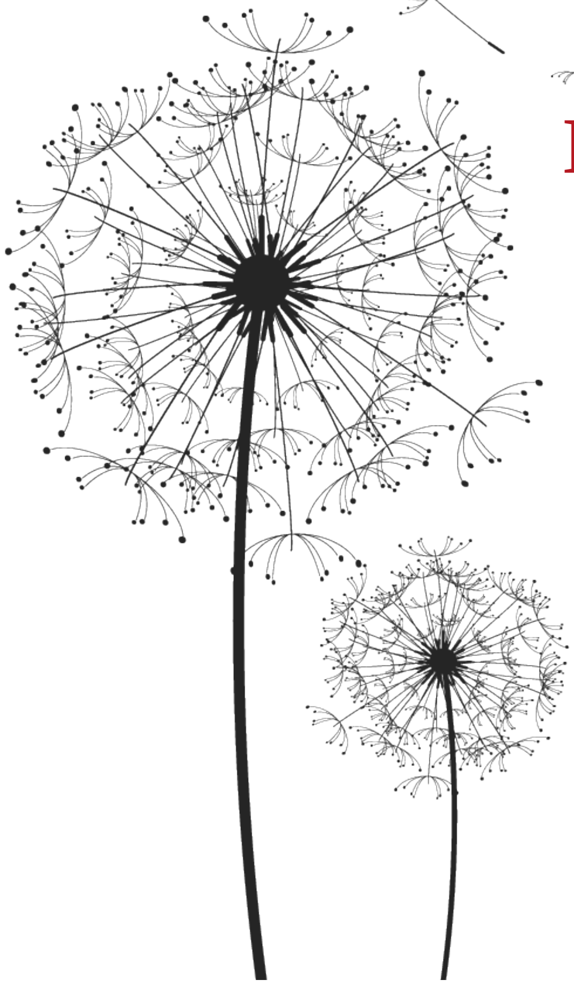

An Improved Nonsingular Fast Terminal Sliding Mode Guidance Law with Impact Angle Constraints

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- Abstract
- Introduction
- Problem Formulation
- Design of Sliding-Mode Guidance Laws
- Simulation
- Conclusion

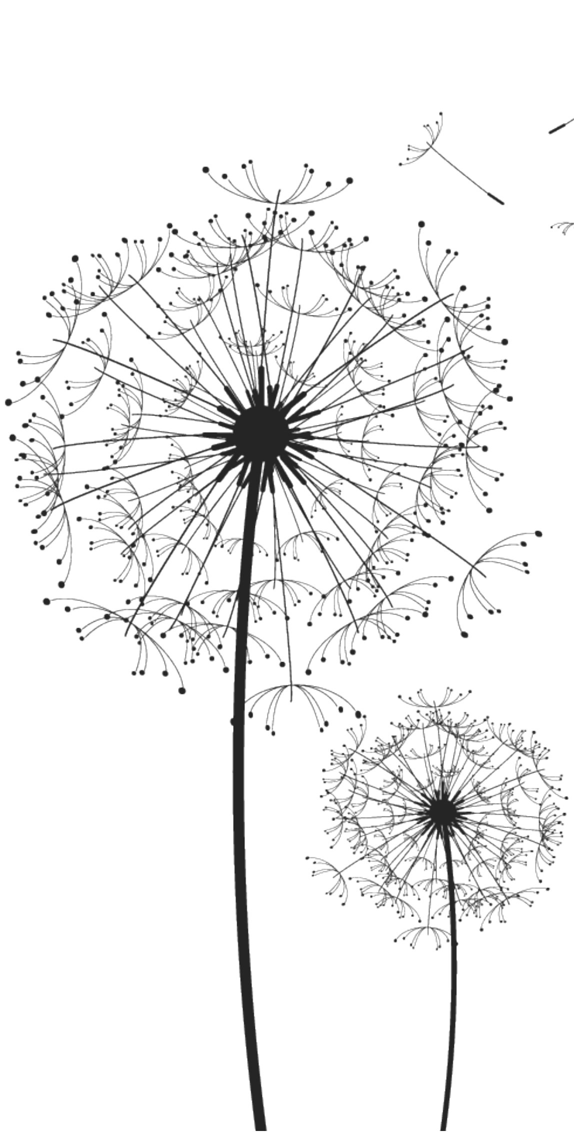
Abstract

The general meaning of the paper

Abstract

In the paper, an improved nonsingular fast terminal sliding mode (INFTSM) guidance law with impact angle constraints is proposed. The guidance law, which employs a double power reaching law and an attractor with negative exponential factor, has a fast speed no matter far from the sliding surface or approach. For the maneuvering targets, extended state observer is designed through which the unknown lateral acceleration of target can be estimated and contributed to make compensation for the motion. Compared with other nonsingular guidance law, the proposed INFTSM guidance law not only has a better performance in impact angle and miss distance, but also show advantage in shorter time and higher accuracy. Numerical simulation results are presented to illustrate the proposed guidance law.

