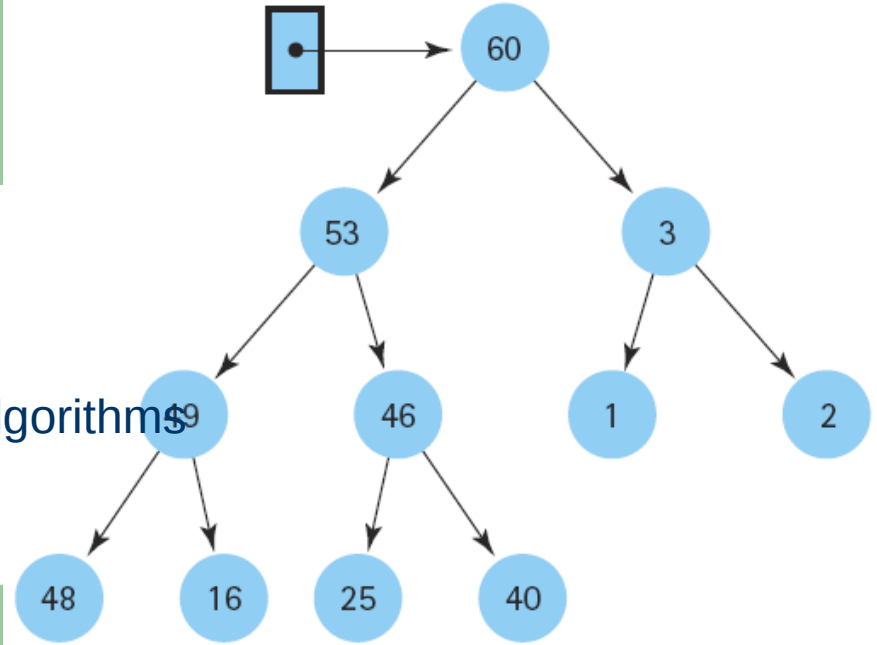
C++ and Data Structures &

- Algorithms



- Week 1 and Week 2

Atomic Variables

- Atomic variables can only store one value at a time.
  - int num;
  - float s;
- A value stored in an atomic variable cannot be subdivided.

- Chapter 0: C++ LANGUAGE

1. ***ADDRESS***
2. ***POINTERS***
3. ***ARRAYS***
4. ***ADDRESS OF EACH ELEMENT IN AN ARRAY***
5. ***ACCESSING & MANIPULATING AN ARRAY USING POINTERS***
6. ***ANOTHER CASE OF MANIPULATING AN ARRAY USING POINTERS***
7. ***TWO-DIMENSIONAL ARRAY***
8. ***POINTER ARRAYS***
9. ***STRUCTURES***
10. ***STRUCTURE POINTERS***

- Chapter 0: C++ LANGUAGE

1. ***ADDRESS***
   - ***For every variable there are two attributes: <u>address</u> and <u>value</u>***

- In memory with address 3: value: 45.
- In memory with address 2: value "Dave"

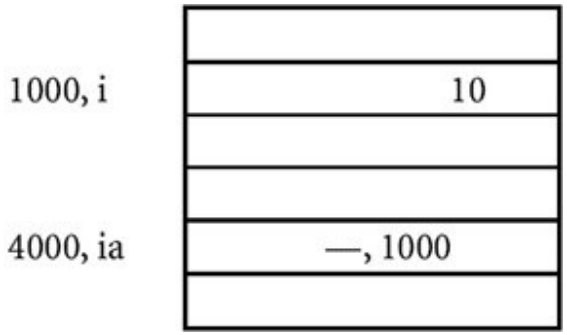
1	4096
2	"Dave"
3	45
4	"Matt"
5	95.5
6	"wbru"
7	0
8	"zero"

```
cout << "Value of 'y' is: " << y << "\n";
cout << "Address of 'y' is: " << &y << "\n\n
";
```

- Chapter 0: C++ LANGUAGE

- ***2. POINTERS***
  - ***is a variable whose value is also an address.***
  - ***A pointer to an integer is a variable that can store the address of that integer***

- **ia: value of variable**
- **&ia: address of ia**
- ***ia means you are printing the value at the location specified by ia**



- Chapter 0: C++ LANGUAGE

```
int i;        //A
 int * ia;     //B
 cout<<"The address of i "<< &i << " value="<<i <<endl;
 cout<<"The address of ia " << &ia << " value = " << ia<< endl;

 i = 10;       //C
 ia = &i;   //D

    cout<<"after assigning value:"<<endl;
 cout<<"The address of i "<< &i << " value="<<i <<endl;
 cout<<"The address of ia " << &ia << " value = " << ia<< " point to: "<<
*ia;
```

Points to Remember

- **Pointers give a facility to access the value of a variable indirectly.**
- **You can define a pointer by including a * before the name of the variable.**
- **You can get the address where a variable is stored by using &.**

- Chapter 0: C++ LANGUAGE

- ***3. ARRAYS***
  - ***An array is a data structure***
  - ***used to process multiple elements with the same data type when a number of such elements are known.***
  - ***An array is a composite data structure; that means it had to be constructed from basic data types such as array integers.***

    - ***int a[5];***
    - ***for(int i = 0;i<5;i++)***
      - ***{a[i]=i; }***

- Chapter 0: C++ LANGUAGE

- ***4. ADDRESS OF EACH ELEMENT IN AN ARRAY***
  - ***Each element of the array has a memory address.***

```
void printdetail(int a[])
{
    for(int i = 0;i<5;i++)
    {
       cout<< "value in array “<< a[i] <<“ at address: “ << &a[i]);
    }
```

- Chapter 0: C++ LANGUAGE

- ***5. ACCESSING & MANIPULATING AN ARRAY USING POINTERS***
  - ***You can access an array element by using a pointer.***
  - ***If an array stores integers->use a pointer to integer to access array elements.***

```
void printarr_usingpointer(int a[])
{
    int *pi;
    pi=a;
     for(int i = 0;i<5;i++)
      {
         cout<<"value array:" << *pi << " address: "<< pi<<endl;
       pi++;
      }
}
```

- Chapter 0: C++ LANGUAGE

- ***6. ANOTHER CASE OF MANIPULATING AN ARRAY USING POINTERS***
  - ***The array limit is a pointer constant : cannot change its value in the program.***

- **int a[5]; int *b;**
- **a=b; *//error***
- **b=a; *//OK***

```
void print_usingptr_a(int a[])
{
    for(int i = 0;i<5;i++)
      {cout<<"a[" << i << "]=" <<*a<<endl;
        a++;}
}
```

- It works correctly even using a++ ???

- Chapter 0: C++ LANGUAGE

- ***7. TWO-DIMENSIONAL ARRAY***
  - ***int a[3][2];***

3	1
5	2
8	7

```
int a[3][2];
for(int i = 0;i<3;i++)
    for(int j=0;j<2 ;j++)
    {
            {
                    a[i][j]=i+j+i*j;
            }
    }
```

```
void print_usingptr(int a[][2])
{
    int *b;
    b=a[0];
    for(int i = 0;i<6;i++)
    {
        cout<<"value :" << *b<<" in address: "<<b<<endl;
        b++; }
}
```

- Chapter 0: C++ LANGUAGE

- ***8. POINTER ARRAYS***
  - ***You can define a pointer array (similarly to an array of integers).***
  - ***In the pointer array, the array elements store the pointer that points to integer values.***

```
int *a[5];
main()
{    int i1=4,i2=3,i3=2,i4=1,i5=0;
     a[0]=&i1;
     a[1]=&i2;
     a[2]=&i3;
     a[3]=&i4;
     a[4]=&i5;

     printarr(a);
     printarr_usingptr(a);
}
```

```
void printarr(int *a[])
{
     printf("Address1\tAddress2\tValue\n");
     for(int  j=0;j<5;j++)
     {
          cout<<*a[j]<<"\t"<<a[j]<<"\t"<<&a[j]<<endl;
     }
}
```

- Chapter 0: C++ LANGUAGE

- ***9. STRUCTURES***
- ***Structures are used when you want to process data of multiple data types***
- ***But you still want to refer to the data as a single entity***
- ***Access data: structurename.membername***

```
struct student
{
    char name[30];
    float marks;
};

main ( )
{
    student student1;
    char s1[30];float f;
    cin>>student1.name;
    cin>>student1.marks;

    cout<<"Name is:"<<student1.name;
    cout<< "Marks are:" << student1.marks;
}
```

- Chapter 1: C++ LANGUAGE

- ***10. STRUCTURE POINTERS***
  - ***Process the structure using a structure pointer***

```
struct student
{   char name[30];
    float marks;    };
main ( )
{
     struct student *student1;
     struct student student2;
     student1 = &student2;
     cin>>student1->name; // cin>>(*student1).name;
     cin>>student2.marks;
     cout<<(*student1).name<<" : " << student2.marks;
}
```

CHAPTER 2: FUNCTION & RECURSION

1. FUNCTION
2. THE CONCEPT OF STACK
3. THE SEQUENCE OF EXECUTION DURING A FUNCTION CALL
4. PARAMETER PASSING
5. CALL BY REFERENCE
6. RESOLVING VARIABLE REFERENCES
7. RECURSION
8. STACK OVERHEADS IN RECURSION
9. WRITING A RECURSIVE FUNCTION
10. TYPES OF RECURSION

CHAPTER 2: FUNCTION & RECURSION

- 1. FUNCTION
  - provide modularity to the software
  - divide complex tasks into small manageable tasks
  - avoid duplication of work

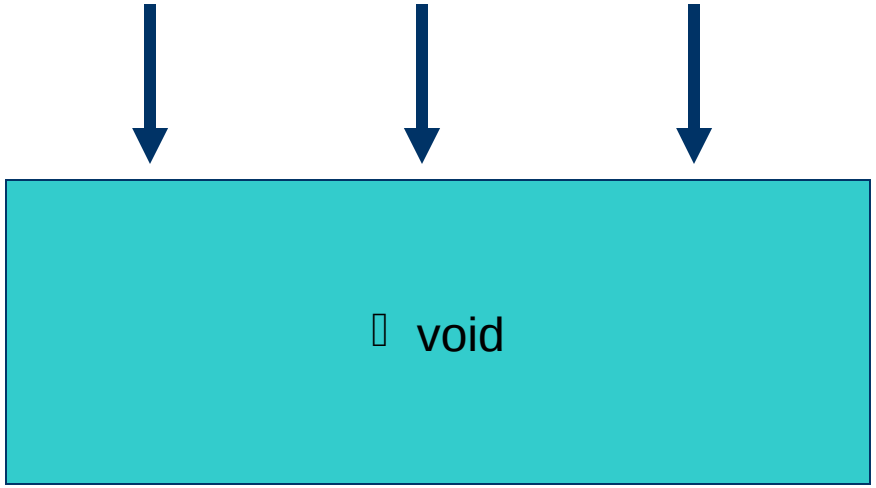
```
int add (int x, int y)
{
    int z;
    z = x + y;
    return z;
}
```

Type of Function: return value & parameters

- *On return value*
  - *Void Functions*
  - Return Function
    - Example: void display(); int max(int a,int b);
- On Parameters
  - value parameters
  - reference parameters
    - Exmple: void swap(int &a, int &b); int BinhPhuong(int n);

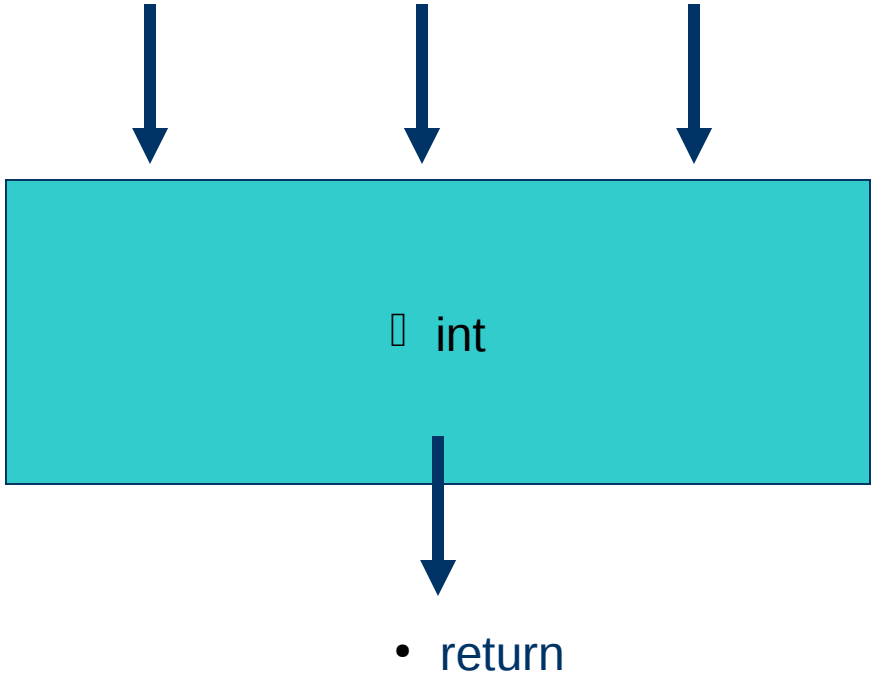
- Week 6: Ôn tập function

- Nothing return



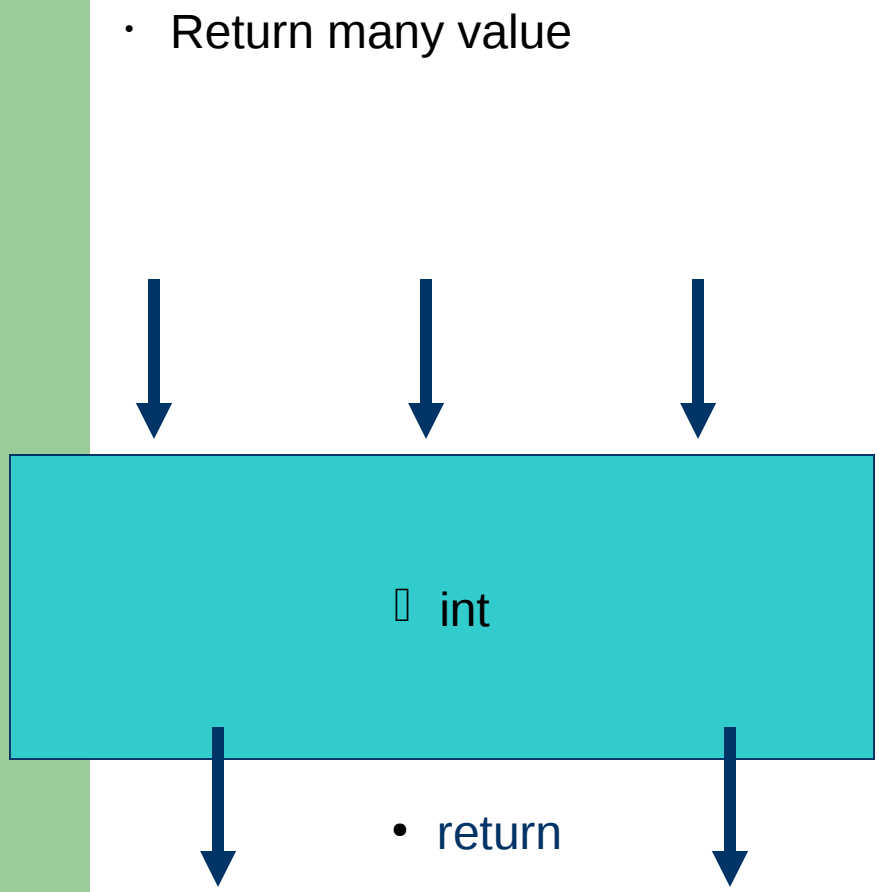
- Week 6: Ôn tập function

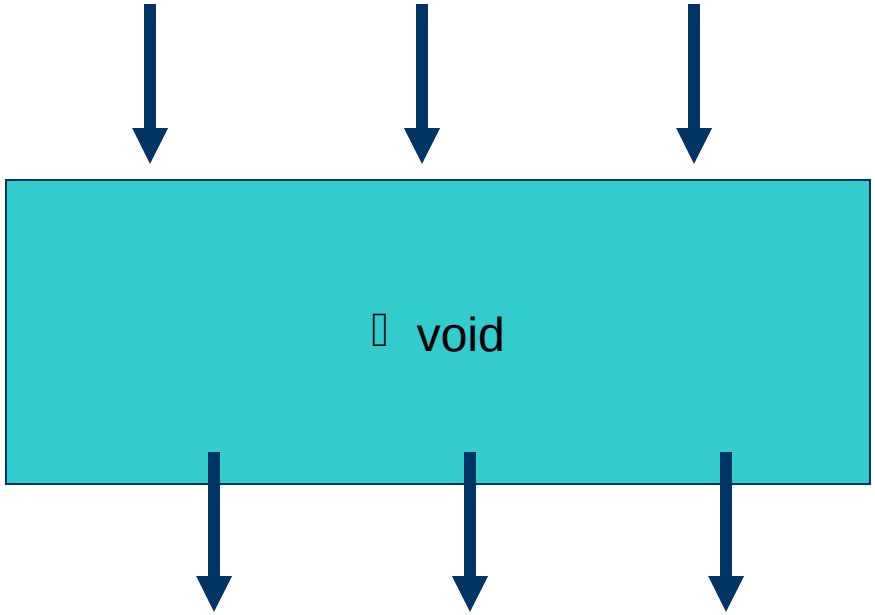
- Return 1 value



- Week 6: Ôn tập function

- Return many value





- On natural way

CHAPTER 2: FUNCTION & RECURSION

- 3. THE SEQUENCE OF EXECUTION DURING A FUNCTION CALL
  - When the function is called, the current execution is temporarily stopped and the control goes to the called function. After the call, the execution resumes from the point at which the execution is stopped.
  - To get the exact point at which execution is resumed, the address of the next instruction is stored in the stack. When the function call completes, the address at the top of the stack is taken.

CHAPTER 2: FUNCTION & RECURSION

- 3. THE SEQUENCE OF EXECUTION DURING A FUNCTION CALL
  - Functions or sub-programs are implemented using a stack.
  - When a function is called, the address of the next instruction is pushed into the stack.
  - When the function is finished, the address for execution is taken by using the pop operation.

CHAPTER 2: FUNCTION & RECURSION

- 3. THE SEQUENCE OF EXECUTION DURING A FUNCTION CALL
- Result:?

```
main ( )
{
     printf ("1 \n");
     printf ("2 \n");
     f1();
     printf ("3 \n");
     printf ("4 \n");
}
void f1()
{
     printf ("f1-5 \n");
     printf ("f1-6 \n");
     f2 ( );
     printf ("f1-7 \n");
     printf ("f1-8 \n");
}
void f2 ()
{
     printf ("f2-9 \n");
     printf ("f2-10 \n");
}
```

CHAPTER 2: FUNCTION & RECURSION

- 4. PARAMETER * REFERENCE PASSING
  - *passing by value*
    - the value before and after the call remains the same
  - *passing by reference*
    - *changed value after the function completes*

```
void (int *k)
{
     *k = *k + 10;
}
```

```
void (int &k)
{
     k = k + 10;
}
```

CHAPTER 2: FUNCTION & RECURSION

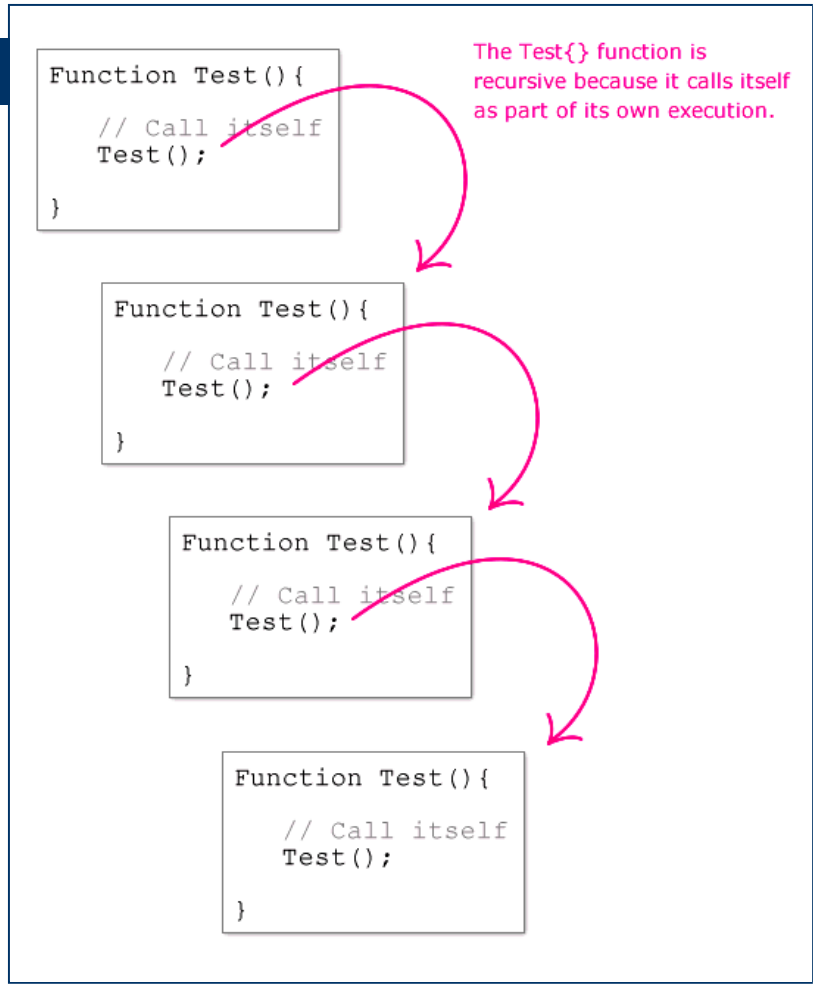
- 6. RESOLVING VARIABLE REFERENCES

  - When a variable can be resolved by using multiple references, the local definition is given more preference

```
int i =0;  //Global variable
main()
{
    int i ;   // local variable for main
    void f1(void) ;
    i =0;
    cout<<"value of i in main: "<< i <<endl;
    f1();
    cout<<"value of i after call:" << i<<endl;
}
void f1()
{
    int i=0;   //local variable for f1
    i = 50;
}
```

CHAPTER 2: FUNCTION & RECURSION

- 7. RECURSION
  - A method of programming whereby a function directly or indirectly calls itself
  - Problems: stop recursion?



CHAPTER 2: FUNCTION & RECURSION

- 7. RECURSION

Example: Factorial

- The factorial is defined in this way:

$$n! = \begin{cases} 1 & n = 0 \\ 1 \times 2 \times \cdots \times n & n > 0 \end{cases}$$

This can be computed by a loop.

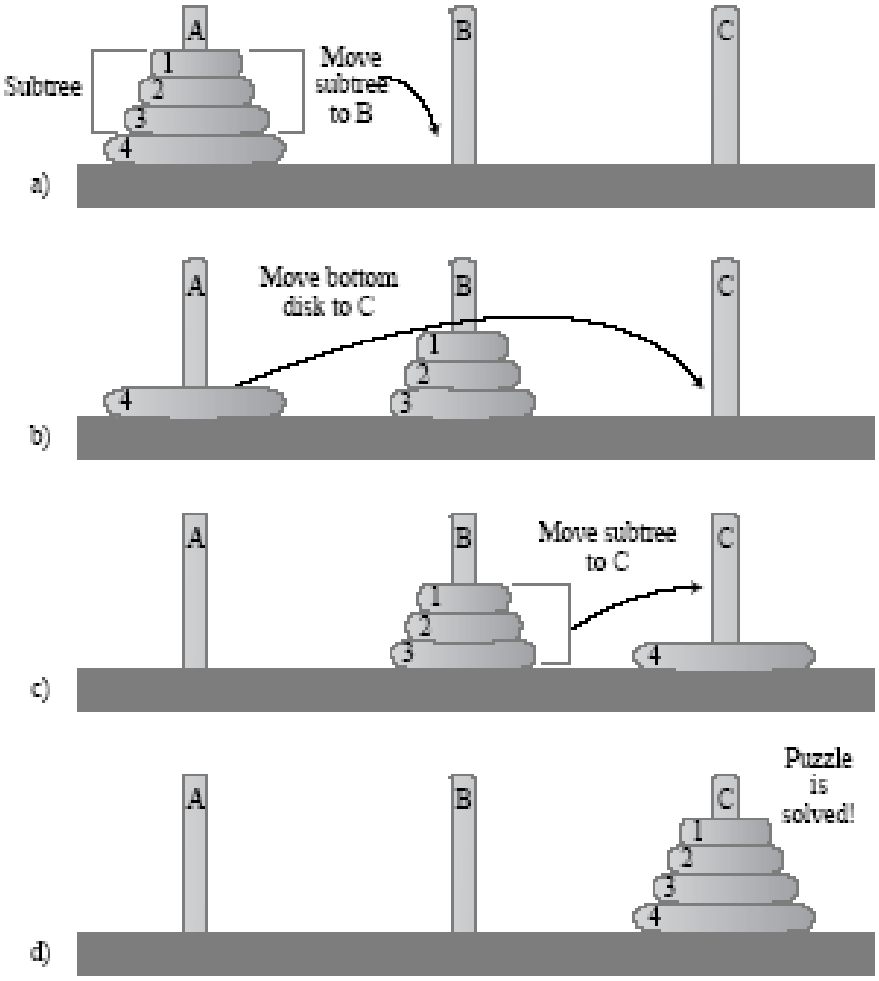
- We can also define the factorial as:

$$n! = \begin{cases} 1 & n = 0 \\ n \times (n-1)! & n > 0 \end{cases}$$

So, if we know how to compute the factorial of $n-1$, we know how to compute the factorial of n.

CHAPTER 2: FUNCTION & RECURSION

- 7. RECURSION: Hanoi tower



CHAPTER 2: FUNCTION & RECURSION

- 7. RECURSION

Example: Factorial

```
int factorial(int n)  // assumes n >= 0
{
  if (n == 0)
    return 1;
  else
    return n * factorial(n-1);
}
```

To see how the computation is done, trace `factorial(3)`:

```
factorial(3) = 3 * factorial(2)
             = 3 * (2 * factorial(1))
             = 3 * (2 * (1 * factorial(0)))
             = 3 * (2 * (1 * 1))
```

CHAPTER 2: FUNCTION & RECURSION

- 9. WRITING A RECURSIVE FUNCTION
  - Recursion enables us to write a program in a natural way. The speed of a recursive program is slower because of stack overheads.
  - In a recursive program you have to specify recursive conditions, terminating conditions, and recursive expressions.

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **LINEAR RECURSION**
  - **TAIL RECURSION**
  - **BINARY RECURSION**
  - **EXPONENTIAL RECURSION**
  - **NESTED RECURSION**
  - **MUTUAL RECURSION**

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **LINEAR RECURSION**
    - **only makes a single call to itself each time the function runs**
    -

```
int factorial (int n)
{
 if ( n == 0 )
   return 1;
 return n * factorial(n-1);
}
```

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **TAIL RECURSION**
    - **Tail recursion is a form of linear recursion.**
    - **In tail recursion, the recursive call is the last thing the function does. Often, the value of the recursive call is returned.**

```
int gcd(int m, int n)
{
        int r;
        if (m < n) return gcd(n,m);
        r = m%n;
        if (r == 0) return(n);
        else return(gcd(n,r));
}
```

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **BINARY RECURSION**
    - **Some recursive functions don't just have one call to themself, they have two (or more).**

```
int choose(int n, int k)
{
        if (k == 0 || n == k) return(1);
        else return(choose(n-1,k) + choose(n-1,k-1));
}
```

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **EXPONENTIAL RECURSION**
    - **An exponential recursive function is one that, if you were to draw out a representation of all the function calls, would have an exponential number of calls in relation to the size of the data set**
    - **(exponential meaning if there were *n* elements, there would be *O*(*a*n) function calls where a is a positive number)**

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **EXPONENTIAL RECURSION**

```
void print_array(int arr[], int n)
{
        int i;
        for(i=0; i<n; i++) printf("%d ", arr[i]);
        printf("\n");
}

void print_permutations(int arr[], int n, int i)
{
        int j, swap;
        print_array(arr, n);
        for(j=i+1; j<n; j++) {
                swap = arr[i]; arr[i] = arr[j]; arr[j] = swap;
                print_permutations(arr, n, i+1);
                swap = arr[i]; arr[i] = arr[j]; arr[j] = swap;
        }
}
```

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **NESTED RECURSION**
    - **In nested recursion, one of the arguments to the recursive function is the recursive function itself**
    - **These functions tend to grow extremely fast.**

```
int ackerman(int m, int n)
{
        if (m == 0) return(n+1);
        else if (n == 0) return(ackerman(m-1,1));
        else return(ackerman(m-1,ackerman(m,n-1)));
}
```

m\n	0	1	2	3	4	n
0	0+1	1+1	2+1	3+1	4+1	$n+1$
1	A(0,1)	A(0,A(1,0))	A(0,A(1,1))	A(0,A(1,2))	A(0,A(1,3))	$n+2=2+(n+3)-3$
2	A(1,1)	A(1,A(2,0))	A(1,A(2,1))	A(1,A(2,2))	A(1,A(2,3))	$2n+3=2*(n+3)-3$
3	A(2,1)	A(2,A(3,0))	A(2,A(3,1))	A(2,A(3,2))	A(2,A(3,3))	$2^{(n+3)}-3$
4	A(3,1)	A(3,A(4,0))	A(3,A(4,1))	A(3,A(4,2))	A(3,A(4,3))	$\underbrace{2^{2^{\cdot^{\cdot^{\cdot^{2}}}}}}_{n+3 \text{ twos}}-3$
5	A(4,1)	A(4,A(5,0))	A(4,A(5,1))	A(4,A(5,2))	A(4,A(5,3))	A(4, A(5, n-1))
6	A(5,1)	A(5,A(6,0))	A(5,A(6,1))	A(5,A(6,2))	A(5,A(6,3))	A(5, A(6, n-1))

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **MUTUAL RECURSION**
    - A recursive function doesn't necessarily need to call itself.
    - Some recursive functions work in pairs or even larger groups. For example, function A calls function B which calls function C which in turn calls function A.

