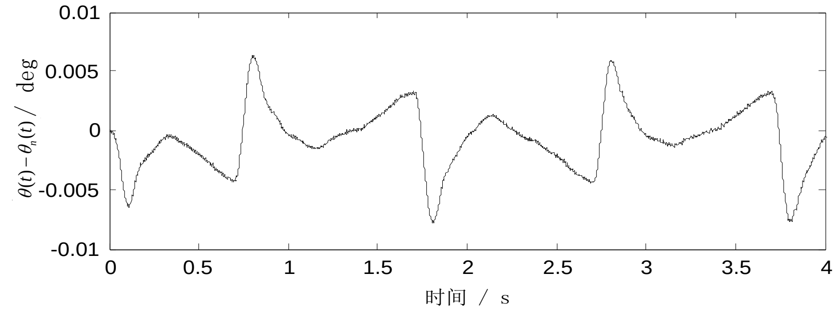
3. Controller

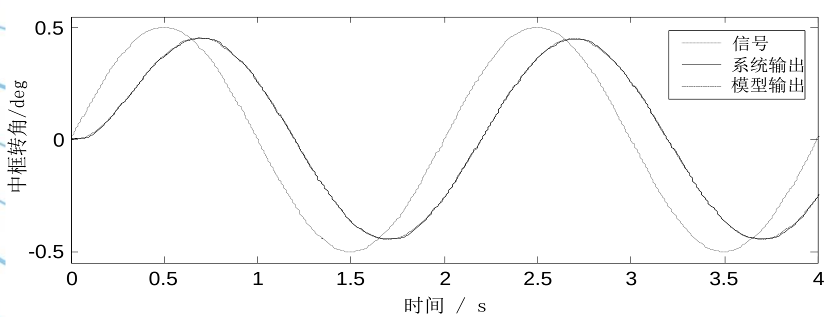
- **3.3 DOB**
- Simulation result



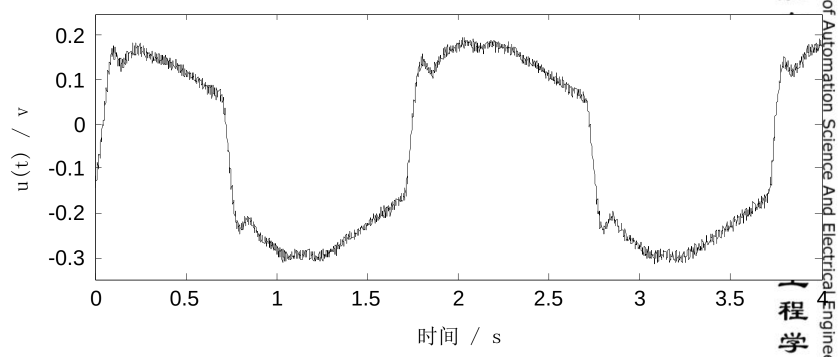
PD controller



Position error



PD+DOB controller



Output of DOB

3. Controller

- ## 3.4 ZPETC

- To meet the dynamic requirements, we designed Zero Phase Error Tracking Controller (ZPETC)

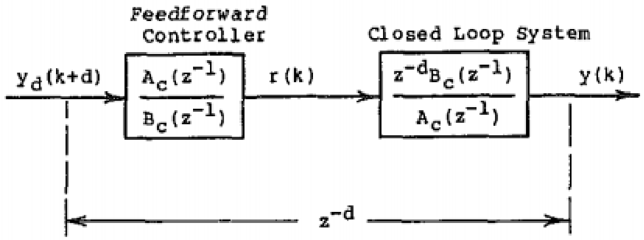


Fig. 1 Feedforward controller for perfect tracking

- The closed loop system can be described as $$G_c(z^{-1}) = \frac{z^{-d}B_c(z^{-1})}{A_c(z^{-1})}$$

$$A_c(z^{-1}) = 1 + a_{c1}z^{-1} + \cdots + a_{cn}z^{-n} \qquad B_c(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_l z^{-l}, b_0 \neq 0$$

- We consider a feedforward tracking controller which provides the reference input in the form

$$y(k) = G_c(z^{-1})r(k)$$

- The controller was designed as $$ZPETC\{G_c(z^{-1})\} = \frac{z^d A(z^{-1})B_u(z)}{\ldots(1)]^2}$$