Keywords—*nonsingular terminal sliding mode; impact angle constraints; extended state observer; finite time convergence*



Introduction

Introduce the background of guidance law

Introduction

Factors: Miss distance, Impact angle, Convergence, Target information

Method	Advantage	Disadvantage
TSM (terminal)	ensured finite time convergence as well as the terminal impact angle	there existed singularity
NTSM (nonsingular terminal)	avoid the singularity, intercept not only stationary targets, but also constant velocity even maneuvering targets.	it may take much time to guarantee convergence
NFTSM (nonsingular fast terminal)	guaranteed a higher precision in a shorter time interval with lower and smoother guidance command	hard to get the information of target lateral acceleration
FTDO (finite-time disturbance observer)	the target acceleration of target is treated as unknown bounded external disturbance .	lower the convergence
ESO (extended state observer)	Treat the target information as a extended state, and estimate it.	lower the convergence

Introduction

Improved Nonsingular Fast Terminal Sliding Mode (INFTSM)

(1) The guidance law, which employs a double power reaching law [9] and an attractor with negative exponential factor [10], has a fast speed no matter far from the sliding surface or approach.

(2) For the maneuvering targets, extended state observer (ESO) was designed, through which the unknown lateral acceleration of target can be estimated and contributed to make compensation for the motion.

(3) Numerical simulation results show that the proposed INFTSM guidance law can not only guarantee the convergence in finite time, but also attack the target in desired impact angle. Besides, it has a better performance in higher accuracy, shorter time and smoother command when compared with the existing guidance laws

Problem Formulation

Set up the intercepting problem in a planar engagement

Problem Formulation

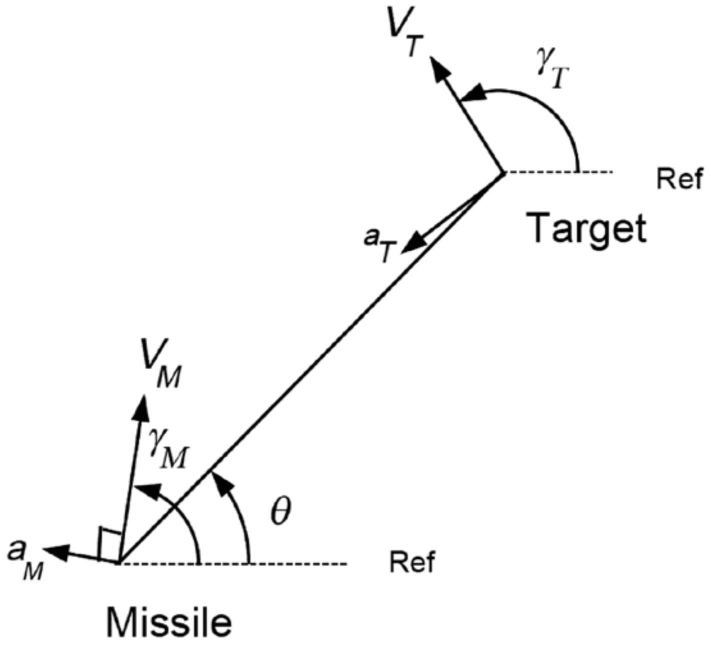
Without losing generality, the engagement between a missile and a target is shown in Fig. 1. V_M and V_T are represented as the missile velocity and target, a_M and a_T donate their lateral accelerations, γ_M and γ_T define their flight-path angles respectively. In this paper, V_M and V_T are assumed as constants. Except these, the relative distance between the missile and the target is defined as r and the line of-sight (LOS) angle as θ. So, the equations of kinematic engagement were described as follows:

$$\dot{r} = V_r = V_T \cos\theta_T - V_M \cos\theta_M$$

$$r\dot{\theta} = V_\theta = V_T \sin\theta_T - V_M \sin\theta_M$$

$$\dot{\gamma}_M = \frac{a_M}{V_M} \qquad \dot{\gamma}_T = \frac{a_T}{V_T}$$

Where $\theta_T = \gamma_T - \theta$ and $\theta_M = \gamma_M - \theta$



With the impact angle constraints, θ_{imp} is defined as the angle between the missile velocity and the target when intercepting occurred.