CHAPTER 2: FUNCTION & RECURSION

- 10. TYPES OF RECURSION
  - **MUTUAL RECURSION**

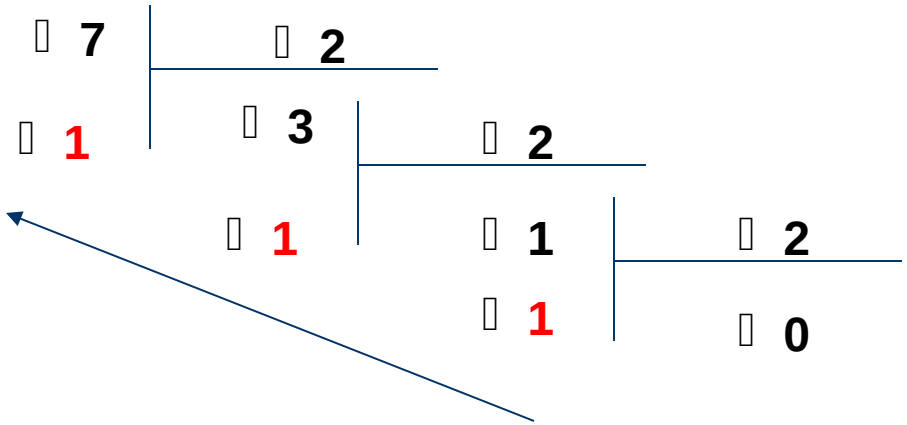
```
int is_even(unsigned int n)
{
        if (n==0) return 1;
        else return(is_odd(n-1));
}

int is_odd(unsigned int n)
{
        return (!is_even(n));
}
```

- Exercises 1: Recursion

```
*
**
***
****
*****
******
*******
********
```

- Exercises 2: Recursion

- Convert number from H10->H2



- Week3: Recursion Excercises (1)

- E1. (44/174) Write a program to compute: S = 1 + 2 + 3 + …n using recursion.

Week3: Recursion Excercises (2-3)

- E3(a). Write a program to print a revert number Example: input n=12345. Print out: 54321.

- E3(b). Write a program to print this number Example: input n=12345. Print out: 12345.

- Week3: Recursion Excercises (4)

- E4. Write a recursion function to find the sum of every number in a int number. Example: n=1980 => Sum=1+9+8+0=18.

- Week3: Recursion Excercises (5)

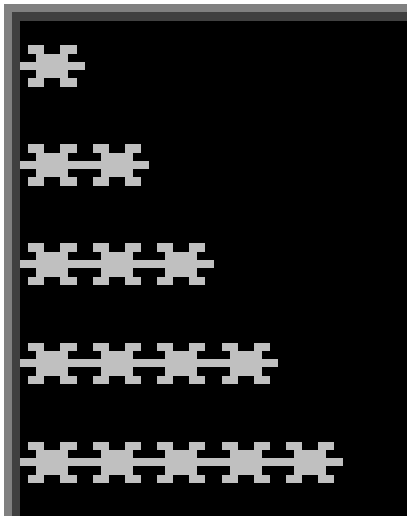
- E4. Write a recursion function to calculate:
  - S=a[0]+a[1]+…a[n-1]
    - A: array of integer numbers

- Week3: Recursion Excercises (6)

- E4. Write a recursion function to find an element in an array (using linear algorithm)

- Week3: Recursion Excercises (7)

- Print triangle



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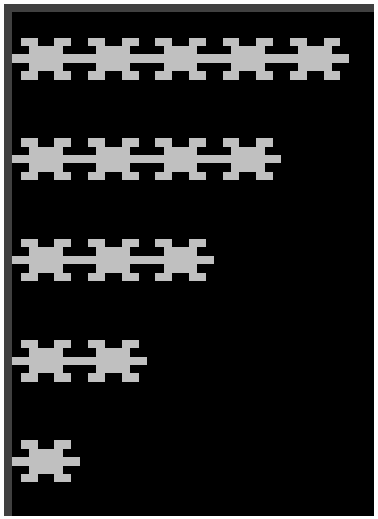
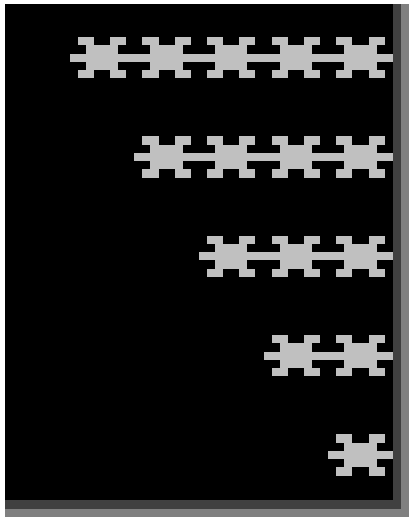


- b

- c



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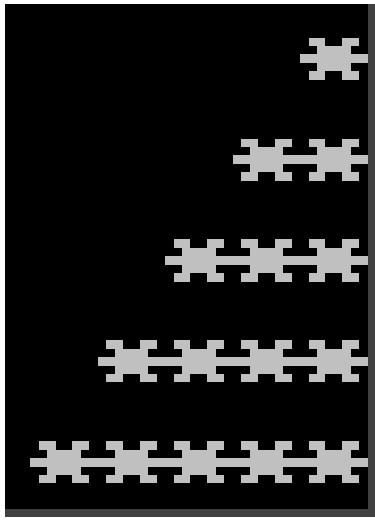
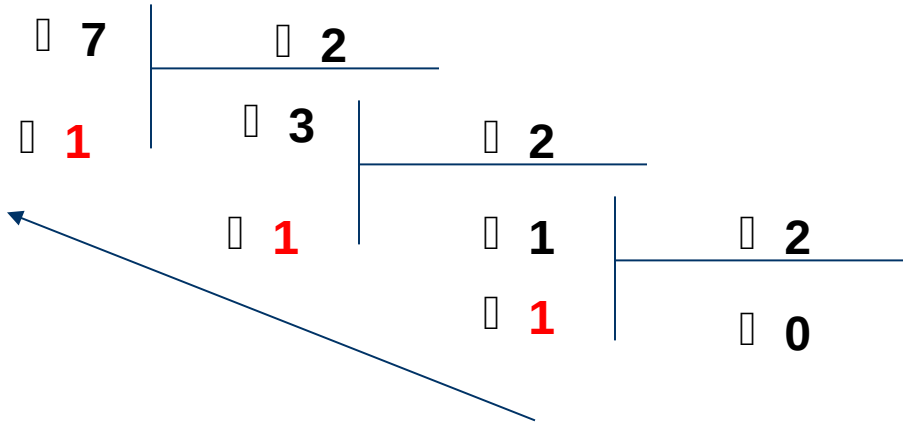


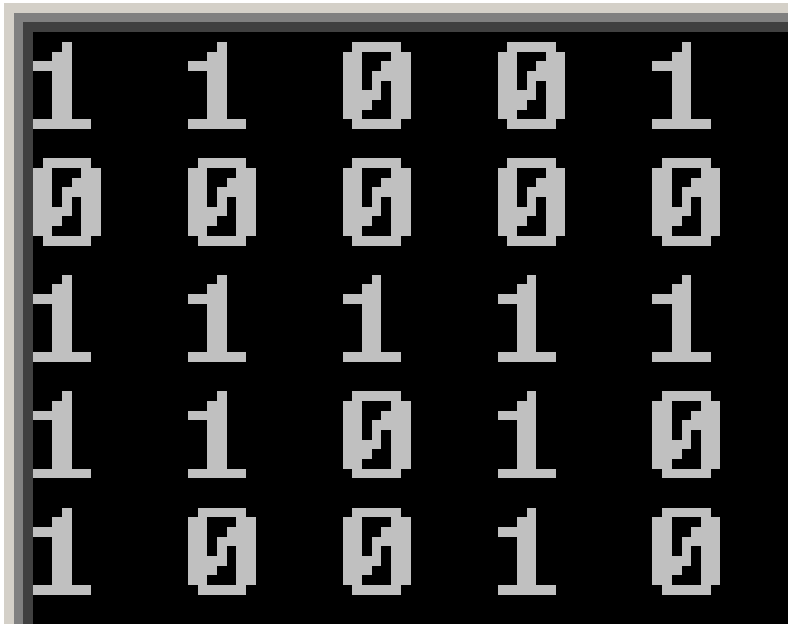
- Week3: Recursion Excercises (8)

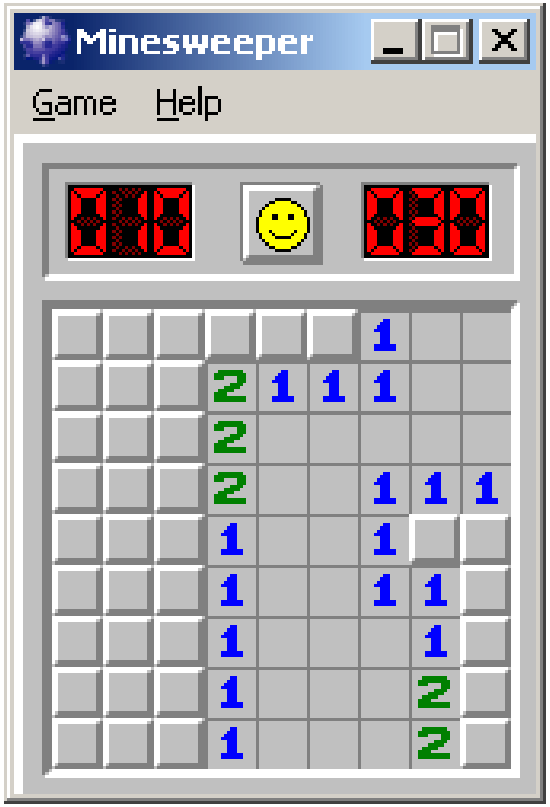
- Convert number from H10->H2



- Week3: Recursion Excercises (9)

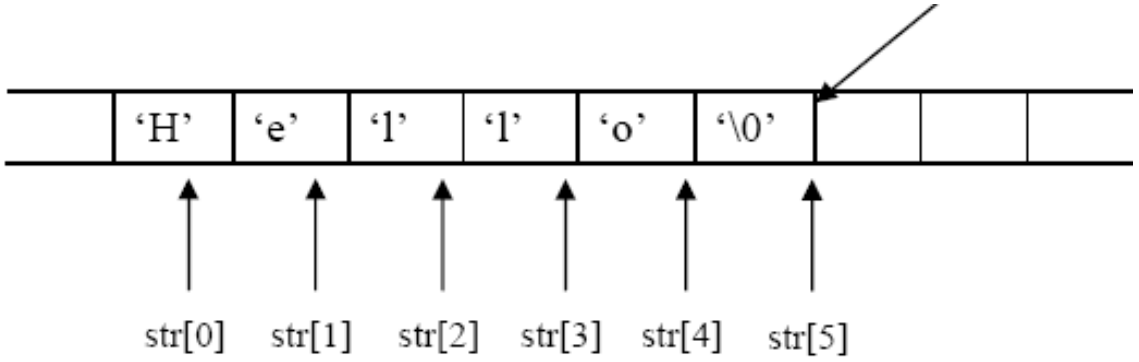
- Minesweeper





1. String: structure

- String
  - is array of char
  - Ending with null char \0 (size +1)
  - Example: store 10 chars:
    - char str[11];
  - “Example”: string const. C/C++ add \0 automayically



1. String: declare

- Declare string
  - Using array of chars
    - char str[] = {‘H’,’e’,’l’,’l’,’o’,’\0’}; //declare with null
    - char str[] = “Hello”; //needn’t null
  - Using char pointer
    - char *str = “Hello”;

1. String: input

- char *gets(char *s);
  - Read every char
  - Until receive Enter
  - Adding Automatically ‘\0’
- cin>>s;

- 1. String: output

- int puts(const char *s);
- cout<<s;

1. String: Problem with buffer?

- Keyboard buffer
  - char szKey[] = "aaa";
  - char s[10];
  - do {
  - cout<<"doan lai di?";
  - gets(s);
  - } while (strcmp (szKey,s) != 0);
  - puts ("OK. corect");
- If user input: aaaaaaaaaaaa???

- 1. String: functions

- #include <string.h>
- strcpy(s1, s2)
- strcat(s1, s2)
- strlen(s1)
- strcmp(s1, s2) -> (-1,0,1)
- strchr(s1, ch)
- strstr(s1, s2)

```
char s1[80], s2[80];
cout << "Input the first string: :";
gets(s1);
cout << "Input the second string: ";
gets(s2);
cout << "Length of s1= " << strlen(s1);
cout << "Length of s2= " << strlen(s2);
if(!strcmp(s1, s2))
cout << "These strings are equal\n";
strcat(s1, s2);
cout << "s1 + s2: " << s1 << endl;;
strcpy(s1, "This is a test.\n");
cout << s1;
if(strchr(s1, 'e')) cout << "e is in " << s1;
if(strstr(s2, "hi")) cout << "found hi in " <<s2;
```

- 2. File: Creating a new file

- #include <io.h>
- FILE *fp;
- fp=fopen(“d:\\test.txt", "wb"))
- fwrite(&Address, sizeof(TYPE), count, fp);
- fclose(fp);

- 2.File: Creating a new file

- int Arr[3]

- Arr
- fwrite(Arr, sizeof(Arr), 1, fp);
- Fwrite(Arr, sizeof(int), 1, fp);
- for (i=0;i<=3;i++)
  - Fwrite(&Arr[i], sizeof(int), 1, fp);

- 2. File: Reading a file

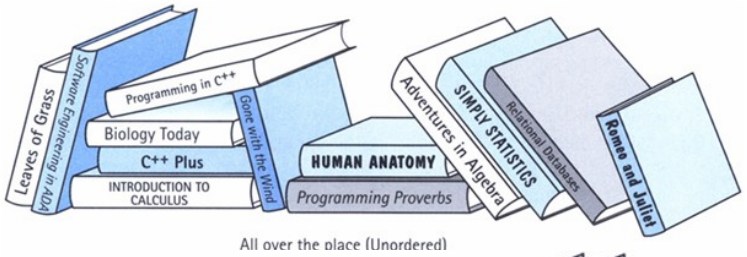
```
#include <io.h>
FILE *fp;
fp=fopen(“d:\\test.txt", “rb"))
while (fwrite(&Address, sizeof(TYPE), count, fp))
{
    ……..
}
fclose(fp);
```

CHAPTER 0: INTRODUTION

- ***What is Data Structures?***
  - ***A data structure is defined by***
    - ***(1) the logical arrangement of data elements, combined with***
    - ***(2) the set of operations we need to access the elements.***

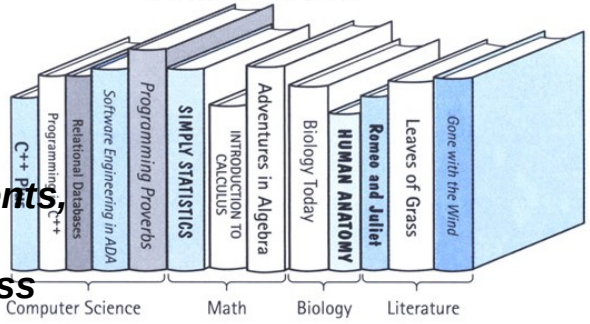
- *What is Data Structures?*

- Example:library
  - is composed of elements (books)
  - Accessing a particular book requires knowledge of the arrangement of the books
  - Users access books only through the librarian



All over the place (Unordered)

Alphebetical order by title



Ordered by subject

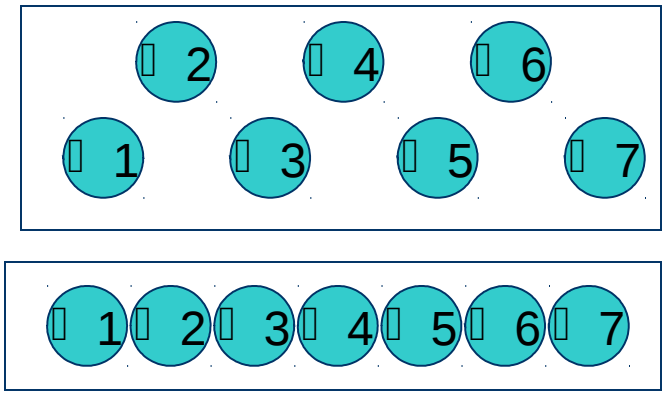
- ***the logical arrangement of data elements, combined with***
- ***the set of operations we need to access the elements.***

Basic Data Structures

- Structures include
  - linked lists
  - Stack, Queue
  - binary trees

What is Algorithm?

- **Algorithm:**
  - A computable set of steps to achieve a desired result
  - Ralationship to Data Structure
    - Example: Find an element



- Sumary

Algorithms + Data Structures = Programs

Algorithms ↔ Data Structures

- SEARCHING TECHNIQUES and Complexity of Algorithm

- CHAPTER 3: SEARCHING TECHNIQUES
- 1. ***LINEAR (SEQUENTIAL) SEARCH***
- ***2. BINARY SEARCH***
- ***3. COMPLEXITY OF ALGORITHMS***

SEARCHING TECHNIQUES

- To finding out whether a particular element is present in the list.
- 2 methods: linear search, binary search
- The method we use depends on how the elements of the list are organized
  - unordered list:
    - linear search: simple, slow
  - an ordered list
    - binary search or linear search: complex, faster

1. *LINEAR (SEQUENTIAL) SEARCH*

- How?
  - Proceeds by sequentially comparing the key with elements in the list
  - Continues until either we find a match or the end of the list is encountered.
  - If we find a match, the search terminates successfully by returning the index of the element
  - If the end of the list is encountered without a match, the search terminates unsuccessfully.

- 1. *LINEAR (SEQUENTIAL) SEARCH*

```
void lsearch(int list[],int n,int element)
{   int i, flag = 0;
   for(i=0;i<n;i++)
   if( list[i] == element)
        { cout<<“found at position”<<i);
          flag =1;
     break;   }
   if( flag == 0)
       cout<<“ not found”;
}
```

- flag: what for???

- 1. *LINEAR (SEQUENTIAL) SEARCH*

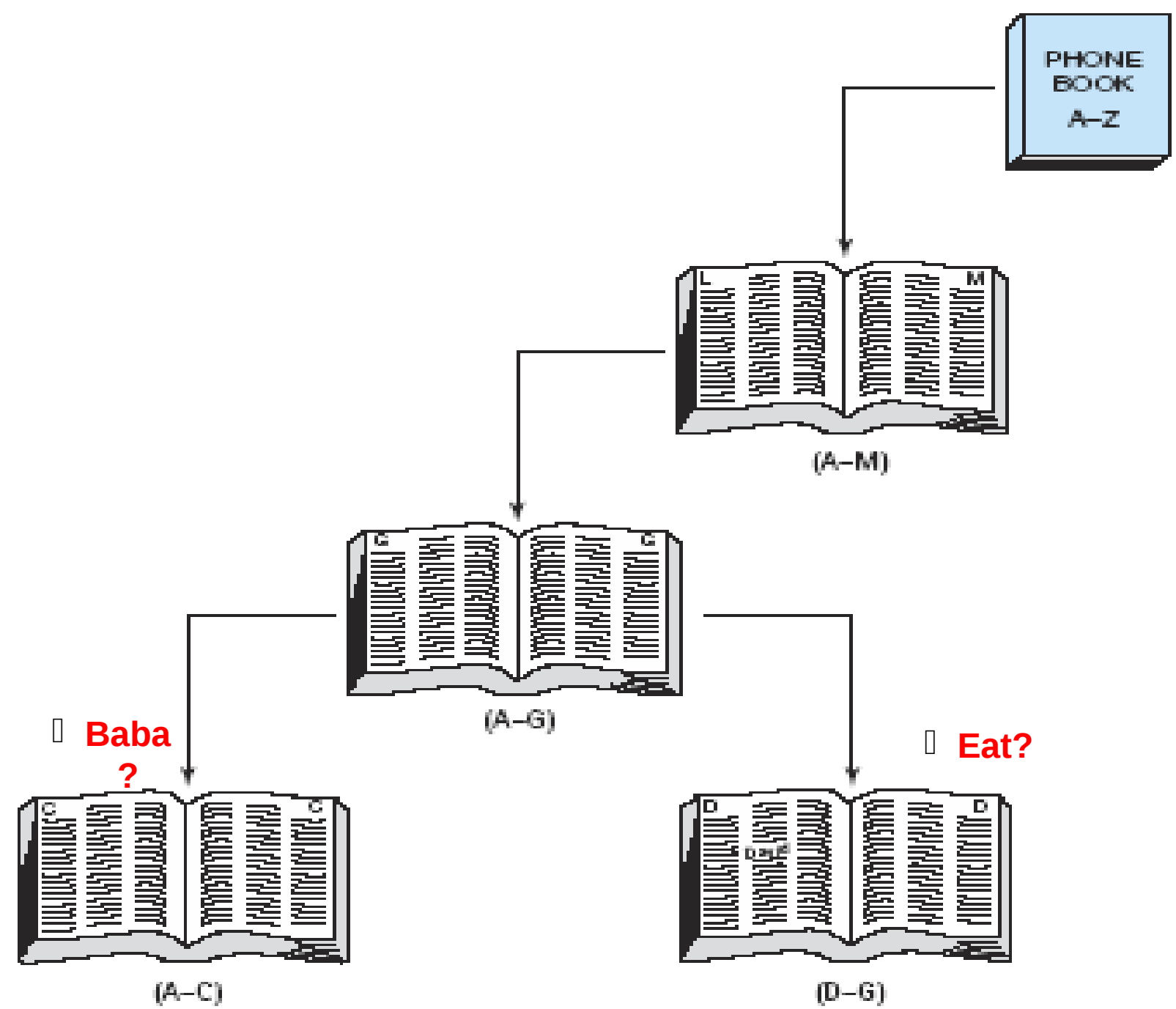
```
int lsearch(int list[],int n,int element)
{   int i, find= -1;
   for(i=0;i<n;i++)
          if( list[i] == element)
                {find =i;
          break;}
     return find;
}
```

- **Another way using flag**

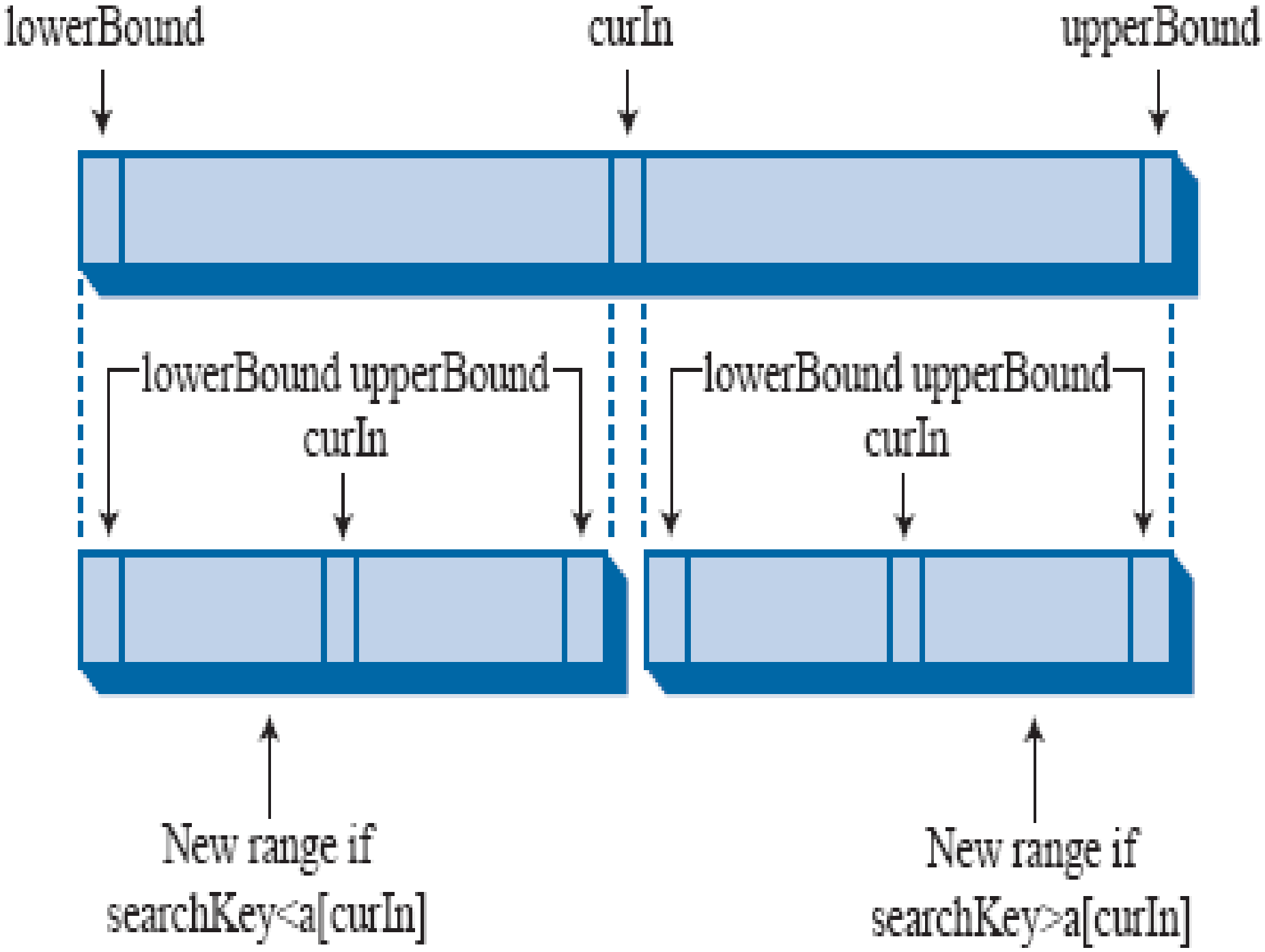
- **average time: O(*n*)**

2. BINARY SEARCH

- List must be a sorted one
- We compare the element with the element placed approximately in the middle of the list
- If a match is found, the search terminates successfully.
- Otherwise, we continue the search for the key in a similar manner either in the upper half or the lower half.



A binary search of the phone book


lowerBound
curIn
upperBound
lowerBound upperBound
curIn
lowerBound upperBound
curIn
New range if
searchKey<a[curIn]
New range if
searchKey>a[curIn]

```
void bsearch(int list[],int n,int element)
{
   int l,u,m, flag = 0;
   l = 0;    u = n-1;
   while(l <= u)
   {  m = (l+u)/2;
      if( list[m] == element)
            {cout<<"found:"<<m;
             flag =1;
             break;}
      else
            if(list[m] < element)
                  l = m+1;
            else
                  u = m-1;
   }
   if( flag == 0)
      cout<<"not found";
}
```

- **average time: O(log2*n*)**

- *BINARY SEARCH: Recursion*

```
int Search (int list[], int key, int left, int right)
    {
    if (left <= right) {
      int middle = (left + right)/2;
      if (key == list[middle])
           return middle;
      else if (key < list[middle])
           return Search(list,key,left,middle-1);
      else     return Search(list,key,middle+1,right);
    }
    return -1;
  }
```

3. COMPLEXITY OF ALGORITHMS

- In Computer Science, it is important to measure the quality of algorithms, especially the specific amount of a certain resource an algorithm needs
- Resources: time or memory storage (PDA?)
- Different algorithms do same task with a different set of instructions in less or more time, space or effort than other.
- The analysis has a strong mathematical background.
- The most common way of qualifying an algorithm is the Asymptotic Notation, also called Big O.

3. COMPLEXITY OF ALGORITHMS

- It is generally written as $O(f(x_1, x_2, \ldots, x_n))$
- Polynomial time algorithms,
  - $O(1)$ --- Constant time --- the time does not change in response to the size of the problem.
  - $O(n)$ --- Linear time --- the time grows linearly with the size (n) of the problem.
  - $O(n2)$ --- Quadratic time --- the time grows quadratically with the size (n) of the problem. In big O notation, all polynomials with the same degree are equivalent, so $O(3n2 + 3n + 7) = O(n2)$
- Sub-linear time algorithms
  - $O(\log n)$ -- Logarithmic time
- Super-polynomial time algorithms
  - $O(n!)$
  - $O(2n)$

3. COMPLEXITY OF ALGORITHMS

- Example1: complexity of an algorithm

```
void f ( int a[], int n )
{
    int i;
    cout<< "N = “<< n;
    for ( i = 0; i < n; i++ )
        cout<<a[i];
    printf ( "n" );
}
```

 2 * O(1) + O(N)

 O(N)

- *3. COMPLEXITY OF ALGORITHMS*

- Example2: complexity of an algorithm

```
void f ( int a[], int n )
{    int i;
     cout<< "N = “<< n;
     for ( i = 0; i < n; i++ )
               for (int j=0;j<n;j++)
                cout<<a[i]<<a[j];

       for ( i = 0; i < n; i++ )
               cout<<a[i];
     printf ( "n" );
}
```

- 2 * O(1) + O(N)+O(N2)

- O(N2)

- *3. COMPLEXITY OF ALGORITHMS*

- Linear Search
  - O(n).
- **Binary Search**
  - **O(log2 N)**

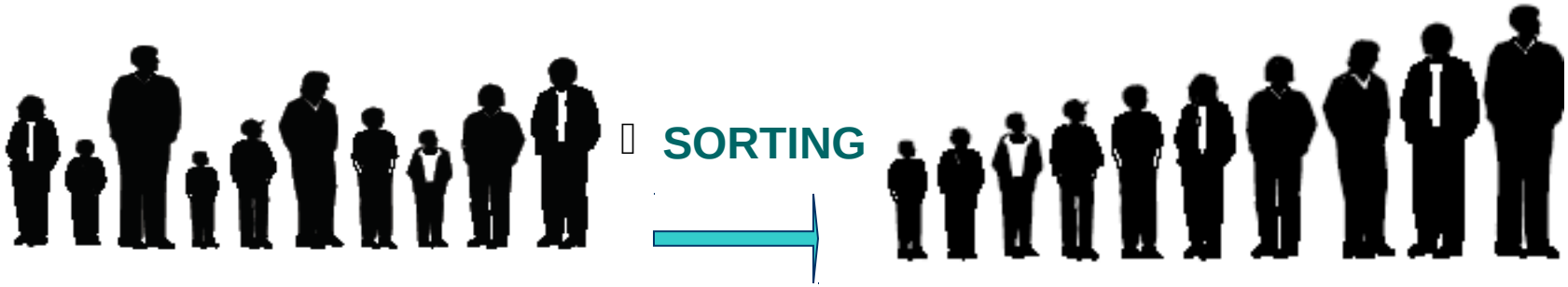
n	$\log_2 n$
10	3
100	6
1,000	9
10,000	13
100,000	16

Exercise

- Exercise:
  - Write a small program
    - Input the number of array
    - Input array of integer
    - Display array
    - Input a value. Using linear search to find position of first match item in array
  - Using 3 function: enterarray, displayarray,linearfind

SORTING TECHNIQUES

- Why?
  - Do binary search
  - Doing certain operations faster



SORTING TECHNIQUES

- Given a set (container) of n elements
  - E.g. array, set of words, etc.
- Suppose there is an order relation that can be set across the elements
- Goal Arrange the elements in ascending order

  - Start 1 23 2 56 9 8 10 100
  - End 1 2 8 9 10 23 56 100

SORTING TECHNIQUES

- Bubble sort, Insertion sort, Selection sort, Quick sort, Heap sort, Merge sort, Exchange sort …
- Focus on
  - Bubble sort
  - Insertion sort
  - Selection sort
  - Exchange sort
  - Quick sort

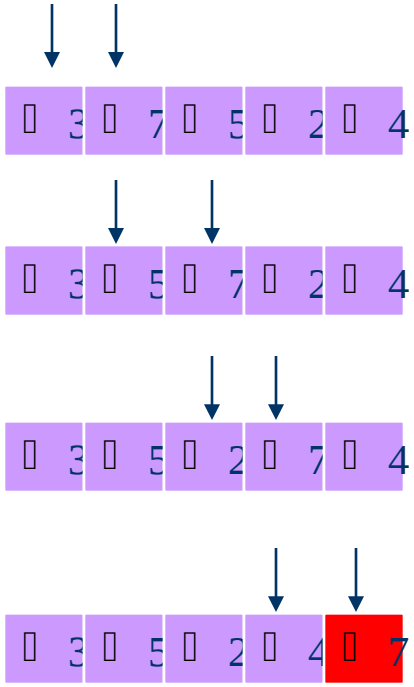
SORTING TECHNIQUES

	Average	Worst
Bubble sort	O(n2)	O(n2)
Exchange sort		
Insertion sort	O(n2)	O(n2)
Selection sort	O(n2)	O(n2)
Quick sort	O(nlogn)	O(n2)

1.Bubble sort: idea

- arrange the elements of the list by forming pairs of adjacent elements.
- The pair of the ith and (i+1)th element.
- If the order is ascending, we interchange the elements of the pair
- This will bring the highest value from among the remaining (*n−1*) values to the (*n*−1)th position.

