数字图像处理
Digital Image Processing

Image as A Function (6)

Discrete Sampling of A Function

Median Filtering

Most slides are courtesy of Juyong Zhang

The Median Filter

- Returns the median value of the pixels in a neighborhood
- Is non-linear
- Is similar to a uniform blurring filter which returns the mean value of the pixels in a neighborhood of a pixel
- Unlike a mean value filter the median tends to preserve step edges

4

Median Filter: General Definition

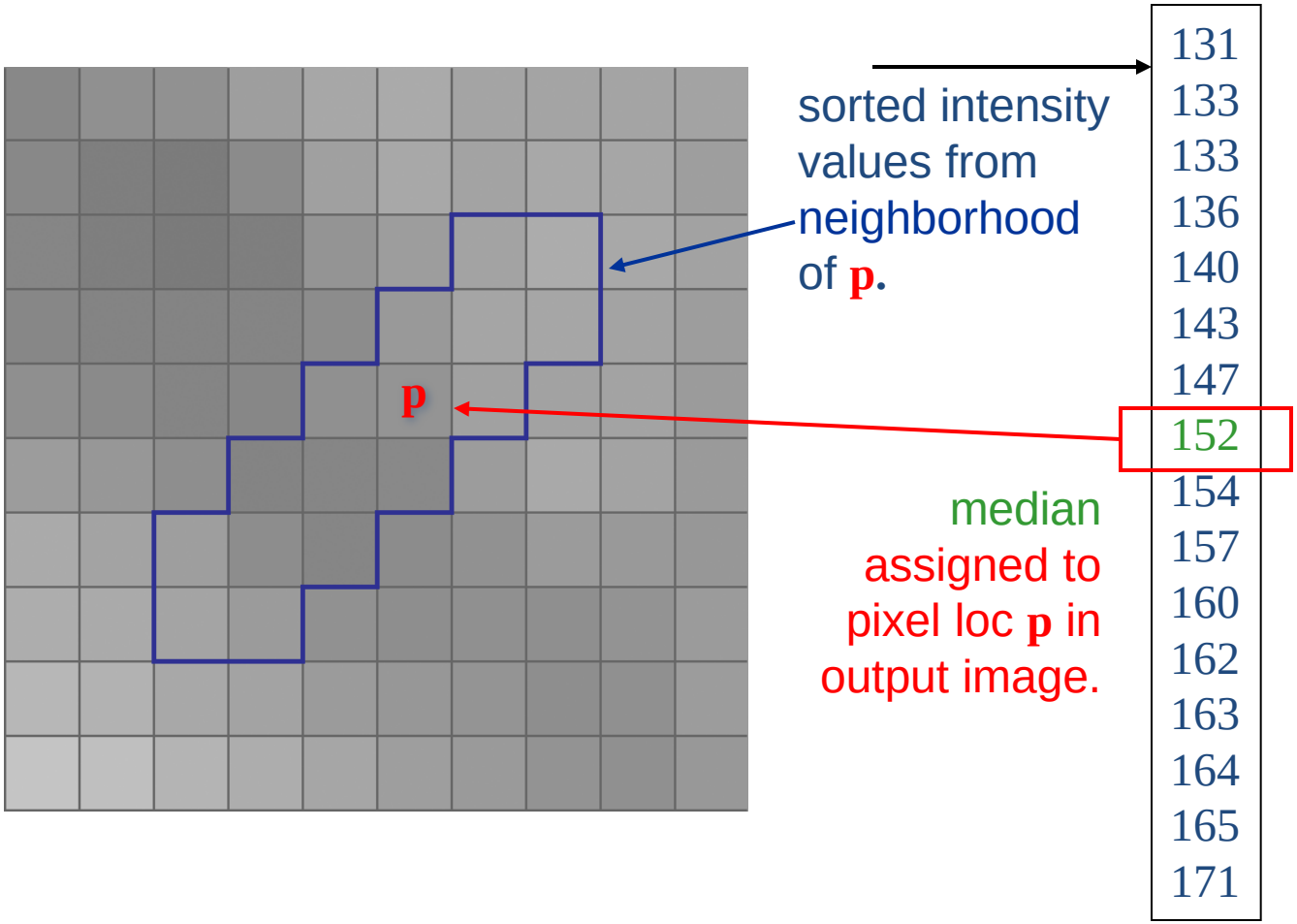
$$\mathrm{med}\{\mathbf{I},\mathbf{Z}\}(\mathbf{p}) = \underset{\mathbf{q}\in\mathrm{supp}(\mathbf{Z}+\mathbf{p})}{\mathrm{median}}\{\mathbf{I}(\mathbf{q})\}$$

This can be computed as follows:

1. Let **I** be a monochrome (1-band) image.
2. Let **Z** define a neighborhood of arbitrary shape.
3. At each pixel location, **p** = (*r*,*c*), in **I** …
4. … select the *n* pixels in the **Z**-neighborhood of **p**,
5. … sort the *n* pixels in the neighborhood of **p,** by value, into a list $L(j)$ for $j = 1,\ldots,n$.
6. The output value at **p** is $L(m)$, where $m = \lfloor \frac{n}{2} \rfloor + 1$

5

Median Filter: General Definition

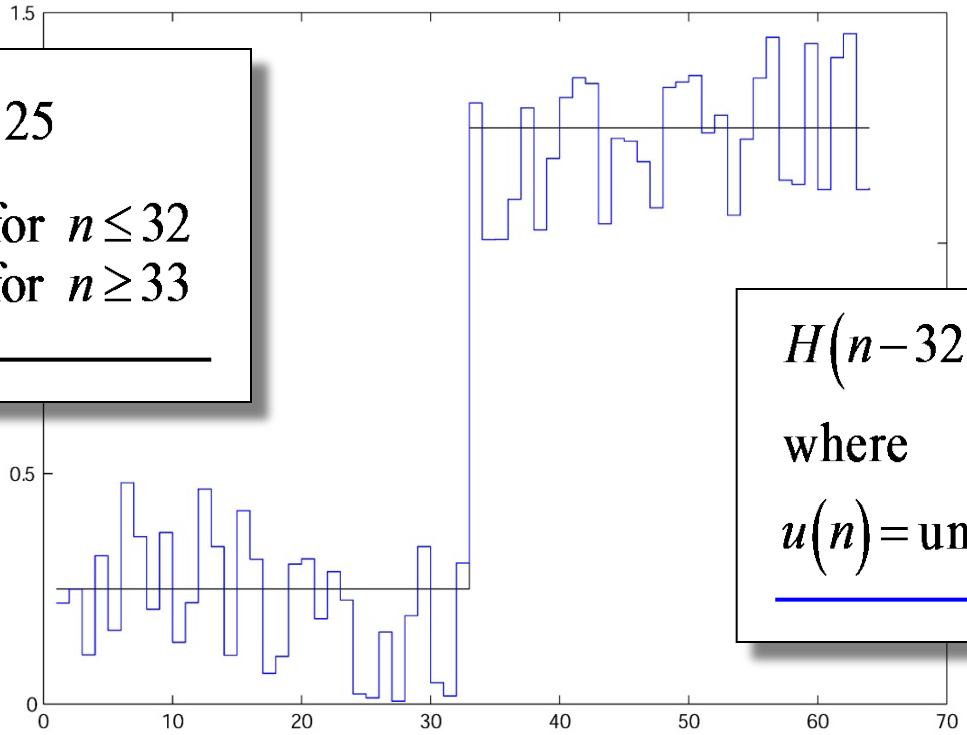


6

A Noisy Step Edge

$$H(n-32.5)+0.25$$

$$= \begin{cases} 0.25 & \text{for } n \le 32 \\ 1.25 & \text{for } n \ge 33 \end{cases}$$



