$$\forall z(Q(z) \to R(y,z))\,[y:=t(q)]$$
$$= \forall z(Q(z)[y:=t(q)] \to R(y,z)[y:=t(q)])$$
$$= \forall z(Q(z[y:=t(q)]) \to R(y[y:=t(q)],\ z[y:=t(q)]))$$
$$= \forall z(Q(z) \to R(t(q), z))$$

$$[\vec{x}:=\vec{t}\,] = [x_1,\dots,x_n := t_1,\dots,t_n]$$

$$\exists z\,\varphi[\overbrace{x,y,t}^{\vec{x}} := \overbrace{f(a),r,q}^{\vec{t}}] \to \exists z\,\varphi[\vec{x}:=\vec{t}\,]$$

$x[\vec{x}:=\vec{t}\,] = t_i$ si $x = x_i$

$x[\vec{x}:=\vec{t}\,] = x$ si para toda i, $x_i \neq x$.

$\bot[\vec{x}:=\vec{t}\,] = \bot$

$\top[\vec{x}:=\vec{t}\,] = \top$

$$P(t_1,\dots,t_m)[\vec{x}:=\vec{t}\,] = P(t_1[\vec{x}:=\vec{t}\,],\dots,t_m[\vec{x}:=\vec{t}\,])$$

$$(\neg\varphi)[\vec{x}:=\vec{t}\,] = \neg(\varphi[\vec{x}:=\vec{t}\,])$$
$$(\varphi_1 * \varphi_2)[\vec{x}:=\vec{t}\,] = (\varphi_1[\vec{x}:=\vec{t}\,] * \varphi_2[\vec{x}:=\vec{t}\,])$$

con $* \in \{\wedge,\vee,\to,\leftrightarrow\}$

$\forall y\,\varphi[\vec{x}:=\vec{t}\,] = \forall y(\varphi[\vec{x}:=\vec{t}\,])$ si $y \notin \vec{x} \cup Var(\vec{t}\,)$

$\exists y\,\varphi[\vec{x}:=\vec{t}\,] = \exists y(\varphi[\vec{x}:=\vec{t}\,])$ si $y \notin \vec{x} \cup Var(\vec{t}\,)$

Ej. 1. $\forall x(Q(x) \to R(z,x))\,[z:=f(x)]$

$\sim_\alpha \forall w(Q(w) \to R(z,w))[z:=f(x)] =$

$\forall w((Q(w) \to R(z,w))[z:=f(x)]) = \forall w((Q(w)[z:=f(x)] \to R(z,w)[z:=f(x)]) = \dots$

α-Equivalencia

Decimos que φ_1 y φ_2 son α-equivalentes si φ_1 y φ_2 difieren a lo más en el nombre de variables ligadas

$$\forall x(Q(x) \to R(z,x)) \sim_\alpha \forall w(Q(w) \to R(z,w))$$
$$\not\sim_\alpha \forall r(Q(r) \to R(t,r))$$

$$\dots = \forall w\,(Q(w[z:=f(x)]) \to R(z[z:=f(x)], w[z:=f(x)]))$$

$$= \forall w\,(Q(w) \to R(f(x), w))$$

2. $$\forall x\,(Q(z,y,x) \land \exists z\, T(f(z), w, y))\ \overbrace{[x, y, z := a, z, g(w)]}^{S}$$

$$\sim_\alpha \forall k\,(Q(z,y,k) \land \exists z\, T(f(z), w, y))\, S$$

$$= \forall k\,((Q(z,y,k) \land \exists z\, T(f(z), w, y))\, S)$$

$$= \forall k\,(Q(z,y,k)\, S \land \exists z\, T(f(z), w, y)\, S)$$

$$= \forall k\,(Q(z S, y S, k S) \land \exists z\, T(f(z), w, y)\, S)$$

$$= \forall k\,(Q(g(w), z, k) \land \exists z\, T(f(z), w, y)\, S)$$

$$\sim_\alpha \forall k\,(Q(g(w), z, k) \land \exists t\, T(f(t), w, y)\, S)$$

$$= \forall k\,(Q(g(w), z, k) \land \exists t\,(T(f(t), w, y)\, S))$$

$$= \forall k\,(Q(g(w), z, k) \land \exists t\, T(f(t) S, w S, y S))$$

$$= \forall k\,(Q(g(w), z, k) \land \exists t\, T(f(t S), w, z))$$

$$= \forall k\,(Q(g(w), z, k) \land \exists t\, T(f(t), w, z))$$