- 1.Bubble sort: idea



- **Why it is called Bubble?**
- compare 3 and 7 ; 7 is > 3 so advance
- compare 7 and 5, 7 > 5 so swap them
- compare 7 and 2, 7 >4 so swap them
- compare 7 and 4, 7 >4 so swap them
- End of pass 1; notice that 7 is in the right place

2.Bubble sort: idea

- Simplest sorting algorithm
- Idea:
  - 1. Set flag = false
  - 2. Traverse the array and compare pairs of two elements
    - 1.1 If E1 ≤ E2 – OK
    - 1.2 Iφ E1 > E2 τηεν Σωιτχη(E1, E2) ανδ σετ φλαγ = τρυε
  - 3. Iφ φλαγ = τρυε γοτο 1.
- Ωηατ ηαππενσ?

- 1.Bubble sort:algorithm idea

```
void bubbleSort (Array S,  length n) {
    boolean isSorted = false;
    while(!isSorted)
    {
        isSorted = true;
        for(i = 0; i<n; i++)
            if(S[i] > S[i+1])
                {
                swap(S[i],S[i+1];)
                isSorted = false;
                }
    }
```

- 1.Bubble sort: implement

```
void bsort(int list[], int n)
{
int count,j;

 for(count=0;count<n-1;count++)
   for(j=0;j<n-1-count;j++)
     if(list[j] > list[j+1])
        swap(list[j],list[j+1]);
}
```

2. Exchange Sorting

- Method : make n-1 passes across the data, on each pass compare adjacent items, swapping as necessary (n-1 compares)
- O(n2)

- 2. Exchange Sorting

```
void Exchange_sort(int arr[], int n)
{
int i,j;

 for(i=0;i<n-1;i++)
   for(j=i+1;j<n;j++)
     if(arr[i] > arr[j])
        swap(arr[i],arr[j]);
}
```

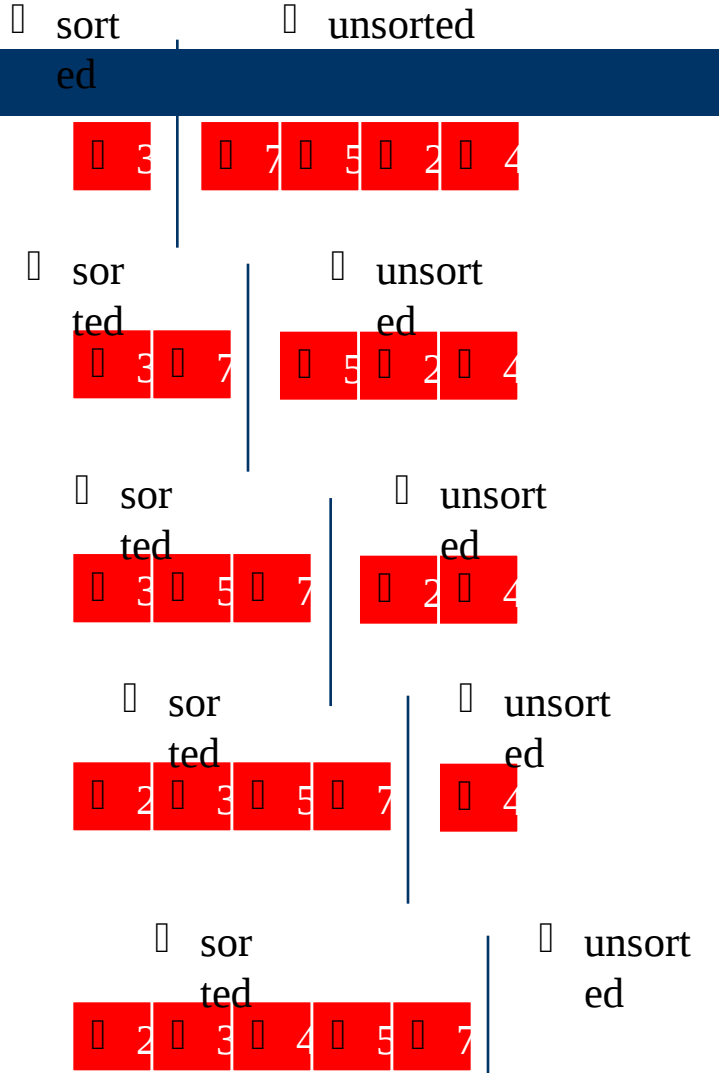
2. Exchange Sorting

- Notes:
  - on each successive pass, do one less compare, because the last item from that pass is in place
  - if you ever make a pass in which no swap occurs, the sort is complete
  - There are some algorithms to improve performance but Big O will remain O(n2)

3. Insertion Sort

- Strategy: divide the collection into two lists, one listed with one element (sorted) and the other with the remaining elements.
- On successive passes take an item from the unsorted list and insert it into the sorted list so the the sorted list is always sorted
- Do this until the unsorted list is empty

3. Insertion Sort



- take an item from the unsorted list (7) and insert into the sorted list
- take next item from the unsorted list (5) and insert into the sorted list
- take next item from the unsorted list (2) and insert into the sorted list
- take next item from the unsorted list (4) and insert into the sorted list

- 3. Insertion Sort

```
void insertionSort(int arr[], int n){
  int j, key;
  for(int i = 1; i < n; i++){
     key = arr[i];
     j = i - 1;
     while(j >= 0 && arr[j] > key)
     { arr[j + 1] = arr[j];
       j = j - 1;
      }
     arr[j + 1] = key;
  }
}
```

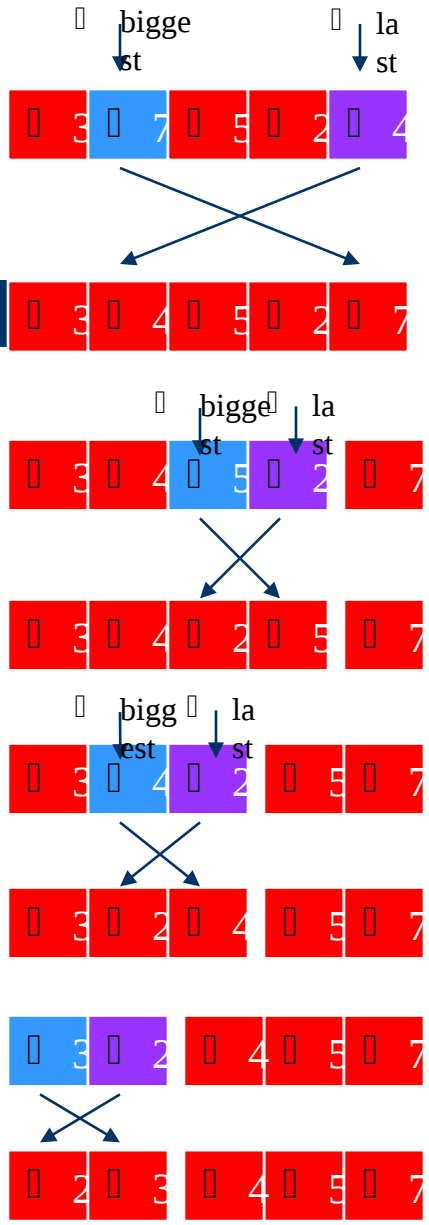
3. Insertion Sort

- Note that each insertion could be O(n-1) and there are n-1 insertions being done therefore Big O is O(n2)
- This is very much like building an ordered linked list except there is more data movement

4. Selection Sort

- Strategy: make a pass across the data looking for the largest item, swap the largest with the last item in the array.
- On successive passes (n-1) assume the array is one smaller (the last item is in the correct place) and repeat previous step

- 4. Selection Sort



- 4. Selection Sort

```
void selection_sort(int arr[], int n)
{int i, j, min;
for (i = 0; i < n - 1; i++)
  {
  min = i;
  for (j = i+1; j < n; j++)
     { if (list[j] < list[min])  min = j;  }
  swap(arr[i],arr[min]);
  }
}
```

4. Selection Sort

- Notice that in selection sort, there is the least possible data movement
- There are still n-1 compares on sublists that become one item smaller on each pass so, Big O is still O(n2)
- This method has the best overall performance of the O(n2) algorithms because of the limited amount of data movement

5. Quick Sort

- This sorting method by far outshines all of the others for flat out speed
- Big O is log2n
- there are problems, worst case performance is when data is already in sorted order or is almost in sorted order (we'll analyze this separately)
- and there are solutions to the problems
- and there is an improvement to make it faster still

5. Quick Sort

- Sorting algorithms that rely on the “DIVIDE AND CONQUER” paradigm
  - One of the most widely used paradigms
  - Divide a problem into smaller sub problems, solve the sub problems, and combine the solutions
  - Learned from real life ways of solving problems

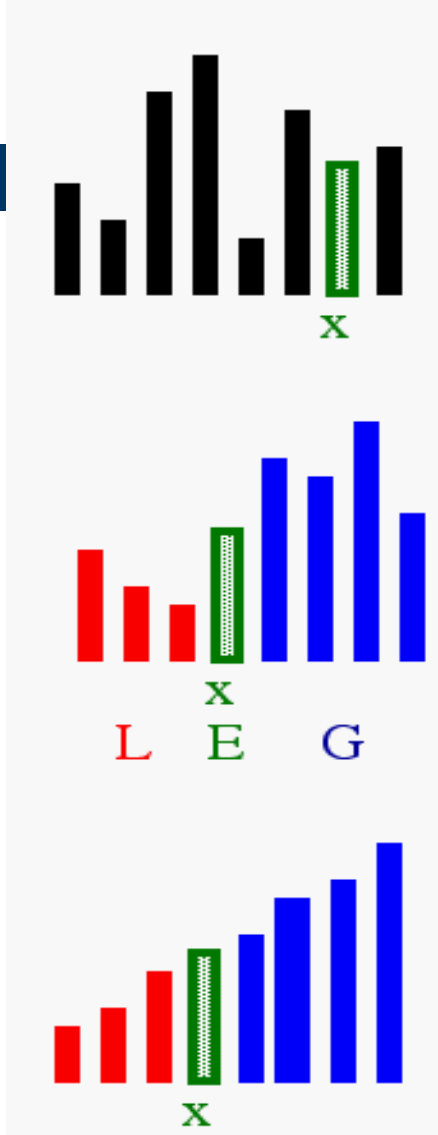
5. Quick Sort

- Another divide-and-conquer sorting algorihm
- To understand quick-sort, let's look at a high-level description of the algorithm

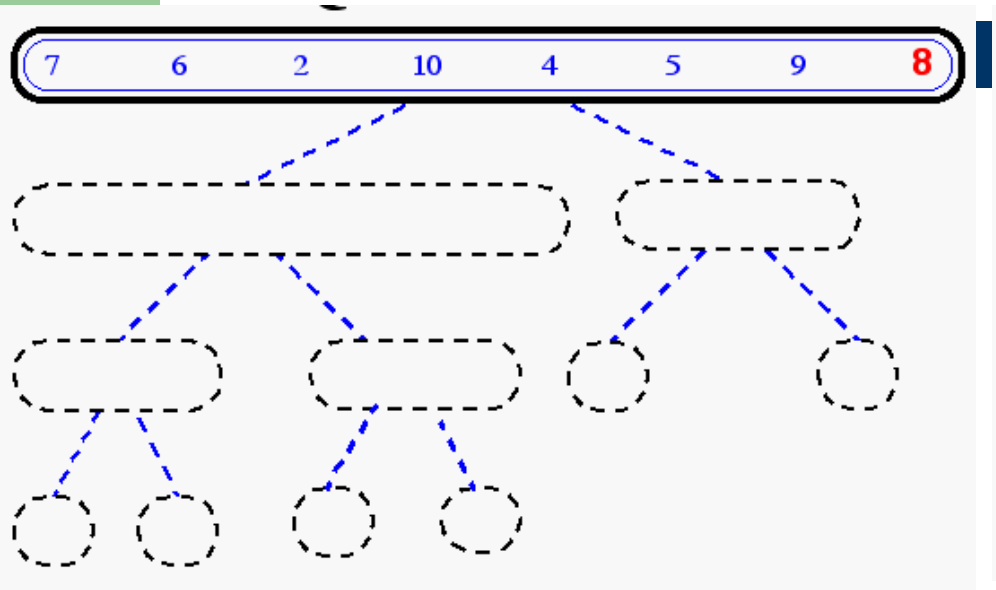
1) **Divide** : If the sequence S has 2 or more elements, select an element x from S to be your **pivot**. Any arbitrary element, like the last, will do. Remove all the elements of S and divide them into 3 sequences:
   - L, holds S's elements less than x
   - E, holds S's elements equal to x
   - G, holds S's elements greater than x
2) **Recurse**: Recursively sort L and G
3) **Conquer**: Finally, to put elements back into S in order, first inserts the elements of L, then those of E, and those of G.

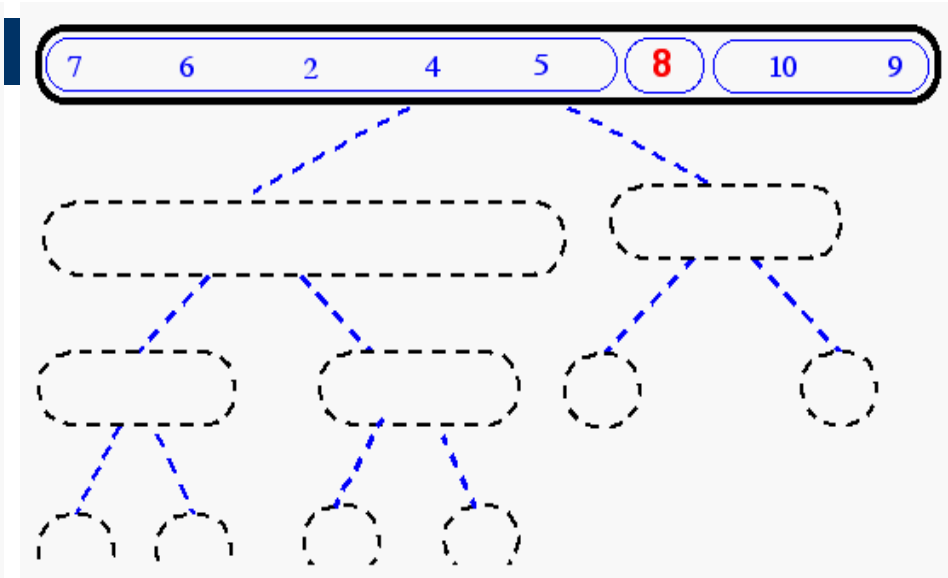
5. Quick Sort: idea

- 1) **Select**: pick an element
- 2**) Divide**: rearrange elements so that x goes to its final position E
- 3) **Recurse and Conquer**: recursively sort
-



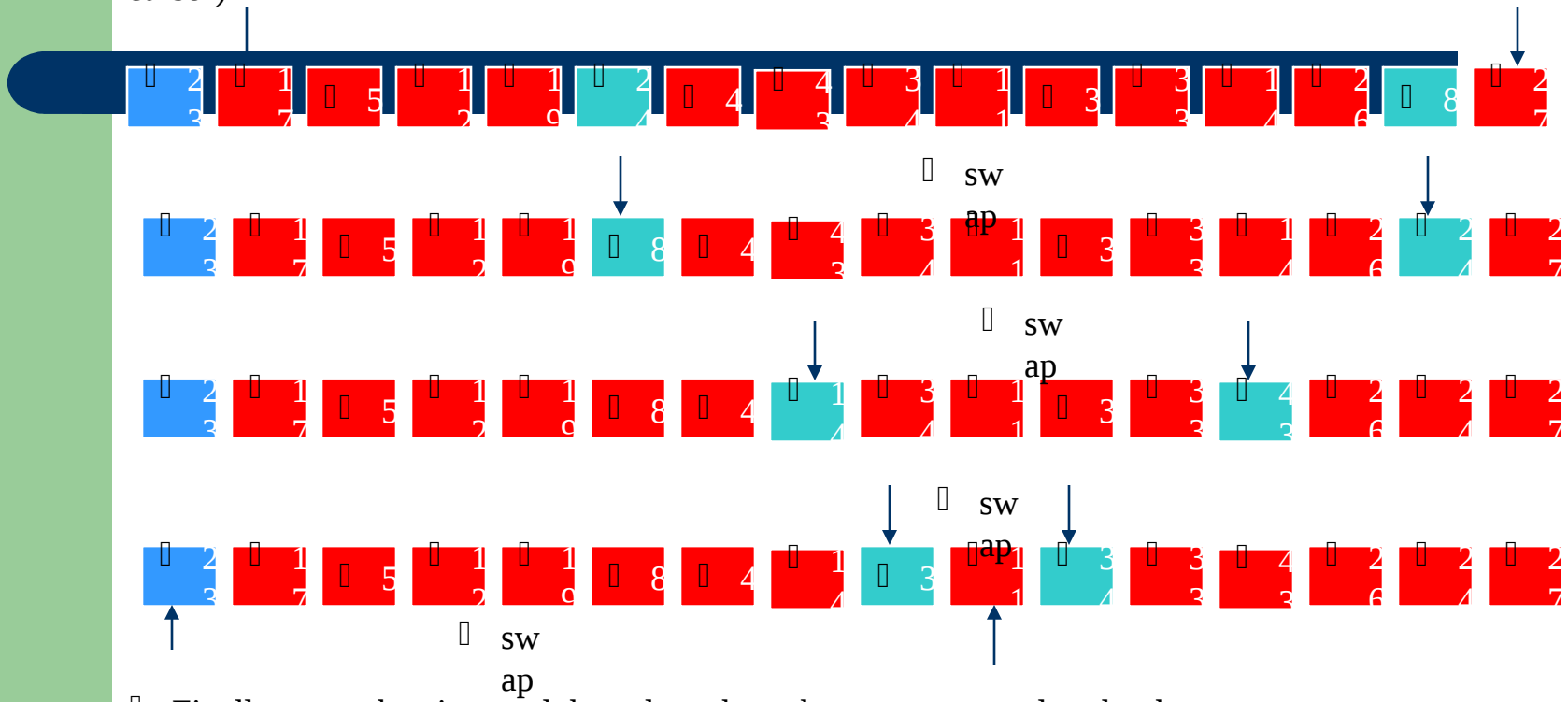
- 5. Quick Sort: idea



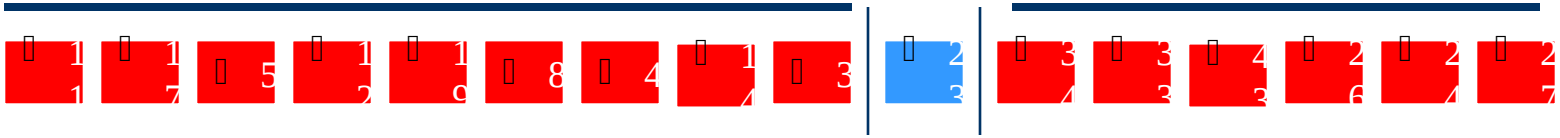


Quick Sort

- Pick the leftmost element as the pivot (23). Now , start two cursors (one at either end) going towards the middle and swap values that are > pivot (found with left cursor) with values < pivot (found with right cursor)



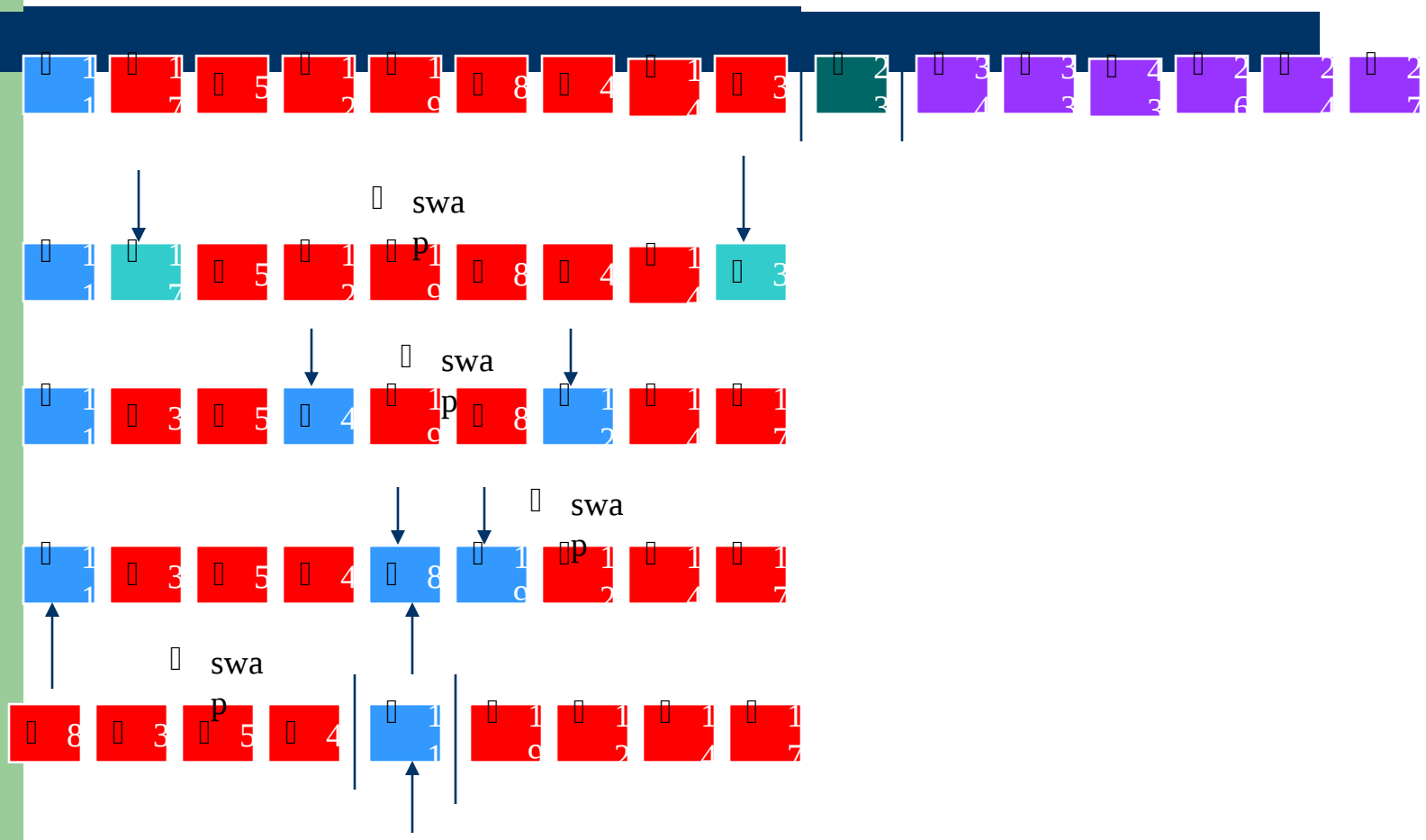
- Finally, swap the pivot and the value where the cursors passed each other



- Note : 23 is now in the right place and everything to its left is < 23 and everything to its right is > 23

Quick Sort

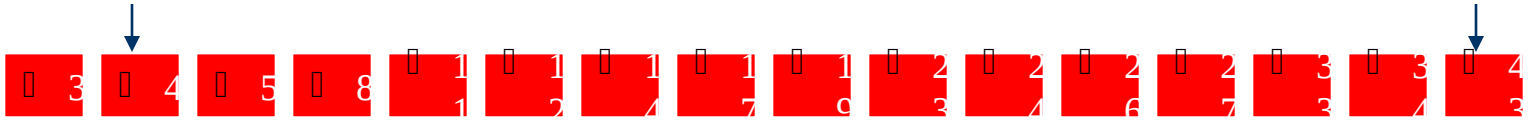
- Now, repeat the process for the right partition



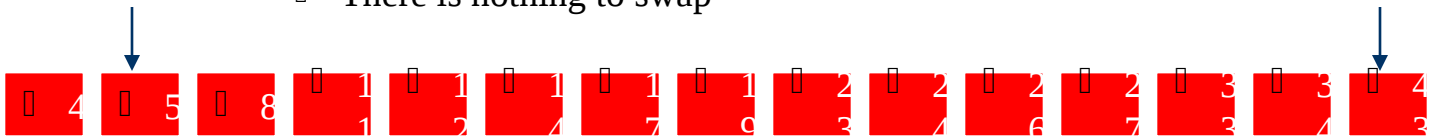
- Note: the 11 is now in the right place, and the left partition is all < pivot and the right partition is all > pivot

Quick Sort (worst case)

- If the data is already sorted watch what happens to the partitions



- There is nothing to swap



- Again, nothing to swap…..
- The partitions are always the maximum size and the performance degrades to O(n2)

Quick Sort

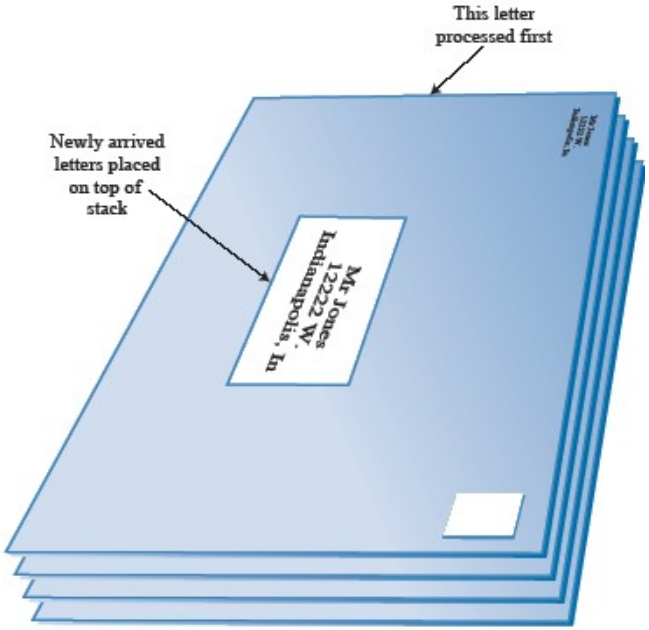
```
void quickSort(int Arr[], int lower, int upper)
{
int x = Arr[(lower + upper) / 2];
int i = lower;  int j = upper;
do{
    while(Arr[i] < x)    i ++;
    while (Arr[j] > x)   j --;
    if (i <= j)
        {
        swap(Arr[i], Arr[j]);
        i ++;  j --; }
 }while(i <= j);

if (j > lower)
    quickSort(Arr, lower, j);
if (i < upper)
    quickSort(Arr, i, upper);
}
```

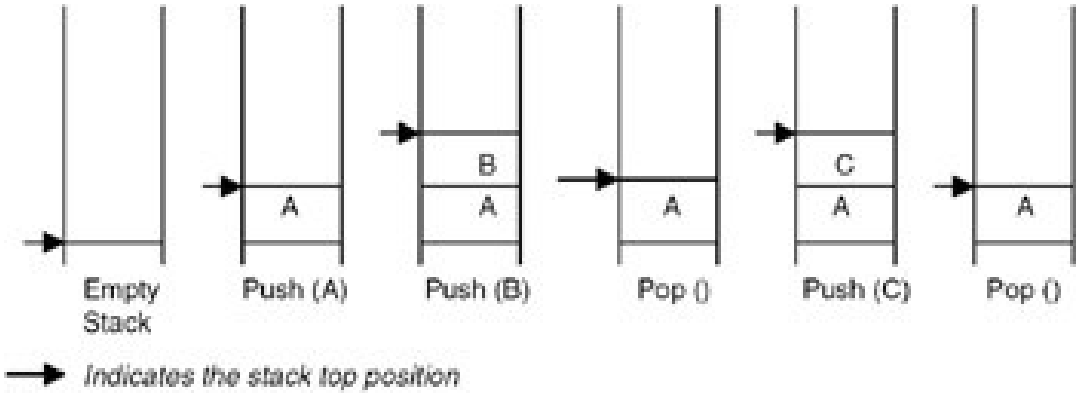
STACKS AND QUEUES

- STACKS: concept
- QUEUES : concept
- STACKS,QUEUES : implement
  - Using array
  - Using Linked List (next chapter)