$$H(n-32.5)+u(n)$$

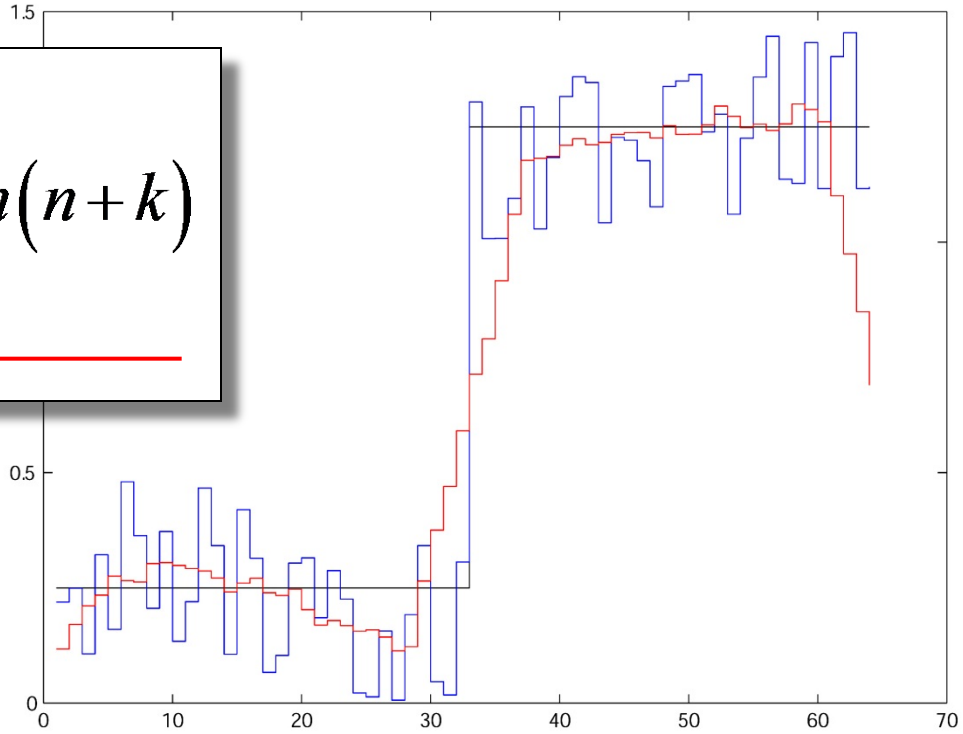
where

$$u(n) = \text{unif}(-0.25, 0.25)$$

7

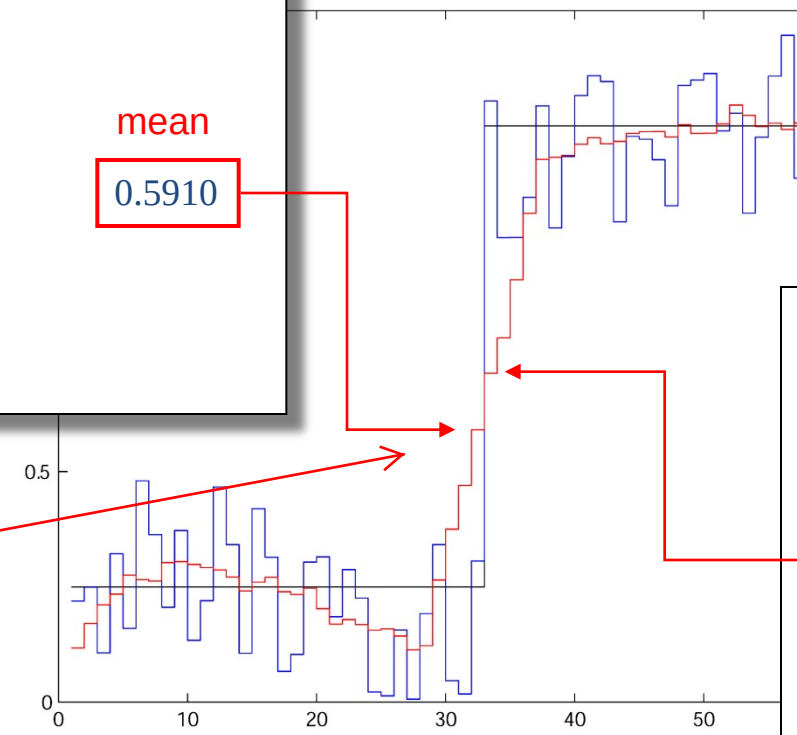
Blurred Noisy 1D Step Edge

$$h(n) = \frac{1}{9}\sum_{k=-4}^{4} h(n+k)$$



8

Blurred Noisy 1D Step Edge



J(32-4:32+4)=
0.1920
0.3416
0.0464
0.0177
0.3062
1.3043
1.0079
1.0082
1.0950

mean

0.5910

J(33-4:33+4)=
0.3416
0.0464
0.0177
0.3062
1.3043
1.0079
1.0082
1.0950
1.2935

mean

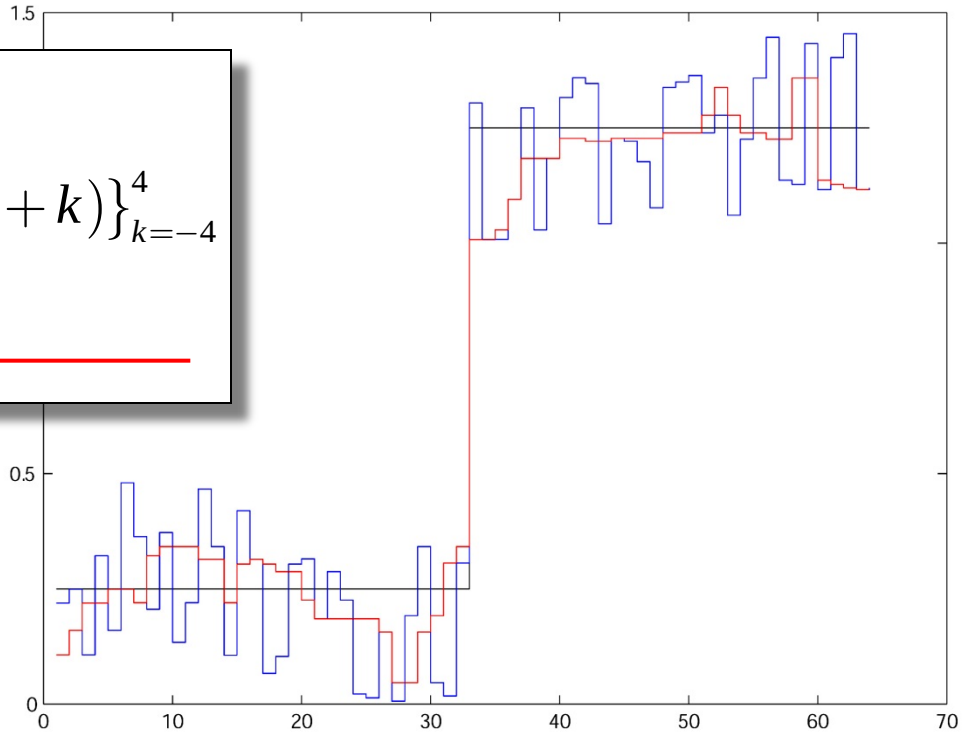
0.7134

注意：阶跃点在32.5，计算离散边缘处均值可以32为中点取邻域，得0.5910；或以33为中点取，得0.7134

9

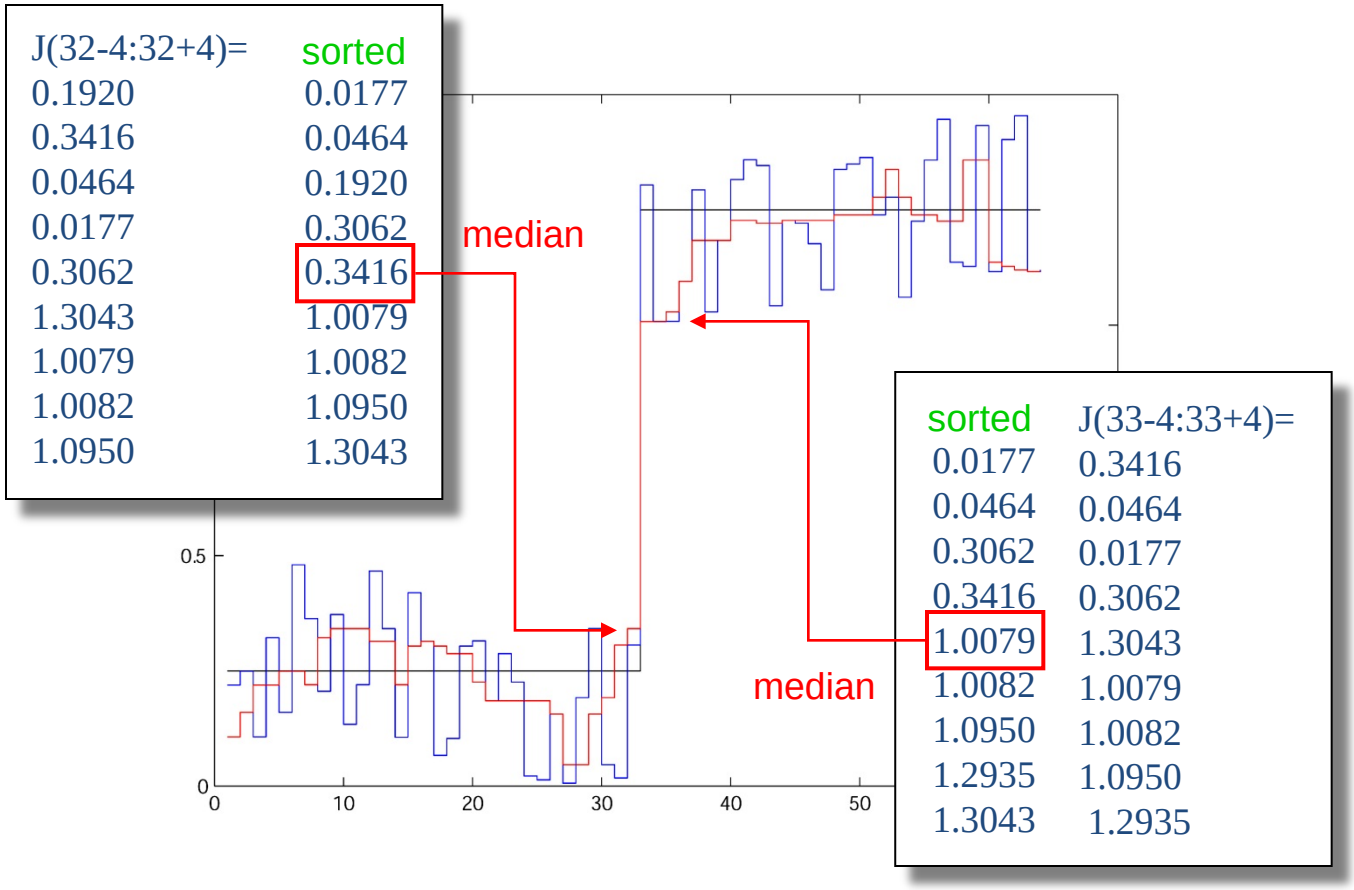
Median Filtered Noisy 1D Step Edge

$$h(n) = \text{med}\{h(n+k)\}_{k=-4}^{4}$$



10

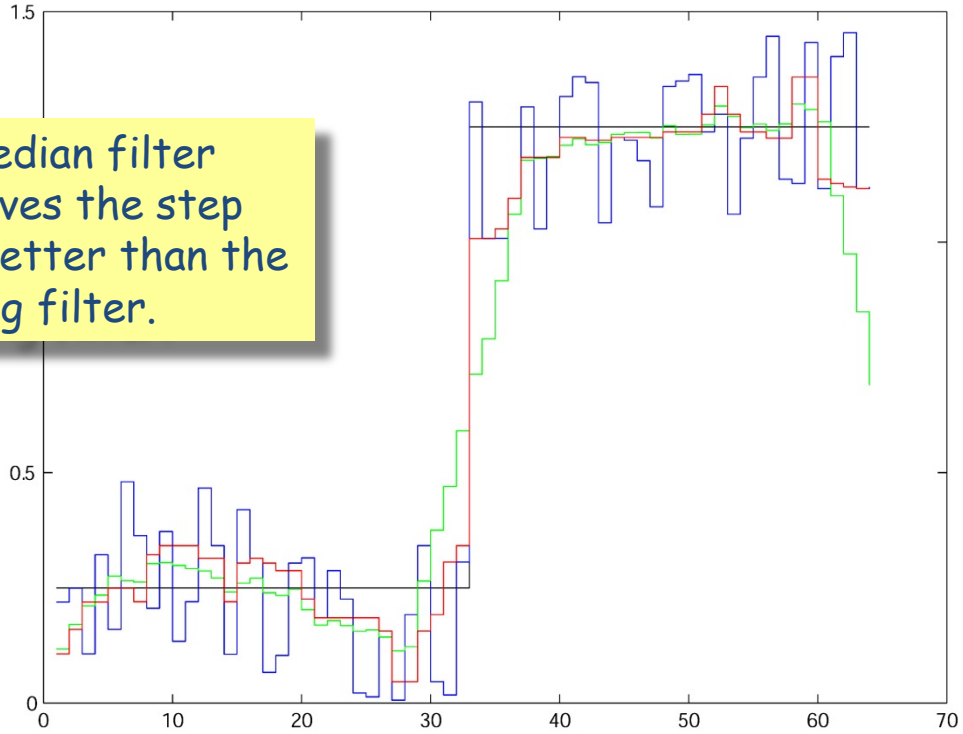
Median Filtered Noisy 1D Step Edge



11

Median vs. Blurred

The median filter preserves the step edge better than the blurring filter.