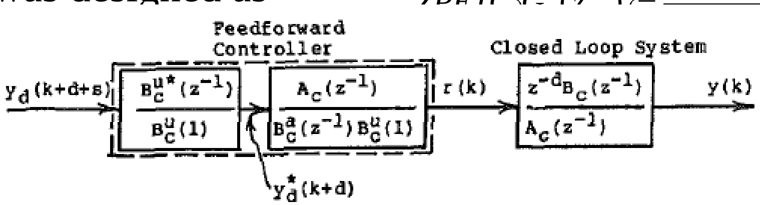


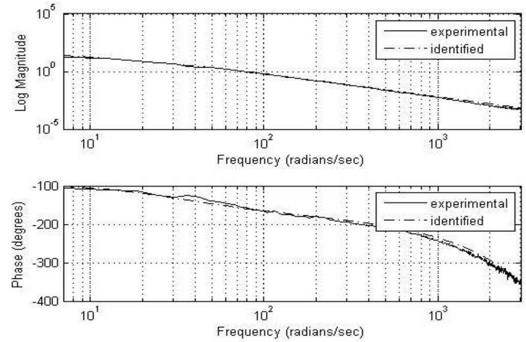
Fig. 2 Feedforward controller for zero phase error tracking

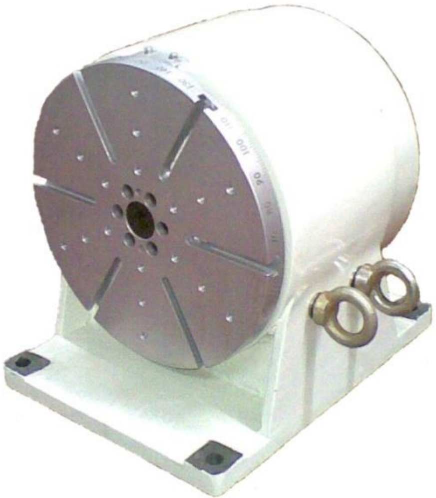
4. Turntable

- **4.1 Turntable with one axis**

  - Transfer function

$$G_p(s) = \frac{6500}{s^2 + 37s}$$



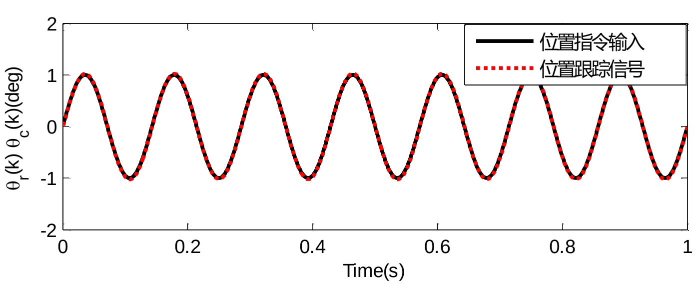


  - Reference model

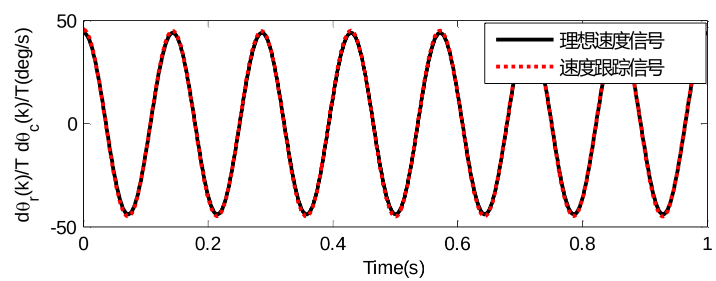
$$G_m(s) = \frac{160000}{s^2 + 565.5s + 160000}$$