1.Stack

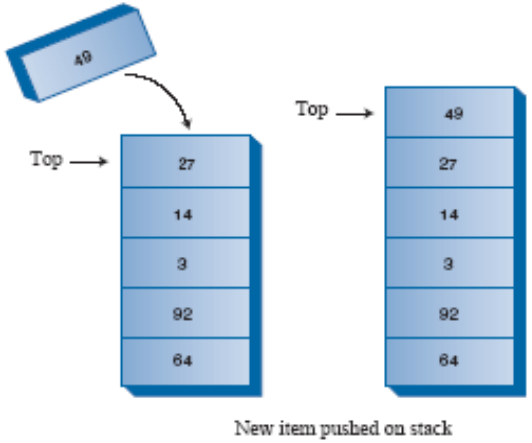
- LIFO (last in first out)



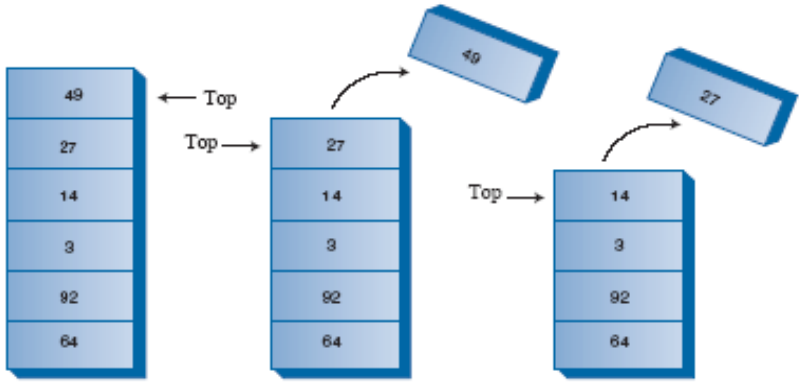


- 1.Stack

- Managing Top element



New item pushed on stack



Two items popped from stack

- 1.Stack: implement using array

```
#define MAX 10
void main()
{
   int stack[MAX];
   int top = -1;
   push(stack,top, 10 );

   pop(stack,top,value);
   int value;

  cout<<value;
}
```

- 1.Stack: implement using array

```
void push(int stack[], int &top, int value)
{
   if(top < MAX )
   {
      top = top + 1;
      stack[top] = value;
   }
  else
      cout<<"The stack is full";
}
```

- 1.Stack: implement using array

```
void pop(int stack[], int &top, int &value)
{
if(top >= 0 )
   {
      value = stack[top];
      top = top - 1;
   }
else
      cout<<"The stack is empty ";
}
```

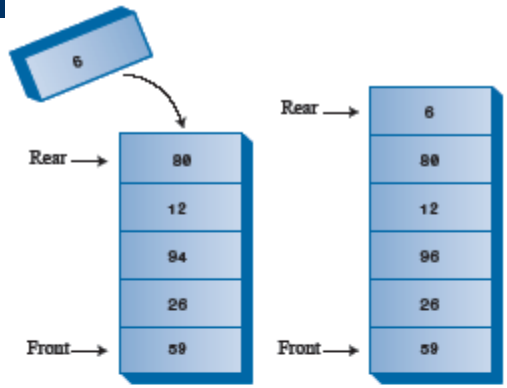
- 2.QUEUE

- *FIFO* (*first in first out*)

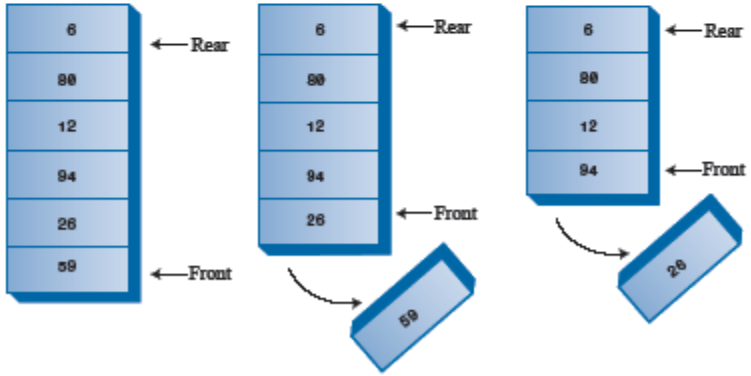
A FIFO queue

- 2.QUEUE: implement using array

- A circular queue



New item inserted at rear of queue



Two items removed from front of queue

- 2.QUEUE: implement using array

```
#define MAX 10
void main()
{
   int queue[MAX];
   int bottom,top,count=0;
   bottom=top=-1;

   enqueue(queue,count,top, 100       );
   int value;
   dequeue(queue,count,bottom,top,value);

}
```

- 2.QUEUE: implement using array

```
void enqueue(int queue[],int &count, int &top, int value)
{
   if(count< MAX)
   {
      count++;
        top= (top +1)%MAX;
      queue[top] = value;
   }
   else
      cout<<"The queue is full";
}
```

- 2.QUEUE: implement using array

```
void dequeue(int queue[], int &count,int &bottom,int top, int &value)
{
   if(count==0)
   {
      cout<<"The queue is empty";
      exit(0);
   }

   bottom = (bottom + 1)%MAX;
   value = queue[bottom];
   count--;
}
```

3. Application of stack, queue

- Stack: Expression evaluation
  - a*(b–c)/d => abc–*d/
- Queue: priority queues

- Exercise:

- Implement: 5 sort algorithms
- Implement stack, queue using array
  - Menu with 4 choices
    - Add, remove, display, exit

- Linked List

- ***THE CONCEPT OF THE LINKED LIST***
- SINGLE LINKED LIST
- DOUBLE LINKED LIST
- CIRCULAR LINKED LIST

THE CONCEPT OF THE LINKED LIST

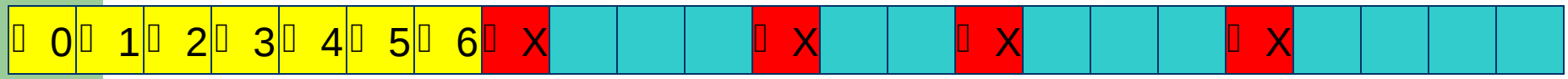
- the size requirement need not be known at compile time
- A *linked list* is a data structure that is used to model such a dynamic list of data items, so the study of the linked lists as one of the data structures is important.

Array and *LINKED LIST*

- ARRAY
  - sequential mapping, elements are fixed distance apart
  - makes insertion or deletion at any arbitrary position in an array a costly operation
- Linked List
  - not necessary that the elements be at a fixed distance apart
  - an element is required to be linked with a previous element of the list
  - done by storing the address of the next element

- Array and *LINKED LIST*

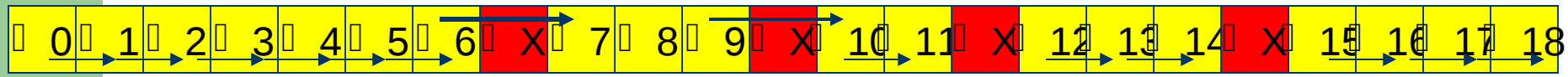
- Array: max length=7



0 1 2 3 4 5 6

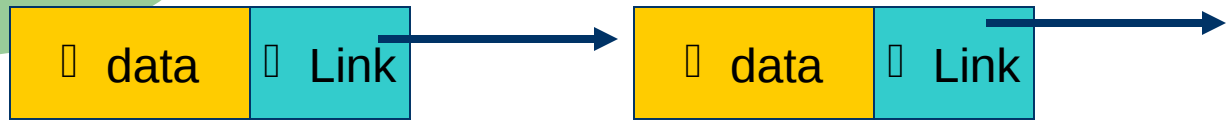
- Get element by order number

- Linked List: max length=18

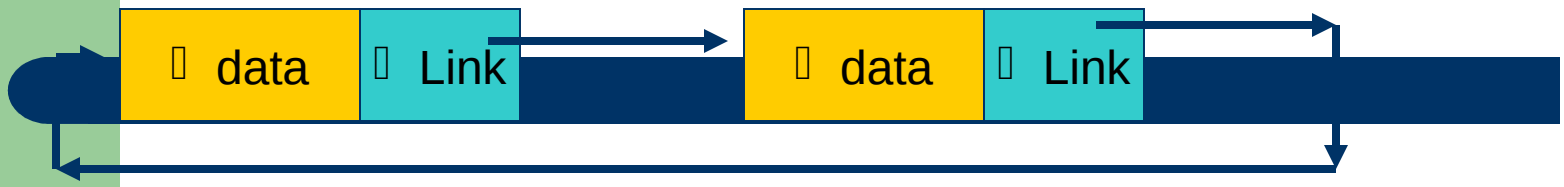


Type of Linked List

1



2



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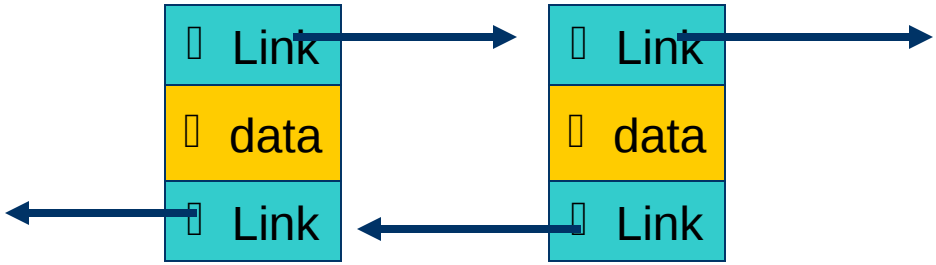
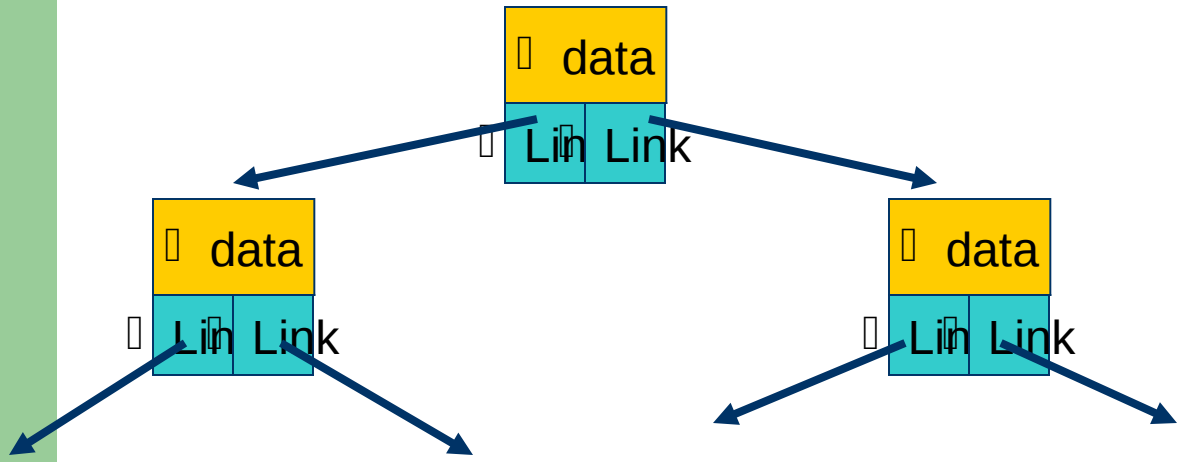


4



- 4 things when building Linked List

- 1. Structure
  - Data element
  - Link field element
- 2. The way to make link between elements
  - First, last, middle element
- 3. **How many node to take all list elements, how to take all list**
- 4. Basic operations:
  - Insert new element (every position)
  - Delete (every position)
  - Find
  - Notes: Check value change in step 3

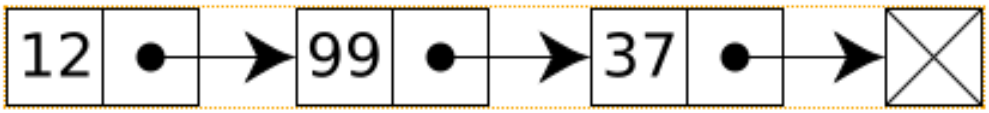
- 2.Singly Linked List

- 1. Structure
  - Data element
  - Link field element
- 2. The way to make link between elements
  - First, last, middle element
- 3. **How many node to take all list elements , how to take all list**
- 4. Basic operations:
  - Insert new element (every location)
  - Delete (every position)
  - Find

- 2.Singly Linked List

- 1. Structure

```
struct Node
{
        int data;
        Node *link;

};
```



A singly-linked list containing two values: the value of the current node and a link to the next node

- 2.Singly Linked List

- 1. Structure: how to use one node

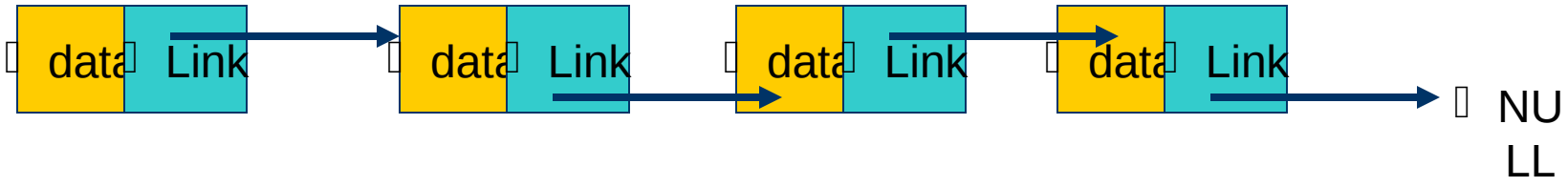
```
Node a;

a.data=10;
a.link=NULL;

cout<<a.data;
```

```
Node *b;
b=new Node;

b->data=20;
b->link=NULL;
cout<<b->data;
delete b;
```

- Compare??? What is the different? Delele and change address

- 2.Singly Linked List

- 2. The way to make link between elements
  - First, last, middle element

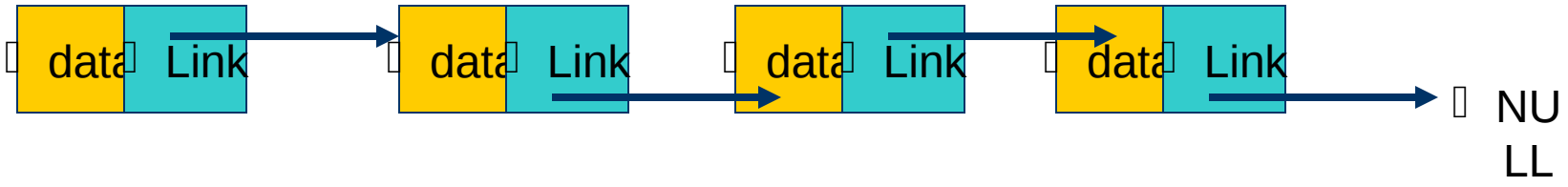


- Head
- Middle
- Last

- 2.Singly Linked List

- 3. **How many node to take all list elements, how to take all li**

pTail

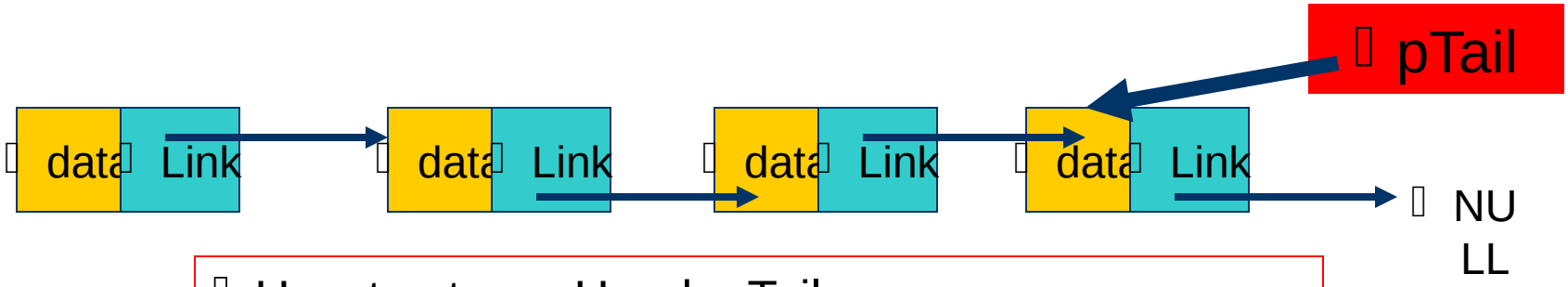


pHead

- Why
- +from pHead, can we list all items?
- +from pHead, can we do everything with list: insert new, delete?
- +Do we need pTail?

- 2.Singly Linked List

- 3. **How many node to take all list elements, how to take all list**



- How to store pHead, pTail

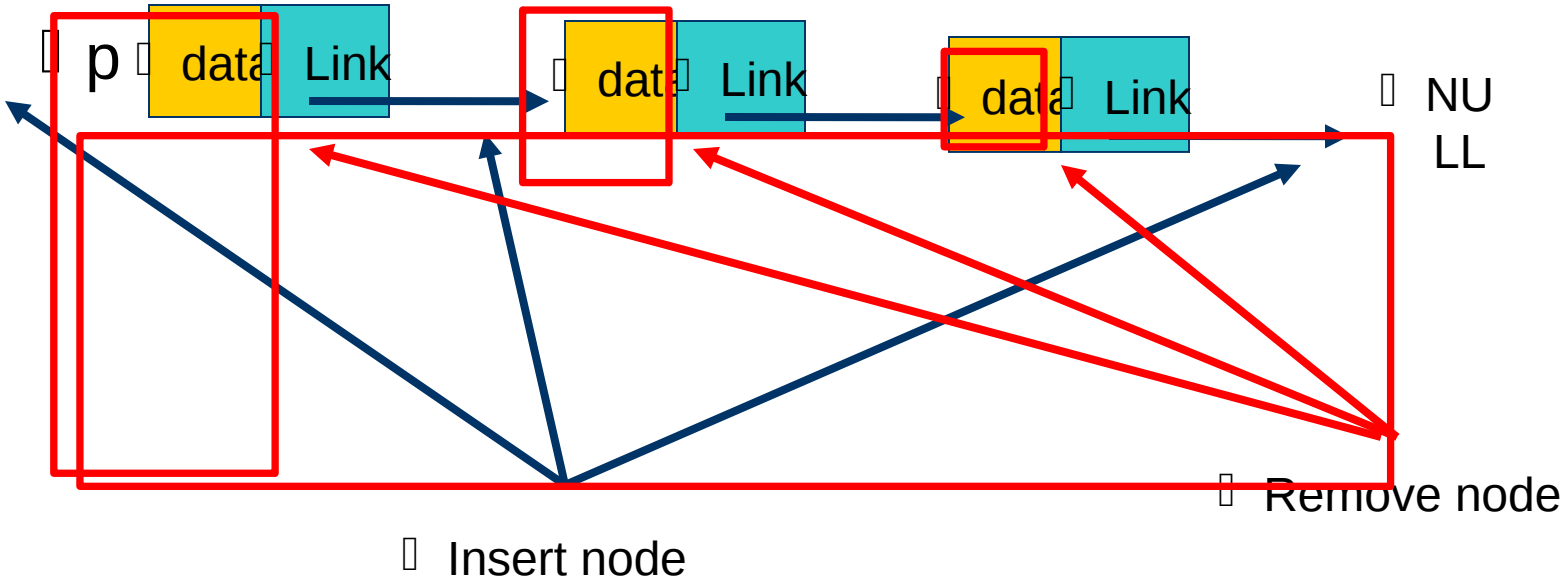
pHead

- Type 1: Node *pHead=NULL, *pTail=NULL;

- Type 2: typedef struct Node *List;
- List pHead, pTail;

- 2.Singly Linked List

- 4. Basic operations:

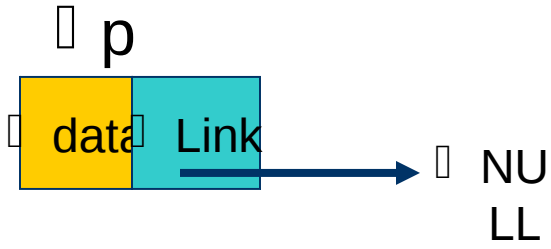


- creating new node

- 2.Singly Linked List

- 4. Basic operations: creating new node

```
Node * createNewNode(int X)
{
Node *p=new Node;
If (p!=NULL)
        {
        p->data=X;
        p->link=NULL;
        }
return p;
}
```



2.Singly Linked List: using Phead only

```
void addnodeatFirst(node *newnode)
{
        if (pHead==NULL)
                {
                        pHead =newnode;
                }
        else
                {
                        newnode->next=pHead;
                        pHead =newnode;
                }
}
```

- Insert Node at First

2.Singly Linked List: using Phead only

```
void displaylist()
{
node *temp=h;
while (temp!=NULL)
        {
                cout<<temp->data<<" ";
                temp=temp->next;
        }
}
```

- Seek Nodes

2.Singly Linked List: using Phead only

```
void RemoveNodeatFirst()
{
if (pHead!=NULL)
                {
                node *t= pHead;
                 pHead = pHead ->next;
                delete t;
                }
}
```

- Remove Node at First

2.Singly Linked List: using Phead only

```
node *find(int key)
{
node *temp=h;
while (temp!=NULL && temp->data!=key)
        temp=temp->next;
return temp;
}
```

- Find Node

2.Singly Linked List: using Phead only

```
void removeatlast()
{
node *t=h;
node *truoc=t;
while (t->next!=NULL)
        {
        truoc=t;
        t=t->next;
        }

truoc->next=NULL;
delete t;
}
```

- Remove Node at Last

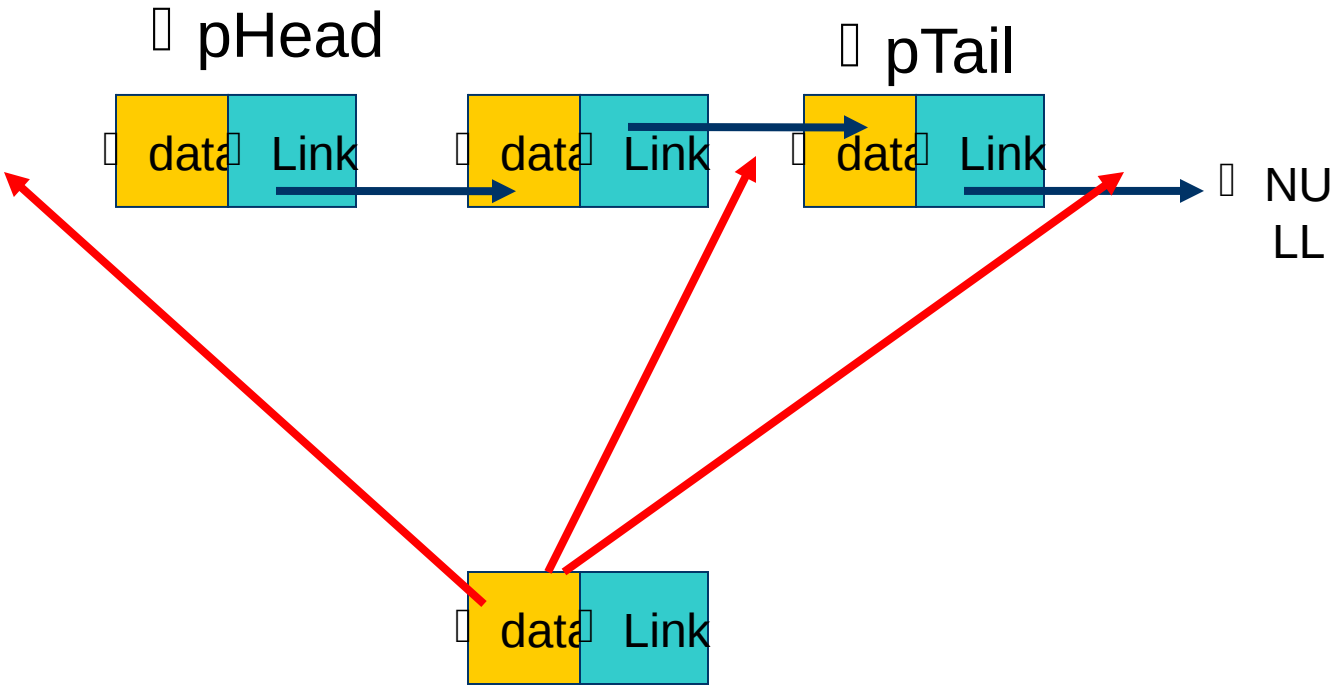
2.Singly Linked List: using Phead only

```
void insertatlast(node *newnode)
{
node *t=h;
while (t->next!=NULL)
        t=t->next;

t->next=newnode;
}
```

- Insert Node at Last

2.Singly Linked List:using pHead & pTail

- 4. Basic operations: Insert new node



- 2.Singly Linked List

- 4. Basic operations: Insert new node at First

```
void Insert_First (node *newnode)
{  if ( pTail == NULL )
        {
          pHead=pTail =newnode ;
         }
      else
        {
         newnode->next=pHead ;
        pHead=newnode;
        }
}
```

- 2.Singly Linked List

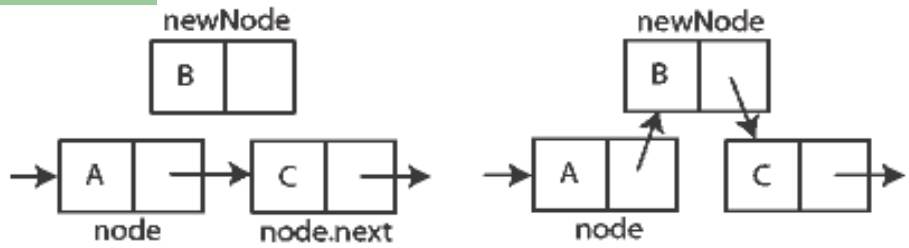
- 4. Basic operations: Insert new node at Last

```
void Insert_Last (node *newnode)
{  if ( pTail == NULL )
        {
           pHead=pTail =newnode ;
         }
      else
        {
         pTail>next=newnode ;
        pTail=newnode;
        }
}
```

- 2.Singly Linked List

- 4. Basic operations: Insert new node after node

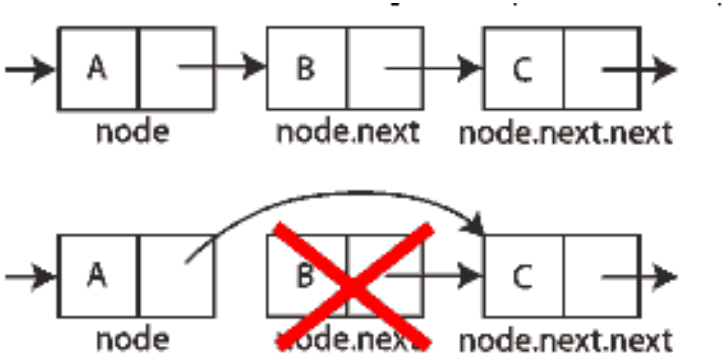
```
void Insert_after (node *newnode,node
*p)
{
If (p!=pTail)
   {
     newnode->next=p>next;
     p->next=newnode;
}
else
   insert_Last (newnode);
 }
```



- 2.Singly Linked List

- 4. Basic operations: remove node at First

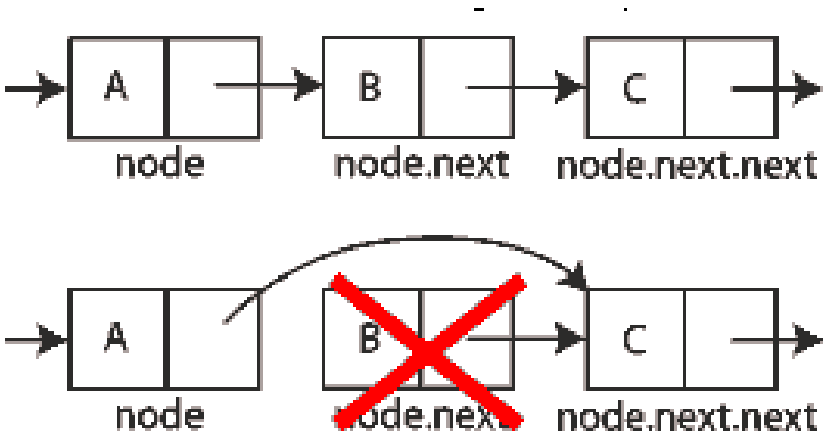
```
void removeNodeAtFirst ()
{
If (pHead!=NULL)
        {
        Node *temp=pHead;
        pHead = pHead >next;
        delete temp;
        }
}
```



- 2.Singly Linked List

- 4. Basic operations: remove node after

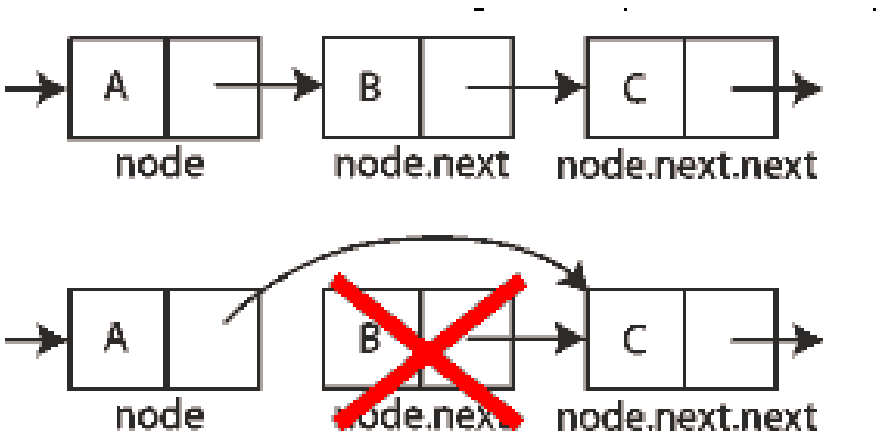
```
void removeNodeAfter (node *p)
{
   Node *temp=p>next;
    p->next=p->next->next;
   delete temp;
}
```



- 2.Singly Linked List

- 4. Basic operations: remove node at Last

  - void removeNodeatLast ()
  - {
  - ???
  - }



- 2.Singly Linked List

- 4. Basic operations: Seek all nodes

```
Void  Display()
{
node *p=pHead;
while (p!=NULL)
        {
        cout<<p->data<<“ “;
        p=p->next;
        }
}
```

- 2.Singly Linked List

- 4. Basic operations: count number of nodes

```
int Count ()
{
int count=0;
node *p=pHead;
while (p!=NULL)
        {
        count+=1;
        p=p->next;
        }
return count;
}
```

- 2.Singly Linked List

- 4. Basic operations: Remove List

  - Remove pHead node
  - Do until pHead is NULL

2.Singly Linked List: Demo

- Write a program for buiding single linked list: using pHead only
  - Display menu
    - Add one node at first
    - Add one node at last
    - Add many node at first
    - Add many node at last
    - Select and display n(th) node
    - Find one node
    - Add one node after select node
    - Display node count
    - Display List
    - Remove one node
    - Remove List
    - Get sum of all nodes

