5 a) $\mathcal{L}\{e^{t}\cos t\}$

$$= \int_0^{\infty} e^{t}(\cos t)\, e^{-st}\, dt$$

$$= \int_0^{\infty} (e^{t-st}) \cos t\, dt$$

$$= \int_0^{\infty} e^{t(1-s)} \cos t\, dt$$

$$= \lim_{K\to \infty} \int_0^{K} e^{(1-s)t} \cos t\, dt$$

• $\int e^{(1-s)t} \cos t = I$ partes

$f = \cos t$ $\qquad g' = e^{(1-s)t}\, dt$

$f' = -\operatorname{sen} t\, dt$ $\qquad g = \frac{1}{1-s} e^{(1-s)t}$

$$fg - gf' = \frac{1}{1-s}\left(e^{(1-s)t}\cos t\right) + \underbrace{\frac{1}{1-s}\int e^{(1-s)t} \operatorname{sen} t\, dt}$$

La segunda integral por partes

$f = \operatorname{sen} t$ $\qquad g' = e^{(1-s)t}\, dt$

$f' = \cos t\, dt$ $\qquad g = \frac{1}{1-s} e^{(1-s)t}$

$$fg - fg' = \frac{1}{1-s}\left(e^{(1-s)t}\operatorname{sen} t\right) - \frac{1}{1-s}\int e^{(1-s)t}\cos t\, dt\,)$$

$$= \frac{1}{1-s}\left(e^{(1-s)t}\operatorname{sen} t\right) - \frac{1}{1-s}\, I$$

Y por lo tanto

$$I = \frac{1}{1-s}\left(e^{(1-s)t}\cos t\right) + \frac{1}{1-s}\left[\frac{1}{1-s}\left(e^{(1-s)t}\operatorname{sen} t\right) - \frac{1}{1-s} I\right]$$

$$\left(1+\frac{1}{(1-s)^2}\right) I = \frac{1}{1-s}\left(e^{(1-s)t}\cos t\right) + \frac{1}{(1-s)^2}\left(e^{(1-s)t}\operatorname{sen} t\right)$$

$$\frac{(1-s)^2+1}{(1-s)^2} I = \frac{1}{1-s}\left(e^{(1-s)t}\cos t\right) + \frac{1}{(1-s)^2}\left(e^{(1-s)t}\operatorname{sen} t\right)$$

$$I = \frac{1}{(1-s)^2+1}\left[(1-s)\left(e^{(1-s)t}\cos t\right) + \left(e^{(1-s)t}\operatorname{sen} t\right)\right]$$

Y se concluye que

$$\mathcal{L}\{e^t\cos t\} = \lim_{K\to\infty} \frac{1}{(1-s)^2+1}\left[(1-s)\left(e^{(1-s)t}\cos t\right) + \left(e^{(1-s)t}\operatorname{sen} t\right)\right]_0^K$$

$$= \frac{1}{(1-s)^2+1}\lim_{K\to\infty}\left[(1-s)e^{(1-s)K}\cos K + e^{(1-s)K}\operatorname{sen} K - (1-s)\right]$$

$$= \frac{1}{(1-s^2)+1}\left(-(1-s)\right)$$

$$= \frac{s-1}{(1-s)^2+1}$$