$$\theta_{imp} = \gamma_{Tf} - \gamma_{Mf}$$

Problem Formulation

$$\dot{r} = V_r = V_T\cos\theta_T - V_M\cos\theta_M$$

$$r\dot{\theta} = V_\theta = V_T\sin\theta_T - V_M\sin\theta_M$$

$$\theta_M = \gamma_M - \theta \qquad \theta_T = \gamma_T - \theta$$

$$\theta_{imp} = \gamma_{Tf} - \gamma_{Mf}$$

$$\Rightarrow$$

$$V_M\sin(\gamma_{Mf} - \theta_{Ff}) = V_T\sin(\gamma_{Tf} - \theta_{Ff})$$

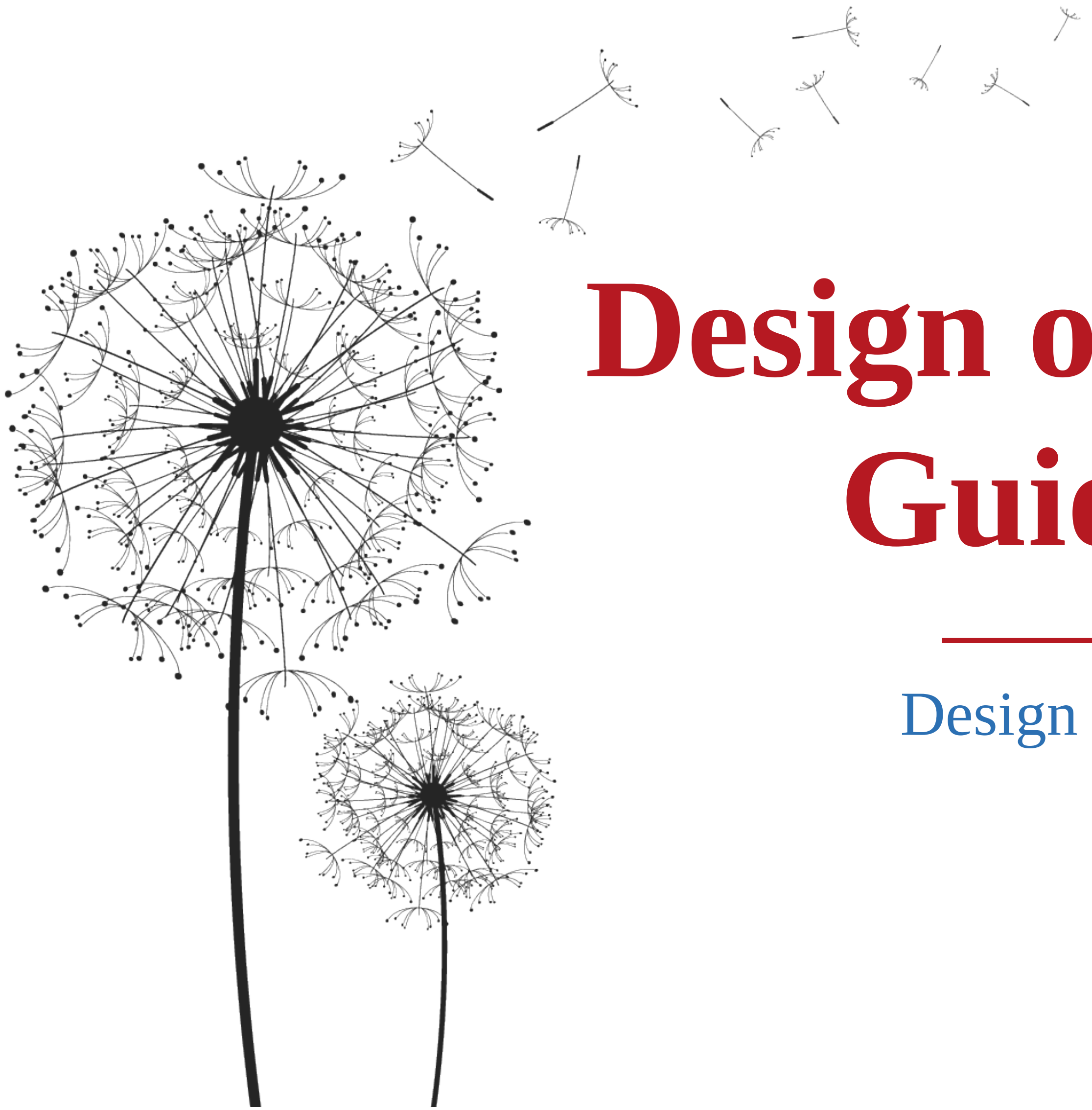
$$\Downarrow$$

$$\theta_{Ff} = \gamma_{Tf} - \tan^{-1}\left(\frac{\sin\theta_{imp}}{\cos\theta_{imp} - V_T/V_M}\right)$$

$$\Downarrow$$

Expanding the relation, a method to make (4) hold from the beginning and until ending is proposed. The relationship can be rewrite as:

$$\theta_F = \gamma_T - \tan^{-1}\left(\frac{\sin\theta_{imp}}{\cos\theta_{imp} - V_T/V_M}\right)$$



Design of Sliding-Mode Guidance Laws

Design the improved nonsingular fast terminal guidance law

Design of Sliding-Mode Guidance Laws

- A. *Nonsingular Terminal Sliding Mode Guidance Law*

$$\dot{r} = V_r = V_T\cos\theta_T - V_M\cos\theta_M$$
$$r\dot{\theta} = V_\theta = V_T\sin\theta_T - V_M\sin\theta_M$$
$$\dot{\gamma}_M = \frac{a_M}{V_M} \qquad \dot{\gamma}_T = \frac{a_T}{V_T}$$