- Cont…

- Find node
- Single linked list: pHead and pTail
- Circular single linked list
- Double Linked List

- Find Node

- Using While
- Using Loop

- Find Node: using while loop

```
Node* temp;  //Node *temp=new Node();???
temp=pHead;
while (temp->data!=Xvalue)
      temp=temp->next;
==========================
=
Node* temp;  //Node *temp=new Node();???
temp=pHead;
while (temp!=NULL && temp->data!=Xvalue)
      temp=temp->next;
```

- Exactly

- May be not found

- Find Node: using for loop

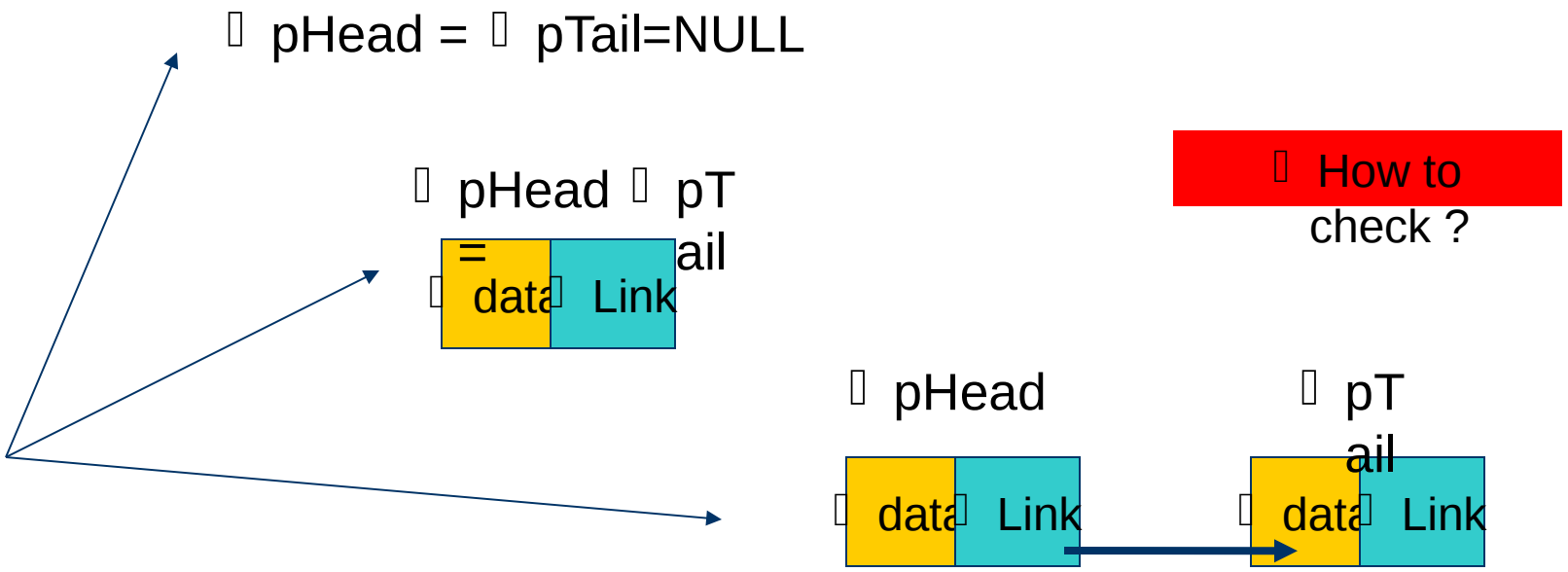
```
for (Node* temp=pHead;temp->data!=Xvalue;
temp=temp->next);
```

3.Singly Linked List: pHead and pTail

- Same to manage list with pHead
- Take care: cases change pTail
  - Add node
  - Remove Node

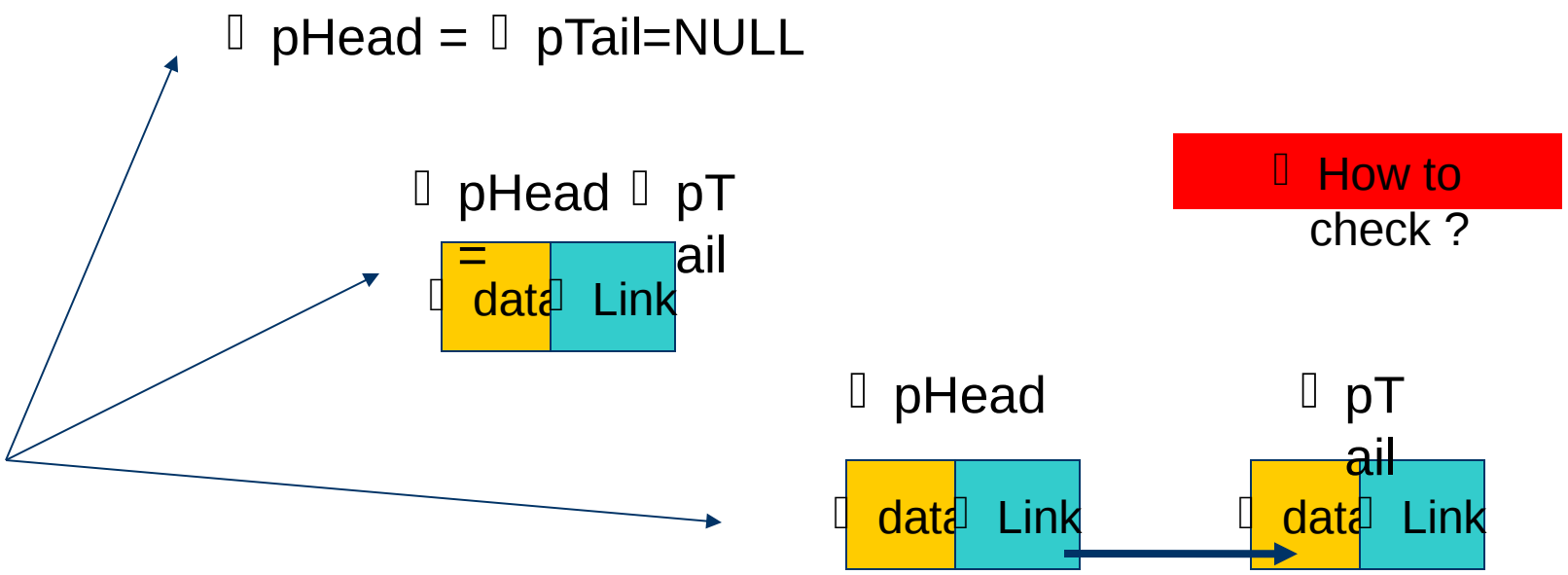
3.Singly Linked List: pHead and pTail

- When pTail is changed?
  - Insert new node at first



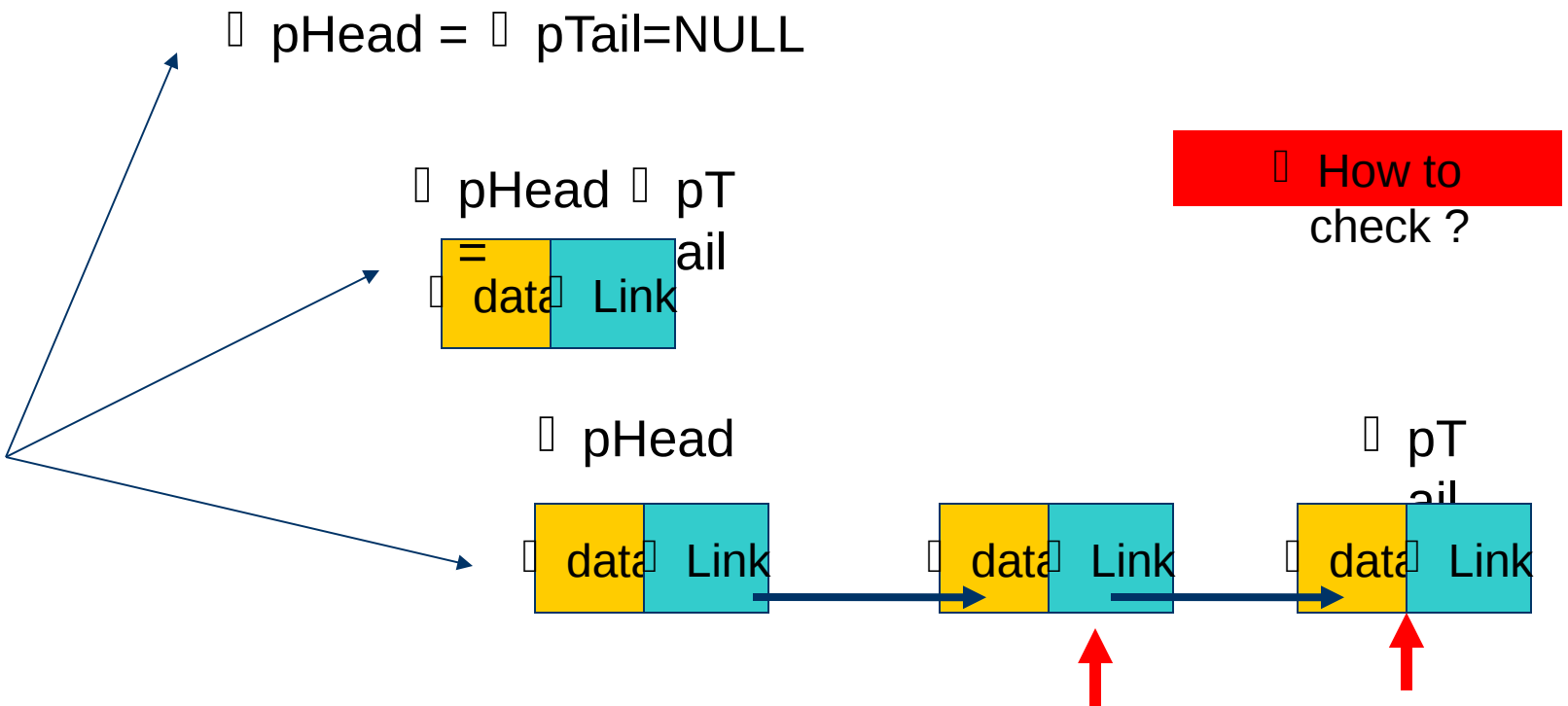
3.Singly Linked List: pHead and pTail

- When pTail is changed?
  - Insert new node at Last



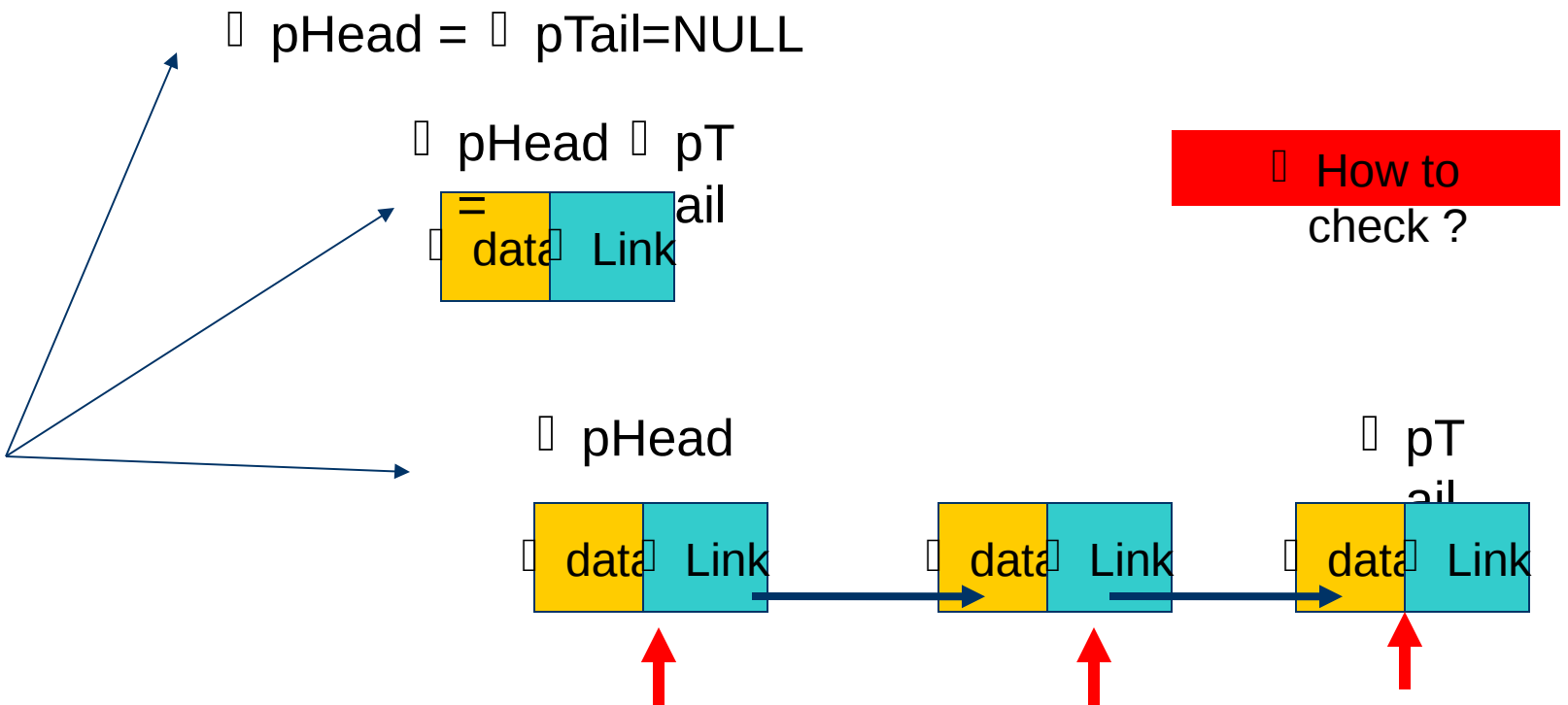
3.Singly Linked List: pHead and pTail

- When pTail is changed?
  - Insert new node after one node



3.Singly Linked List: pHead and pTail

- When pTail is changed?
  - Remove node

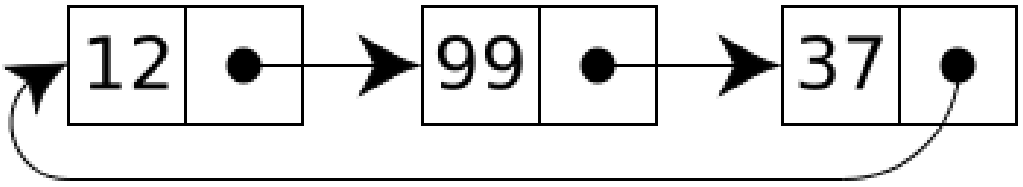


3.Singly Linked List: pHead and pTail

- Example:
  - Write function to insert at last
  - Single linked list with pHead and pTail

4. Circular single linked list

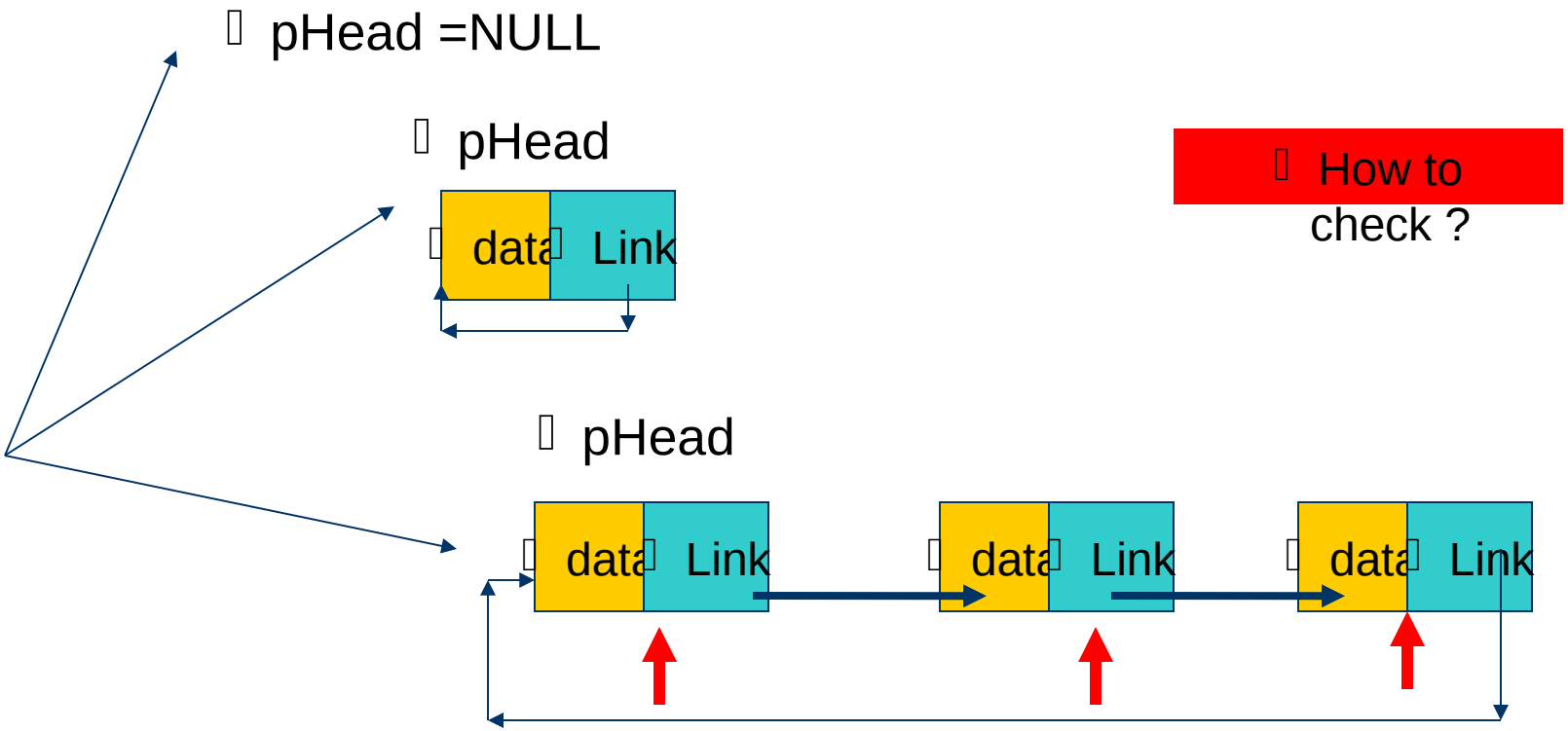
- Circular
  - Last node point to first node
  - Draw like Circle
- When using Circular single linked list
  - Every node in list had the same position
  - Needn’t Head, Tail



A circularly-linked list containing three integer values

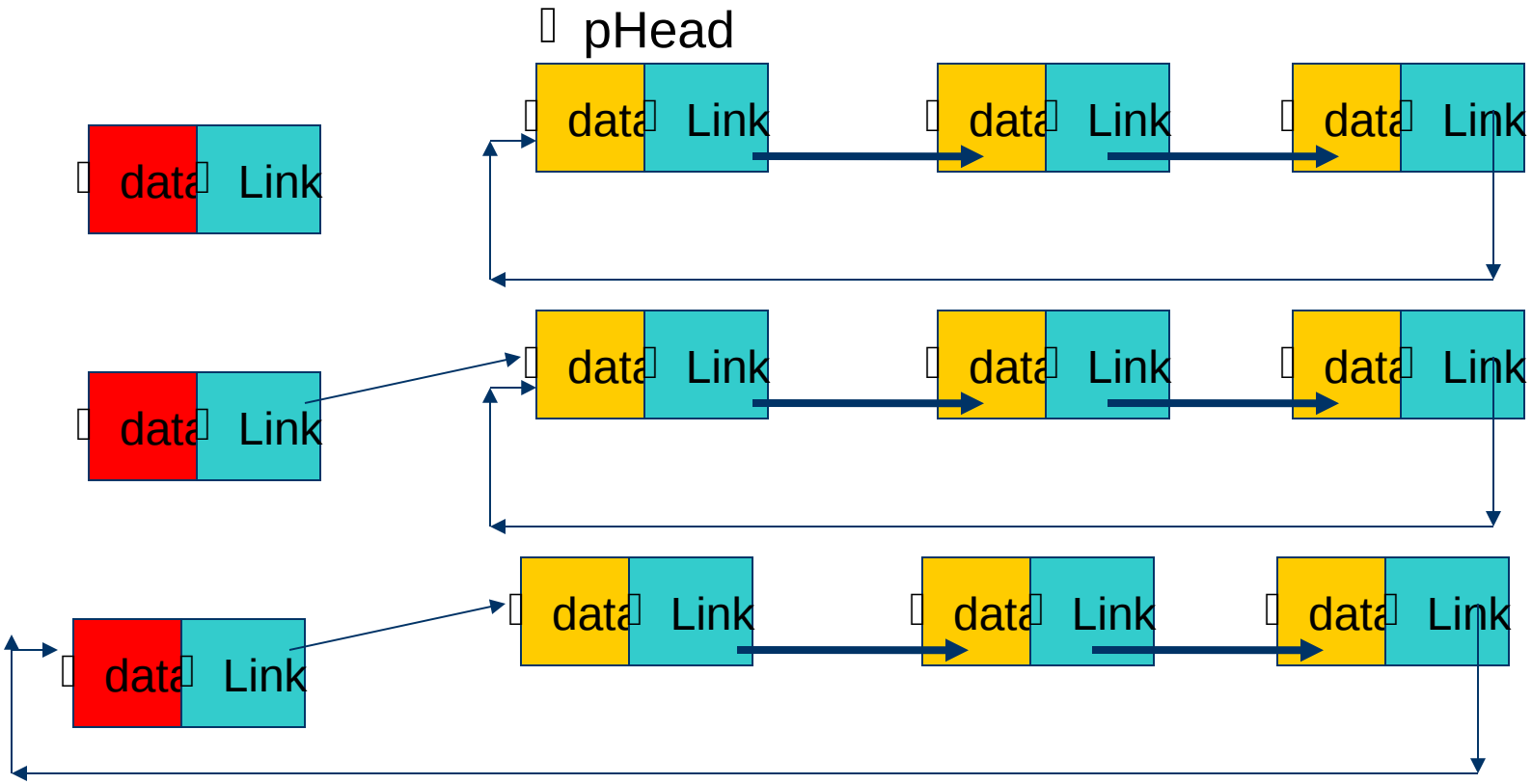
- 4. Circular single linked list

- Control Circular single linked list: Insert node



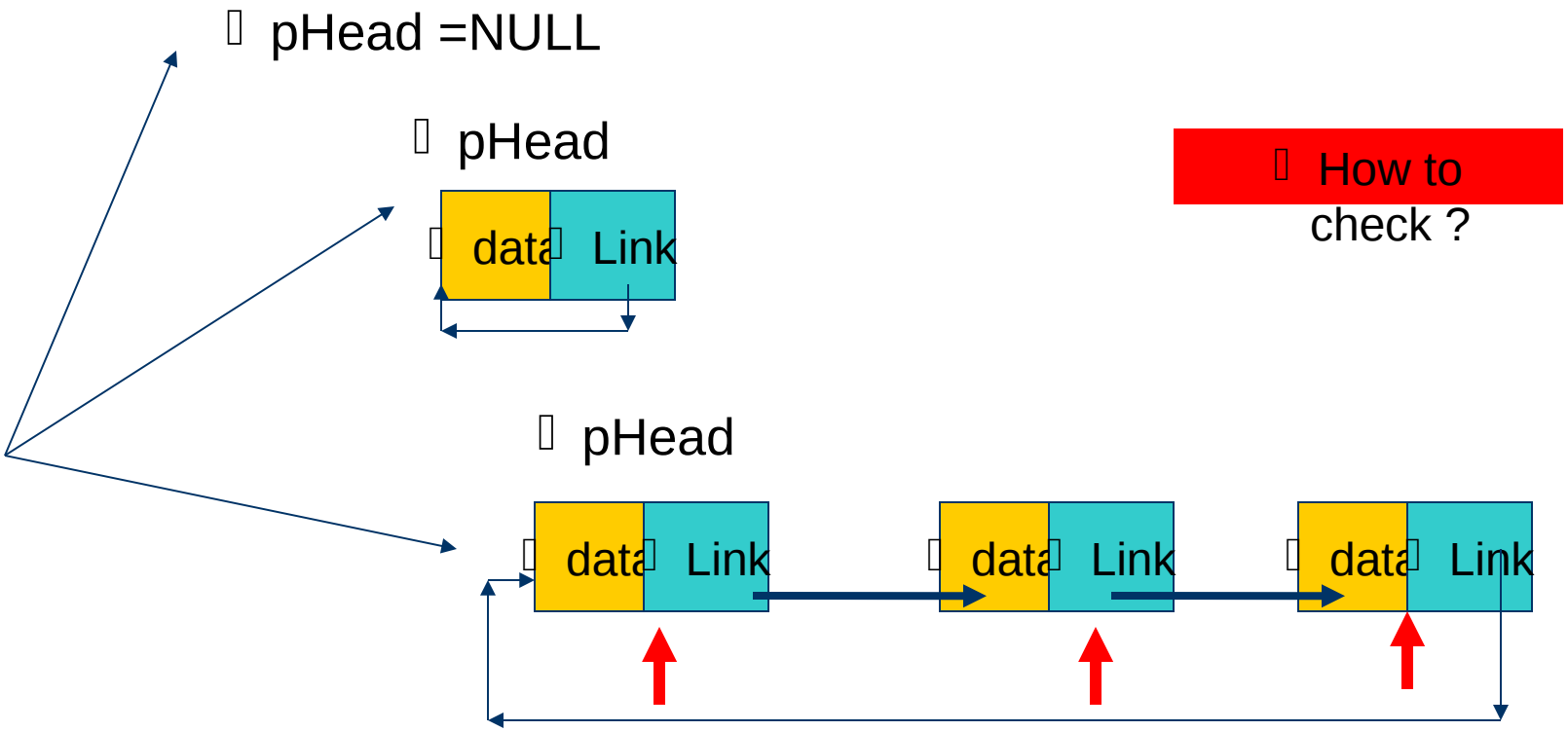
- 4. Circular single linked list

- Control Circular single linked list: Insert node steps



- 4. Circular single linked list

- Control Circular single linked list: Remove node

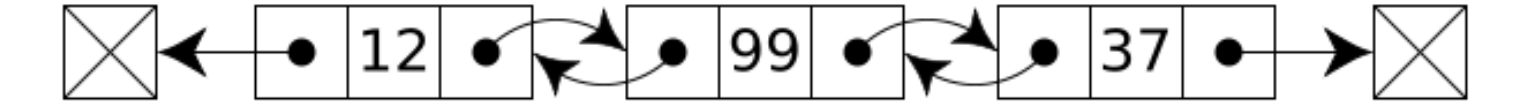


4. Circular single linked list

- Example:
  - Write function to remove a node
  - Circular single linked list with pHead and pTail

- 4. Double Linked List

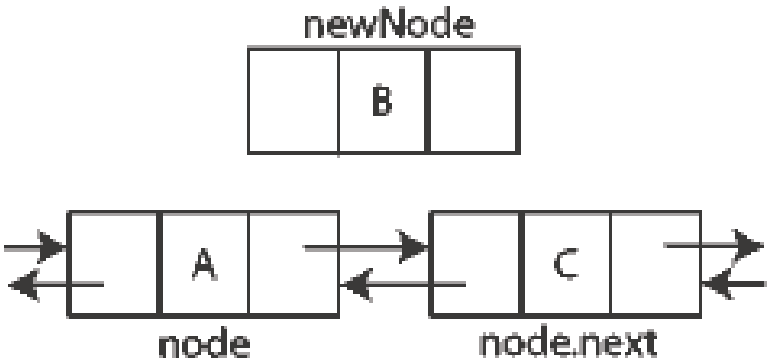
- Struct Node
- {
  - Int data;
  - Node *next;
  - Node * pre;
- };

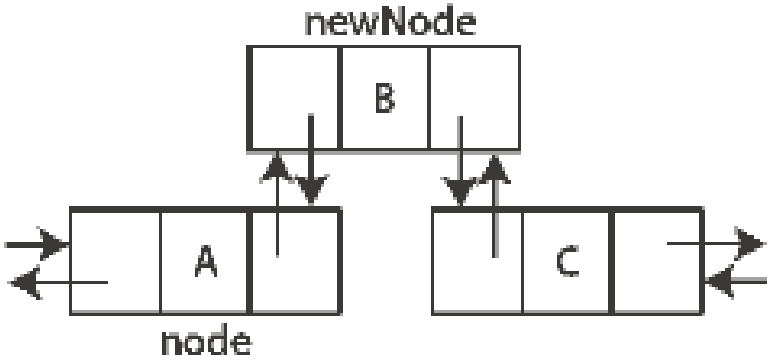


A doubly-linked list containing three integer values: the value, the link forward to the next node, and the link backward to the previous node

4. Double Linked List

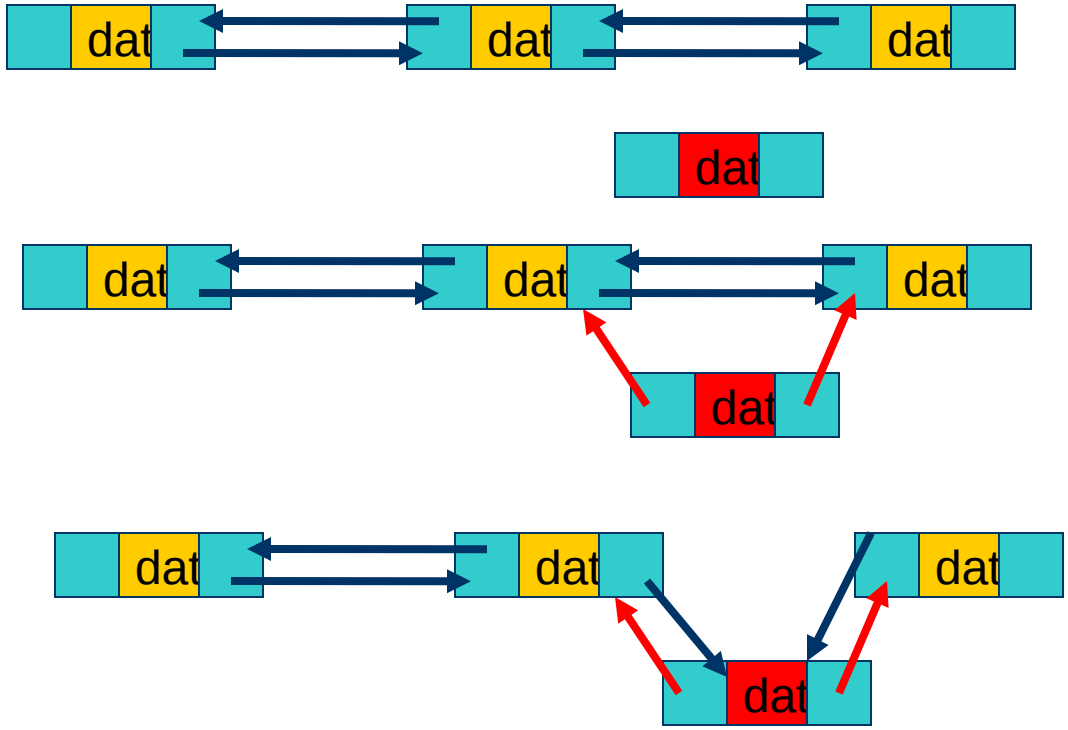
- Insert new node
  - First, Last, after node
- Remove node
  - First,Last, at one middle node





