⑦ a) $c(t) = \langle e^t \cos t,\ e^t \operatorname{sen} t,\ 2 \rangle$

- $c'(t) = \langle e^t \cos t - e^t \operatorname{sen} t,\ e^t \operatorname{sen} t + e^t \cos t,\ 0 \rangle$

$= \langle e^t(\cos t - \operatorname{sen} t),\ e^t(\operatorname{sen} t + \cos t),\ 0 \rangle$

Luego $\| c'(t) \| = \left[ e^{2t}(\cos t - \operatorname{sen} t)^2 + e^{2t}(\operatorname{sen} t + \cos t)^2 + 0^2 \right]^{1/2}$

$= \left[ e^{2t}(\cos^2 t - 2 \operatorname{sen} t \cos t + \operatorname{sen}^2 t) + e^{2t}(\operatorname{sen}^2 t + 2 \operatorname{sen} t \cos t + \cos^2 t) \right]^{1/2}$

$= (2e^{2t})^{1/2}$

$= \sqrt{2} e^t$

Entonces $T(t) = \dfrac{c'(t)}{\| c'(t) \|}$

$= \dfrac{1}{\sqrt{2} e^t} \langle e^t(\cos t - \operatorname{sen} t),\ e^t(\operatorname{sen} t + \cos t),\ 0 \rangle$

$$\boxed{= \frac{1}{\sqrt{2}} \langle \cos t - \operatorname{sen} t,\ \cos t + \operatorname{sen} t,\ 0 \rangle} \quad T(t)$$

- $T'(t) = \dfrac{1}{\sqrt{2}} \langle -\operatorname{sen} t - \cos t,\ \cos t - \operatorname{sen} t,\ 0 \rangle$

Ahora $\| T'(t) \| = \left| \dfrac{1}{\sqrt{2}} \right| \left[ [-(\operatorname{sen} t + \cos t)]^2 + (\cos t - \operatorname{sen} t)^2 \right]^{1/2}$

$= \dfrac{1}{\sqrt{2}} \left[ \operatorname{sen}^2 t + 2 \operatorname{sen} t \cos t + \cos^2 t + \cos^2 t - 2 \operatorname{sen} t \cos t + \operatorname{sen}^2 t \right]^{1/2}$

$$= \frac{1}{\sqrt{2}} \cdot (2)^{1/2}$$

$$= 1$$

Por lo tanto.

$$N(t) = \frac{T'(t)}{\|T'(t)\|}$$

$$\boxed{= \frac{1}{\sqrt{2}} \langle -\text{sen}t - \cos t \,,\ \cos t - \text{sen}t,\ 0 \rangle} \quad N(t)$$

- Recuerde que $B(t) = T(t) \times N(t)$

Nota: Si $u, v \in \mathbb{R}^3$ y $\alpha, \beta \in \mathbb{R}$ entonces

$$(\alpha \vec{u}) \times (\beta \vec{v}) = \alpha\beta (\vec{u} \times \vec{v})$$

Entonces

$$B(t) = \frac{1}{\sqrt{2}} \langle \cos t - \text{sen}t,\ \cos t + \text{sen}t,\ 0 \rangle \times \frac{1}{\sqrt{2}} \langle -\text{sen}t - \cos t,\ \cos t - \text{sen}t,\ 0 \rangle$$

$$= \frac{1}{2} \left[ \langle \cos t - \text{sen}t,\ \cos t + \text{sen}t,\ 0 \rangle \times \langle -\text{sen}t - \cos t,\ \cos t - \text{sen}t,\ 0 \rangle \right]$$

$$= \frac{1}{2} \begin{vmatrix} \vec{i} & \vec{j} & k \\ \cos t - \text{sen}t & \cos t + \text{sen}t & 0 \\ -\text{sen}t - \cos t & \cos t - \text{sen}t & 0 \end{vmatrix}$$

Desarrollar el determinante por la columna 3

$$B(t) = \frac{1}{2}\left[ K \begin{vmatrix} \cos t - \operatorname{sen} t & \cos t + \operatorname{sen} t \\ -\operatorname{sen} t - \cos t & \cos t - \operatorname{sen} t \end{vmatrix} \right]$$

$$= \frac{1}{2}\left( (\cos t - \operatorname{sen} t)^2 + (\cos t + \operatorname{sen} t)^2 \right) K$$

$$= \frac{1}{2}(2)K$$

$$= \vec{K}$$

$$\boxed{= \langle 0, 0, 1 \rangle} \quad B(t)$$

- Calculo de curvatura.

Recuerde que

$$K(t) = \frac{\| T'(t) \|}{\| c'(t) \|}$$

$$= \boxed{\frac{1}{\sqrt{2}\, e^t}} \quad K(t)$$