median

blurred

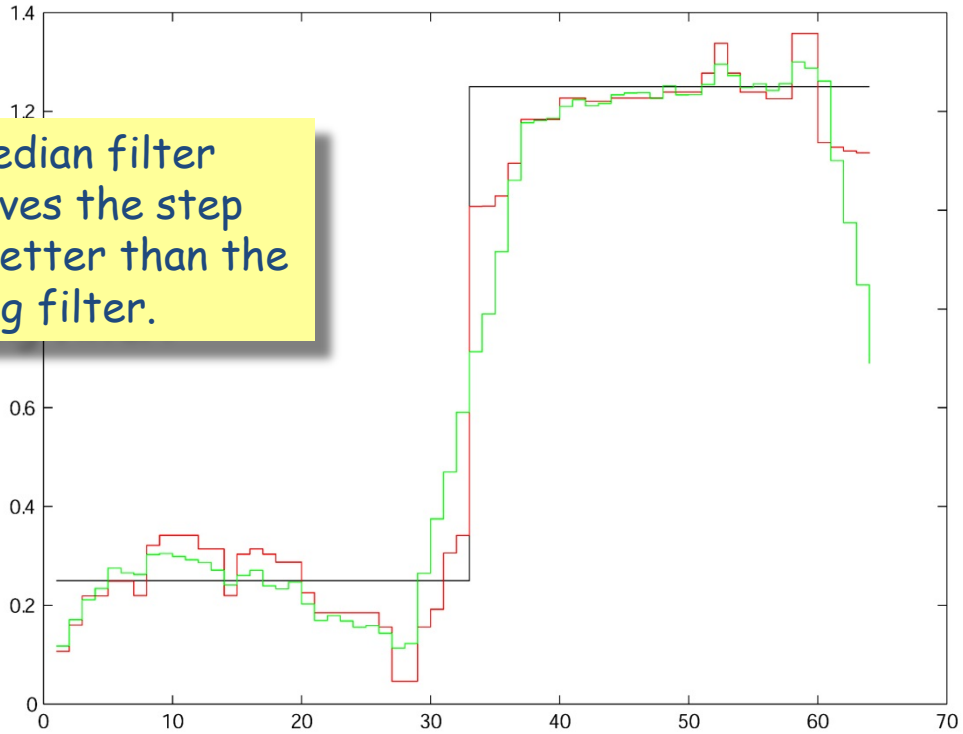
noisy

step

12

Median vs. Blurred

The median filter preserves the step edge better than the blurring filter.



blurred

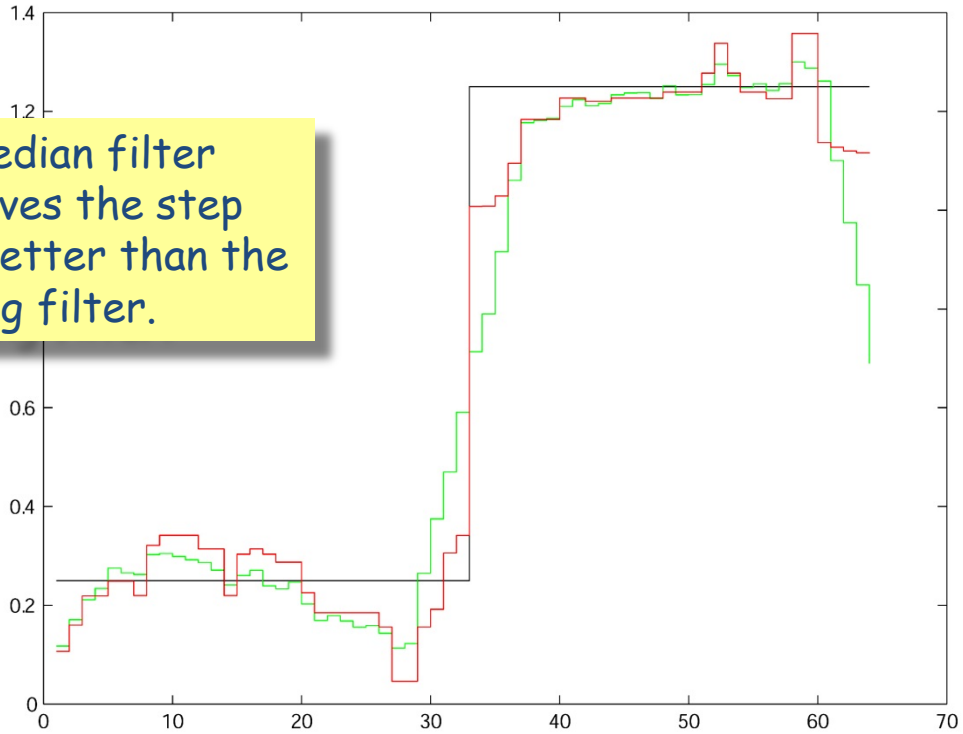
median

step

13

Median vs. Blurred

The median filter preserves the step edge better than the blurring filter.