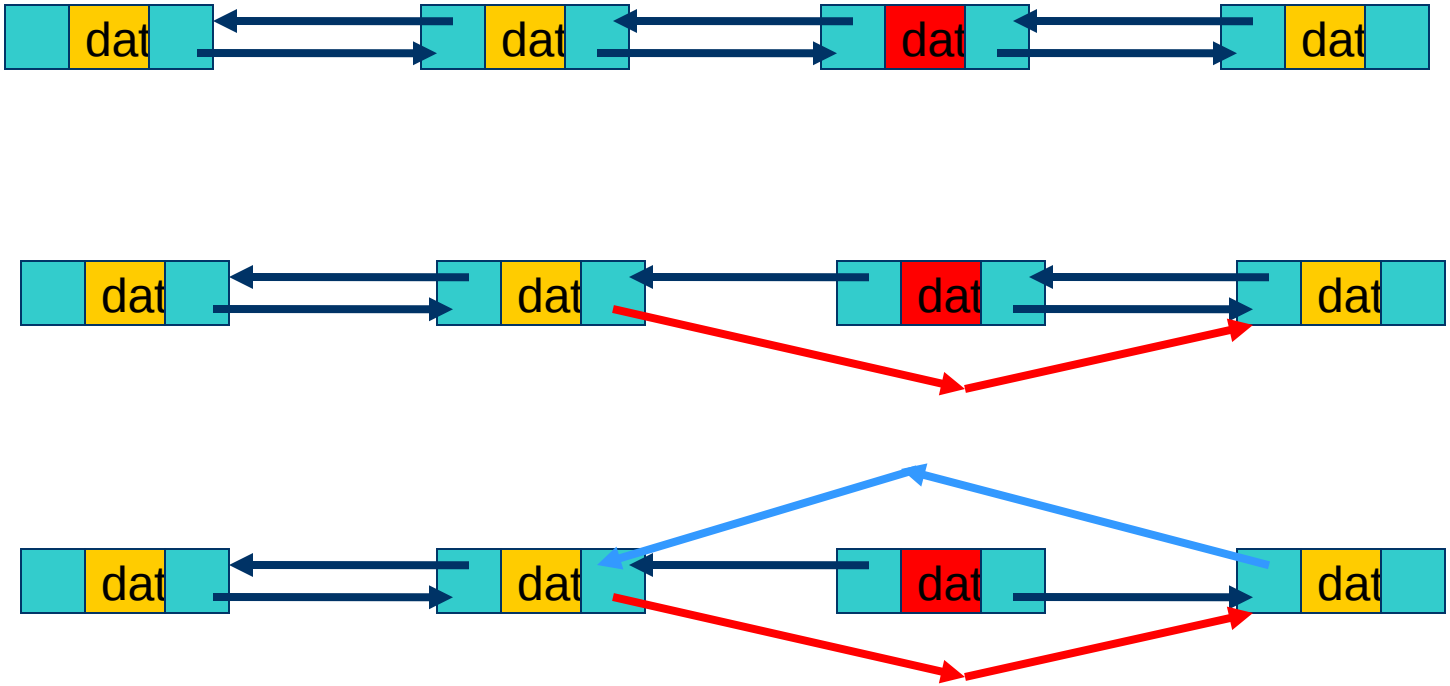
4. Double Linked List

- Insert new node: after one Node First steps



- 4. Double Linked List

- Remove node: steps

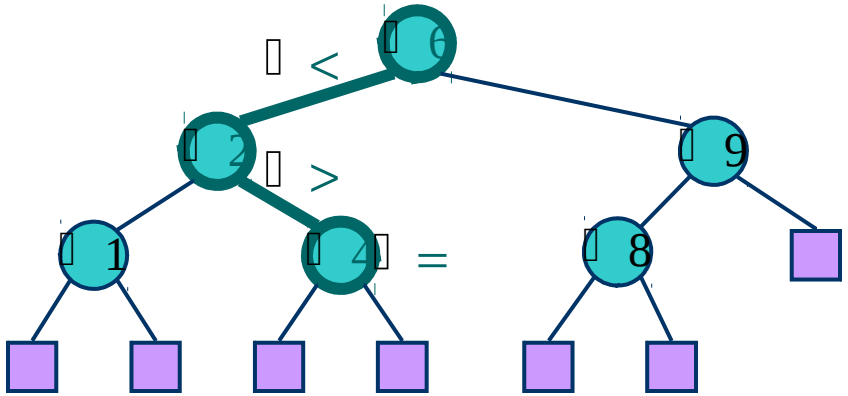


- 4. Double Linked List

- Example
  - Write function to remove first node (pHead)
  - Write function to insert a node after another node

- Tree

- ***1. THE CONCEPT OF TREES***
- *2. BINARY TREE AND REPRESENTATION*
- *3. BINARY TREE TRAVERSAL*
- 4. *BINARY SEARCH TREE*



1. *THE CONCEPT OF TREES*

- A tree is a set of one or more nodes T:
  - there is a specially designated node called a root
  - The remaining nodes are partitioned into *n disjointed* set of nodes T1, T2,…,T*n*, each of which is a tree.

- 1. *THE CONCEPT OF TREES*

- Example

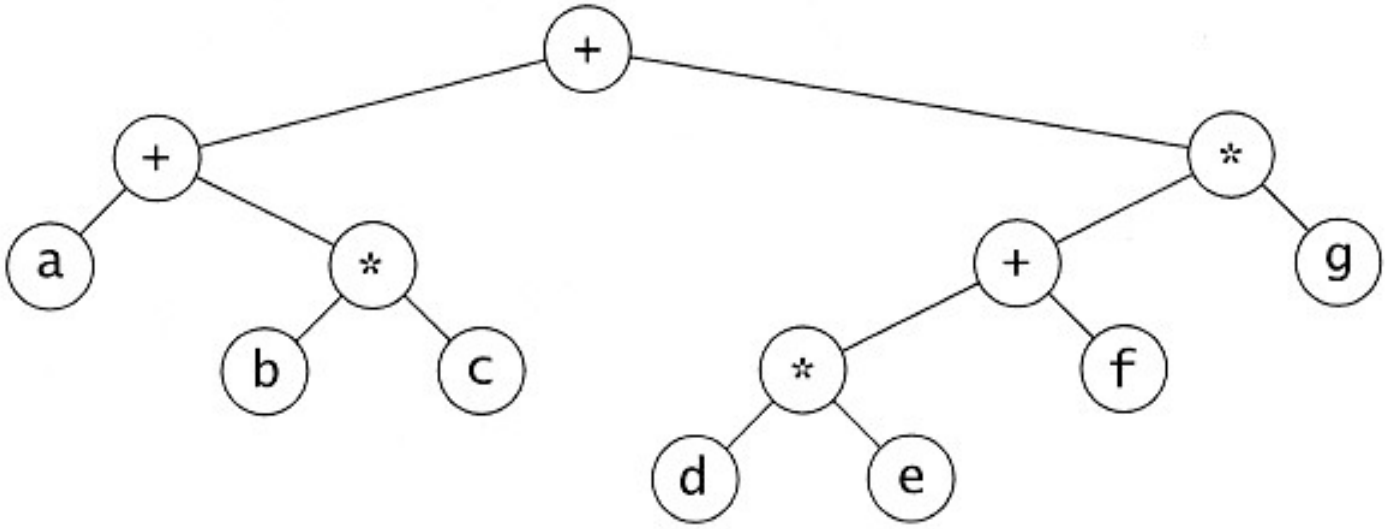
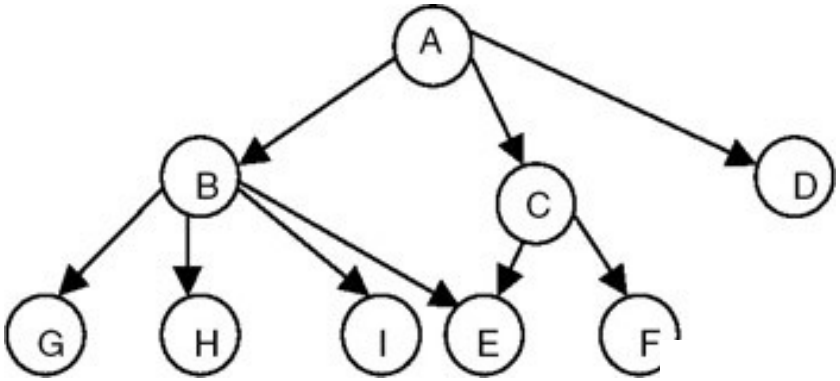


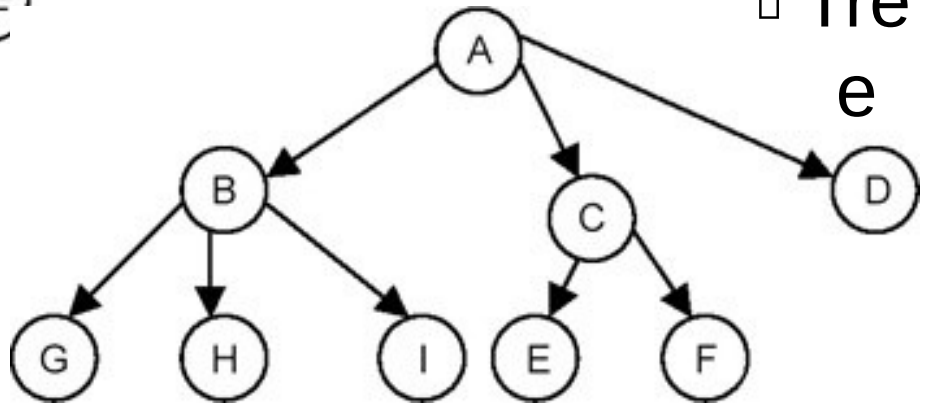
Figure 4.14 Expression tree for (a + b * c) + ((d * e + f) * g)

- 1. *THE CONCEPT OF TREES*

- It’s not a tree

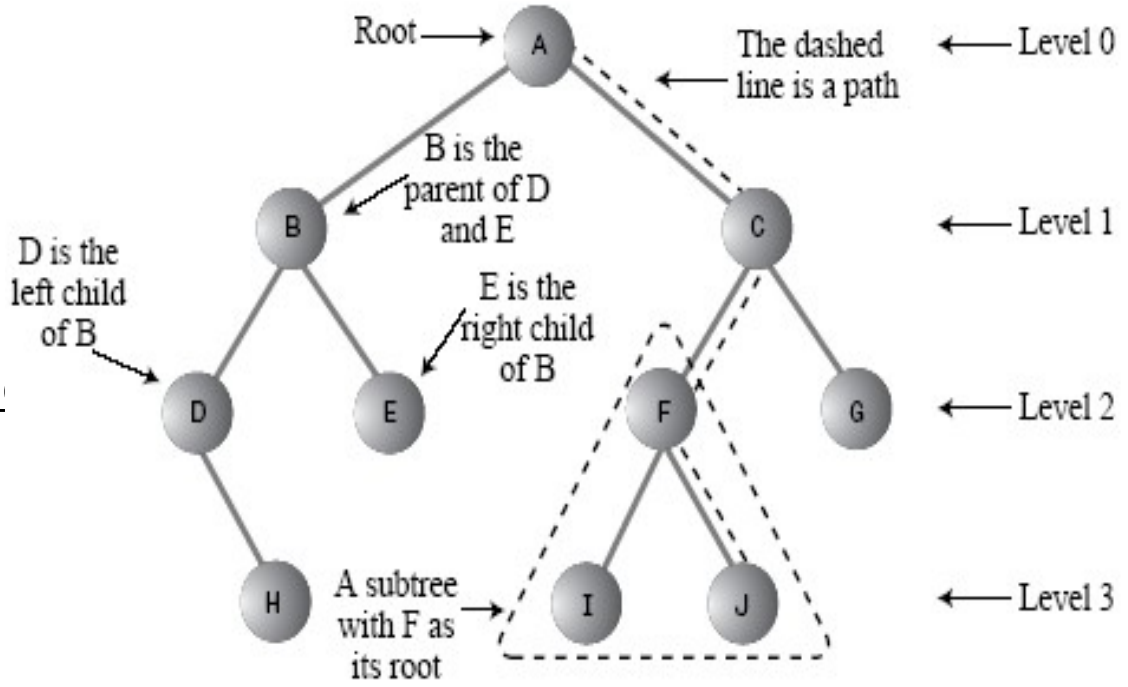


- Tree



1. *THE CONCEPT OF TREES: Some* terminology

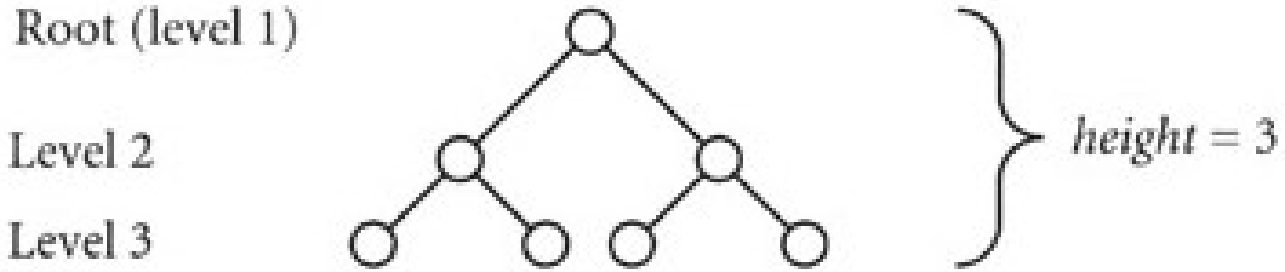
- Root
- Child (left,right)
- Parent
- Leaf node
- Subtree
- Ancestor of a node
- Descendant of a nod



H, E, I, J, and G are leaf nodes

- 1. *THE CONCEPT OF TREES*

- **Degree of a Node of a Tree**
  - The *degree of a node of a tree* is the number of subtrees having this node as a root.
- **Degree of a Tree**
  - The *degree of a tree* is defined as the maximum of degree of the nodes of the tree
- **Level of a Node**
  - level of the root node as 1, and incrementing it by 1 as we move from the root towards the subtrees.

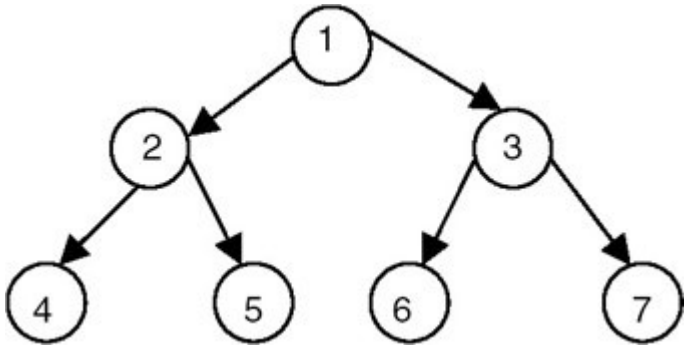


2. BINARY TREE AND REPRESENTATION

- *BINARY TREE*
  - *no node can have a degree of more than 2.*
  - *The maximum number of nodes at level i will be 2^{i-1}*
  - *If k is the depth of the tree then the maximum number of nodes that the tree can have is*
  - *$2^k - 1 = 2^{k-1} + 2^{k-2} + \ldots + 2^0$*

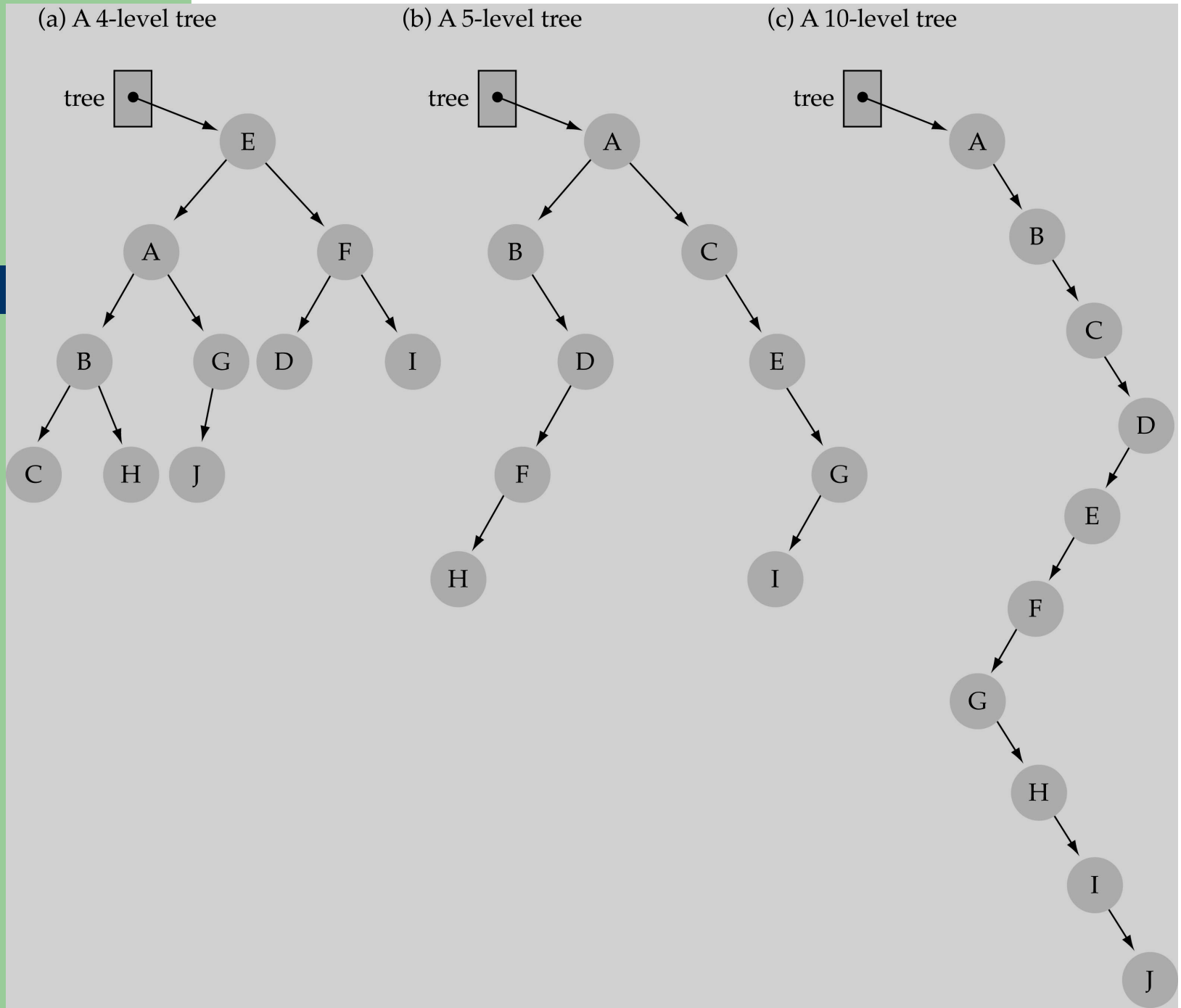
2. BINARY TREE AND REPRESENTATION

- *BINARY TREE*
  - *A full binary tree is a binary of depth k having 2k − 1 nodes.*
  - *If it has < 2k − 1, it is not a full binary tree*



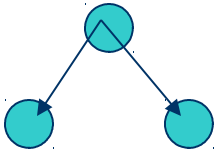
- What is the height h of a full tree with N nodes?

- The max height of a tree with *N* nodes is *N* (same as a linked list)
- The min height of a tree with *N* nodes is *log(N+1)*

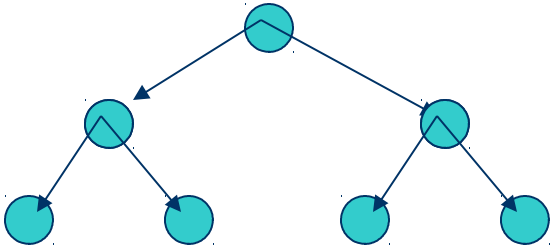

(a) A 4-level tree
tree
E
A
F
B
G
D
I
C
H
J
(b) A 5-level tree
tree
A
B
C
D
E
F
G
H
I
(c) A 10-level tree
tree
A
B
C
D
E
F
G
H
I
J

2. BINARY TREE AND REPRESENTATION

- *full binary*

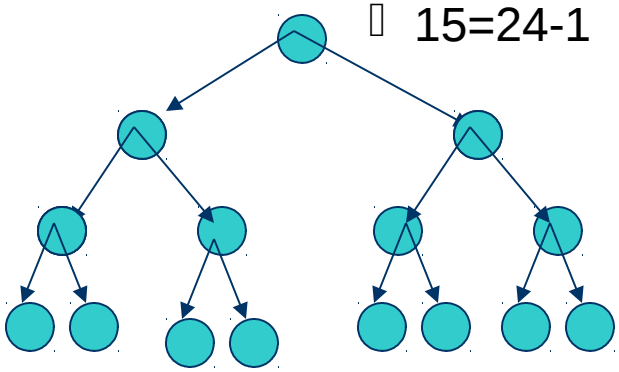


- 3=22-1



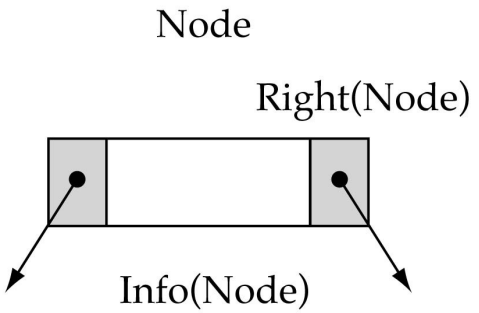
- 7=23-1

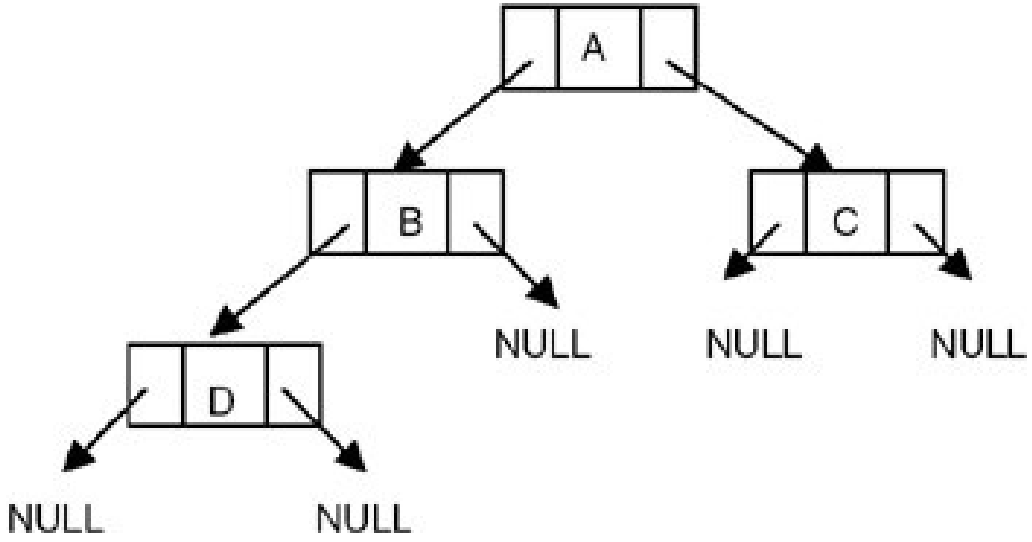
- 15=24-1



2. BINARY TREE AND REPRESENTATION

- struct node
  - { int data;
  - node *left;
  - node *right;
  - };





- Tree traversal

- Used to print out the data in a tree in a certain order
  - inorder (LDR)
  - Postorder (LRD)
  - preorder (DLR)
- Pre-order traversal
  - Print the data at the root
  - Recursively print out all data in the left subtree
  - Recursively print out all data in the right subtree

Preorder, Postorder and Inorder

- Preorder traversal
  - node, left, right
  - prefix expression
    - ++a*bc*+*defg

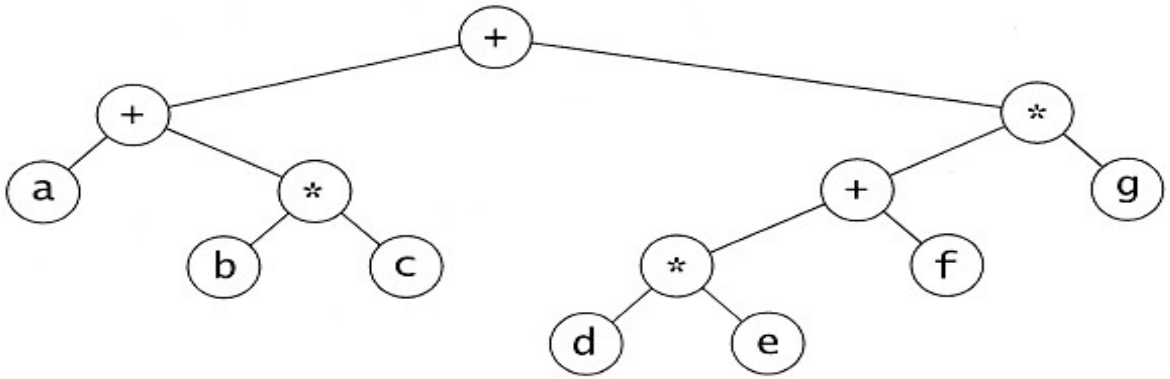


Figure 4.14 Expression tree for (a + b * c) + ((d * e + f) * g)

Preorder, Postorder and Inorder

- Postorder traversal
  - left, right, node
  - postfix expression
    - abc*+de*f+g*+
- Inorder traversal
  - left, node, right.
  - infix expression
    - a+b*c+d*e+f*g

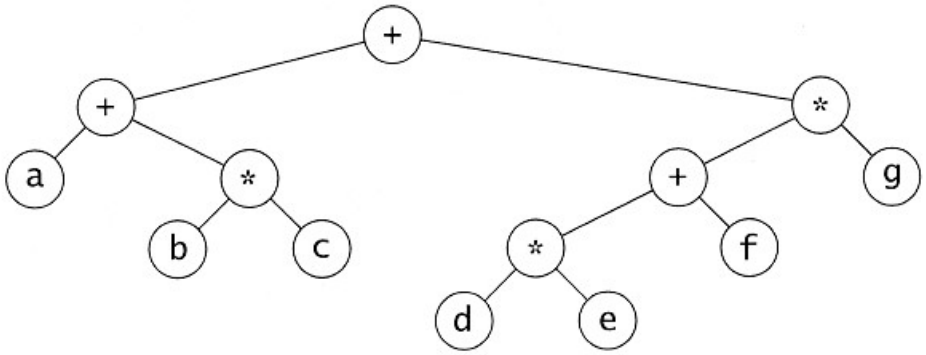
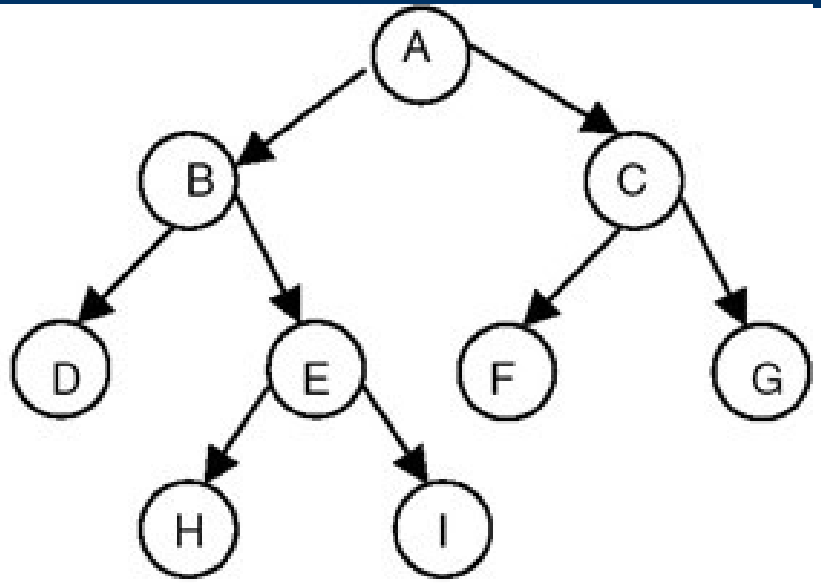


Figure 4.14 Expression tree for (a + b * c) + ((d * e + f) * g)

