Lógico Minimal

$$\neg A =_{def} A \to \bot$$

$$\frac{}{A\to B,\ B\to\bot,\ A \vdash_m A}\ \text{Hip}$$

$$\frac{}{A\to B,\ B\to\bot,\ A \vdash_m B}\ \to L$$

$$\frac{}{A\to B,\ B\to\bot,\ A \vdash_m \bot}\ \to L$$

$$\frac{}{A\to B,\ B\to\bot \vdash_m A\to\bot}\ \to R$$

$$\frac{}{A\to B,\ \neg B \vdash_m \neg A}\ \text{Def}\ \neg$$

$$\frac{}{A\to B \vdash_m \neg B\to\neg A}\ \to R$$

$$\frac{}{\vdash_m (A\to B)\to(\neg B\to\neg A)}\ \to R$$

⇐ Teoremas

$$\frac{}{A\to\bot,\ \neg B,\ A \vdash_m A}\ \text{Hip}$$

$$\frac{}{A\to\bot,\ \neg B,\ A \vdash_m \bot}\ \to I$$

$$\frac{}{\neg A,\ \neg B,\ A \vdash_m \bot}\ \text{Def}\ \neg$$

Análogo.

$$\frac{}{\neg A,\ \neg B,\ B \vdash_m \bot}$$

$$\frac{}{\neg A,\ \neg B,\ A\vee B \vdash_m \bot}\ \vee I$$

$$\frac{}{\neg A\wedge\neg B,\ A\vee B \vdash_m \bot}\ \wedge I$$

$$\frac{}{\neg A\wedge\neg B \vdash_m (A\vee B)\to\bot}\ \to R$$

$$\frac{}{\neg A\wedge\neg B \vdash_m \neg(A\vee B)}\ \text{Def}\ \neg$$

$$\frac{}{\vdash_m (\neg A\wedge\neg B)\to\neg(A\vee B)}\ \to R$$

$$\frac{}{(\neg\neg A)\to\bot,\ A,\ A\to\bot \vdash_m A}\ \text{Hip}$$

$$\frac{}{(\neg\neg A)\to\bot,\ A,\ A\to\bot \vdash_m \bot}\ \to I$$

$$\frac{}{(\neg\neg A)\to\bot,\ A,\ \neg A \vdash_m \bot}\ \text{Def}\ \neg$$

$$\frac{}{(\neg\neg A)\to\bot,\ A \vdash_m (\neg A)\to\bot}\ \to R$$

$$\frac{}{(\neg\neg A)\to\bot,\ A \vdash_m \neg\neg A}\ \text{Def}\ \neg$$

$$\frac{}{(\neg\neg A)\to\bot,\ A \vdash_m \bot}\ \to I$$

$$\frac{}{(\neg\neg A)\to\bot \vdash_m A\to\bot}\ \to R$$

$$\frac{}{\neg\neg\neg A \vdash_m \neg A}\ \text{Def}\ \neg$$

$$\frac{}{\vdash_m (\neg\neg\neg A)\to(\neg A)}\ \to R$$

$$\dfrac{}{\neg A \to B,\ A \to B,\ B \to \bot,\ A \vdash_m A}\ \text{Hip}$$

$$\dfrac{}{\neg A \to B,\ A \to B,\ B \to \bot,\ A \vdash_m B}\ \to I$$

$$\dfrac{}{\neg A \to B,\ A \to B,\ B \to \bot,\ A \vdash_m \bot}\ \to I$$

$$\dfrac{}{\neg A \to B,\ A \to B,\ B \to \bot \vdash_m A \to \bot}\ \to R$$

$$\dfrac{}{\neg A \to B,\ A \to B,\ B \to \bot \vdash_m \neg A}\ \text{Def } \neg$$

$$\dfrac{}{\neg A \to B,\ A \to B,\ B \to \bot \vdash_m B}\ \to L$$

$$\dfrac{}{\neg A \to B,\ A \to B,\ B \to \bot \vdash_m \bot}\ \to L$$

$$\dfrac{}{\neg A \to B,\ A \to B,\ \neg B \vdash_m \bot}\ \text{Def } \neg$$

$$\dfrac{}{\neg A \to B,\ A \to B \vdash_m \neg B \to \bot}\ \to R$$

$$\dfrac{}{\neg A \to B,\ A \to B \vdash_m \neg\neg B}\ \text{Def } \neg$$

$$\dfrac{}{\neg A \to B \vdash_m (A \to B) \to \neg\neg B}\ \to R$$

$$\vdash_m (\neg A \to B) \to (A \to B) \to \neg\neg B\quad \to R$$

Lógica Intuicionista

$$\dfrac{}{\Gamma,\ A,\ \neg A \vdash C}\ \text{Exp}$$

$$\dfrac{\dfrac{}{\neg A,\ A \vdash_i B}\ \text{Exp} \qquad \dfrac{}{B,\ A \vdash_i B}\ \text{Hip}}{\neg A \vee B,\ A \vdash_i B}\ \vee I$$

$$\dfrac{}{\neg A \vee B \vdash_i A \to B}\ \to R$$

$$\vdash_i (\neg A \vee B) \to (A \to B)\quad \to R$$