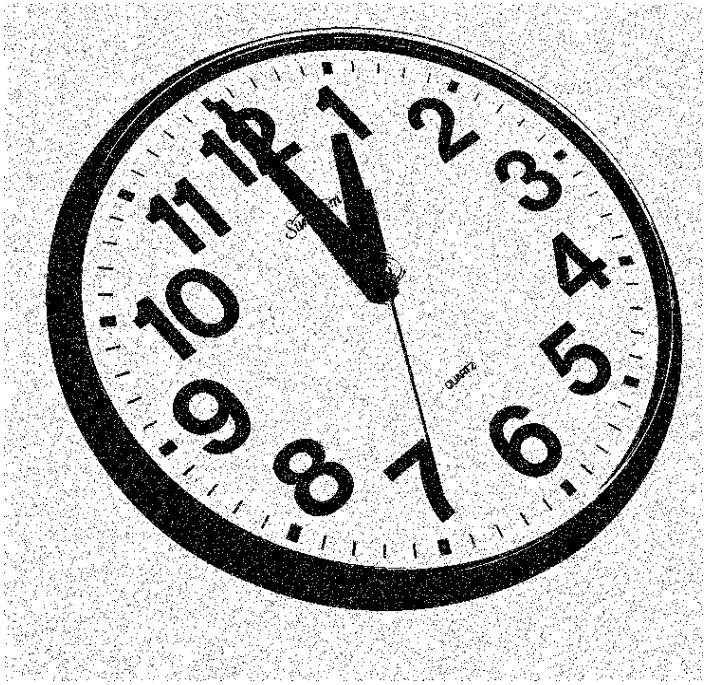
median

blurred

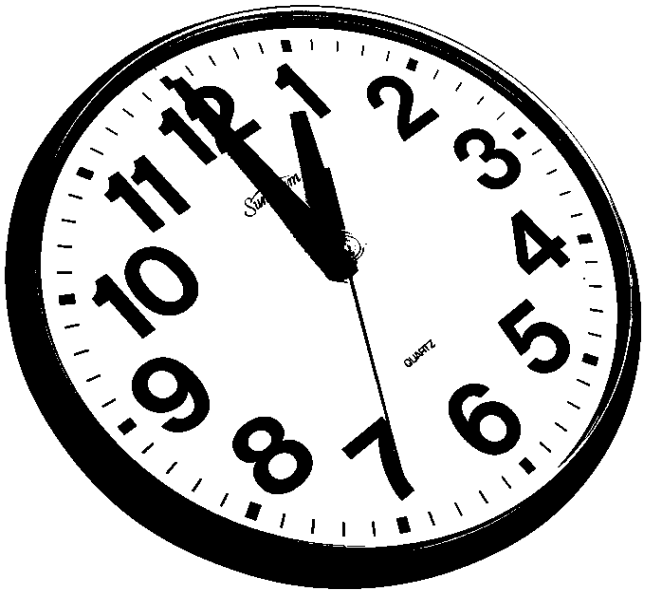
step

14

Median Filtering of Binary Images



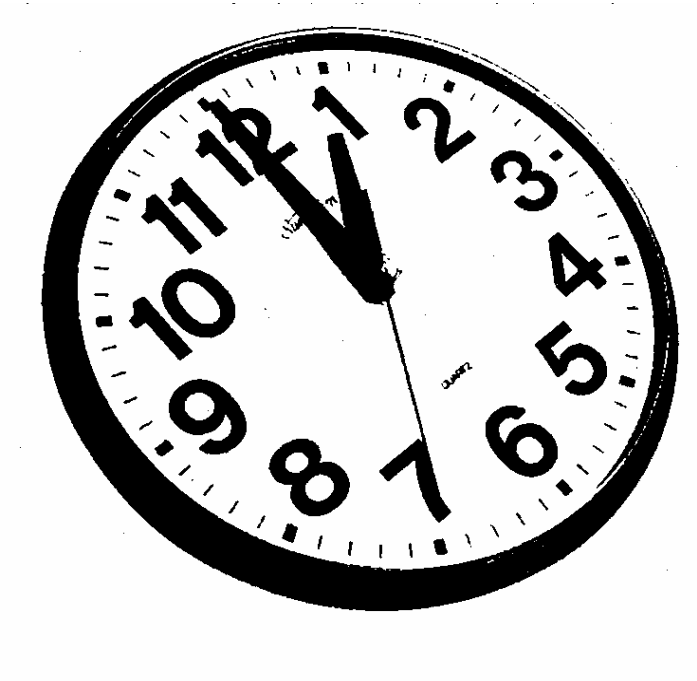
Noisy



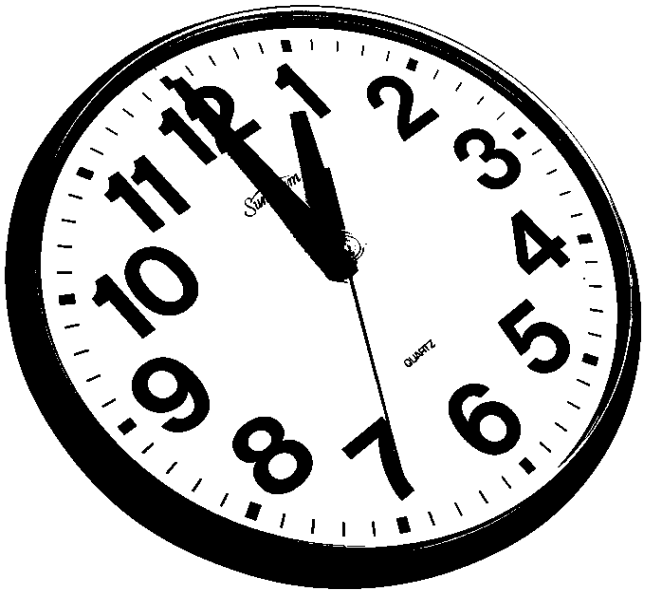
Original

15

Median Filtering of Binary Images



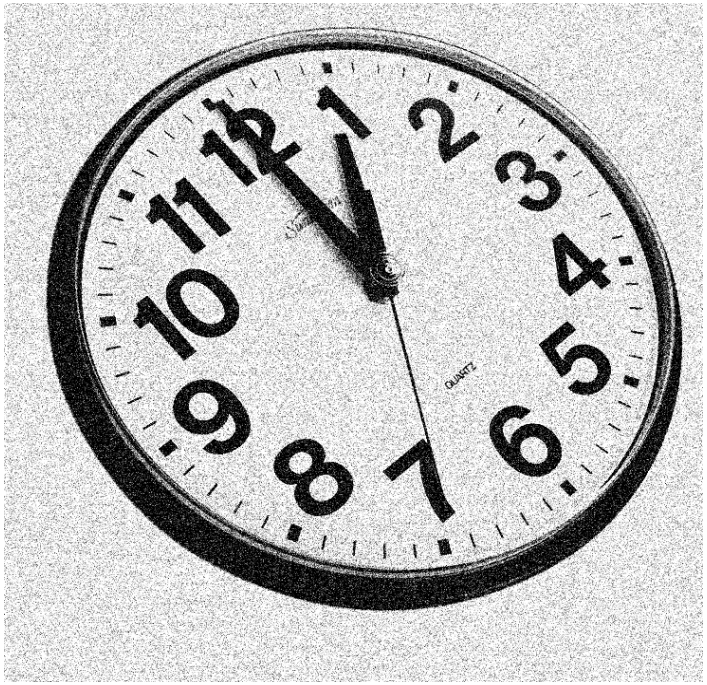
Median Filtered Noisy



Original

16

Filtering of Grayscale Images

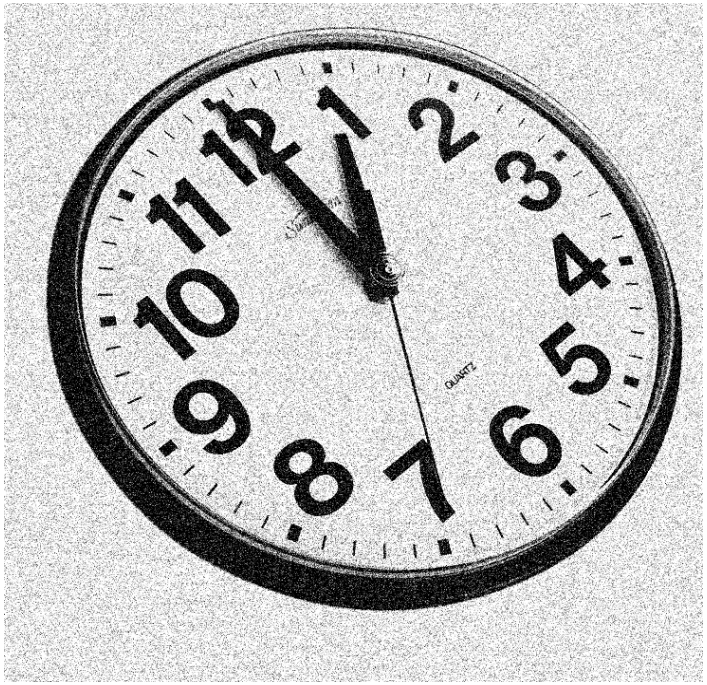


Noisy

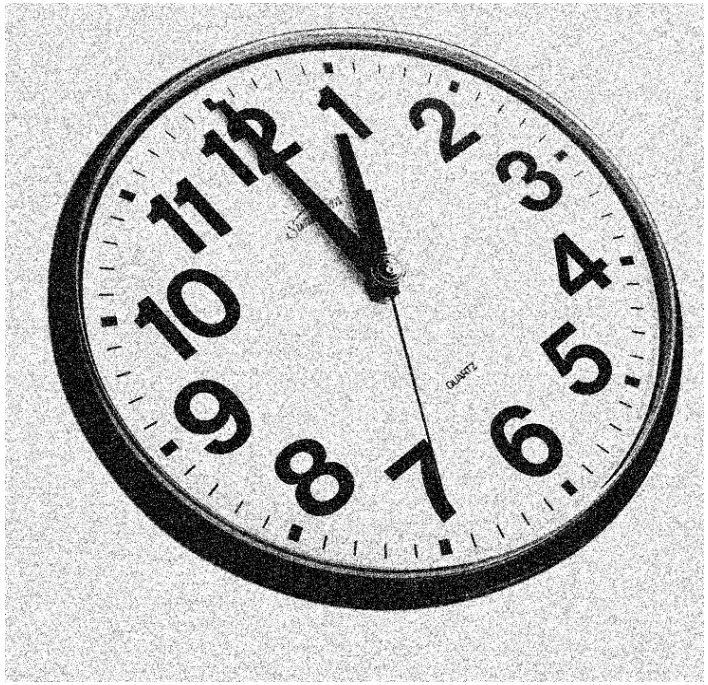
Original

17

Filtering of Grayscale Images



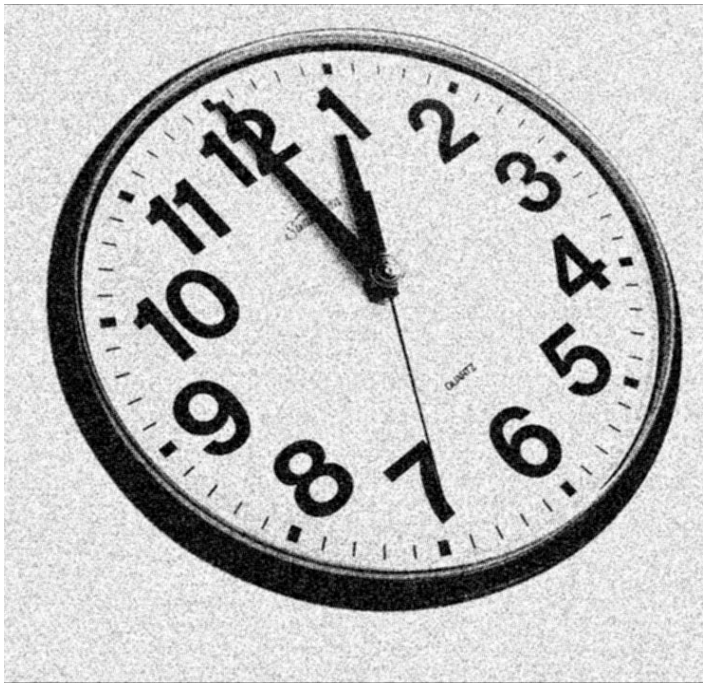
Noisy



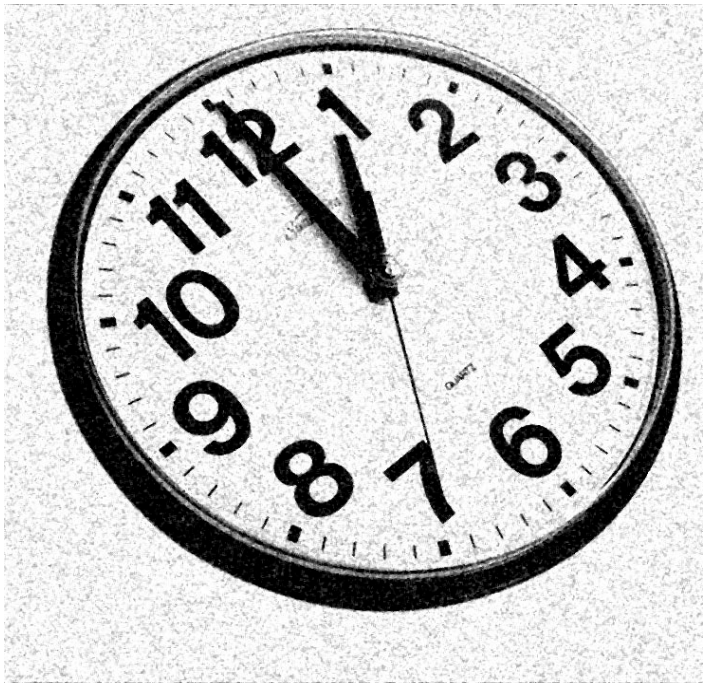
Noisy

18

Filtering of Grayscale Images



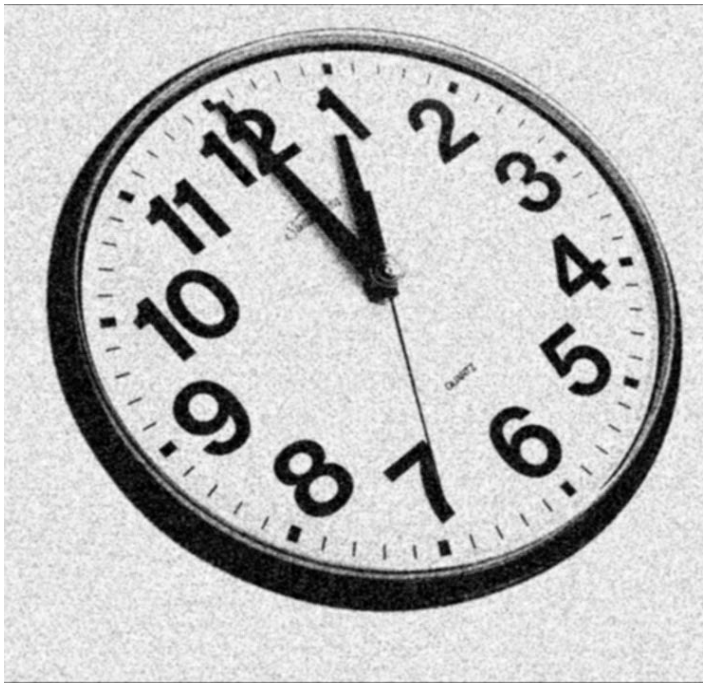
3x3-blur x 1



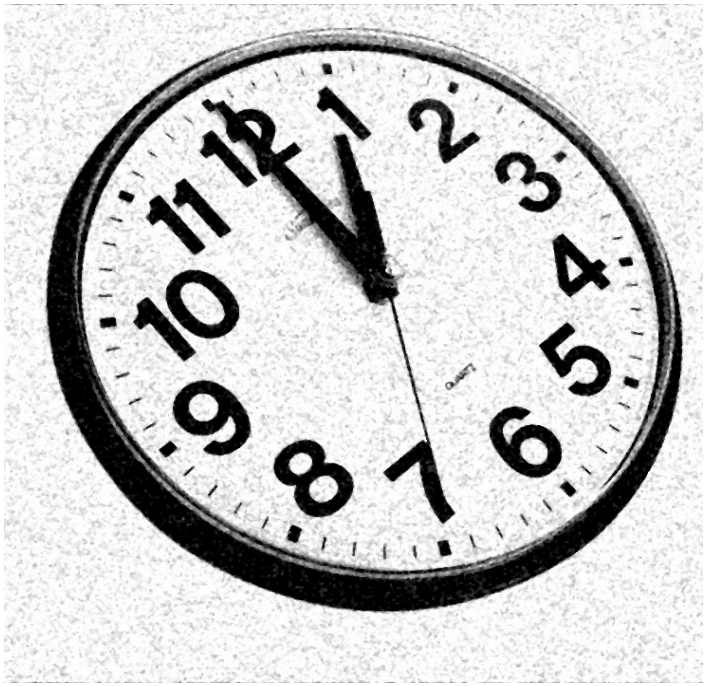
3x3-median x 1

19

Filtering of Grayscale Images



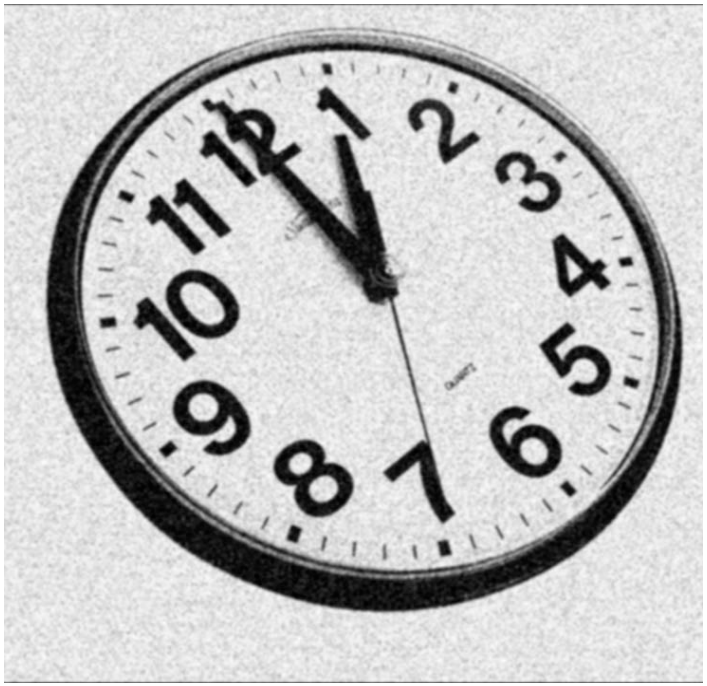
3x3-blur x 2



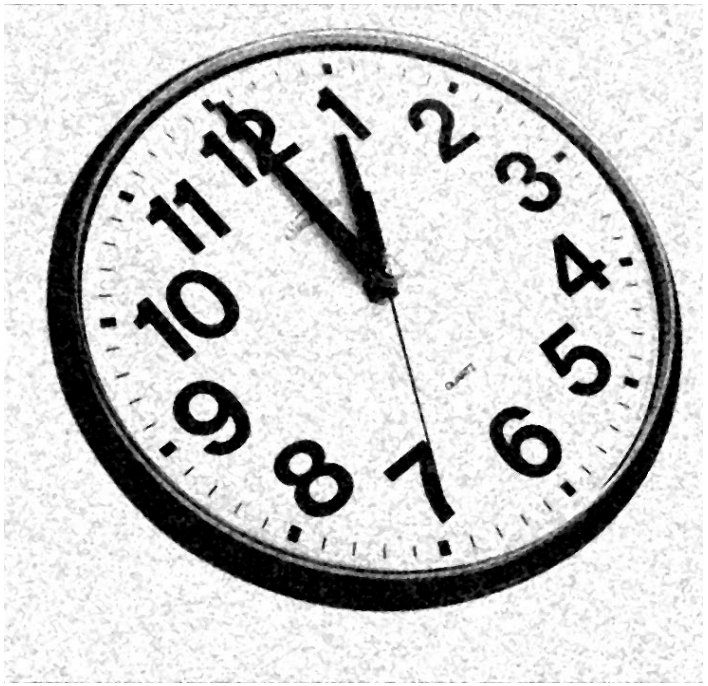
3x3-median x 2

20

Filtering of Grayscale Images



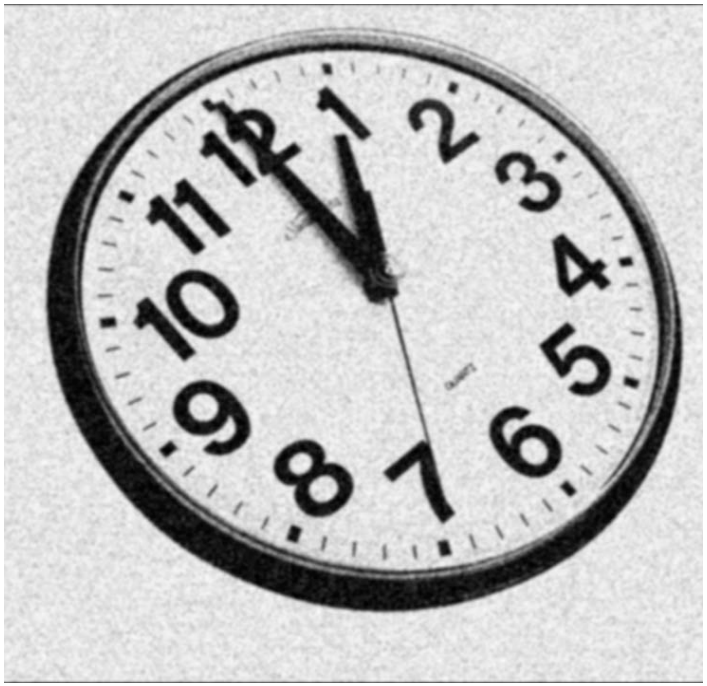
3x3-blur x 3



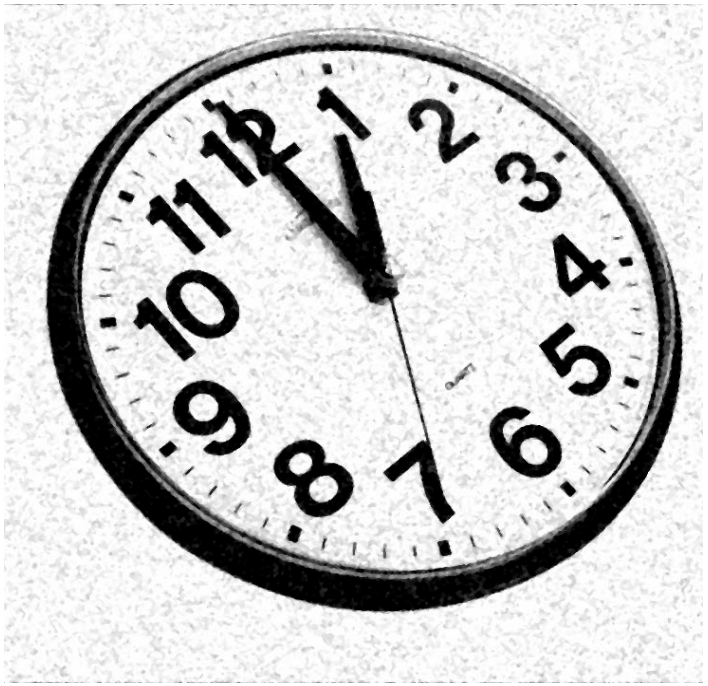
3x3-median x 3

21