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} + \left(\frac{\cos\theta_T}{r}\right)a_T - \left(\frac{\cos\theta_M}{r}\right)a_M$$

$$e_1 = \theta - \theta_F \qquad e_2 = \dot{\theta} - \dot{\theta}_F$$

- The error states equation can be expressed as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = -\frac{2\dot{r}}{r}e_2 + \left(\frac{\cos\theta_T}{r}\right)a_T - \left(\frac{\cos\theta_M}{r}\right)a_M \end{cases}$$

- Like INTSM, the switching surface is chosen as

$$s = e_1 + k_1|e_1|^{a_1} + k_2|e_2|^{a_2}$$

$k_1 > 0, k_2 > 0,$
$a_1 > a_2, 1 < a_2 < 2$

- The reaching law was designed as follows:

$$\dot{s} = \boxed{\left(-\alpha|s|^{b_1}sign(s) - \beta|s|^{b_2}sign(s)\right)}\boxed{|e_2|^{a_2-1}}$$

Double reaching law

an attractor with negative exponential factor

$\alpha > 0, \beta > 0,$
$b_1 > 1, 0 < b_2 < 1$

$$a_{M0} = \frac{r}{\cos\theta_M}\left(\frac{1}{k_2a_2}|e_2|^{2-a_2}(1 + k_1a_1|e_1|^{a_1-1}) - \frac{2\dot{r}\dot{\theta}}{r} + \left(\frac{\cos\theta_T}{r}\right)a_T - \left(-\alpha|s|^{b_1}sign(s) - \beta|s|^{b_2}sign(s)\right)\right)$$

Design of Sliding-Mode Guidance Laws

- B. *Design of Extend State Observer*

$$\ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} + \left(\frac{\cos\theta_T}{r}\right)a_T - \left(\frac{\cos\theta_M}{r}\right)a_M \xrightarrow{d(t) = (\cos\theta_T/r)a_T} \begin{cases} \ddot{\theta} = -\frac{2\dot{r}\dot{\theta}}{r} - \frac{\cos\theta_M a_M}{r} + d(t) \\ \dot{d}(t) = g(t) \end{cases}$$

- Similar to [7], the second order ESO is proposed as:

$$\begin{cases} \varepsilon = z_1 - \dot{\theta} \\ \dot{z}_1 = z_2 - \beta_1\varepsilon - 2\dot{r}\dot{\theta}/r - \cos\theta_M a_M/r \\ \dot{z}_2 = -\beta_2 fal(\varepsilon, \sigma, \delta) \end{cases}$$

$$0 < \sigma < 1, 0 < \delta < 1 \quad \beta_1 > 0, \beta_2 > 0$$

$$fal(\varepsilon, \sigma, \delta) = \begin{cases} |\varepsilon|^\sigma sgn(\varepsilon), & |\varepsilon| > \delta \\ \varepsilon/\delta^{1-\sigma}, & otherwise \end{cases}$$

- Note that ε and z_2 are estimation error and value of term $(\cos\theta_T/r)a_T$

$$a_{M0} = \frac{r}{\cos\theta_M}\left(\frac{1}{k_2 a_2}|e_2|^{2-a_2}(1 + k_1 a_1 |e_1|^{a_1-1}) - \frac{2\dot{r}\dot{\theta}}{r} + \left(\frac{\cos\theta_T}{r}\right)a_T - \left(-\alpha|s|^{b_1}sign(s) - \beta|s|^{b_2}sign(s)\right)\right)$$

$$a_{M1} = \frac{r}{\cos\theta_M}\left(\frac{1}{k_2 a_2}|e_2|^{2-a_2}(1 + k_1 a_1 |e_1|^{a_1-1}) - \frac{2\dot{r}\dot{\theta}}{r} + z_2 - \left(-\alpha|s|^{b_1}sign(s) - \beta|s|^{b_2}sign(s)\right)\right)$$

Design of Sliding-Mode Guidance Laws

- C. *Convergence analysis*

- Step 1: Prove the estimated state z_2 converges to the actual term $(\cos\theta_T/r)a_T$

$$\varepsilon_2 = z_2 - (\cos\theta_T/r)a_T \qquad \begin{cases} \dot{\varepsilon}_1 = \varepsilon_2 - \beta_1\varepsilon_1 \\ \dot{\varepsilon}_2 = -g(t) - \beta_2 fal(\varepsilon,\sigma,\delta) \end{cases}$$

- If the parameters are selected appropriately, the derivatives of the vector are approaching zero

- Step 2: Prove the guidance law guarantees convergence

$$z_2 = (\cos\theta_T/r)a_T$$

$$\dot{V} = s\dot{s} = s\left(-\alpha|s|^{b_1}sign(s) - \beta|s|^{b_2}sign(s)\right)|e_2|^{a_2-1} \le 0 \qquad \alpha > 0, \beta > 0, b_1 > 1, 0 < b_2 < 1$$

- As , the above inequality can be demonstrated easily. What’s more, singularity can be avoided and a fast speed no matter far from or approaching the sliding surface is achieved. Besides, there are more advantages about the method was showed in [10].

