for given loss function, get (β_m, G_m) and $f_m(x)$

$$L(y, f(x)) = \exp(-yf(x))$$
$$f_m(x) = f_{m-1}(x) + \beta_m G_m(x)$$
$$(\beta_m, G_m) = \underset{\beta, G}{\operatorname{argmin}} \sum_{i=1}^{N} \exp(-y_i(f_{m-1}(x_i) + \beta G(x_i)))$$ $* G_m(x) \in \{-1, 1\}$

[풀이]

i) $(\beta_m, G_m) = \operatorname{argmin} \sum \exp(-y_i(f_{m-1}(x_i) + \beta G(x_i)))$
$= \operatorname{argmin} \sum \exp(-y_i f_{m-1}(x_i)) \exp(-y_i \beta G(x_i))$

현재함수의 웨이트로 볼수있음

$= \operatorname{argmin} \sum w_i^{(m)} \exp(-y_i \beta G(x_i))$

ii) finding G_m

$(\beta_m, G_m) = \operatorname{argmin} \sum w_i^{(m)} \exp(-\beta y_i G(x_i))$

β, G와 상관 x

[올바른 예측 $y_i = G(x_i)$ → 부분 음수 → loss 감소
  틀린 예측 $y_i \neq G(x_i)$ → 부분 양수 → loss 증가

$\therefore G_m = \underset{G}{\operatorname{argmin}} \sum w_i^{(m)} I(y_i \neq G(x_i))$

*loss가 최소가 되려면 예측이 틀릴때의 가중합이 최소가 되어야 함

iii) finding β_m

$\operatorname{argmin} \sum w_i^{(m)} \exp(-\beta y_i G(x_i))$

예측 맞으면 1
틀리면 -1

$= \operatorname{argmin} \{ \underbrace{e^{\beta} \sum w_i^{(m)} I(y_i \neq G(x_i))}_{\text{틀렸을때}} + \underbrace{e^{-\beta} \sum w_i^{(m)} I(y_i = G(x_i))}_{\text{맞았을때}} \}$

$= \operatorname{argmin} \{ e^{\beta} \sum w_i^{(m)} I(y_i \neq G(x_i)) + e^{-\beta} \sum w_i^{(m)} (1 - I(y_i \neq G(x_i))) \}$

$= \operatorname{argmin} \{ (e^{\beta} - e^{-\beta}) \sum w_i^{(m)} I(y_i \neq G(x_i)) + e^{-\beta} \sum w_i^{(m)} \}$

$\rightarrow \beta_m = \frac{1}{2} \log \frac{1 - err_m}{err_m}$

where $err_m = \dfrac{\sum_{i=1}^{N} w_i^{(m)} I(y_i \neq G(x_i))}{\sum w_i^{(m)}}$

iv) finding $f_m(x)$

$f_m(x) = f_{m-1}(x) + \beta_m G_m(x)$