- Preorder, Postorder and Inorder

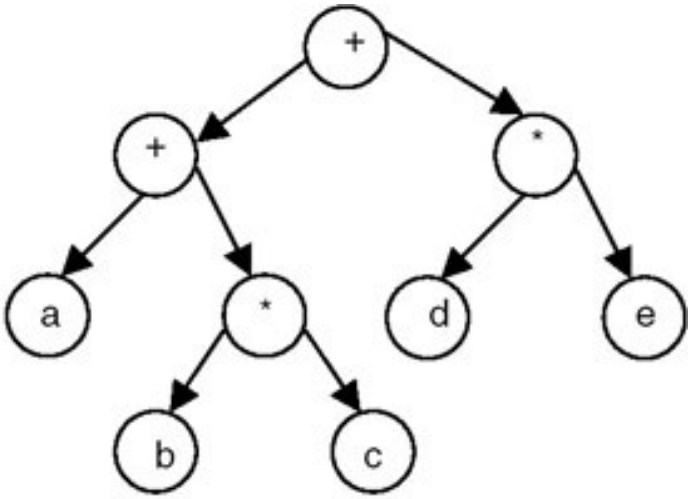
- *3. BINARY TREE TRAVERSAL*



Inorder : DBHEIAFCG

Preorder : ABDEHICFG

Postorder : DHIEBFGCA

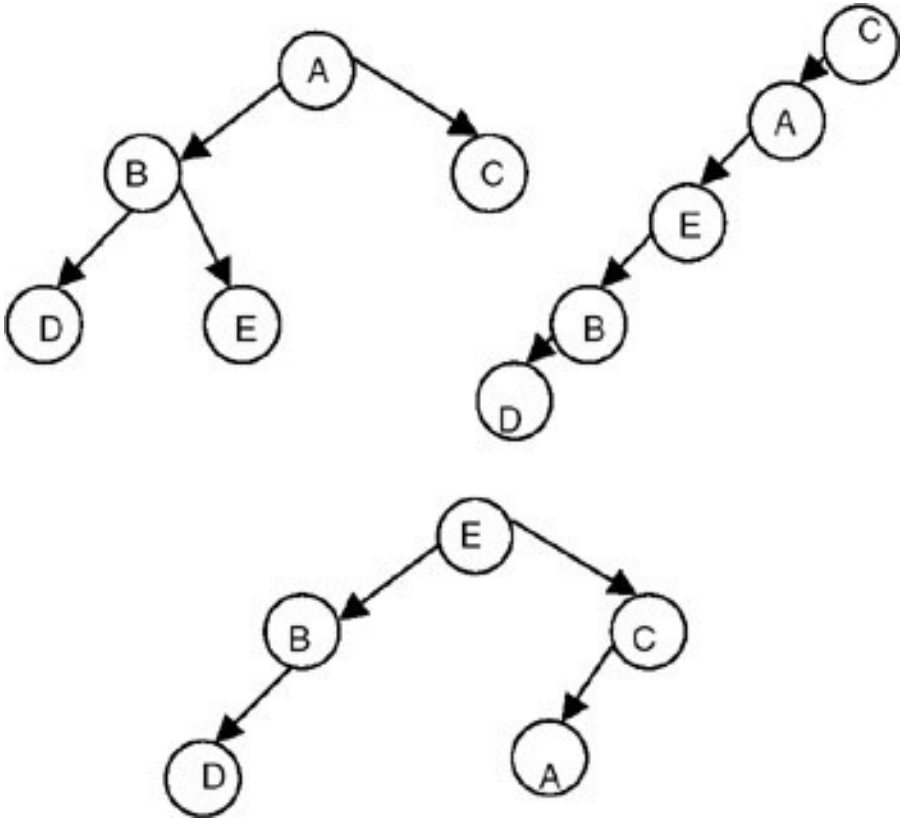


Inorder : a + b * c + d * e

postorder : abc*+de*+

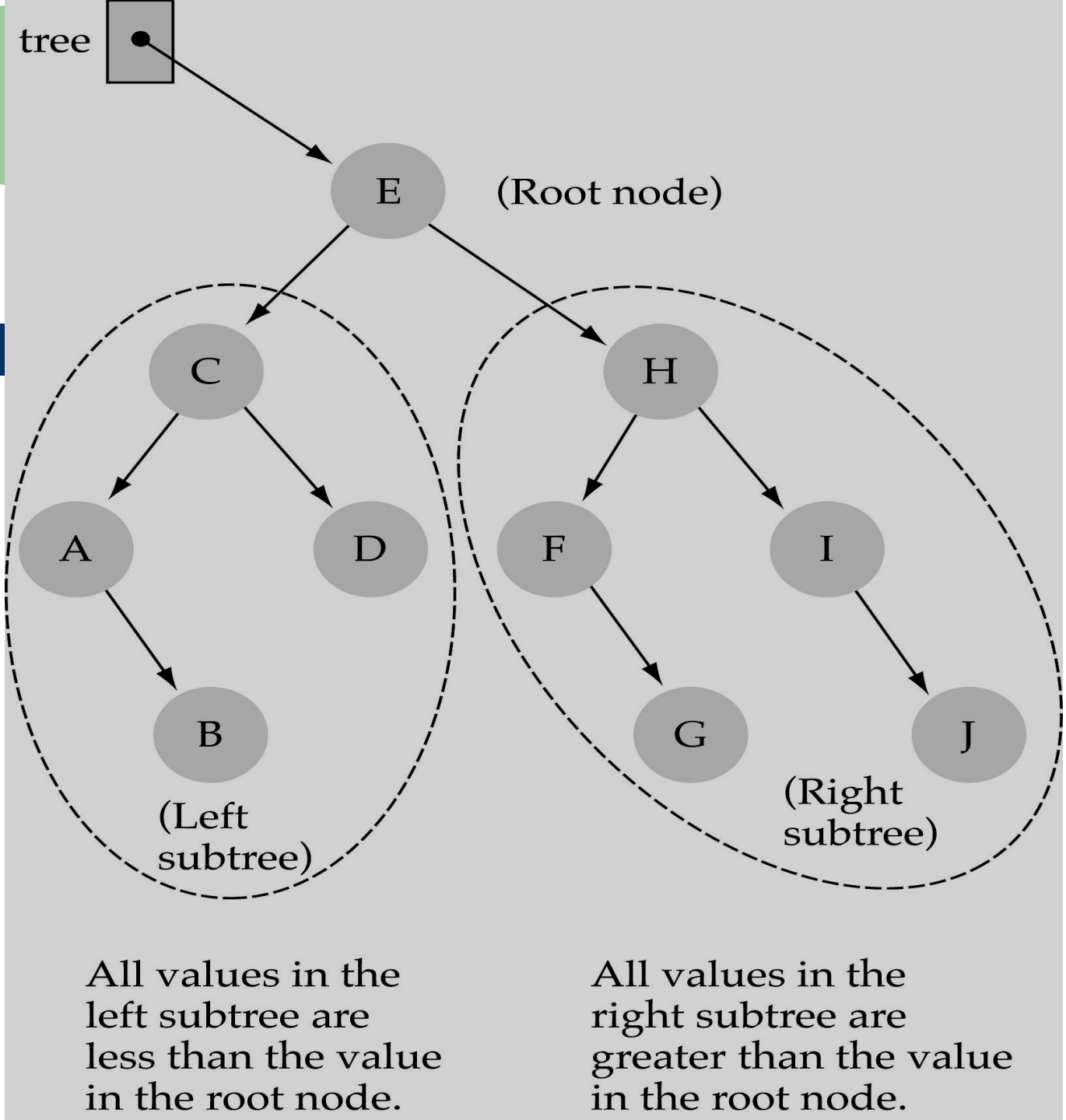
3. BINARY TREE TRAVERSAL

- Inorder = DBEAC
- Many trees



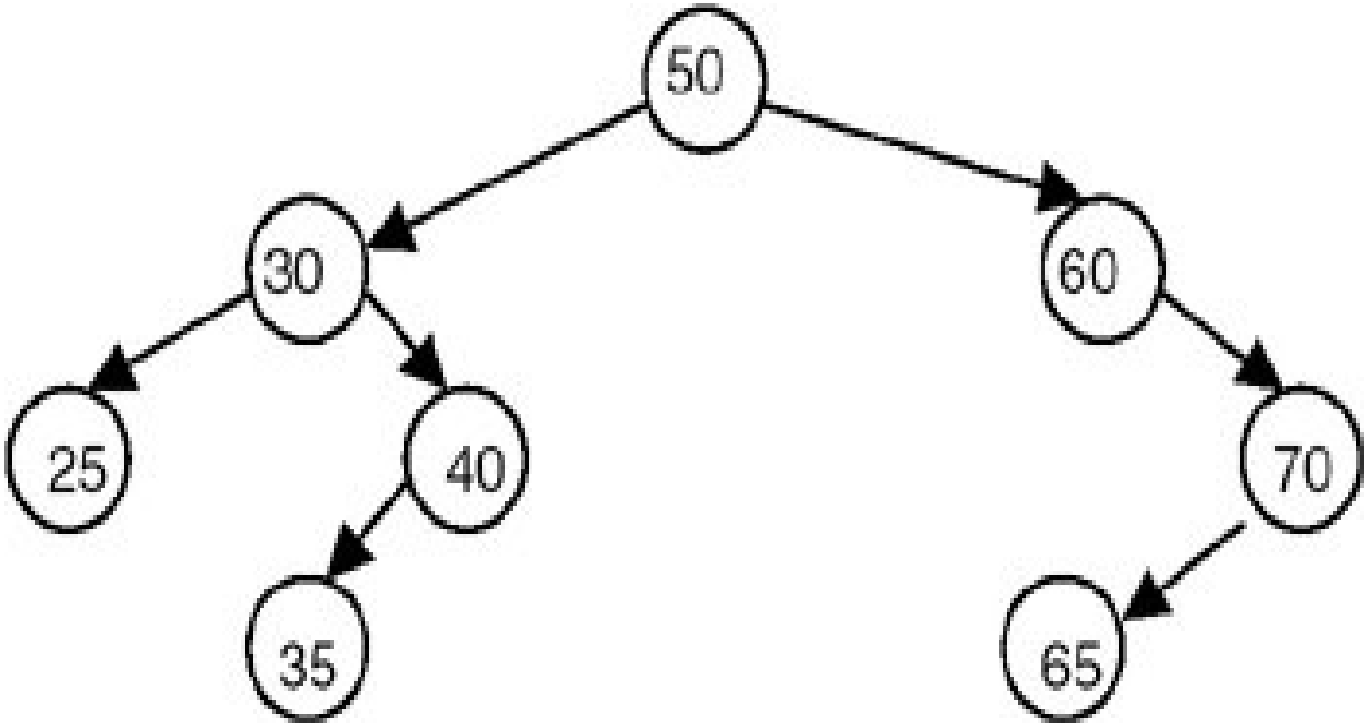
4. *BINARY SEARCH TREE*

- A *binary search tree*
  - is a binary tree (may be empty)
  - every node must contain an identifier.
  - An identifier of any node in the left subtree is less than the identifier of the root.
  - An identifier of any node in the right subtree is greater than the identifier of the root.
  - Both the left subtree and right subtree are binary search trees.


tree
E
(Root node)
C
H
A
D
F
I
B
G
J
(Left subtree)
(Right subtree)
All values in the left subtree are less than the value in the root node.
All values in the right subtree are greater than the value in the root node.

- 4. *BINARY SEARCH TREE*

- binary search tree.



- Binary Search Trees

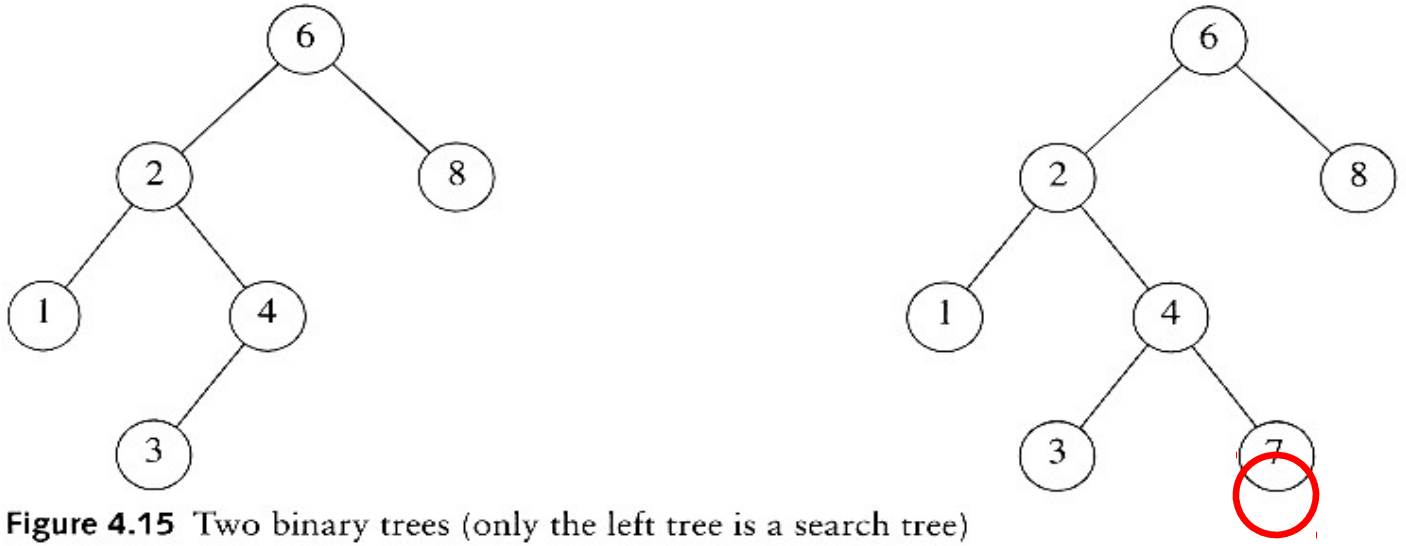


Figure 4.15 Two binary trees (only the left tree is a search tree)

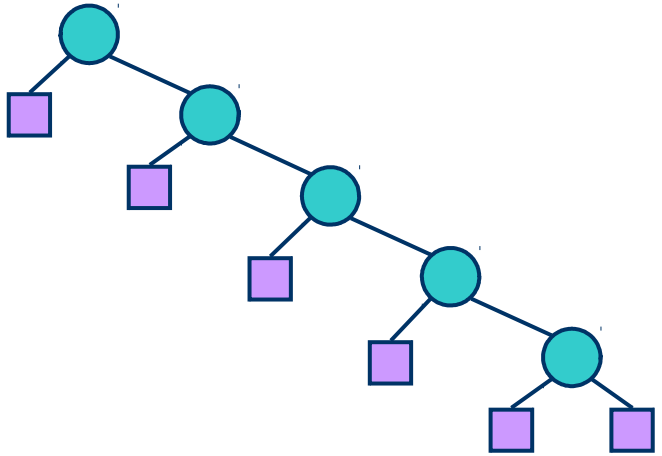
- **A binary search tree**
- **Not a binary search tree**

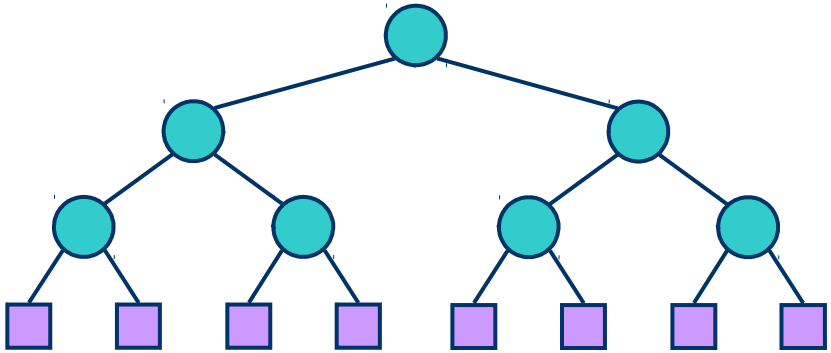
- Binary search trees

- **Two binary search trees representing**
- **the same set: Why?**

Performance

- Consider a dictionary with n items implemented by means of a binary search tree of height h
  - the space used is $O(n)$
  - methods find, insert and remove take $O(h)$ time
- The height h is $O(n)$ in the worst case and $O(\log n)$ in the best case





4. *BINARY SEARCH TREE*

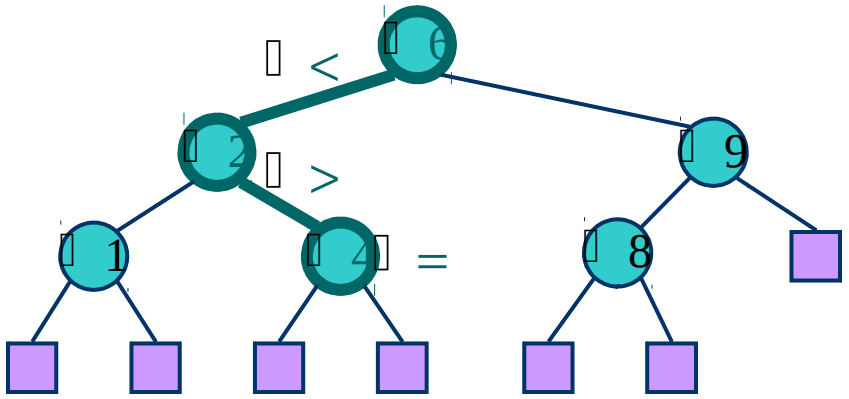
- Why using binary search tree
  - traverse in inorder: sorted list
  - searching becomes faster

- But..
  - Insert, delete: slow
- Important thing: Index in Database system
  - Using the right way of Index property

Search

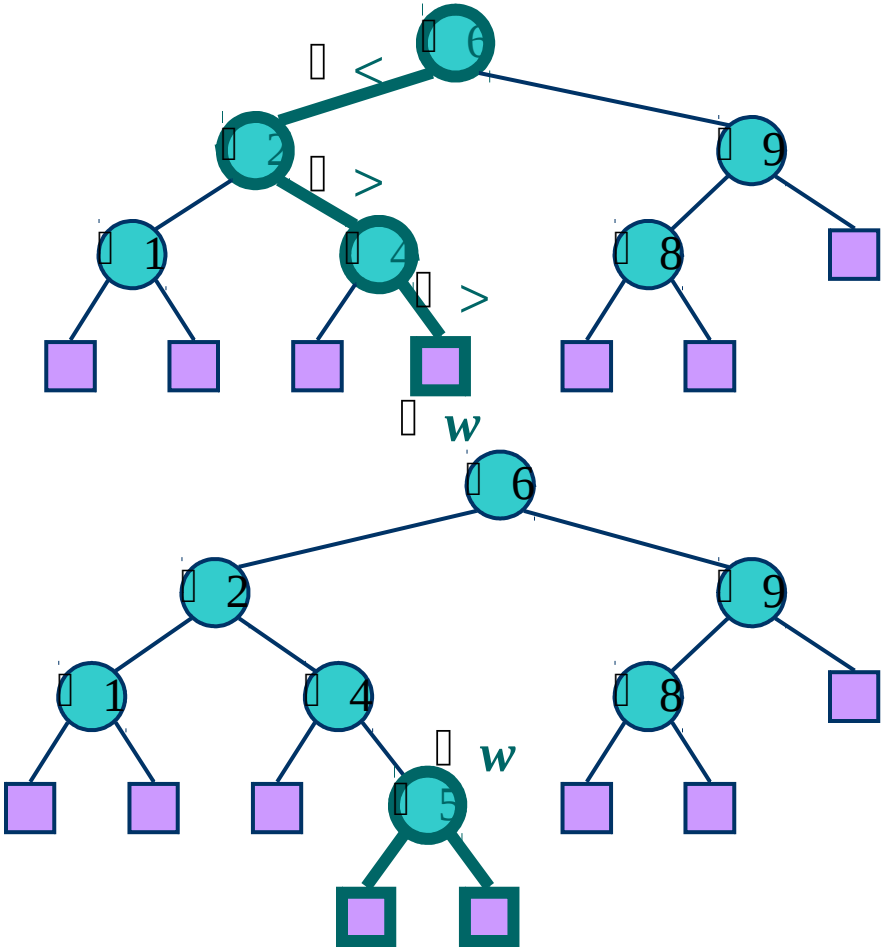
- To search for a key ***k***, we trace a downward path starting at the root
- The next node visited depends on the outcome of the comparison of ***k*** with the key of the current node
- If we reach a leaf, the key is not found and we return nukk
- Example: find(4):
  - Call TreeSearch(4,root)

```
Algorithm TreeSearch(k, v)
    if (v ==NULL)
        return v
    if k < key(v)
        return TreeSearch(k, T.left(v))
    else if k = key(v)
        return v
    else { k > key(v) }
        return TreeSearch(k, T.right(v))
```



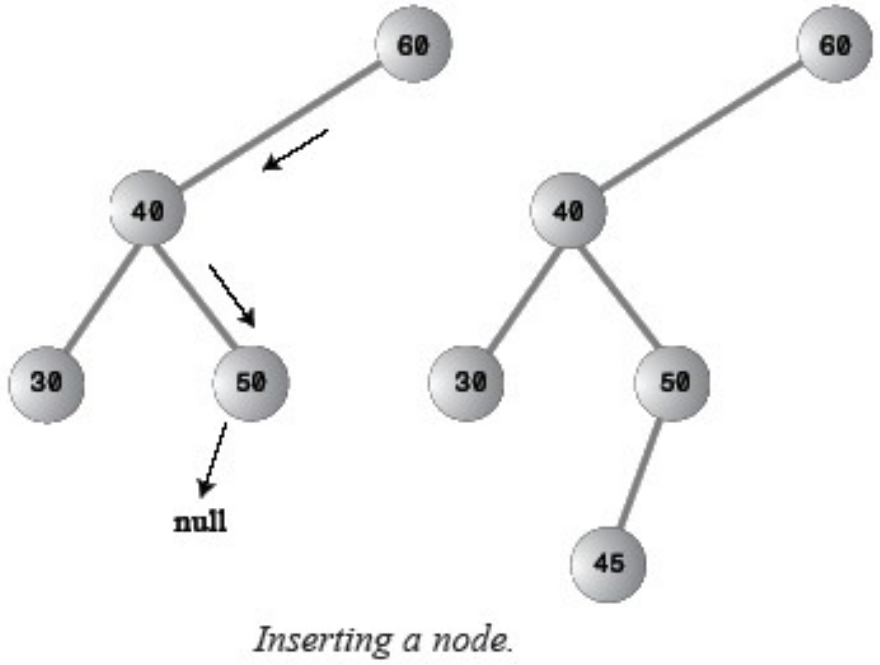
- Insertion

- To perform operation inser(k, o), we search for key k (using TreeSearch)
- Assume k is not already in the tree, and let let w be the leaf reached by the search
- We insert k at node w and expand w into an internal node
- Example: insert 5



- 4. *BINARY SEARCH TREE*

- Insert new node



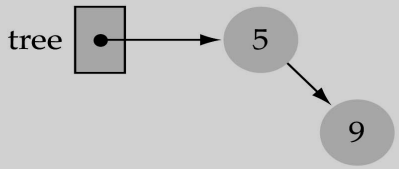
Inserting a node.

a) Before insertion **b) After insertion**

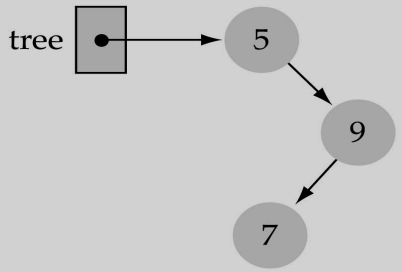
(a) tree

(b) Insert 5

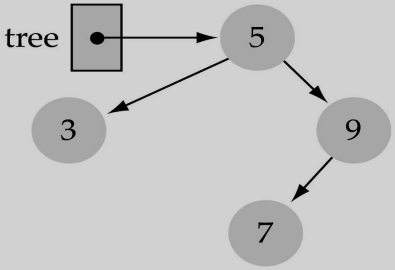
(c) Insert 9



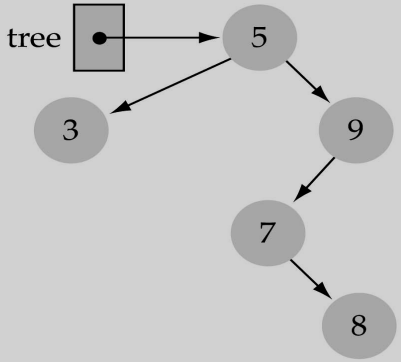
(c) Insert 7



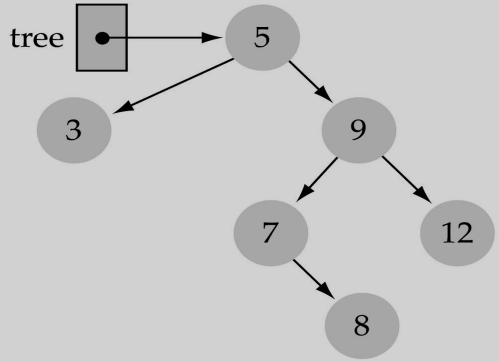
(e) Insert 3



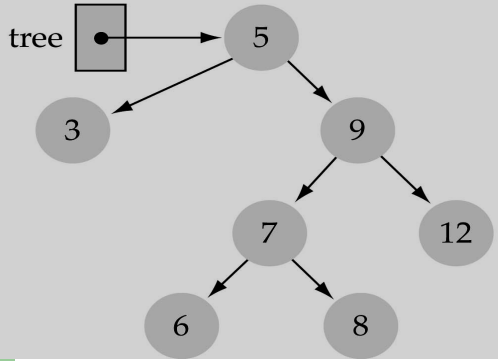
(f) Insert 8



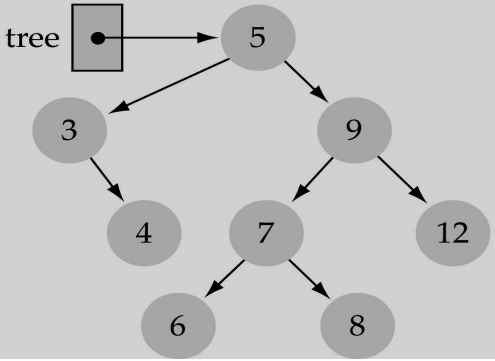
(g) Insert 12



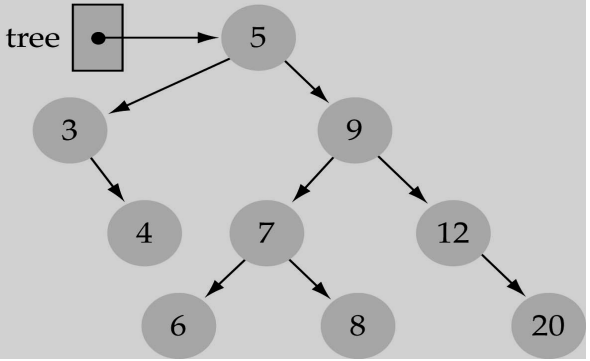
(h) Insert 6



(i) Insert 4



(j) Insert 20



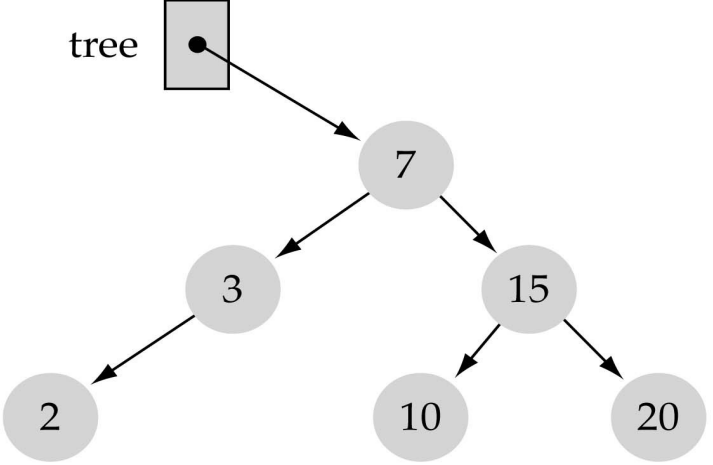
- 4. *BINARY SEARCH TREE*

- Insert new node

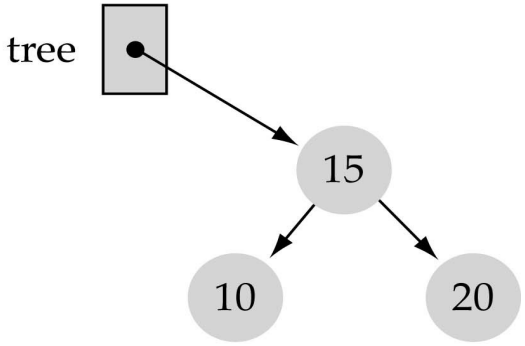
```
void InsertNode(node* &root,node *newnode)
{
      if (root==NULL)
            root=newnode;
      else
            if (root->data>newnode->data)
                  InsertNode(root->l,newnode);
            else
                  if (root->data<newnode->data)
                        InsertNode(root->r,newnode);
}
```