Design of Sliding-Mode Guidance Laws

- D. *Extension to large heading angle error*

  - In some practical issues are discussed. When large heading errors and $Vr > 0$ occurs, the interceptor–target range goes to zero, and the magnitude of the missile lateral acceleration is out of control. To avoid miss, it's necessary to modify the guidance law as:

$$a_{M1} = \frac{r}{\cos\theta_M}\left(\frac{1}{k_2 a_2}|e_2|^{2-a_2}\left(1 + k_1 a_1 |e_1|^{a_1-1}\right) - \frac{2\dot{r}\dot{\theta}}{r} + z_2 - \left(-\alpha|s|^{b_1}sign(s) - \beta|s|^{b_2}sign(s)\right)\right)$$

$$\boxed{a_{M2} = \frac{r}{|\cos\theta_M|}\left(\frac{1}{k_2 a_2}|e_2|^{2-a_2}\left(1 + k_1 a_1 |e_1|^{a_1-1}\right) + \frac{2|\dot{r}|\dot{\theta}}{r} + z_2 - \left(-\alpha|s|^{b_1}sign(s) - \beta|s|^{b_2}sign(s)\right)\right)}$$

  - in the practical situation, the acceleration of missile cannot be infinity, so it should be bounded with the saturation function:

$$a_M = \begin{cases} a_{M\max} sign(a_M), & |a_M| \geq a_{M\max} \\ a_M, & |a_M| < a_{M\max} \end{cases} \qquad a_T = \begin{cases} a_{T\max} sign\left(\dfrac{z_2}{\cos\theta_T}\right), & \left|\dfrac{z_2}{\cos\theta_T}\right| \geq a_{T\max} \\ \dfrac{z_2}{\cos\theta_T}, & otherwise. \end{cases}$$

Simulation

Validate the performance of the proposed guidance law

Simulation

- In this section, through several numerical simulations, the performance of proposed guidance law can be validated.
- three kinds of targets:
  stationary, velocity-constant and maneuvering targets.
- three kinds of guidance law (make comparisons)
  NTSM, NFTSM, INFTSM

Parameter	value
Relative distance	1000m
Initial LOS angle	30 deg
Velocity of missile	500m/s
Allowable acceleration of missile	200m/(s^2)

- The parameters of sliding mode surface is selected as

$$k_1 = 1 \quad k_2 = 2 \quad a_1 = 3 \quad a_2 = 1.5$$

- The parameters of reaching law is designed as

$$\alpha = 500 \quad \beta = 300 \quad b_1 = 0.6 \quad b_2 = 1.2$$

- The parameters on extended state observer are

$$\beta_1 = 50 \quad \beta_2 = 100 \quad \sigma = 0.5 \quad \delta = 0.01$$

$$\boxed{a_{M2} = \frac{r}{|\cos\theta_M|}\left(\frac{1}{k_2 a_2}|e_2|^{2-a_2}\left(1 + k_1 a_1 |e_1|^{a_1-1}\right) + \frac{2|\dot{r}|\dot{\theta}}{r} + z_2 - \left(-\alpha|s|^{b_1} sign(s) - \beta|s|^{b_2} sign(s)\right)\right)}$$