  - Input $r(k)=\sin(2\pi FkT)$

  - output



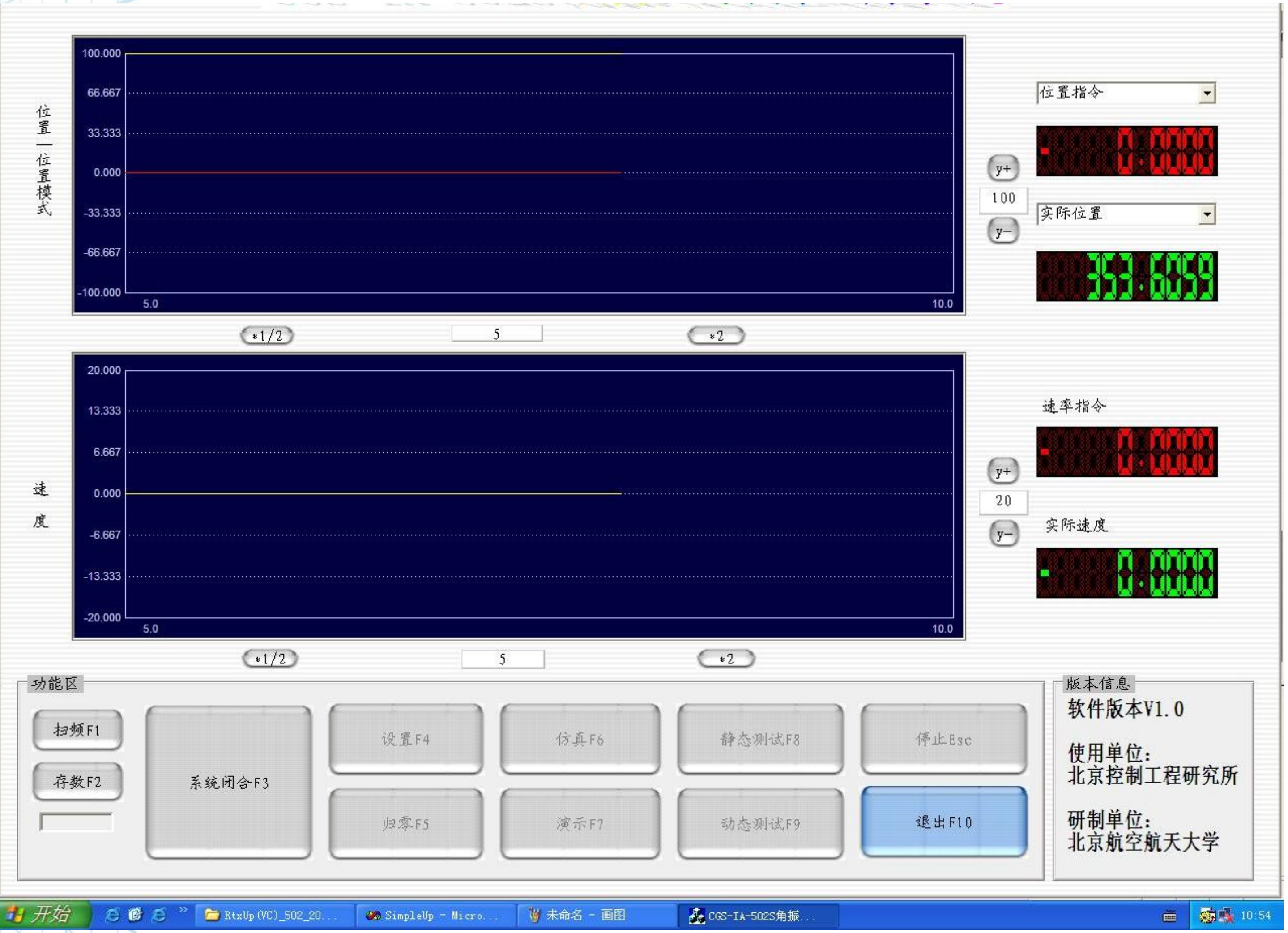
position

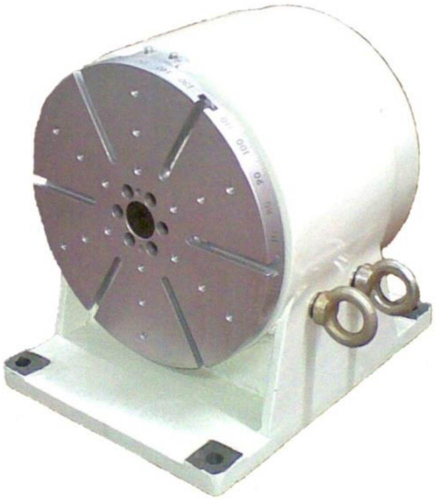


speed

4. Turntable

- **4.1 Turntable with one axis**
- GUI





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4. Turntable

- **4.1 Turntable with three axis**