Convexity adjustments with a bit of Malliavin

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August 17, 2022

Abstract

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1 Introduction

Mathematical finance aims to find a methodology to price consistently all the instruments quoted in the market. When working with fixed income derivatives, a classic research topic is the introduction of a price adjustment to achieve this. This adjustment is called convexity adjustment. It is non-linear and depends on the interest rate model.

There are several reasons to include this type of adjustment. One of them is to incorporate futures on the yield curve construction. Futures and other fixed-income instruments are quoted differently. The firsts are linear against the yield, but the others are not. Therefore, the changes in value and yield of different contracts are different. This difference will depend on the volatility and correlation of the yield curve.

But it is not the only one. The fixed-income market has several features changing the schedule of payments. For example, in a swap in arrears, the floating coupon fixing and payment are on the same date. Or in a CMS swap, the floating rate is linked to a rate longer than the floating length. Any customization of an interest rate product based on changing time, currency, margin, or collateral will require a convexity adjustment. Deep down, by making these changes, we are mixing the martingale measures.

Convexity adjustments have become popular again. Not only by the increase in volatility in the markets. In addition, as a consequence of the transition in risk-free rates from the IBOR (InterBank Offered Rates) indices to the ARR (Alternative Reference Rates) indices, also called RFR. Both indices try to represent the same thing, the risk-free rate, but they are fundamentally different. While the former represents the average rate at which Panel Banks believe they could borrow money, the latter is calculated backward based on transactions. Therefore, these new products need their corresponding convexity adjustment.

The first references on the convexity adjustment were Ritchken and Sankarasubramanian (1993), Flesaker (1993) and Brotherton-Ratcliffe and Iben (1993), published almost simultaneously. A convexity formula for averaging contracts was found in Ritchken and Sankarasubramanian (1993). Flesaker derived a convexity adjustment for computing the expected Libor rate under the Ho-Lee model in a continuous and discrete setting in Flesaker (1993). Brotherton-Ratcliffe and Iben (1993) used the Taylor expansion on the inverse function for calculating the convexity

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adjustment. In the following years, several improvements were made. For example, the convexity adjustment was extended to other payoffs in Hull (2006). Hart (1997) improved the Taylor expansion. Kirikos and Novak (1997) derived the convexity adjustment for the Hull-White model. Afterwards, we can find papers that extend the convexity adjustment to different payoffs, see Benhamou (2000b) or Hagan (2003). Or by applying alternative techniques such as the change of measure in Pelsser (2001), a martingale approach in Benhamou (2000a) or the effects of stochastic volatility in Piterbarg and Renedo (2006) and Hagan and Woodward (2020).

In the present paper, we find an alternative way to calculate the convexity adjustment for a general interest rate model. The idea is to use the Itô representation theorem. Unfortunately, the theorem does not give an insight into how to calculate the elements therein. Therefore, it is necessary to introduce basic concepts of Malliavin calculus to apply the Clark-Occone representation formula.

The structure of the paper is as follows. In Section 2, we give the basic preliminaries and our notation related to Interest Rates models. This notation will be used throughout the paper without being repeated in particular theorems unless we find it useful to do so in order to guide the reader through the results. In Section 3, we make an introduction to Malliavin calculus. In Section 4, In Section 5, In Section 6

2 Preliminaries and notation

3 Malliavin

4 Convexity Adjustment

5 Numerical Results

6 Conclusion

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