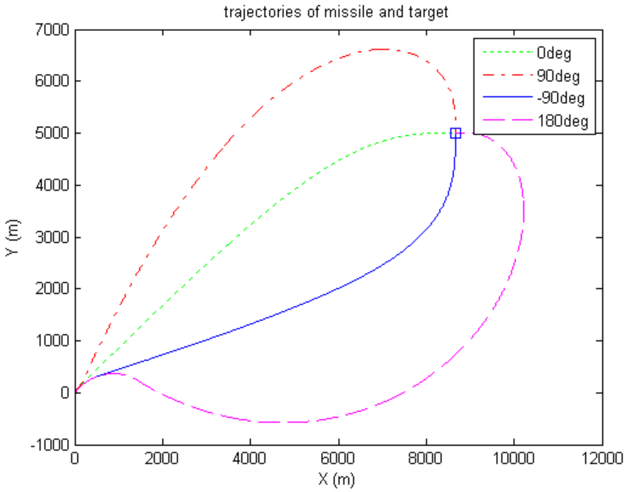
Simulation

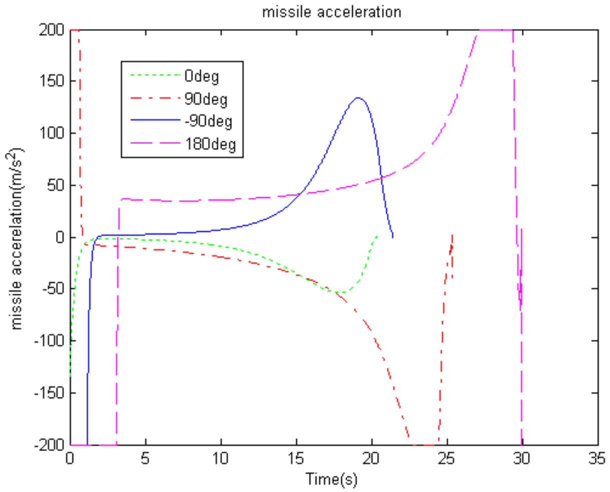
- **1) *Stationary Targets***

$$\gamma_T = V_T = a_T = 0$$

- the flight-path angle :60deg
- the impact angles :

  0deg, 90deg, -90deg, and 180deg, respectively.





Impact angle (deg)	The main parameters of the system		
	Time(s)	Miss distances(m)	Impact angle error(deg)
0	20.49	1.87	0.01
-90	21.43	1.86	0.01
90	25.36	0.84	0.05
180	29.95	2.28	0.87

Simulation

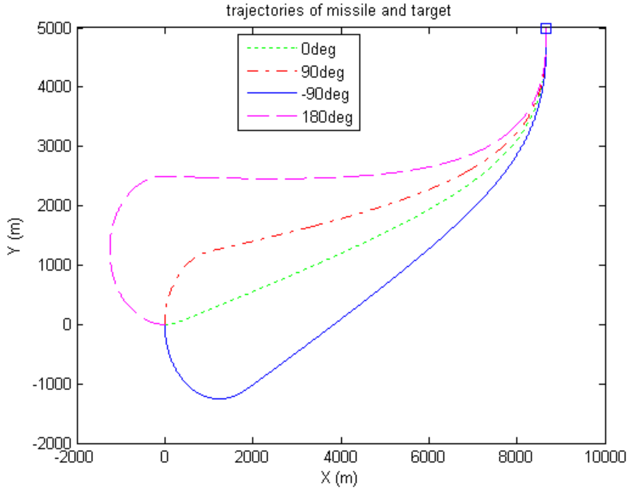
➤ ***1) Stationary Targets***

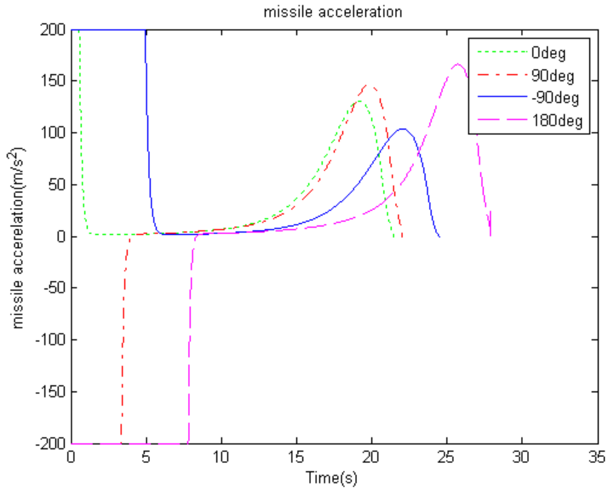
$$\gamma_T = V_T = a_T = 0$$

➤ the flight-path angle :

0deg, 90deg, -90deg, and 180deg, respectively.

➤ the impact angles : -90deg





Initial angle(deg)	**The main parameters of the system**		
	Time(s)	Miss distances(m)	Impact angle error(deg)
0	21.44	1.26	0.02
-90	24.48	0.89	0.01
90	22.05	0.36	0.05
180	27.87	1.26	0.10

Result:
For these cases, we can see that when the missile intercept the target, the INFTSM guidance law can not only guarantee the impact angle but also justify the all-aspect capability against stationary targets. By the way, the miss distances, impact angle error and the lateral acceleration of missile are acceptable.