Filtering of Grayscale Images



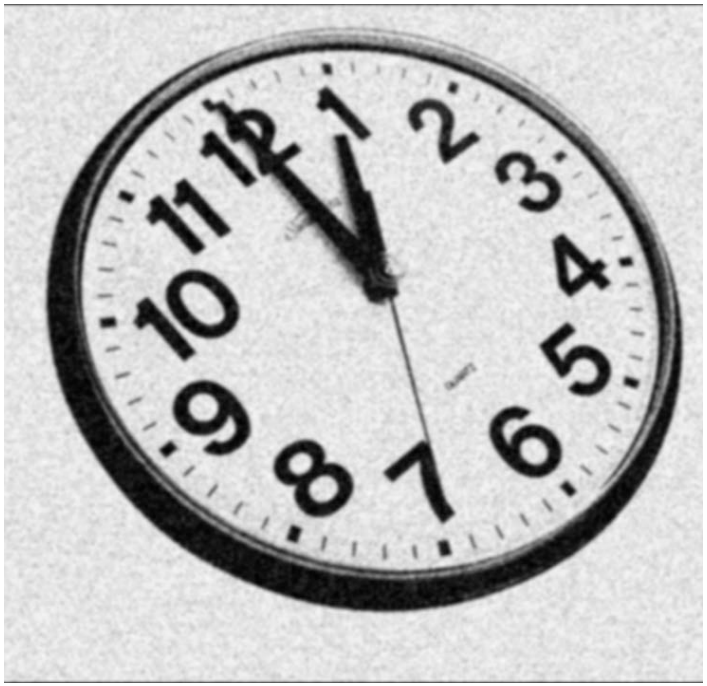
3x3-blur x 4



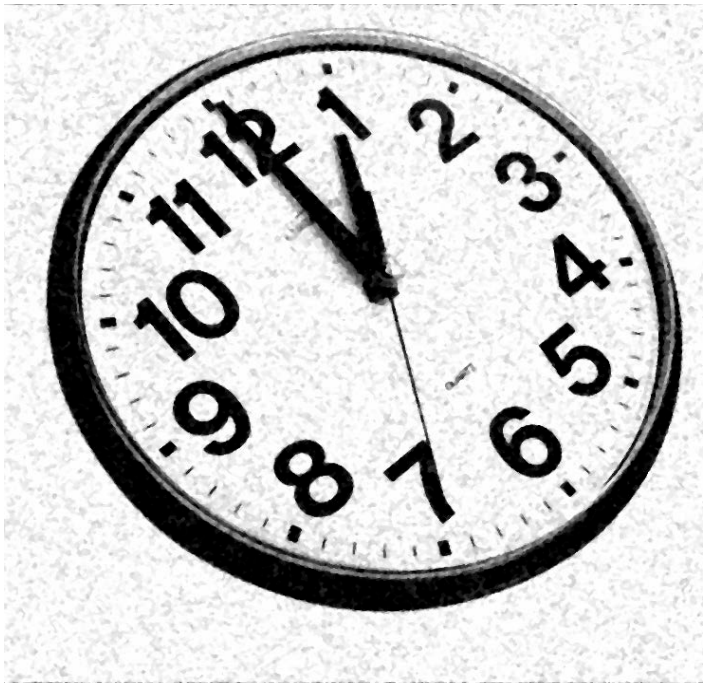
3x3-median x 4

22

Filtering of Grayscale Images



3x3-blur x 5

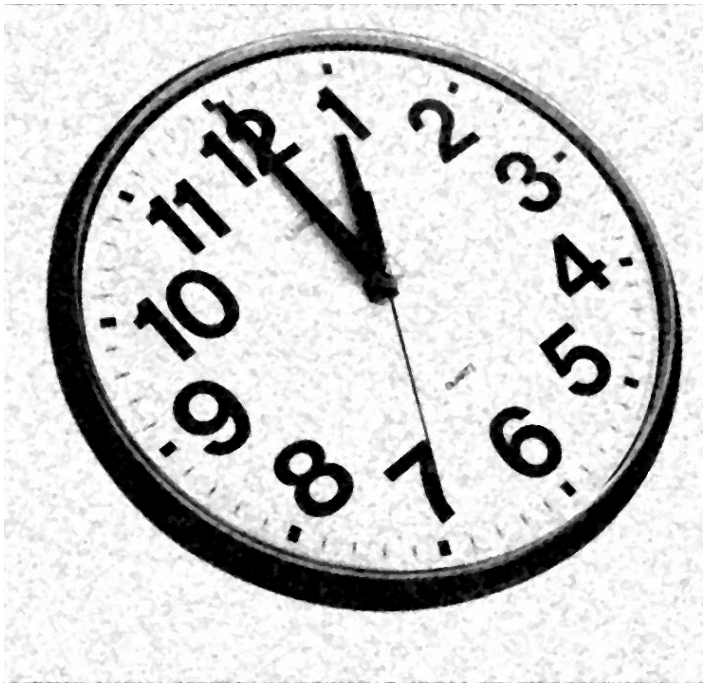


3x3-median x 5

23

Filtering of Grayscale Images

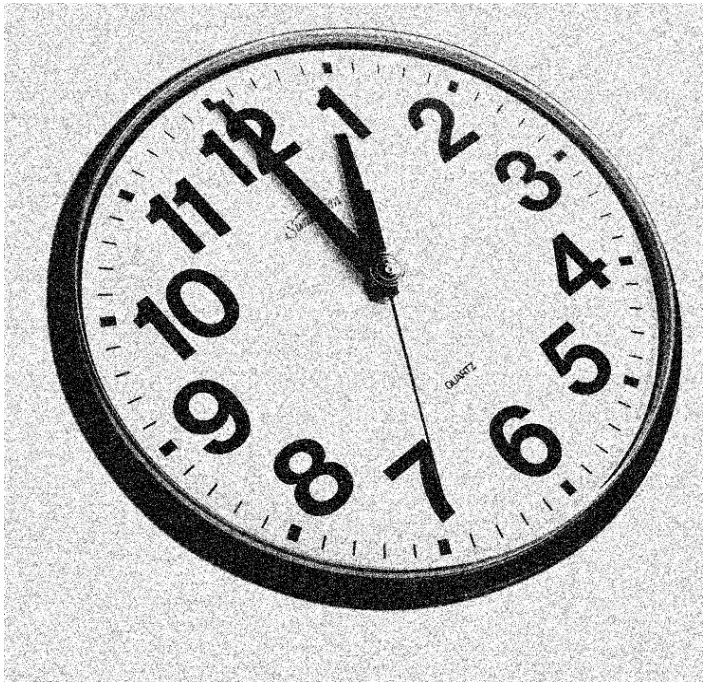
3x3-blur x 10



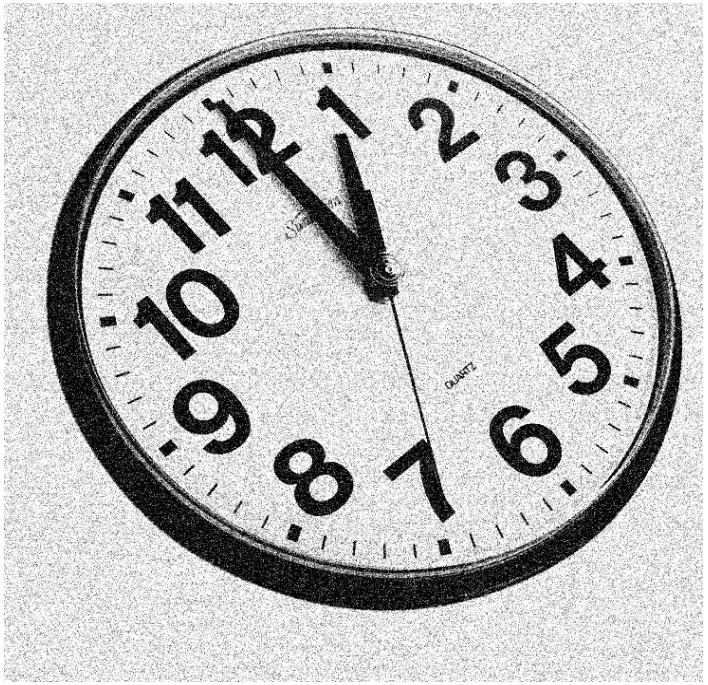
3x3-median x 10

24

Filtering of Grayscale Images



Noisy



Noisy

25

Limit and Root Images

Fact: if you repeatedly filter an image with the same blurring filter or median filter, eventually the output does not change. That is, let

$$\mathbf{I}\Big[*\mathbf{h}\Big]^k \equiv \Big(\big(\big(\mathbf{I}*\mathbf{h}\big)*\mathbf{h}\big)\cdots*\mathbf{h}\Big), \quad k \text{ times, and}$$

$$\mathbf{I}\Big[\operatorname{med}\mathbf{Z}\Big]^k \equiv \Big(\big(\big(\mathbf{I}\ \operatorname{med}\mathbf{Z}\big)\ \operatorname{med}\mathbf{Z}\big)\cdots\operatorname{med}\mathbf{Z}\Big), \quad k \text{ times.}$$

Then

$$\lim_{k\to\infty}\mathbf{I}\Big[*\mathbf{h}\Big]^k = \mathbf{I}\Big[*\mathbf{h}\Big]^n = \mathbf{I}_0, \quad \text{and}$$