- Insert node

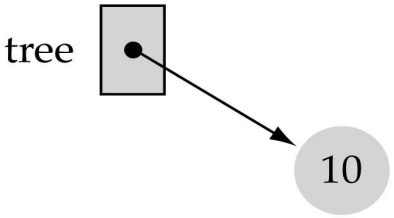
(a) The initial call



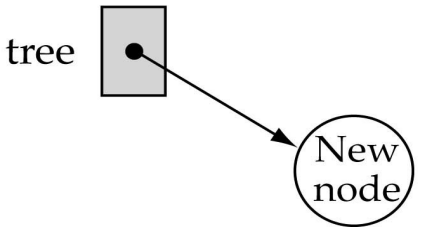
(b) The first recursive call



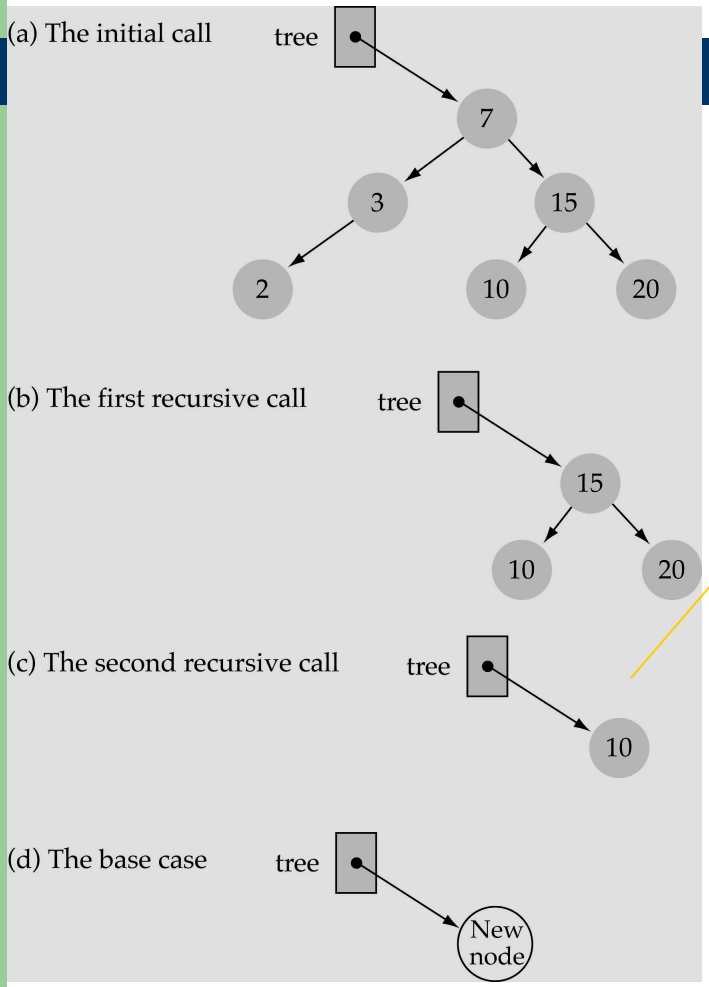
(c) The second recursive call

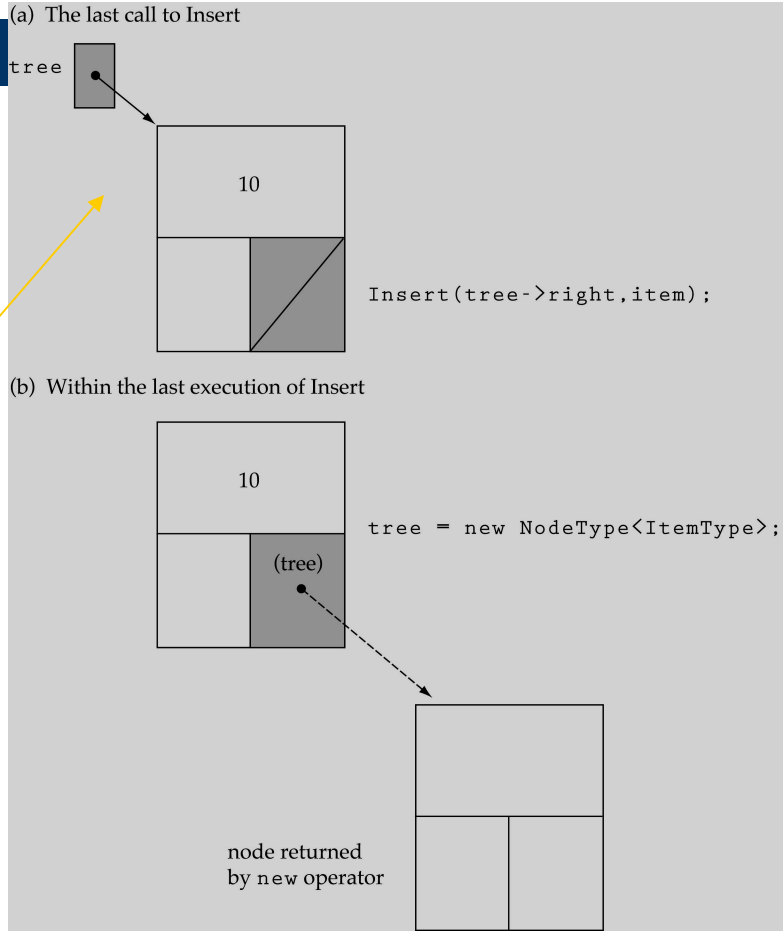


(d) The base case

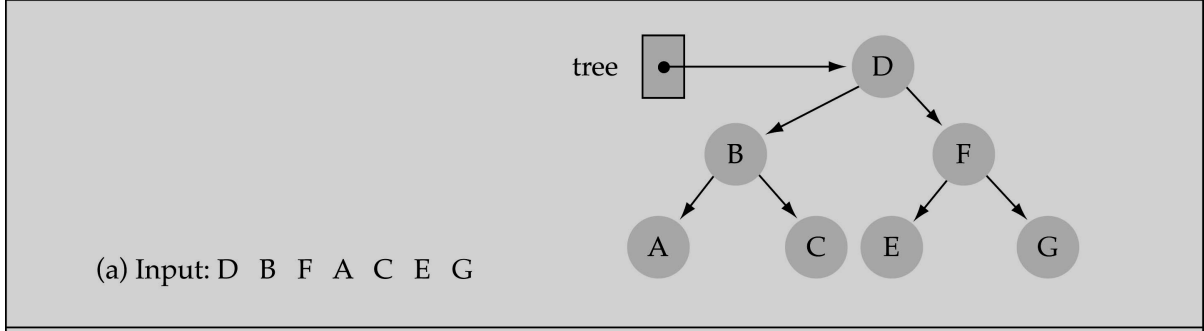


- Insert node

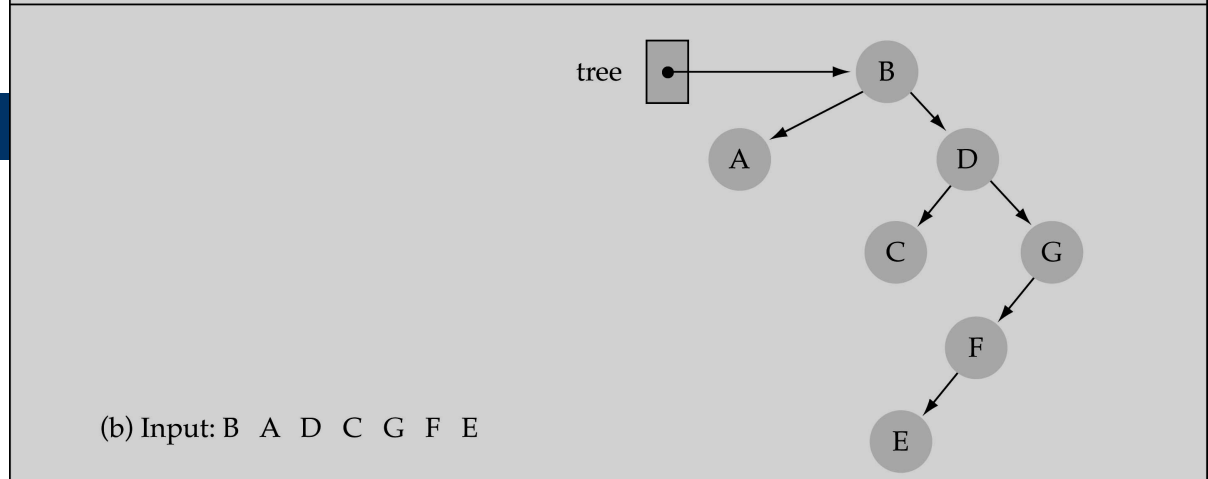

(a) The initial call
tree
7
3
15
2
10
20
(b) The first recursive call
tree
15
10
20
(c) The second recursive call
tree
10
(d) The base case
tree
New node


(a) The last call to Insert
tree
10
Insert(tree->right,item);
(b) Within the last execution of Insert
10
(tree)
tree = new NodeType<ItemType>;
node returned
by new operator

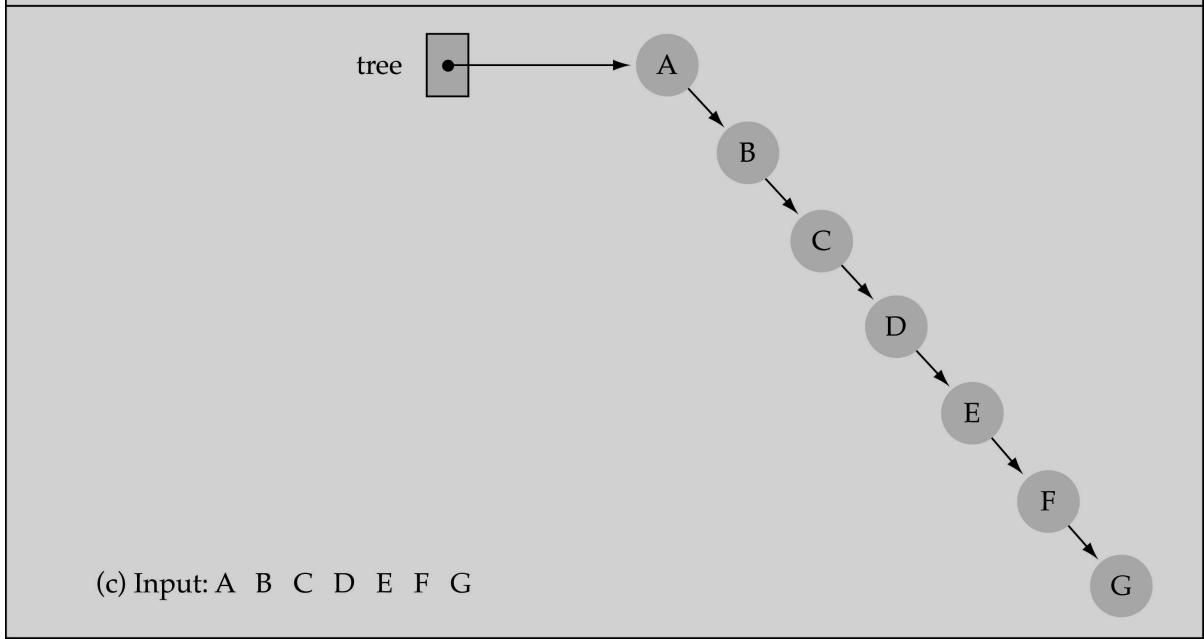
Insert Order



(a) Input: D B F A C E G



(b) Input: B A D C G F E



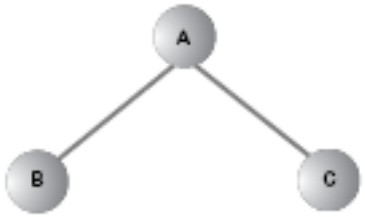
(c) Input: A B C D E F G

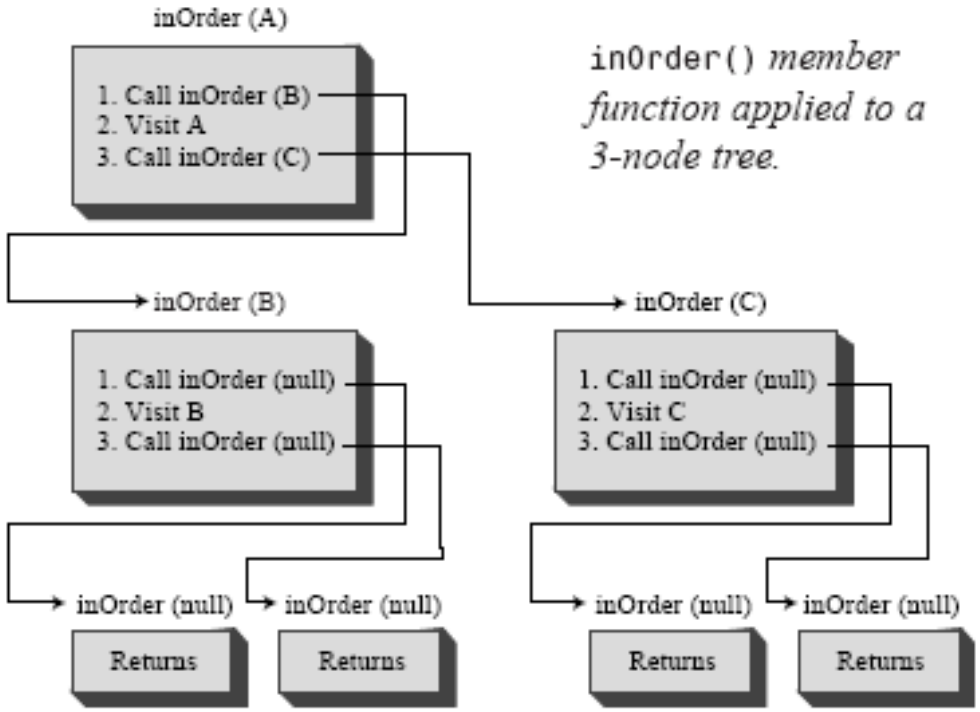
- 4. *BINARY SEARCH TREE*

- traverse node
  - void preorder(node* r)
  - {
  - if (r!=NULL)
  - { cout<<r->data<<" ";
  - inorder(r->l);
  - inorder(r->r);
  - }
  - }

- 4. *BINARY SEARCH TREE*

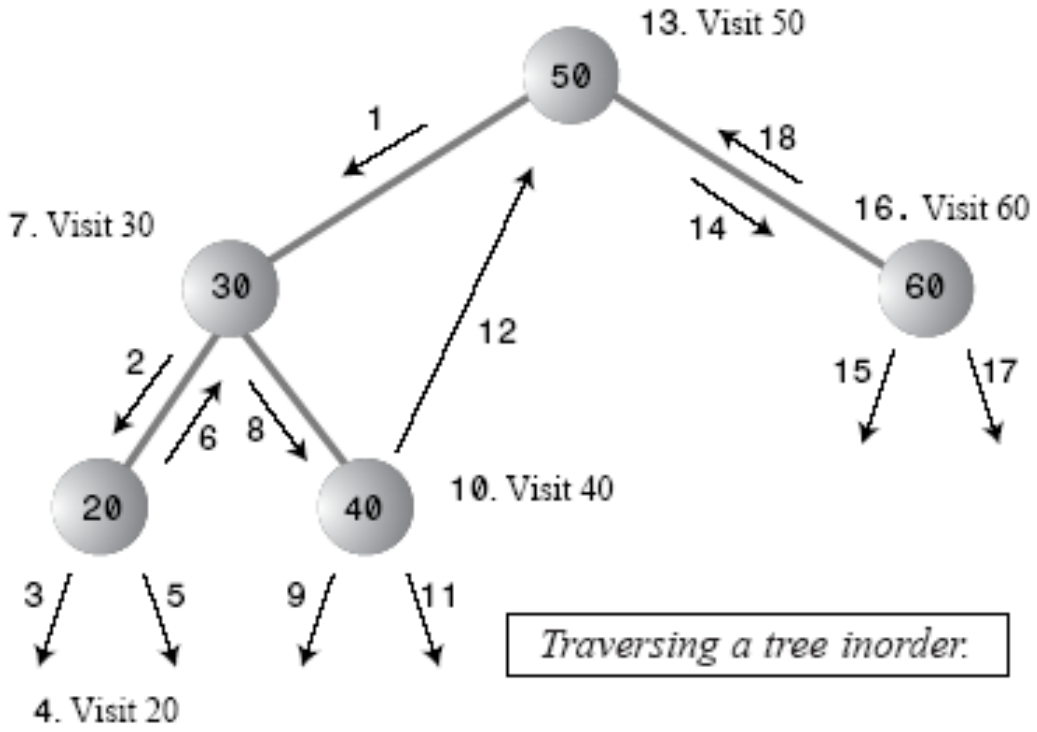
- traverse node





- 4. *BINARY SEARCH TREE*

- traverse node

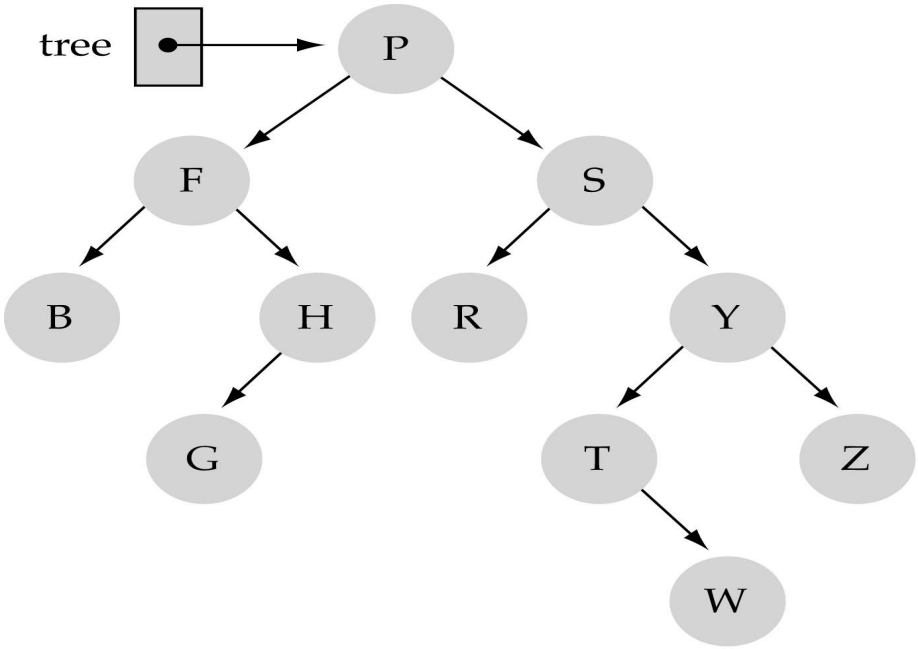


- Exercise 1

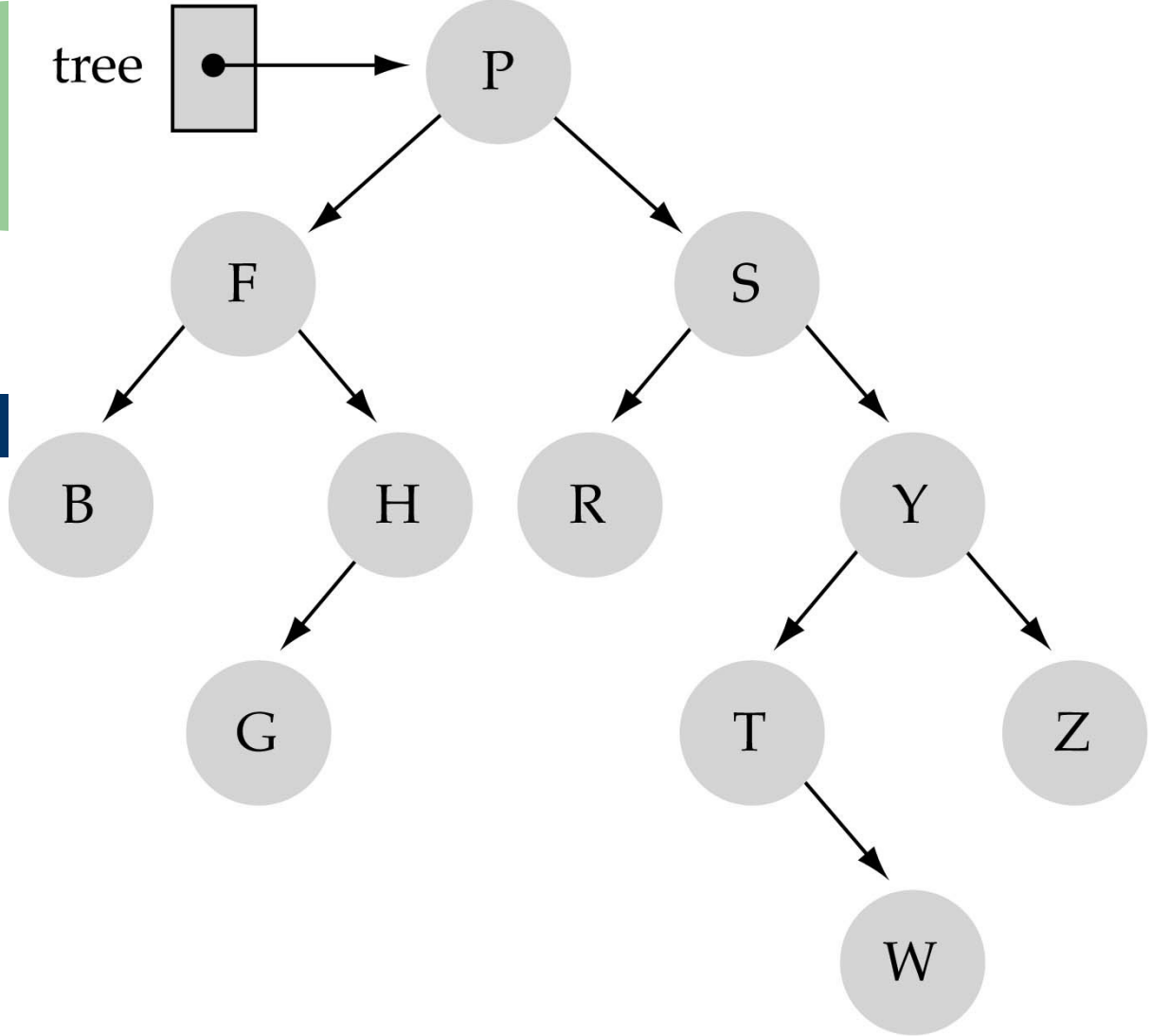
- 1. Build Binary Search Tree from list
  - 10 4 7 12 16 20 30 5 2 26 15
  - 24 12 89 4 32 50 10 6 36 79 5 9 11

- Exercise 2

- 1. Order of: inoder, postorder, preorder of



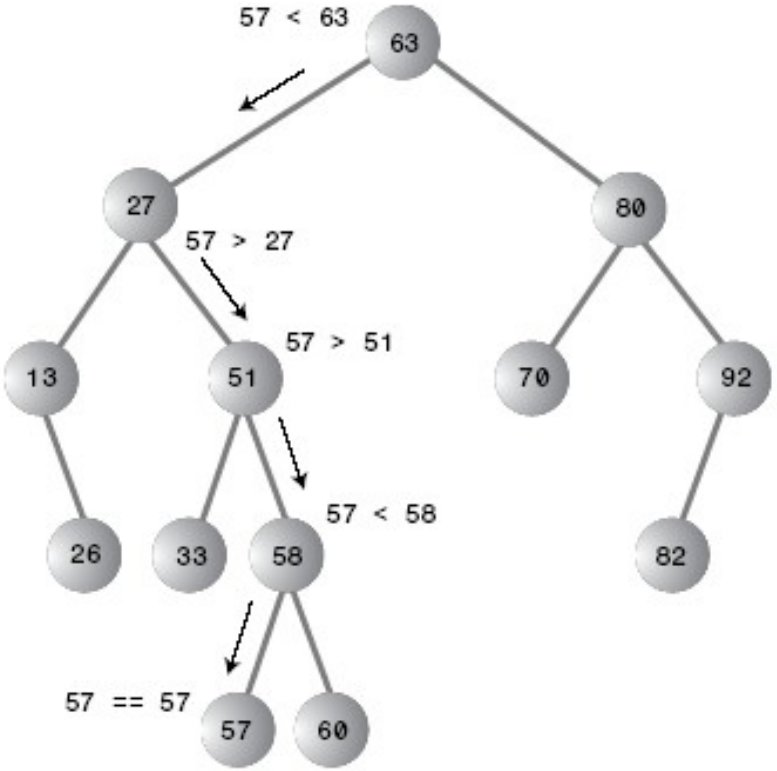
Inorder:	B	F	G	H	P	R	S	T	W	Y	Z
Preorder:	P	F	B	H	G	S	R	Y	T	W	Z
Postorder:	B	G	H	F	R	W	T	Z	Y	S	P



Inorder:	B	F	G	H	P	R	S	T	W	Y	Z
Preorder:	P	F	B	H	G	S	R	Y	T	W	Z
Postorder:	B	G	H	F	R	W	T	Z	Y	S	P

- Exercise 3

- 1. Order of: inoder, postorder, preorder of



- Week 10

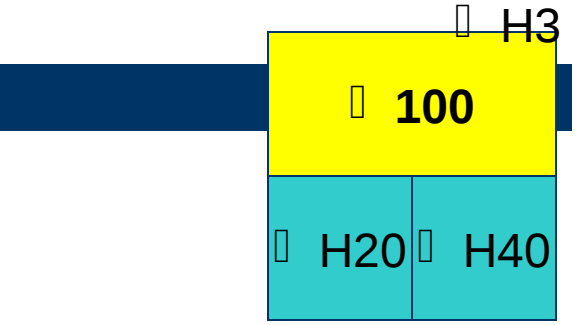
- Search node
- Count
  - Even/Odd
  - Leaf
- Sum
  - Even/Odd
- Height
- **Delete node**

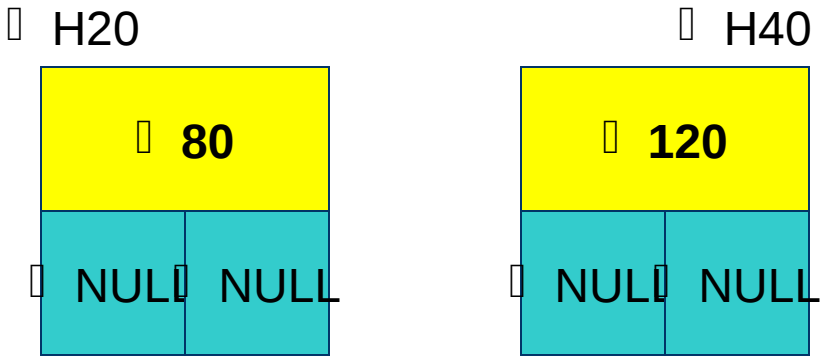
- *1. SEACRCHING NODE*

```
node* search(node* &r, int data)
{
        if (r==NULL)
                return NULL;
        else
                if (r->data==data)
                        return r;
        else
                if (data<r->data)
                        return search (r->l,data);
                else
                        if (data>r->data)
                         return seach(r->r,data);

}
```

- *1. SEACRCHING NODE*

```
node* search(node* &r, int data)
{
        if ( (r==NULL) || (r->data==data) )
                return r;
        else
                if (data<r->data)
                        return search (r->l,data);
                else
                        if (data>r->data)
                         return seach(r->r,data);


}
```





- Node* S=search(r,80)
- What does S stand for?

2. COUNTING THE NUMBER OF NODES

- Without Recursion

- With Recursion

```
int count(struct tnode *p)
    {
        if( p == NULL)
            return(0);
        else
            if( p->lchild == NULL && p->rchild == NULL)
                return(1);
            else
                return(1 + (count(p->lchild) + count(p->rchild)));

    }
```

2. COUNTING THE NUMBER OF NODES

```
int count(struct tnode *p)
    {
        if( p == NULL)
            return(0);
        else
          return(1 + (count(p->lchild) + count(p->rchild)));

    }
```

- 3. Sum of all nodes

```
int sum(node *p)
    {
        if( p == NULL)
            return(0);
        else
          return( p->data+sum(p->l)+sum(p->r) );
    }
```

4. COUNTING THE NUMBER OF EVEN (ODD) NODES

```
int counteven(node* r)
{
    if (r!=NULL)
        if (r->data%2==0)
            return   1+ counteven(r->l)+counteven(r->r);
        else
            return   counteven(r->l)+counteven(r->r);
    else
        return 0;

}
```

5. SUM OF EVEN (ODD) NODES

```
int counteven(node* r)
{
    if (r!=NULL)
        if (r->data%2==0)
            ??????????????????
        else
            ??????????????????
    else
        return 0;

}
```

- 6. Count number of leaf nodes

- Exercise
  - Count number of leaf nodes

- 6. Count number of leaf nodes

```
int countleaf(node* r)
{
      if (r!=NULL)
            if (r->l==NULL && r->r==NULL)
                  return    1;
            else
                  return    countleaf(r->l)+countleaf(r->r);
      else
            return 0;

}
```

6. Count number of node had 1 child

```
int count1child(node* r)
{
	if (r!=NULL)
		if (??????????????????????????)
			return	1+count1child(r->l)+count1child(r->r);
		else
			return	count1child(r->l)+count1child(r->r);
	else
		return 0;

}
```

6. Count number of node had 2 children

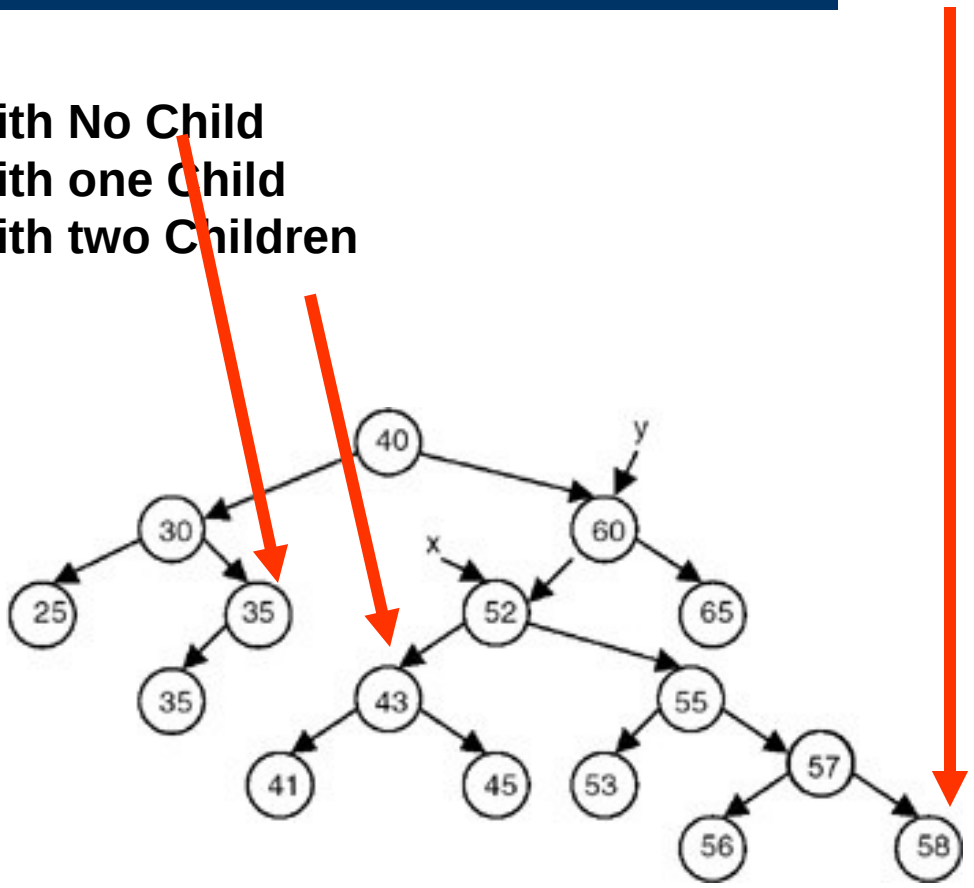
```
int count2child(node* r)
{
	if (r!=NULL)
		if (??????????????????????)
			return	1+count2child(r->l)+count2child(r->r);
		else
			return	count2child(r->l)+count2child(r->r);
	else
		return 0;

}
```

- 7. Find height of tree

```
int Height (node* n)
{
if(n==NULL) return 0;
else return 1+max(Height (n->l)),
 Height (n->r));
 }
```

- 8. Delete node

- **Divide into 3 cases**
  - **Deletion of a Node with No Child**
  - **Deletion of a Node with one Child**
  - **Deletion of a Node with two Children**



- 8. Delete node

- **Deletion of a Node with No Child**
  - **Set the left of y to NULL**
  - **Dispose of the node pointed to by x**