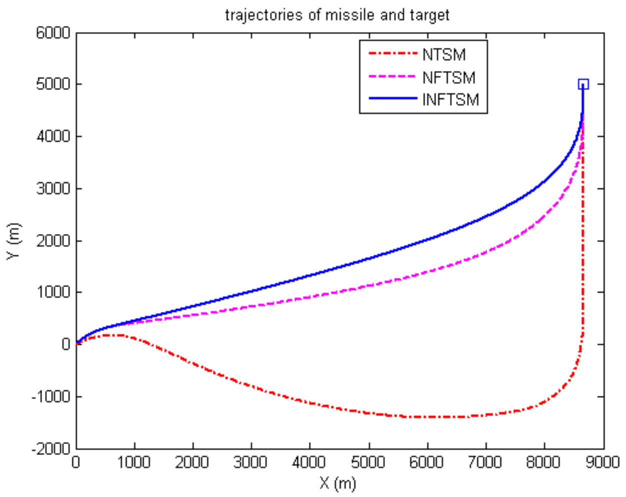
Simulation

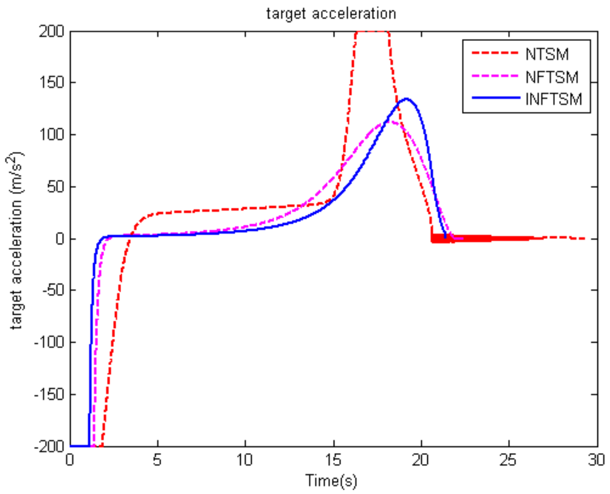
➢ ***1) Stationary Targets***

$$\gamma_T = V_T = a_T = 0$$

- the flight-path angle :45deg
- the impact angles : -90deg
- Guidance law:

  NTSM, NFTSM and INFTSM.





Different Guidance Law	The main parameters of the system		
	Time(s)	Miss distances(m)	Impact angle error(deg)
NTSM	29.14	0.09	0.01
NFTSM	22.40	0.01	0.01
INFTSM	21.75	0.03	0.01

Result:

1) all of the sliding mode can intercept the stationary target with desired impact angle
2) With the proposed guidance law, the missile finds the optimal trajectory, and it takes the shortest time to intercept the target. By applying the same sliding mode surface, the reaching law of INFTSM responds much more quickly than NFTSM.
3) Besides, the lateral acceleration of target changes smoothly in Fig. 4, and still there is no chattering in the system. Note that the miss distances are longer than others, because the guidance law sacrifices the accuracy for short time.

Simulation

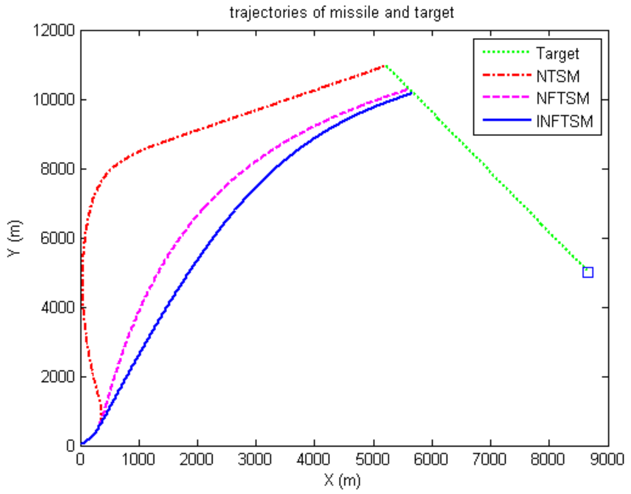
- ***2) Constant Velocity Targets***

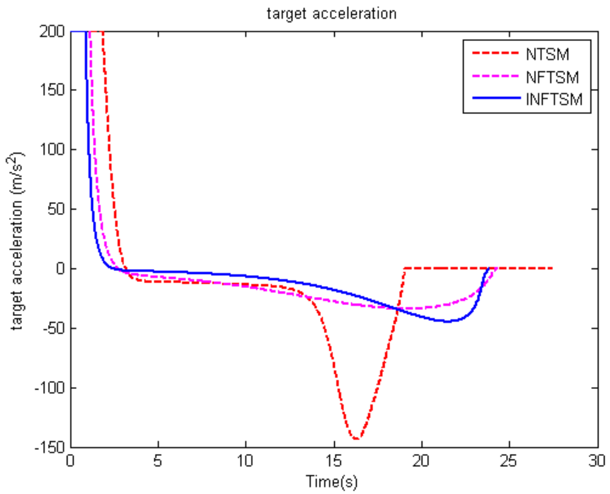
$$\gamma_{T0} = 120^\circ \quad V_T = 250\, m/s \quad a_T = 0\, m/s^2$$

- the flight-path angle : 45deg
- the impact angles : 90deg
- Guidance law:

  NTSM, NFTSM and INFTSM.