$$\lim_{k\to\infty}\mathbf{I}\Big[\operatorname{med}\mathbf{Z}\Big]^k = \mathbf{I}\Big[\operatorname{med}\mathbf{Z}\Big]^m = \mathbf{I}_r,$$

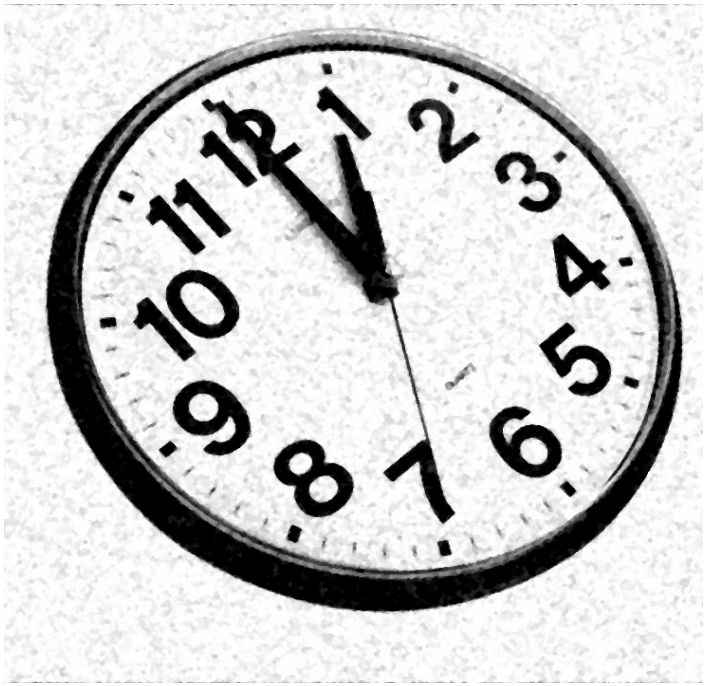
where n and m are integers ($< \infty$) , $\mathbf{I}_0$ is a single-valued image and $\mathbf{I}_r$ is called the *median root* of $\mathbf{I}$.

灰度值为单一常数的图像

26

Limit and Root Images

3x3-blur x 10

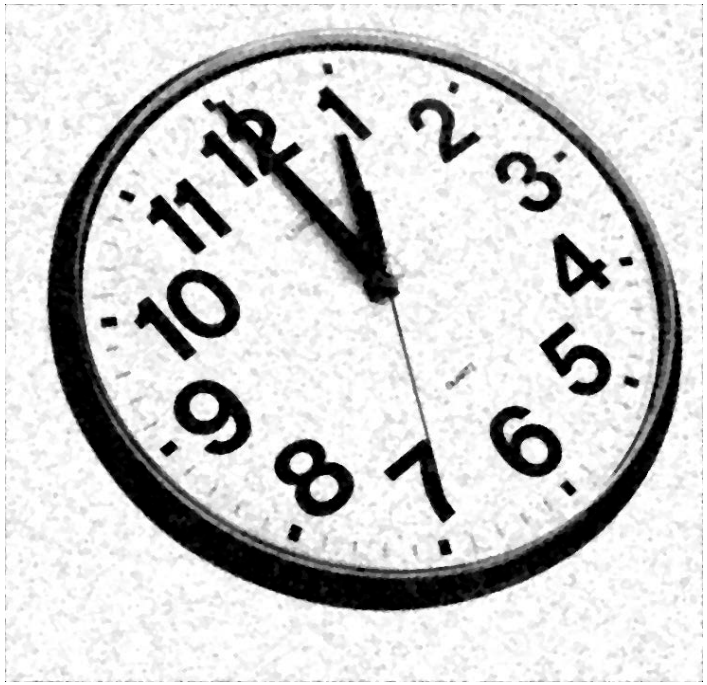


3x3-median x 10

27

Limit and Root Images

3×3-blur $\times n \rightarrow \infty$



3x3-median root

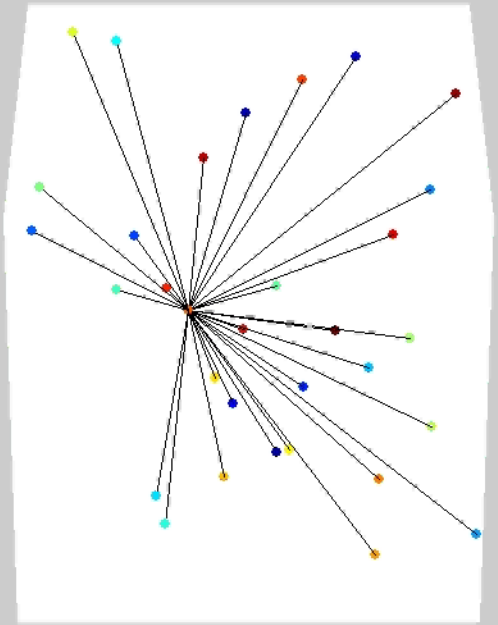
28

Vector Median Filter

A vector median filter selects from among a set of vectors, the one vector that is closest to all the others.

That is, if S is a set of vectors, in $\mathbb{F}^n$ the median, $\overline{\mathbf{v}}$, is

$$\overline{\mathbf{v}} = \arg\min_{k \neq j} \left\{ \left\| \mathbf{v}_k - \mathbf{v}_j \right\| \;\middle|\; \mathbf{v}_k, \mathbf{v}_j \in S \right\}.$$



◁$\mathbb{F}^n$ is an n-dimensional linear vector space over the field, $\mathbb{F}$.)