Different Guidance Law	The main parameters of the system		
	Time(s)	Miss distances(m)	Impact angle error(deg)
NTSM	27.49	2.67	0.01
NFTSM	24.42	2.61	0.01
INFTSM	23.97	2.01	0.01





Result:

1) NFTSM and INFTSM guidance responds more quickly than NTSM. Because the missile's velocity is constant, the more trajectory optimized, the shorter time is.
2) Note that in this case, the intercept time of NFTSM and INFTSM is much shorter than NTSM.
3) Note that the miss distances of INFTSM are much longer than NFTSM, but it can become smaller by selecting other parameter, which may cost a long time.

Simulation

➢ ***3) Maneuvering Targets***

$$\gamma_{T0} = 120^{\circ} \qquad V_T = 250\, m/s$$

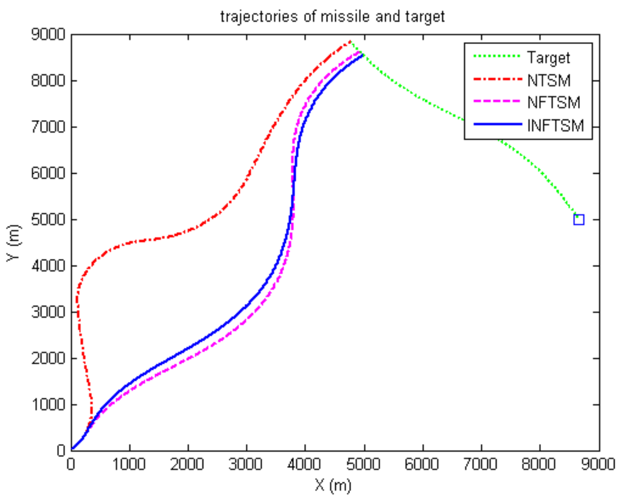
$$a_T = 15\sin(0.25t)\, m/s^2$$

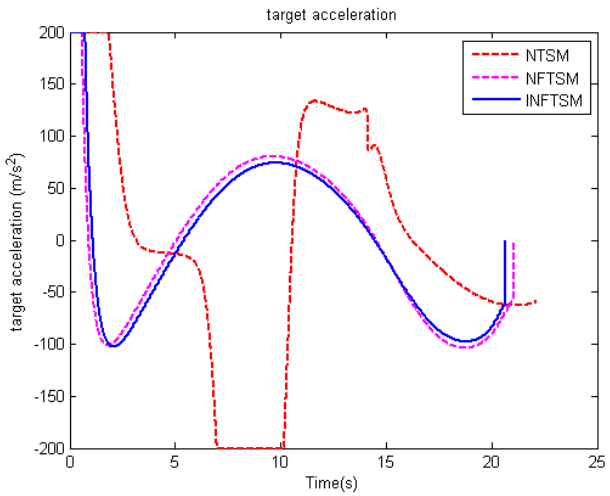
➢ the flight-path angle : 45deg

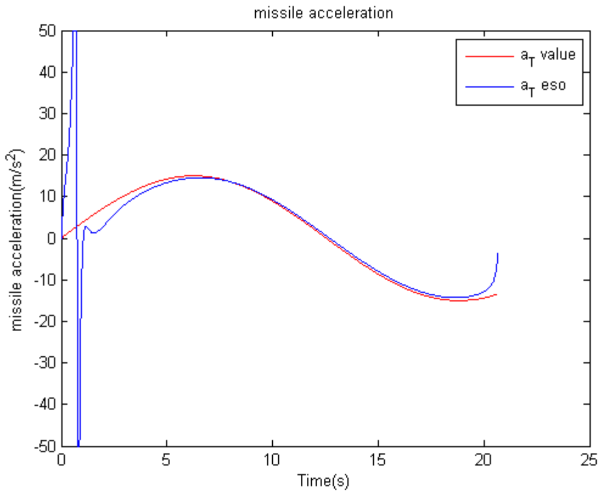
➢ the impact angles : 90deg

➢ Guidance law: NTSM, NFTSM and INFTSM.

1) compared to stationary and constant velocity targets, intercepting maneuvering targets take a longer time, and the miss distances and impact error may worse.
2) The performances of the extended states observer to estimate the target acceleration *aT* are given. It is obvious that the estimated states are approximately approach to the target acceleration *aT*







Different Guidance Law	The main parameters of the system		
	Time(s)	Miss distances(m)	Impact angle error(deg)
NTSM	22.11	4.34	5.26
NFTSM	21.04	2.14	0.02
INFTSM	20.68	2.07	0.01

Conclusion

Make a conclusion about the paper

Conclusion

01

Set up a new guidance law:

INFTSM = SMC + double power reaching law + an attractor with negative exponential factor + ESO

02

Analysis:

Prove that INFTSM grantee the convergence, and make modify.

03

Verify the effectiveness

three kinds of target : stationary, velocity constant, and maneuvering targets.

For stationary target :different initial heading angles, and different desired impact angles

To make comparison: NTSM, NFTSM, INFTSM

04

Result:

From the results, the INFTSM guidance law not only has a fast speed in finite time convergence, but also shows advantage in shorter time and higher precise.

Thank you!