选择一个和其他向量距离和最小的向量作为中值滤波的输出

30

向量中值滤波算法:

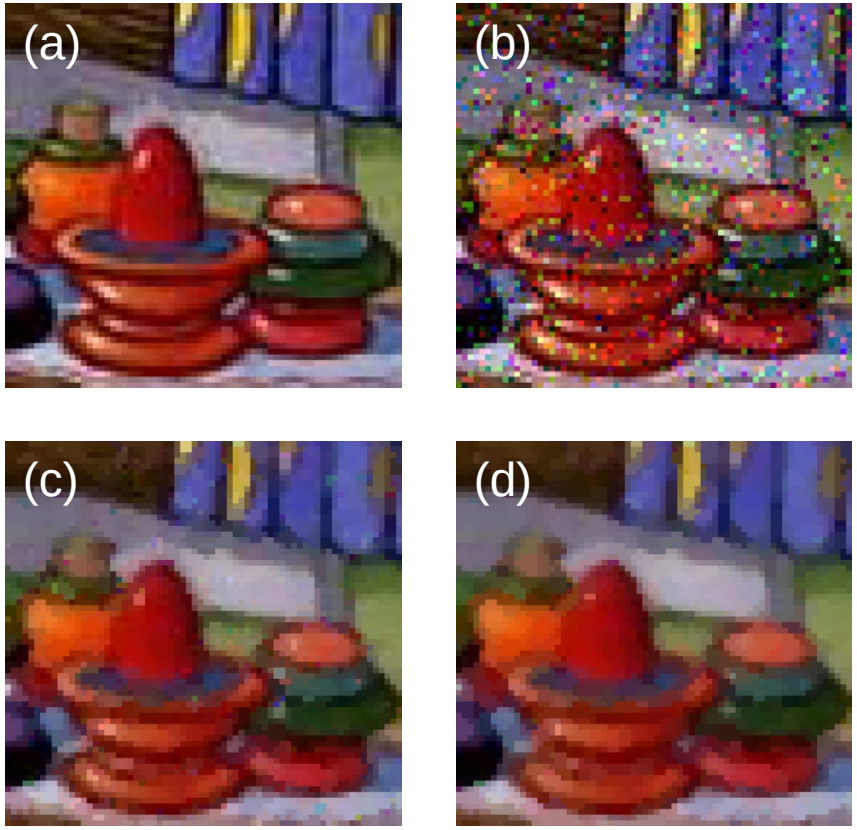
设$\mathbf{x}_1,\mathbf{x}_2,\cdots,\mathbf{x}_N \in R^k$，是对应滤波器窗口的输入向量，计算每个向量$\mathbf{x}_i$到其他向量的距离和:

$$s(\mathbf{x}_i)=\sum_{j=1,\,j\neq i}^{N}\left\|\mathbf{x}_i-\mathbf{x}_j\right\|_L$$

其中$\|\bullet\|_L$为 1-或者 2-范数，寻找$s(\mathbf{x}_i)$ 的最小值，并作为为向量中值滤波的输出

Color Median Filter

If we let $\mathbb{F}^n = \mathbb{R}^3$ then the vector median can be used as a color median filter.



(a) original image
(b) image (a) with sparse noise
(c) image (b) color median filtered
(d) image (c) color median filtered

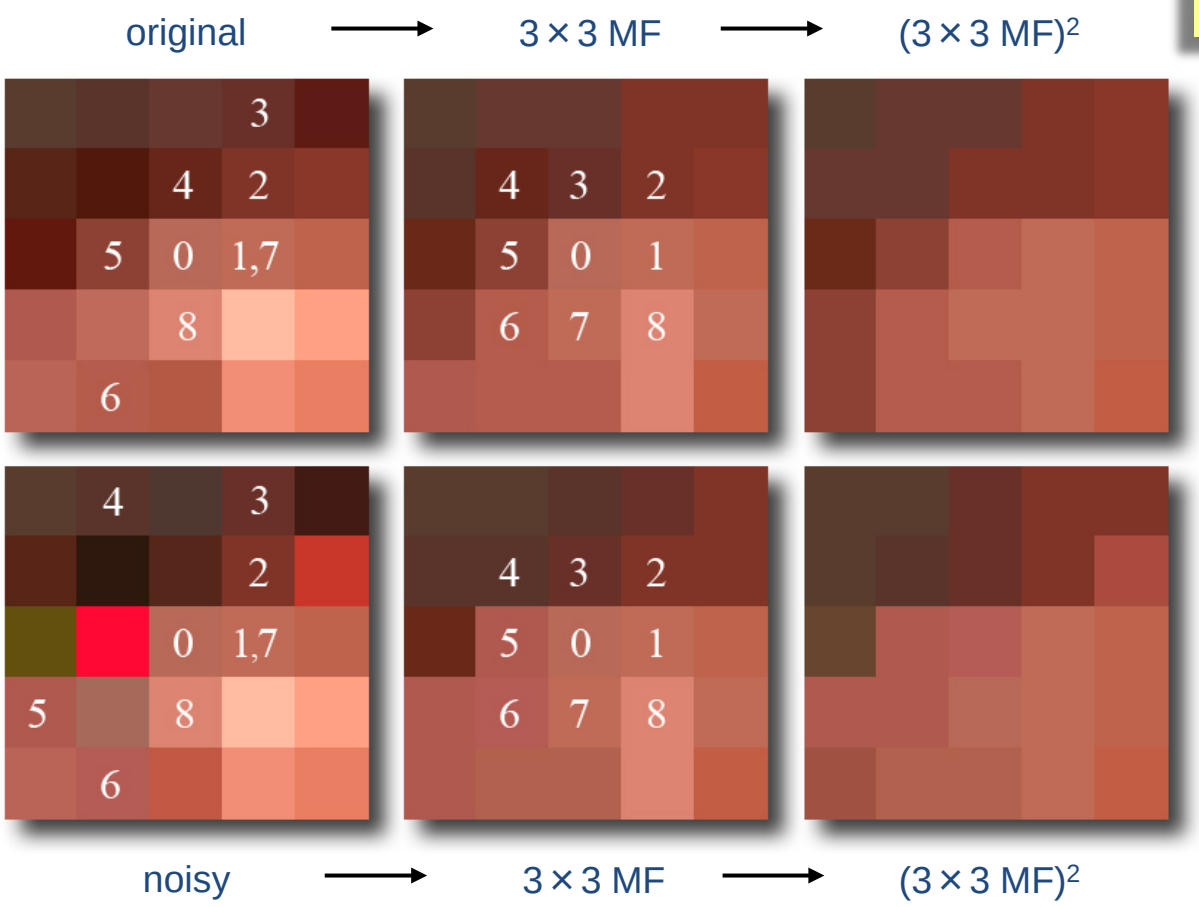
Median filter performed on 3×3 nbhd.

original image: Jim Woodring
www.jimwoodring.com

31

Color Median Filter

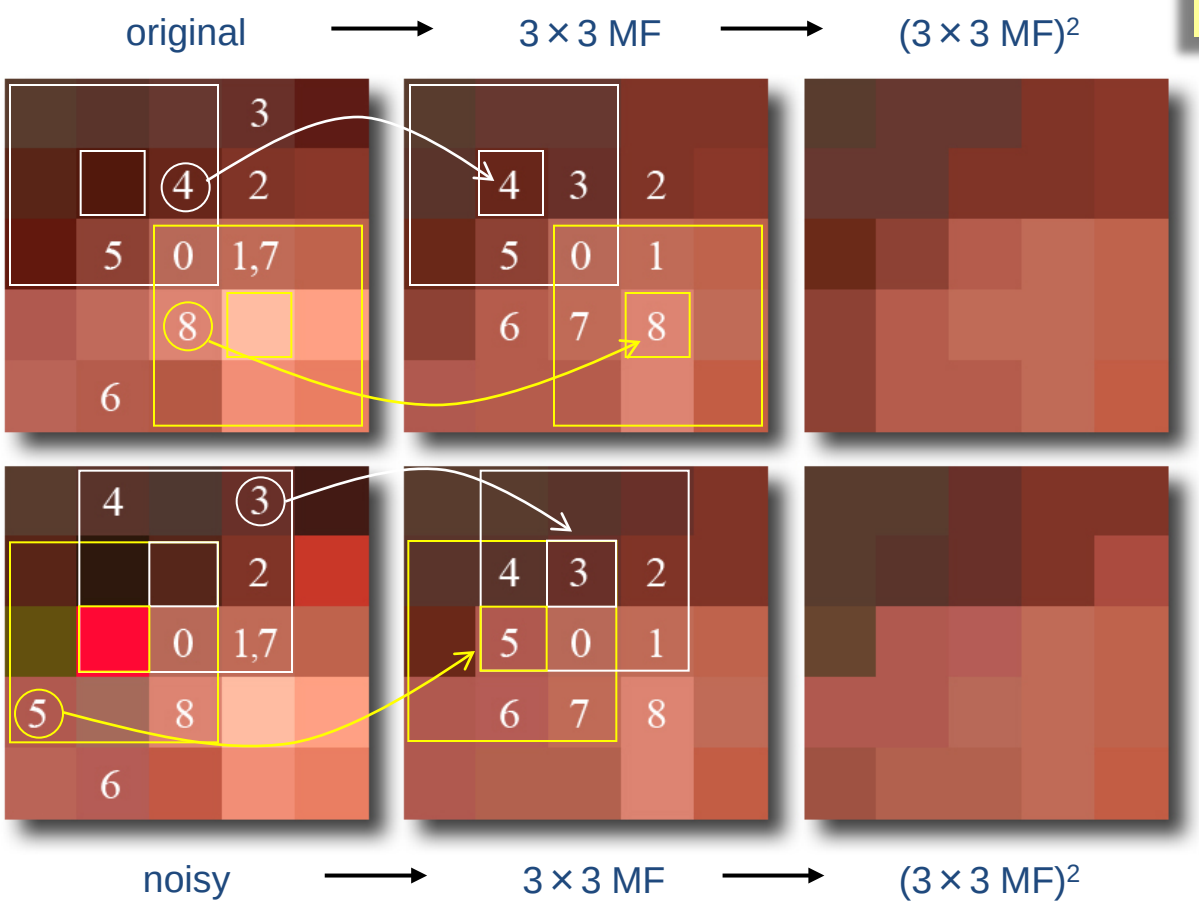
The output color at (r,c) is always selected from a nbhd of (r,c) in the input image.



32

Color Median Filter

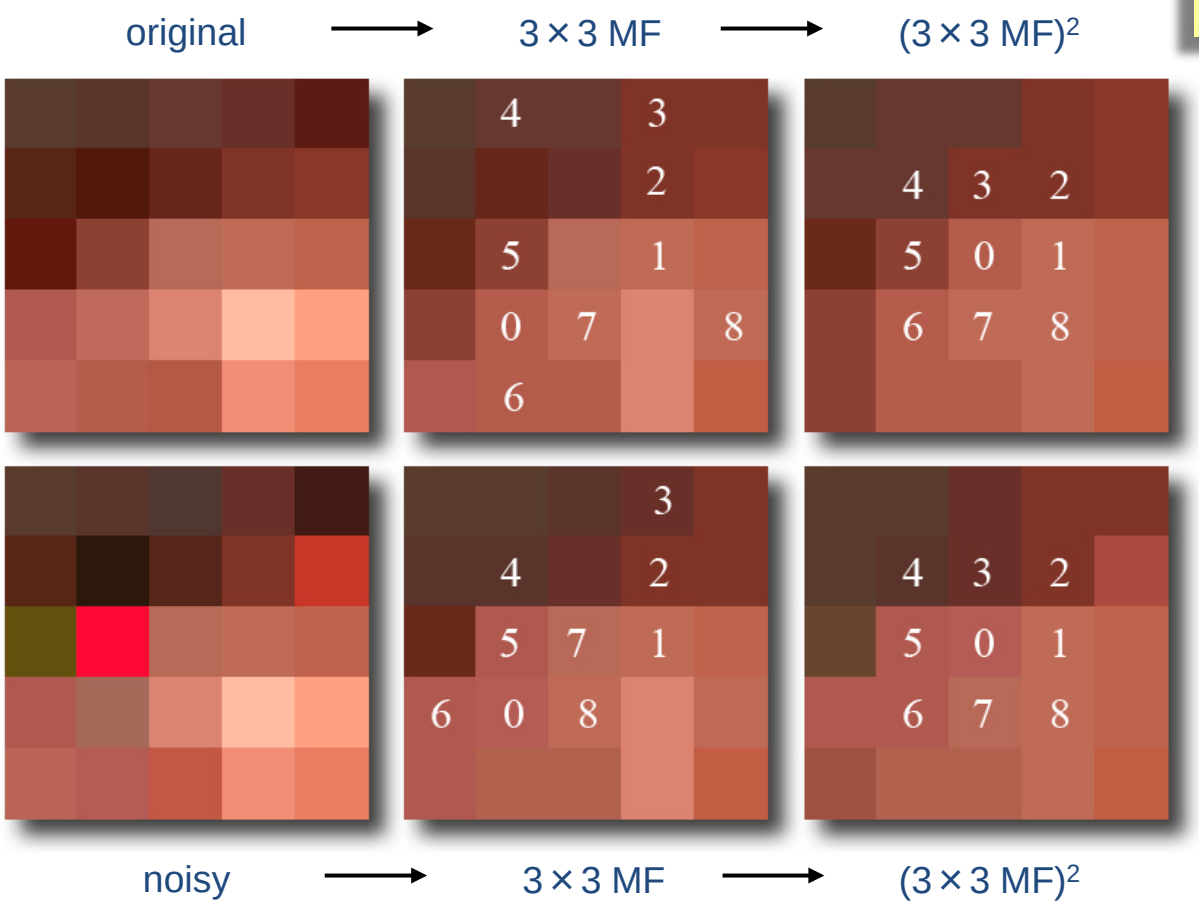
The output color at (r,c) is always selected from a nbhd of (r,c) in the input image.



33

Color Median Filter

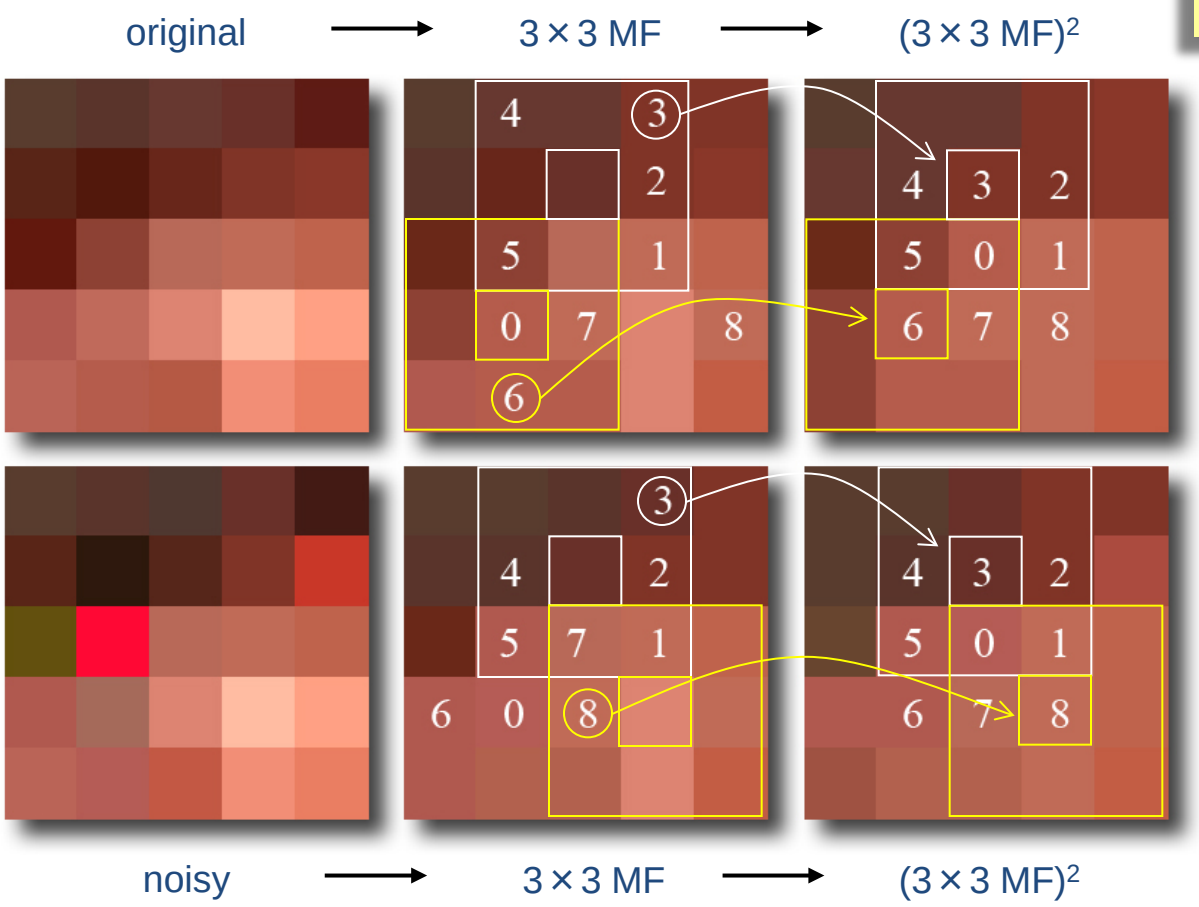
The output color at (r,c) is always selected from a nbhd of (r,c) in the input image.



34

Color Median Filter

The output color at (r,c) is always selected from a nbhd of (r,c) in the input image.



35

Color Median Filter

Jim Woodring – A Warm Shoulder

www.jimwoodring.com

Sparse noise, 32% coverage in each band

36

Color Median Filter

3 × 3 color median filter applied once

3 × 3 color median filter applied twice

original image: Jim Woodring
www.jimwoodring.com

37

Color Median Filter

Sparse noise, 32% coverage in each band

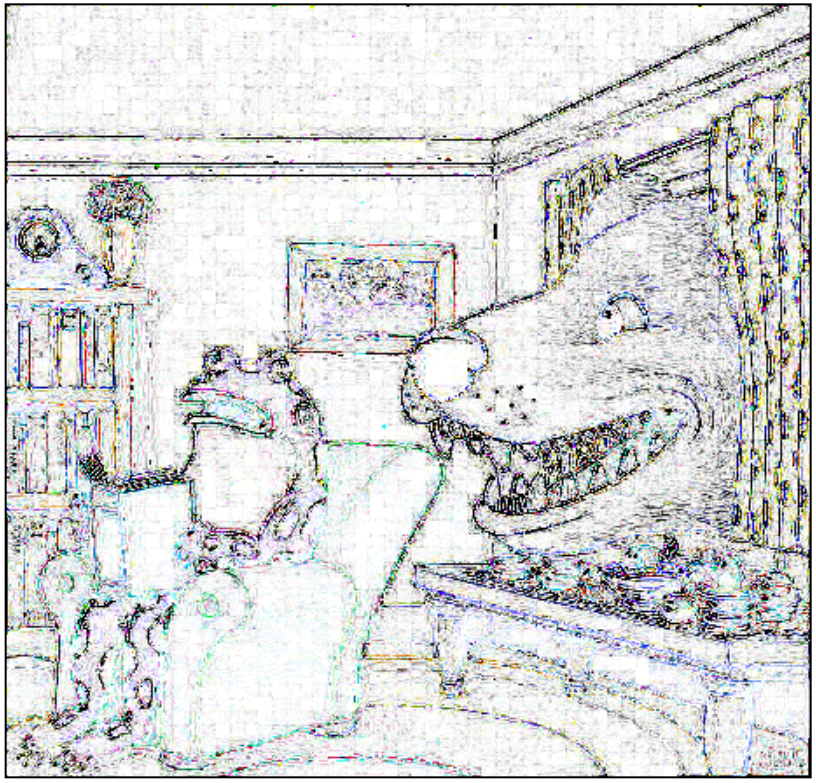
Jim Woodring – A Warm Shoulder

www.jimwoodring.com

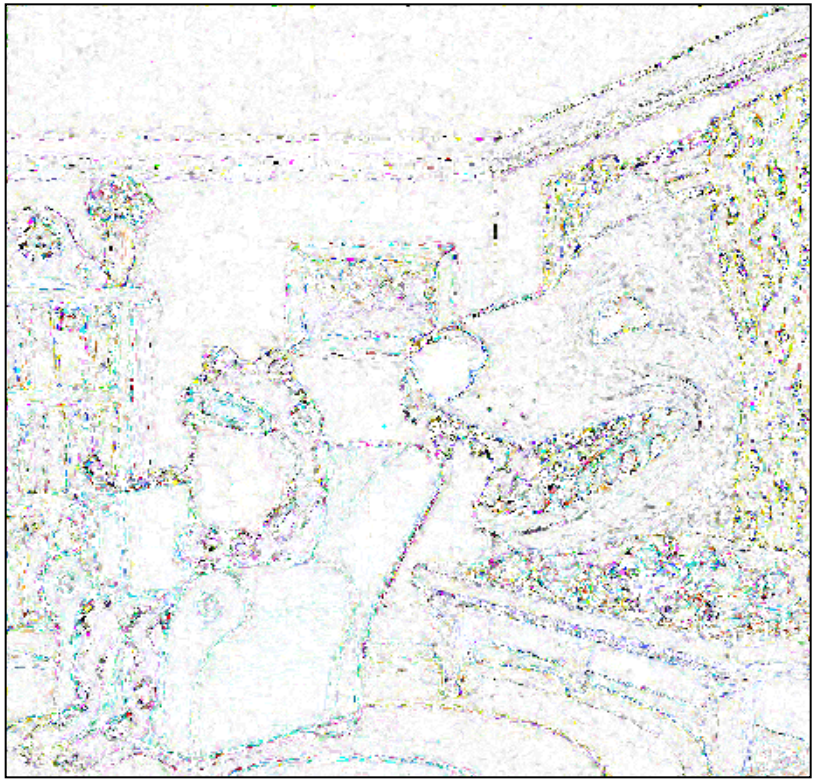
38

Color Median Filter

Absolute differences displayed as negatives to enhance visibility



(3 × 3 CMF^2 of noisy) – original



(3 × 3 CMF^2 of noisy) – (3 × 3 CMF^2 of original)

original image: Jim Woodring
www.jimwoodring.com

39

CMF vs. Standard Median on Individual Bands

A color median filter has to compute the distances between all the color vectors in the neighborhood of each pixel. That's expensive computationally.

Q: Why not simply take the 1-band median of each color band individually?

A: The result at a pixel could be a color that did not exist in the pixel's neighborhood in the input image. The result is not the median of the colors – it is the median of the intensities of each color band treated independently.

Q: Is that a problem?

A: Maybe. Maybe not. It depends on the application. It may make little difference visually. If the colors need to be preserved, it could be problematic.

40

CMF vs. Standard Median on Individual Bands

Jim Woodring – A Warm Shoulder

www.jimwoodring.com

Sparse noise, 32% coverage in each band

41

CMF vs. Standard Median on Individual Bands

3 × 3 color median filter applied once

3 × 3 color median filter applied twice

original image: Jim Woodring
www.jimwoodring.com

42

CMF vs. Standard Median on Individual Bands

3 × 3 median filter applied to each band once

3 × 3 median filter applied to each band twice

43

CMF vs. Standard Median on Individual Bands

Sparse noise, 32% coverage in each band

Jim Woodring – A Warm Shoulder

44

Thanks!